POLITECNICO
MILANO 1863

DIPARTIMENTO DI FISICA

Diffuse Optical Tomography Project - Phase 2

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Scenario

We are considering an infinite medium made of an unperturbed homogeneous background with absorption coefficient $\mu_a = 0.1\ cm^{-1}$ and scattering coefficient $\mu_s = 10\ cm^{-1}$, which is then in **diffusion regime**, and a point-like absorbing defect characrerized by the absorption coefficient $\Delta\mu_a = 0.1\ cm^{-1}$.
A cubical volume space of $8\ cm^3$ is discretized in 512 voxels of $1\ cm^3$. 64 source-detector (SD) pairs, assumed at null distance from one and other, are placed onto each pixel of the bottom surface of the space, each of them perform a time resolved measure from 0 to $10\ ns$ in 11 temporal gates of duration $t_{gate} = 1\ ns$ .This measure is the contrast $M(\vec{r}_{SD}, \vec{r}_P, t_{gate})$, defined as the ratio of the unperturbed fluence of the background medium $\Phi_0(\vec{r}_{SD}, t_{gate})$ and the perturbation $\delta\Phi(\vec{r}_{SD}, \vec{r}_P, t_{gate})$ due to the absorbing defect, i.e.:$M \triangleq \frac{\delta\Phi}{\Phi_0}$

The solution of this equation is found by applying the **Born approximation** to the **Diffusion Equation** and is:

$$M(\vec{r}_{SD}, \vec{r}_P, t_{gate}) = \frac{V \cdot \delta\mu_a}{2\pi \cdot D \cdot |r_p - r_{SD}|} exp(\frac{|r_p - r_{SD}|^2}{D \cdot c \cdot t_{gate}})$$

where V is the volume of a voxel, $D = \frac{1}{3 \cdot \mu_s}$ is the diffusion coefficient and $|\vec{r}_p - \vec{r}_{SD}\ | = \sqrt{(x_{SD} - x_p)^2 + (y_{SD} - y_p)^2 + (z_{SD} - z_p)^2}$ is the distance between particular SD pair and perturbation.

Problem

The goal of this project is to retrieve the vector of measures M from a vector A representing the perturbation position though the use of the sensitivity matrix W of the presented setup, i.e. solving the vectorial problem: $M = W \cdot A$

Solution

Voxel Space A:

To represent the perturbation position as a one dimensional vector **compression of indexes** must be used. In this case we pass from a 3D vector with 3 indexes for x,y,z coordinates to a 1D vector where positions in space are stored sequentially and labelled with increasing integers numbers.

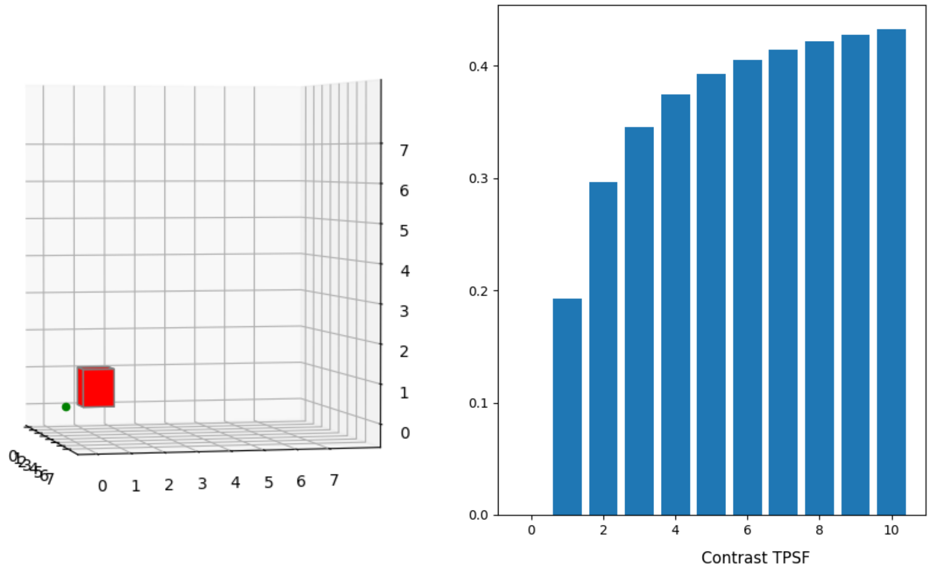
$$A[\vec{r}_p] = A[x_p, y_p, z_p] \implies A[r'_p] \qquad \begin{aligned} xy_p &= x_p + y_p \cdot N_{x_p} \\ r'_p &= xy_p + z_p \cdot N_{xy_p} \end{aligned}$$

