Aalto University

Distributed Algorithms 2020

6a Randomized coloring

(Δ = maximum degree)

Vertex coloring with Δ+1 colors

Week 4: deterministic, $O(\Delta + \log^* n)$

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Vertex coloring with Δ+1 colors

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Today: randomized, w.h.p. $O(\log n)$

Simplest possible idea:

- everyone tries to pick a ***random free color***

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- stop if successful

Pretty simple idea:

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Lemma: A node that is still running, will stop in this round with probability ≥ 0.25

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Corollary: The node is still running after T rounds with probability $\leq 0.75^T$

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Corollary: The node will stop after $O(\log n)$ rounds w.h.p.

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Corollary: ***All*** nodes will stop after $O(\log n)$ rounds w.h.p.

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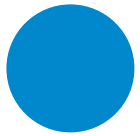
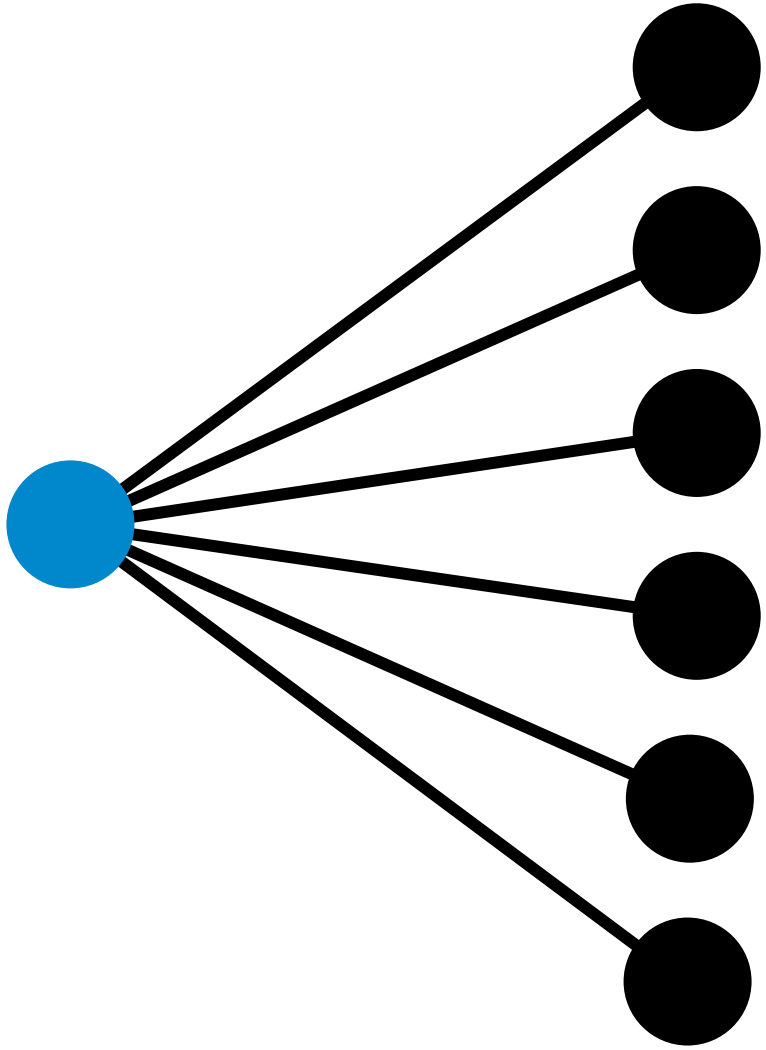
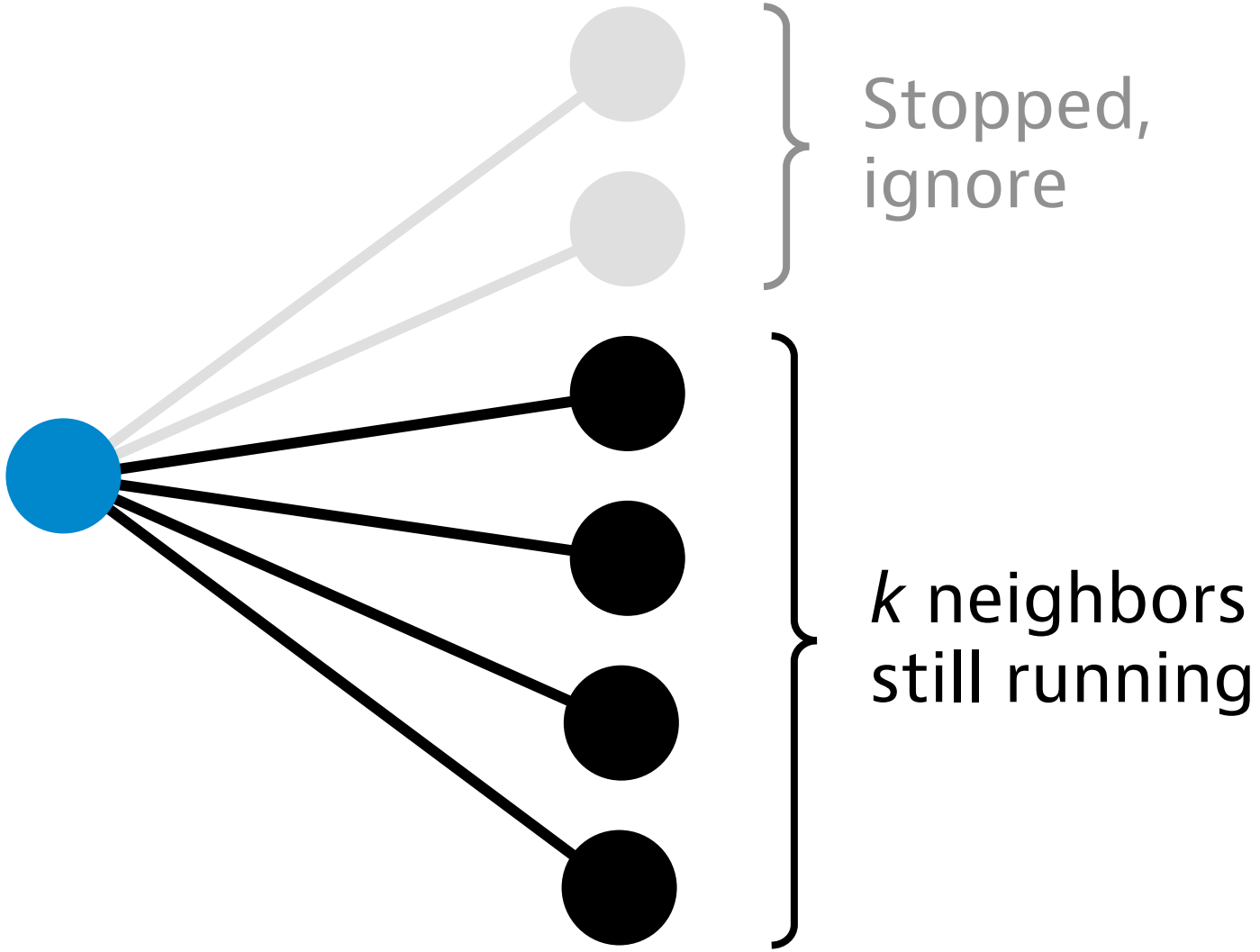
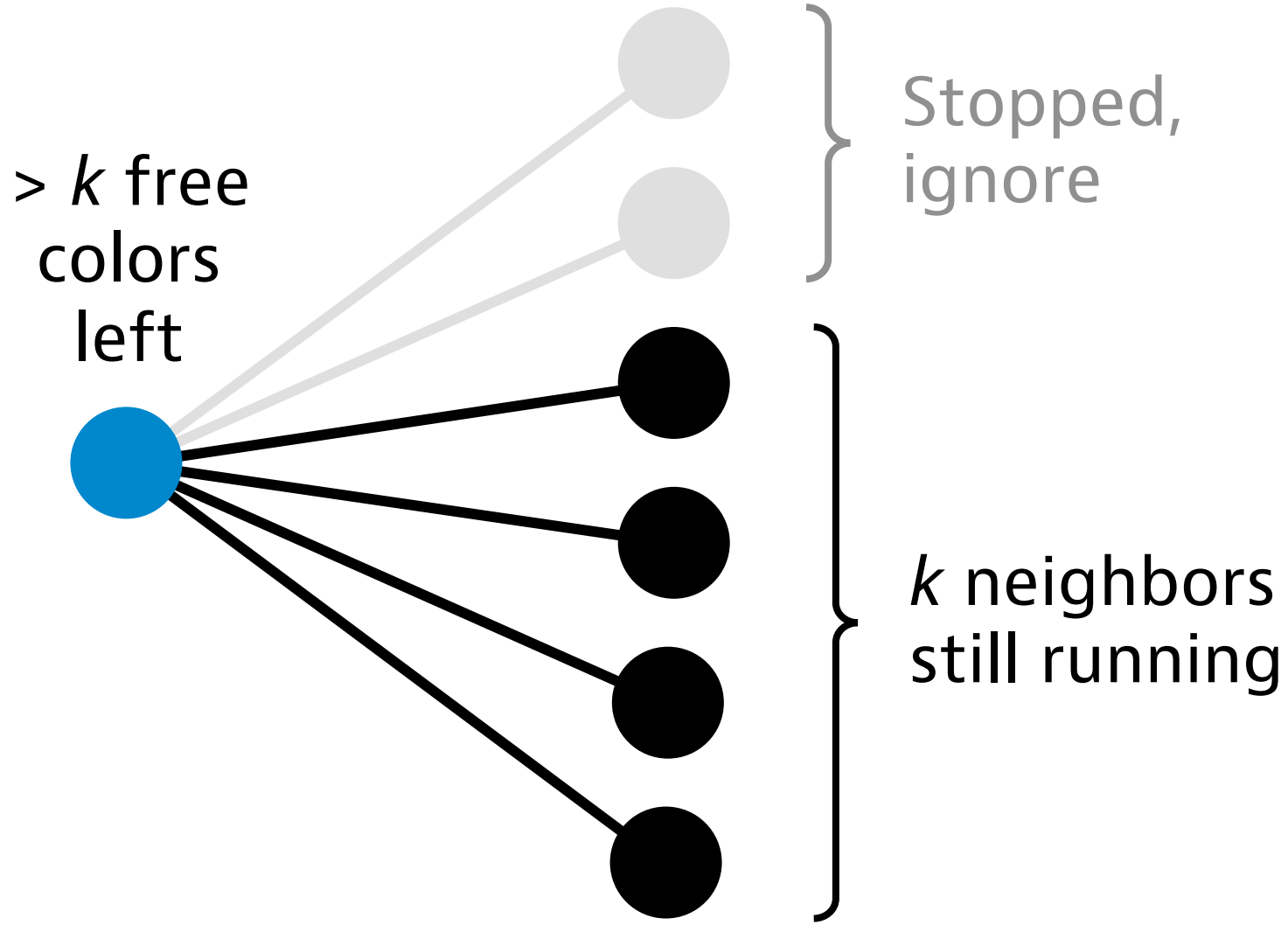
Proof…?

