A
$\mathcal{N}(A) =$
$\{x \in$
$R^n \mid Ax =$
$0\}$
$\mathcal{N}(A) \leq$
R^n/R
$\mathcal{N}(A)$
A
??
$\mathcal{N}(A)$
$x, x' \in$
$\mathcal{N}(A) \Rightarrow$
$Ax =$
$0, Ax' =$
0

$$A(x+x') = Ax+Ax' = 0+0 = 0 \Rightarrow x+x' \in \mathcal{N}(A)$$

$$A(\alpha x) = \alpha Ax = \alpha \cdot 0 = 0 \Rightarrow \alpha x \in \mathcal{N}(A), \forall \alpha \in R \mathcal{N}(A) \leq R^n/R$$

$Ax =$
0
$Ax =$
b
R^n/R

$$x, x' \longrightarrow Ax = b, Ax' = b$$

$$A(x+x') = Ax+Ax' = 2b \neq b$$

$$(1)\,42536\,(u)\,v = (0)\,00 \Longrightarrow \mathcal{N}(A) = \{(0)\,0\}$$

$$(1)\,45257369\,(u)\,vw = (0)\,00 \Longrightarrow \mathcal{N}(A) = \{(t)\,t - t, t \in (-\infty, \infty)\}$$

$\{$
$\}$
$\leq$
R^m/R
$\{$
$\in$
$R^n \mid Ax =$
$0\}$
$=$
$nullspaceof A \leq$
R^n/R
m
n
$ax =$
$b, a, b, x \in$
R
$a \neq$
$0 \Rightarrow$
$x =$
$\frac{b}{a}$
$a =$
0
$b =$
$0 \Rightarrow$
$a =$
0
$b =$
$0 \Rightarrow$
x
$Ax =$
b
A
A^{-1}
$\Rightarrow$
$x =$
$A^{-1}b$
A
A
$x =$
$A^{-1}b$

$m \times n$
R
( ) 00000

$m \times n$
R
1
( ) 1000100001

$m \times n$
A
P
L
$m \times n$
$U \ni$
$PA =$
LU
OR
$m \times n$
A

$b_{m\times 1} =$
0

$Ax = 0$

x

$(\ )\, 133200310000\, (\, u\,)\, vwy = (\, 0\,)\, 00, \{\ basicvariables : u, w freevariables : v, y$

$\{\, 3\ w{+}y = 0u{+}3v{+}3w{+}2y = 0 \Longrightarrow \{\, w\ = -\frac{1}{3}yu = -3v{-}y$

$x = (\, u\,)\, vwy = (\, -\,)\, 3v{-}yv{-}\frac{1}{3}yy = v\, (\, -\,)\, 3100{+}y\, (\, -\,)\, 10{-}\frac{1}{3}1$

$(\, -\,)\, 3$
1
0
0
x
$\{\, v\ =$
1
$y =$
0
$(\, 1\,)$
0
$-$
$\frac{1}{3}$
0
x
$\{\, v\ =$
0
$y =$
1
$A_{m\times n}x =$
0
$nunknows$
$mequations$
$(A_{m\times n}) \longrightarrow (A_{m\times n})$

m
m
$(n-$
$m)$

$b \neq$
0

$Ax = b \to Ux = c where c = L^{-1}b$

$(1)\,3322695{-}1{-}330\,(x)_1\,x_2x_3x_4 = (b)_1\,b_2b_3$

$\Longrightarrow (\ )\,133200310000\,(u)\,vwy = (b)_1\,b_2{-}2b_1b_3{-}2b_2{+}5b_1 \longrightarrow b_3{-}2b_2{+}5b_1 = 0$

$Ax =$
b
$\Rightarrow$
$b \in$
$\mathcal{C}(A)$
$\mathcal{C}(A) =$
(1)
2
$-$
1
(3)
9
3
$\{(b)_1\,b_2b_3 \mid b_3 - 2b_2 + 5b_1 = 0\} \perp (5)\,-21$

$b = (1)\,55$

$(\ )\,133200310000\,(u)\,vwy = (1)\,30 \Longrightarrow \{\, w = 1{-}\frac{1}{3}yu = -2{-}3v{-}y$

$x = (u)\,vwy = (-)\,2{-}3v{-}yv1{-}\frac{1}{3}yy = shift(-)\,2010{+}solution to Ax = 0(nullspace)v\,(-)\,3100 + y\,(-)\,10 - \frac{1}{3}1$

$Ax =$
b

$x_{general} = x_{particular} + x_{homogeneous}; x_g = x_p = x_h$

A
GRAPH HERE
$Ax = b$
$Ax = b$
$Ux = c$
x_p
x_h

$\Longrightarrow x_g = x_p + x_h$

$A_{m\times n}$
r
$n - r$
$A_{m\times} x = b$
$Ux = c$
r
$(m - r)$
U
$(m - r)$
c
$r = m$
$r = n$
$x = 0$
r
A
$A_{m\times n} x = b$
$r = n \rightarrow$
$\rightarrow$
$\mathcal{N}(A) = \{x \in R^n \mid Ax = 0\} = \{0\}$
$r = m \rightarrow$
$U \rightarrow$
$\mathcal{C}(A) = R^m \Rightarrow$
$\exists solution for all b$
A
spanning
basis
dimension