Aalto University

Distributed Algorithms 2020

10a Sinkless orientation

Last week:

- ***round elimination*** technique

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- toy examples

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- ***round elimination*** technique
- toy examples

This week:

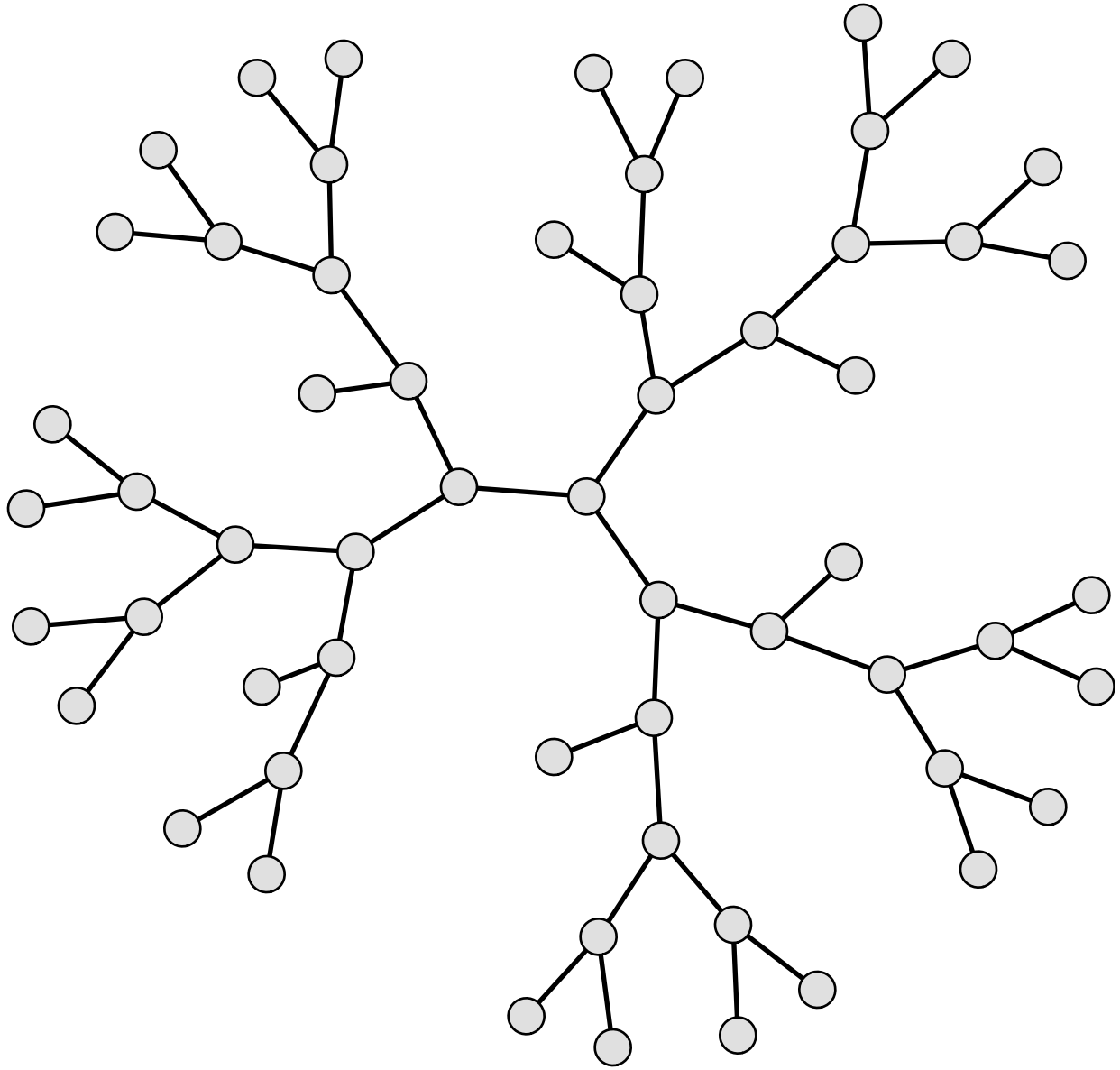
- using round elimination we can show that ***sinkless orientation*** is hard to solve

