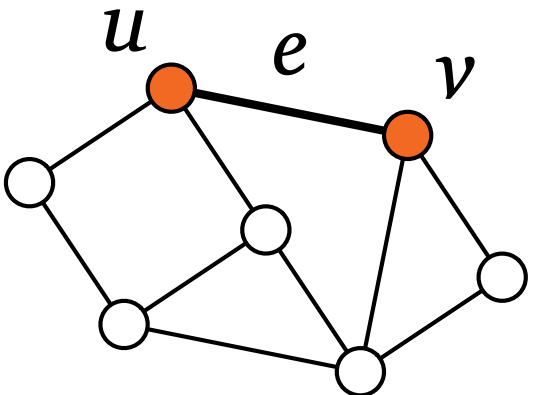
u
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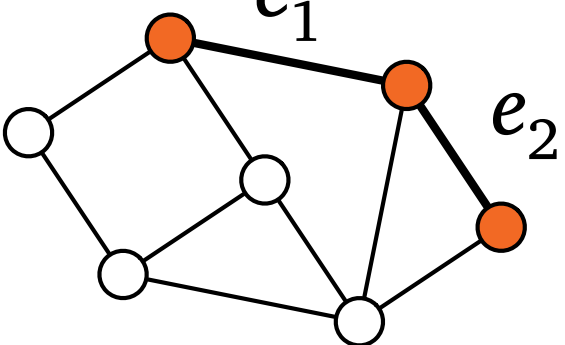
v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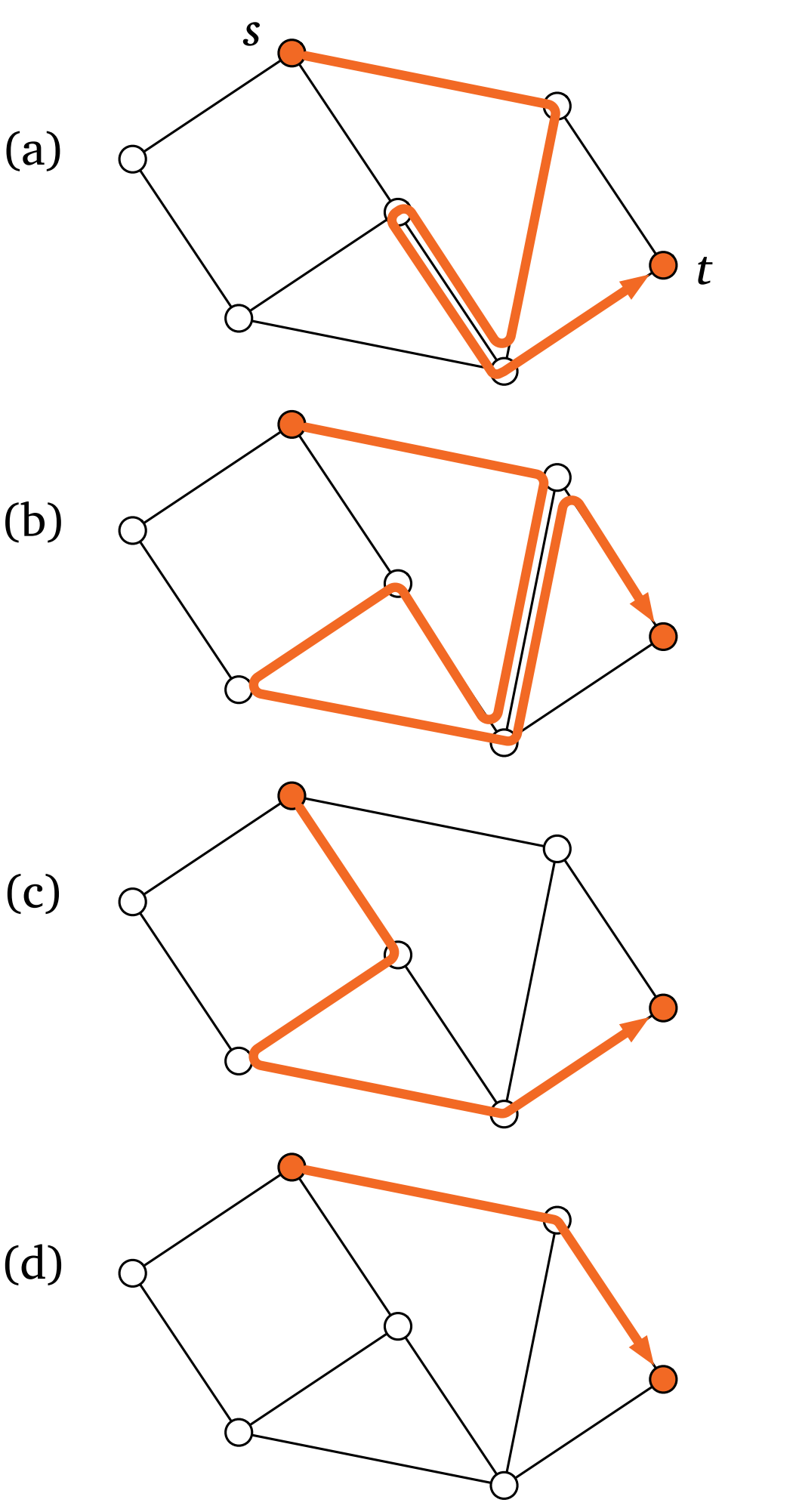
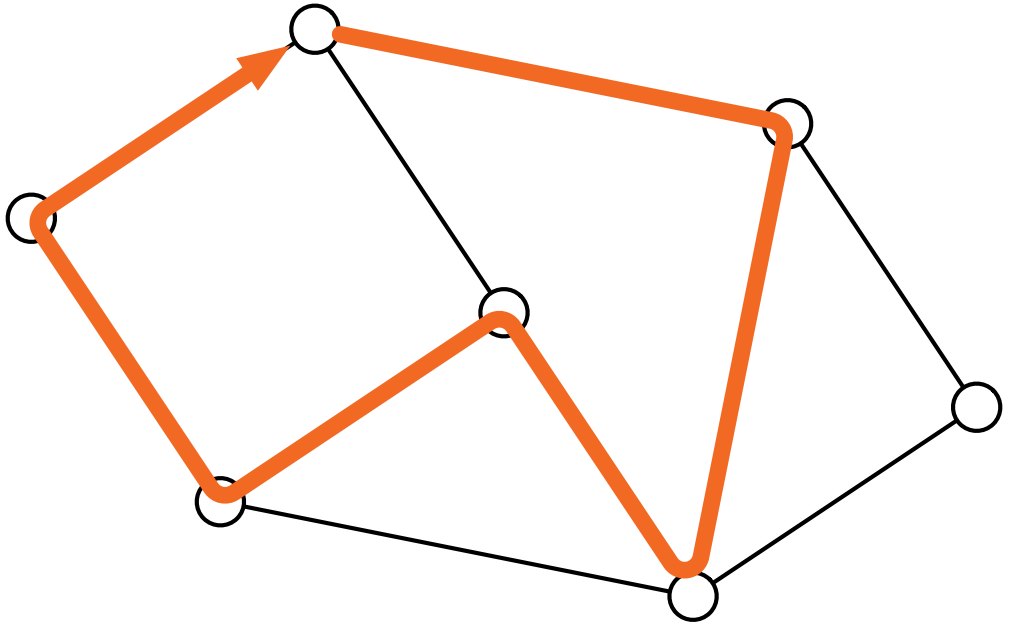
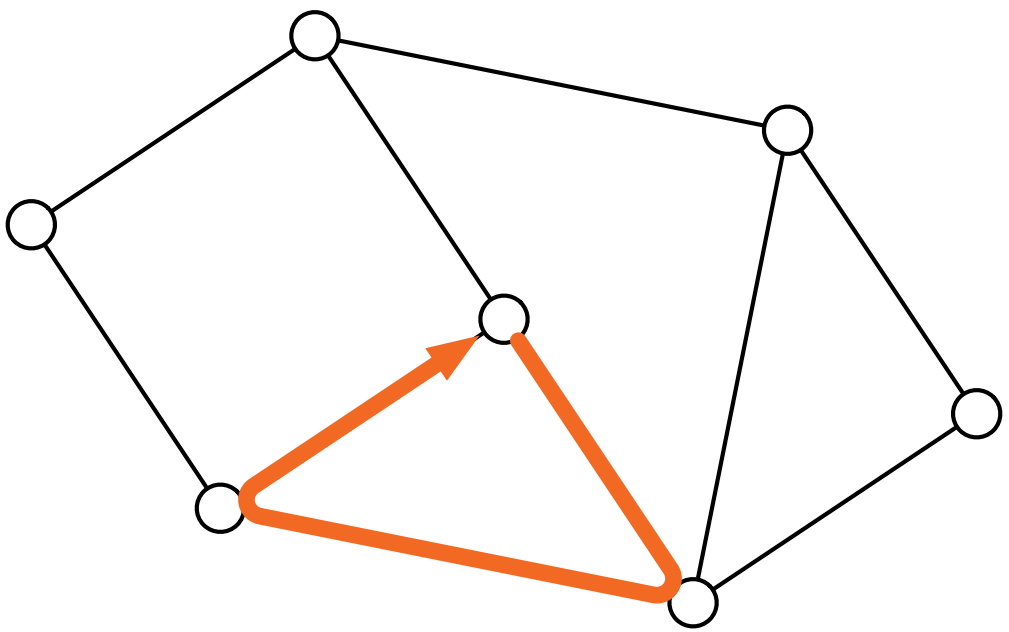
e_2


s
t
(a)
(b)
(c)
(d)

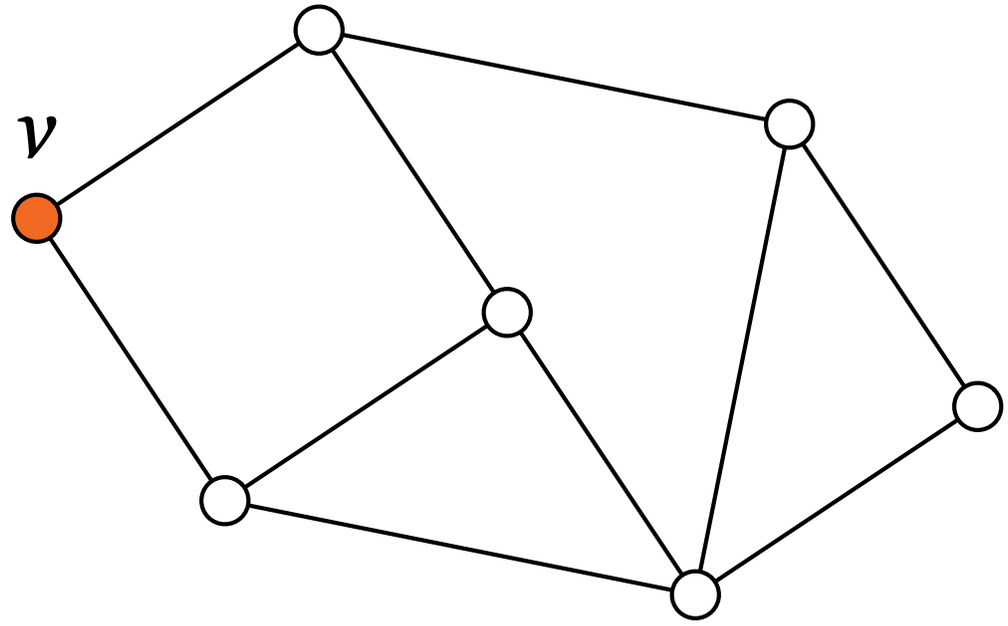
(a)



(b)



$\text{ball}_G(v, 0)$:



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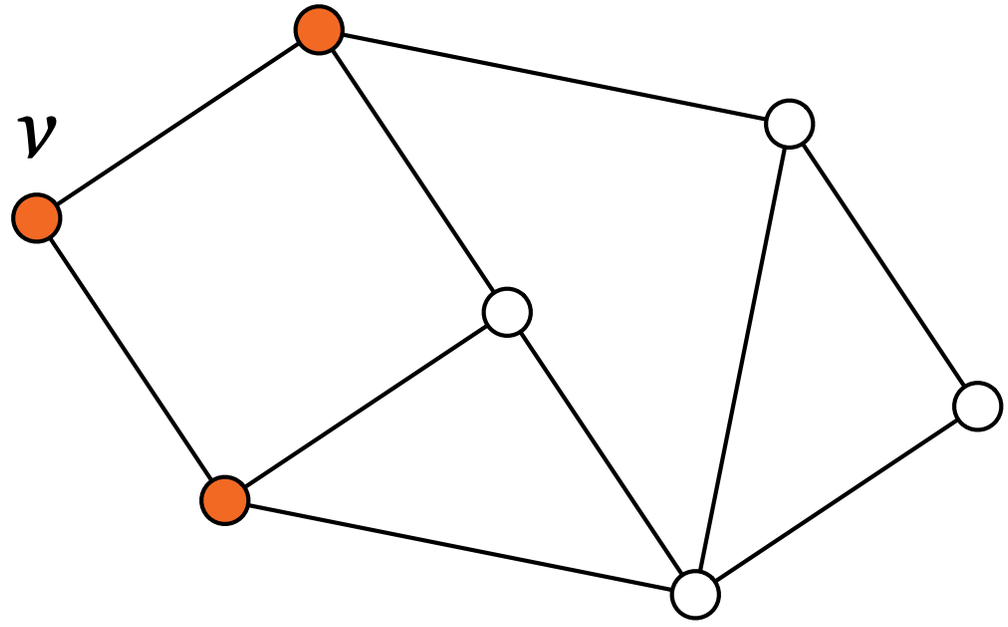

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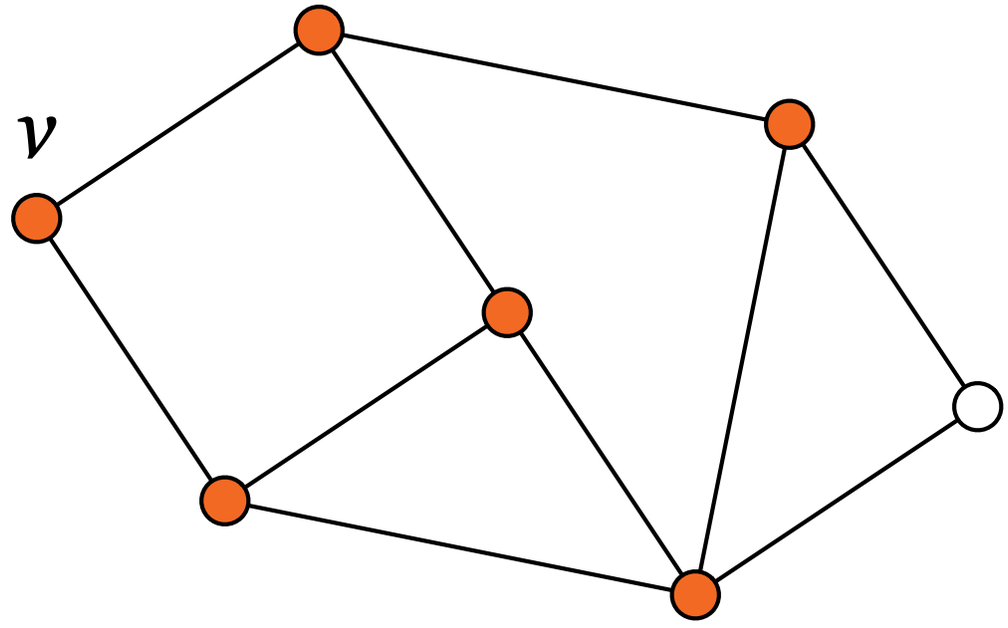
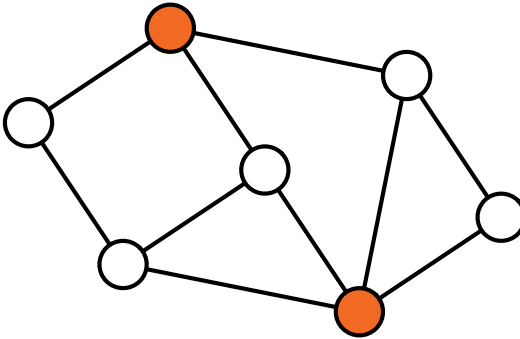
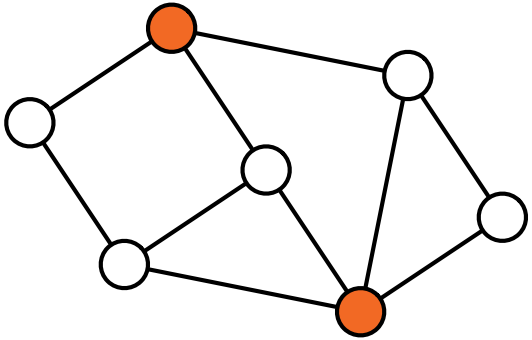




(a)



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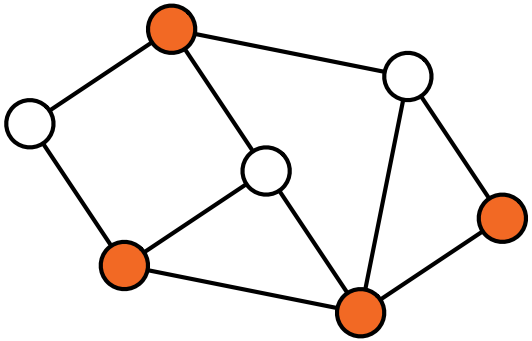
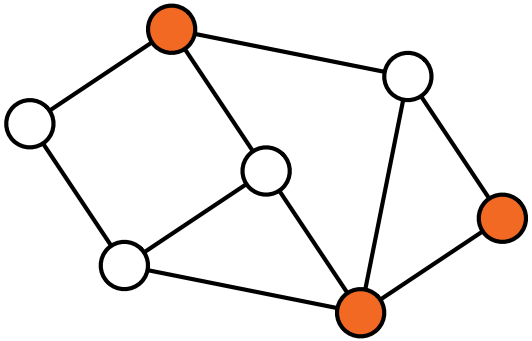
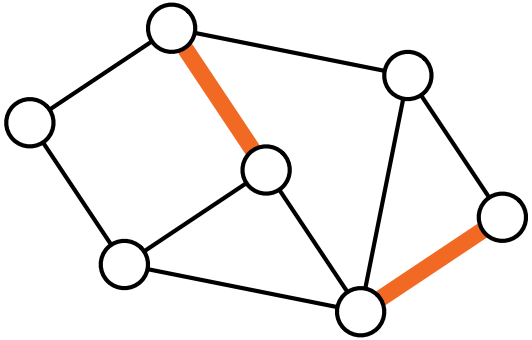


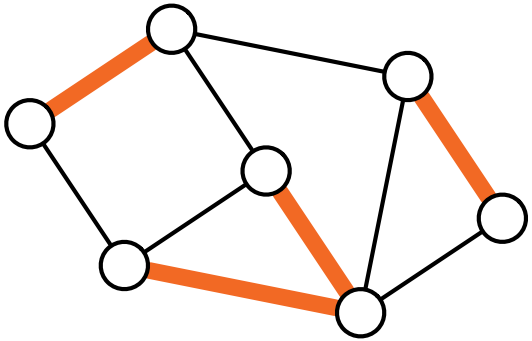
(c)



(d)



(e)



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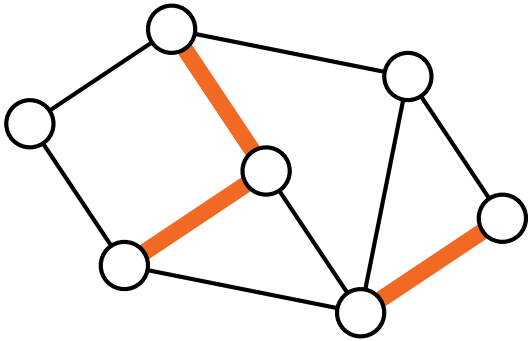
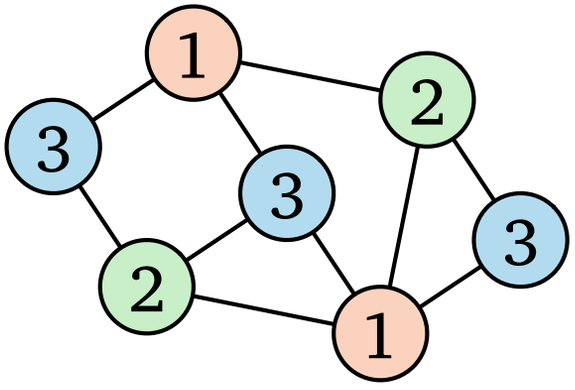
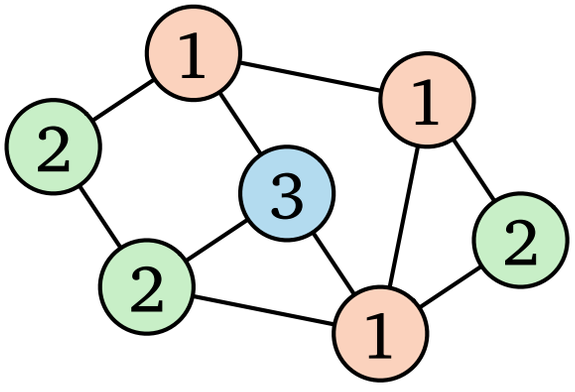
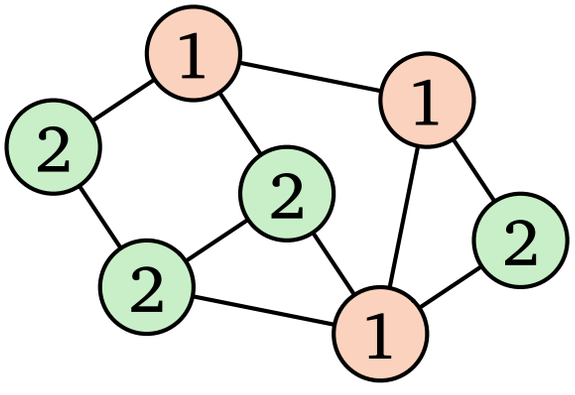


(a) 

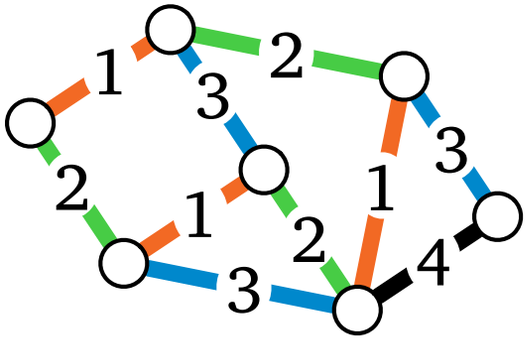
3-colouring

(b) 

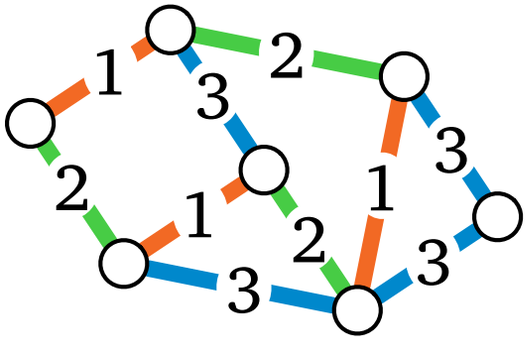
weak 3-colouring

(c) 

domatic partition (size 2)

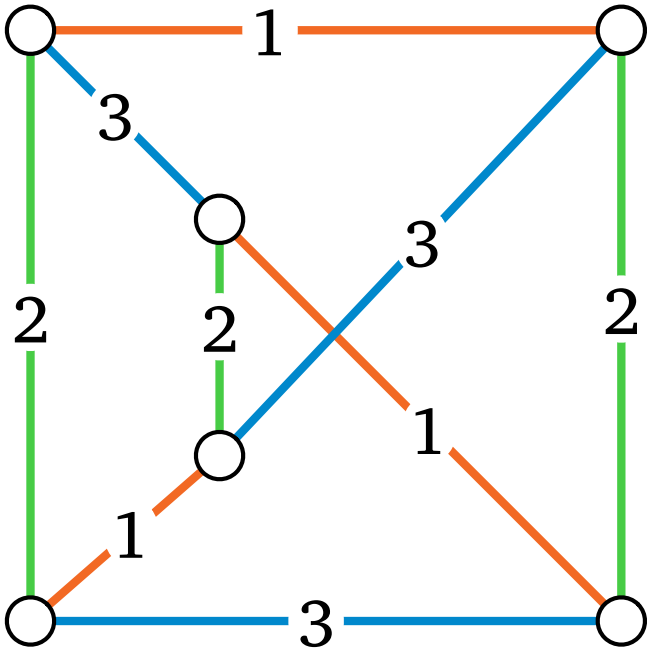
(d) 

4-edge colouring

(e) 

edge domatic partition (size 3)

(a)



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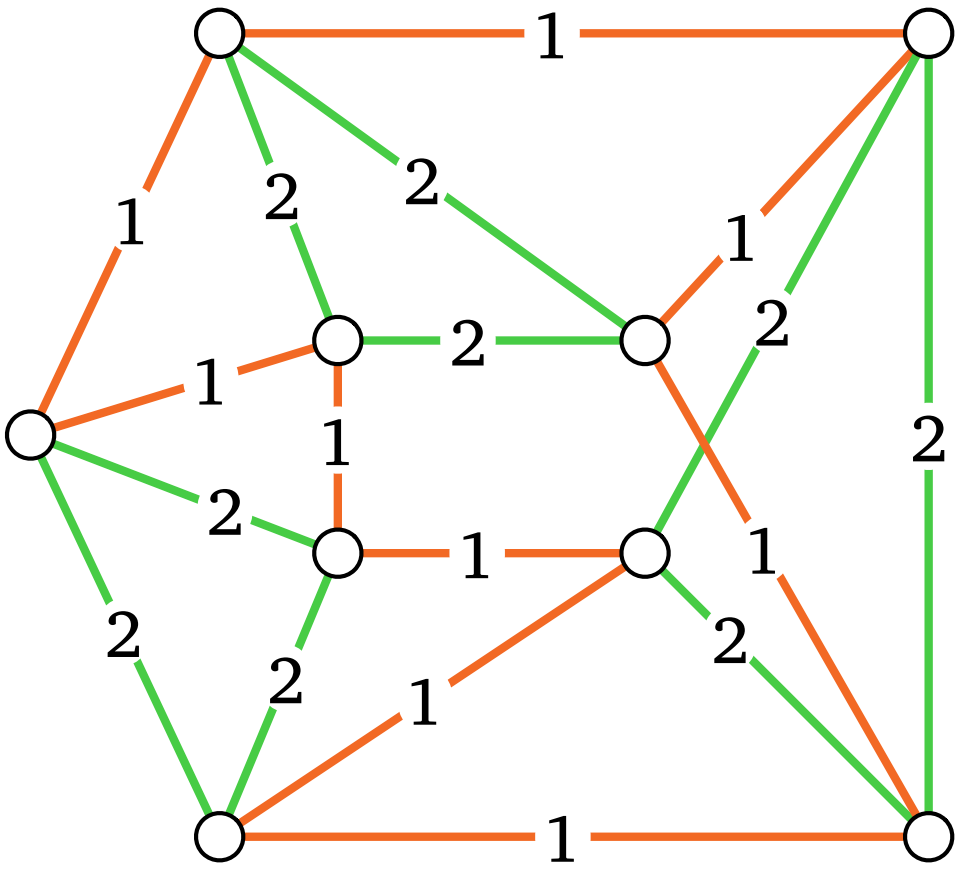
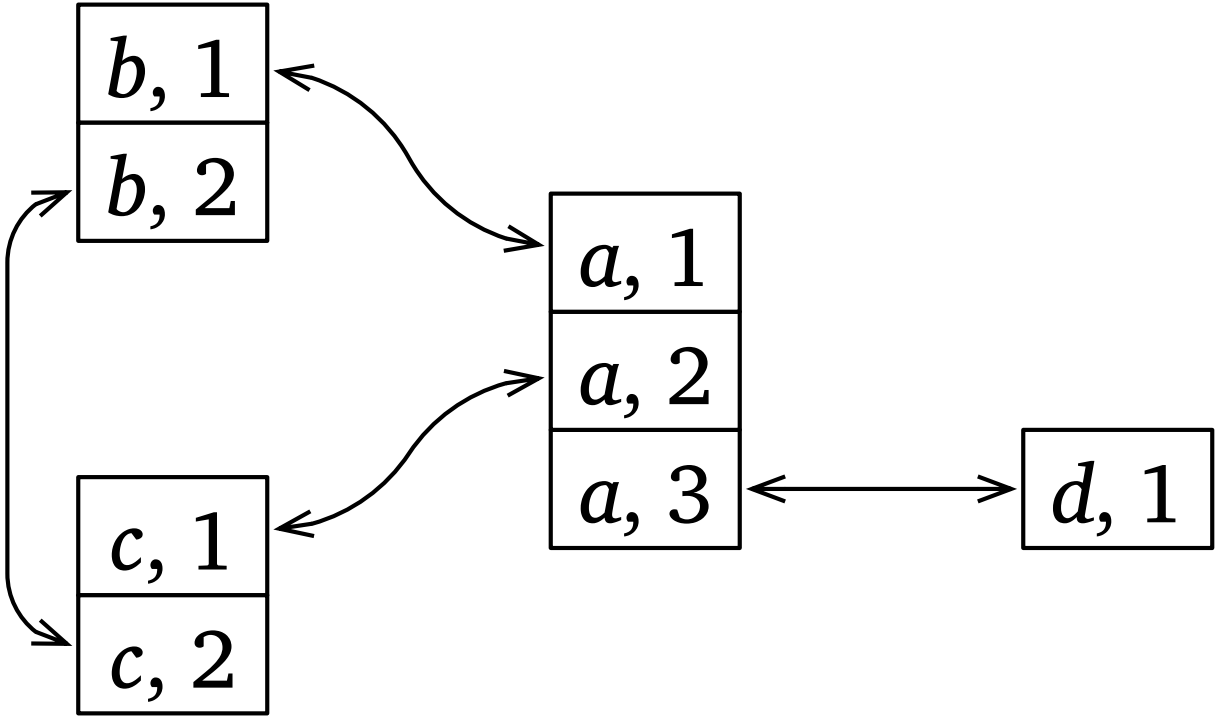
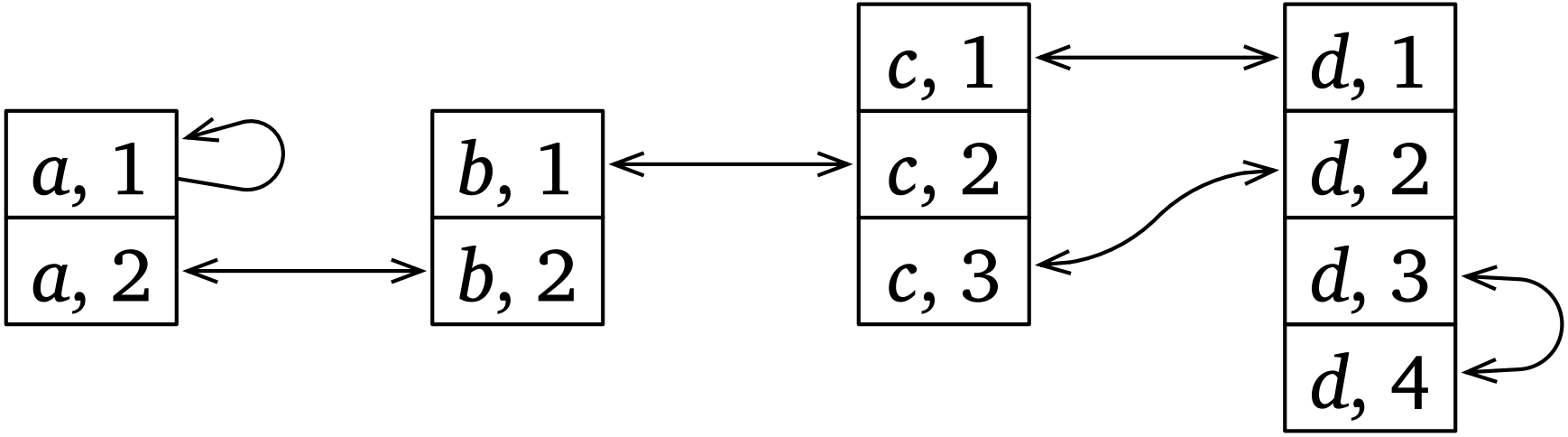
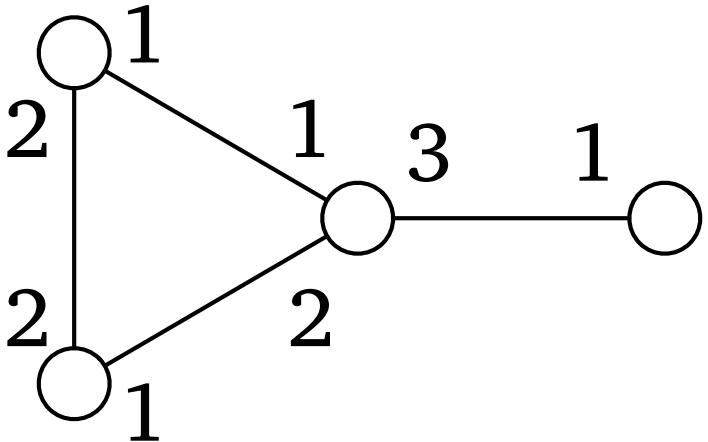
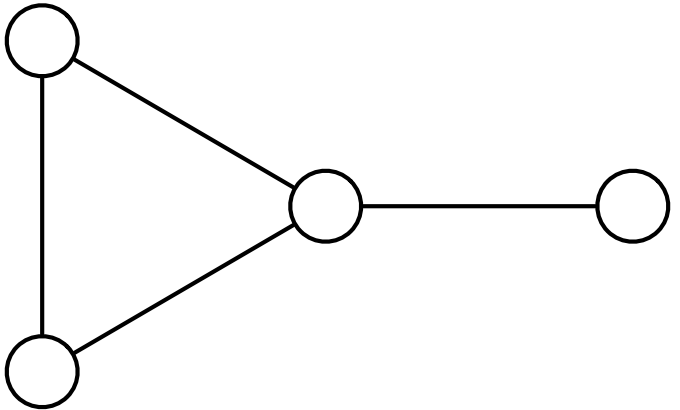



b, 1
b, 2
a, 1
a, 2
a, 3
d, 1
c, 1
c, 2


a, 1
a, 2
b, 1
b, 2
c, 1
c, 2
c, 3
d, 1
d, 2
d, 3
d, 4



(a)



