Introduction to Counterparty Credit Risk (CCR) and Valuation Adjustments (XVA)

Introduzione all’Ingegneria Finanziaria
Seminario Professionalizzante

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Counterparty Credit Risk (CCR) definition

Counterparty Credit Risk (CCR) is the risk which economic agents face due to the possible default of their over the counter (OTC) counterparts occurring prior to the full compliance of contractual payments.

We have learned from the global financial crisis, that no banking or corporate entity can be any more considered entirely default-free. In other words, CCR *bilaterally* affects OTC derivatives trades.

Differently from loans, in the case of derivatives the future exposures toward the counterpart are not known today. Therefore, potential **future losses are stochastic**.

Consequences of XVAs on fair pricing

Economic reasoning suggests that the fair value of a CCR risky contract has to be lower than the one of an identical default-free contract.

The deviation between these two prices is known as the **Total Valuation Adjustment (XVA)**[1] and is a correction on the fair value due to the presence of default risk.

In terms of *fair valuation*, the XVA represents the **fair price of Counterparty Risk**.

Model set-up

In a bilateral counterparty risk perspective, let me introduce two **defaultable** counterparts involved in a OTC derivative deal denoted by the investment bank B (the dealer) and its client C (which might be a corporate firm or another bank).

The market is modelled through the assignment a **filtered probability space** $(\Omega, \mathbb{Q}, \mathcal{G}_t)$ in which Ω is the set of possible events, $\mathbb{Q}$ is some *risk-neutral* martingale measure and the enlarged filtration is defined as $\mathcal{G}_t := \mathcal{F}_t \vee \mathcal{H}_t$.

$\{\mathcal{F}_t\}_{0 \leq t \leq T}$ is the reference filtration which contains all market information except of default events up to time t, while $\{\mathcal{H}_t\}_{0 \leq t \leq T}$ is the algebra $\sigma(\tau \wedge t)$ generated by default history. The random *first-to-default* time is defined as $\tau := \tau_B \wedge \tau_C$.

Firm equity dynamics

In this framework the NIG process X_t is the relevant *risk driver* for the uncertain dynamics of the underlying asset S_t, whose price at time t under the risk neutral measure $\mathbb{Q}$ is:

$$S_t = S_0 e^{(r-q-\varphi x(-\imath))t+X(t)} \tag{4}$$

where $r > 0$ is the risk-free rate[2] and $a > 0$ is the constant *dividend yield* paid by the underlying stock

Default event

Suppose equation 4 describes the equity value of firms $\{B, C\}$ in addition to that of the underlying commodity asset. Then, the default of the firm is modelled as first passage time at the level of the barrier:

$$\tau_i := \inf\{t \in (0, T] : S_i(0)e^{(r - q_i - \varphi_{X_i}(-\imath))t + X_i(t)} \leq K_i\}$$

or equivalently,

$$= \inf\{t \in (0, T] : X_i(t) \leq \log\left(\frac{K_i}{S_i(0)}\right) - (r - q_i - \varphi_{K_i}(-\imath))t\}$$

Let me define the default indicators $\{D_t^i\}_{0 \leq t \leq T}$ denoting the occurrence of the respective credit events:

$$D_t^i := \mathbb{1}_{\{\tau_i \leq t\}}, \quad i \in \{B, C\}$$

ISDA Master Agreements

In presence of Counterparty Credit Risk it is used distinguish between $V(t, S)$, which denotes the economic value at time t of the *default-free* derivative contract, and $\hat{V}(t, S, D_B, D_C)$, which denotes the value of a correspondent counterparty risky claim.

In case of no default before the maturity of the contract, at time T the buyer will receive or make the flow of contractual payments correspondent to the derivative payoff $\hat{\Phi}(S_T)$, while the dealer earns the opposite cash flows $-\hat{\Phi}(S_T)$.

According to **2002 ISDA Master Agreement** in case of premature default, the surviving counterpart would pay all its debt if it is *out of the money* or, in case of being *in the money*, it could claim just a recovery fraction of its credit.

Bilateral CCR pricing

Let me denote by ε_t *the exposure toward the counterpart at time* t*.*
Suppose that in compliance with the **ISDA 2002 close-out agreements** *at the stopping times* τ_C *and* τ_B*, the following border conditions hold:*

$$\begin{aligned} \hat{V}_{\tau_C} &= R_C(\varepsilon^{+}_{\tau_C}) - \varepsilon^{-}_{\tau_C} \\ \hat{V}_{\tau_B} &= \varepsilon^{+}_{\tau_B} - R_B(\varepsilon^{-}_{\tau_B}) \end{aligned} \tag{5}$$

then, according to the Asset Pricing Theorem (APT) the **value at time** t **of a counterparty-risky derivative claim** *is:*

$$\hat{V}_t = V_t - \underbrace{\mathbb{E}_t[\mathbb{1}_{\{\tau=\tau_C\}} D(t,\tau_C)(\varepsilon^{+}_{\tau_C} - R_C(\varepsilon^{+}_{\tau_C}))]}_{CVA} + \underbrace{\mathbb{E}_t[\mathbb{1}_{\{\tau=\tau_B\}} D(t,\tau_B)(\varepsilon^{-}_{\tau_C} - R_B(\varepsilon^{-}_{\tau_B}))]}_{DVA} \tag{6}$$

The price of CCR: economic interpretation

By assuming that both the recovery functions take deterministically values in [0,1]:

- the **Credit Valuation Adjustment (CVA)** appears to be a call option with zero strike and random maturity issued on the uncollaterized exposure and represents the **expected loss on banks's credits** due to counterparty deafault risk.
- Conversely, the **Debt Valuation Adjustment (DVA)** appears to be a put option with zero strike and random maturity issued on the uncollateratized exposure and represents the **expected gain on bank's debts** due to its own default risk.