Portfolio Optimization

For MIT MFin Application - Fall 2018

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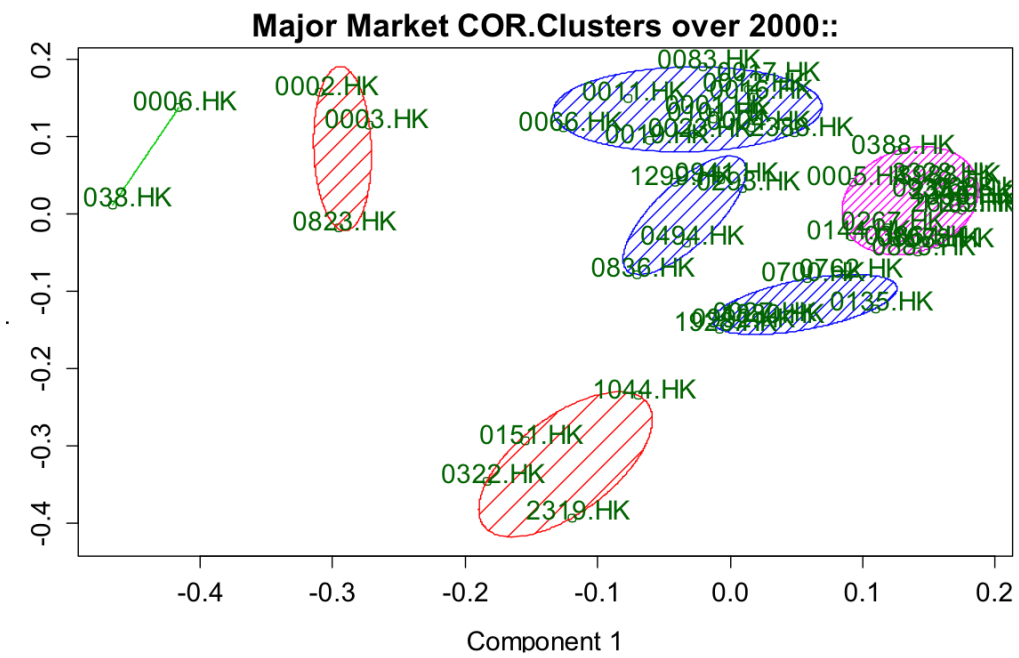
Following are selected parts from my research findings...

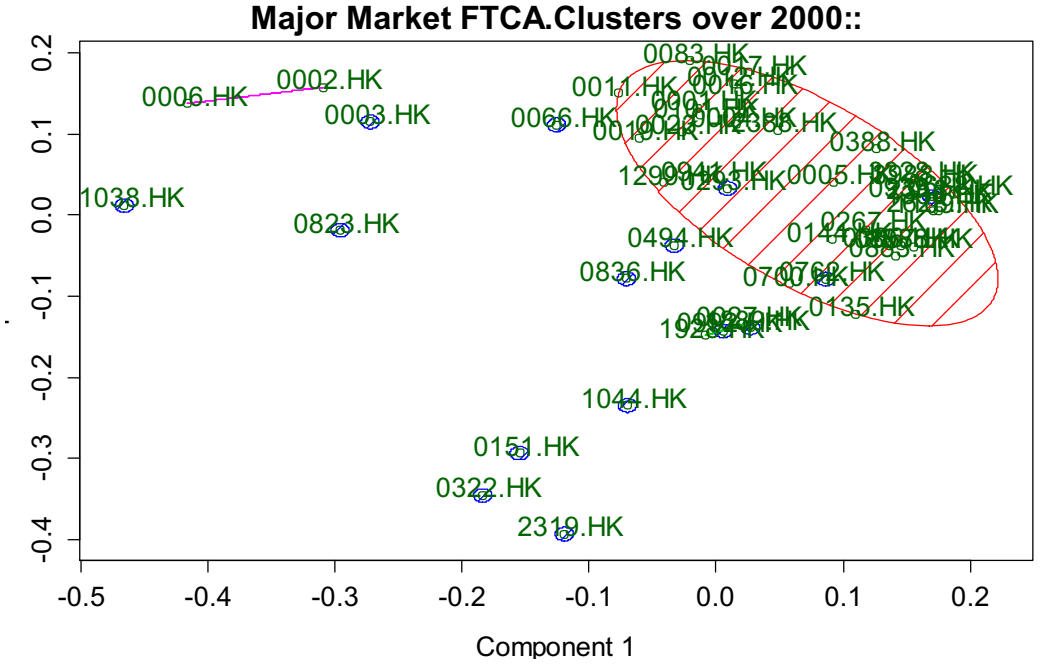
Cardinality constraint portfolio optimization problem is of **popular concern** in recent years in the area of portfolio optimization. **Transaction costs** such as *brokerage fees* make the diversification proposed by Markowitz not feasible in the real world. Thus there is a genie interest in solving the **cardinality constraint portfolio optimization** problems.