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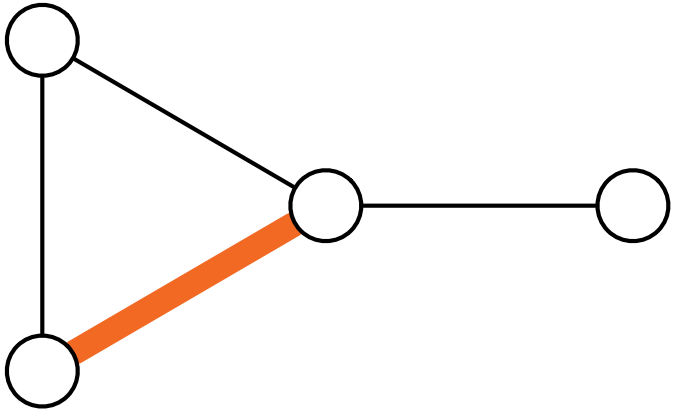
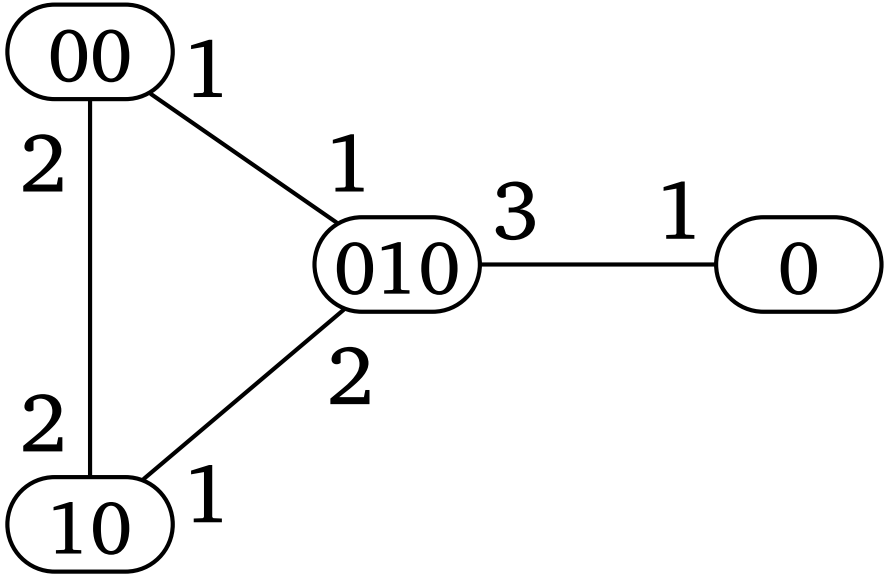


(a)



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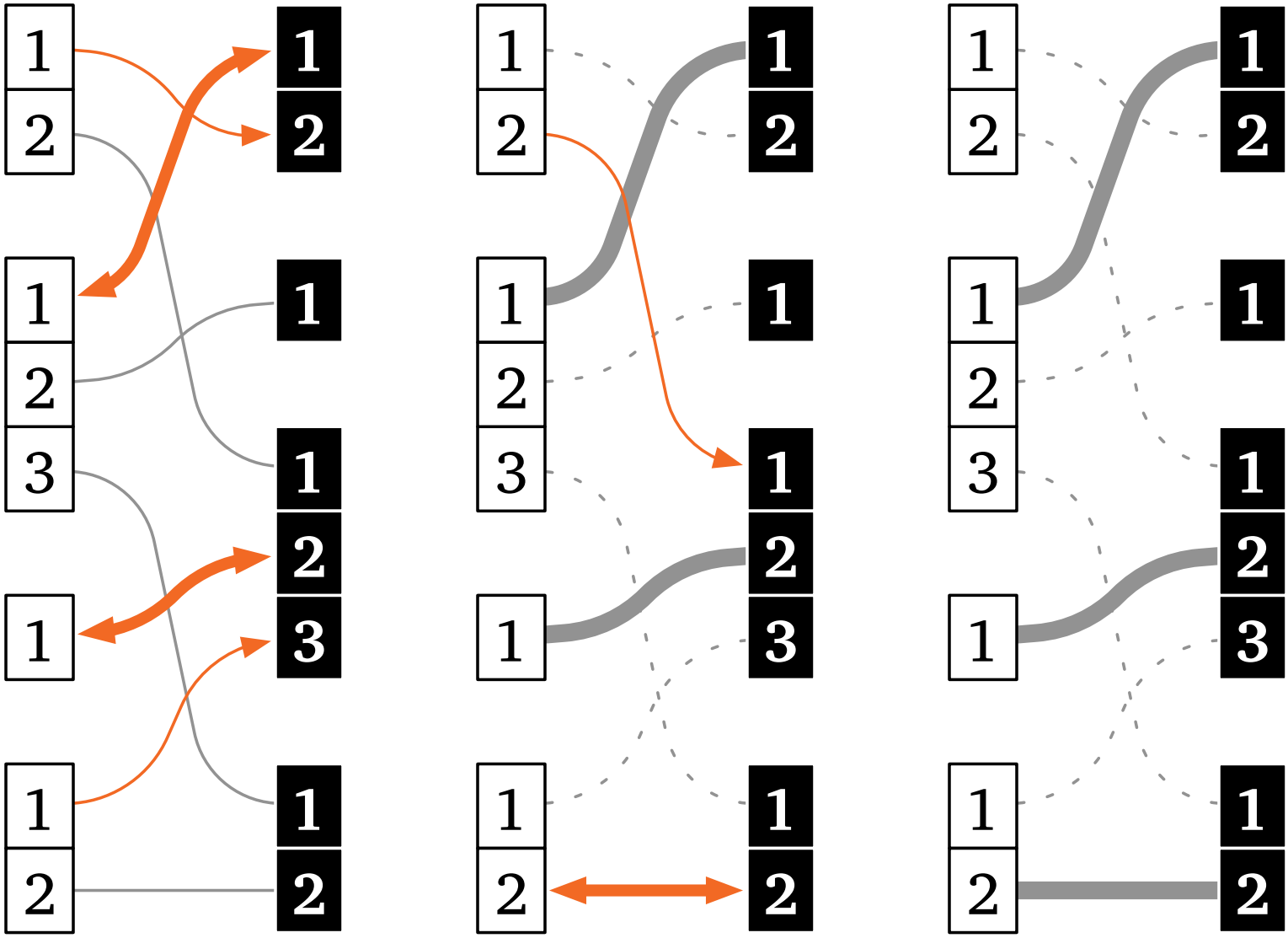
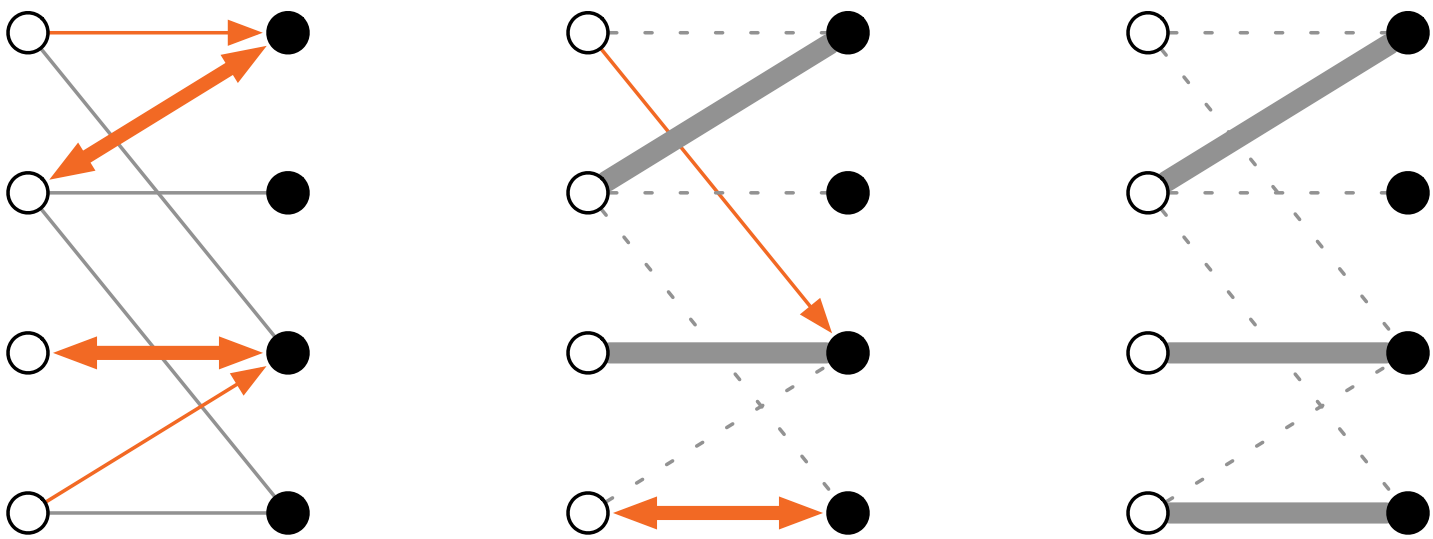
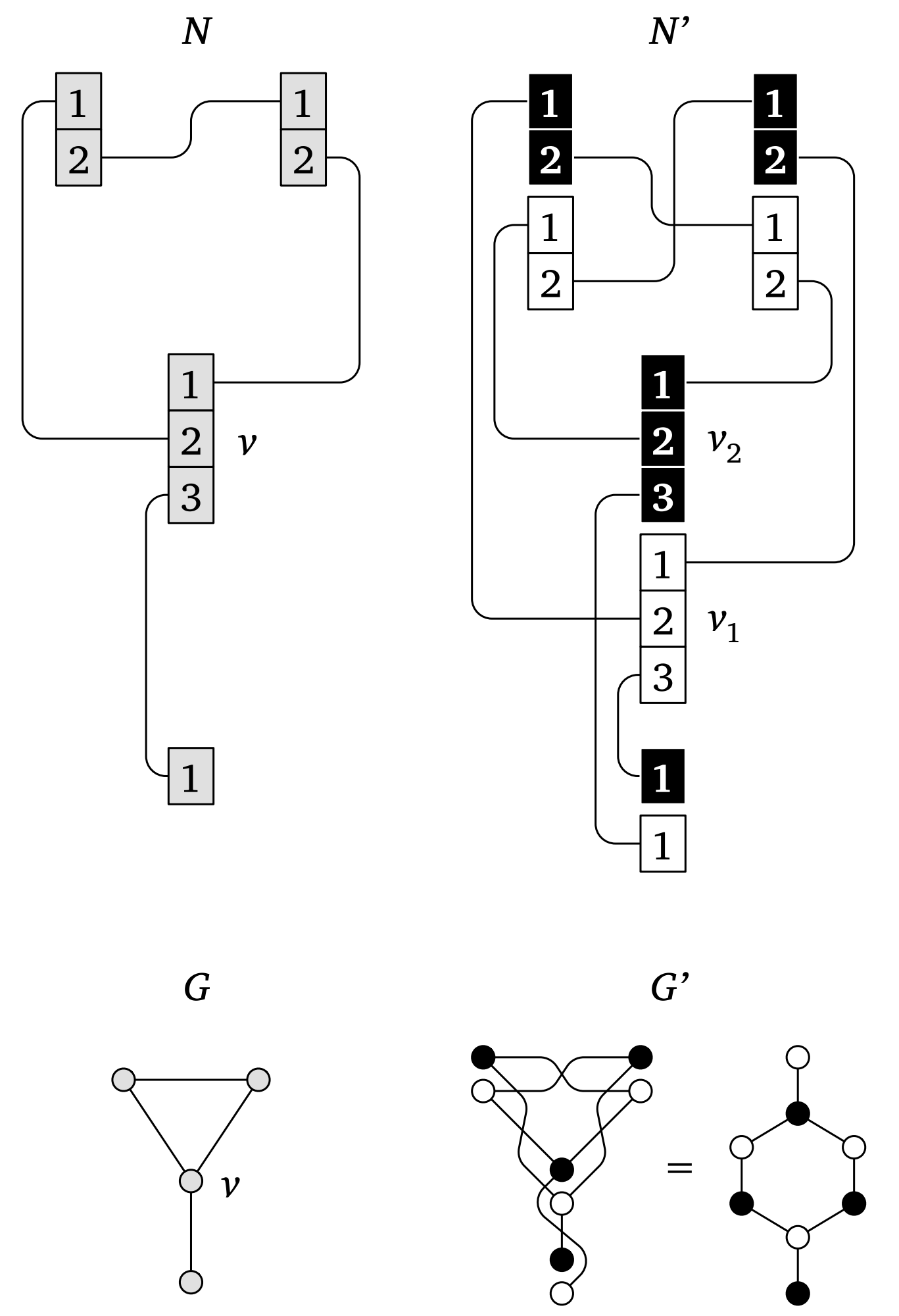
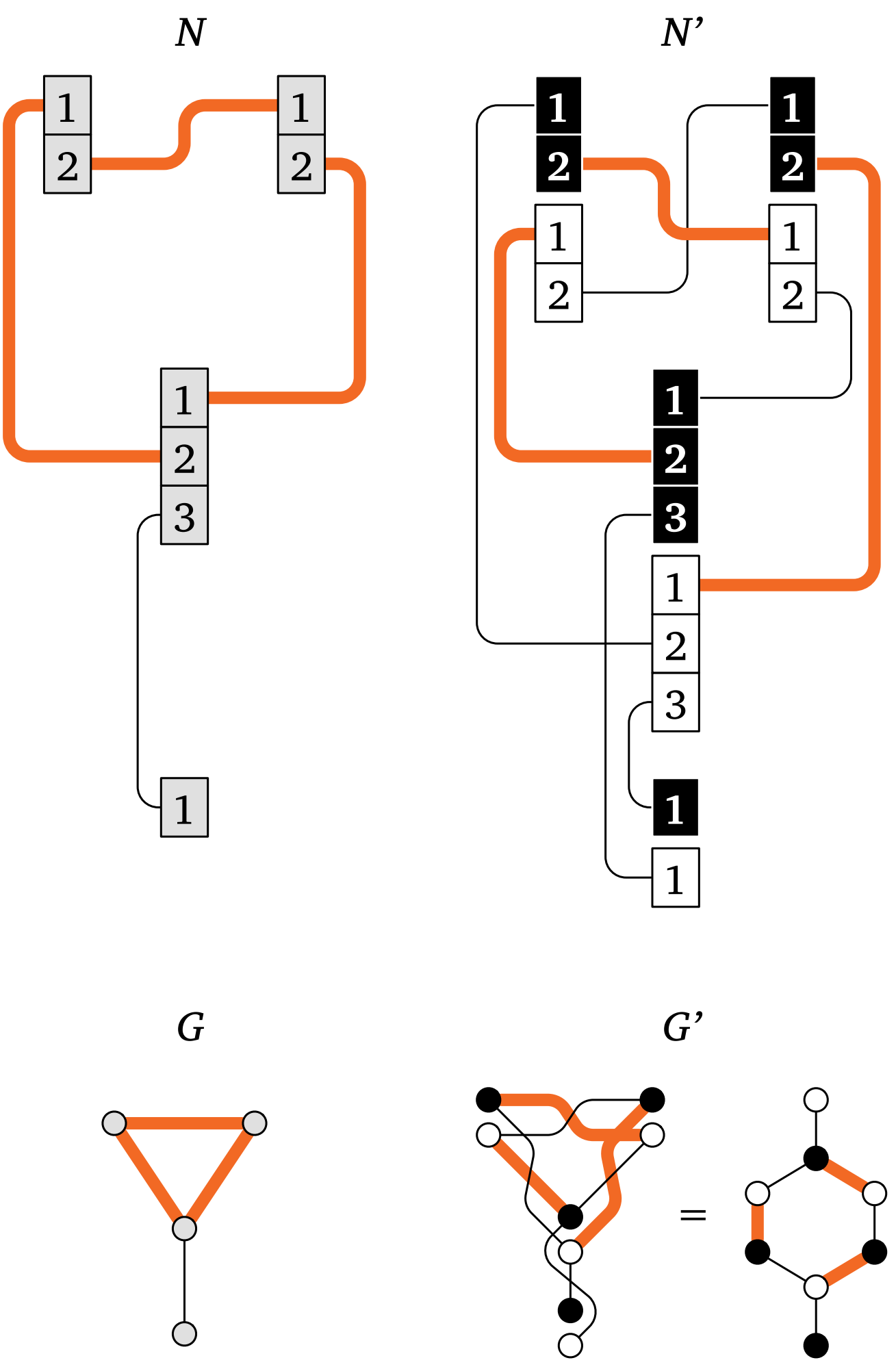
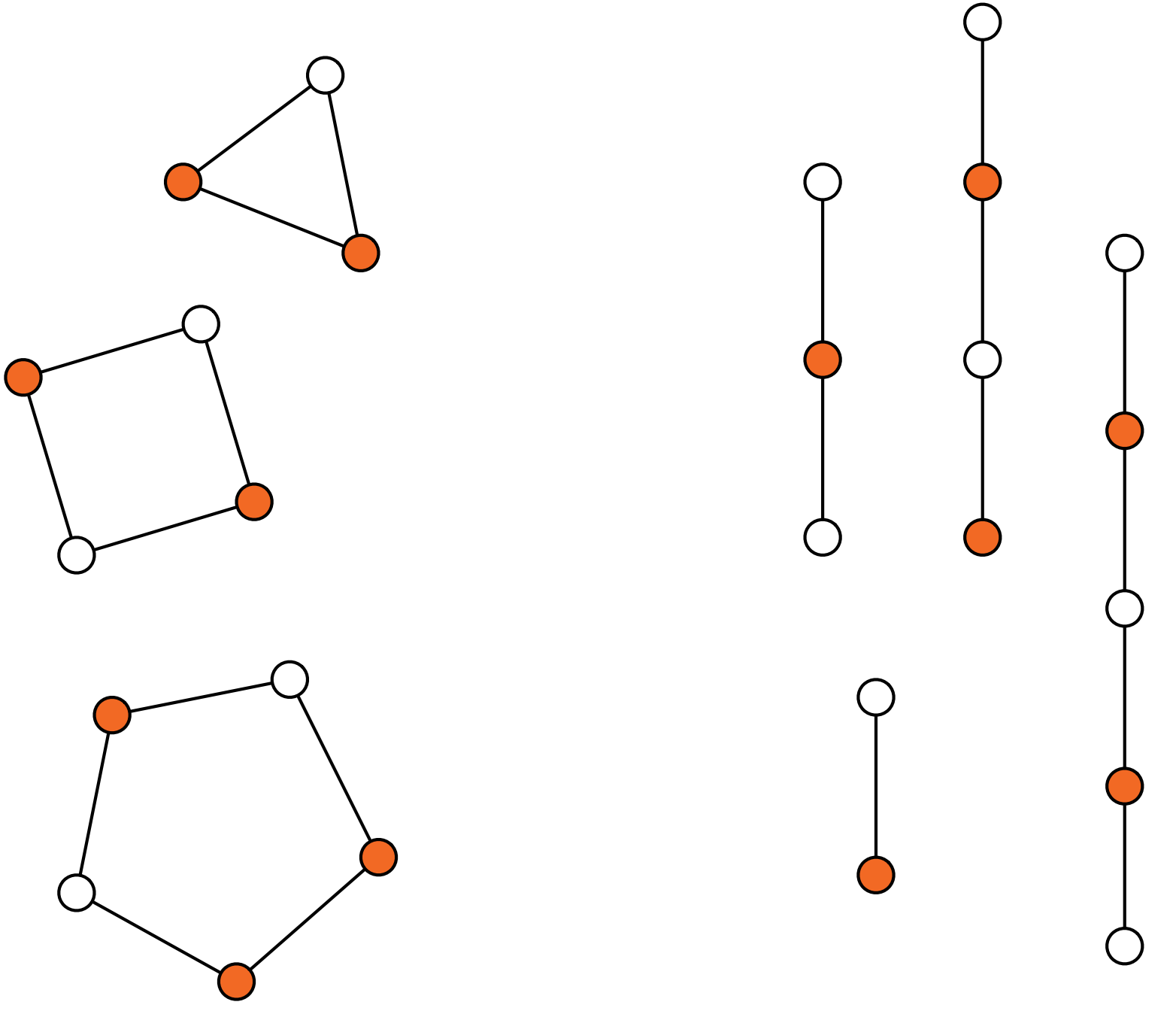


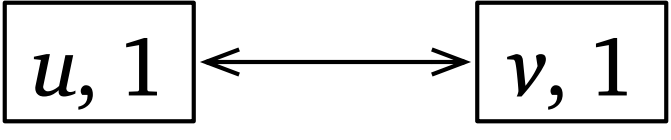
rounds 1–2 rounds 3–4 rounds 5–6

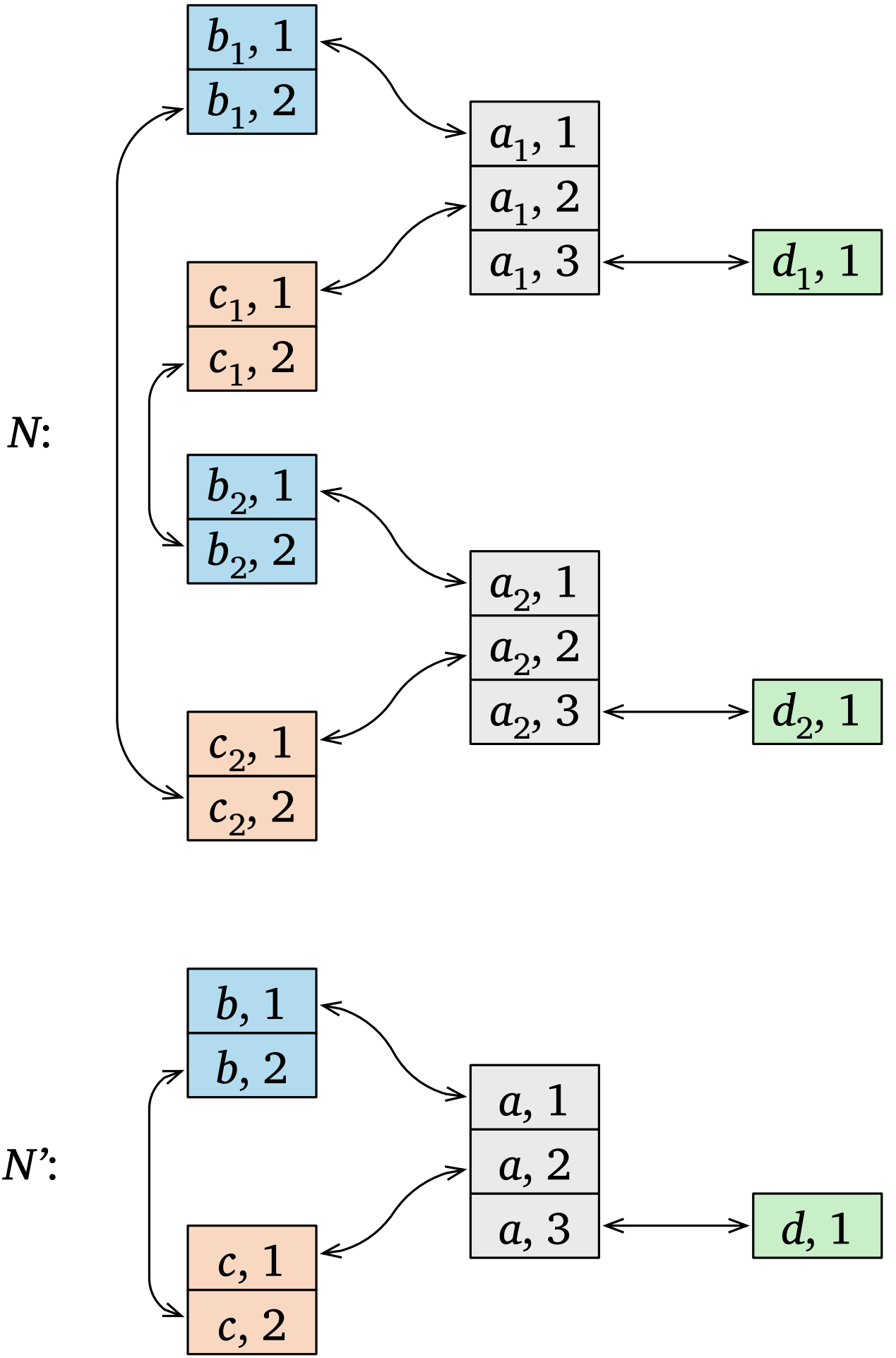



N
1
2
1
2
1
2
3
v
1
N'
1
2
1
2
1
2
1
2
1
2
3
v2
1
2
3
v1
1
1
G
v
G'
=


N
N'
G
G'
=


(a)
(b)


u, 1
v, 1


N:
$b_1, 1$
$b_1, 2$
$a_1, 1$
$a_1, 2$
$a_1, 3$
$d_1, 1$
$c_1, 1$
$c_1, 2$
$b_2, 1$
$b_2, 2$
$a_2, 1$
$a_2, 2$
$a_2, 3$
$d_2, 1$
$c_2, 1$
$c_2, 2$
N':
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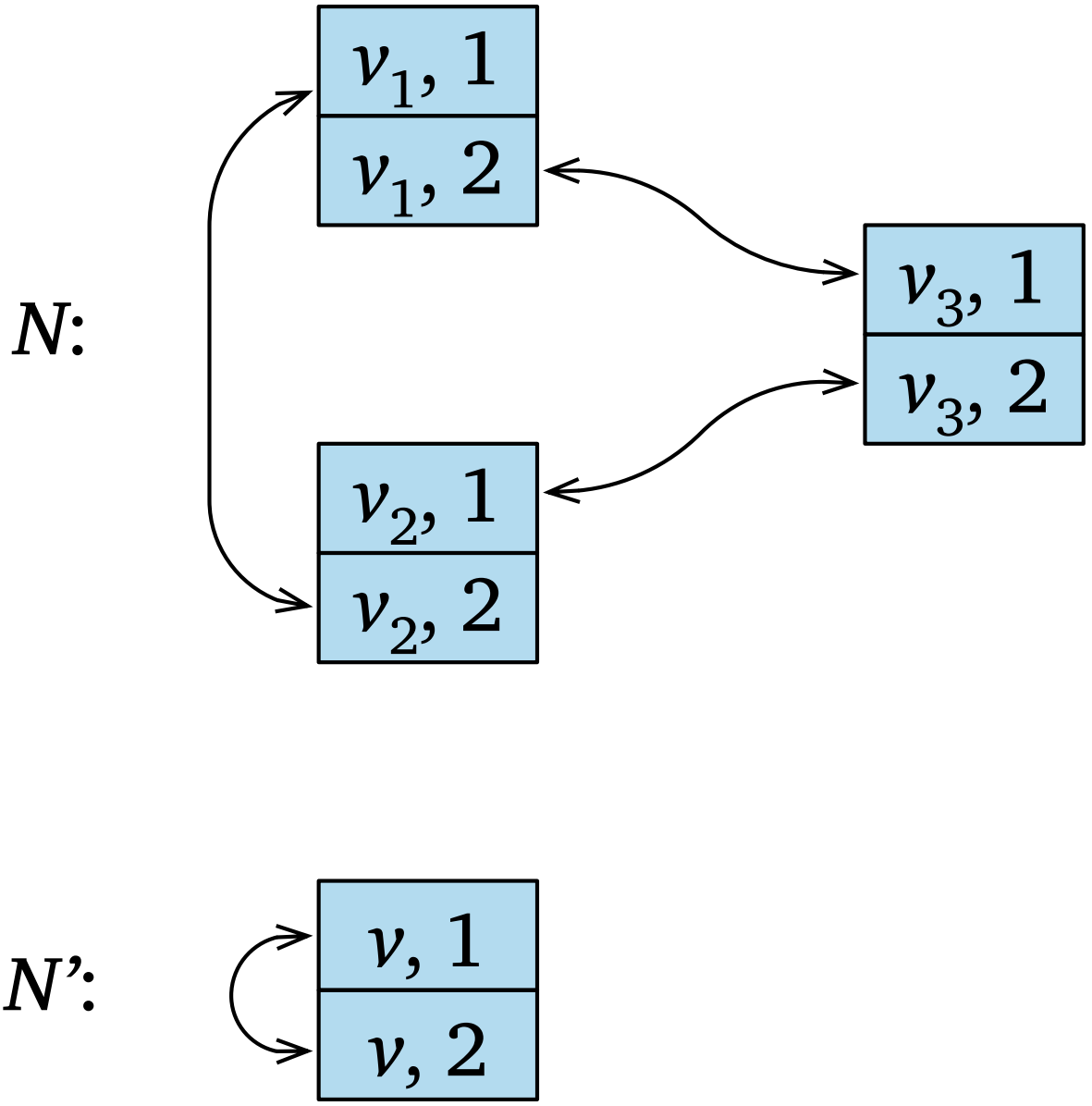
$b, 2$
$a, 1$
$a, 2$
$a, 3$
$d, 1$
$c, 1$
$c, 2$