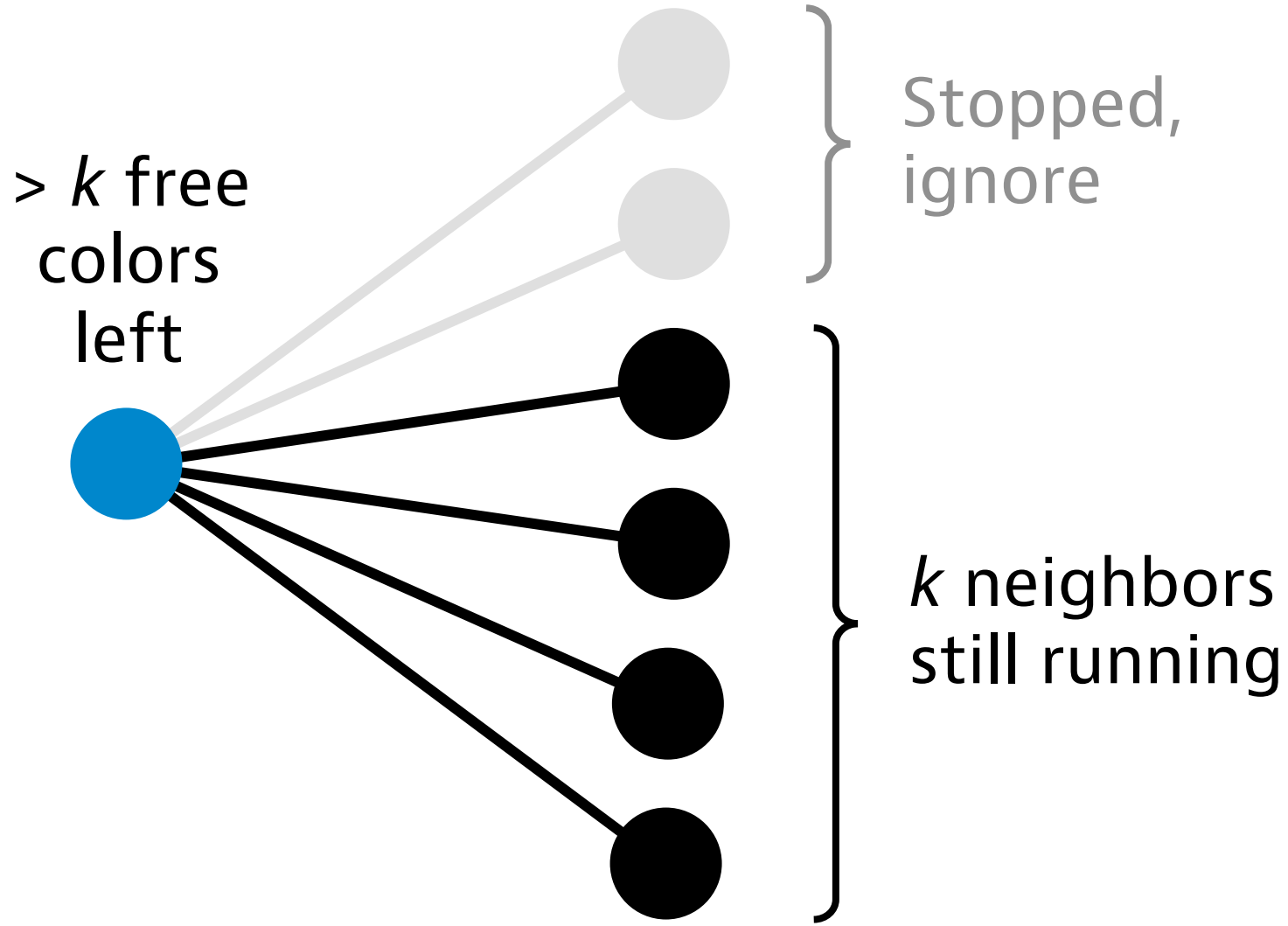





Stopped,
ignore
k neighbors
still running


> k free colors left
Stopped, ignore
k neighbors still running

