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**Title:**

Optimal Allocation of Multiple Emergency Service Resources for Protection of Critical Transportation Infrastructure

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**Publication Date:**

02-01-2008

**Series:**

[Recent Work](#)

**Permalink:**

<http://escholarship.org/uc/item/83m87669>

**Keywords:**

UCD-ITS-RP-08-11

**Abstract:**

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# Optimal Allocation of Multiple Emergency Service Resources for Protection of Critical Transportation Infrastructure

Yongxi Huang, Yueyue Fan, and Ruey Long Cheu

Optimal deployment of limited emergency resources in a large area is of interest to public agencies at all levels. In this paper, the problem of allocating limited emergency service vehicles including fire engines, fire trucks, and ambulances among a set of candidate stations is formulated as a mixed integer linear programming model, in which the objective is to maximize the service coverage of critical transportation infrastructure (CTI). On the basis of this model, the effects of demand at CTI nodes and of transportation network performance on the optimal coverage of CTI are studied. In addition, given a fixed total budget, the most efficient distribution of investment among the three types of emergency service vehicles is identified. To cope with the uncertainty involved in some of the model parameters such as traffic network performance, formulations based on various risk preferences are proposed. The concept of regret is applied to evaluate the robustness of proposed resource allocation strategies. The applicability of the proposed methodologies to high-density metropolitan areas is demonstrated through a case study that uses data from current practice in Singapore.

Critical transportation infrastructure (CTI) facilities, such as bus terminals and interchanges, mass rapid transit stations, tunnels, airports, and seaports, are vital in maintaining normal societal functionality, especially in metropolitan areas. Such facilities are vulnerable to natural and man-made disasters because of the density of people and traffic at these locations and their high costs of repair and maintenance. Therefore, developing an effective protection mechanism for CTI is important in disaster mitigation and the protection of large urban areas.

One way to protect CTI is to improve emergency response readiness. This requires emergency service resources sufficient to serve CTI within an acceptable time. In this paper, the focus is on optimal allocation of fire engines, fire trucks, and ambulances to protect CTI, but the methods presented are applicable to other emergency service resources as well (for example, to the location of emergency medical service vehicles). The ideal would be to deploy unlimited resources to protect the CTI. However, in practice service resources are often limited. Thus the question becomes how to allocate limited resources to a set of possible stations in order to serve as much CTI

as possible (i.e., to maximize the service coverage of the CTI). A CTI node is considered to be covered only if all three types of vehicle can simultaneously reach the node within the required time window (service standard) and are available at a specified reliability level (service reliability). Detailed definitions of service standard and reliability are given in the next section.

The problem being addressed in this paper belongs to the general category of facility location, whose formulations and solution algorithms have been studied extensively in operations research over several decades. A thorough review of strategic facility location problems is provided by Schilling et al. (1) and Owen and Daskin (2). Static and deterministic facility location problems can be further classified into three basic types in terms of their different objectives and constraints and are summarized in Table 1 (3). A comprehensive review and implementations of the three types of problems are given by Jia et al. (21).

The covering model (the first type of model in Table 1) is adopted in this study to locate emergency service vehicles (fire engines, fire trucks, and ambulances). The reasons are as follows. In practice, acceptable service standards in terms of travel time for fire engines, fire trucks, and ambulances are usually predetermined by emergency management agencies and naturally become the constraints in the model; this study is targeted to maximize coverage of CTI nodes (a CTI node is said to be covered if it can be served within a specified time) with limited resources. Earlier models as summarized in Table 1 do not consider the possibility of a server being unavailable when it is busy serving other demand. Later, additional constraints are imposed to guarantee that the probability of at least one vehicle being available to serve each demand node must be greater than or equal to a predefined constant  $\alpha$ . Daskin (22) proposed the “systemwide busy fraction” concept in his model to maximize the expected coverage within a time standard, given  $p$  facilities (stations) to be located in the network. Bianchi and Church (23) proposed a hybrid model incorporating the concepts of MEXCLP (22) and FLEET (5) to site stations and allocate ambulances. This model has been applied to locate emergency medical service vehicles in Fayetteville, North Carolina (24). ReVelle and Marianov (25) then applied the busy fraction of servers concept to the problem of allocating multiple types of emergency resources. ReVelle and Hogan (20, 26) examined different aspects of a similar problem, where the objective is to minimize the total number of utilized servers subject to server availability constraints.