N:
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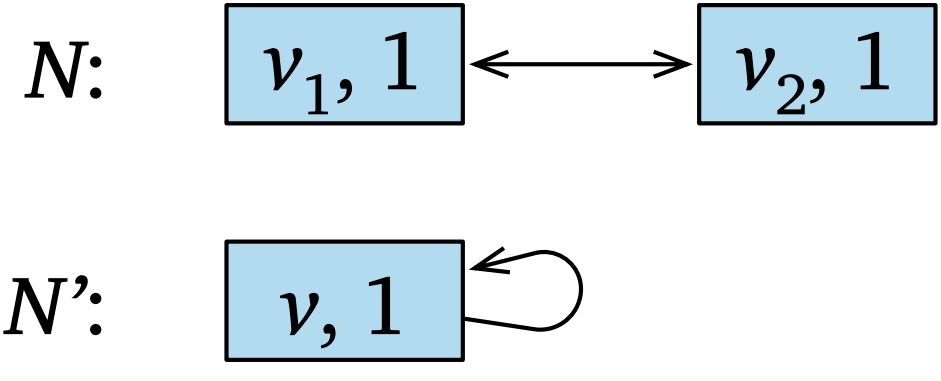
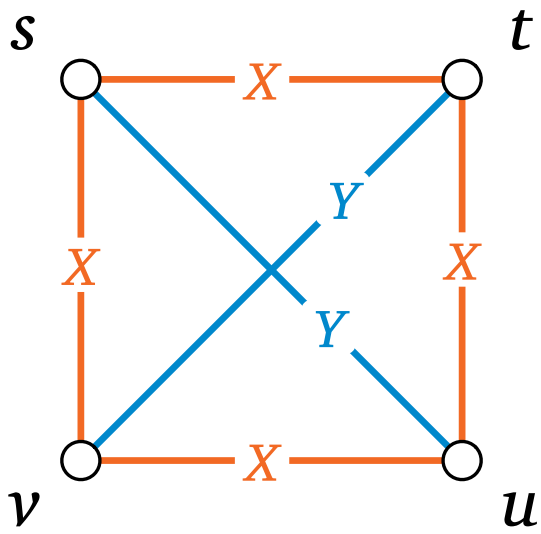
v_1, 2
v_3, 1
v_3, 2
v_2, 1
v_2, 2
N':
v, 1
v, 2


N:
v1, 1
v2, 1
N':
v, 1

G:



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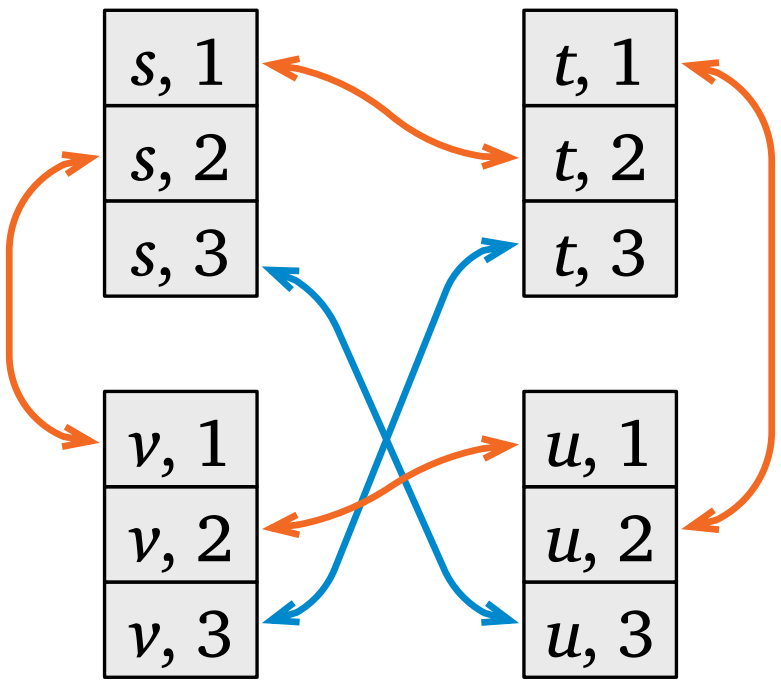

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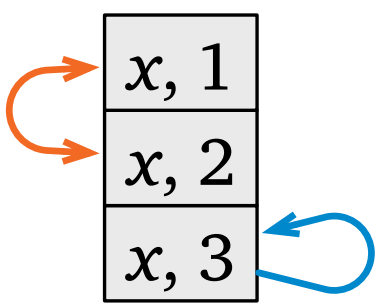
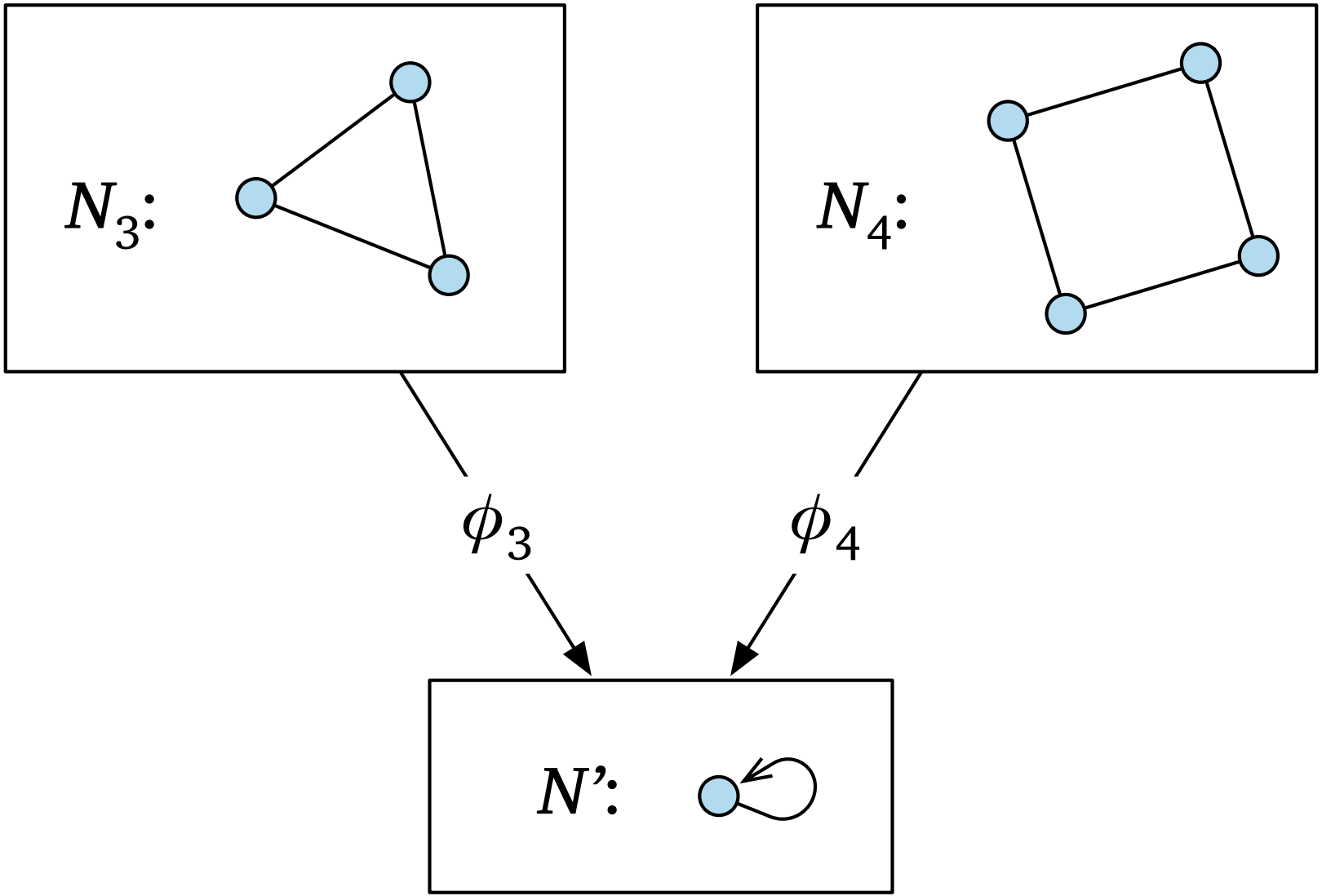
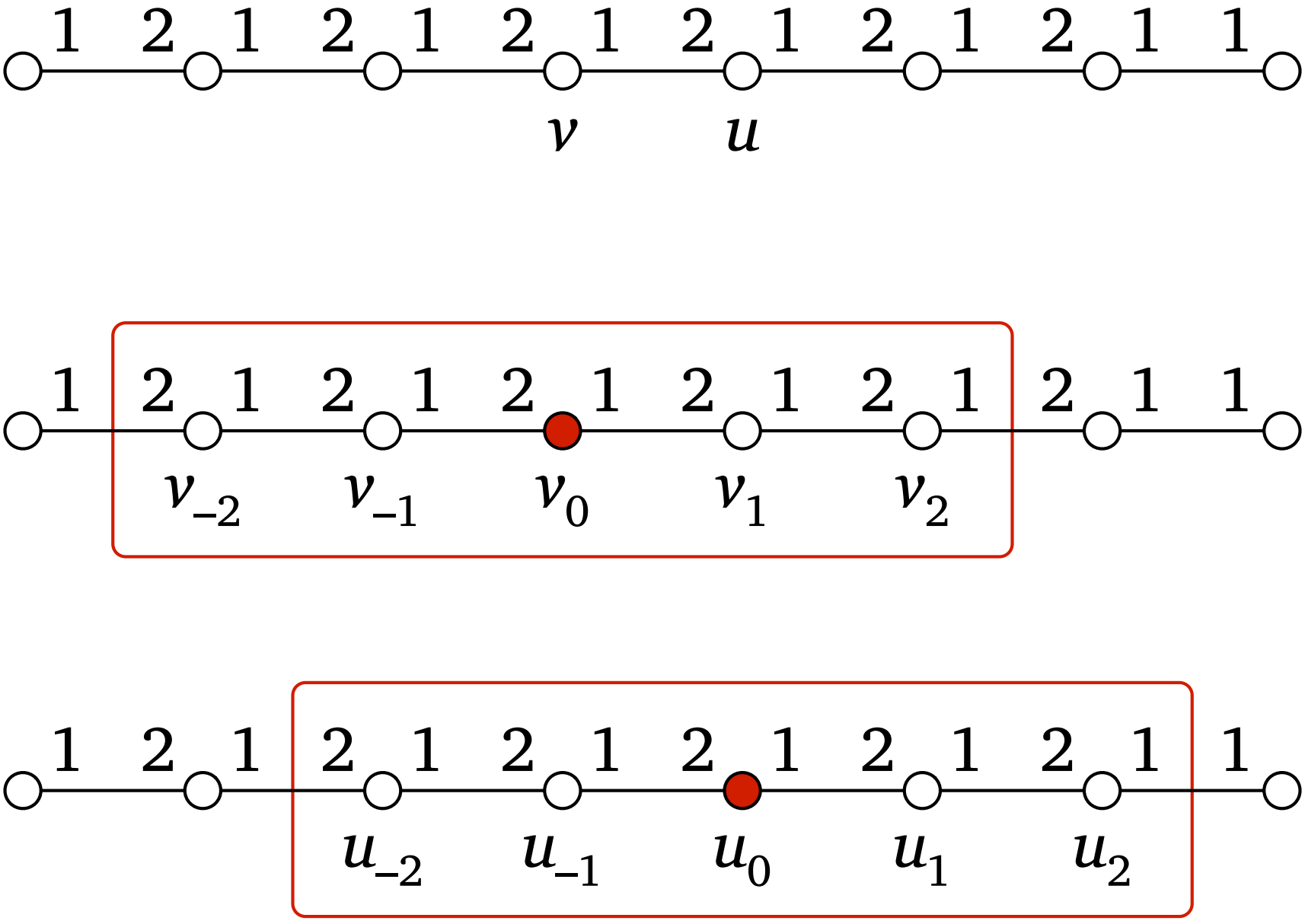
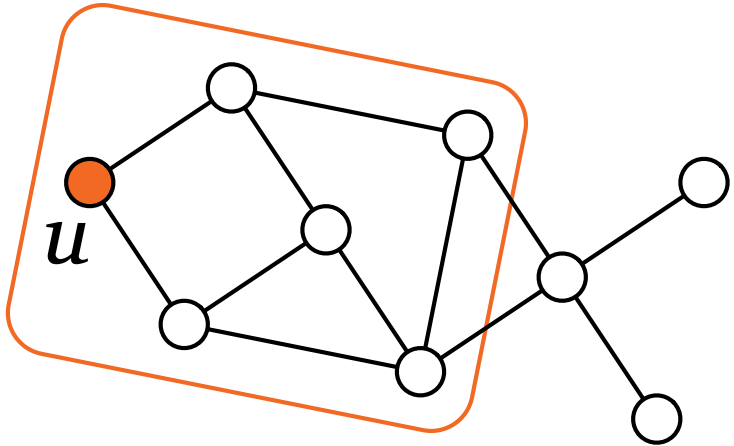


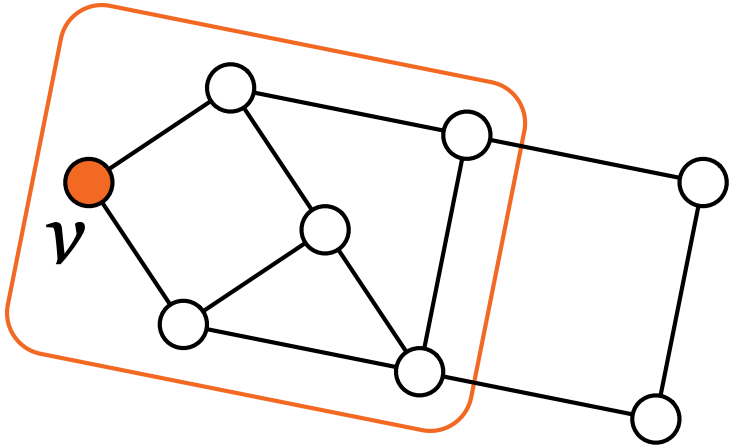

N_3:
N_4:
ϕ_3
ϕ_4
N':

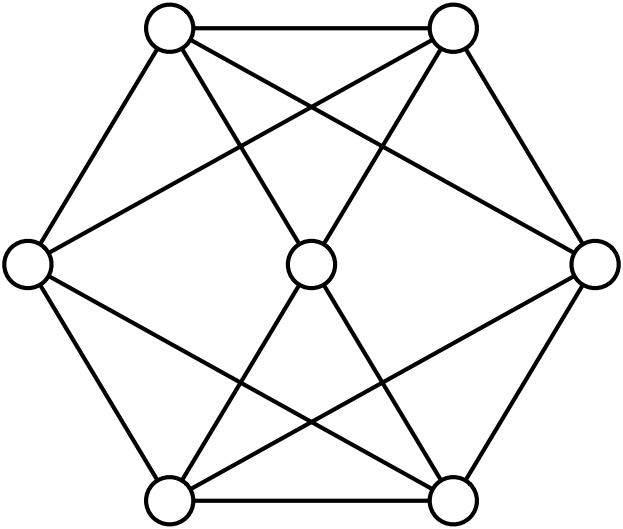
1
2
1
2
1

1
2
1
2
1
2

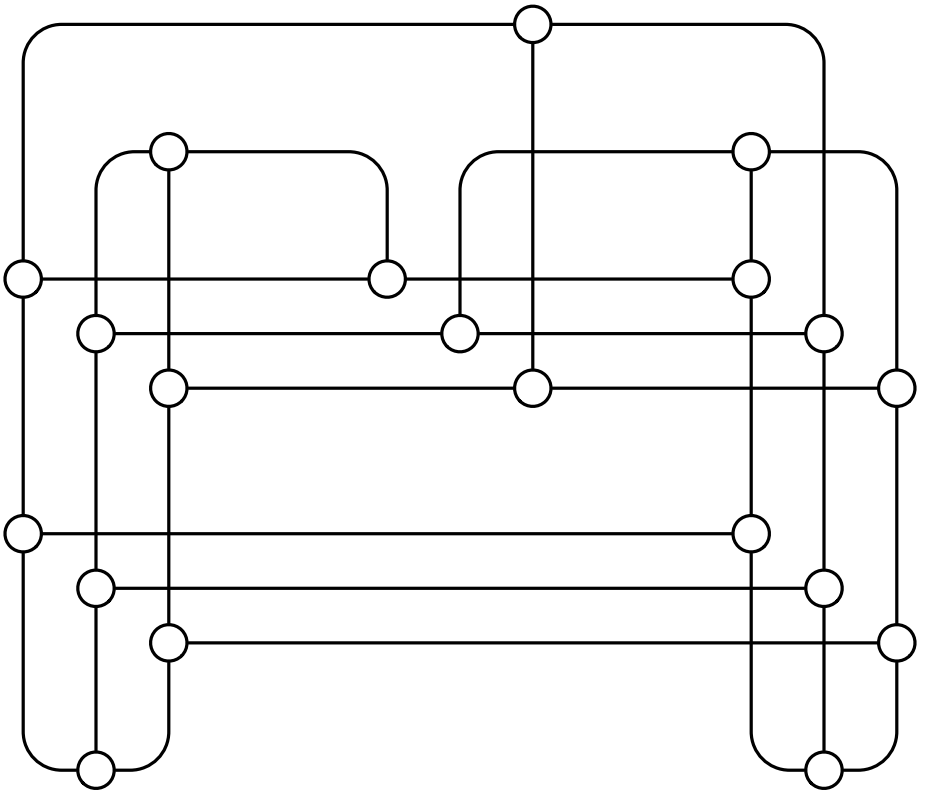

1 2 1 2 1 2 1 2 1 2 1 2 1 2 1 1
v
u
1 2 1 2 1 2 1 2 1 2 1 2 1 1
v_-2 v_-1 v_0 v_1 v_2
1 2 1 2 1 2 1 2 1 2 1 2 1 1
u_-2 u_-1 u_0 u_1 u_2


u


v



G:



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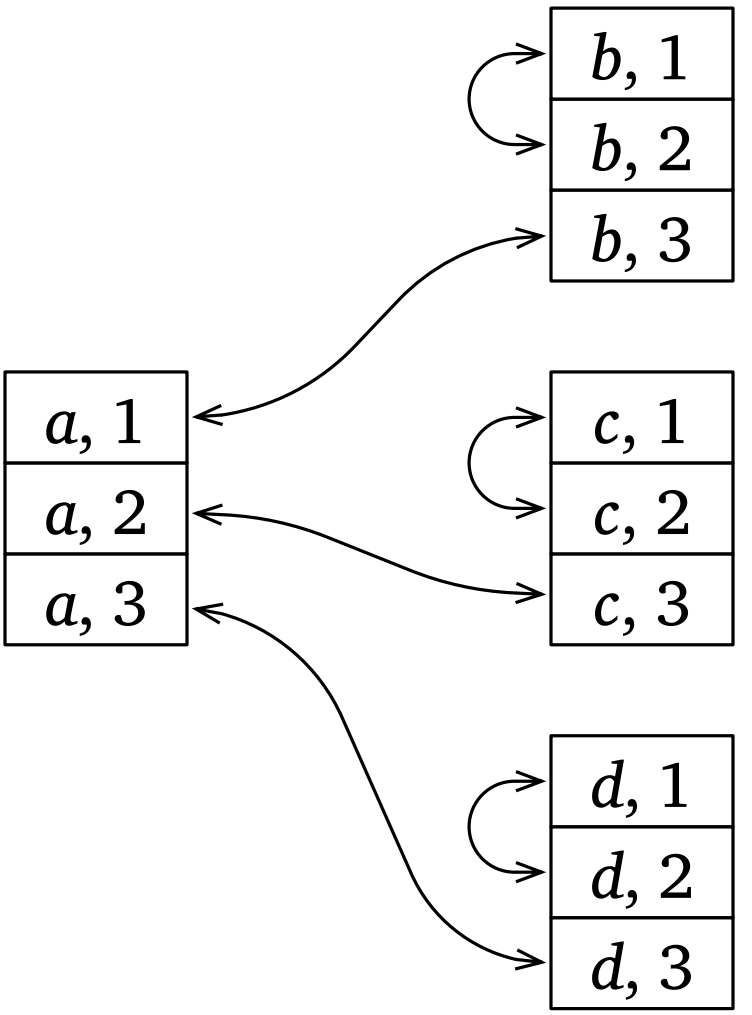
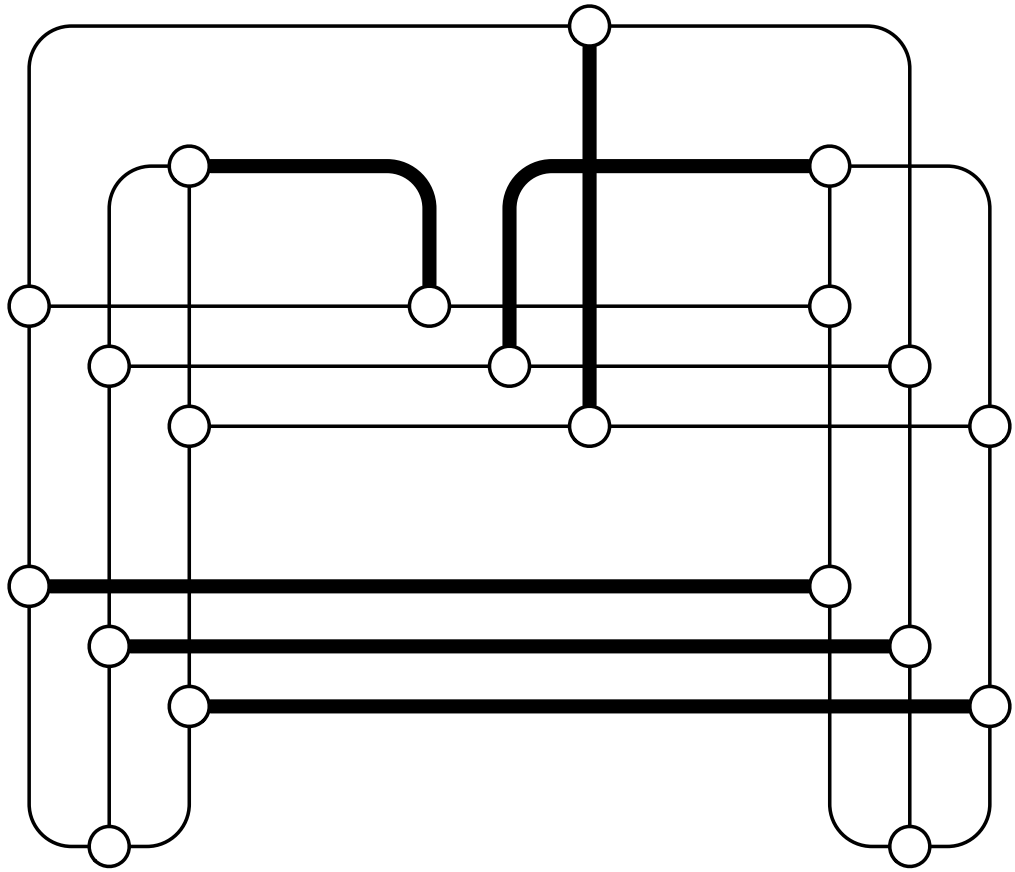
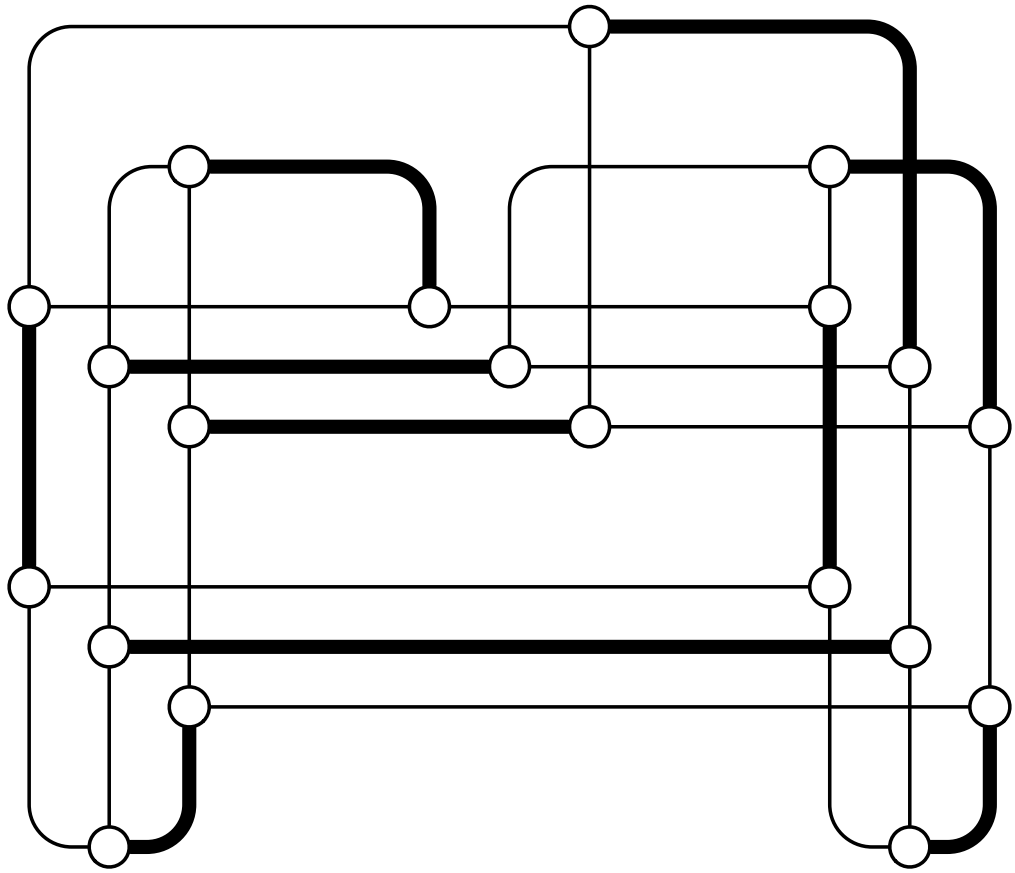
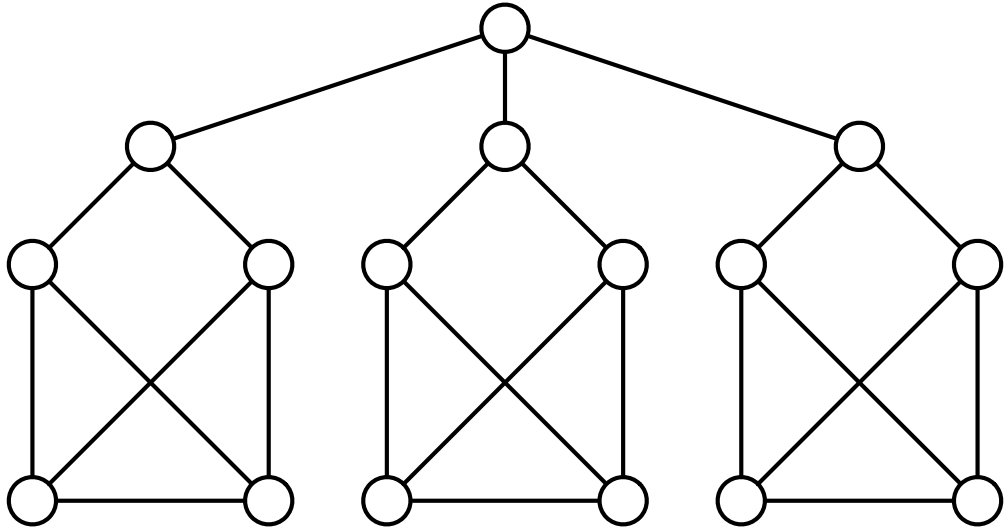
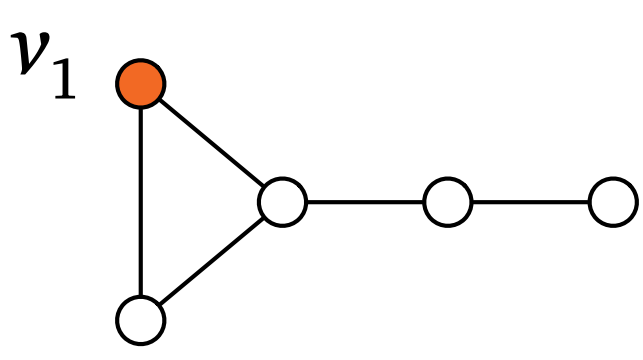