Many other problems are at least as hard as **sinkless orientation**

Sinkless orientation

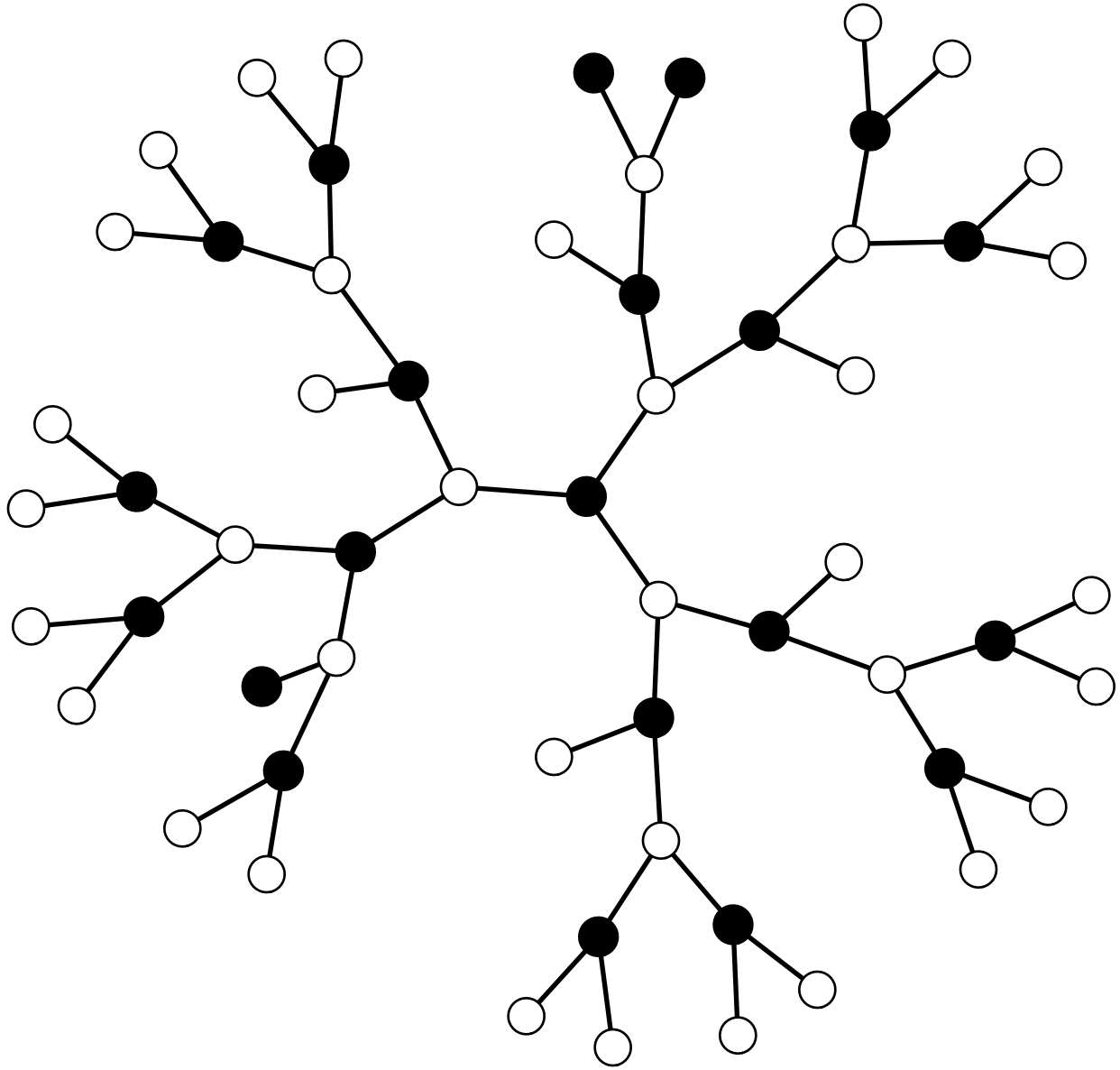
Sinkless orientation

- 3-regular trees
  - nodes have degree 1 or 3



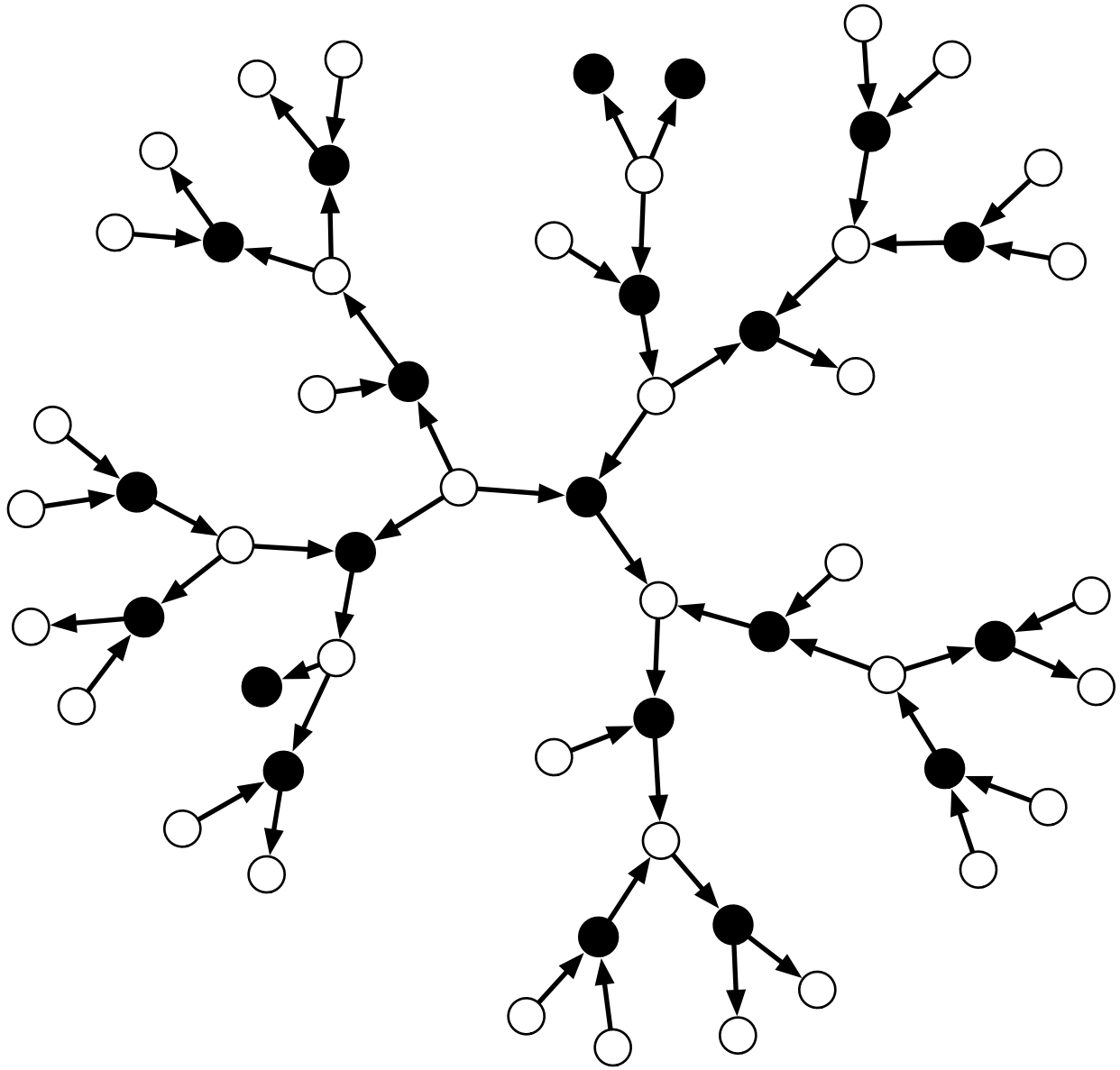
Sinkless orientation

- 3-regular trees
  - nodes have degree 1 or 3
- 2-coloring
  - black = active
  - white = passive