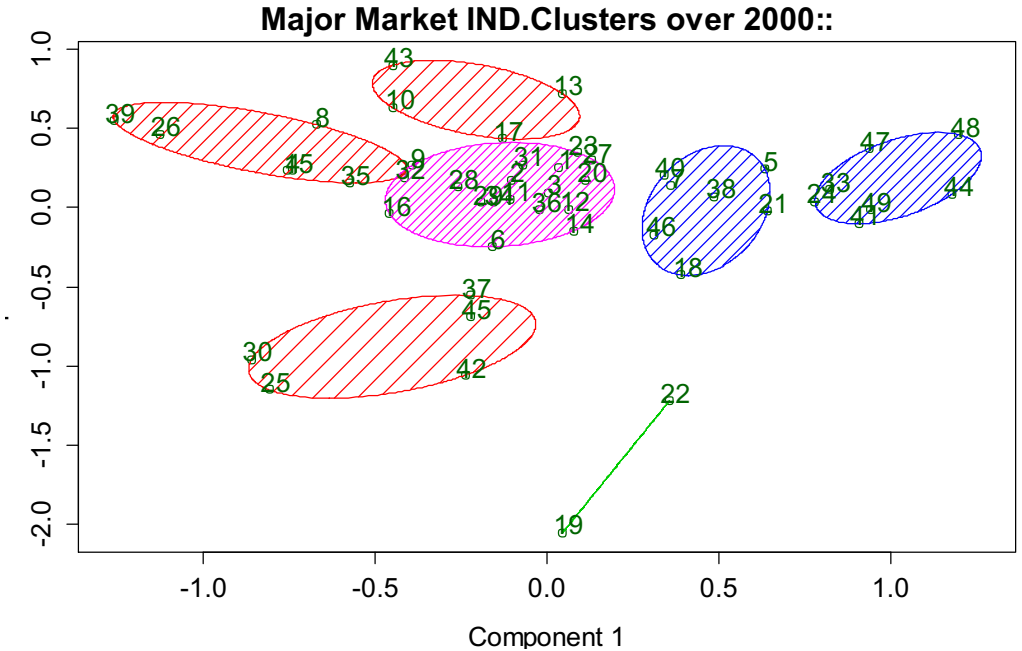


←k = 7

COR

k =18 →





←IND

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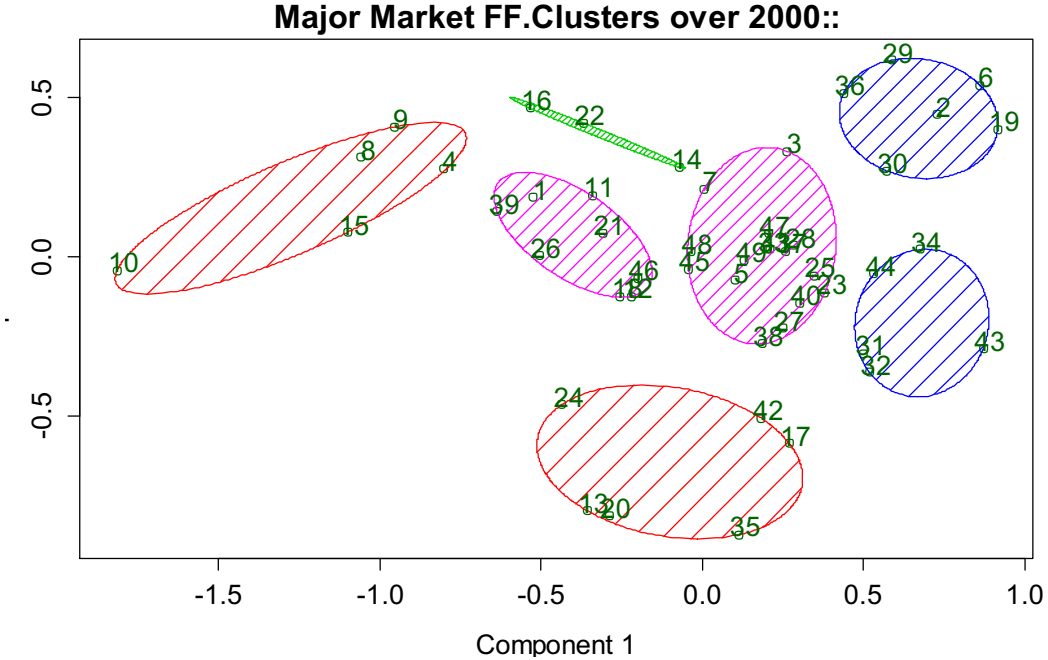


DYNAMIC REBALANCING MODEL

Symbols and Notations ii

Input in prior to rebalancing time point t

$\mu_{t,i}$: the expected return vector

$\sigma_{t,ij}$: the covariance matrix

$P_{t,i}$: the current market price of per share of asset i at time point t

$c_{t,i}$: the transaction cost rate if any trading of asset i is incuured, $c_{t,i} = fc_{t,i}$ for fixed transaction cost and $c_{t,i} = lc_{t,i}$ for linear cost type

$l_{t,i}$: lower bound on portfolio weight

$u_{t,i}$: upper bound on portfolio weight

$X_{t,i}$: the number of shares hold in the current portfolio of asset i (i = 1,. . . ,N) at time point t

Symbols and Notations iv

Decision variables for investment period [t, t+1]

$x_{t,i}$: the number of shares to hold in the portfolio of asset i in the new portfolio at time point t

$a_{t,i}$: binary variable representing whether asset i is to be included in the new portfolio at time point t, equals to 1 if it is, 0 otherwise

$t_{t,i}$: the number of shares to trade in the position on asset i in order to get the optimal portfolio at time point t

