Numerical Method

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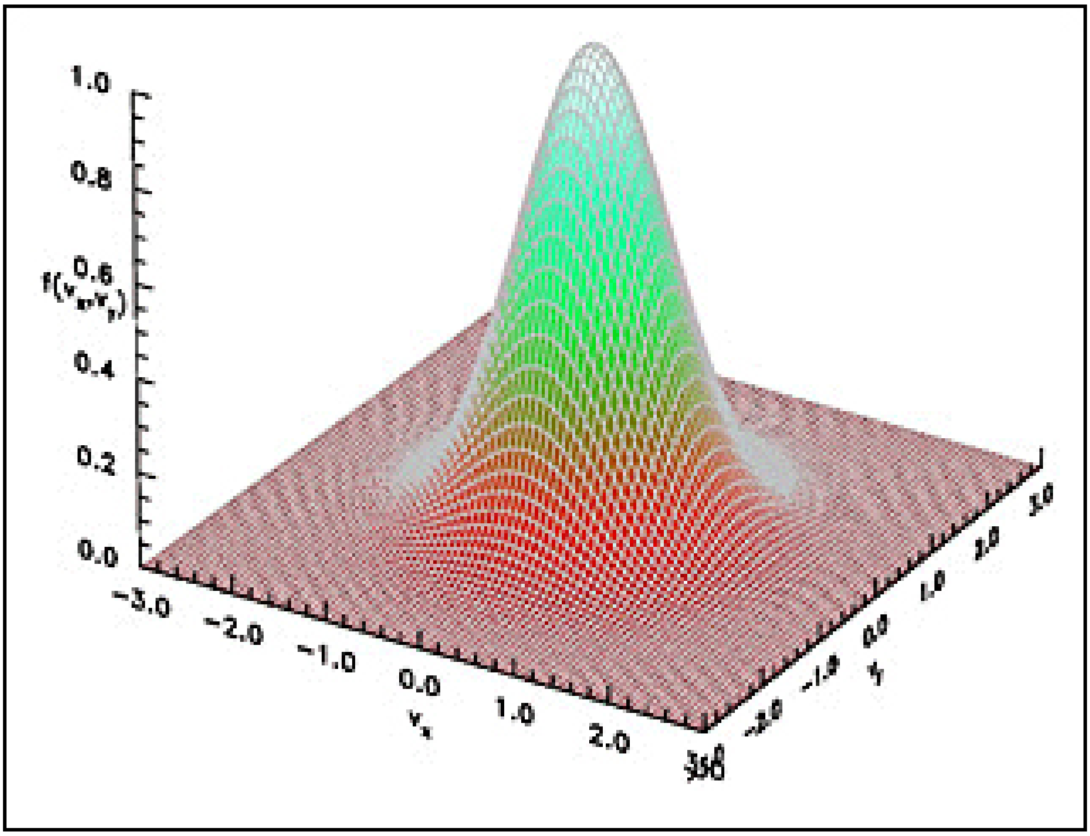
Abstract

The lecture note of 2023 Spring Numerical Method by professor 林智仁.



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Lecture 1

1.1 Floating-point basics

2025.12.09

This chapter is mainly based on the science of floating-point arithmetics which is based on the IEEE standard 754.

1.1.1 Why learning floating-point operations?

Example. A one-variable problem

$$\min_{x} f(x) \quad \text{where } x \geq 0$$

In the normal program, we should set an upper bound of x, or x may be wrongly increased to ∞. We have to find the largest representable number in the computer

Example. A ten-variable problem

$$\min_{\mathbf{x}} f(\mathbf{x}) \quad \text{where } x_i \geq 0, i = 1, 2, \ldots, 10$$

We want to know how many are zeros, we may use

```
for (int i = 0; i < 10; i++)
    if (x[i] == 0) count++;
```

But people say that don't do the comparison of floating-point

```
double epsilon = 1.0e-12;
for (int i = 0; i < 10; i++)
    if (x[i] <= epsilon) count++;
```

Which is better? How to chose `epsilon`? Can't do the comparison of floating-point? We need to understand the floating-point representation.

1.1.2 Floating-point Format

We know `float` (single precision): 4 bytes and `double` (double precision) 8 bytes in C/C++.

Definition 1.1.1 (format). A floating-point system requires a base β, precision p, significand (mantissa) $d.d\dots d$