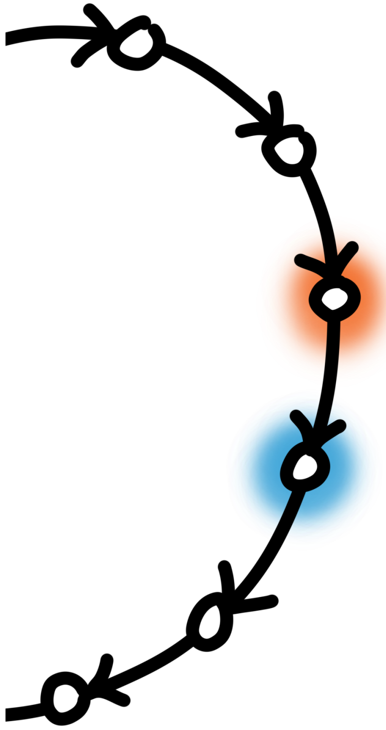
Aalto University

Distributed Algorithms 2020

11a Hardness of coloring

Week 1: fast coloring



$(\underset{0-7}{2}, \underset{0-1}{0}) = \underbrace{010}_{3\text{ bits}}\underbrace{0}_{1\text{ bit}}$

$\cancel{123} = 01111011$

$47 = 00101111$

bit 2

3-coloring

$O(\log^* n)$
rounds

– LOCAL model
– directed cycles

$n^{O(1)}$-coloring

0 rounds

(unique IDs)

– LOCAL model

– directed cycles

$n^{O(1)}$-coloring
0 rounds
(unique IDs)

$\rightarrow \rightarrow \rightarrow \rightarrow \rightarrow$

3-coloring
$O(\log^* n)$
rounds

– LOCAL model
– directed cycles

$n^{O(1)}$-coloring → → → → → **3-coloring**

0 rounds *(unique IDs)* | $O(\log^* n)$ rounds

2^k-coloring → **$2k$-coloring** *Fast color reduction*

$n^{O(1)}$-coloring

0 rounds

(unique IDs)

$O(\log^* n)$ steps

3-coloring

$O(\log^* n)$

rounds

2^k-coloring **$2k$-coloring**

Fast color reduction

$n^{O(1)}$-coloring

0 rounds

(unique IDs)

$\rightarrow \rightarrow \rightarrow \rightarrow \rightarrow$

$O(\log^* n)$ steps

3-coloring

$O(\log^* n)$

rounds

2^k-coloring $\rightarrow$ **$2k$-coloring**

Fast color reduction

Is this optimal?

Could we find 3-coloring faster?

$n^{O(1)}$-coloring
0 rounds
(unique IDs)

$O(\log^* n)$ steps

3-coloring
$O(\log^* n)$ rounds

2^k-coloring → **$2k$-coloring**

Fast color reduction

Round elimination

$n^{O(1)}$-coloring

0 rounds

(unique IDs)

$O(\log^* n)$ steps

3-coloring

$O(\log^* n)$ rounds

2^k-coloring

$T - 1$ rounds

$2k$-coloring

T rounds

Fast color reduction

Round elimination

$n^{O(1)}$-coloring

0 rounds

(unique IDs)

$O(\log^* n)$ steps

3-coloring

$O(\log^* n)$ rounds

2^k-coloring → **$2k$-coloring**

$T - 1$ rounds | T rounds

Fast color reduction

***k*-coloring**

Round elimination

***n*$^{O(1)}$-coloring** → → → → → **3-coloring**

0 rounds
(unique IDs)

$O(\log^* n)$ steps

$O(\log^* n)$ rounds

2^k-coloring → **$2k$-coloring** *Fast color reduction*

$T - 1$ rounds T rounds

2^k-coloring ← RE **k-coloring** *Round elimination*

$n^{O(1)}$-coloring → → → → → **3-coloring**

0 rounds
(unique IDs)

$O(\log^* n)$ steps

$O(\log^* n)$
rounds

2^k-coloring → **$2k$-coloring**

Fast color reduction

$T - 1$ rounds

T rounds

2^k-coloring ← RE **k-coloring**

Round elimination

3-coloring

$T << \log^* n$
rounds

$n^{O(1)}$-coloring

0 rounds

(unique IDs)

