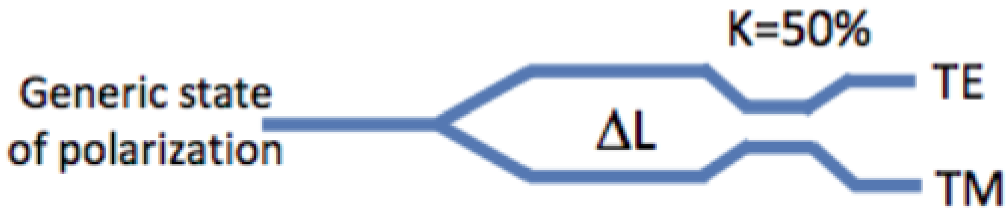
**Task:** Calculate the transfer matrix of the Mach-Zehnder in figure. Design the Mach-Zehnder to achieve the splitting of the two polarizations. Assume no losses and ideal Y-branches.




**Analytical solution:**

To calculate the transfer matrix is possible to split the device in three stages, the first is the Y-branches, the second the phase shifter and the last a directional coupler, that since it has a coupling efficiency of 50% is a -3dB coupler. The 3 transfer functions are:

$$T_{PS} = \begin{pmatrix} e^{-i\phi_1} & 0 \\ 0 & e^{-i\phi_2} \end{pmatrix} \quad T_Y = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \quad T_{3dB} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -i \\ -i & 1 \end{pmatrix}$$

At this point, multiplying from right to left the three terms, is possible to find the complete transfer matrix of the Mach-Zehnder:

$$T_{MZ} = T_{3dB} \cdot T_{PS} \cdot T_Y = \frac{e^{-i\phi_1}}{2} \begin{pmatrix} 1 & -ie^{i\Delta\phi} \\ -i & e^{i\Delta\phi} \end{pmatrix}$$

In the form of a system of equations that relate the amplitude of input and amplitude of the outputs, it becomes:

$$\begin{cases} E_{out1} = \frac{E_{in}}{2} \cdot e^{-i\phi_1} \left(1 - ie^{i\Delta\phi}\right) \\ E_{out2} = \frac{E_{in}}{2} \cdot e^{-i\phi_1} \left(-i + e^{i\Delta\phi}\right) \end{cases}$$

To express these relations in terms of optical power the square modulus has to be calculated. Firstly transforming the complex exponentials in goniometric form:

$$e^{i\Delta\phi} = cos(\Delta\phi) + i \cdot \sin(\Delta\ \phi\ )$$
$$-ie^{i\Delta\phi} = \sin(\Delta\phi\ ) - i \cdot cos(\Delta\phi\ )$$

And then finishing the operation:

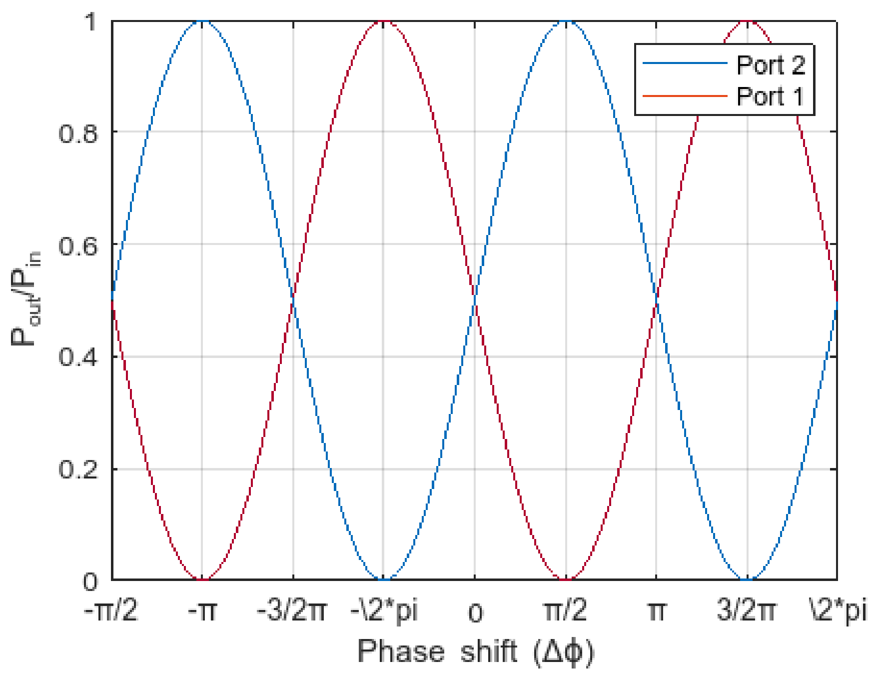
$$P_{out1} = \frac{1}{4}|1 + \sin(\Delta\phi) - i \cdot cos(\Delta\phi)|^2 \cdot P_{in}$$

$$P_{out1} = \frac{1}{4}|-i + cos(\Delta\phi) + i \cdot sin(\Delta\phi)|^2 \cdot P_{in}$$