Sinkless orientation

- Degree 3 nodes: outdegree ≥ 1
- Degree 1 nodes: unconstrained



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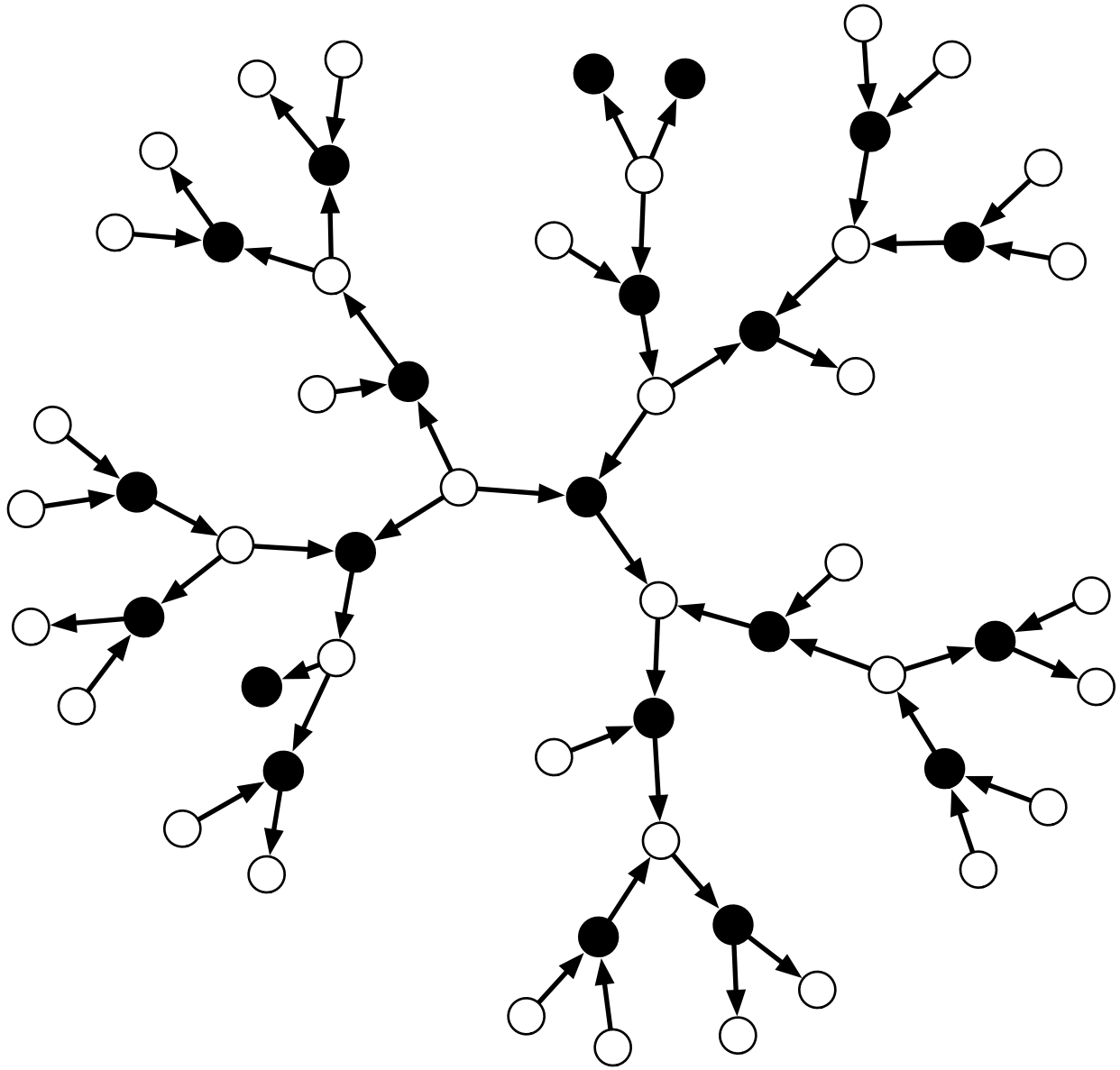

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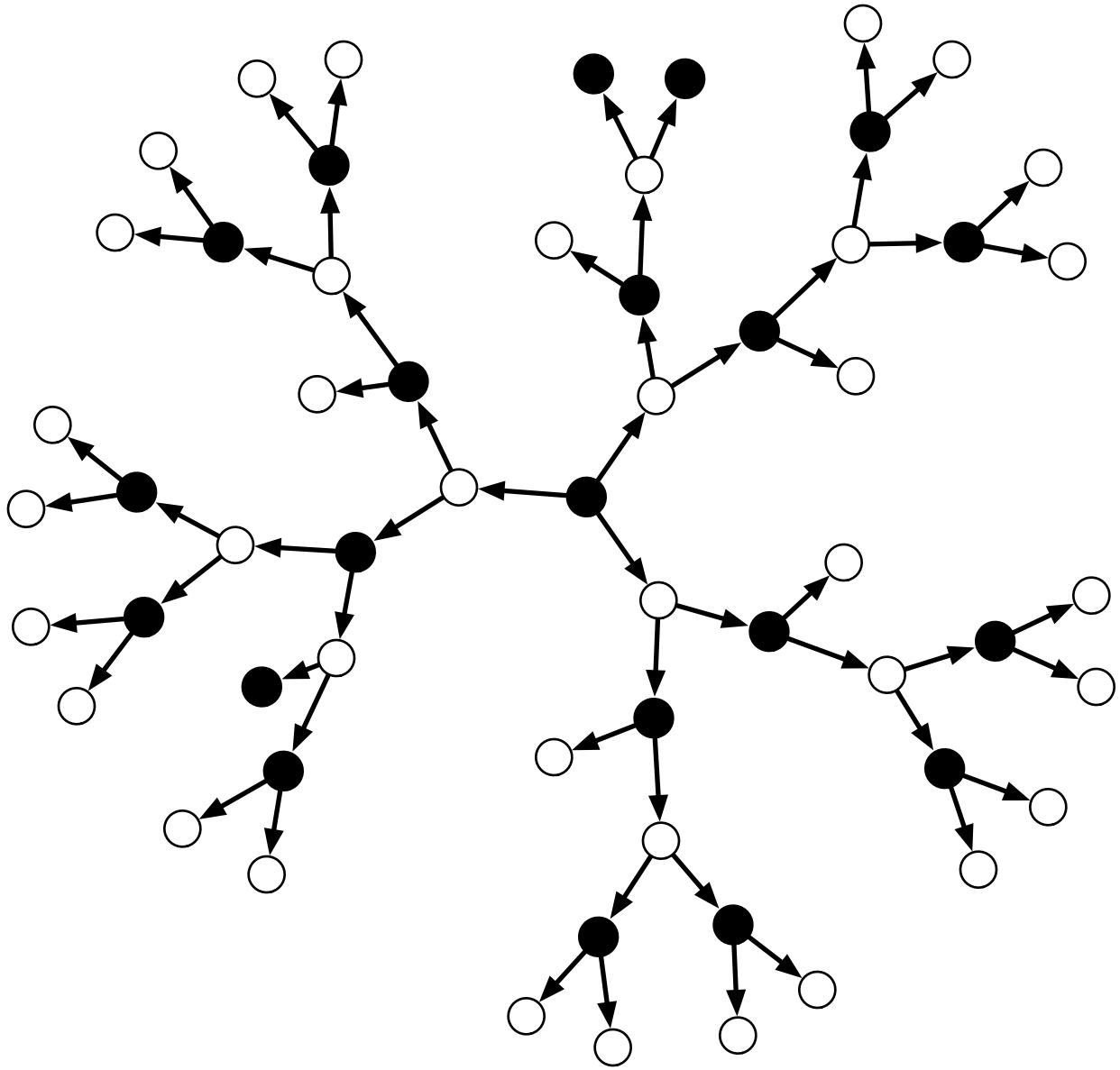