$O(\log^* n)$ steps

3-coloring

$O(\log^* n)$

rounds

2^k-coloring **$2k$-coloring**

$T - 1$ rounds T rounds

Fast color reduction

2^k-coloring RE **k-coloring**

Round elimination

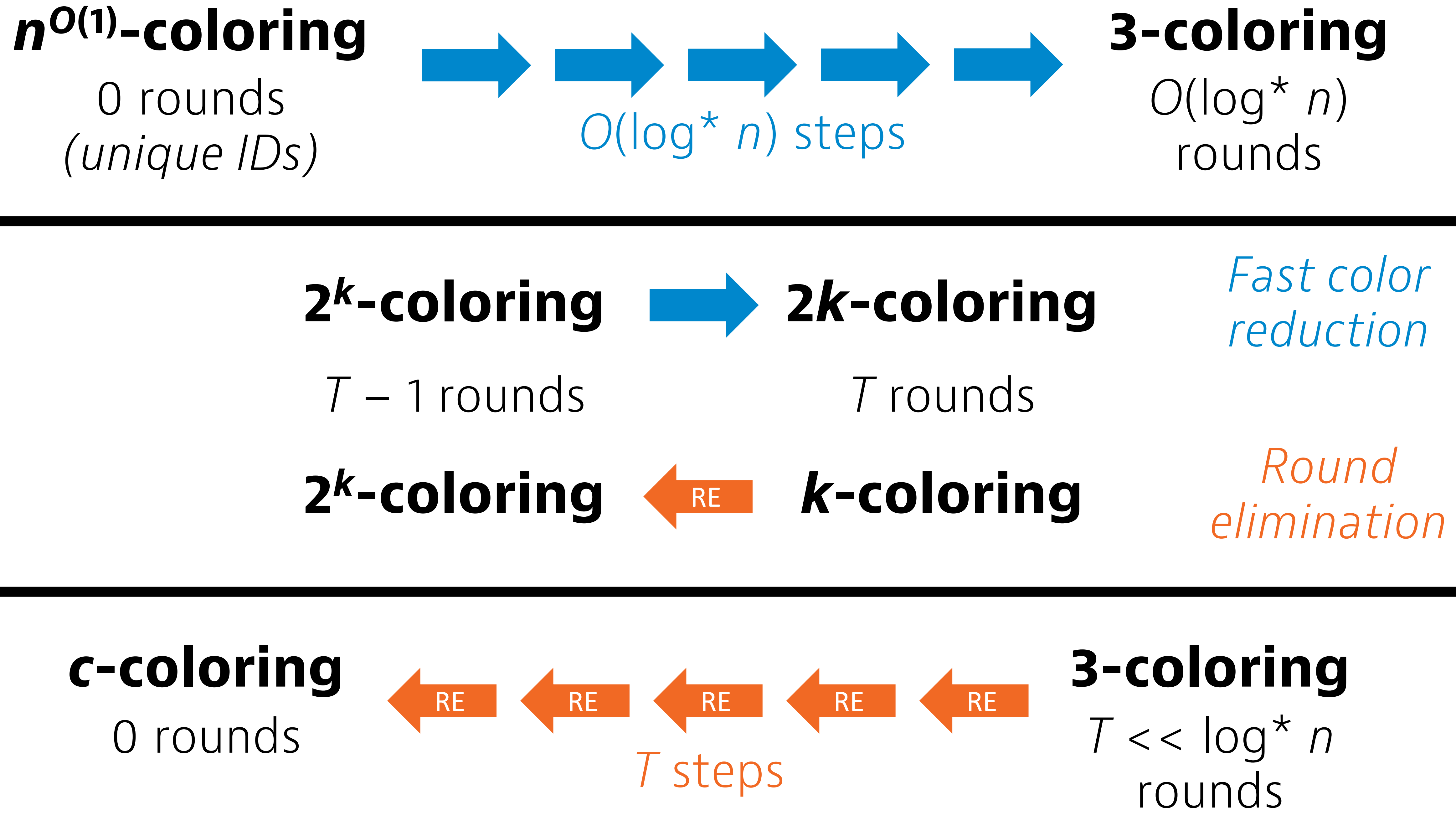
RE RE RE RE RE

3-coloring

$T << \log^* n$

rounds

T steps



$n^{O(1)}$-coloring
0 rounds
(unique IDs)
$O(\log^* n)$ steps
3-coloring
$O(\log^* n)$
rounds
2^k-coloring
$2k$-coloring
Fast color
reduction
$T - 1$ rounds
T rounds
2^k-coloring
RE
k-coloring
Round
elimination
c-coloring
0 rounds
RE
RE
RE
RE
RE
T steps
3-coloring
$T << \log^* n$
rounds

$n^{O(1)}$-coloring

0 rounds

(unique IDs)

$O(\log^* n)$ steps

3-coloring

$O(\log^* n)$

rounds

c = power tower of height T

2^k-coloring

$T - 1$ rounds

$2k$-coloring

T rounds

Fast color reduction

2^k-coloring

RE

k-coloring

Round elimination

c-coloring

0 rounds

RE RE RE RE RE

T steps

3-coloring

$T << \log^* n$

rounds

$n^{O(1)}$-coloring

0 rounds

(unique IDs)

$O(\log^* n)$ steps

3-coloring

$O(\log^* n)$ rounds

---

c = power tower of height T $<< n$

2^k-coloring

$T - 1$ rounds

$2k$-coloring

T rounds

Fast color reduction

2^k-coloring

RE

k-coloring

Round elimination

---

c-coloring

0 rounds

RE RE RE RE RE

T steps

3-coloring

$T << \log^* n$ rounds

$n^{O(1)}$-coloring
0 rounds
(unique IDs)

