MATH-4007 Calculus 2 class 14 , Homework 11 常見錯誤

B13902126 資工二胡允升

For the writing style:

再再再再再再次提醒拜託各位寫題目的時候

- 如果是用電子檔寫題目可以「新開一頁」，把答案寫在下一頁
- 如果是用紙本，可以拿一張新的紙把題目標清楚
- 標示清楚計算過程

以減少助教眼壓。

For studying（關於課程網上的影片）:

教授上課不一定能夠 cover 到所有題目，助教們在 NTUCOOL 上都會放上「詳解」影片，**請務必要觀看**，在這次題目中，有不少題都是教授上課沒有 cover 到的講義內容變化題，明顯的很多人並沒有觀看助教們錄好的影片，希望大家能夠確實觀看，才能在期考拿下高分。

For questioning:

有任何問題都可以來信問助教，WebWork 題目也可以在寄信給助教的時候一併詢問。

Problem 2-(c)

Description. Find the following the indefinite integral

$$\int \frac{\mathrm{d}x}{\sqrt{x(x-6)}}$$

Correct Ans.

$$\ln\left|\left(\frac{x-3}{3}\right)+\sqrt{\left(\frac{x-3}{3}\right)^2-1}\right|+C$$

Reason. Note that by completing the square（配方法）, we have

$$x(x-6)=(x-3)^2-9$$

Therefore, we do the substitution:

$$x-3=3\sec\theta\ (\mathrm{d}x=3\sec\theta\tan\theta\ \mathrm{d}\theta)$$

and hence,

$$\begin{aligned}
\int \frac{\mathrm{d}x}{\sqrt{x(x-6)}} &= \int \frac{3\sec\theta\tan\theta\ \mathrm{d}\theta}{\sqrt{(3\sec\theta)^2-9}} \\
&= \int \frac{3\sec\theta\tan\theta\ \mathrm{d}\theta}{3\sqrt{\sec^2\theta-1}} \\
&= \int \sec\theta\ \mathrm{d}\theta \\
&= \ln|\sec\theta+\tan\theta|+C \\
&= \ln\left|\left(\frac{x-3}{3}\right)+\sqrt{\left(\frac{x-3}{3}\right)^2-1}\right|+C
\end{aligned}$$

□

Note. We can reduce the expression a bit more:

$$\begin{aligned}
\ln\left|\left(\frac{x-3}{3}\right)+\sqrt{\left(\frac{x-3}{3}\right)^2-1}\right|+C &= \ln\left|\frac{x-3+\sqrt{(x-3)^2-9}}{3}\right|+C \\
&= \ln\left|x-3+\sqrt{x(x-6)}\right|-\ln 3+C \\
&= \ln\left|x-3+\sqrt{x(x-6)}\right|+C'
\end{aligned}$$

□

Problem 4-(a)

Description. Let $I_n = \displaystyle\int sec^n(x)\ \mathrm{d}x$. Prove that

$$I_n = \frac{1}{n-1}\tan x \sec^{n-2}(x) + \frac{n-2}{n-1} I_{n-2}$$

Reason. First imply the integration by parts:

	D	I
+	$\sec^{n-2}(x)$	$\sec x \tan x$
-	$(n-2)\sec^{n-3}(x)\sec x \tan x$	$\sec x$

Then we get

$$\begin{aligned}
I_n &= \int \sec^{n-2}(x)\ \sec^2(x)\,dx \\
&= \tan(x)\sec^{n-2}(x) - (n-2)\int \sec^{n-2}(x)\tan^2(x)\,dx \\
&= \tan(x)\sec^{n-2}(x) - (n-2)\int \sec^{n-2}(x)(\sec^2 x - 1)\,dx \\
&= \tan(x)\sec^{n-2}(x) - (n-2)\int \underbrace{\sec^n(x)}_{I_n}\,dx + (n-2)\int \underbrace{\sec^{n-2}(x)}_{I_{n-2}}\,dx \\
&= \tan(x)\sec^{n-2}(x) - (n-2)I_n + (n-2)I_{n-2}
\end{aligned}$$

Rearranging the equation, we have

$$I_n = \frac{1}{n-1}\tan x \sec^{n-2}(x) + \frac{n-2}{n-1} I_{n-2}$$

$\square$

Problem WebWork

Description. Find the following the indefinite integral

$$\int \frac{5}{[(ax)^2 + b^2]^{3/2}} \mathrm{d}x$$

Correct Ans.

$$-\frac{5}{ab\sqrt{b^2 - a^2x^2}} + C$$

Reason. We have to convert it into a standard form

$$\sec^2\theta = 1 + \tan^2\theta$$

So we let $x = \frac{b}{a}\sec\theta$ and we get

$$\begin{aligned}
\int \frac{5}{[(ax)^2 + b^2]^{3/2}} \mathrm{d}x &= \int \frac{5}{[a^2(\frac{b}{a}\sec\theta)^2 + b^2]^{3/2}} \cdot \frac{b}{a}\sec\theta\tan\theta \; \mathrm{d}\theta \\
&= \int \frac{5}{[b^2\sec^2\theta + b^2]^{3/2}} \cdot \frac{b}{a}\sec\theta\tan\theta \; \mathrm{d}\theta \\
&= \int \frac{5}{b^3(\sec^2\theta)^{3/2}} \cdot \frac{b}{a}\sec\theta\tan\theta \; \mathrm{d}\theta \\
&= \frac{5}{ab^2}\int \frac{\cos\theta}{\sin^2\theta} \; \mathrm{d}\theta
\end{aligned}$$

then, let $u = \sin\theta$ ($\mathrm{d}\theta = \frac{1}{\cos\theta}\mathrm{d}u$), we have

$$\begin{aligned}
\frac{5}{ab^2}\int \frac{\cos\theta}{\sin^2\theta} \; \mathrm{d}\theta = \frac{5}{ab^2}\int \frac{\cos\theta}{u^2} \cdot \frac{1}{\cos\theta}\mathrm{d}u &= \frac{5}{ab^2}\int u^{-2}\mathrm{d}u \\
&= \frac{5}{ab^2} \cdot (-u^{-1}) + C \\
&= -\frac{5}{ab^2\sin\theta} + C
\end{aligned}$$

Finally, we have to convert back to x:

$$\sin\theta = \sqrt{1 - \cos^2\theta} = \sqrt{1 - \left(\frac{ax}{b}\right)^2} = \frac{\sqrt{b^2 - a^2x^2}}{b}$$

Thus, the final answer is

$$-\frac{5}{ab^2} \cdot \frac{b}{\sqrt{b^2 - a^2x^2}} + C = -\frac{5}{ab\sqrt{b^2 - a^2x^2}} + C$$

$\square$