$w_{t,i}$: portfolio weight vector of asset i in the new portfolio

Formulation of $w_{t,i}$ and Balance of portfolio value

$$w_{t,i} = \frac{P_{t,i} x_{t,i}}{v_t}, \tag{1}$$

where

$$V_t = \sum_{i=1}^{N} P_{t,i} X_{t,i}$$

$$v_t = V_t - TC_t$$

$$x_{t,i} = X_{t,i} + t_{t,i}, \quad i = 1, \ldots, N$$

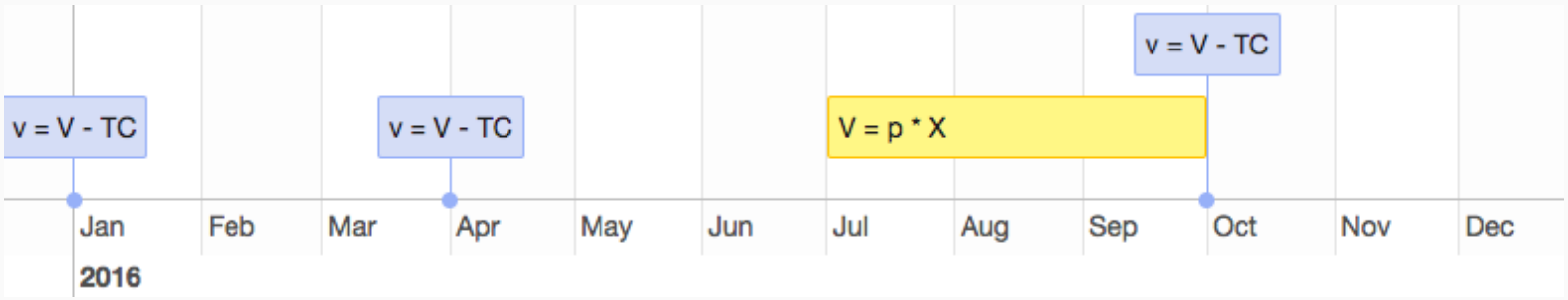


Figure 2: Evolution of a quarterly rebalanced portfolio value

Position Change Constraint

$$\sum_{i=1}^{N} |a_{t,i} - A_{t,i}| = \Delta \tag{2}$$

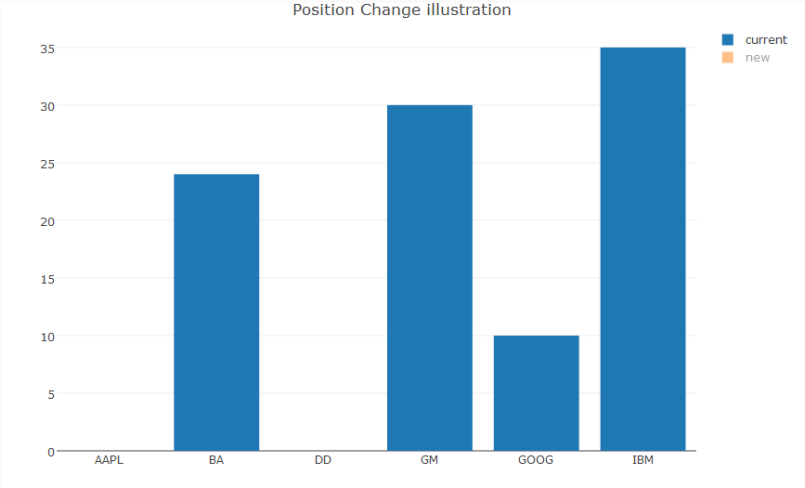
where

$$\Delta = 0, 2, 4, \ldots, 2K \tag{3}$$

Table 1: Example to illustrate how position change constraint works

position index	AAPL	BA	GM	IBM	DD	GOOG
current position index	0	1	0	1	1	1
new position index	1	1	0	0	1	1