$O(\log^* n)$ steps

3-coloring
$O(\log^* n)$
rounds

---

c = power tower of height T << n

2^k-coloring → **$2k$-coloring**

$T - 1$ rounds | T rounds

Fast color reduction

2^k-coloring ← RE — **k-coloring**

Round elimination

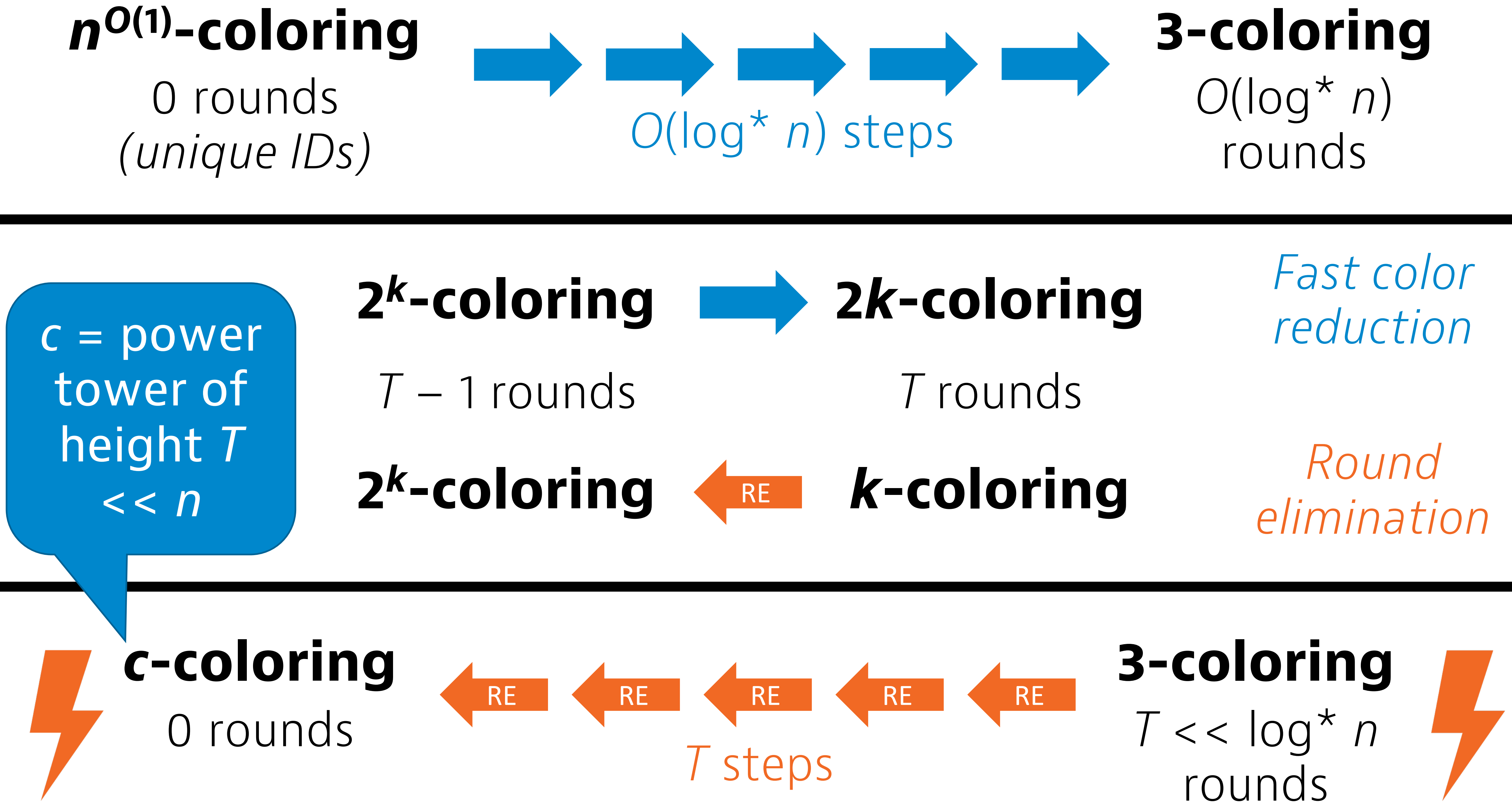
---

c-coloring
0 rounds

RE ← RE ← RE ← RE ← RE

T steps

3-coloring
T << $\log^* n$
rounds



$n^{O(1)}$-coloring
0 rounds
(unique IDs)