Obviously, allocating emergency service resources is a planning problem involving prediction of model parameters such as incident rate at demand sites and transportation network performance. In practice, these parameters are often random. How to treat uncertainty

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*Transportation Research Record: Journal of the Transportation Research Board*, No. 2022, Transportation Research Board of the National Academies, Washington, D.C., 2007, pp. 1–8.  
DOI: 10.3141/2022-01

TABLE 1 Examples of Facility Location Problems Using Different Modeling Methods

Type	Objective	Constraints	Examples
Covering problem	Maximize coverage of demands (4–6)	Given acceptable service distance/time Limited resources	Locate EMS vehicles (7–9) Locate rural health care workers (10) Place a fixed number of engine and truck companies (11,12) Identify EMS vehicle locations (15,16)
	Set covering: minimize the cost of facility location (13,14)	Specified level of coverage obtained Given acceptable service distance/time	
<i>P</i> -median problem	Minimize the total travel distance/time between demands and facilities (17)	Full coverage obtained Limited resources	Ambulance position for campus emergency service (18) Locate fire stations for emergency services in Barcelona (19)
<i>P</i> -center problem	Minimize the maximum distance between any demand and its nearest facility	Full coverage obtained Limited resources	Locate EMS vehicles with reliability requirement (20)

NOTE: EMS = emergency medical services.

in the location problem has recently become a matter of interest. A comprehensive review of facility location problems under uncertainty is provided by Snyder (27). In the field of stochastic system optimization, various decision criteria have been introduced to cope with uncertainty, including expectation, reliability, and robustness, and chance constraints and penalty functions are used. The most commonly used criterion is the average performance (expectation). In general, expectation-based strategies are risk-neutral and tend to perform well in the long run in a repetitive environment. However, sometimes random events do not repeat themselves often. A good example of such one-time decision making is planning against the occurrence of a large-scale disaster involving infrastructure. Hence, robust approaches are introduced to handle decision making under environments of extreme uncertainty. The concept of regret is used in robust optimization to measure the difference between the best possible result if everything could be predicted and the actual result (28). Since robust optimization focuses on the worst-case scenario, it is usually more conservative than techniques focusing on expectation. Location problems studied in the robust optimization framework can be found in work by Serra and Marianov (19) and Jia et al. (21). In general, the choice of decision criteria usually reflects and depends on decision makers' risk preferences. Later in this paper the performance of different risk models in various uncertain environments will be analyzed and the effect of different risk preferences on the usefulness of emergency service resource allocation decisions will be explored.

This study is built on the foundation of a previous deterministic model (29). The previous model has been revised from a binary integer linear programming formulation to a mixed integer linear program. This change speeds model computation and facilitates extensive sensitivity analysis. More important, a thorough sensitivity and robustness analysis of the model is provided in order to explore the applicability of the model in practice, especially in large metropolitan areas. Sensitivity analysis allows determination of how changes in some predefined model parameters, such as the amount of emergency service resources, affect maximum coverage. This type of cost–benefit analysis is critical to decision makers, since all requests for federal or state funding need to be justified. Robust analysis places more emphasis on parameters describing the uncertain environment (for example, roadway travel time and demand frequency). The key goals of robust analysis are to provide broader decision support under environments of different uncertainty levels and to inform decision makers of the effect of different risk preferences.

## MATHEMATICAL MODELS

Given predicted demand for emergency services at CTI nodes, the goal is to find an optimal strategy for allocating the limited number of fire engines, fire trucks, and ambulances to a set of predefined candidate stations so as to maximize coverage of the CTI. A CTI node is covered if it is served within the required time by at least one fire engine, one fire truck, and one ambulance. In addition, the service reliability, defined as the probability of at least one vehicle of each type being available at any time, is required to be no less than  $\alpha$ . This is a maximum expected covering problem. Models considering service reliability are categorized as probabilistic models in the review by Owen and Daskin (2). However, in this analysis, the difference between standard stochastic models and the proposed model will be realized. Standard stochastic models usually involve an explicit probability distribution of random parameters or possible scenarios, while the proposed maximum expected covering model preprocesses input parameters on the basis of the reliability requirement and historical demand quantity and then inputs all parameters into the core model as known deterministic values.

### Base Model Formulation

First, the formulation of the base scenario is considered. The following assumptions are made:

1. The total number of available emergency vehicles of each type is given.
2. A set of candidate stations is predefined, and their locations are known.
3. A restriction on capacity is imposed at each station.
4. Incident occurrence rates at demand nodes are estimated on the basis of historical data.
5. Emergency service vehicles are assumed to travel at their free-flow speeds. Note that the free-flow speeds of emergency service vehicles are higher than those of normal vehicles.

The main purpose of introducing the base model is to explain the concept and computation of service reliability.