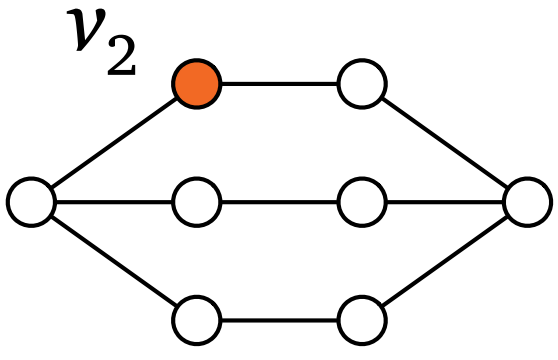




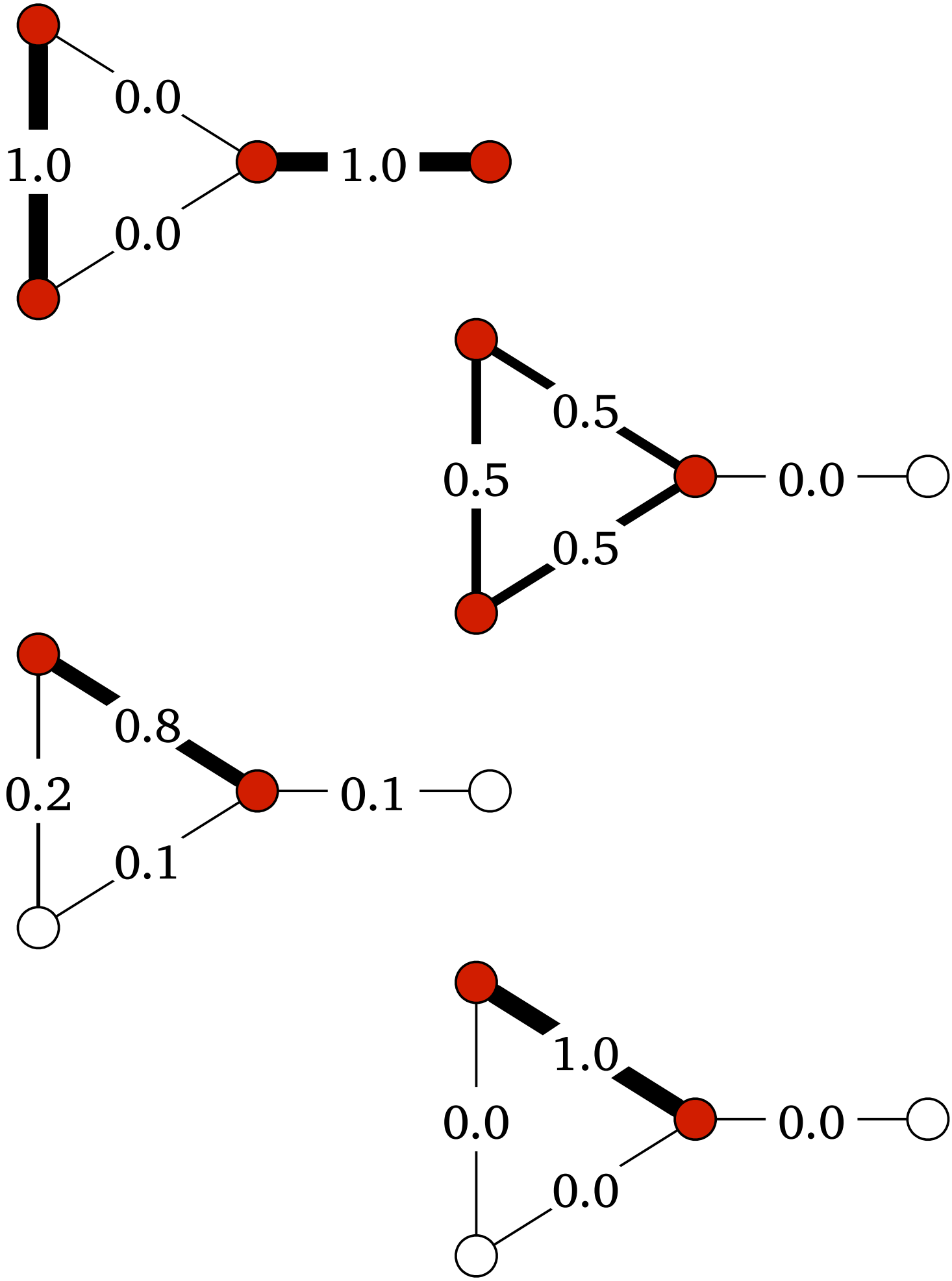




G_1



G_2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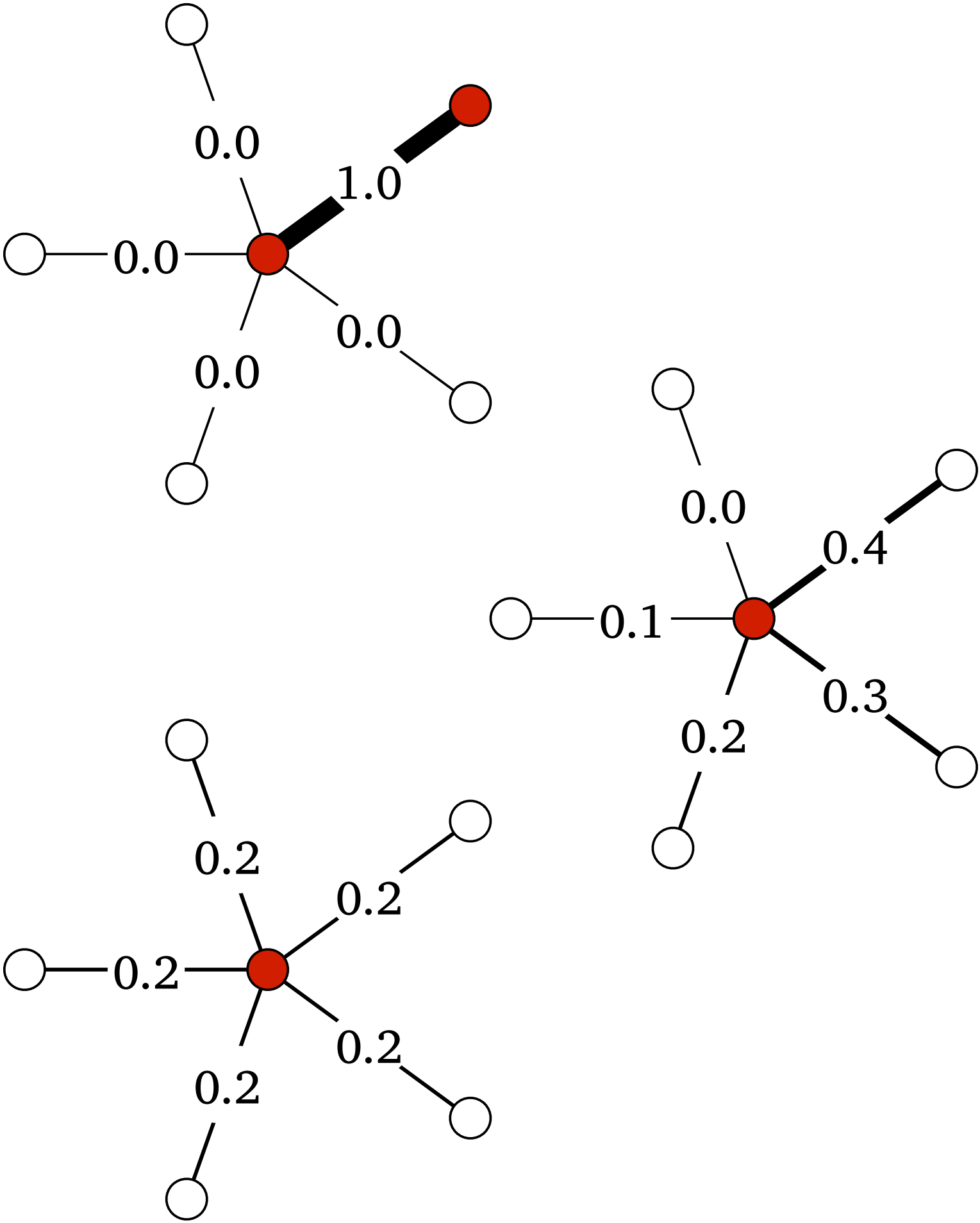
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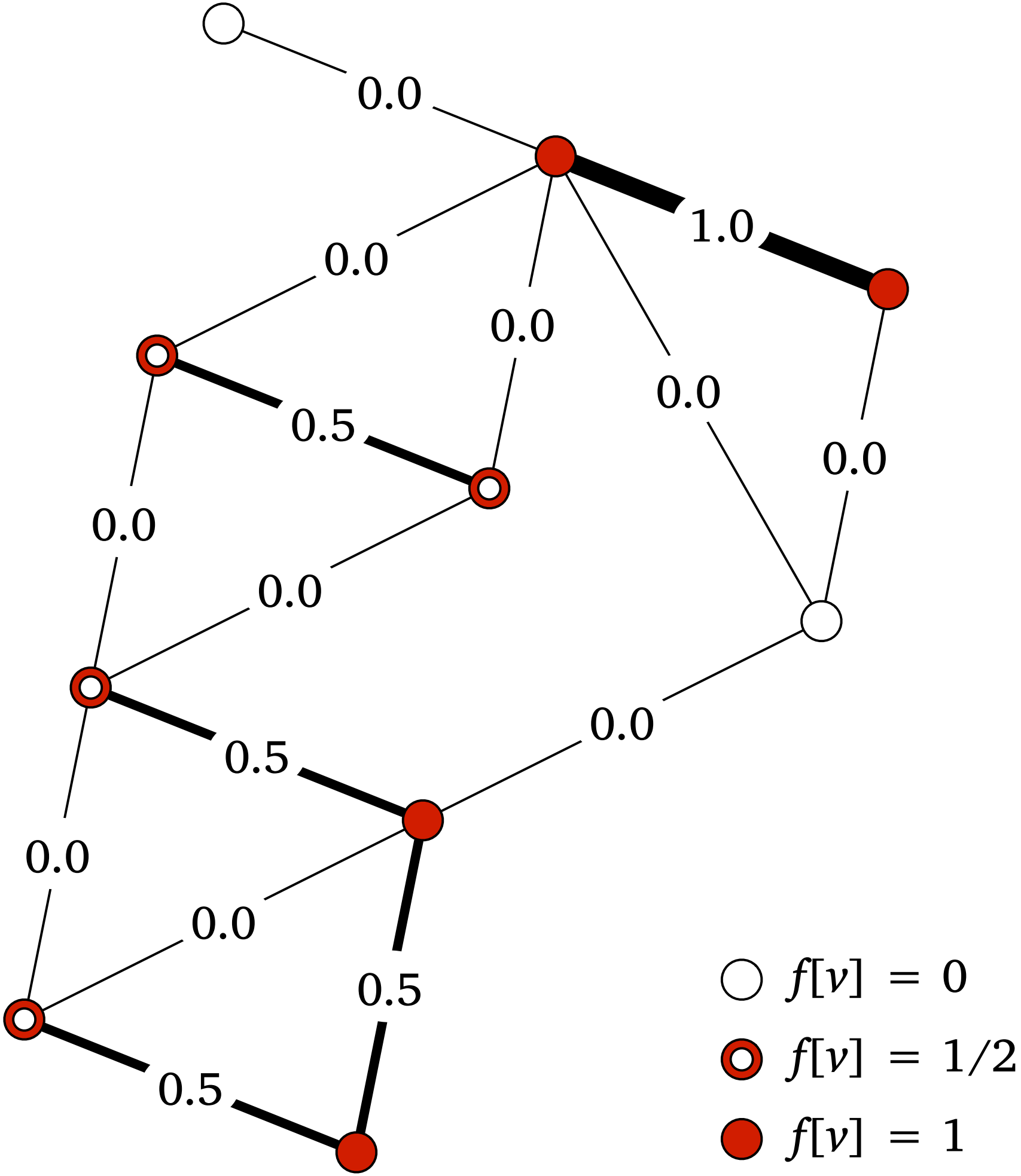
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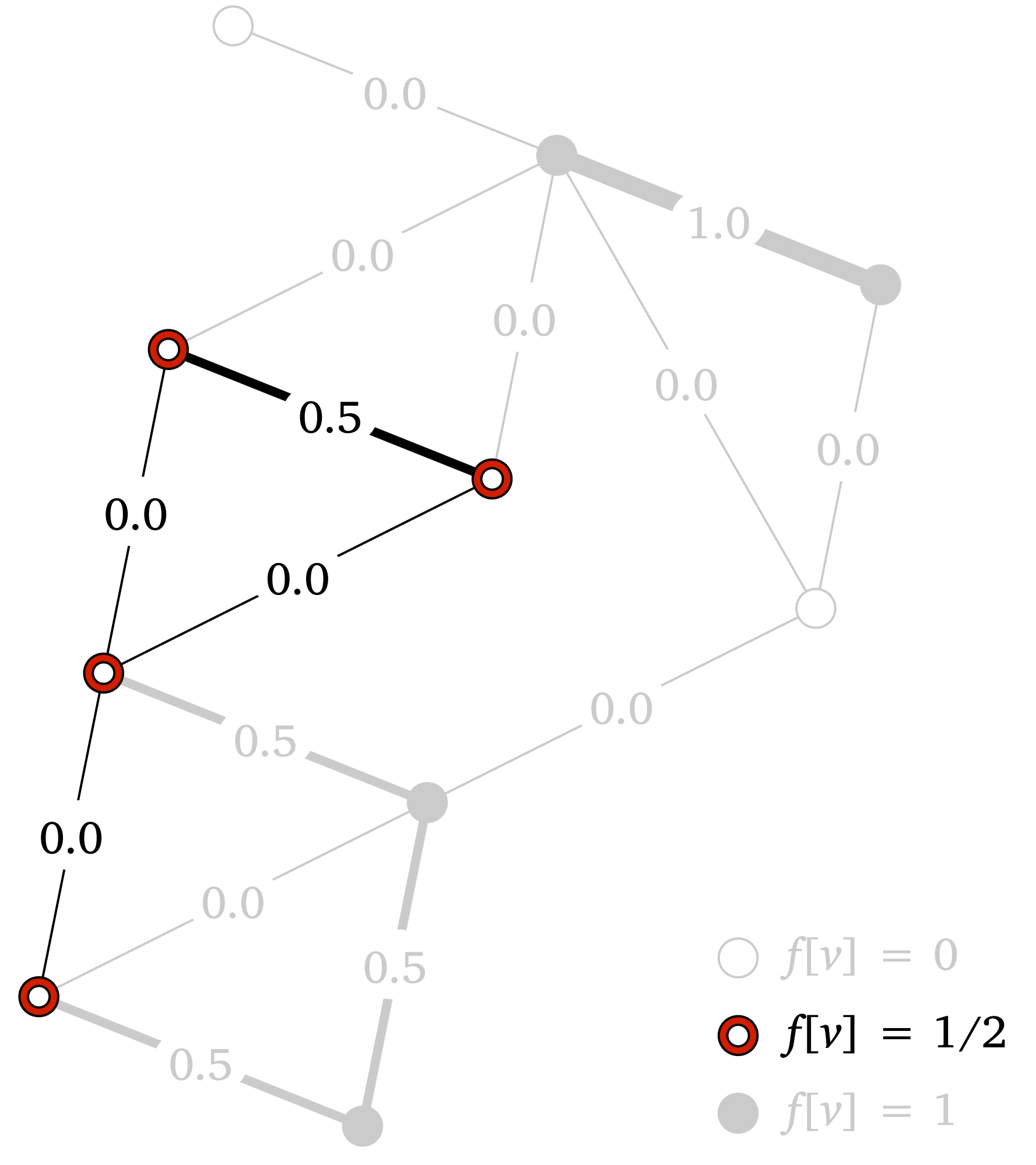
0.5
$f[v] = 0$
$f[v] = 1/2$
$f[v] = 1$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0.0
0.0
0.0
0.0
0.0
0.0
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0.0
0.5
0.5
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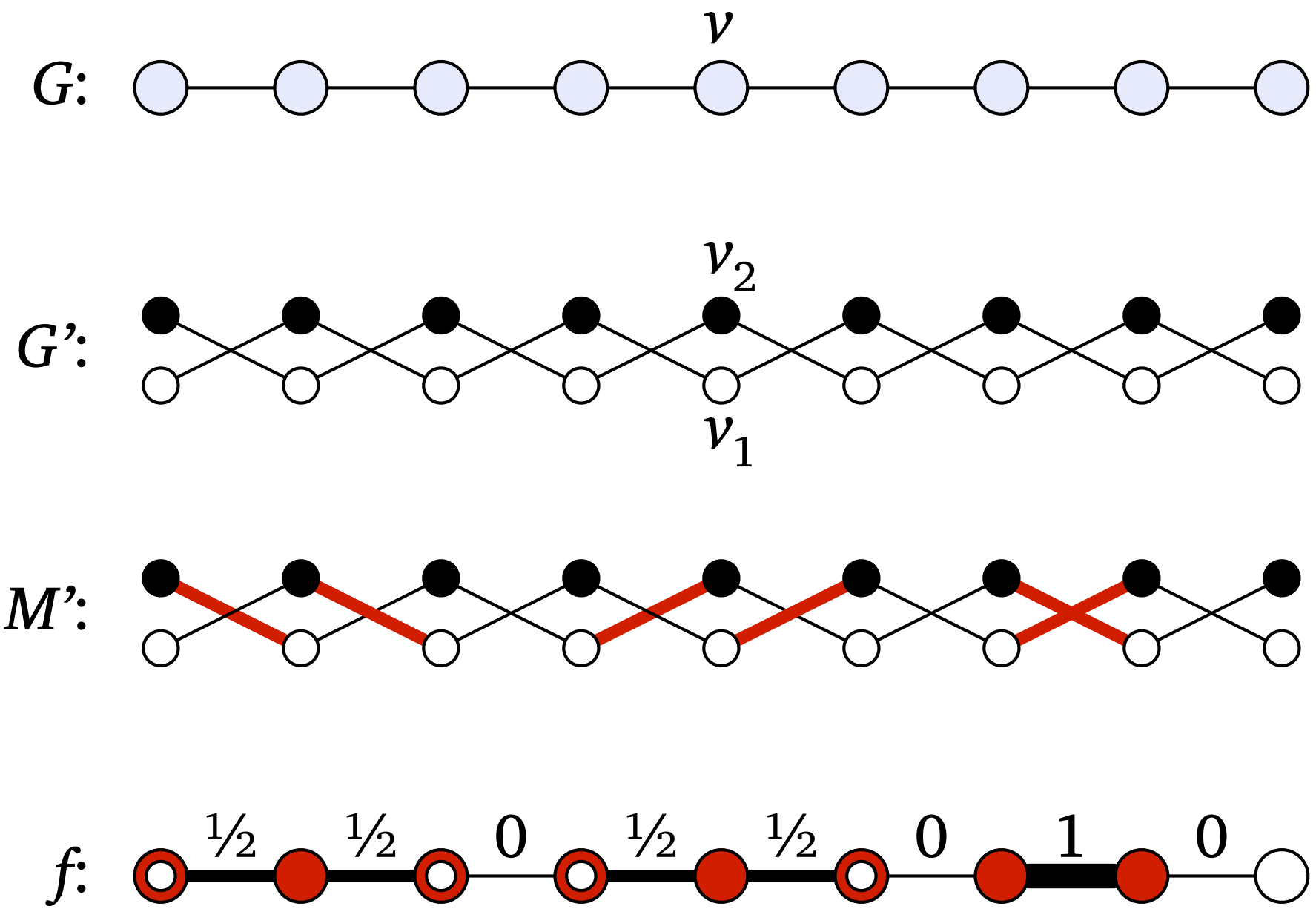
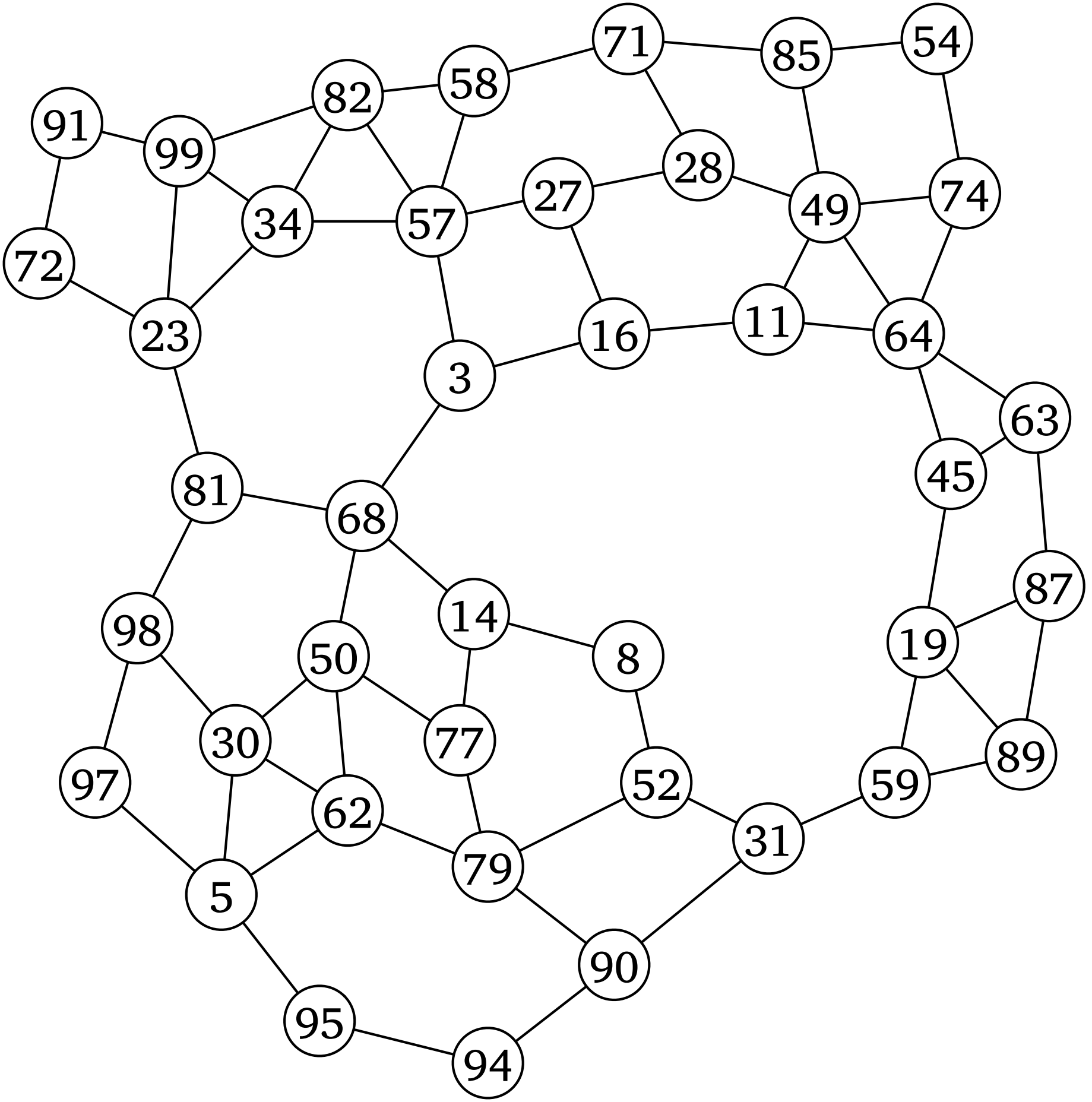
$f[v] = 1/2$
$f[v] = 1$