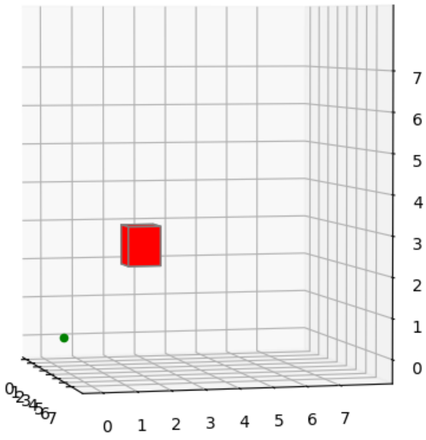
The perturbation vector A is then one dimensional and has $N_{voxels} = 512$ elements. For a point-like source all elements are equal to zero besides one whose value is the absorption coefficient in cm^{-1} of the defect and its position in 3D space is indicated with a single index.

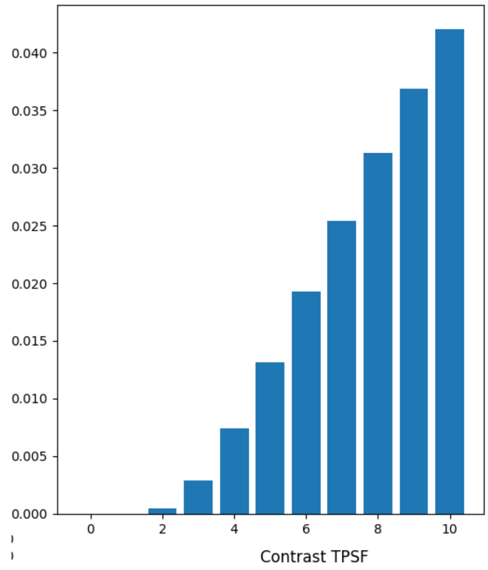
Measurements Space M:

The measurement vector M is computed by applying the perturbation vector A to the sensitivity matrix W. Using compression of indexes it results in a one dimensional vector of size $N_{SD} \cdot N_{t_{gate}}$, where each element represents the value of measured contrast by a particular SD pair in a particular time gate for a point-like defect placed in a certain position. Each value is as an adimensional number comprised between 0 and 1 and expresses a percentage (photon density due to perturbation with respect to the homogeneous photon density distribution).

Using clever methods of vector slicing is possible to extract from M the **Time Point Spread Function** for each SD pair or a 2D map of the medium imaged by all the detectors at each time gate. In the figures below is possible to see two TPSFs for a single SD pair (green dot) for two different locations of the inhomogeneity (red cube):







Sensitivity Matrix:

The sensitivity matrix W, also called **Jacobian Matrix**, relates the changes in the measurements to the perturbations in the optical properties. Every element of the matrix represents how the contrast measured by a particular SD pair at a particular time gate would change if a point-like defect were located at a particular position in space. W is two dimensional with $N_{col} = N_{vox}$ and $N_{rows} = N_{meas} = N_{SD} \cdot N_{gates}$. When multiplied by A every element of a row of W is multiplied by the corresponding value in A and all result summed togheter, representing the superposition of source terms located at every position in space, this operation is equal to a discrete convolution integral. In case of a point-like source only one contribution is non-null.
The problem can now be reformulated in vectorial notation as:

$$\begin{bmatrix} M_{SD_0,gate_0} \\ M_{SD_0,gate_1} \\ \dots \\ M_{SD_0,gate_{10}} \\ M_{SD_1,gate_0} \\ \dots \end{bmatrix} = \begin{bmatrix} W_{SD_0,gate_0;vox_0} & \dots & W_{SD_0,gate_0;vox_{511}} \\ W_{SD_0,gate_1;vox_0} & \dots & W_{SD_0,gate_1;vox_{511}} \\ \dots & \dots & \dots \\ W_{SD_0,gate_{10};vox_0} & \dots & W_{SD_0,gate_{10};vox_{511}} \\ W_{SD_1,gate_0;vox_0} & \dots & W_{SD_1,gate_0;vox_{511}} \\ \dots & \dots & \dots \end{bmatrix} \cdot \begin{bmatrix} A_{vox_0} \\ A_{vox_1} \\ \dots \\ \dots \\ A_{vox_{510}} \\ A_{vox_{511}} \end{bmatrix}$$