Denote as  $I$  the set of demand nodes (CTI nodes in this paper) and as  $J$  the set of possible stations. First consider the availability of each type of emergency service vehicle at station  $j$  ( $j \in J$ ) when there is a request for emergency services at demand node  $i$  ( $i \in I$ ). Theoretically, the probability of a server being busy should depend

on the features of the server and its competing neighboring demand nodes. Thus it is more realistic to “make use of server-specific busy fractions” (30). However, because of the computational burden involved in the use of server-specific busy fractions, an intermediate approach was introduced by ReVelle and Hogan (20): the “use of demand-area-specific busy fractions.” The busy fraction in the service region around demand node  $i$  for a particular type of emergency service vehicle (e.g., fire engines) is defined as the required service time in the region divided by the available service time in the region (30). Thus,

$$q_i^E = \frac{\bar{t}^E \sum_{j \in ME_i} f_j}{24 \sum_{j \in NE_i} x_j^E} = \frac{\rho_i^E}{\sum_{j \in NE_i} x_j^E} \quad \forall i \quad (1)$$

where

$q_i^E$  = local busy fractions for fire engines, centered at demand node  $i$ ;

$\bar{t}^E$  = average service time of fire engines on site (hours per call);

$f_j$  = frequency of requests for service at demand node  $i$  (calls per day);

$x_j^E$  = number of fire engines located at fire station  $j$ ;

$S^E$  = service standards in terms of travel time for fire engines;

$ME_i = \{\forall i \mid t_{ji} \leq S^E\}$ , the set of demand nodes competing for services by fire engines located within  $S^E$  of demand node  $i$ ;

$NE_i = \{\forall j \mid t_{ji} \leq S^E\}$ , the set of fire stations located within  $S^E$  of demand node  $i$ ;

$t_{ji}$  = travel time between station  $j$  and demand node  $i$ ; and

$\rho_i^E$  = utilization ratio of fire engines at demand node  $i$ , as defined by ReVelle and Snyder (31).

The local estimate of busy fraction for fire trucks and ambulances can be similarly expressed by using their associated service standards.

In accordance with the work by ReVelle and Hogan (20), it is assumed that the requests for services from different nodes are independent and that all demand nodes within  $S^E$  have the same  $q_i^E$  value. The probability of one or more servers being busy thus follows a binomial distribution. The probability of having at least one fire engine available is therefore

$$1 - P(\text{all engines within } S^E \text{ of node } i \text{ are busy}) \\ = 1 - (q_i^E)^{\sum_{j \in NE_i} x_j^E} = 1 - \left( \frac{\rho_i^E}{\sum_{j \in NE_i} x_j^E} \right)^{\sum_{j \in NE_i} x_j^E} \quad (2)$$

To meet the server availability requirement—that the probability of having at least one fire engine available within  $S^E$  of node  $i$  when node  $i$  is requesting service must be larger than or equal to  $\alpha$ —the following must hold:

$$1 - \left( \frac{\rho_i^E}{\sum_{j \in NE_i} x_j^E} \right)^{\sum_{j \in NE_i} x_j^E} \geq \alpha \quad (3)$$

This probabilistic constraint does not have an analytical linear equivalent. However, a numerical linear equivalent can be found by defining the parameters  $e_i$  as the smallest integers satisfying the following:

$$1 - \left( \frac{\rho_i^E}{e_i} \right)^{e_i} \geq \alpha \quad (4)$$

Similar expressions can be written for trucks and ambulances by substituting  $T$  and  $A$ , respectively, for  $E$ . Correspondingly, new parameters  $t_i$  and  $a_i$  are defined as the minimum number of fire trucks and ambulances that must be located within  $S^T$  and  $S^A$  of node  $i$  to ensure that node  $i$  is covered at reliability level  $\alpha$ .

The complete mixed integer model formulation for the maximum coverage problem is as follows:

$$\text{maximize } \sum_{i \in I} y_i \quad (5)$$

subject to

$$\sum_{j \in NE_i} x_j^E \geq e_i y_i \quad \forall i \in I \quad (6)$$

$$\sum_{j \in NT_i} x_j^T \geq t_i y_i \quad \forall i \in I \quad (7)$$

$$\sum_{j \in NA_i} x_j^A \geq a_i y_i \quad \forall i \in I \quad (8)$$

$$\sum_{j \in J} x_j^E \leq P^E \quad (9)$$

$$\sum_{j \in J} x_j^T \leq P^T \quad (10)$$

$$\sum_{j \in J} x_j^A \leq P^A \quad (11)$$

$$0 \leq x_j^E \leq B \quad \forall j \in J \quad (12)$$

$$0 \leq x_j^A \leq B \quad \forall j \in J \quad (13)$$

$$0 \leq x_j^T \leq B \quad \forall j \in J \quad (14)$$

$$y_i = 0, 1 \quad \forall i \in I \quad (15)$$

where

$y_i = 1$  if demand node  $i$  is covered by  $e_i$  fire engines within  $S^A$ ,  $t_i$  fire trucks within  $S^T$ , and  $a_i$  ambulances within  $S^A$ ; otherwise,  $y_i = 0$ ;

