# MATH-4006 Calculus 1 class 14 , Homework 1 常見錯誤

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**For the writing style**:

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拜託各位寫題目的時候

- 如果是用電子檔寫題目可以新開一頁，把答案寫在下一頁
- 如果是用紙本，可以拿一張新的紙把題目標清楚

以減少助教眼壓

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## Problem 1-(b)-(ii)

**Description.** Solve the following equations for x.

$$\ln x + \ln(x-1) = 1$$

**Wrong Ans.**

$$x = \frac{1 \pm \sqrt{1+4e}}{2}$$

**Correct Ans.**

$$x = \frac{1 + \sqrt{1+4e}}{2}$$

**Reason.** For $\ln(x-1)$ to be defined, we need $x-1>0$, or $x>1$.

> **Note.** The domain of $\ln(x)$ is $(0,\infty)$

Therefore,

$$x = \frac{1 - \sqrt{1+4e}}{2}$$

is rejected.

## Problem 2-(a)

**Description.** Simplify the following expressions.

$$\sin^{-1}\left(\sin\frac{5\pi}{4}\right)$$

**Wrong Ans.**

$$\frac{5\pi}{4}, \frac{7\pi}{4}$$

**Correct Ans.**

$$-\frac{\pi}{4}$$

**Reason.** Follow the definition from the textbook the definition $\sin^{-1}$ is

**Note.** In page 62 of the textbook

$$\sin^{-1} x = y \iff \sin y = x, \quad \text{and} \quad -\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$$

The range of $\sin^{-1}$ can only be in $[-\frac{\pi}{2}, \frac{\pi}{2}]$

## Problem 2-(c)

**Description.** Simplify the following expressions.

$$\sin(\sec^{-1}(x)) \text{ for } x \leq -1$$

**Wrong Ans.**

$$-\frac{\sqrt{x^2-1}}{x}$$

**Correct Ans.**

$$\frac{\sqrt{x^2-1}}{x}$$

**Reason.** By the Pythagoras Theorem, we can know that the answer might be

$$\pm\frac{\sqrt{x^2-1}}{x}$$

To get the "sign" of answer, we can first check the definition of $\sec^{-1}$ on the textbook

> **Note.** In page 64 of the textbook
>
> $$y = \sec^{-1} x \ (|x| \geq 1) \iff \sec y = x, \quad \text{and} \quad y \in (0, \pi/2] \cup [\pi, 3\pi/2)$$

When $x \leq -1$, $\sec^{-1}$ is between $[\pi, 3\pi/2)$. Thus, $\sec^{-1}$ lies in the third quadrant (第三象限), which will let sin function be negative. Hence, the final answer is

$$\frac{\sqrt{x^2-1}}{x}$$

which is negative when $x \leq -1$.

**請務必對照課本，別忘記有課本**

## Problem 5-(b)

**Description.** Let
$$g(x) = \frac{x^2 - 5x + 6}{|x - 2|}$$
Does the limit $\lim_{x\to 2} g(x)$ exist ? Explain.

**Wrong Ans.** Since $g(2)$ does not exist, the limit $\lim_{x\to 2} g(x)$ does not exist.

**Correct Ans.** Since $\lim_{x\to 2^+} g(x) \neq \lim_{x\to 2^-} g(x)$, the limit $\lim_{x\to 2} g(x)$ does not exist.

**Reason.** A function being discontinuous at a point, or even undefined there, does not necessarily mean that the limit at that point does not exist. Here is the counter example.
$$f(x) = \frac{x^2 - 1}{x - 1}$$
In this situation, $f(1)$ does not exist but
$$\lim_{x\to 1^+} f(x) = \lim_{x\to 1^-} f(x) = \lim_{x\to 1} f(x)$$
the limit $\lim_{x\to 1} f(x)$ exists.

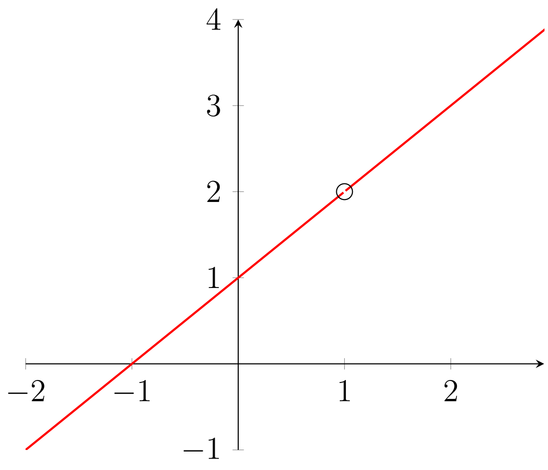


Figure 1: Graph of $f$