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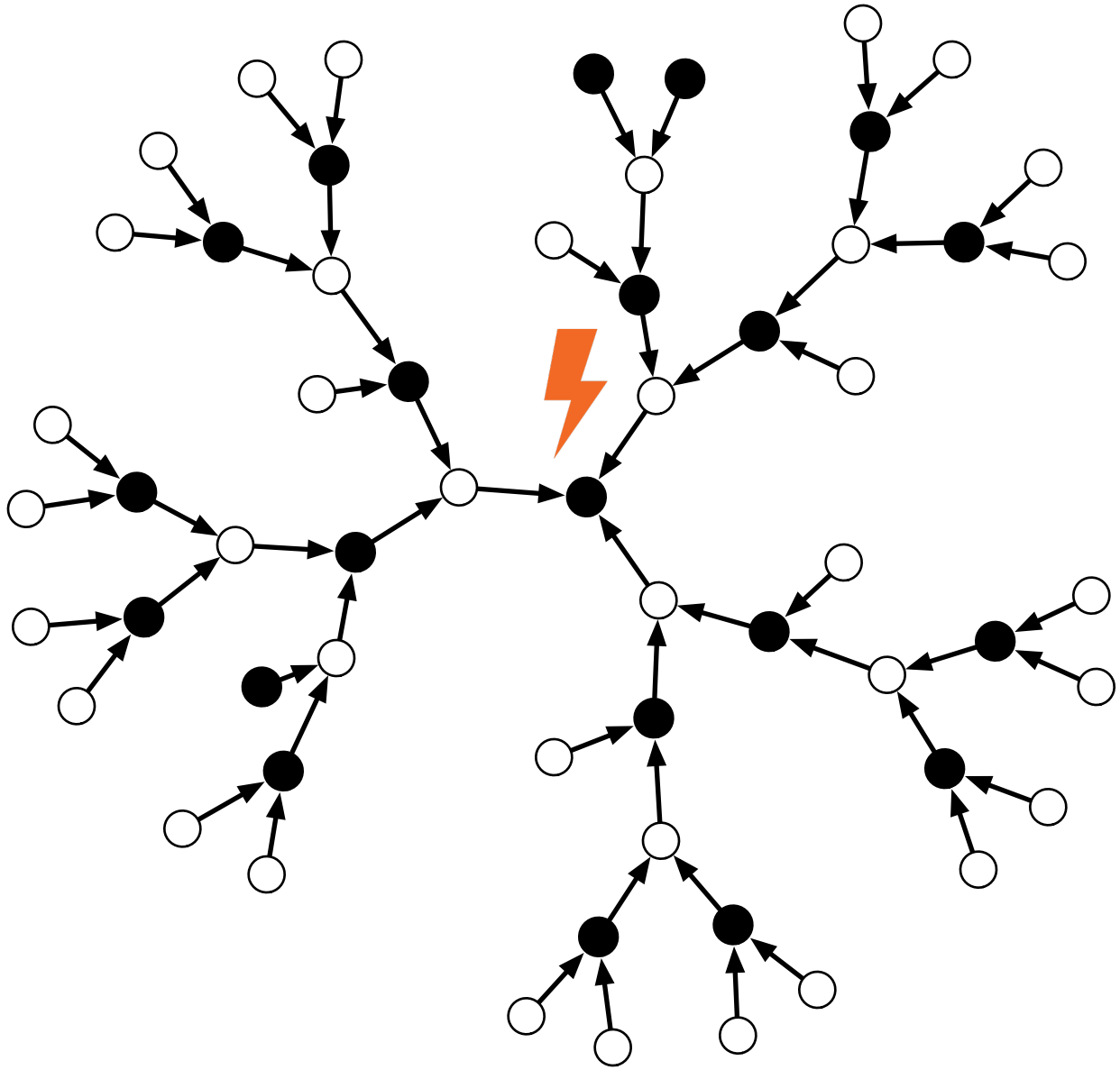
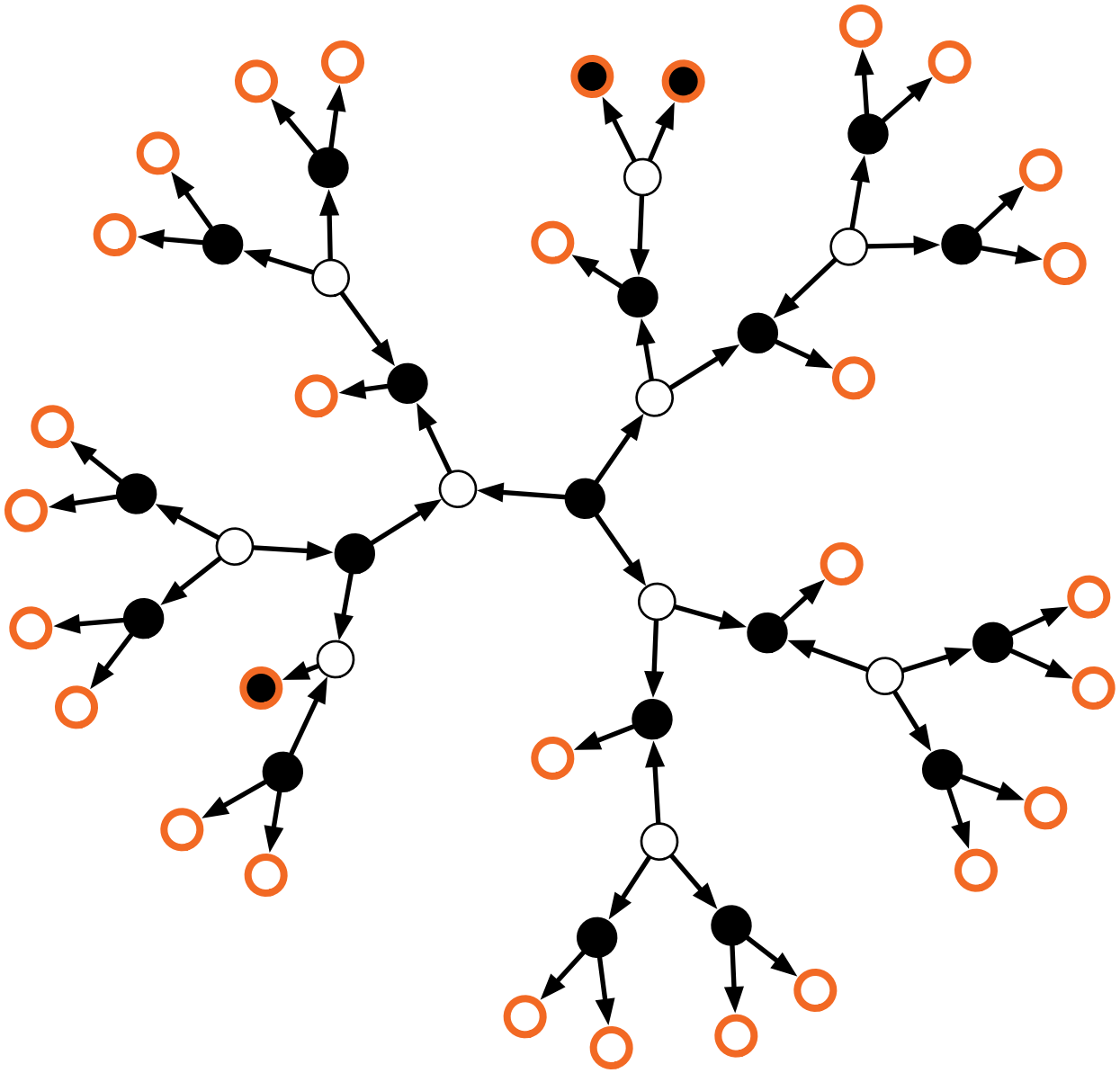


Sinkless orientation

- Simple algorithm: ***orient towards nearest leaf***
  - break ties with port numbers, colors
- **$O(\log n)$ rounds**