I'm active: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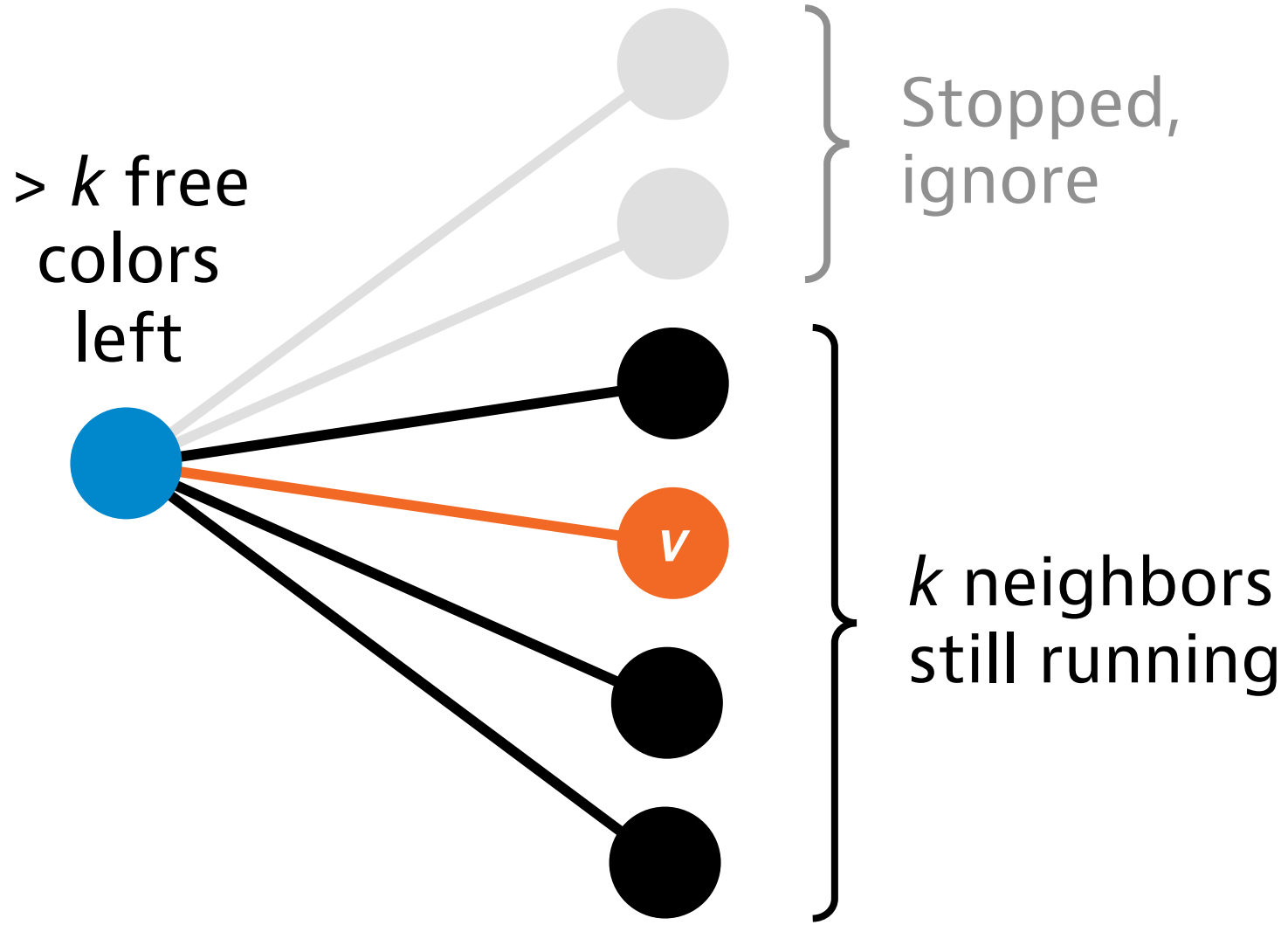
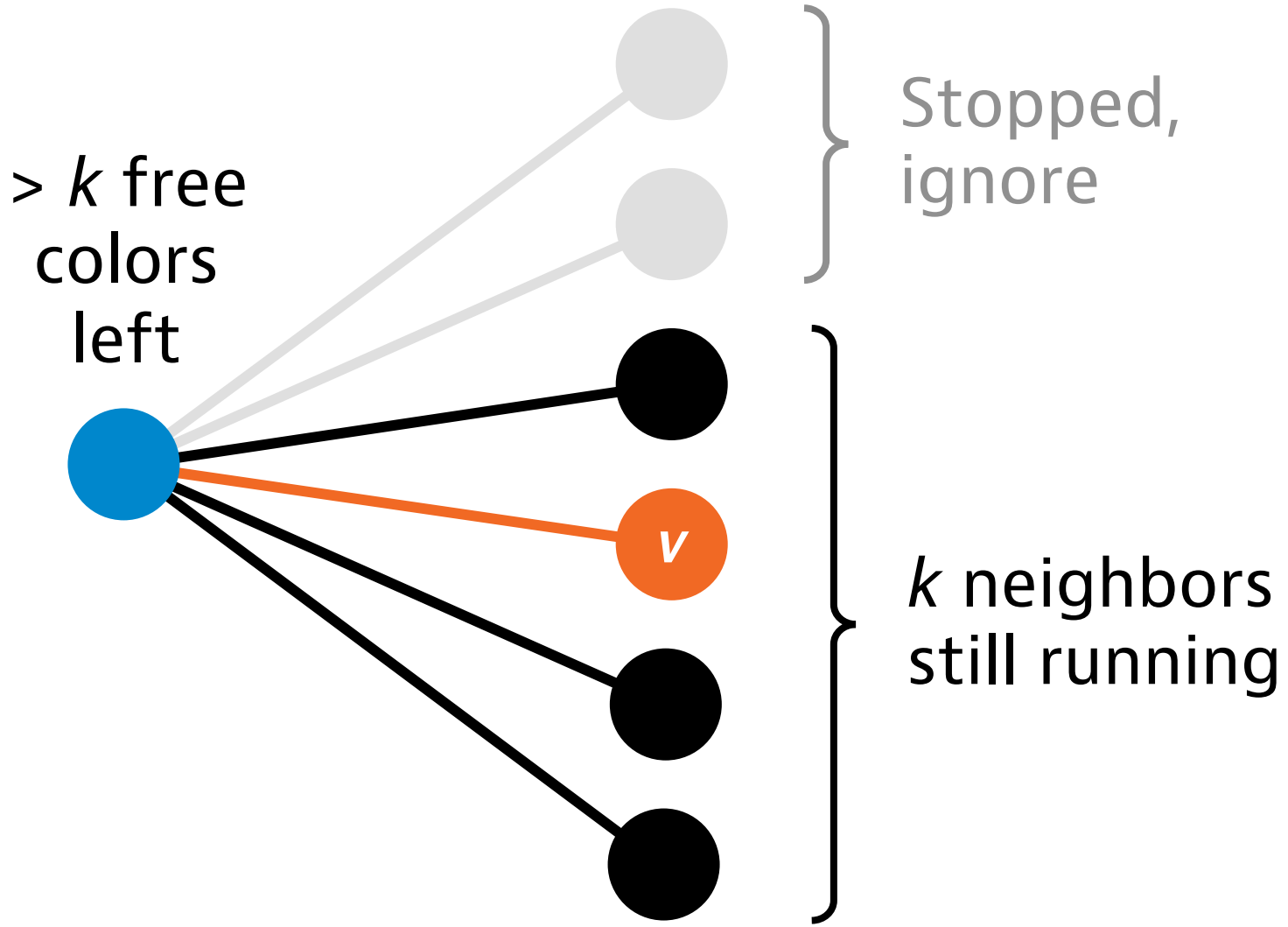