$O(\log^* n)$ steps
3-coloring
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c = power
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c-coloring
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RE
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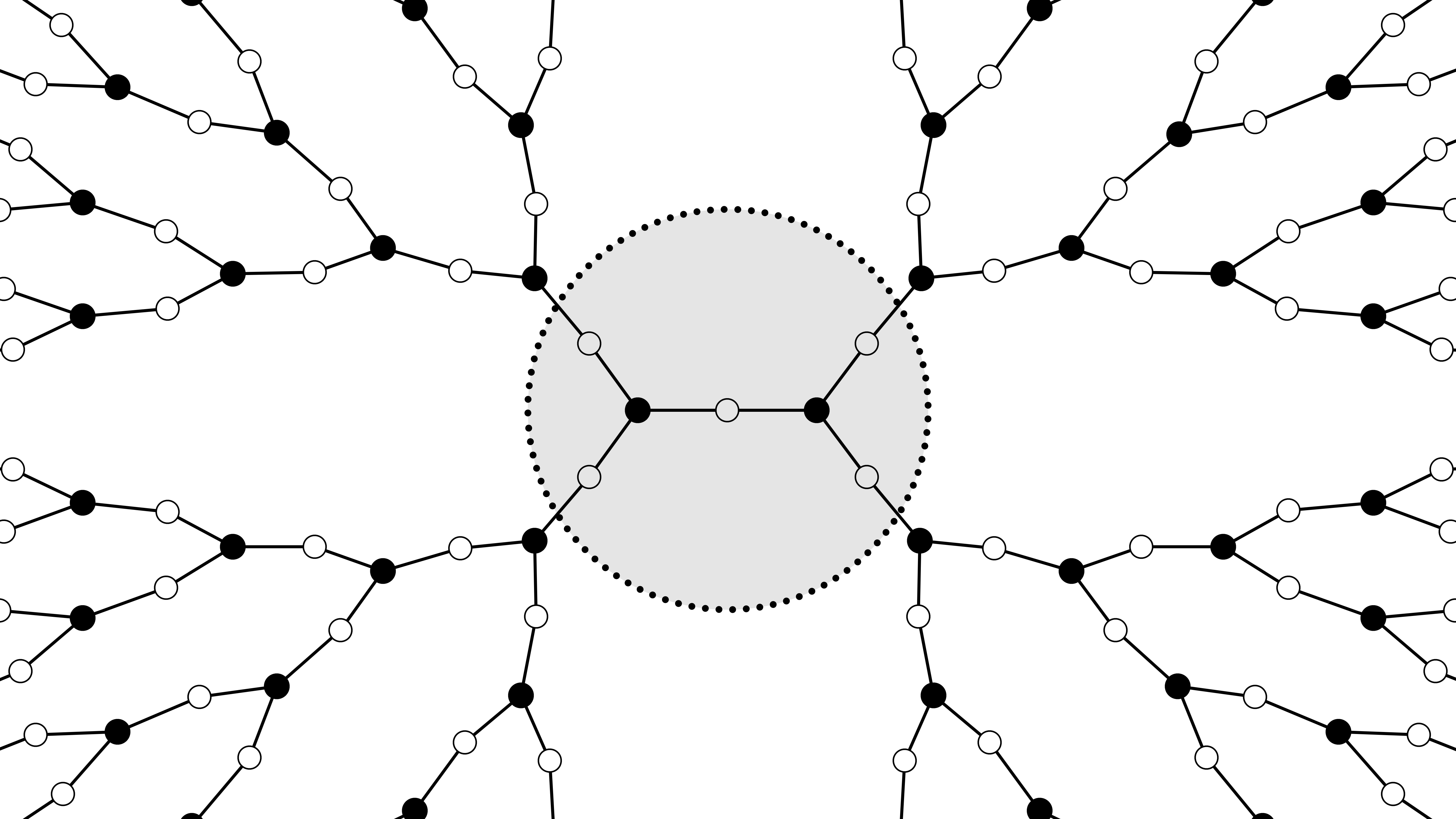
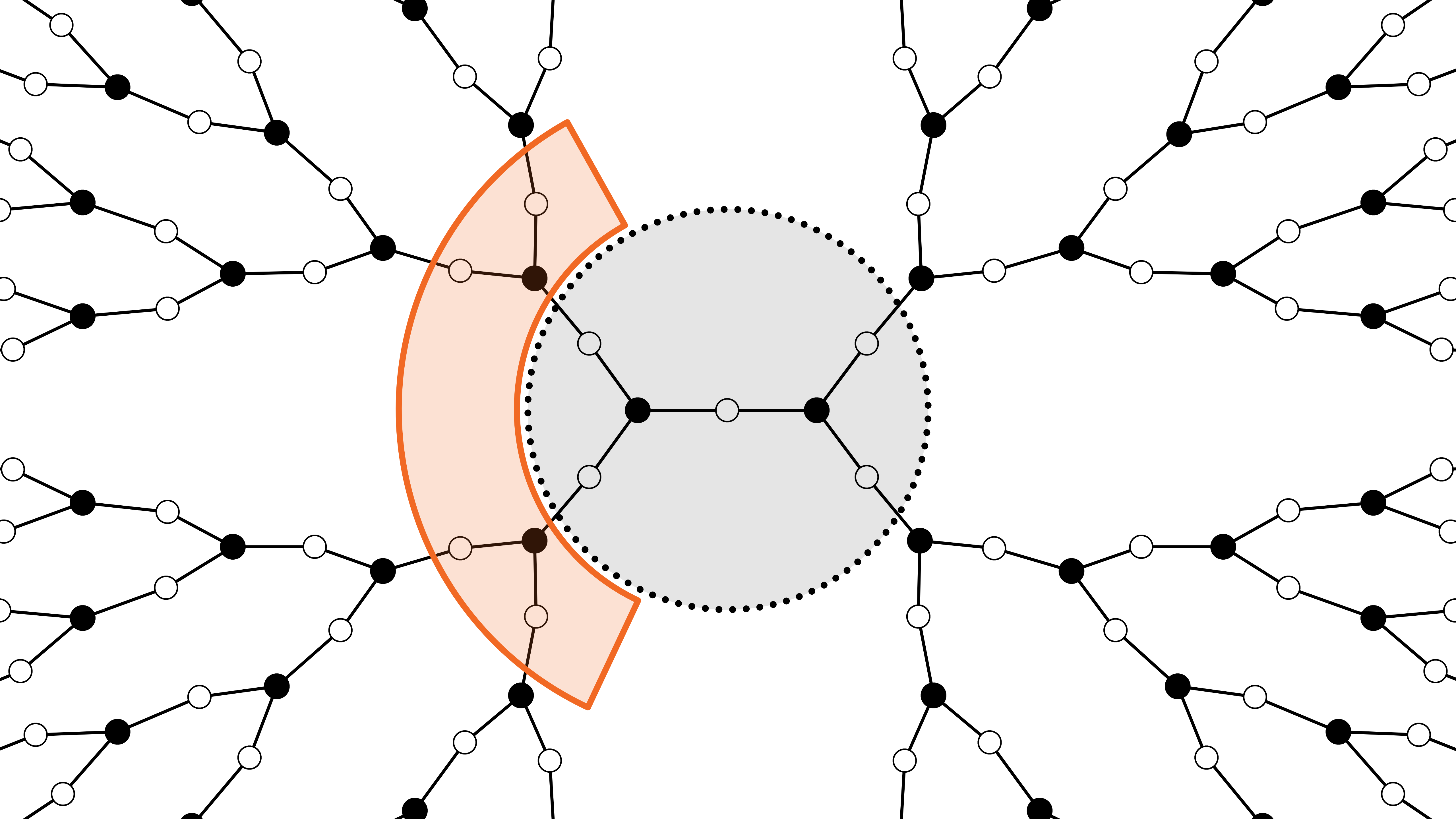
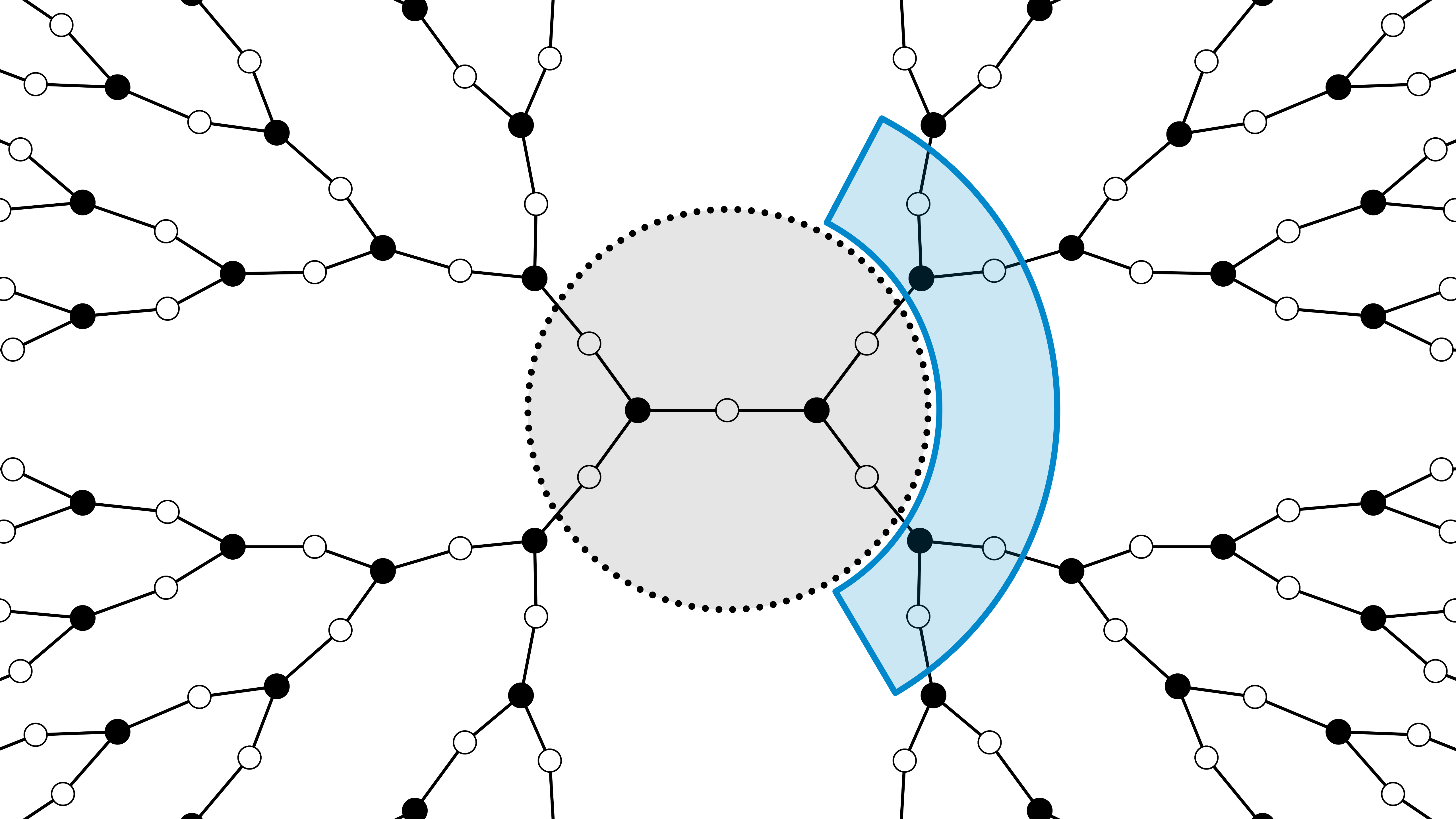
3-coloring
$T << \log^* n$
rounds