G:
v
G':
v2
v1
M':
f:
½
½
0
½
½
0
1
0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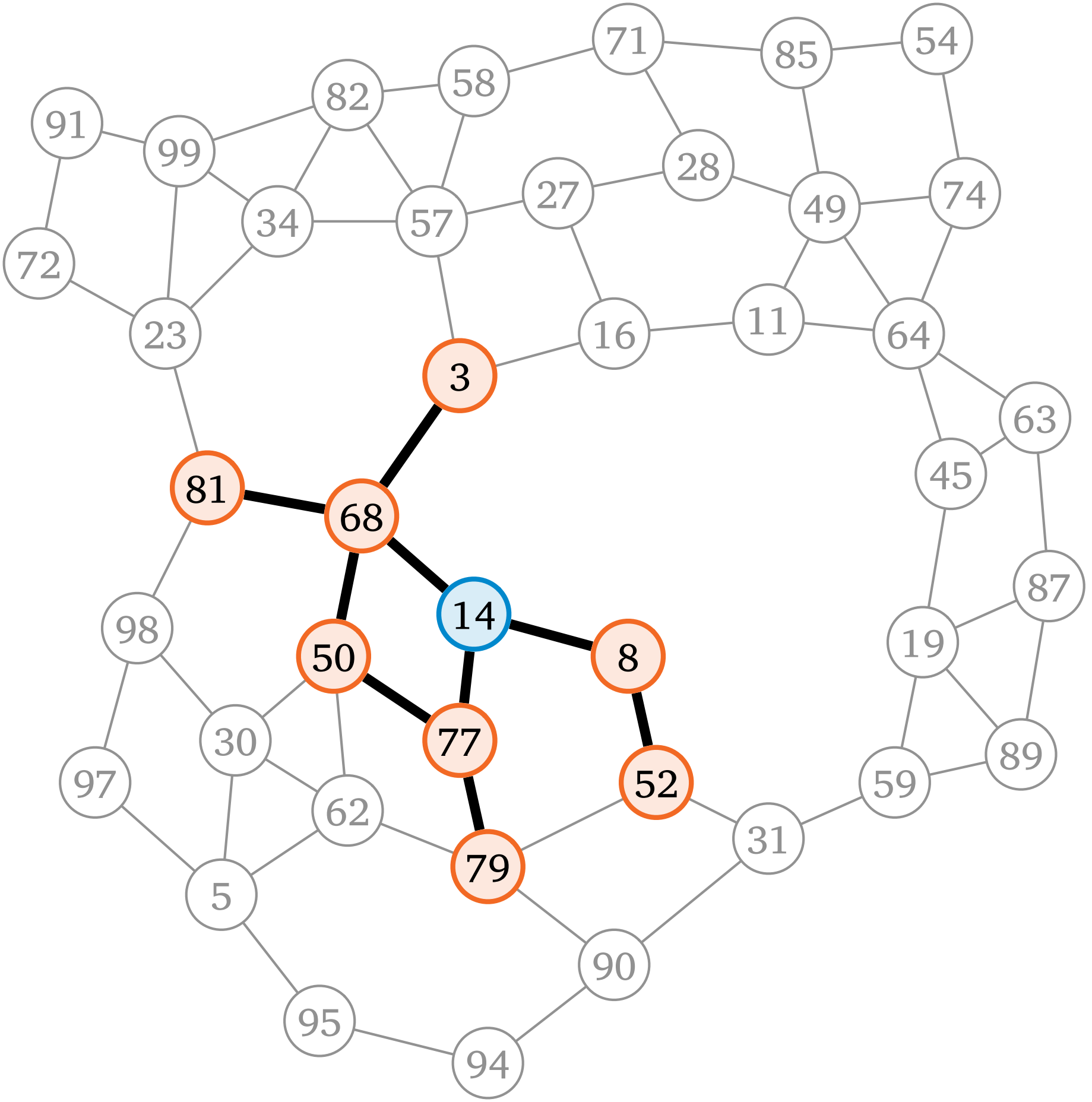
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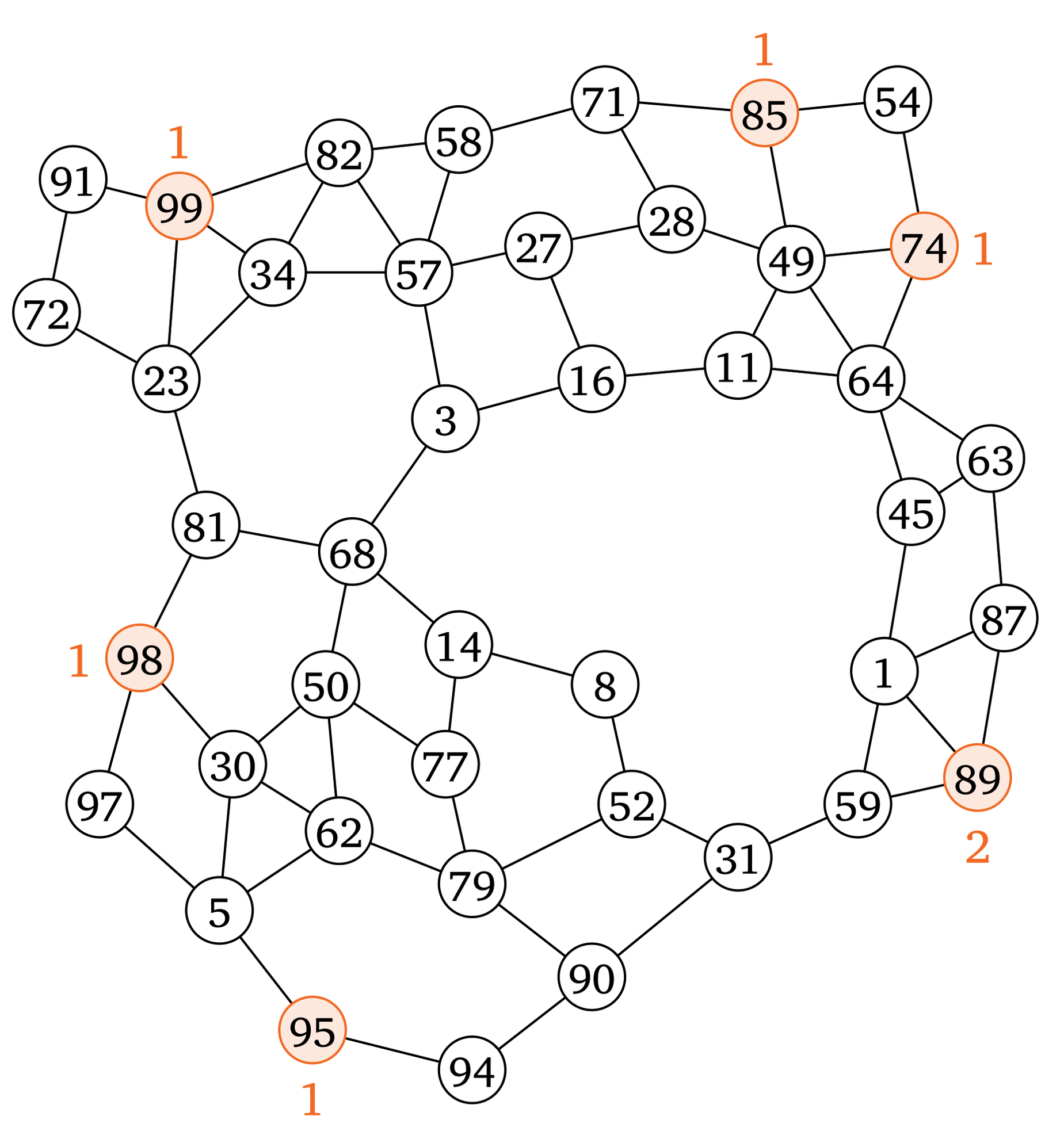
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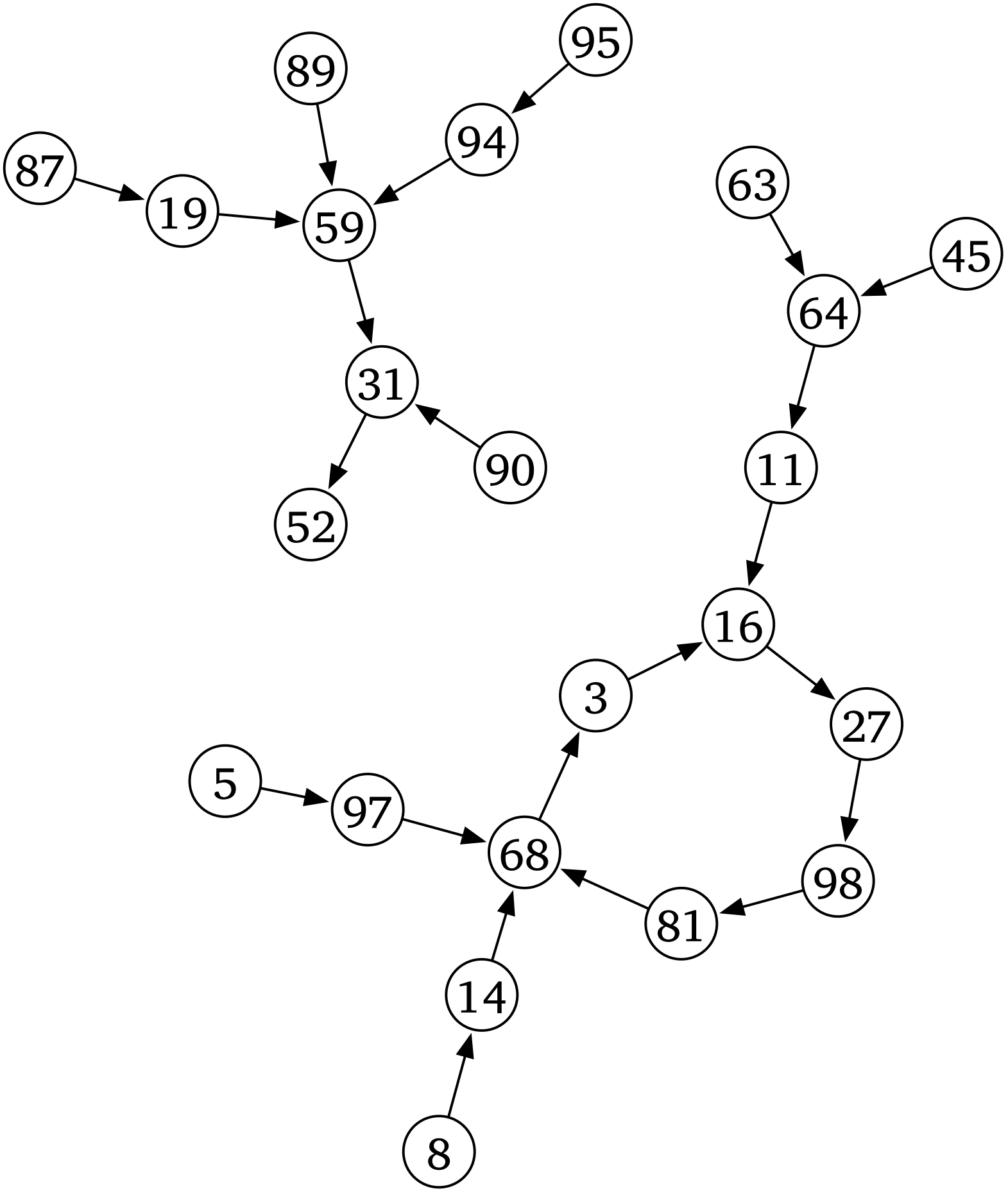
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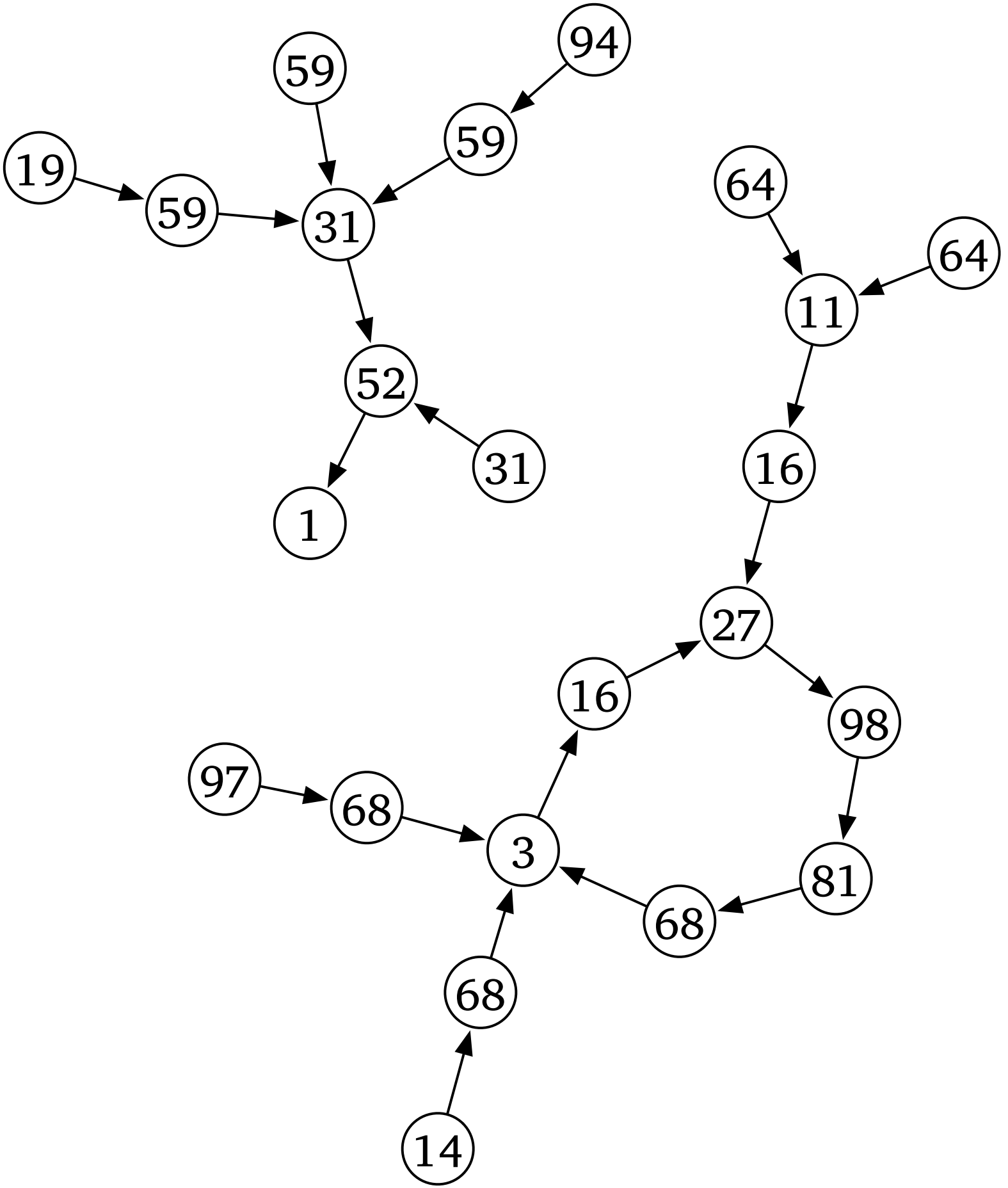
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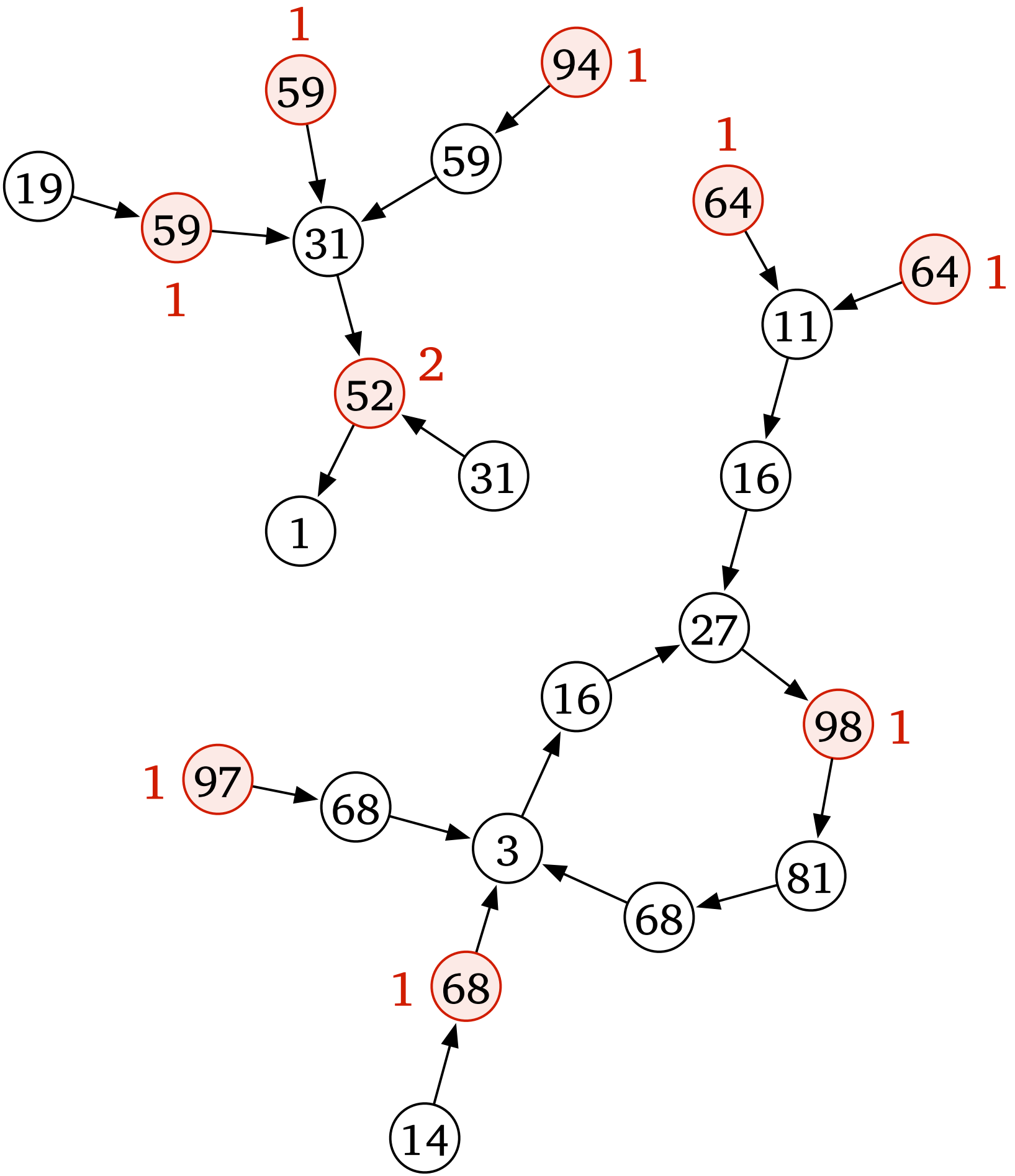
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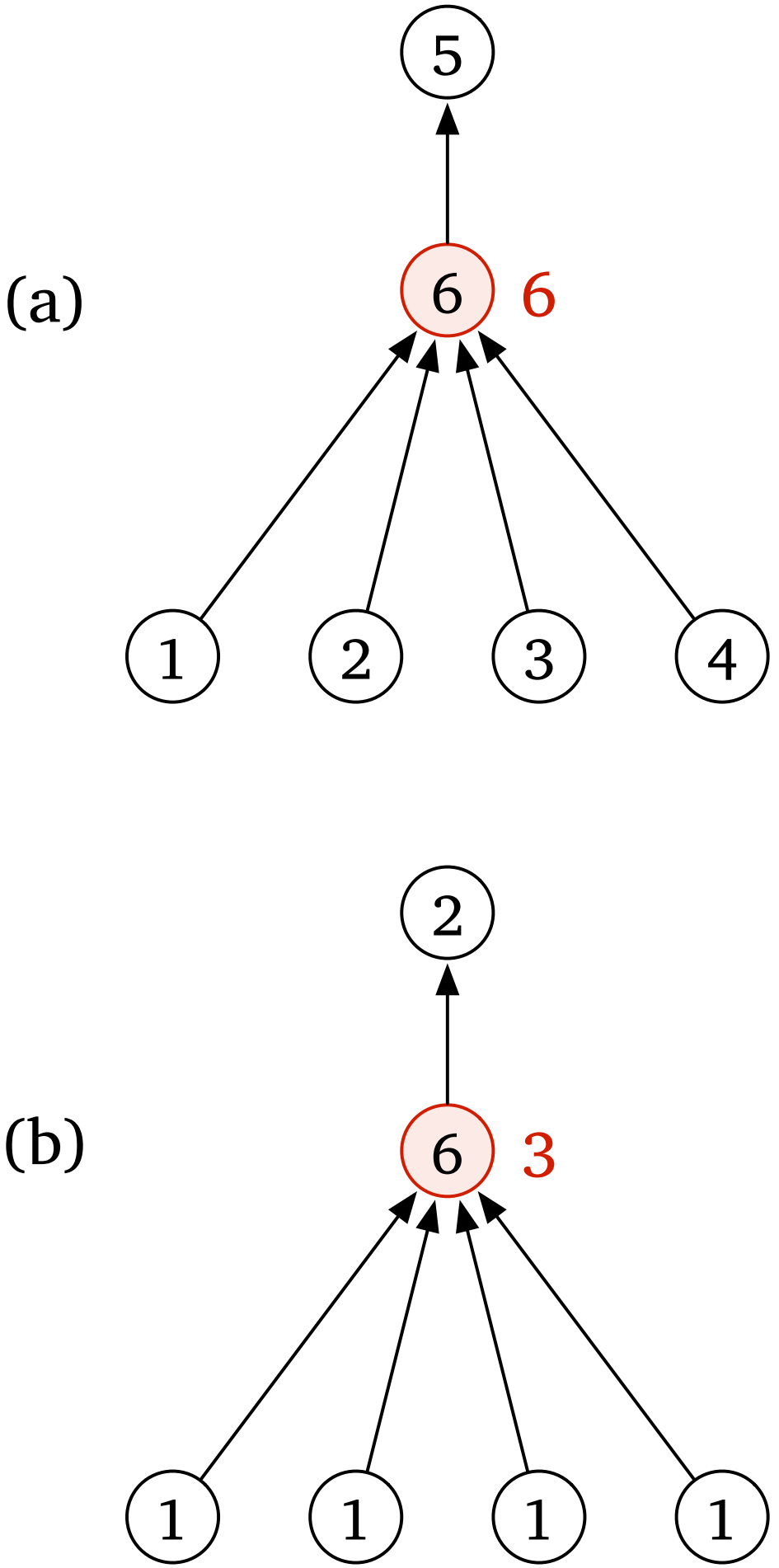
(a)
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6
6
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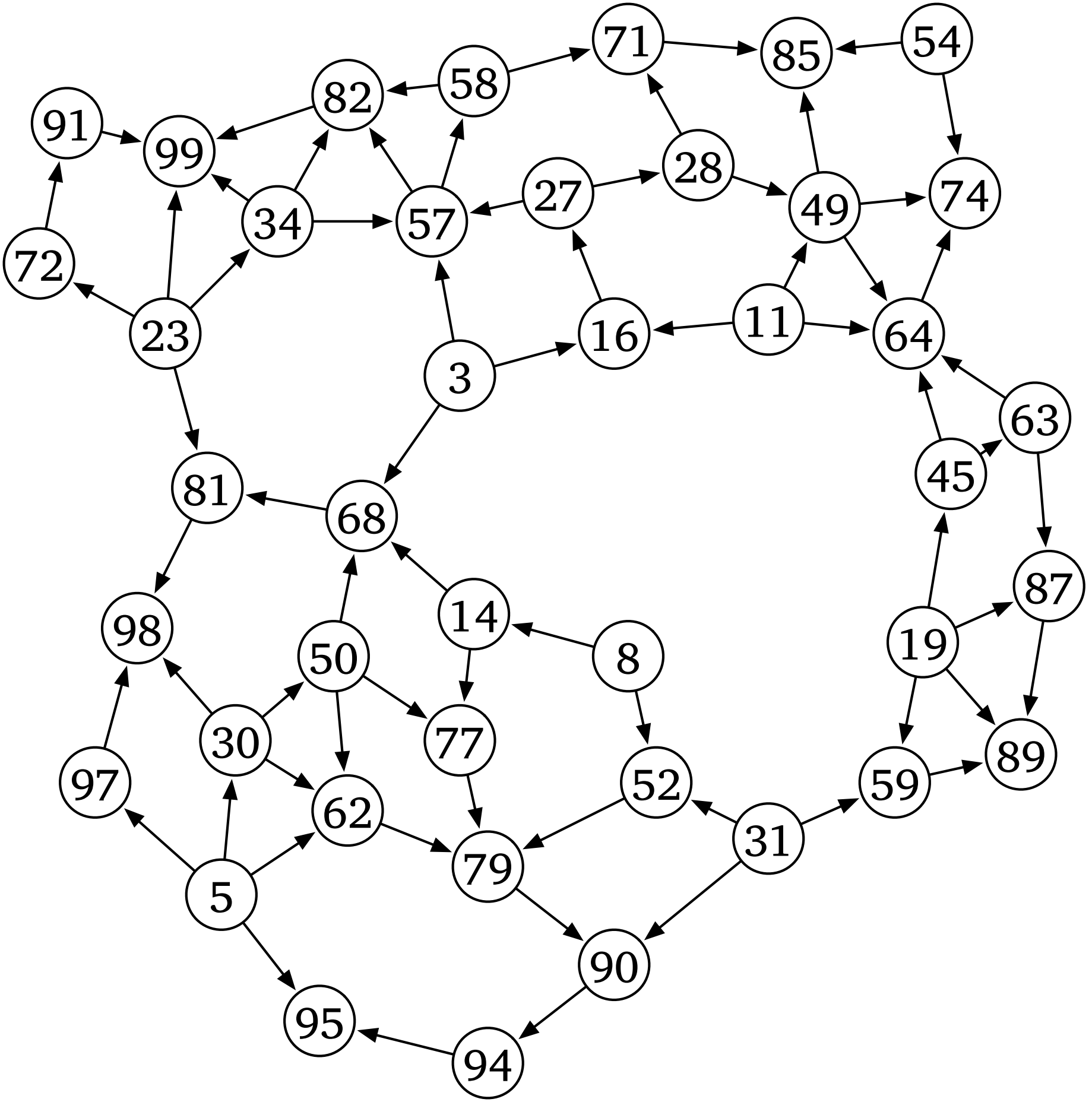
4
(b)
2
6
3
1
1
1
1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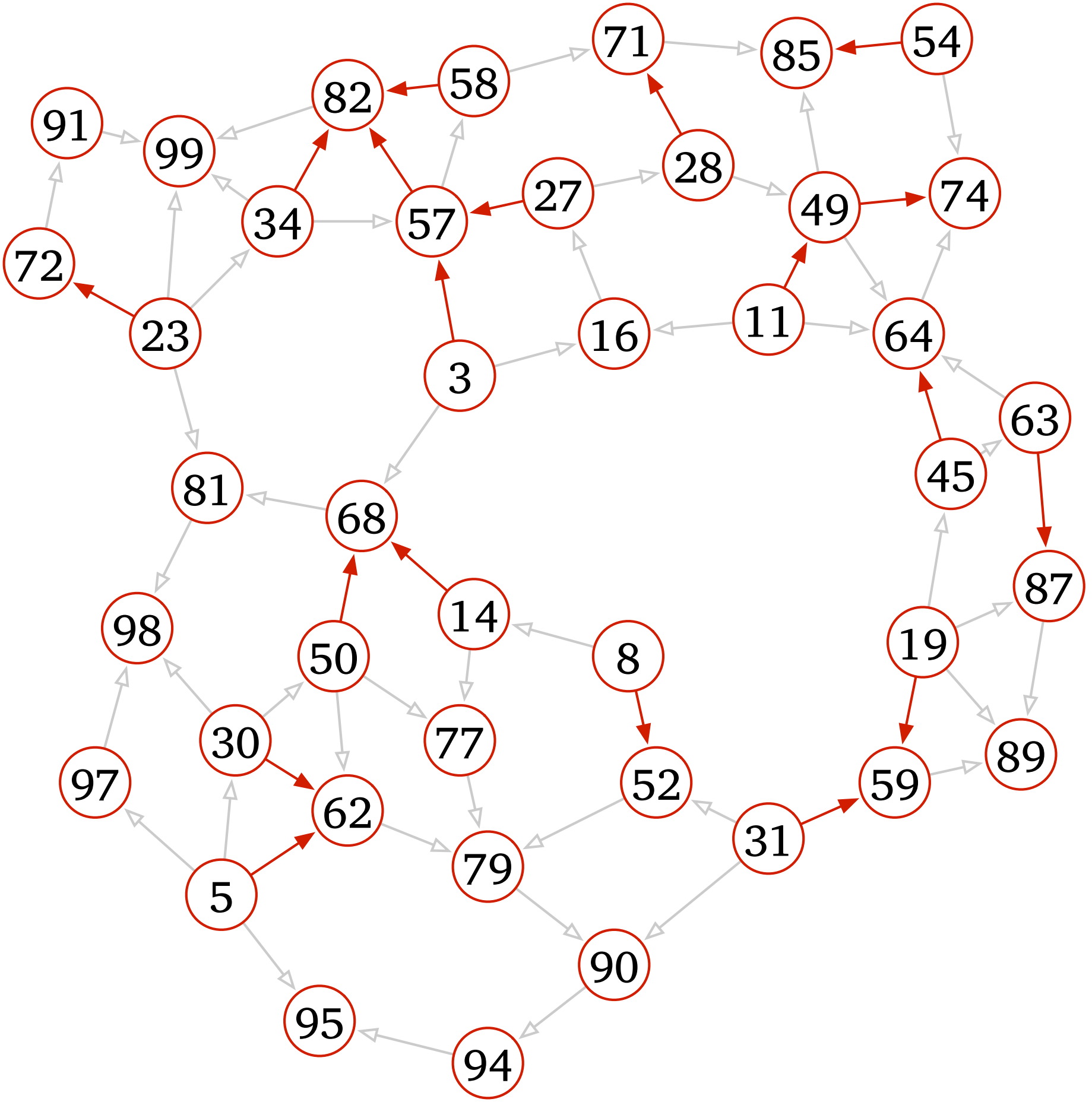
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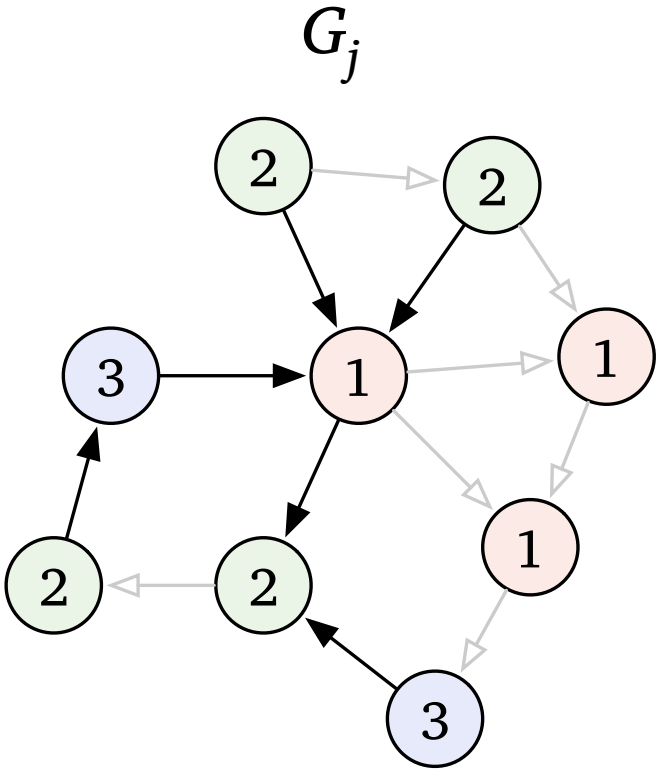
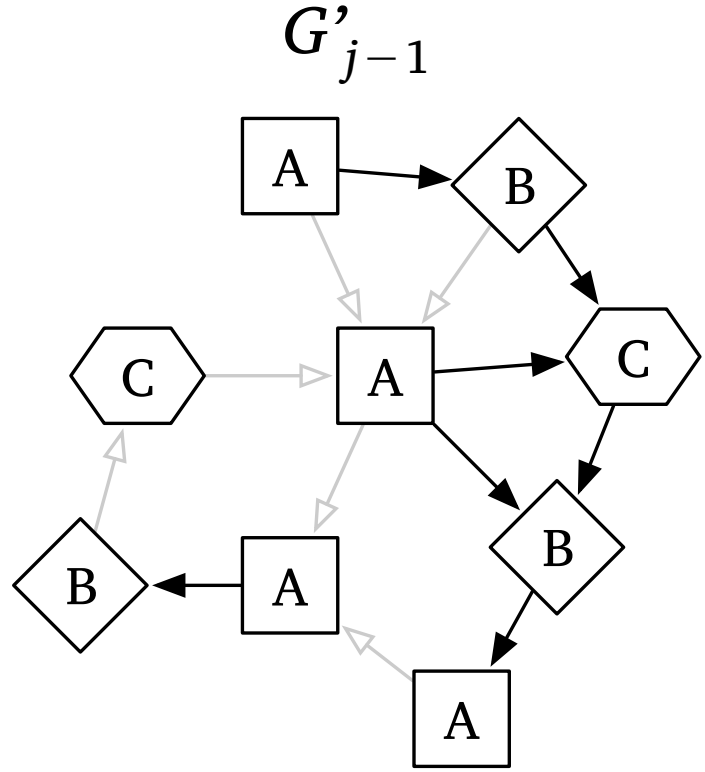
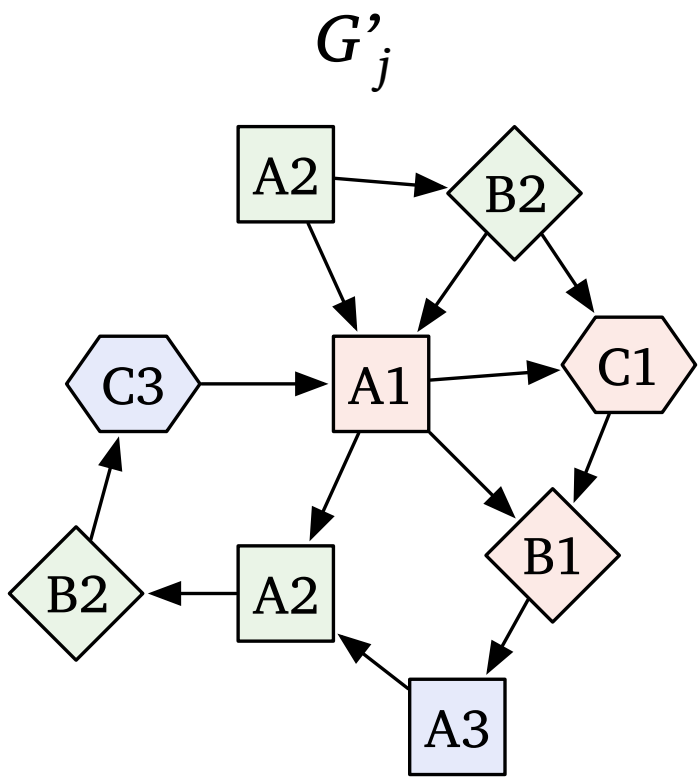


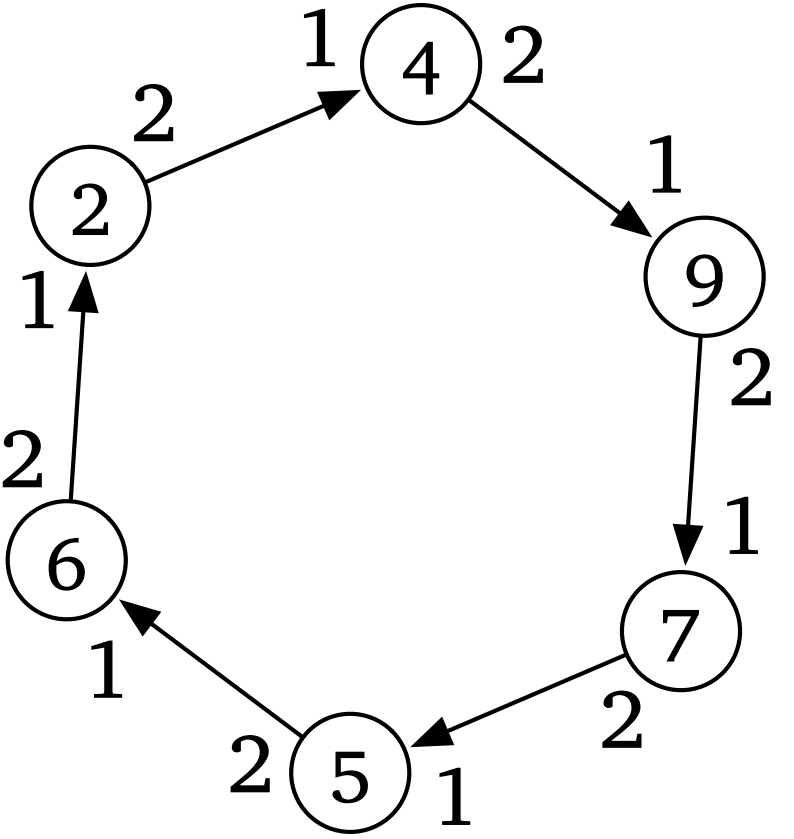
3-colouring g_j



$(\Delta+1)$-colouring g'_{j-1}



$3(\Delta+1)$-colouring h_j


4
2
9
7
5
6
2
1
1
2
1
2
2
1
2
1
2
1

f:

$\{1,2,3\} \mapsto 1$	$\{1,2,4\} \mapsto 1$	$\{1,2,5\} \mapsto 1$
$\{1,2,6\} \mapsto 2$	$\{1,2,7\} \mapsto 1$	$\{1,3,4\} \mapsto 1$
$\{1,3,5\} \mapsto 1$	$\{1,3,6\} \mapsto 1$	$\{1,3,7\} \mapsto 1$
$\{1,4,5\} \mapsto 1$	$\{1,4,6\} \mapsto 2$	$\{1,4,7\} \mapsto 1$
$\{1,5,6\} \mapsto 1$	$\{1,5,7\} \mapsto 1$	$\{1,6,7\} \mapsto 2$
$\{2,3,4\} \mapsto 2$	$\{2,3,5\} \mapsto 1$	$\{2,3,6\} \mapsto 1$
$\{2,3,7\} \mapsto 1$	$\{2,4,5\} \mapsto 2$	$\{2,4,6\} \mapsto 1$
$\{2,4,7\} \mapsto 2$	…	$\{4,5,6\} \mapsto 2$
$\{4,5,7\} \mapsto 1$	…	$\{5,6,7\} \mapsto 1$

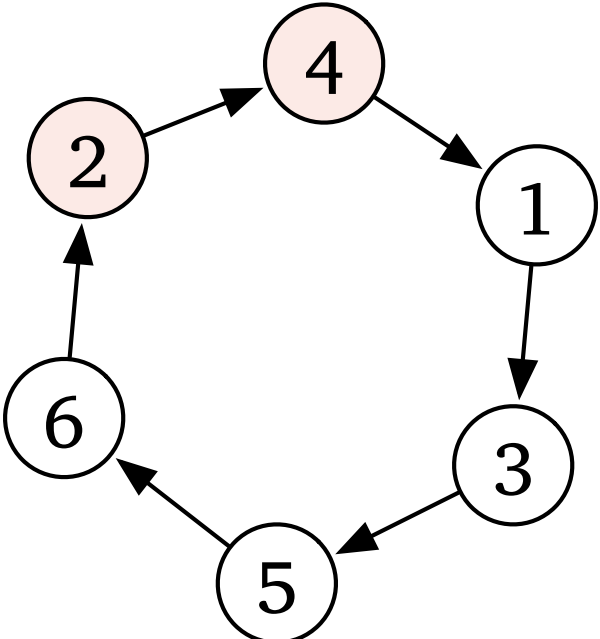
f_2:

$\{2,3\} \mapsto 1$	$\{2,4\} \mapsto 1$	$\{2,5\} \mapsto 1$
$\{2,6\} \mapsto 2$	$\{2,7\} \mapsto 1$	$\{3,4\} \mapsto 1$
$\{3,5\} \mapsto 1$	$\{3,6\} \mapsto 1$	$\{3,7\} \mapsto 1$
$\{4,5\} \mapsto 1$	$\{4,6\} \mapsto 2$	$\{4,7\} \mapsto 1$
$\{5,6\} \mapsto 1$	$\{5,7\} \mapsto 1$	$\{6,7\} \mapsto 2$

$X_2 = \{2,3,4,5,7\}$, monochromatic in f_2

$W_2 = \{2,4,5,7\}$, almost monochromatic in f

$W = \{1,2,4,5,7\}$, almost monochromatic in f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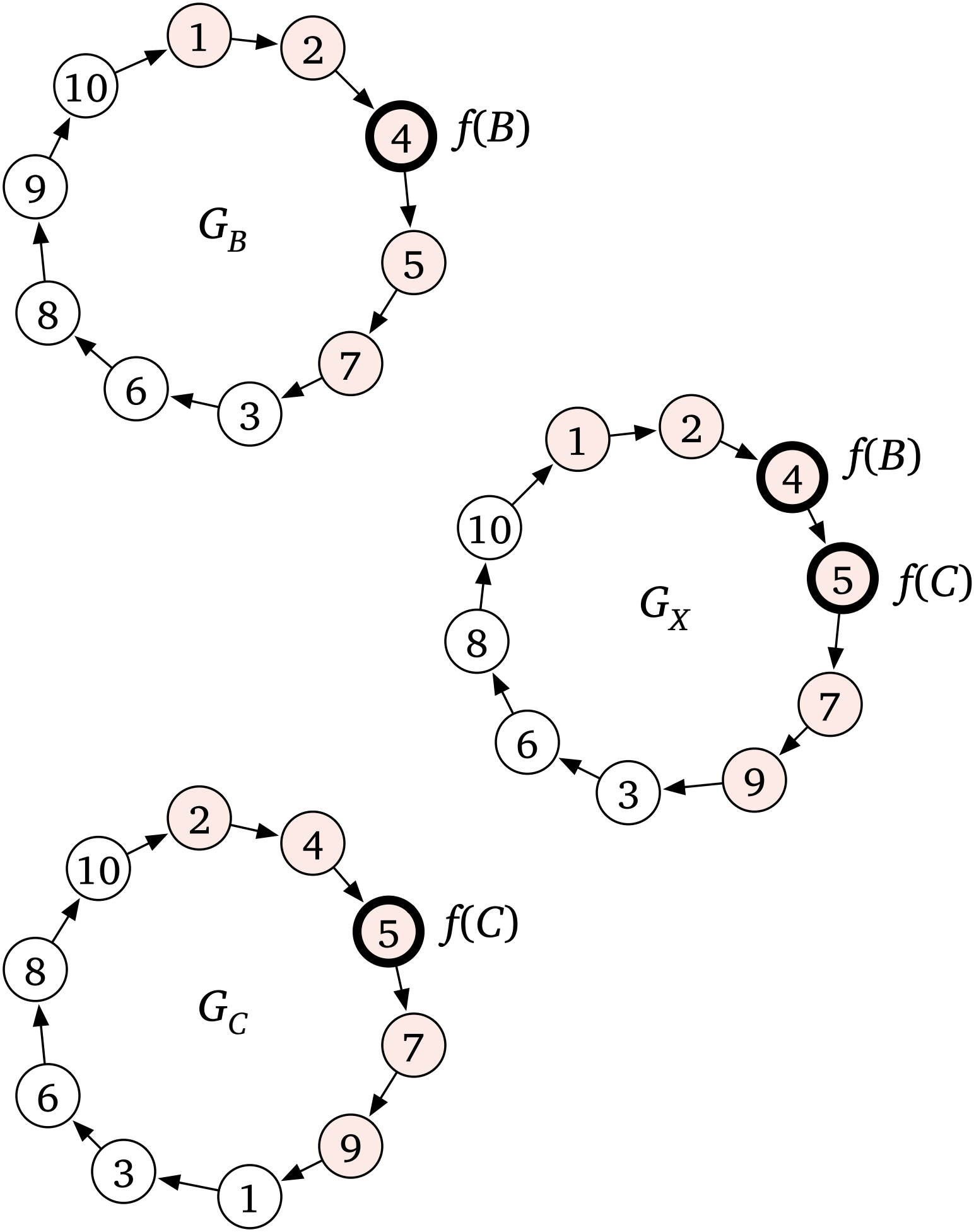
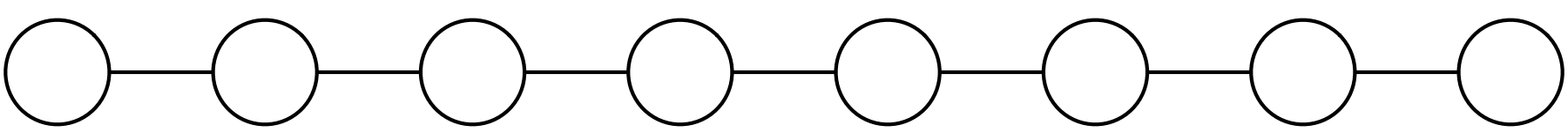
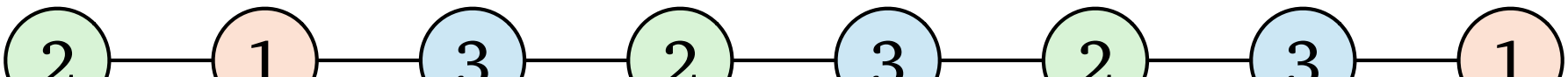
1
6
3
5


1
2
4
f(B)
5
7
3
6
8
9
10
G_B
1
2
4
f(B)
5
f(C)
7
9
3
6
8
10
G_X
2
4
5
f(C)
7
9
1
3
6
8
10
G_C

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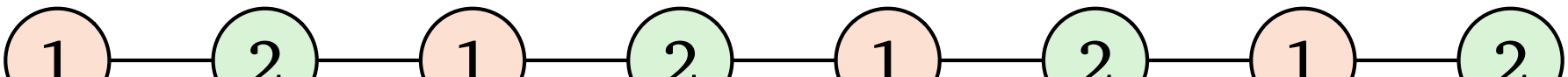
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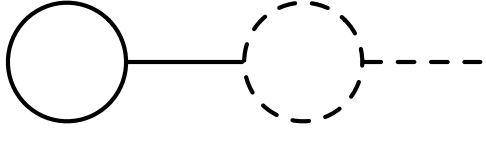

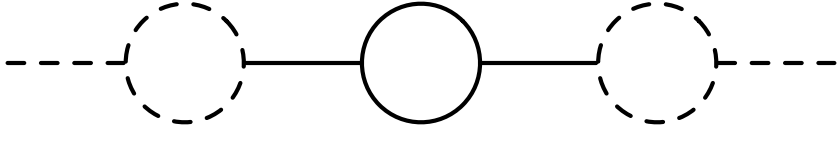

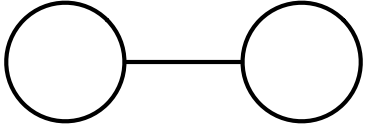

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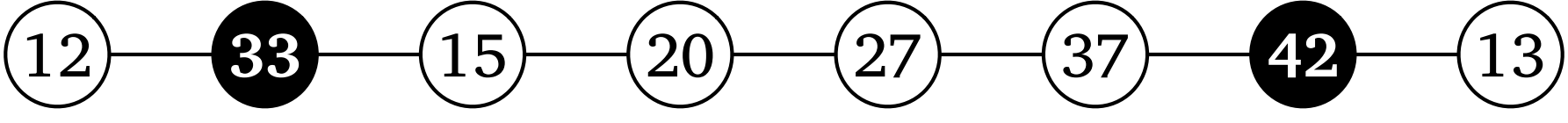
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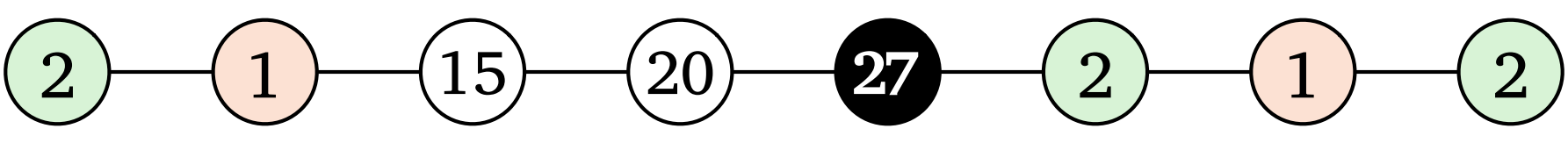
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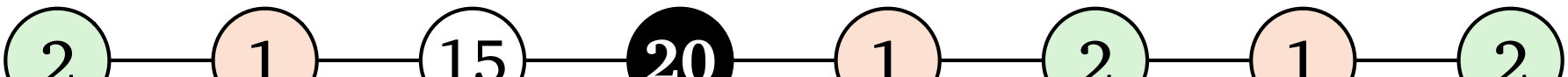
2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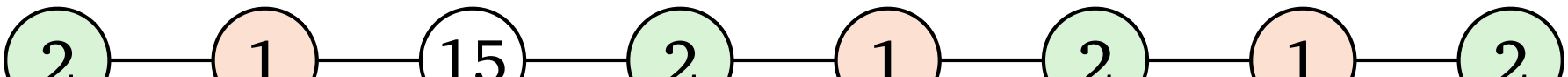
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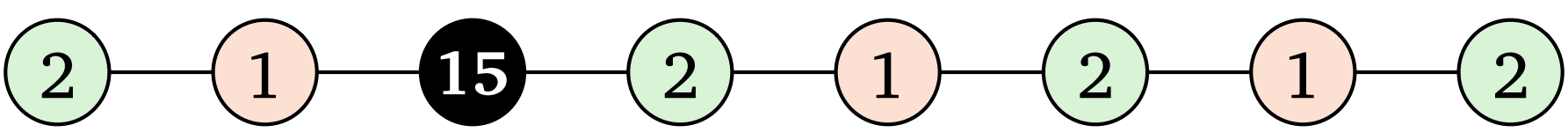
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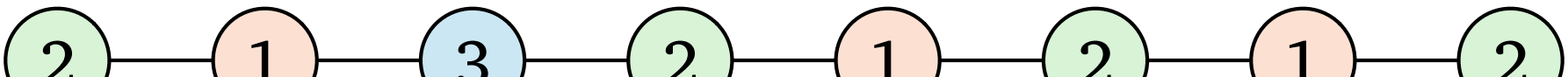
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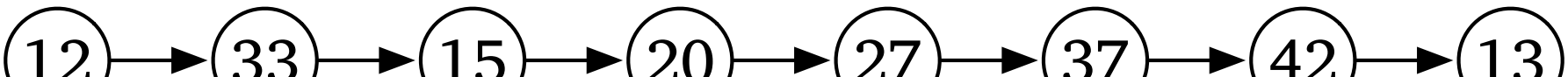
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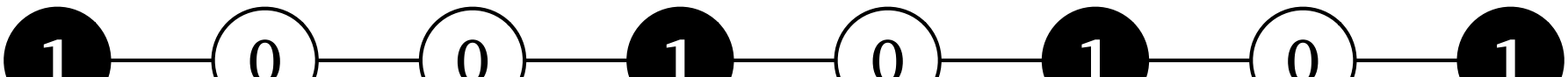
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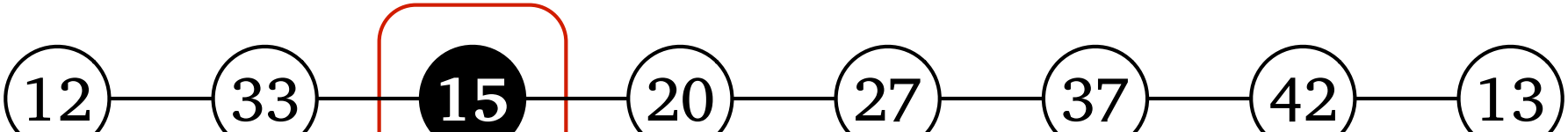
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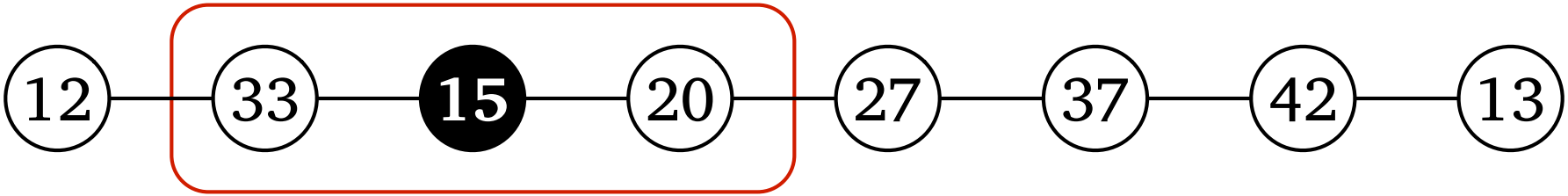
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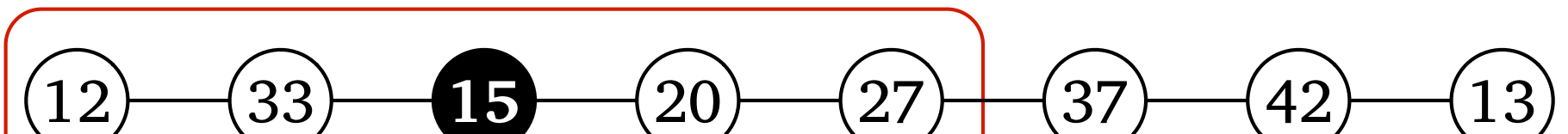
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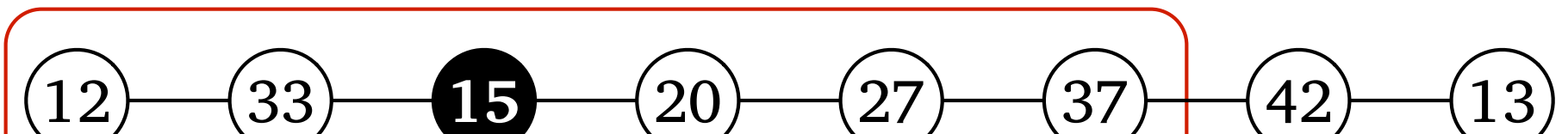
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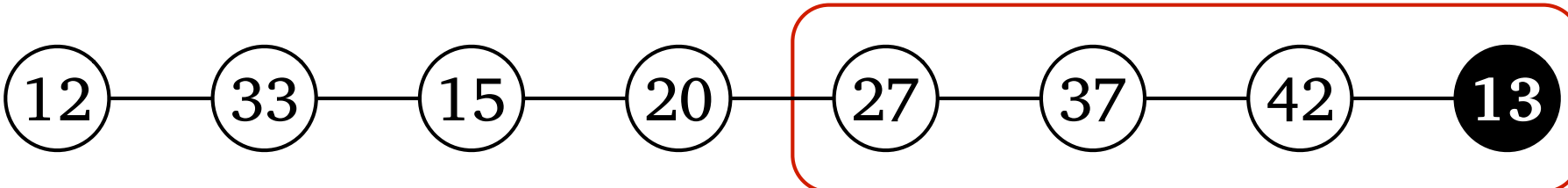
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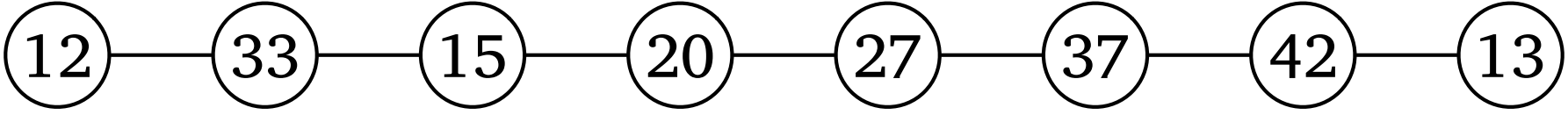
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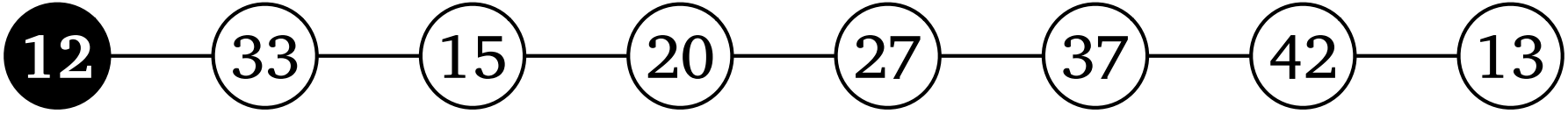
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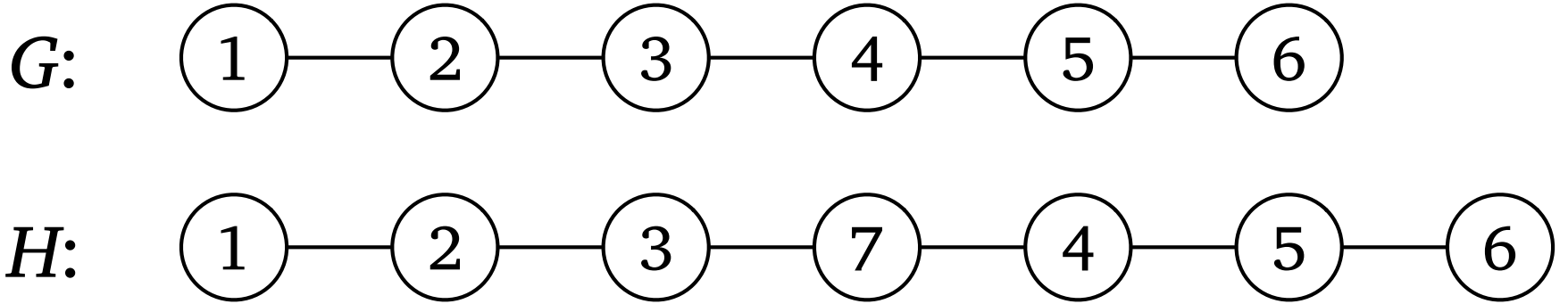
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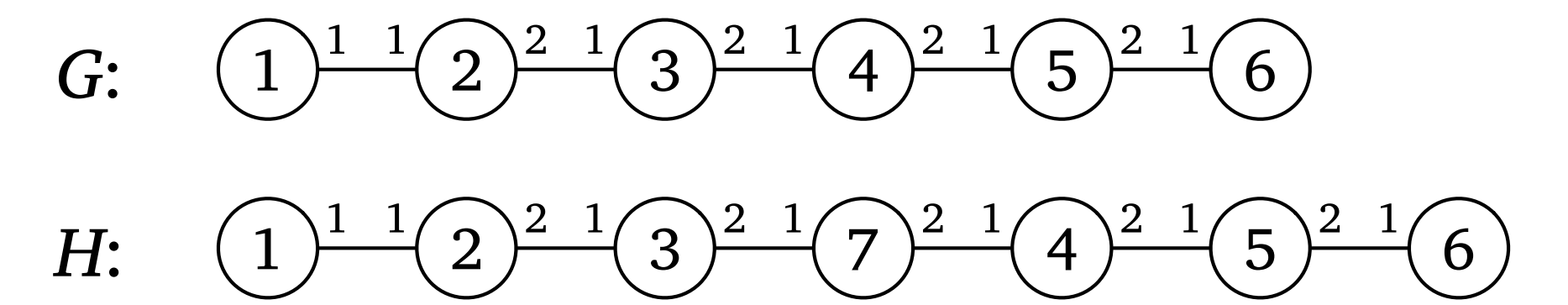
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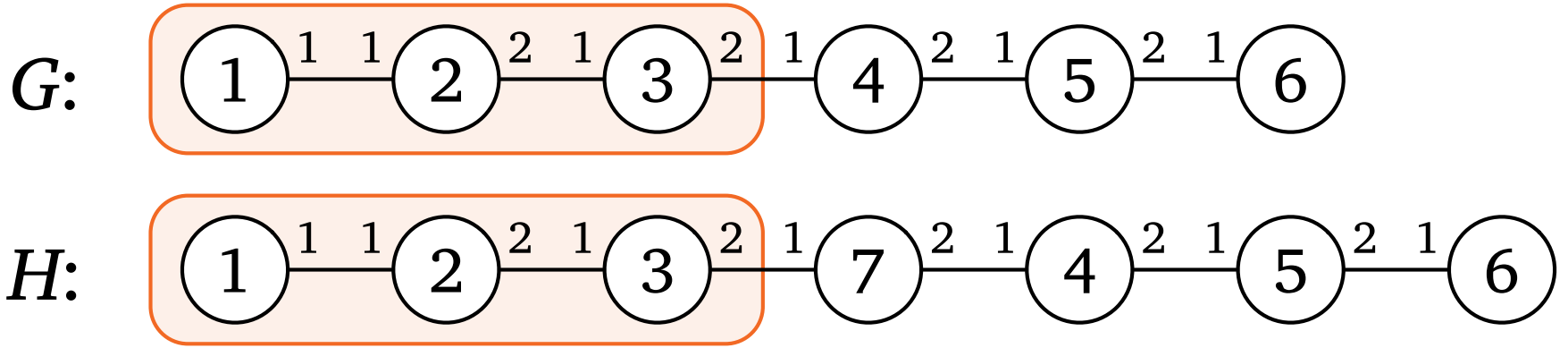
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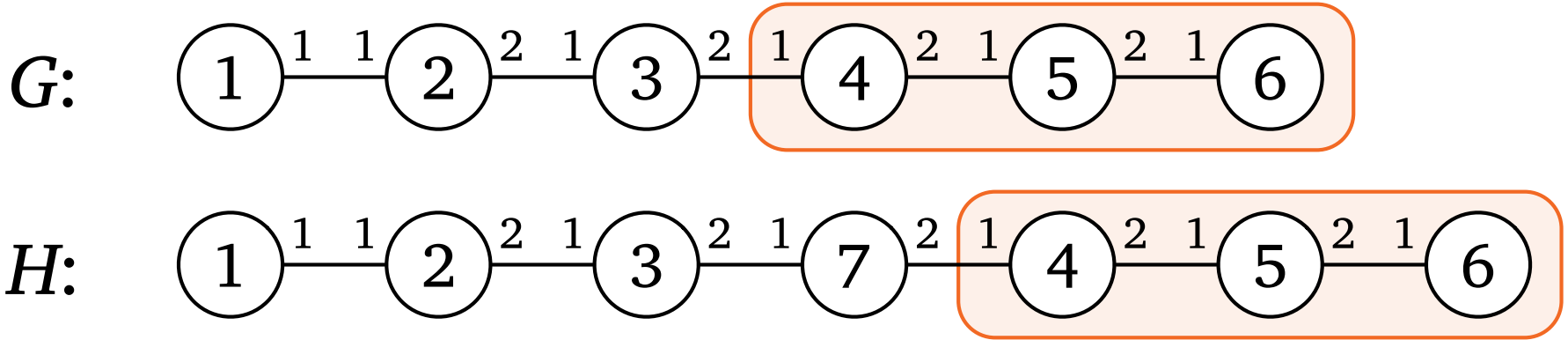
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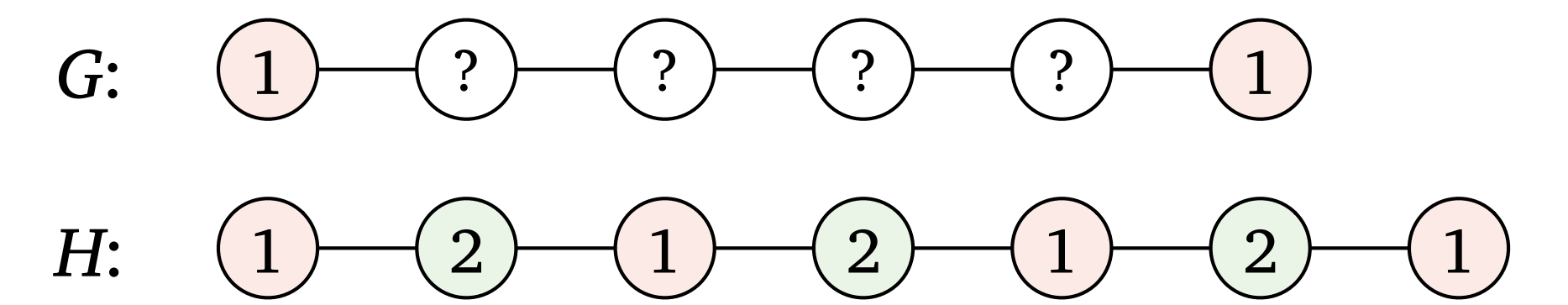
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H:
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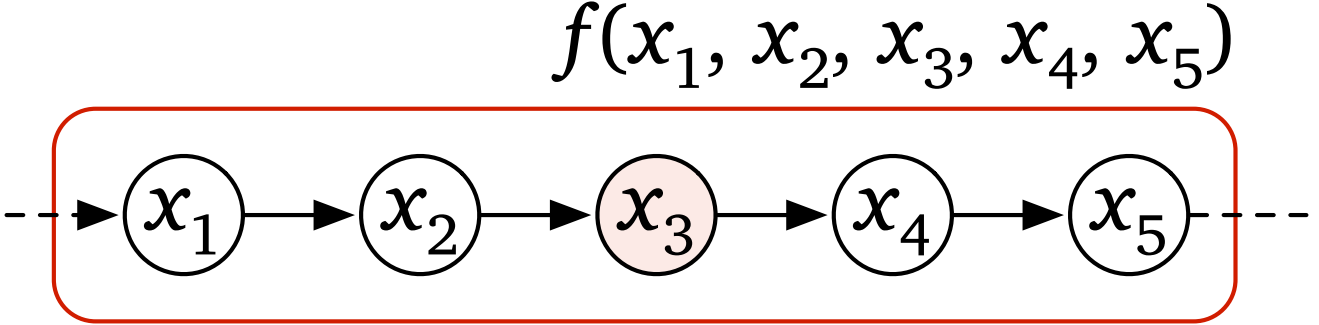
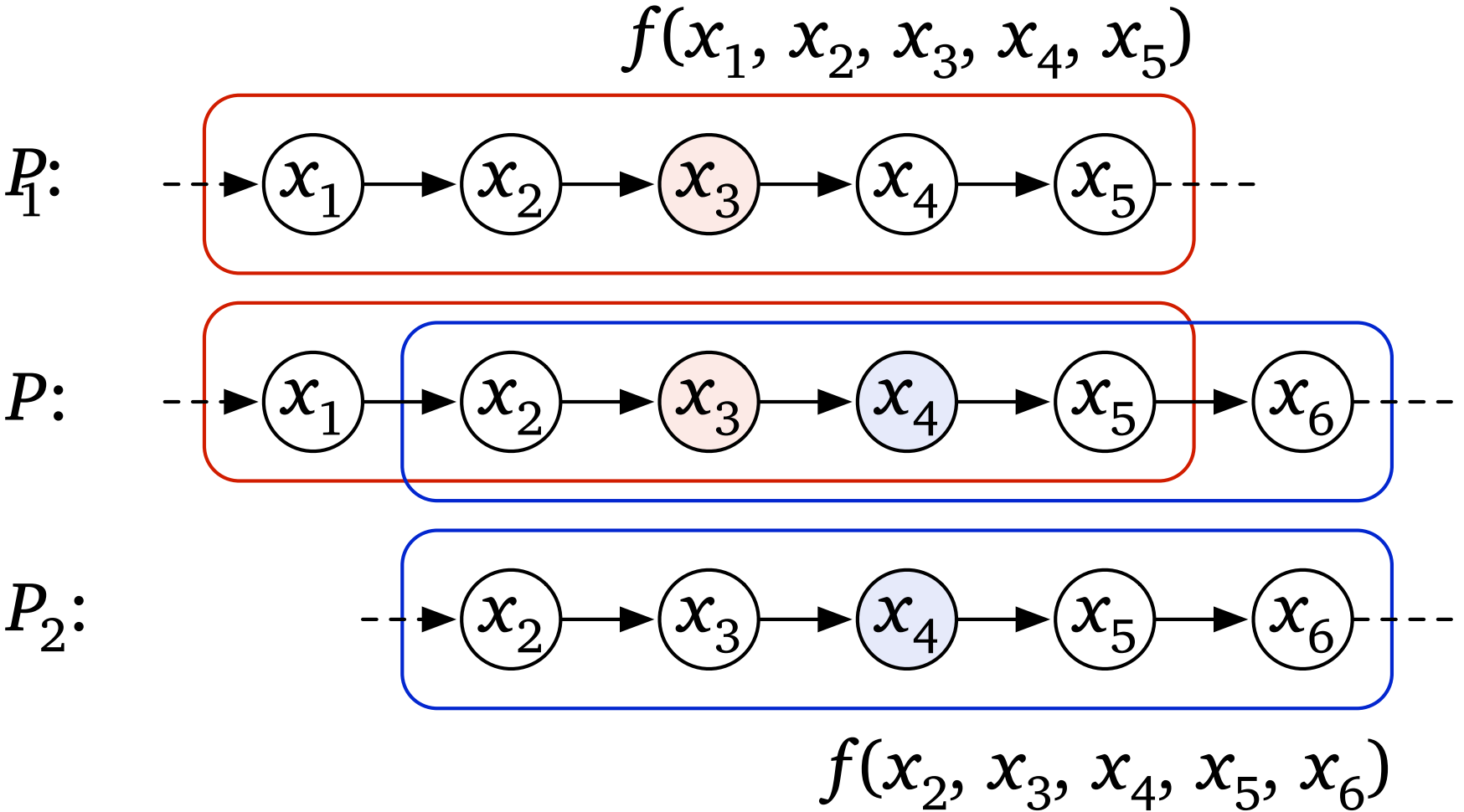
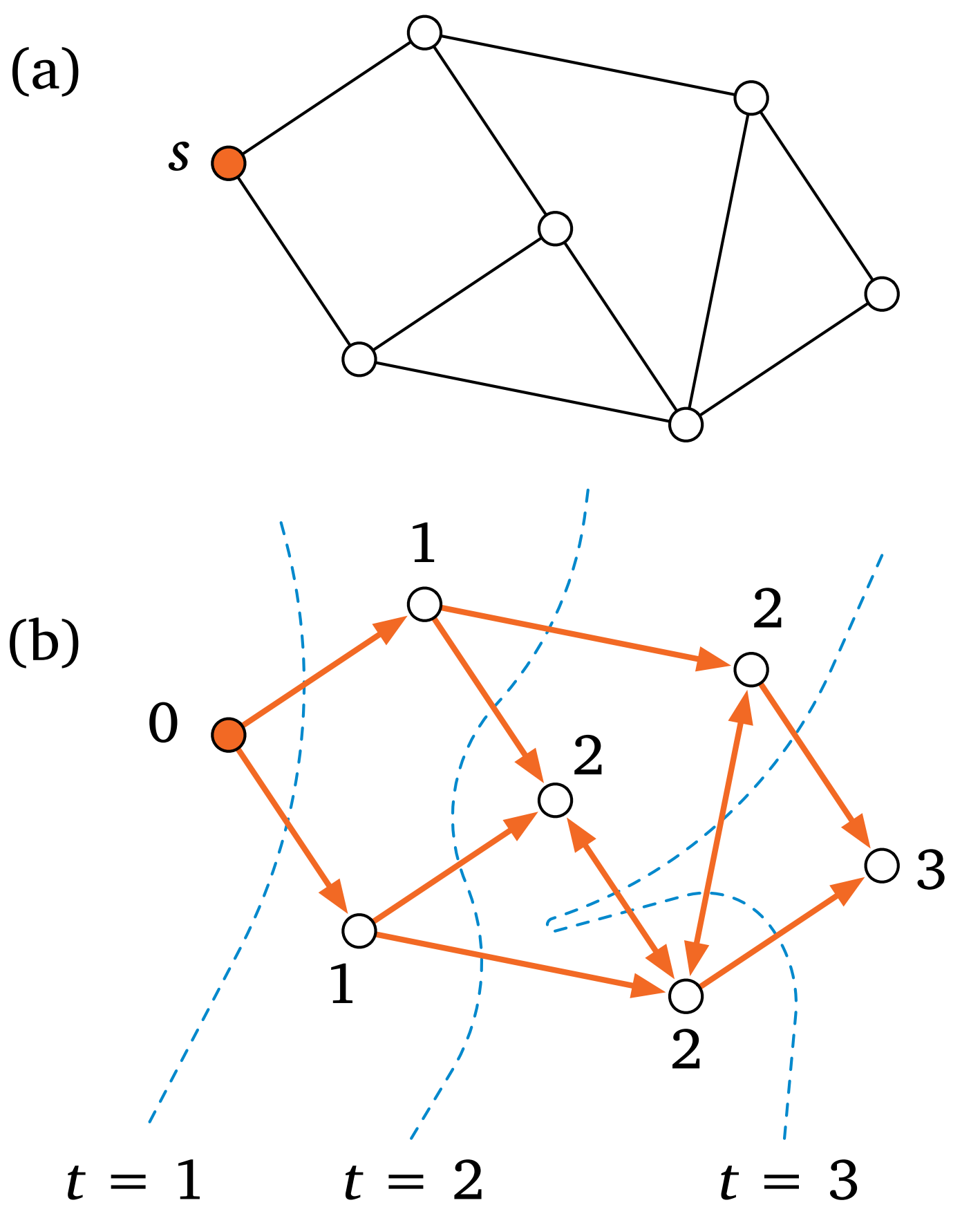
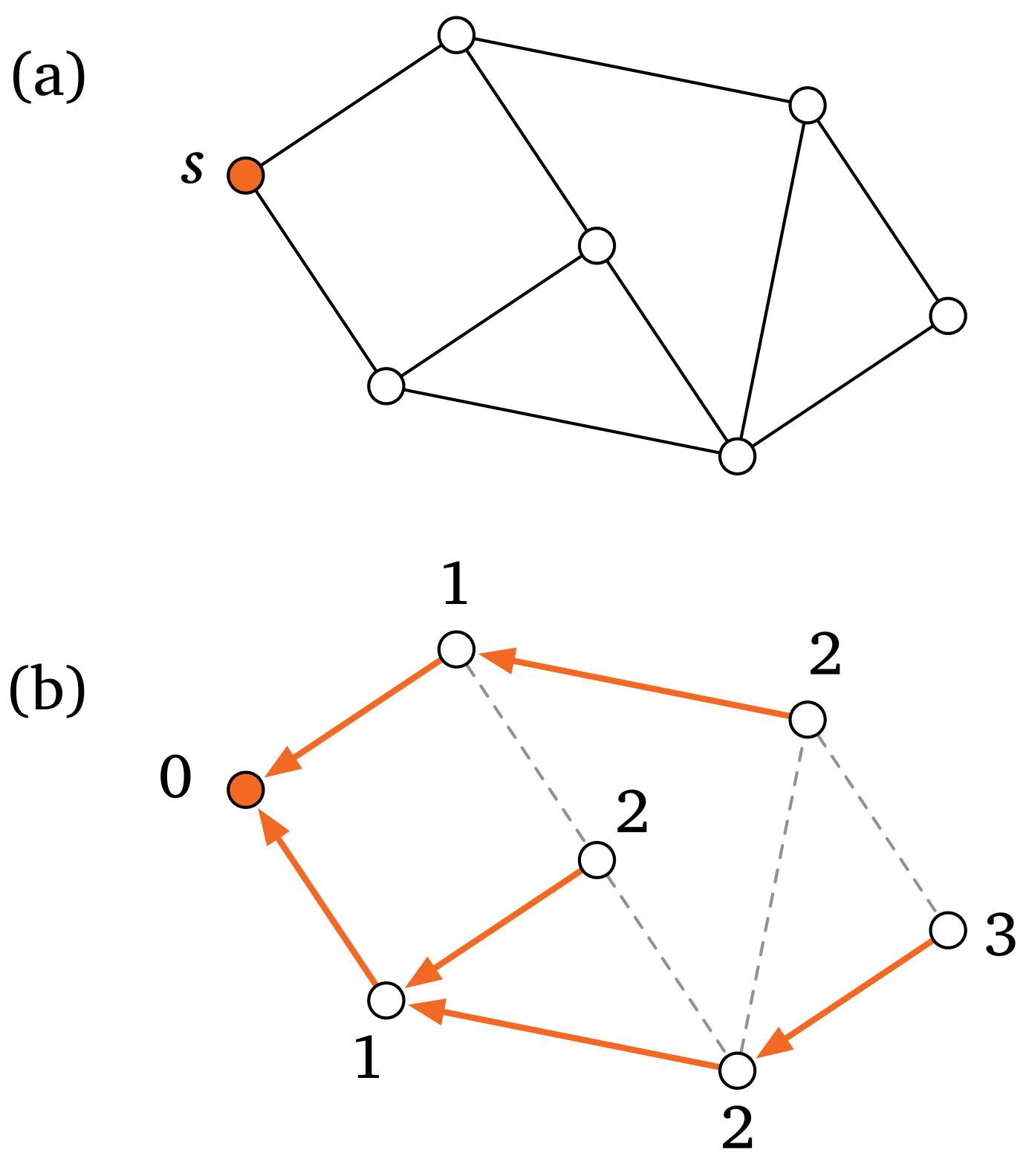
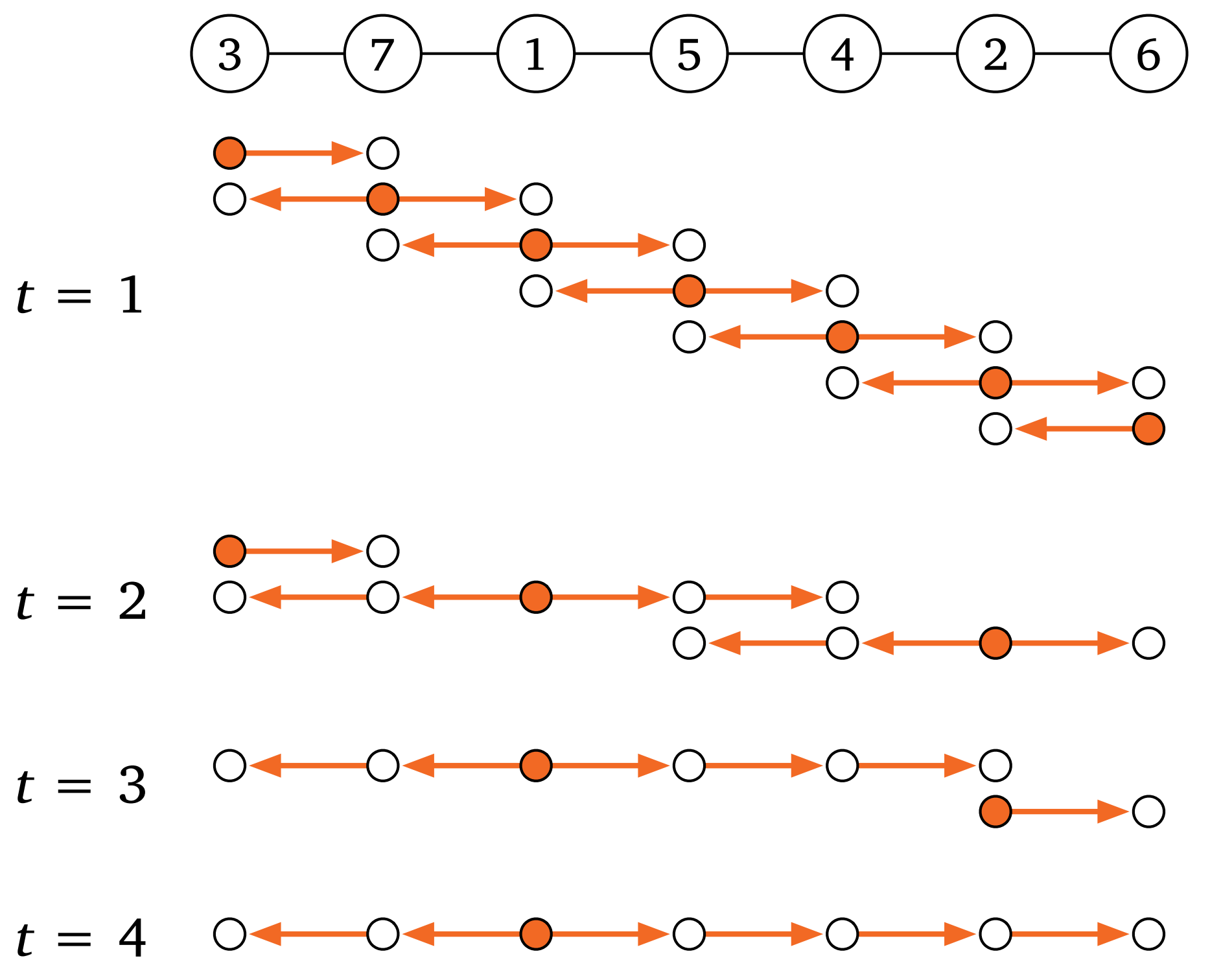

