

## Practical Session 5

### Optimization

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#### 1 Convexity

**Problem 1:** Show that affine functions of the form  $\mathbf{w}^T \mathbf{x} + b$  are both convex and concave.

**Problem 2:** Show that a twice differentiable function  $f(x)$  is convex if and only if its domain is convex and its Hessian or second derivative exists and is positive semidefinite:  $\nabla^2 f(x) \geq 0$  for all  $x \in \text{dom}(f)$ .

#### 2 Logistic Regression

**Problem 3:** Prove that the objective function of logistic regression

$$E(\mathbf{w}) = -\ln p(\mathbf{y} \mid \mathbf{w}, \mathbf{X}) = -\sum_{i=1}^N (y_i \ln \sigma(\mathbf{w}^T \mathbf{x}_i) + (1 - y_i) \ln(1 - \sigma(\mathbf{w}^T \mathbf{x}_i))) \quad (1)$$

is convex. What is the benefit of having a convex function for optimization?

#### 3 Optimization methods

**Problem 4:** Discuss the following topics:

- Condition number
  - Stability, consistency, convergence
  - Stiffness
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