$x_j^E$  = number of fire engines located at station  $j$ ;

$x_j^A$  = number of ambulances located at station  $j$ ;

$x_j^T$  = number of fire trucks located at station  $j$ ;

$NE_i$ ,  $NT_i$ , and  $NA_i$  = sets of stations located within  $S^E$ ,  $S^T$ , and  $S^A$  of demand node  $i$ , respectively (e.g.,  $NE_i = \{\forall j \mid t_{ji} \leq S^E\}$ );

$B$  = maximum number of vehicles of each type that can be accommodated by each station (it is assumed that every candidate station has the same capacity to house  $B$  fire engines,  $B$  fire trucks, and  $B$  ambulances); and

$P^E$ ,  $P^T$ , and  $P^A$  = total number of available fire engines, fire trucks, and ambulances, respectively.

The objective presented in Expression 5 maximizes the total number of covered CTI facilities. Constraint 6 states that at each node  $i$ , the number of fire engines located at fire stations within  $S^E$  of node  $i$  must be greater than or equal to the number of fire engines needed within  $S^E$  of node  $i$  to meet the reliability requirement. Constraints 7 and 8 can be similarly explained for fire trucks and ambulances, respectively. Constraints 9 through 11 restrict the total number of fire engines, fire trucks, and ambulances to be assigned. Inequalities 12 through 14 impose the capacity constraints at each fire station.

A detailed procedure for parameter preparation in the base model is provided as follows:

1. Use geographic information system software (e.g., ArcGIS 8.0 in this study) to locate CTI nodes and to integrate these nodes in the transportation network.
2. Generate travel time matrices containing travel times between each pair of CTI nodes and between each CTI node and candidate fire station location. For the purpose of this study, a script to carry out this task was written in ArcView 3.1.
3. Generate node sets  $NE_i$ ,  $NT_i$ , and  $NA_i$  for each CTI node by using travel time information given in the matrices obtained from Step 2. These sets of nodes indicate the set of candidate fire stations within the service range of CTI node  $i$ .
4. Generate node sets  $ME_i$ ,  $MT_i$ , and  $MA_i$  for each CTI node by using travel time information given in the matrices obtained from Step 2. The sets of nodes indicate the set of CTI nodes competing for emergency service with CTI node  $i$ .
5. Compute the local server busy fraction for each CTI node according to Equation 1. Once the busy fraction is computed, the minimum number of emergency service vehicles can be computed according to Inequality 4.

## Robust Optimization Model

The parameters in the base model are all assumed known. However, some of the parameters, such as travel time over the traffic network and the incident rate at demand nodes, may be uncertain in practice. An optimal policy can be computed from a mathematical model by using forecast most likely parameters. However, the future realized values of the parameters are often different from the forecast values, while the effectiveness of a decision is often evaluated in the aftermath of a realized scenario as if all the parameters were known in advance. Significant data uncertainty of the decision environment and the strong desire of decision makers to avoid extremely bad consequences naturally lead to the consideration of robust approaches, which emphasize the worst-case scenario.

If the model parameters such as travel time over the traffic network were known in advance, their values could be input into the base model, and the best possible coverage could be achieved. The difference between the best possible objective value and the realized objective value from a chosen strategy is called the “regret” of the strategy in that realization (27). Some robust optimization approaches deal directly with the objective values across all possible realizations. In this case, the criterion is to find a strategy that maximizes the worst benefit across all possible realizations, also called the absolute robustness criterion. Some robust approaches use the robust deviation criterion, which is to minimize the largest regret (28). In this section, the facility location problem is modeled on the basis of the absolute robustness criterion (i.e., an allocation strategy is produced so as to maximize the minimum coverage for CTI nodes across all possible realizations).