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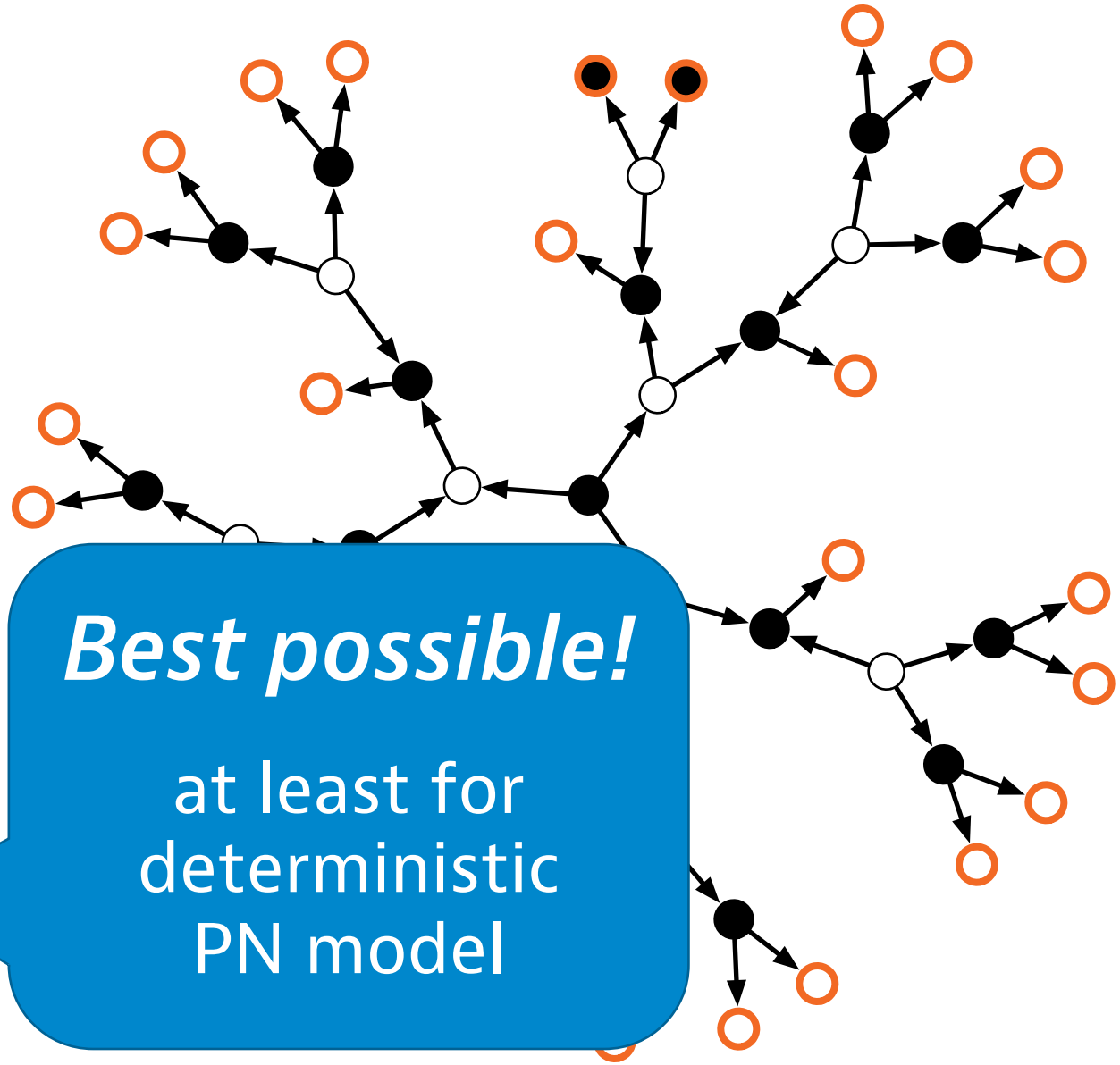

Example: vertex coloring

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  - *4-coloring???*

4-coloring vertices in graphs of degree ≤ 4

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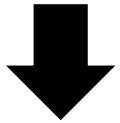
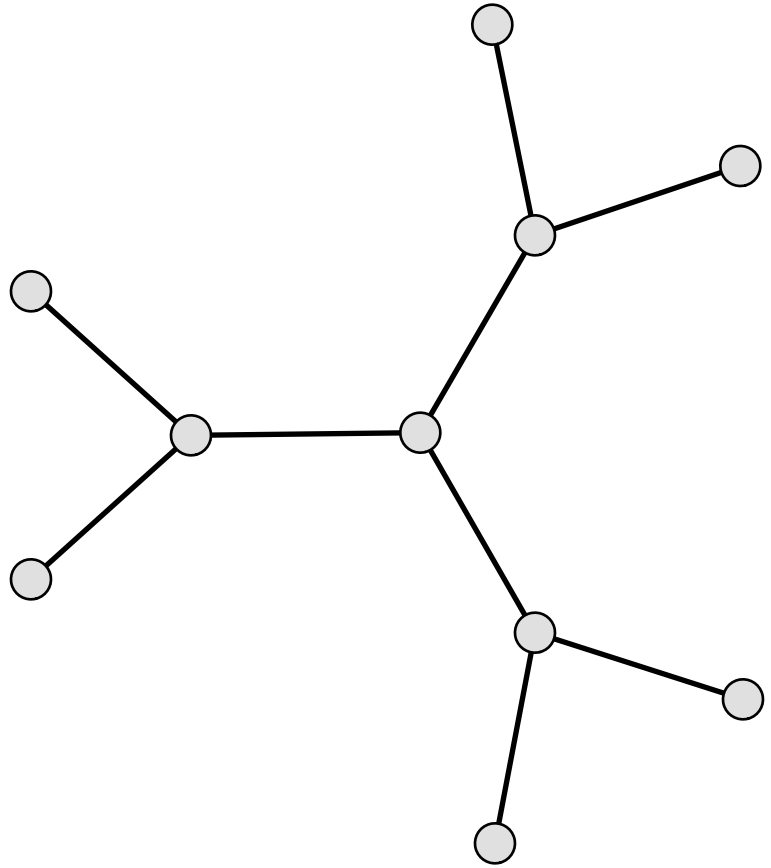