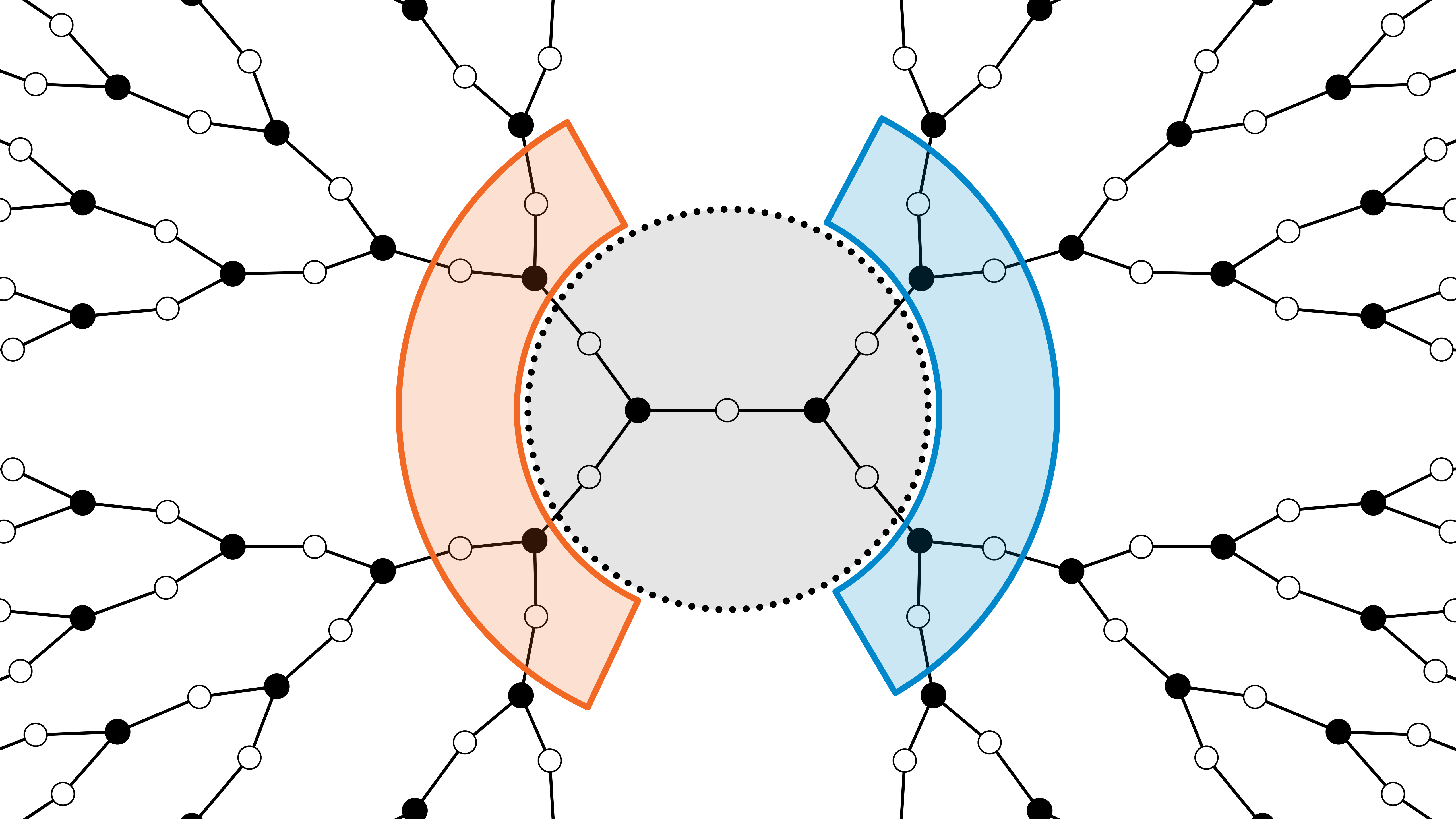
Round elimination
assumes the **PN** model