$f(x_1, x_2, x_3, x_4, x_5)$
x_1
x_2
x_3
x_4
x_5

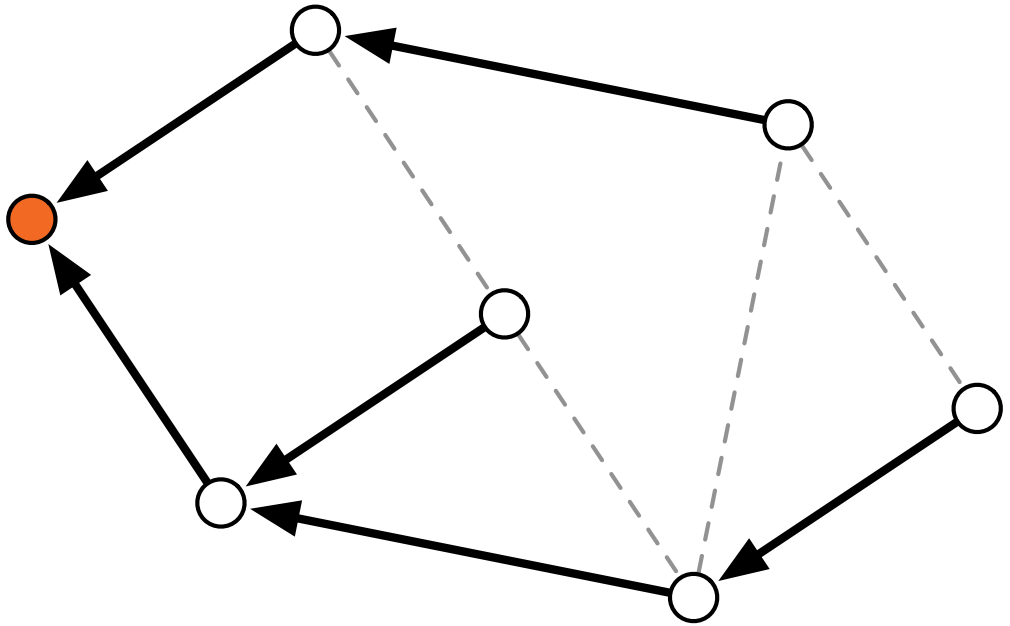

$f(x_1, x_2, x_3, x_4, x_5)$
P_1:
x_1
x_2
x_3
x_4
x_5
P:
x_1
x_2
x_3
x_4
x_5
x_6
P_2:
x_2
x_3
x_4
x_5
x_6
$f(x_2, x_3, x_4, x_5, x_6)$


(a)
s
(b)
0
1
2
2
1
2
3
t = 1
t = 2
t = 3

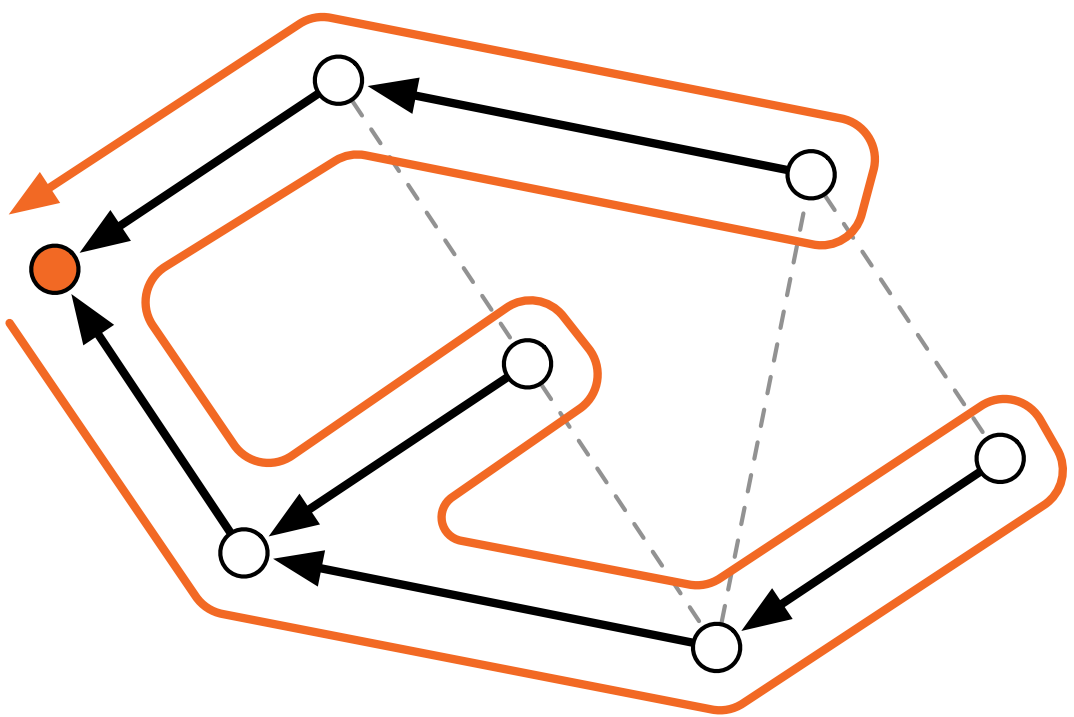

(a)
s
(b)
1
2
0
2
3
1
2

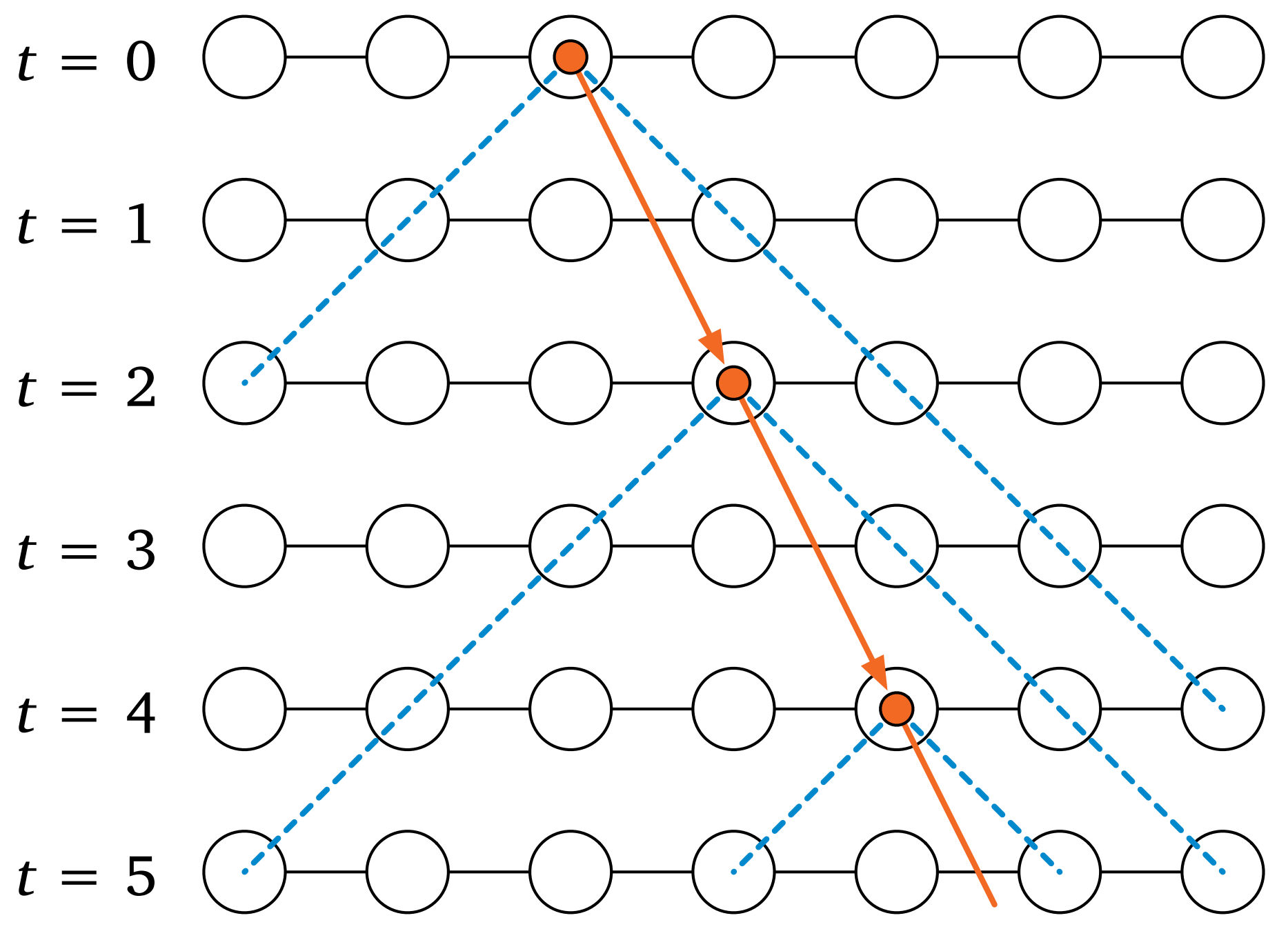

3
7
1
5
4
2
6
t = 1
t = 2
t = 3
t = 4

(a)



(b)

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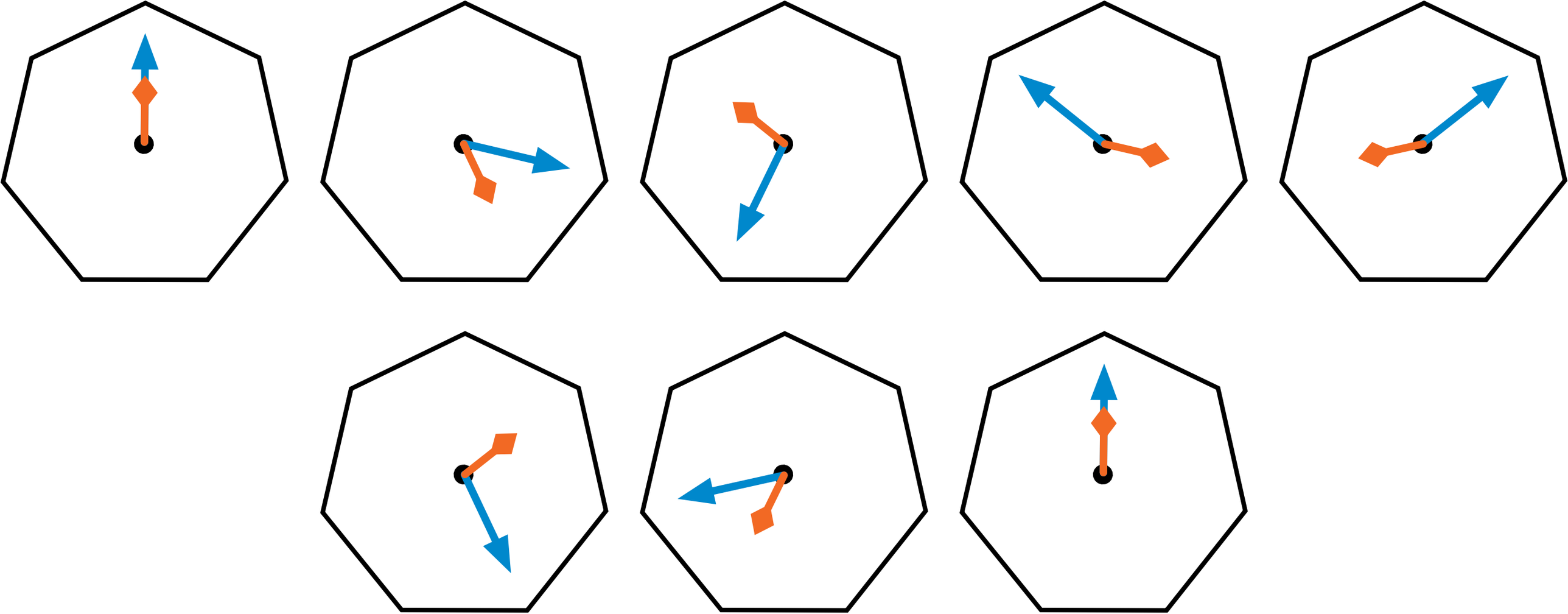
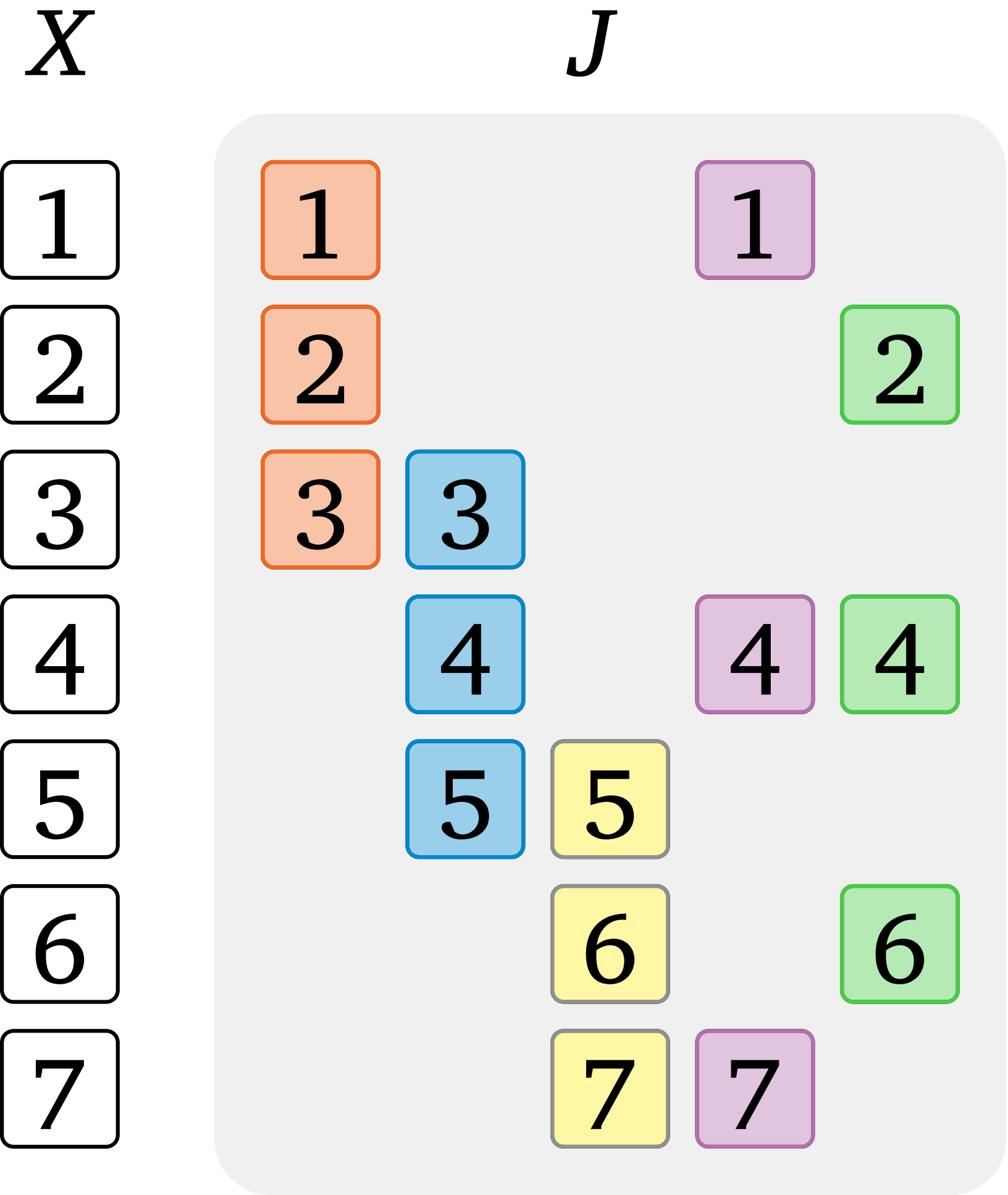
$t = 1$
$t = 2$
$t = 3$
$t = 4$
$t = 5$