4-coloring edges in 3-regular trees

4-coloring vertices in graphs of degree ≤ 4



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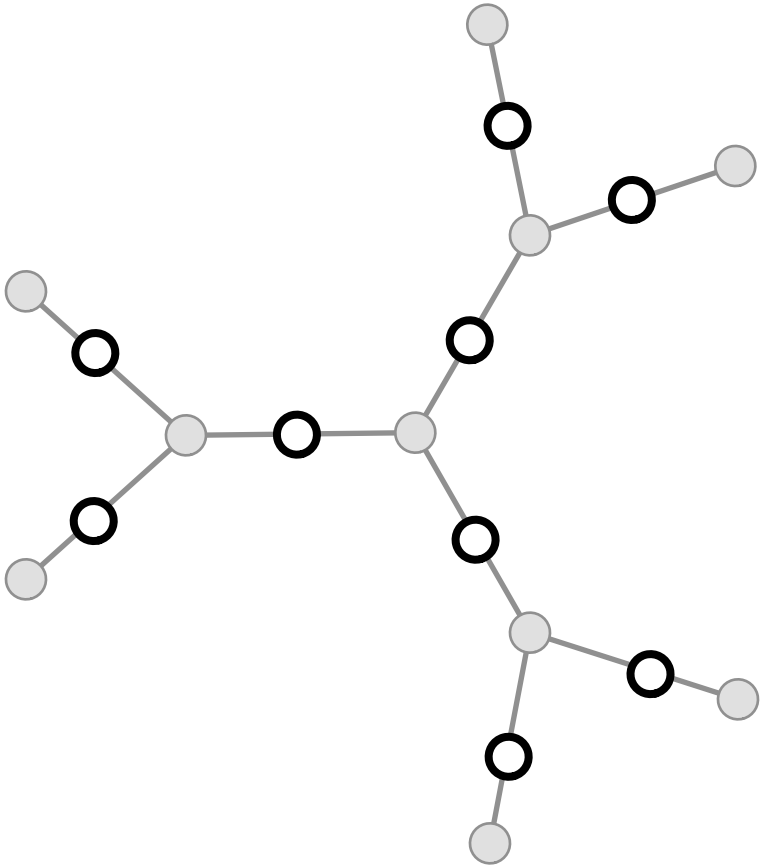

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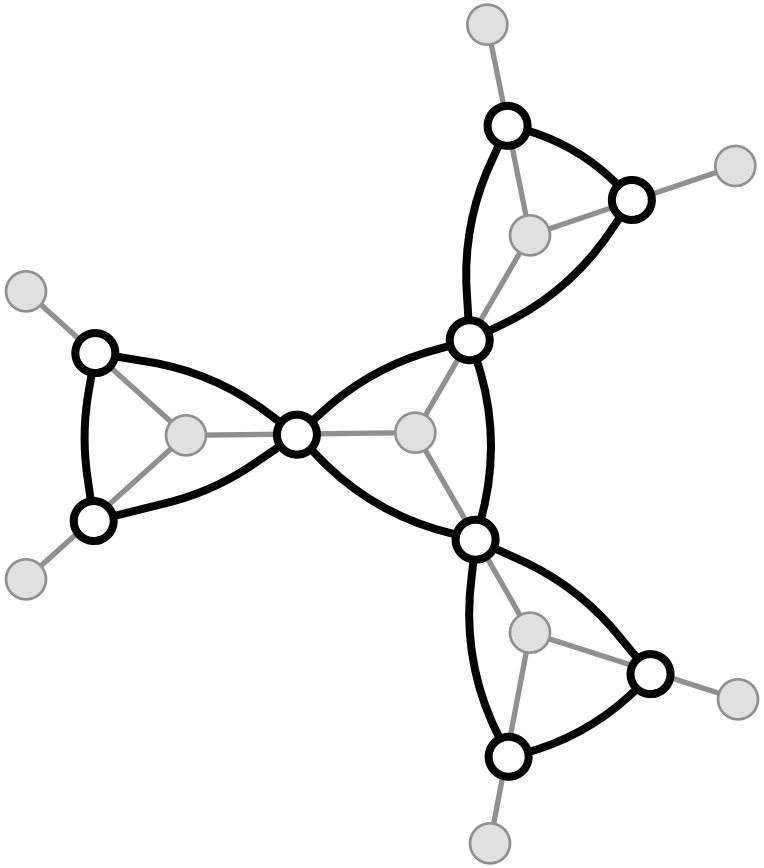

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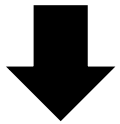

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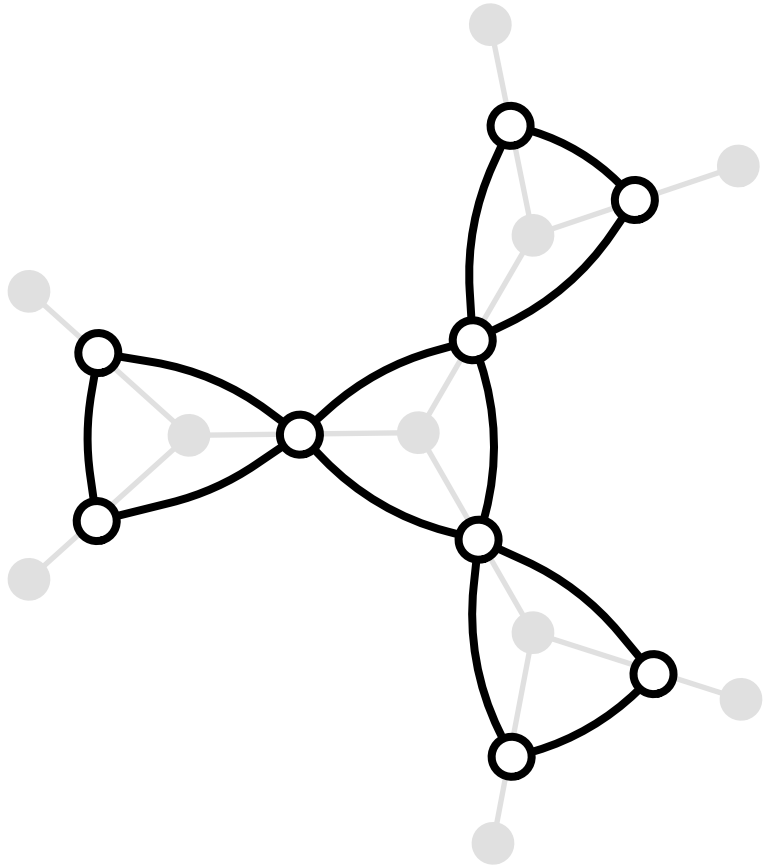

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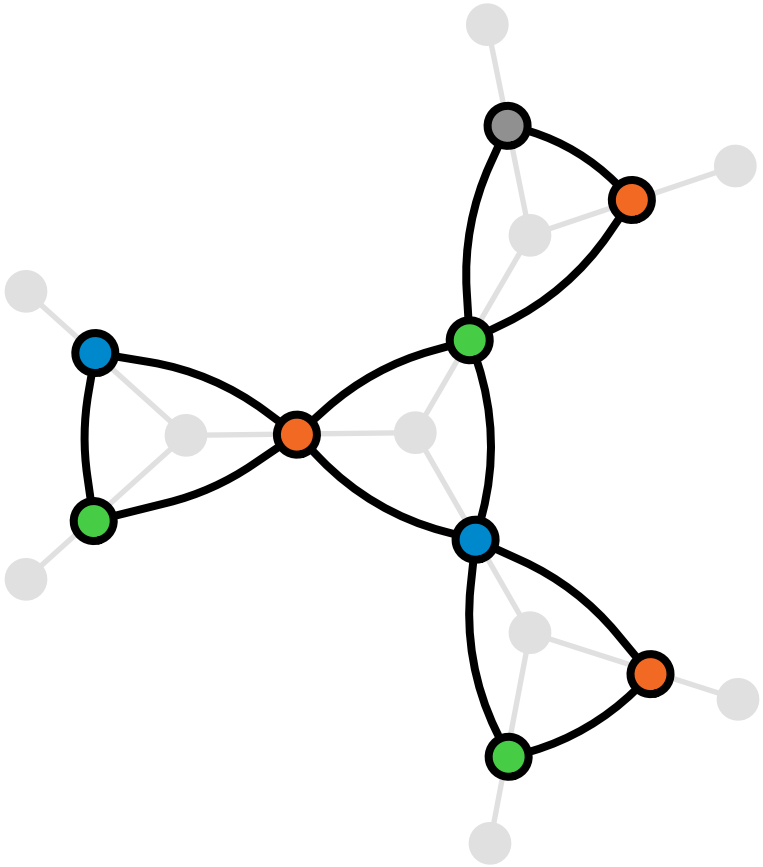