Several parameters in the base model can be uncertain. Here, only uncertain travel times over the network will be considered as an illustration. The mathematical formulation is as follows:

$$\text{maximize } M \quad (16)$$

subject to Constraints 9 through 15 of the base model formulation and

$$\sum_{j \in NE_i^k} x_j^E \geq e_i y_i^k \quad \forall i \in I \quad \forall k \in K \quad (17)$$

$$\sum_{j \in NT_i^k} x_j^T \geq t_i y_i^k \quad \forall i \in I \quad \forall k \in K \quad (18)$$

$$\sum_{j \in NA_i^k} x_j^A \geq a_i y_i^k \quad \forall i \in I \quad \forall k \in K \quad (19)$$

$$\sum_{i \in I} y_i^k \geq M \quad \forall k \in K \quad (20)$$

where

- $t_{ji}^k$  = travel time between  $i$  and  $j$  in realization  $k$ ;
- $NE_i^k$ ,  $NT_i^k$ , and  $NA_i^k$  = sets of stations located within  $S^E$ ,  $S^T$ , and  $S^A$  of demand node  $i$  in realization  $k$ , respectively (e.g.,  $NE_i^k = \{j \mid t_{ji}^k \leq S^E\}$ );
- $y_i^k$  = 1 if demand node  $i$  is covered by  $e_i$  fire engines within  $S^A$ ,  $t_i$  fire trucks within  $S^T$ , and  $a_i$  ambulances within  $S^A$  under realization  $k$ ; otherwise,  $y_i = 0$ ;
- $M$  = smallest total coverage achieved in any realization; and
- $K$  = set of possible realizations.

All other variables have the same meanings as in the base model formulation. The left side of Constraint 17 represents the supply of fire engines around demand node  $i$  in scenario  $k$ . The right side of Constraint 17 is the demand for fire engines at demand node  $i$ , which is assumed to be deterministic. Constraints 17 through 19 guarantee that the computed  $y_i$  is the worst coverage of demand node  $i$  across all possible scenarios. The objective of the model is still to maximize the total coverage of CTI nodes, but because of the changing meaning of  $y_i$ , the objective becomes to find an allocation strategy that maximizes the minimum total coverage achieved across all scenarios.

## CASE STUDY

### Background

Singapore is used as an example of the application of the proposed model to a high-density metropolitan area. Singapore has 15 fire stations and 151 CTI facilities (see Figure 1). The CTI facilities include mass rapid transit stations, transit or bus interchanges, bus terminals, expressway tunnels and interchanges, and seaport and airport terminals.

The Singapore Civil Defense Force (SCDF) is the government agency responsible for providing emergency response services. SCDF operates fire engines and trucks and a fleet of 30 ambulances. The three types of vehicles are based at fire stations. Current published service standards of SCDF are 8 min for fire engines and fire trucks and 11 min for ambulances to reach the incident site (32).



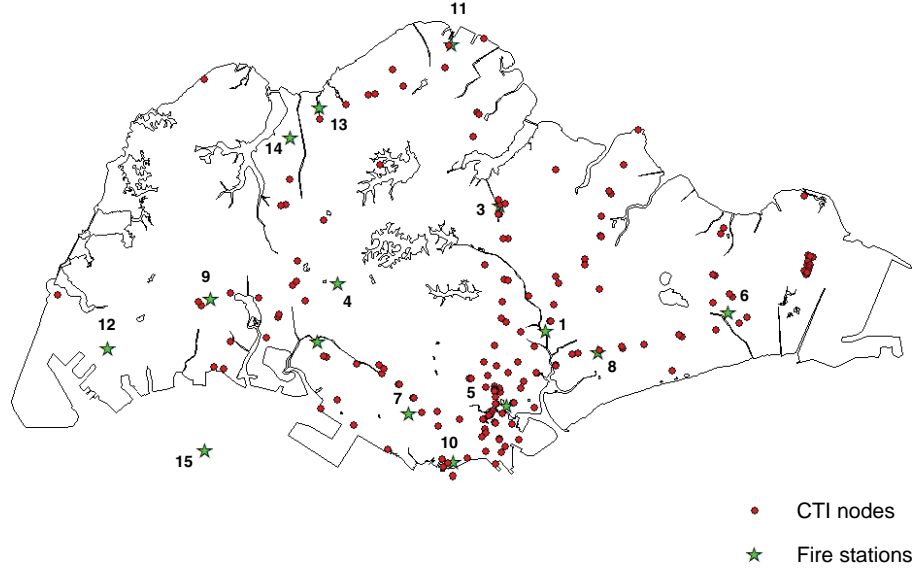


FIGURE 1 Map of Singapore with candidate fire stations.

The average service times for fire engines, fire trucks, and ambulances are set to be 2, 2, and 1.5 h, respectively, which are adopted from ReVelle and Marianov (25). A total of 3,912 fire cases during the period from January 2003 to December 2003 (33) are used as historical data to estimate  $f_i$ , the incident frequency at each demand node. The service reliability required by SCDF is 90%.