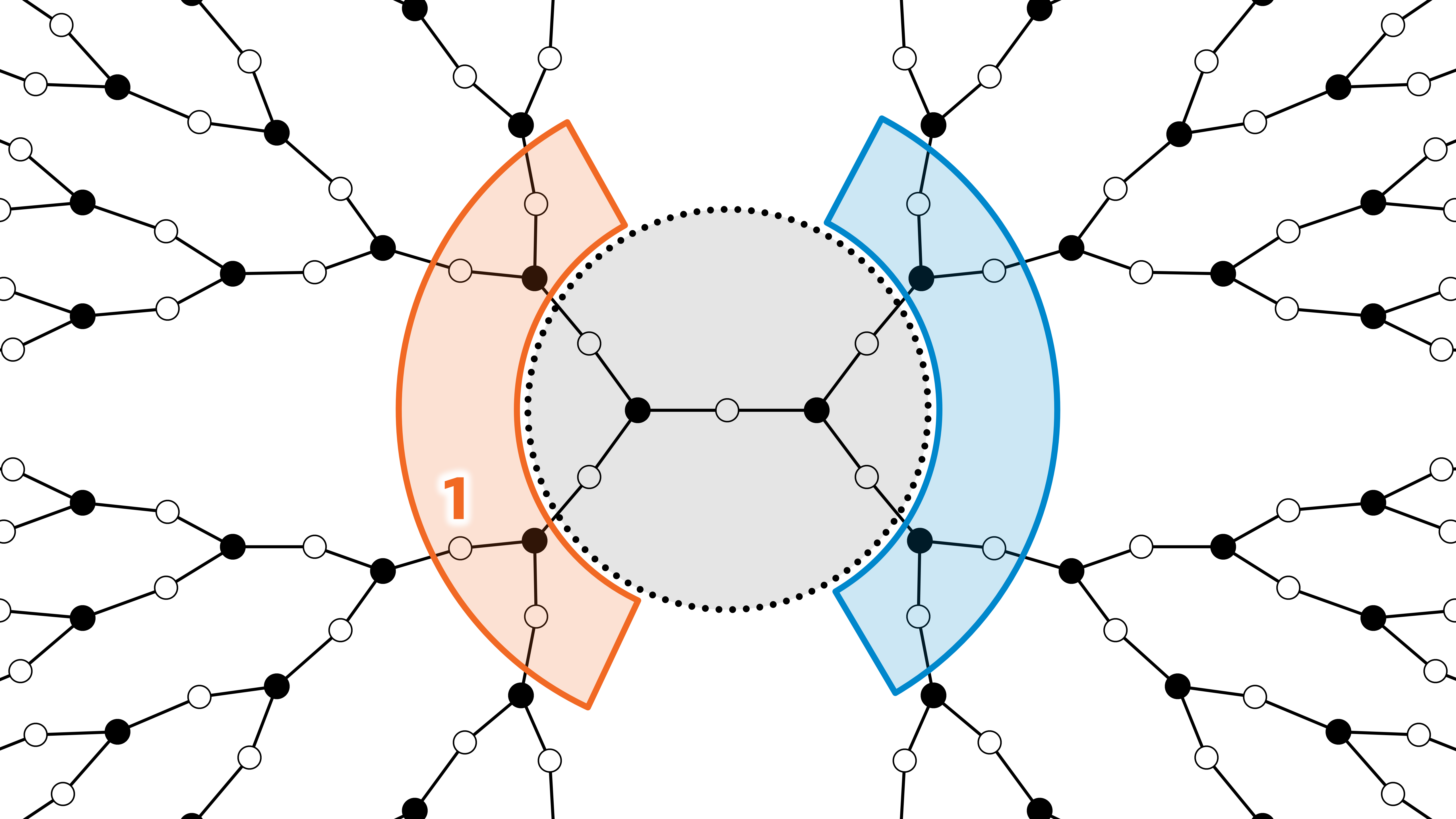
Round elimination assumes the **PN** model, it does **not** work in the **LOCAL** model