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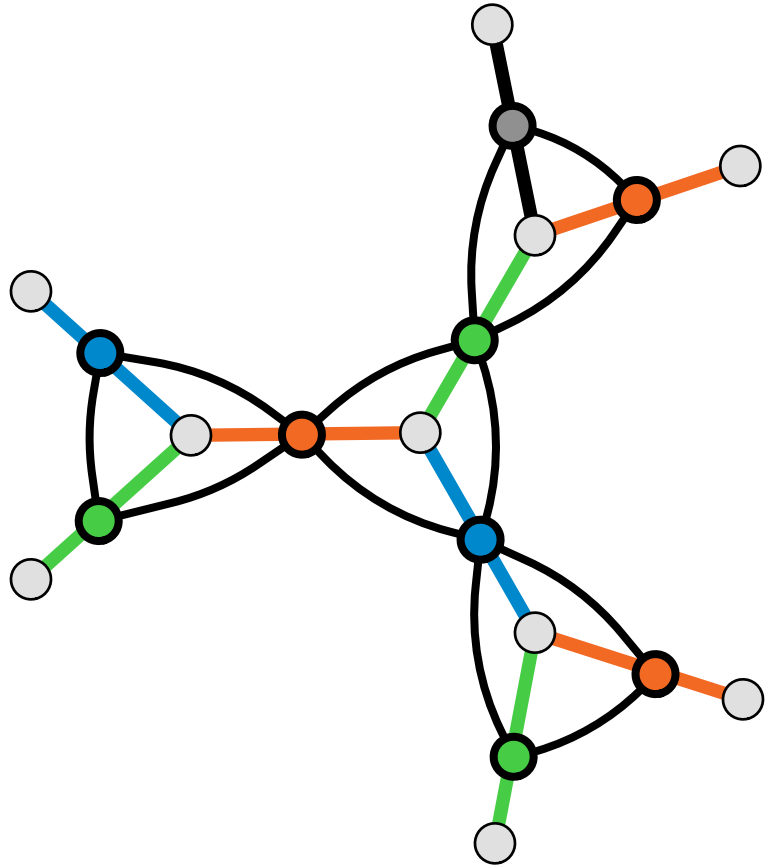

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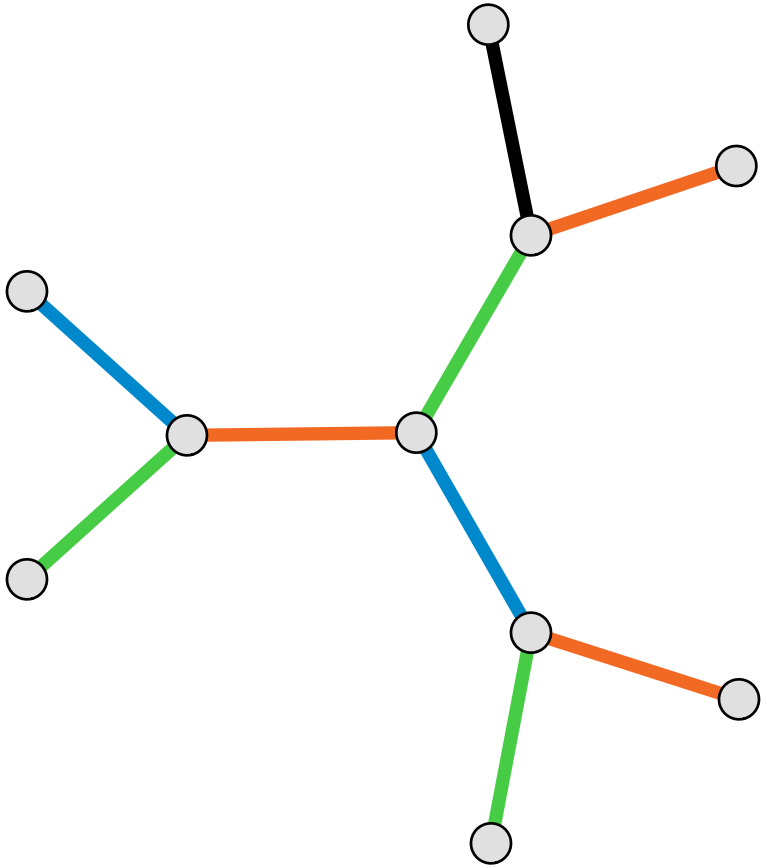

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sinkless orientation
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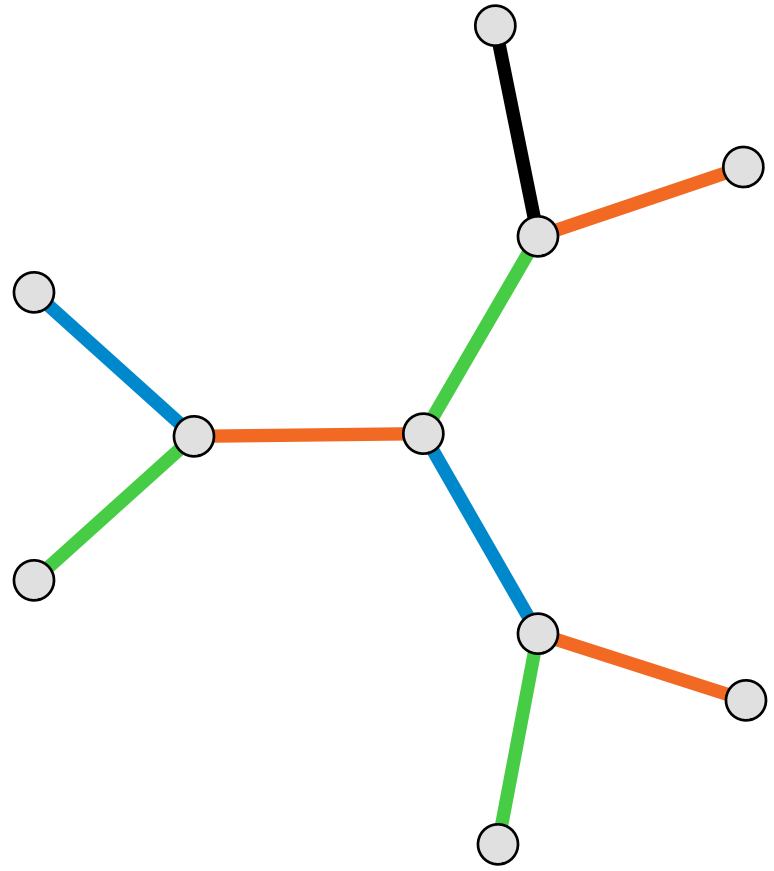