### Base Scenario

The base scenario is based on the existing practice of SCDF, in which no more than 15 fire engines, 15 fire trucks, and 30 ambulances (i.e.,  $p^E = 15$ ,  $p^T = 15$ , and  $p^A = 30$ ) are allocated among the 15 candidate fire stations to maximize the coverage of the 151 CTI nodes. An optimal solution to the base scenario is given in the table below, where, for example,  $1^2$  means that two fire engines should be allocated at Fire Station 1.

Fire Engines (total = 15)	Fire Trucks (total = 15)	Ambulances (total = 21)
$1^2, 2^2, 3^2, 6^2, 7, 9,$ $11, 12, 13^2, 14$	$1^2, 2^2, 3^2, 4, 6^2, 7,$ $9, 11, 12, 13, 14$	$1^2, 3^4, 4, 5^2, 6^4, 7,$ $8^4, 11, 12, 13$

The existing emergency service vehicles can cover at most 126 CTI nodes if resources are allocated optimally. Several observations are made on the basis of these results. First, the existing setting of service resources and candidate stations in Singapore is not sufficient to cover all CTI nodes under the SCDF requirements for service standard and service reliability. Second, all fire engines and trucks are fully utilized in the optimal solution, but there is some redundancy with regard to the number of available ambulances. This indicates a need for redistributing the share of the three types of vehicles to achieve better system coverage. Finally, the results indicate multiple optimal allocation strategies that lead to the same maximum coverage. Note that all the constraints and unit benefits for fire engines and trucks are the same. Thus identical allocation strategies are expected for these two types of vehicles. However, there is a slight difference between the second and third columns of the table for Stations 4 and 13. The two allocation strategies were later switched for the two types of vehicles, and the same maximum coverage of 126 was obtained.

Multiple optimal solutions usually provide alternatives and thus more flexibility in the actual allocation of service resources.

### Sensitivity Analysis in Resource Budget

As observed in the base scenario, on the one hand, the total existing service resources are not sufficient to cover all CTI nodes. On the other hand, some existing resources are not fully utilized. In this section, the effects of the total resource budget on maximum coverage are studied on the basis of a sensitivity analysis. In addition, changing the budget constraints to allow redistribution of the three types of resources is examined. Usually, different types of service vehicle resources are planned by different agencies or divisions. It is demonstrated that, through a more efficient budget allocation among the agencies or divisions, a higher coverage can be achieved.

Constraints on multiple types of service vehicles (Constraints 9 through 11) are combined into the following single monetary resource constraint:

$$c^E * \sum_{j \in J} x_j^E + c^T * \sum_{j \in J} x_j^T + c^A * \sum_{j \in J} x_j^A \leq I \quad (21)$$

where  $I$  is total investment and  $c^E$ ,  $c^T$ , and  $c^A$  are unit purchasing prices of fire engines, fire trucks, and ambulances, respectively. According to SCDF, these parameters take the value of \$325,000, \$700,000, and \$200,000 (U.S.), respectively.

In the base scenario, the optimal strategy requires a total of 15 fire engines, 15 fire trucks, and 21 ambulances. The corresponding cost is \$19.6 million (U.S.), and the corresponding coverage is 126. When redistribution of resources is allowed, only \$16.9 million is needed to achieve the same coverage. Furthermore, the coverage can be improved to 127 with \$18.7 million. The ability to cover more demand with less money shows the benefit of allowing redistribution of the three types of resources.

Policy makers are often more interested in what funding level they should request than in detailed allocation strategies. Justification of a certain funding level requires a cost–benefit study that can be used to measure the marginal change of coverage as the total funding

changes. The range of total investments from \$2.5 million to \$19 million in increments of \$0.3 million is examined. The relationship between the total investment and the maximum coverage is illustrated in Figure 2.

In Figure 2, every data point denotes the coverage corresponding to an investment. As the total investment increases, the optimal coverage reaches a maximum of 127 and does not increase beyond that no matter how much investment is made. An investigation of the spatial relationship between the potential station sites and demand sites suggests that some demand nodes are beyond the service areas (in terms of service standard and reliability) of the existing stations. Simply purchasing more vehicles does not improve the coverage of those demand nodes. New stations must be planned to cover those remote areas. Another observation from Figure 2 is that the marginal benefit of increasing one unit of investment varies across the investment levels. Therefore, in making investment decisions, a balance between safety and efficiency must be sought.