The resulting equations for the power output at each port is:

$$P_{out1} = \left[\frac{1}{2} + \frac{sin(\Delta\phi)}{2}\right] \cdot P_{in}$$
$$P_{out2} = \left[\frac{1}{2} - \frac{sin(\Delta\phi)}{2}\right] \cdot P_{in}$$

It can be seen that the two functions are in opposition of phase:



At a value of Δφ equal to π/2 the power is completely directed on the port 1, and for a phase shift of -π/2 it is directed on the port 2. So to split the polarizations it is necessary to dimension the two arms of the interferometer so that the TE mode experiences a phase shift of π/2 and the TM mode one of -π/2.
Starting from the phase equations:

$$\Delta\phi_{TE} = \frac{2\pi}{\lambda} \cdot \left(n_{eff1}L_1 - n_{eff2}L2\right) = \frac{\pi}{2}$$

$$\Delta\phi_{TM} = \frac{2\pi}{\lambda} \cdot \left(n_{eff1}L_1 - n_{eff2}L2\right) = -\frac{\pi}{2}$$

In the scope of this project it has been decided to make the two arms of equal length and induce the phase shift only by changing the effective index of refraction, the solutions of the previous equations are:

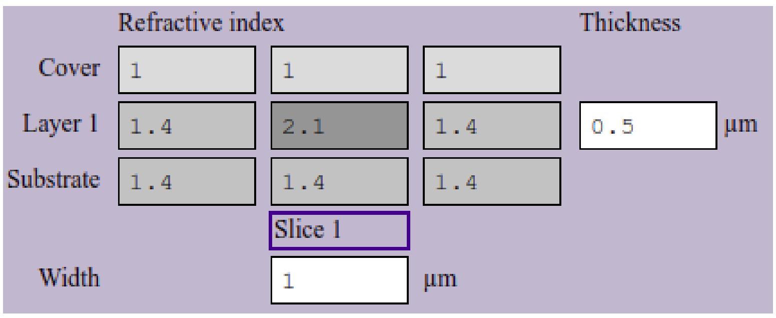
$$\Delta\phi_{TE} = \Delta L_{eff_{TE}} = \frac{\lambda}{4} \rightarrow \left(n_{eff1_{TE}} - n_{eff2_{TE}}\right) = \frac{\lambda}{4L}$$

$$\Delta\phi_{TM} = \Delta L_{eff_{TM}} = -\frac{\lambda}{4} \rightarrow \left(n_{eff2_{TM}} - n_{eff1_{TM}}\right) = \frac{\lambda}{4L}$$

## **Realization:**

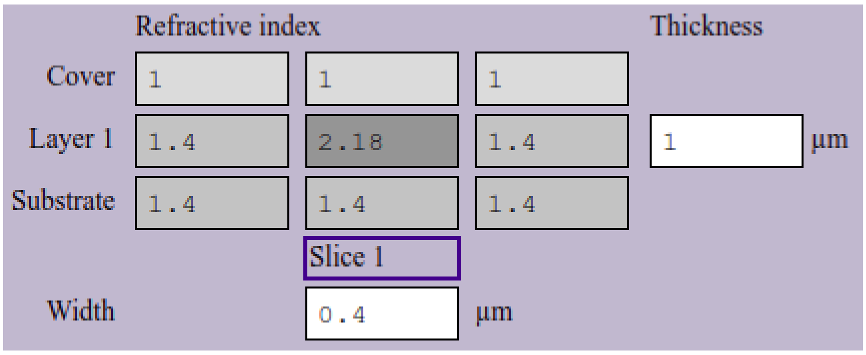
To achieve this condition the effective indexes of TE and TM modes of one arm must have opposite values of the ones in the other arm. This is fulfilled dimensioning the two wavwguides in shapes that are 90° rotated one with respect to the other.
One solution that has been found, for a vacuum wavelength of 1.55µm, is:

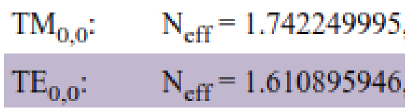
Guide 1:




| | |
|---|---|
| $TE_{0,0}$: | $N_{eff}$ = 1.743402742, |
| $TM_{0,0}$: | $N_{eff}$ = 1.612134274, |

Guide 2:




Knowing these parameters it is possible to decide the length of the arms, that in this case is L=2,9243µm.
For this configuration the phase shifts for the two polarizations are:

$$\Delta\phi_{TE} = 1{,}56999592$$

$$\Delta\phi_{TM} = -1{,}541441923$$

So the extintion ratios for the two ports are:

$$r_{e_1} = 0,\ 000016\% \ = \ -67{,}96\ dB$$

$$r_{e_2} = 0,\ 0215451\% \ = \ -36{,}66\ dB$$

Other configurations could have been exploited, maybe reducing the difference between refractive indexes of the arms in order to reduce the length of the arms, or making the values refractive indexes closer to the opposite ones to achieve better extintion ratios, or employing multimodal waveguides. The aim of this project was only to demonstrate how to split the polarizations and a technique to realize this effect, for devices with better performances more complex techniques have to be used.