> k free colors left
v
Stopped, ignore
k neighbors still running

I'm active:

case 1:
v is passive

conflict
probability 0



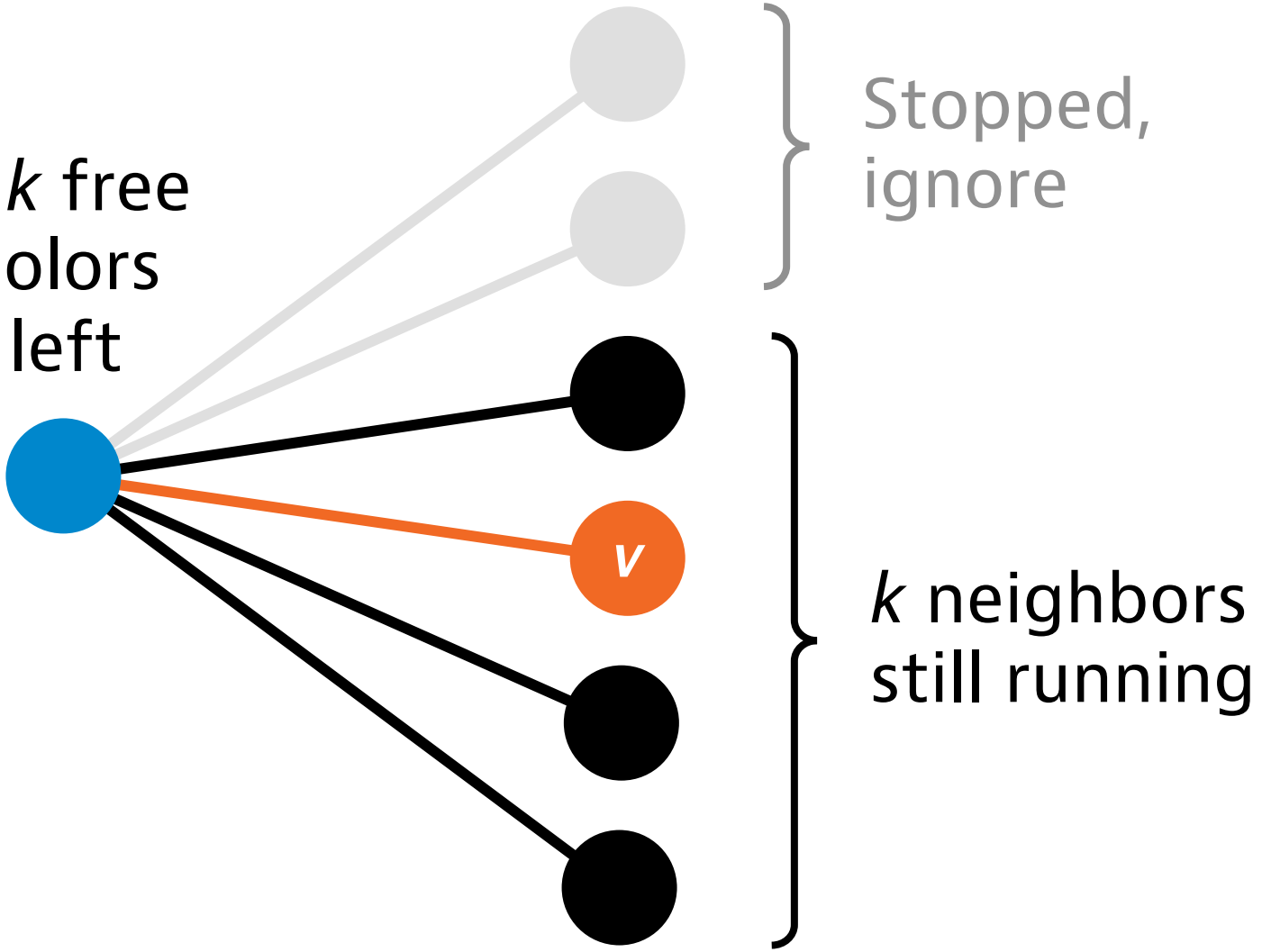
I'm active:

case 1:
v is passive

conflict
probability 0

case 2:
v is active

conflict
probability < $1/k$



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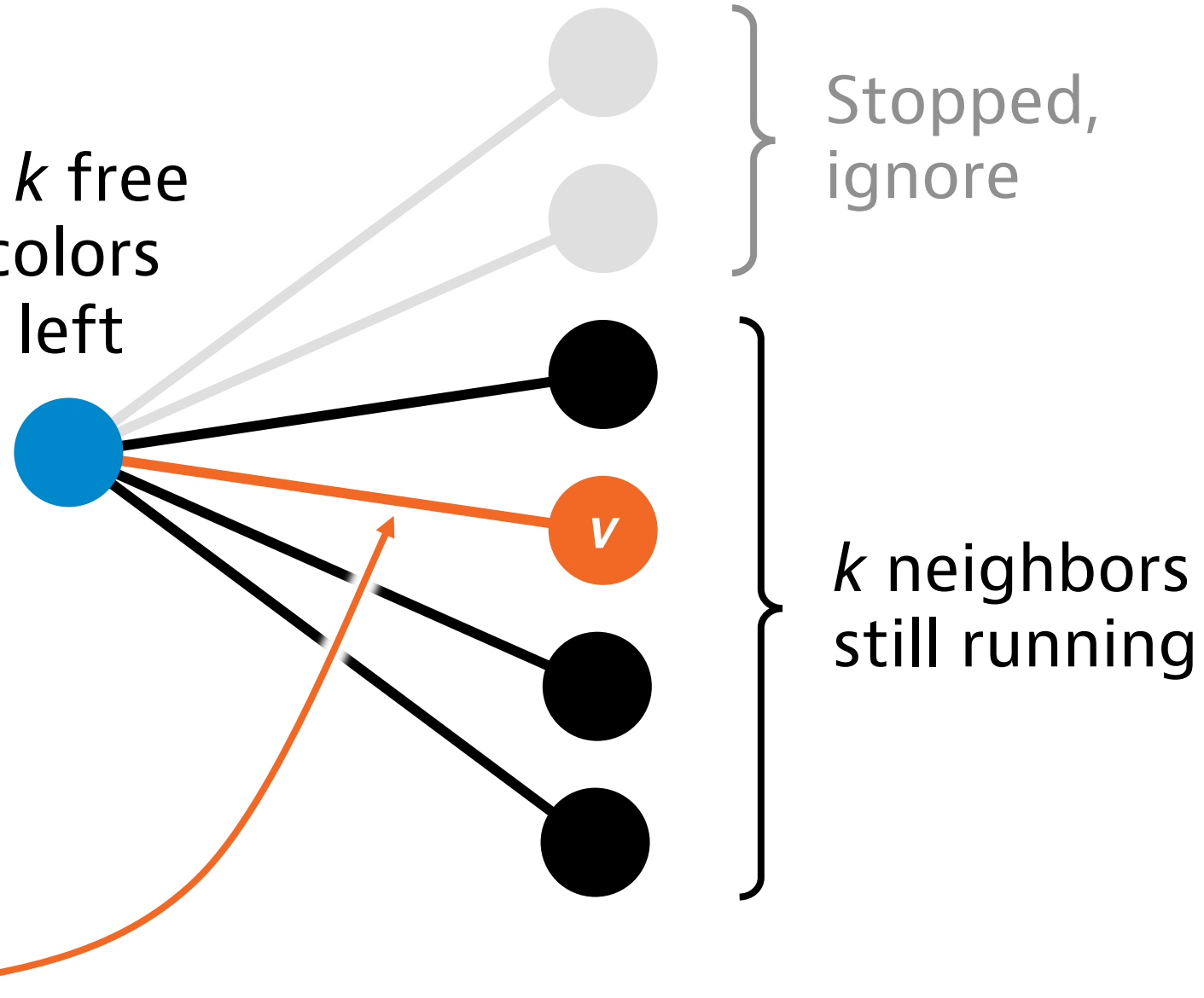
overall:

conflict
probability < $1/(2k)$



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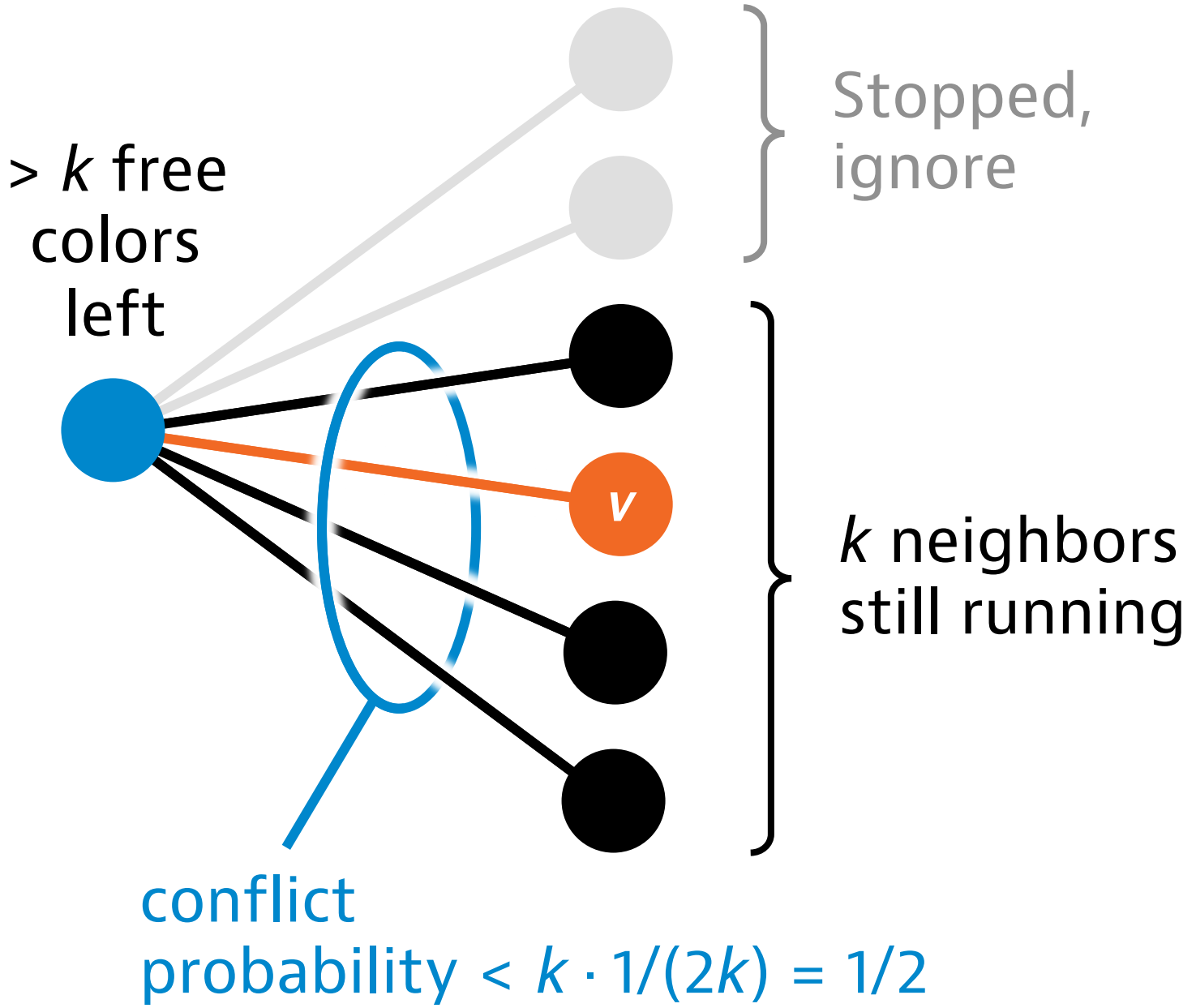
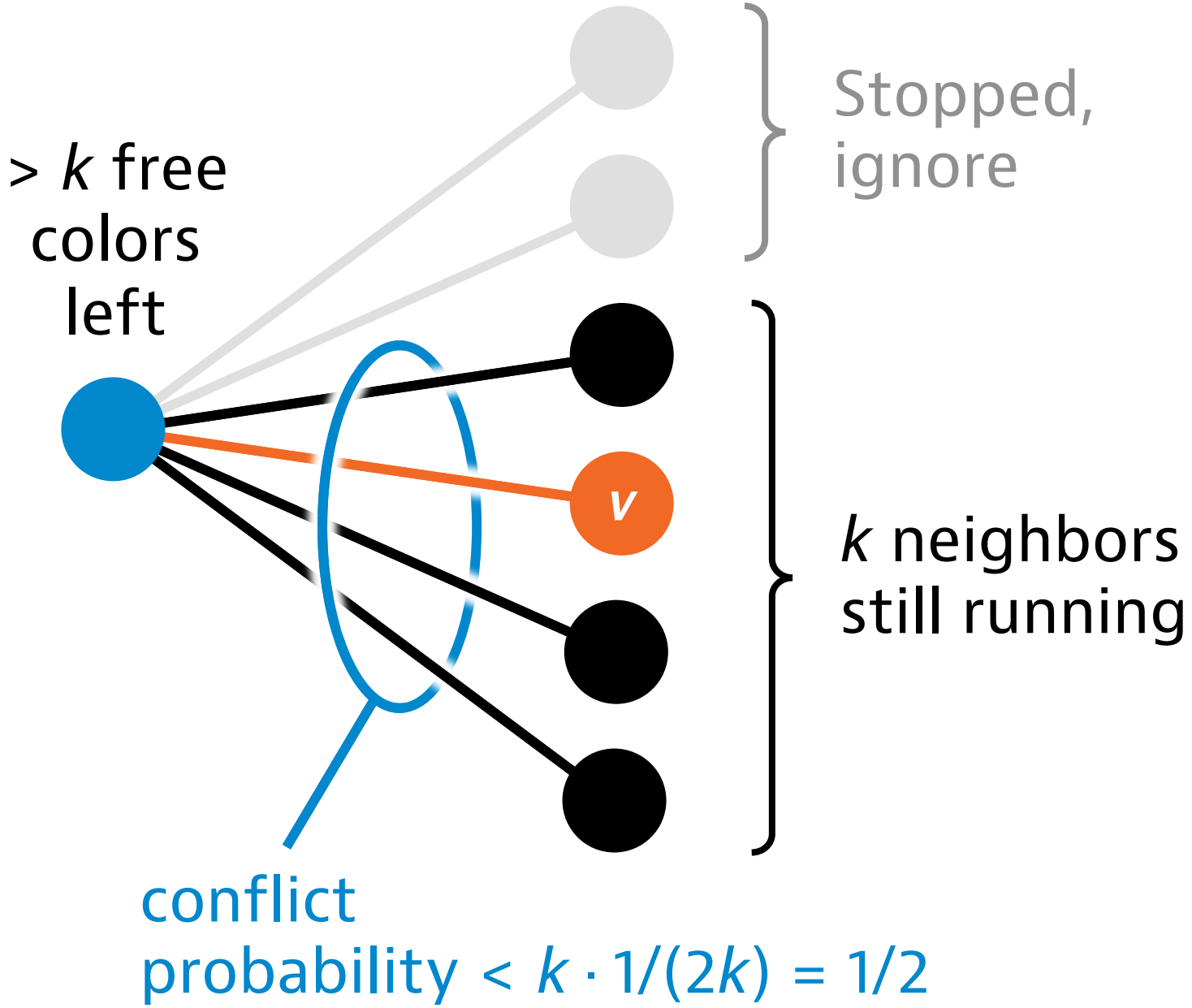