### Robust Analysis

The sensitivity analysis given above is conducted in a deterministic environment in which travel times of emergency vehicles are assumed to be known and equal to their free-flow speeds. In this section, the travel times between CTI nodes and fire stations are allowed to fluctuate. The fluctuation may be caused by congestion or the unavailability of some road segments after natural or human-induced disasters. In general, noise can be positive or negative. However, since the travel times used in the base scenario are already based on free-flow speed, only positive noise with a uniform distribution over  $[0, 1]$  is considered—that is, travel time between  $[t, t(1 + n)]$ , where  $t$  is the free-flow travel time and  $n$  is the noise level expressed in percent. Three levels of noise (20%, 50%, and 100%) are considered. A higher noise level implies a more congested traffic network.

For each noise level, 100 realizations with random travel times are simulated. Assume that the actual travel times in each realiza-

tion are known in advance. The best resource allocation strategy and the corresponding maximum coverage from the base model are computed. The maximum coverage in each realization is plotted in Figure 3, and the statistics are given in the second through fourth columns of Table 2.

Two different strategies are then presented: expected strategy and robust strategy with their associated regrets. For each given noise level, 100 independent sets of travel time matrices were simulated. The average travel times of the 100 realizations were entered into the maximum expectation model to find the optimal expected strategy. The same sets of realized travel times were also used to compute the robust strategy. The regrets of the two strategies and their statistics are given in Table 2.

The effects of the uncertain environment on the quality of decision strategies are now considered. Whether a model is sensitive to a change of its parameters can be examined by observing the change in objective value caused by the change in model parameters. In this regard, the proposed base model is not sensitive to the fluctuation in traffic network performance in the sense that the average coverage only drops by 5.9%  $[(121.38 - 114.24)/121.38]$  as the noise level of travel time increases from 20% to 50%. However, it is consistently observed that the performance of both strategies degrades as the level of uncertainty of the environment increases. Furthermore, as the uncertainty level becomes higher (the noise level reaches 50% or 100%), the overall performance of the robust strategy is better than that of the expected strategy.

Conceptually, regret can be considered as a measure of the value of perfect information (i.e., the benefit of having perfect information of uncertain model parameters). A decision maker who prefers the base model for its conceptual simplicity should be willing to pay more for data forecasting and calibration, since the benefit from better data quality is significant.

However, these observations are based on a relatively small sample. Much more computer simulation must be conducted to draw a more representative conclusion with regard to the uncertain environment.

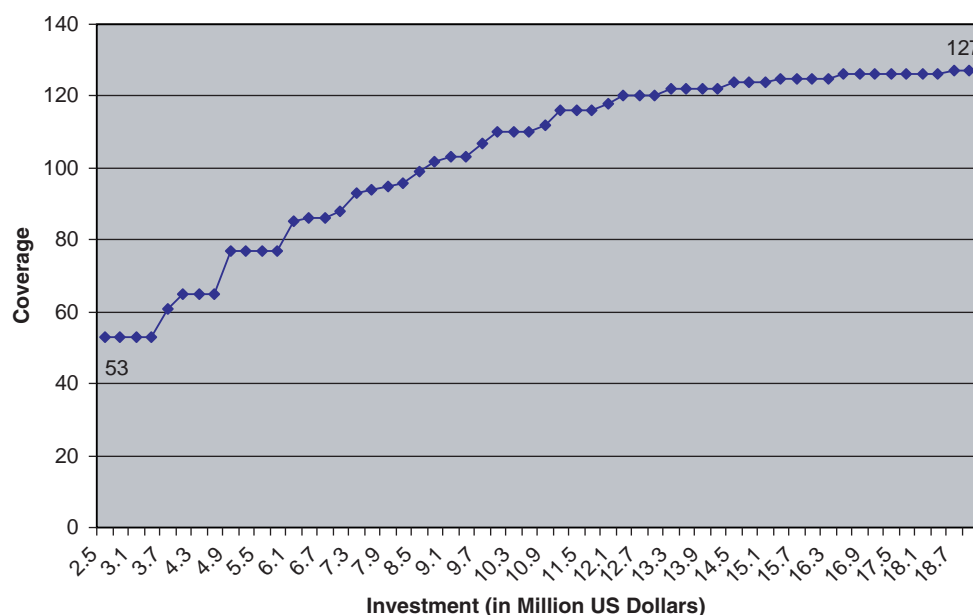


FIGURE 2 Relationship between total investment and maximum coverage: sensitivity analysis.

