$\mathcal{N}(A)$
$A_{m\times n}$
$\{x \mid$
$Ax =$
$0\} =$
$\{x \mid$
$Ux =$
$0\}$
$Thenullspaceof$

$isthesameasthenullspaceof[2N]Thenullspaceisofdimension$
$Abasisofcanbeconstructedbyreducingtowhichhasfreevariablescorrespondingtothecolumnsifthatdonotcontainpivots.Le$
$.Thevectorsproducedinthismannerwillbeabasisof.$
$\dim(\mathcal{N}(A)) =$
$n-$
$rTheisalsocalledthekernelof, \ker(A)$
A

$\ker(A) = \mathcal{N}(A)$

$\mathcal{C}(A)$
$\mathcal{R}$
$\mathcal{R}(A)$
range
f
$f(x) =$
$A_{m\times n}x_{n\times 1}$
f
R^n
f
$\{b \in$
$R^m \mid$
$Ax =$
$b\} =$
$\mathcal{C}(A) =$
$\mathcal{R}(A)$
f
$\{x \in$
$R^n \mid$
$Ax =$
$f(x) =$
$0\} =$
$\mathcal{N}(A) =$
$\ker(A)$
U
A
$\mathcal{C}(A) \neq$
$\mathcal{C}(U)$
U
$\mathcal{C}(U)$
A
$\mathcal{C}(A)$
$Ax =$
0
$Ux =$
0
A
x
U
A
U
A
$\mathcal{C}(A)$
A
U
$=$
$\#ofindependentcolumns$

$\#ofindependentrows$
$ormoreformally, (A) =$
$r =$
$rowrank =$
$columnrank$

$\mathcal{N}(A^T)$

$n \times mA^T m \times 1y = n \times 10 = (1 \times my^T m \times nA)^T$

$(\#ofbasicvariables)+(\#offreevariables) = (\#ofvariables) = n$

$\dim(\mathcal{C}(A)) + \dim(\mathcal{N}(A)) = \#ofcolumnsofA$

A^T
m
A^T
A
(A)
$\dim(\mathcal{N}(A^T)) = m-(A)$

$\dim(\mathcal{C}(A^T)) + \dim(\mathcal{N}(A^T)) = \#ofcolumnsofA^T$

$\mathcal{N}(A^T)$
$m-$
r
A
$\ni$
$y^T A =$
0
$SupposethatPA =$
$LU \longrightarrow$
$m \times mL^{-1}P\, m \times nA =$
$m \times nU$
$m-$
r
$L^{-1}P$
$m-$
r
$L^{-1}P$
$\dim(\mathcal{N}(A^T))$
$m-$
r
$\to$
$\mathcal{N}(A^T)$
A
$m\times$
n
$\dim(\mathcal{C}(A)) = \dim(\mathcal{C}(A^T)) = (A)$

$\dim(\mathcal{N}(A)) = n-(A); \dim(\mathcal{N}(A^T)) = m-(A)$

$A = (1) 01023414361 \longrightarrow U = ( ) 1010012/31/30000 r = 2$

0°
$\mathcal{C}(A)$

$\mathcal{B} = \{(1) 24, (0) 33\} \dim(\mathcal{C}(A)) = r = 2$

$\mathcal{N}(A)$

$Ax = 0 \longrightarrow Ux = 0 \longrightarrow U (x)_1 x_2 x_3 x_4 = (0) 00$

$\{ x_1 + x_3 = 0 x_2 + \frac{2}{3} x_3 + \frac{1}{3} x_4 = 0$

$x_3 =$
1
$x_4 =$
0
$\longrightarrow$
$(-) 1$
$-$
$2/3$
1
$0 =$
v_2
$x_3 =$
0
$x_4 =$
1
$\longrightarrow$
(0)
$-$
$1/3$
0
$1 =$
v_2
$\mathcal{B} =$
$\mathcal{N}(A)$
$\{v_1, v_2\}$
$\dim(\mathcal{N}(A)) = n - r = 4 - 2 = 2$

$\mathcal{C}(A^T)$

$U = (1) 010012/31/30000 = (S)_1 S_2 0 \longrightarrow \mathcal{B} = \{S_1^T, S_2^T\}, \dim(\mathcal{C}(A^T)) = r = 2$

$\mathcal{N}(A^T) \longrightarrow$
$\mathcal{N}(B)$

$B = (1) 24033146011 = A^T \longrightarrow ( ) 124011000000 (y)_1 y_2 y_3 = (0) 000 \longrightarrow \{ y_1 + 2y_3 = 0 y_2 + y_3 = 0$

$z = 1 \longrightarrow (-) 2 - 11 \mathcal{B} = \{(-) 2 - 11\}, \dim(\mathcal{N}(A^T)) = m - r = 3 - 2 = 1$

$Ax =$
b
Existence
$Ax =$
b
x
b
$R^m (r =$
$m)$
$\exists\, n \times m \text{“}right'' inverse C \ni AC = I$

$m \leq$
n
Uniqueness
$Ax =$
b
x
b
$(r =$
$n)$
$\exists\, n \times m \text{“}left'' inverse B \ni BA = I$

$m \geq$
n
Existence

$Ax = b has a solution for each b \iff b \in \mathcal{C}(A), \forall b \in R^m \mathcal{C}(A) = R^m$

$e_1, e_2, \cdots, e_m$
R^m
$\exists\, x_1, x_2, \cdots, x_m \ni$
$Ax_i =$
$e_i, \forall i =$
$1, 2, \cdots, m$
$C =$
$(x_1 \mid$
$x_2 \mid$

$A_{n\times n}$
A
A
R^n
$Ax =$
b
$\forall b \in$
R^n
A
$Ax =$
0
$x =$
0
A
R^n
A
$PA =$
LDU
$d_i \neq 0$
$\exists\ A^{-1} \ni$
$AA^{-1} =$
$A^{-1}A =$
I_n
A
$\det(A) \neq 0$
A
$A^T A$
A
$\mathcal{N}(A)$
$\mathcal{C}(A)$
A
$n\times$
n
$x \in$
R^n
A
x
$Ax \in$
$\mathcal{C}(A)$
0°

$A = (\, c \,)\, 00cA \,(\, x \,)_1\, x_2 = (\, c \,)\, x_1 c x_2 = c \,(\, x \,)_1\, x_2 (scaling by$

$)$

$A = (\, 0 \,) {-110} A \,(\, x \,)_1\, x_2 = (\, - \,)\, x_2 x_1 (rotation by$

$\circ$

$A = (\, 0 \,)\, 110A \,(\, x \,)_1\, x_2 = (\, x \,)_2\, x_1 (reflection about$

$_1 =$
x_2

$A = (\, 1 \,)\, 000A \,(\, x \,)_1\, x_2 = (\, x \,)_1\, 0 (projection onto$

$_1$