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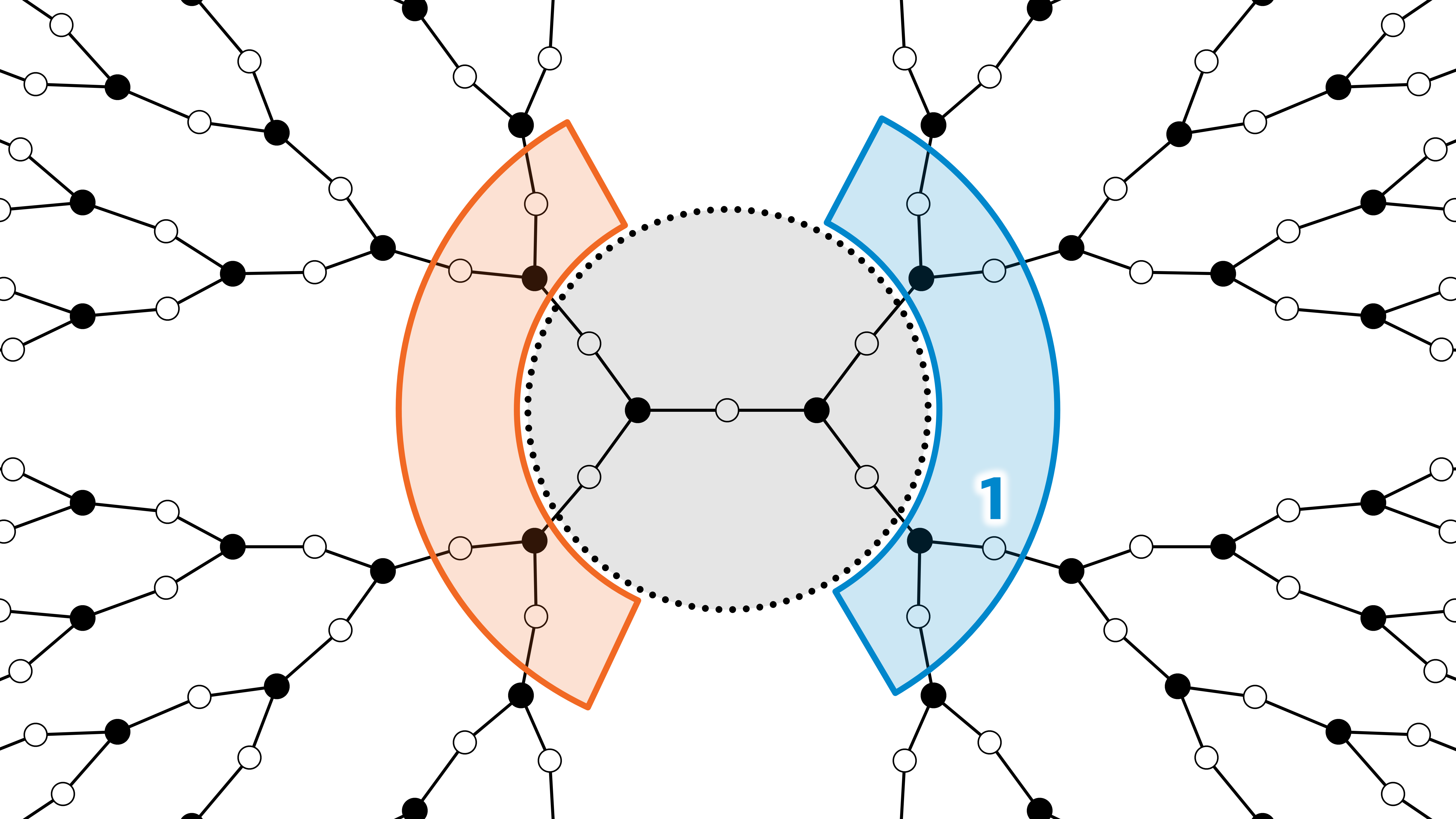
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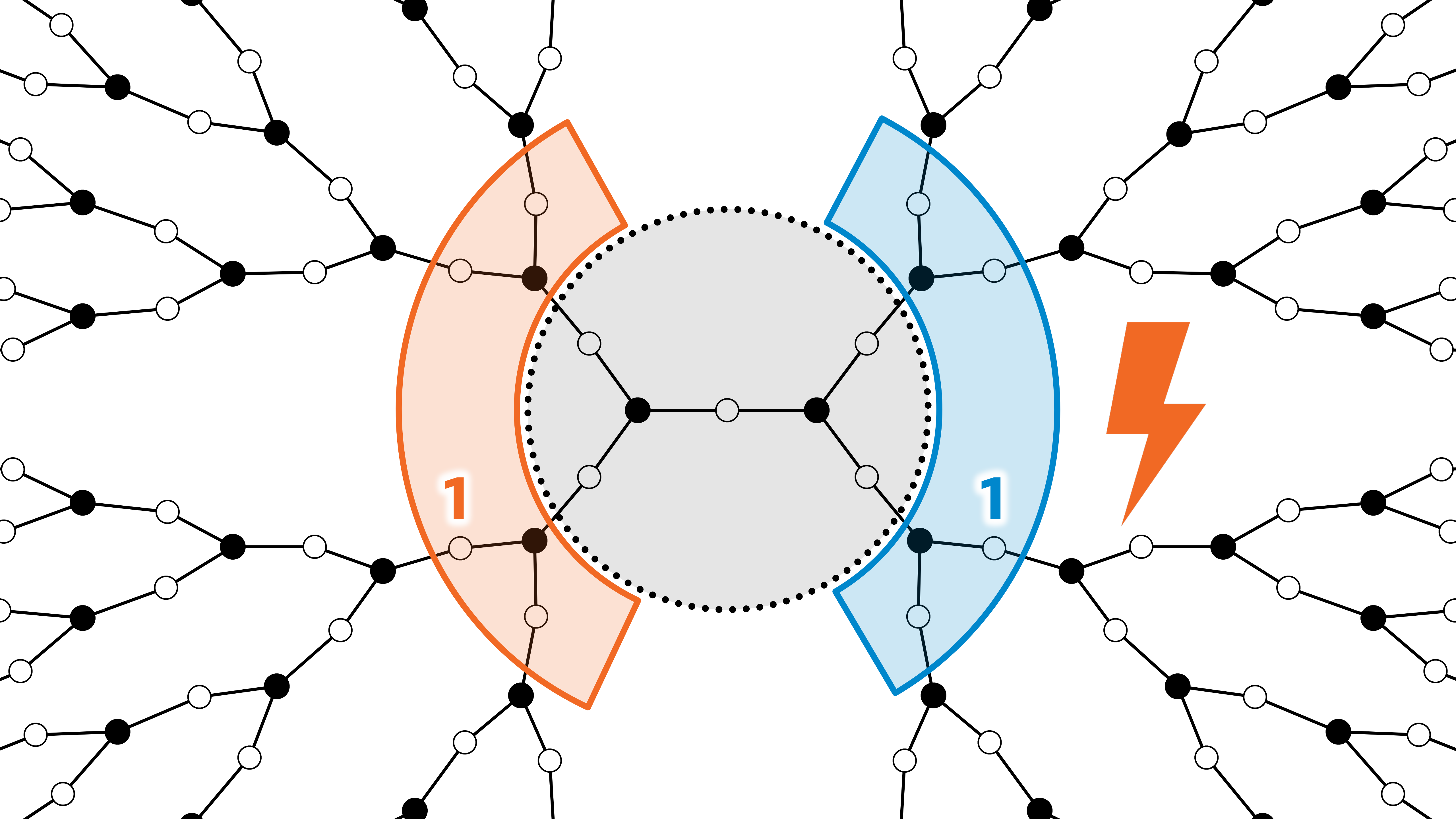
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1