Illustration



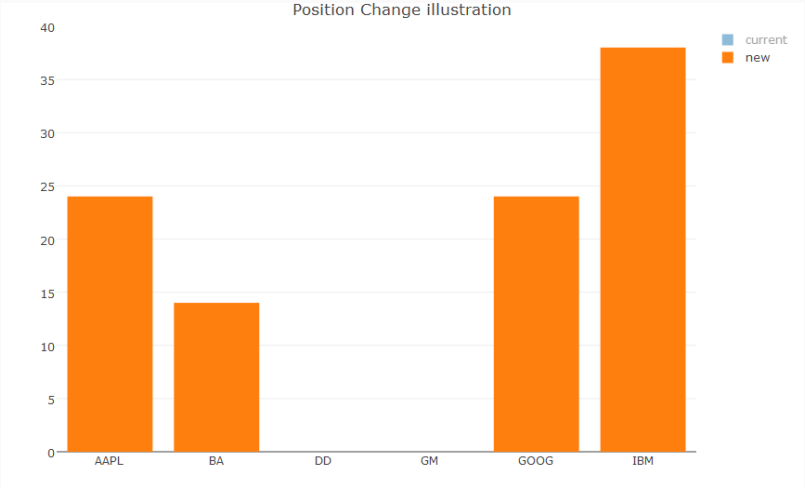
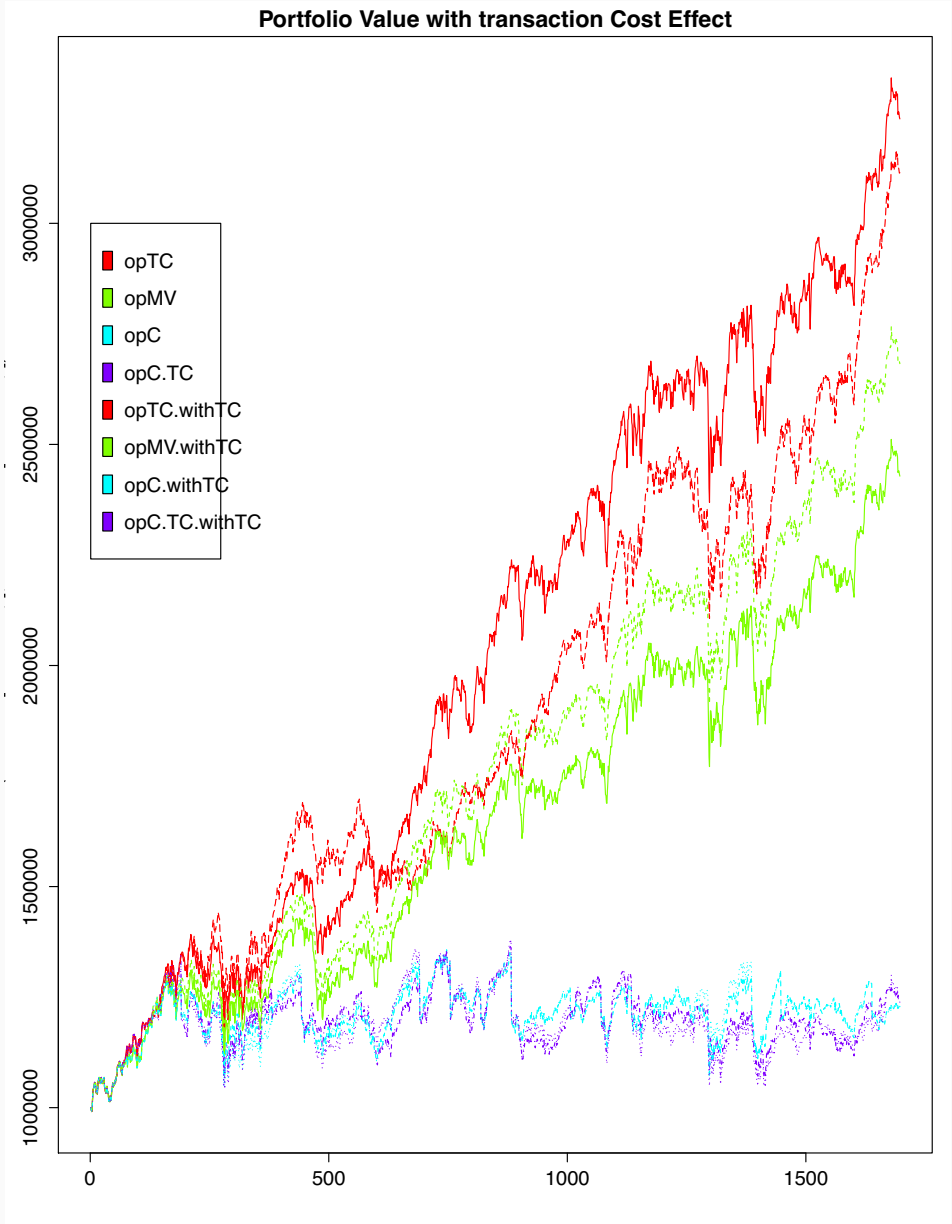


Illustration Example: Current and new positions in a simple portfolio

Closer to the effect of transaction costs. . .



Find out more …

Do not *hesitate to contact with me if you are interested*
And I am willing to ***share*** *the research findings~*