X
J
1
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3
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3
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5
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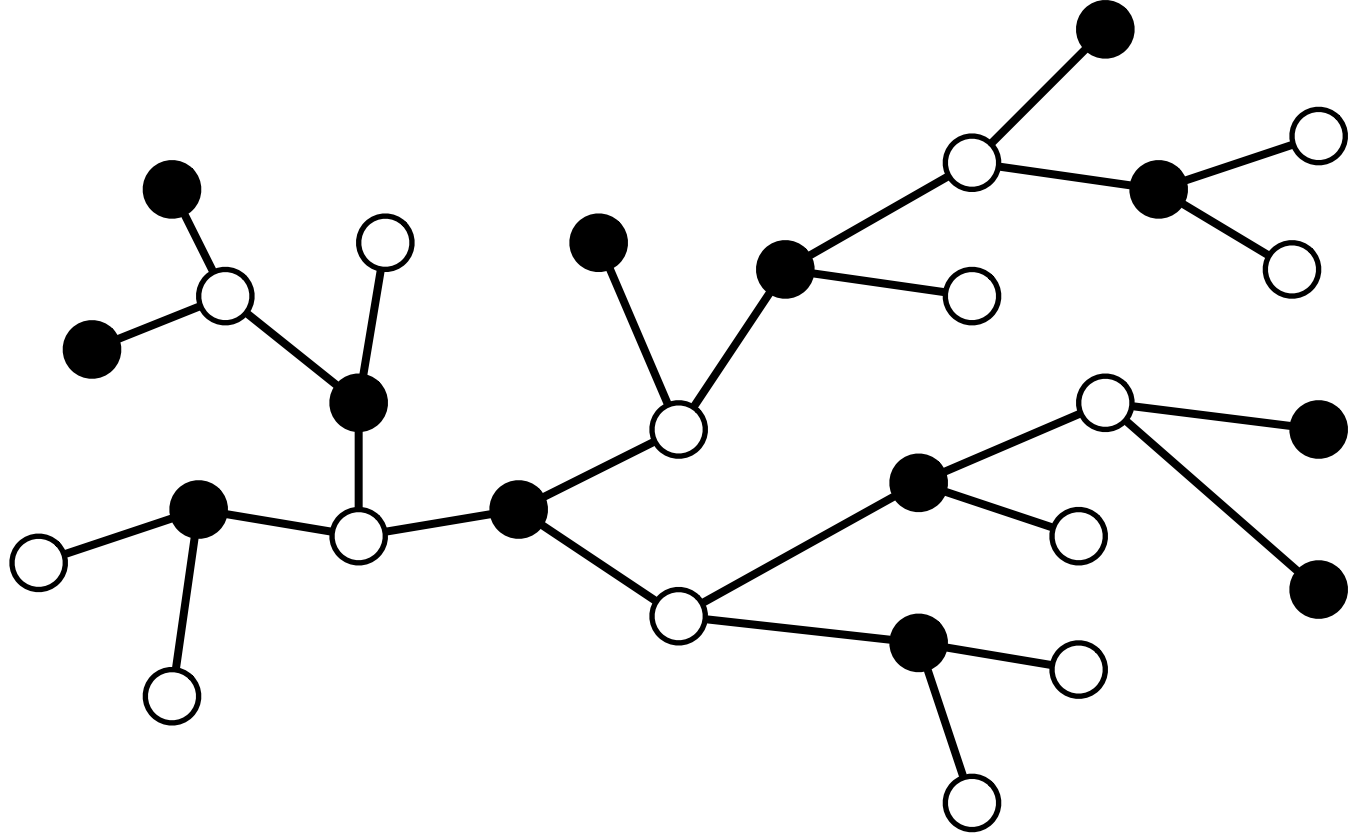
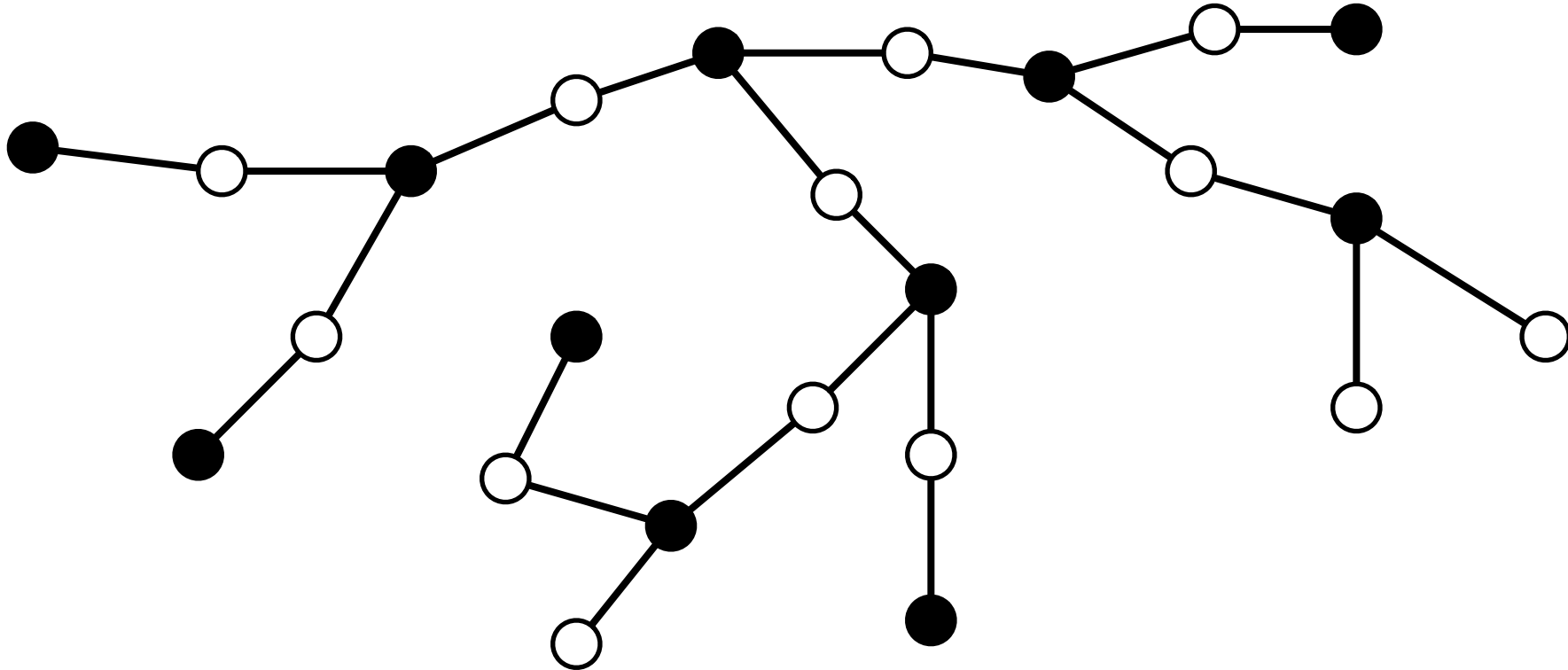
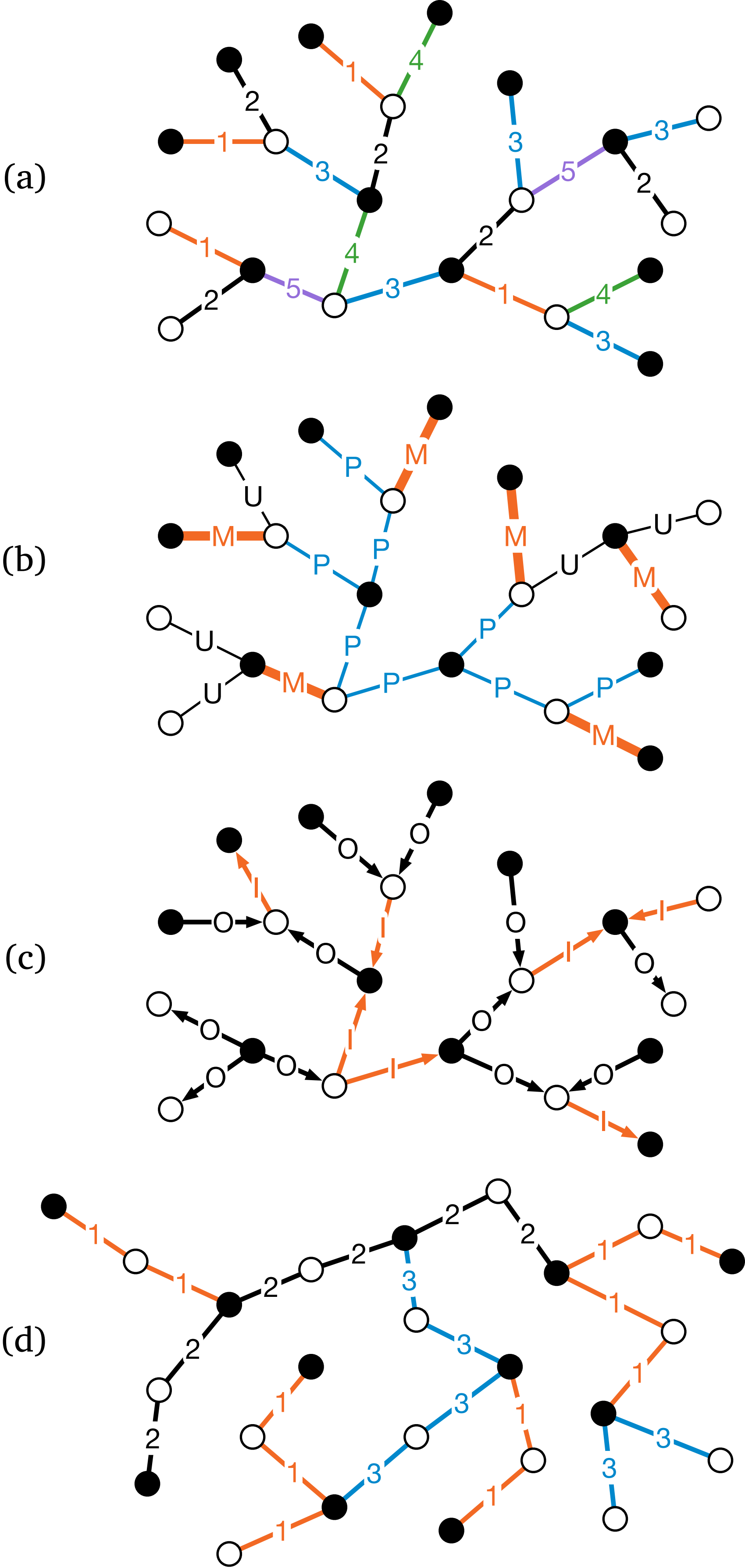
1
4
4
2
6
6
7
7

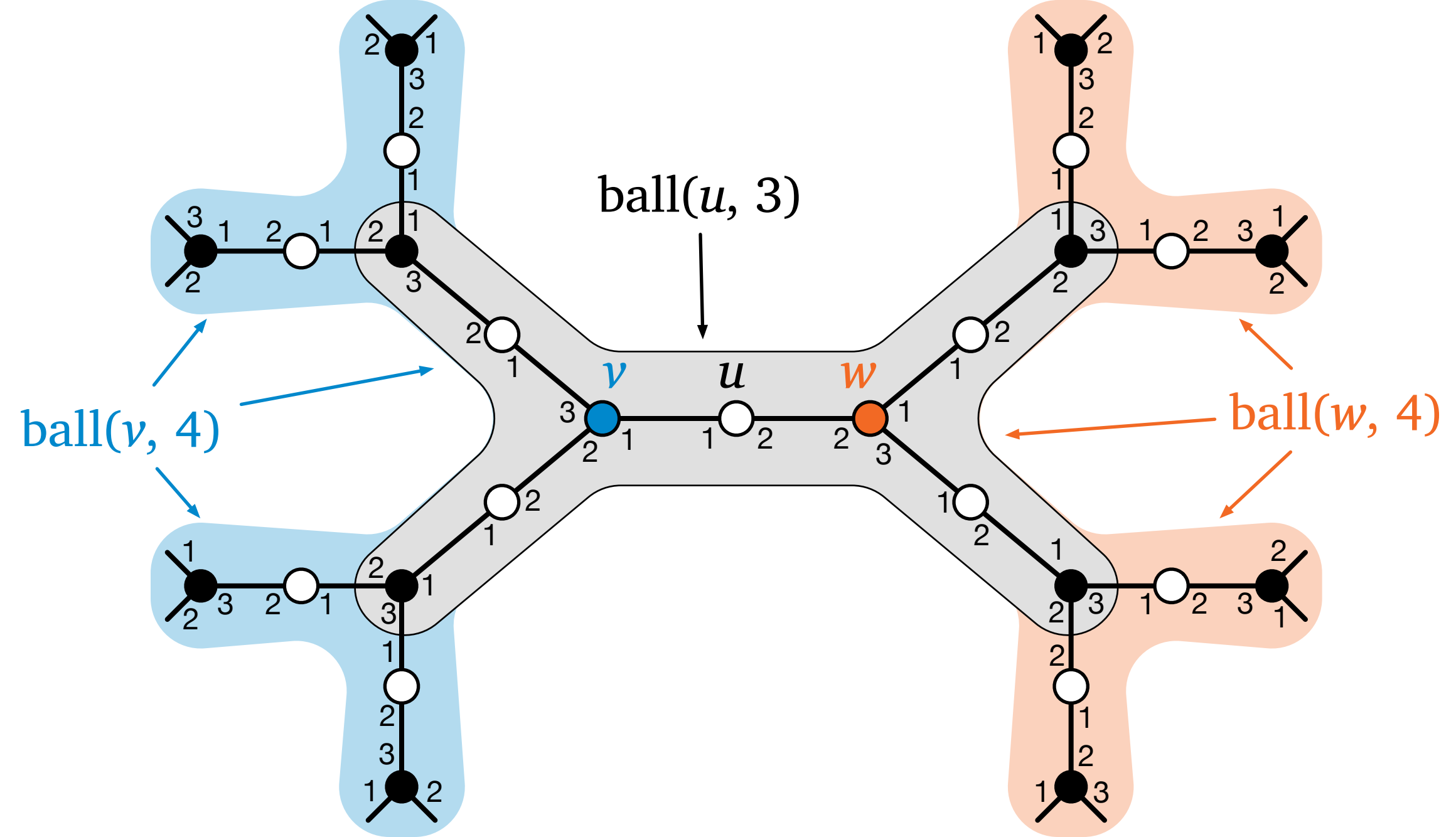
(a)

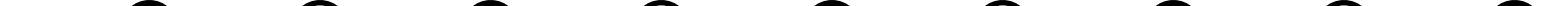


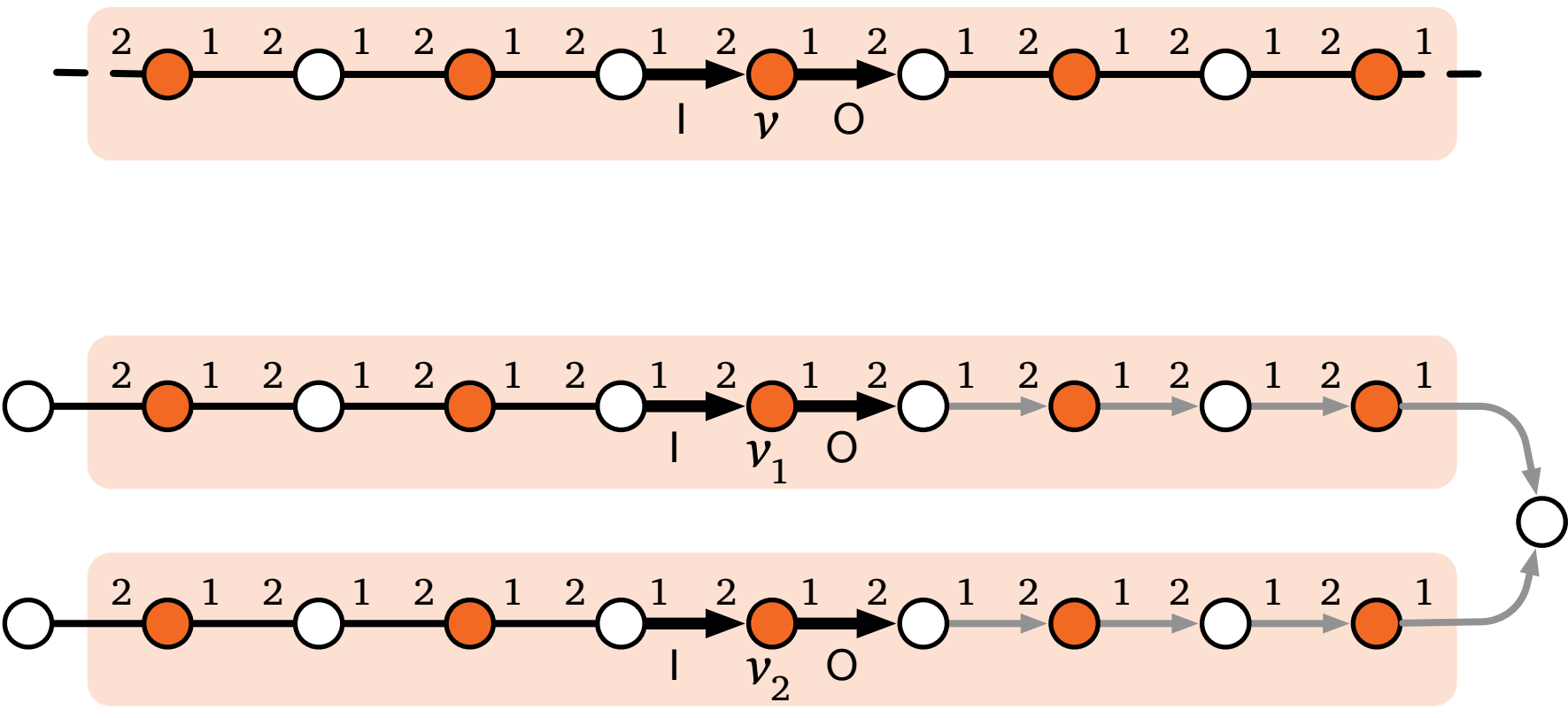
(b)



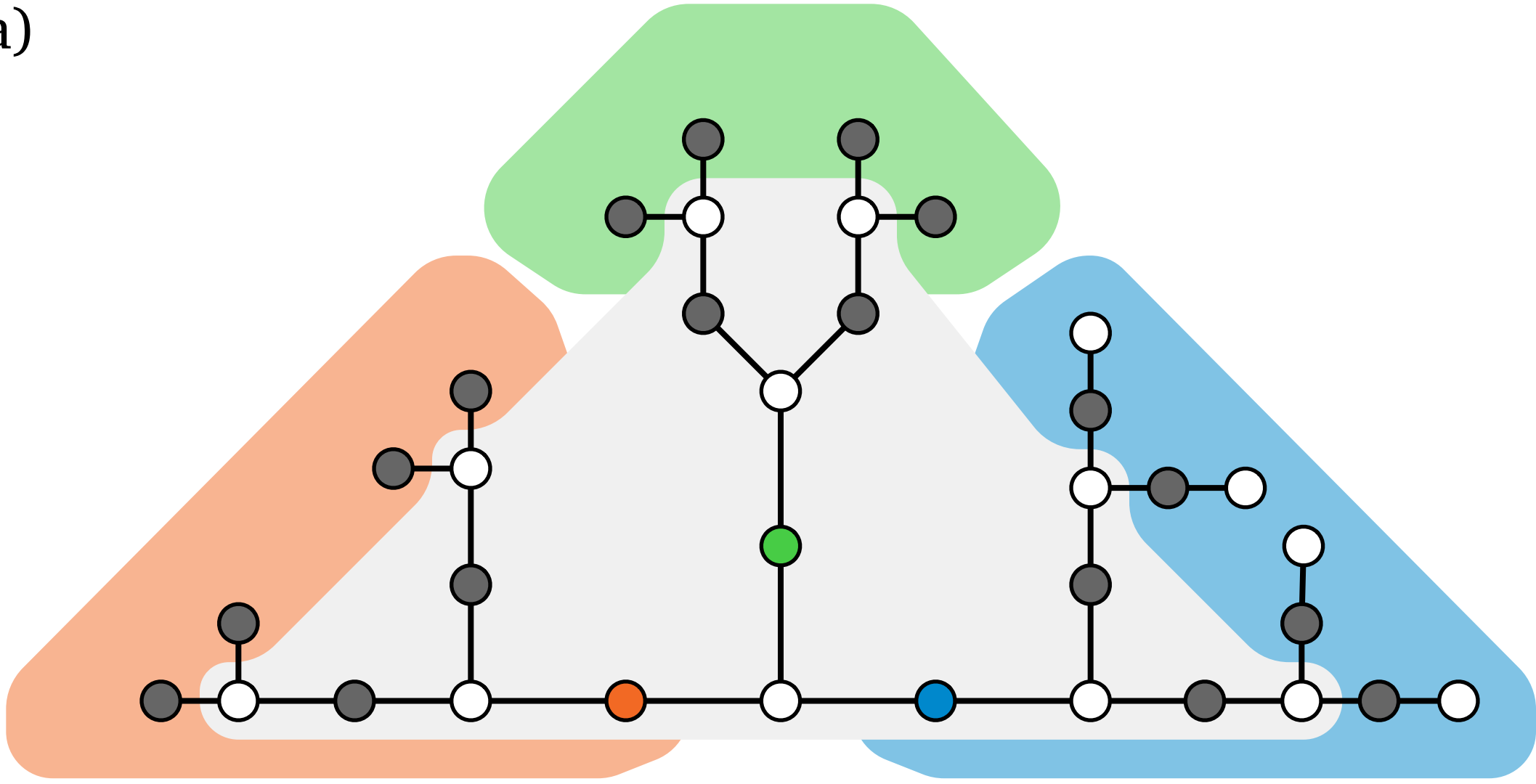

(a)
(b)
(c)
(d)


ball(u, 3)
ball(v, 4)
ball(w, 4)
v
u
w

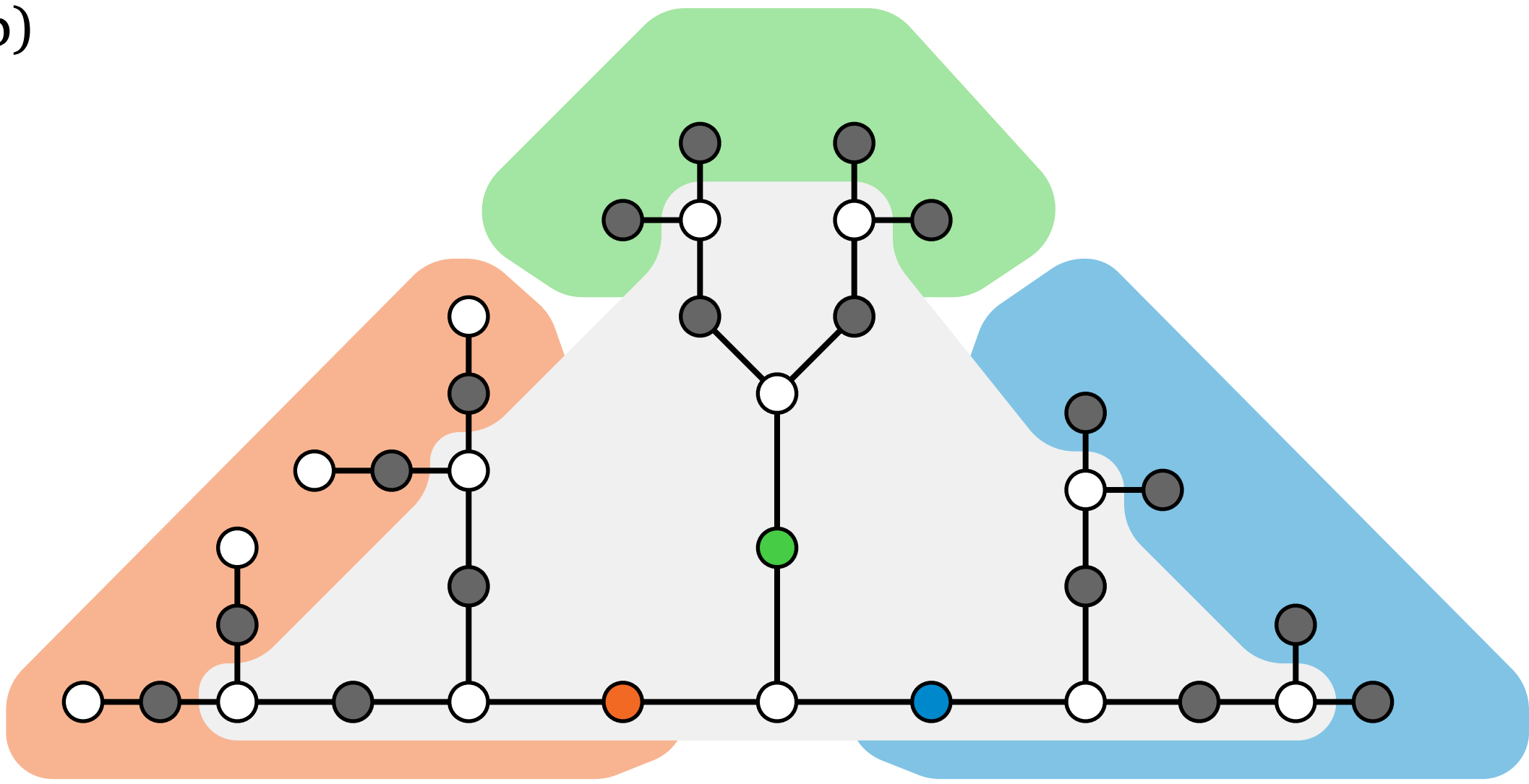



2 1 2 1 2 1 2 1 2 1 2 1 2 1 2 1 2 1 2 1
I
v
O
2 1 2 1 2 1 2 1 2 1 2 1 2 1 2 1 2 1 2 1
I
v1
O
2 1 2 1 2 1 2 1 2 1 2 1 2 1 2 1 2 1 2 1
I
v2
O

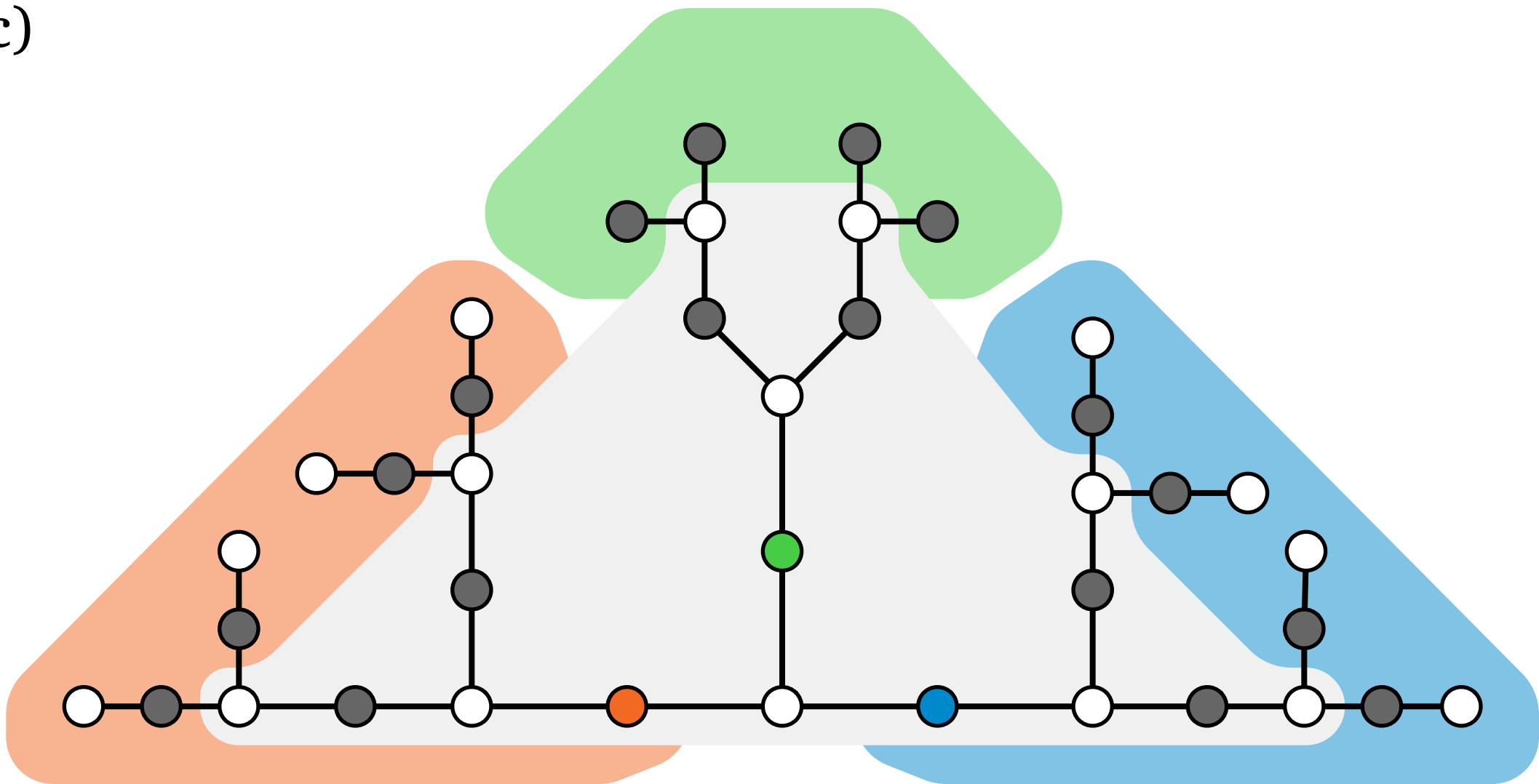
(a)

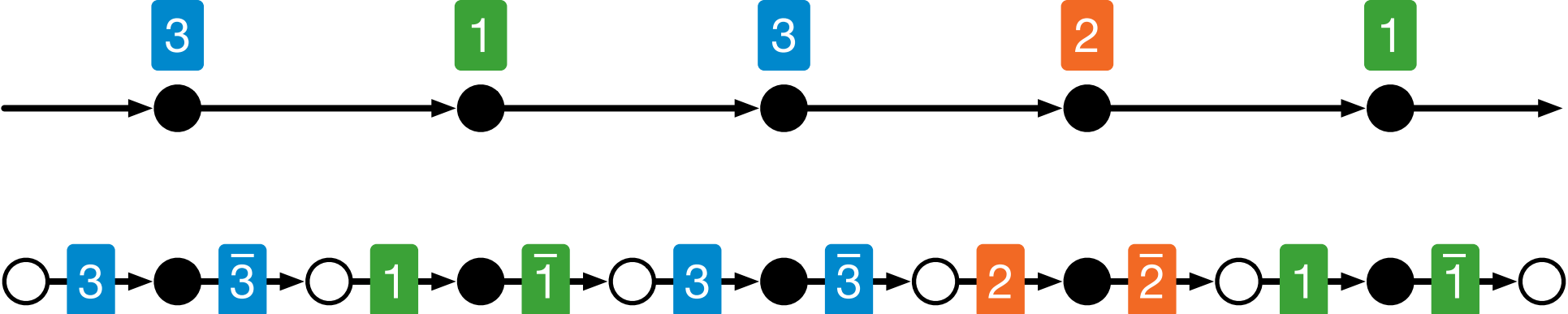


(b)



(c)

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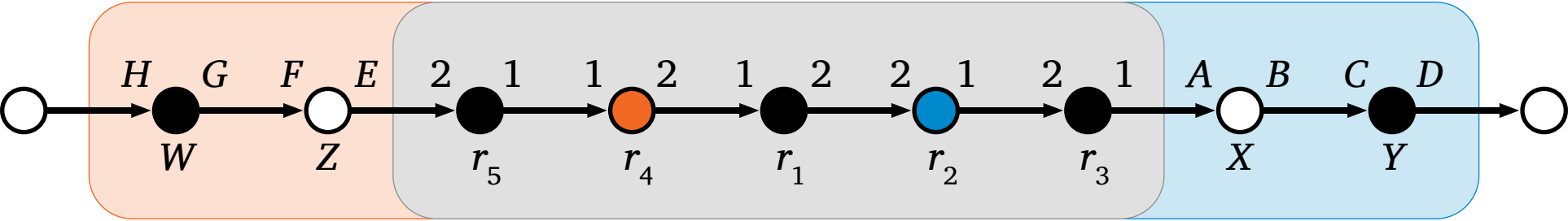
3̄
1
1̄
3
3̄
2
2̄
1
1̄


H
G
F
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1
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A
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C
D
W
Z
r5
r4
r1
r2
r3
X
Y