I'm active:

I can stop with probability > 1/2 if I'm active



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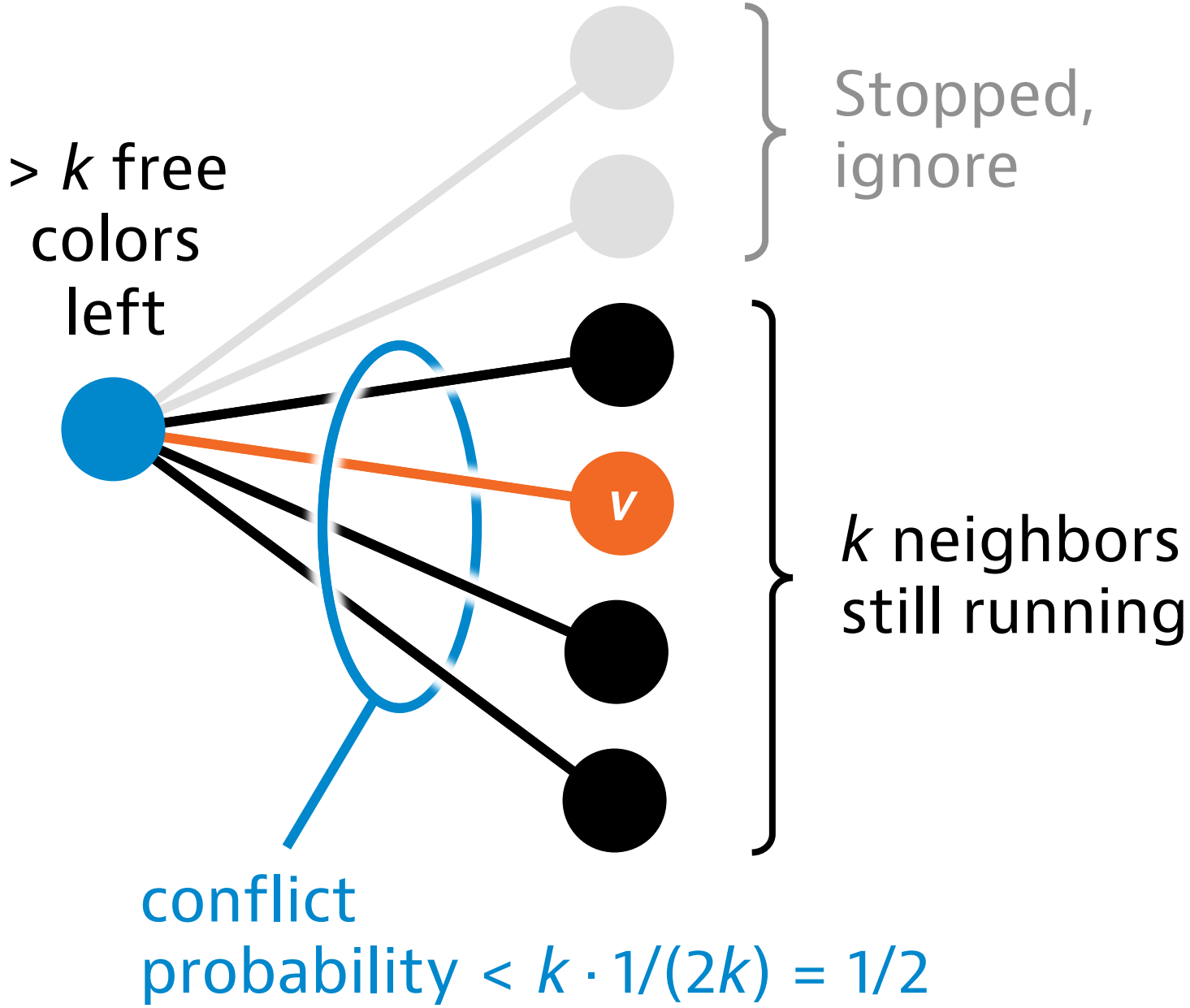
I'm active with probability 1/2



I'm active:

I can stop with probability > 1/2 if I'm active

I'm active with probability 1/2

Overall:

I can stop with probability > 1/4