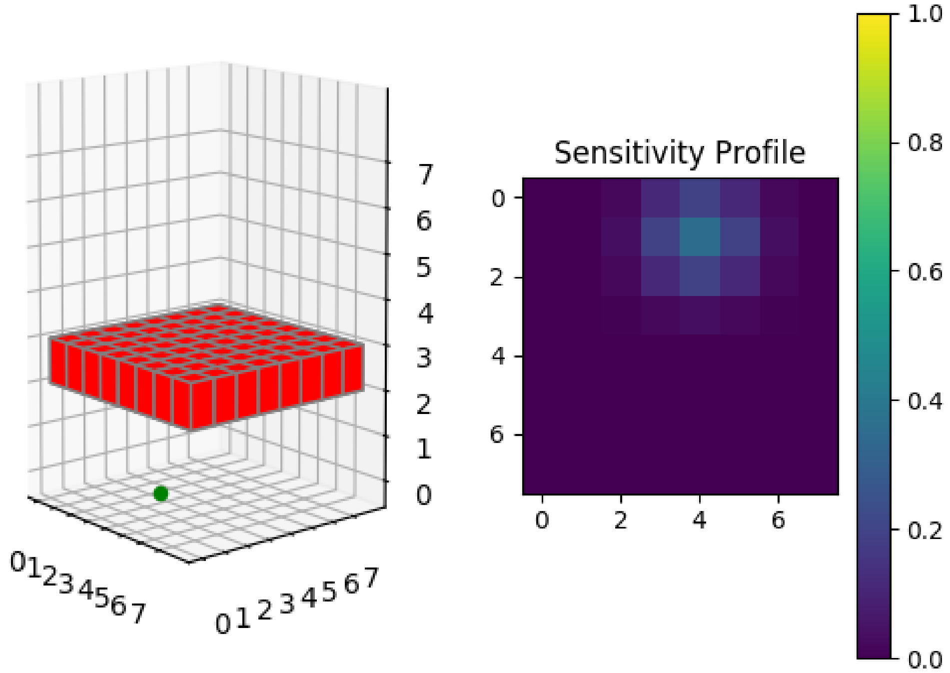
Where each element of matrix W is given by:

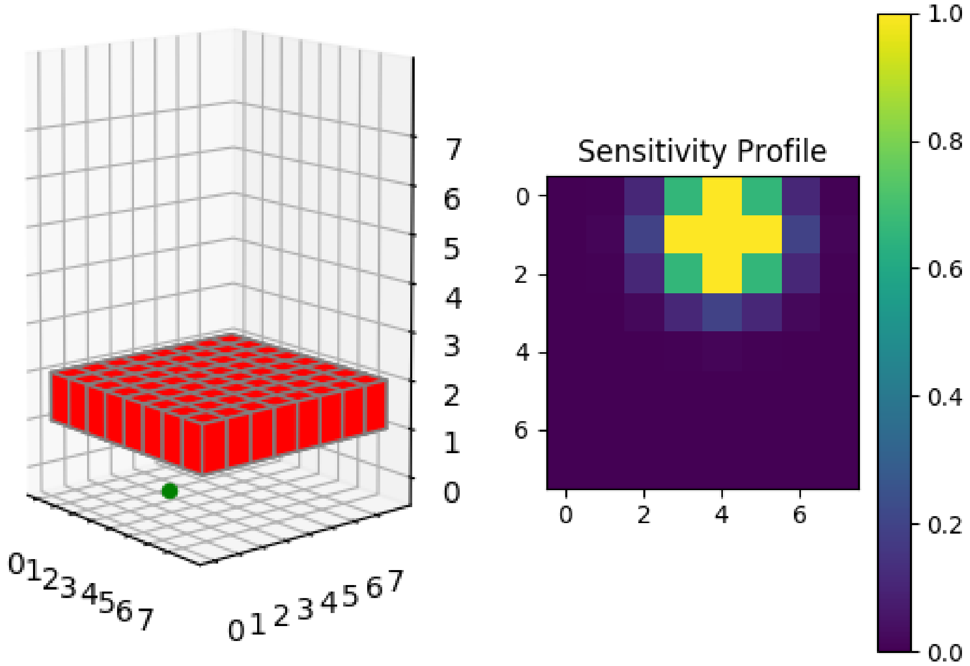
$$W_{(SD;gate),vox} = \frac{V}{2\pi \cdot D \cdot |r_{vox} - r_{SD}|} exp(\frac{|r_{vox} - r_{SD}|^2}{D \cdot c \cdot t_{gate}})$$

And the i-th element of vector M is given by the operation:

$$M_i = \sum_{j=0}^{N_{vox}} W_{i,j} \cdot A_j = W_{i,0} \cdot A_0 + W_{i,1} \cdot A_1 + \dots$$

To illustrate a representation of the sensitivity matrix is possible to cut a transversal slice of the voxel space at a certain depth and represent the sensitivity of the system at every point of the choosen plane. In the figures below are shown two sensitivity profiles of the system at two different depths:


Sensitivity Profile
0
2
4
6
0 2 4 6
1.0
0.8
0.6
0.4
0.2
0.0
7
6
5
4
3
2
1
0
01234567
01234567


Sensitivity Profile
0
2
4
6
0 2 4 6
1.0
0.8
0.6
0.4
0.2
0.0
7
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5
4
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