Negative result for randomized PN ?

Negative result for randomized PN

- randomized round elimination

Negative result for randomized PN

- randomized round elimination:
  - A_0 solves X_0 in T rounds

Negative result for randomized PN

- randomized round elimination:
  - A_0 solves X_0 in T rounds
  - A_1 solves X_1 in $T - 1$ rounds

Negative result for randomized PN

- randomized round elimination:
  - A_0 solves X_0 in T rounds with probability p_0
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w.h.p.

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w.h.p.

c-coloring
0 rounds

RE RE RE RE RE
T steps

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$T << \log^* n$
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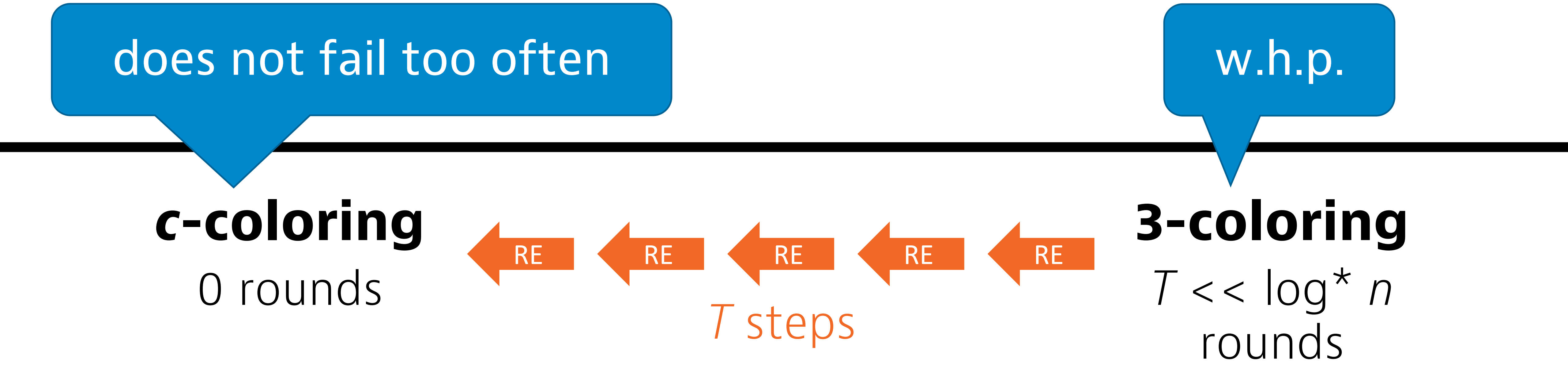

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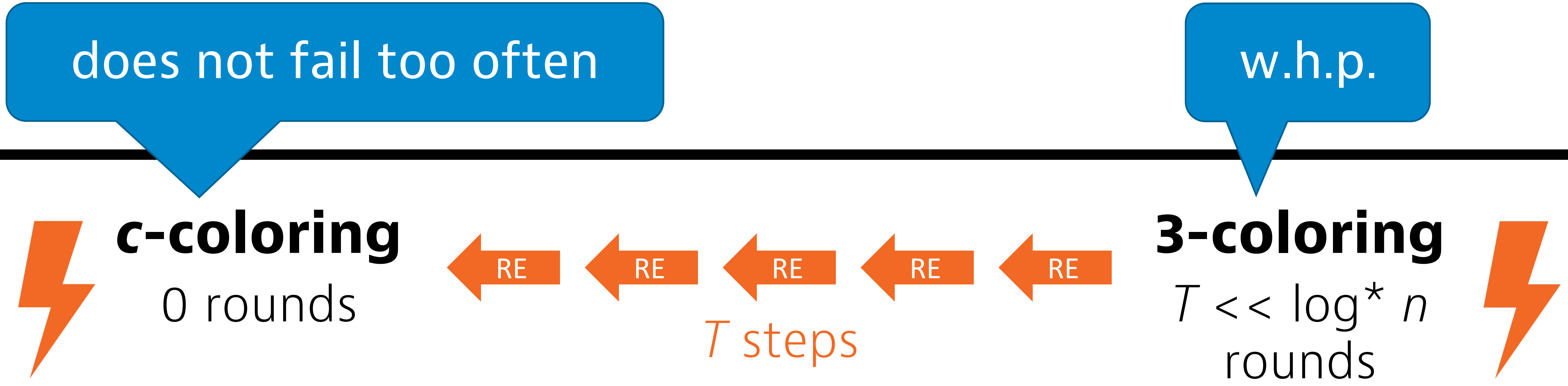

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Negative result for randomized LOCAL

- randomized PN can simulate randomized LOCAL

use randomness to construct identifiers that are unique w.h.p.

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Negative result for randomized LOCAL

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Negative result for deterministic LOCAL

- deterministic is a special case of randomized

Negative result for deterministic PN

Negative result for deterministic PN

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