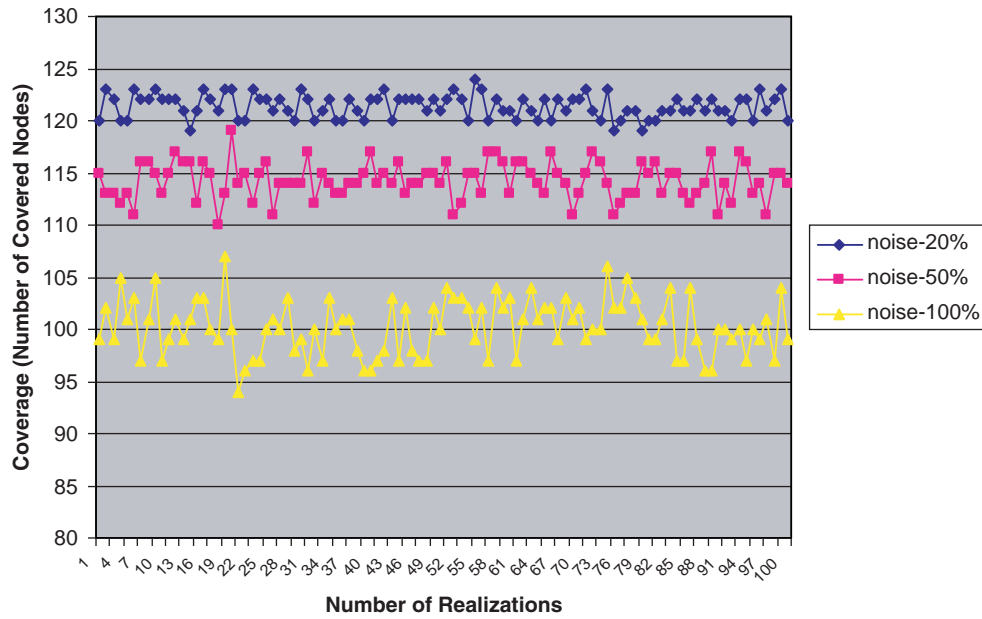


FIGURE 3 Best possible coverage in each realization.

## DISCUSSION OF RESULTS

In this paper, the facility location and resource allocation problem has been considered in the context of emergency management. According to practitioners in the field of emergency management, the role of location problem models is often underplayed in practice because it is considered unrealistic to relocate existing emergency service stations except in cases of new areas under development. The case study indicates that a location model with a resource allocation feature can be used to identify not only the best location for potential stations but also the distribution of service resources among the stations. Therefore, even for developed areas already equipped with service stations, reexamining whether the limited resources are distributed most efficiently may still be valuable. The sensitivity analysis of resource constraint also provides a quantitative method for justifying the investment level for equipping emergency stations in a given region. The regret analysis in computer simulations provides the value of improving prediction of uncertain model parameters and helps justify the need for improving data quality.

From a modeling viewpoint, the focus has been on handling resource availability and uncertainty of the environment. A base model

using expected values of the model parameters and a robust model focusing on the worst-case scenario are studied in parallel. Both models are approximations of reality and thus involve simplification and assumptions. The base model is simpler, both conceptually and computationally. However, when significant uncertainty is involved in model parameters, following the robust approach tends to be the safer choice.

In this work, the focus has been on allocation of three types of emergency resources among fire stations. However, the methods are suitable for other types of emergency services and management centers, such as planning for shelters following large-scale urban disasters and allocating inspection or medical treatment resources and personnel. Despite the intense data processing and modeling efforts involved in this work, several important issues have not been addressed. First, even though the concept of service reliability is one way to handle uncertainty in demand, the minimum service resource requirements at demand nodes are computed on the basis of historical data. The fluctuation of future demand due to population, unknown risk, or spatial features of the study area should be considered explicitly in the model. Robust analysis of the proposed model against demand fluctuation is necessary. Second, the current work only

TABLE 2 Statistics of Regrets of Maximum Expected Strategy and Robust Strategy

	Maximum Coverage in Simulated Realizations for Noise Level			Regret of Expected Strategy for Noise Level			Regret of Robust Strategy for Noise Level		
	20%	50%	100%	20%	50%	100%	20%	50%	100%
Average	121.38	114.24	100.22	0.42	3.15	3.83	0.43	2.03	2.38
Min	119	110	94	0	0	1	0	0	0
Max	124	119	107	3	8	7	1	5	6
SD	1.12	1.77	2.67	0.68	1.74	1.54	0.50	1.31	1.17
Range	5	9	13	3	8	6	1	5	6



provides single-layer coverage. However, for highly critical infrastructure, single-layer coverage may not be sufficient in an extreme environment. The introduction of backup coverage for such infrastructure with implementation of robust optimization approaches would be an interesting extension of this work.

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*The Critical Transportation Infrastructure Protection Committee sponsored publication of this paper.*