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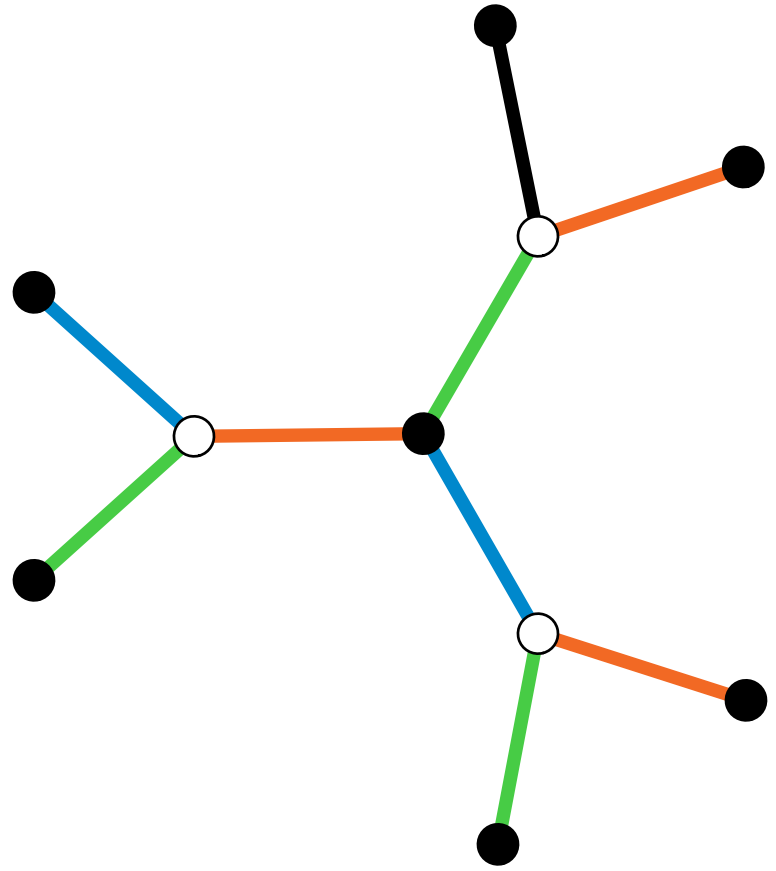

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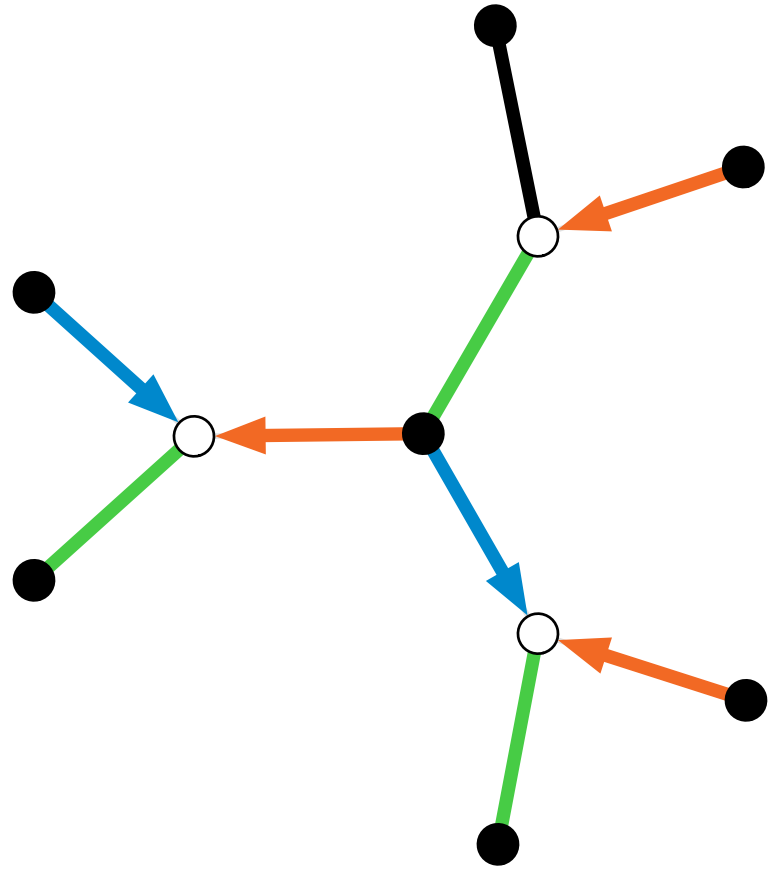

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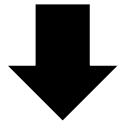

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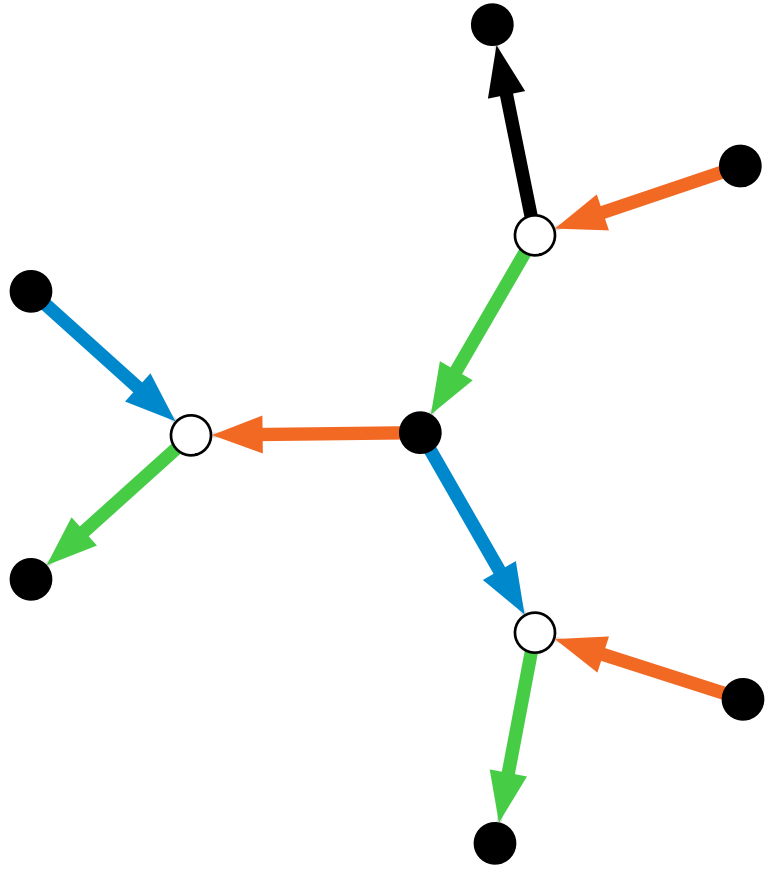

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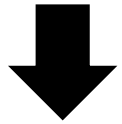

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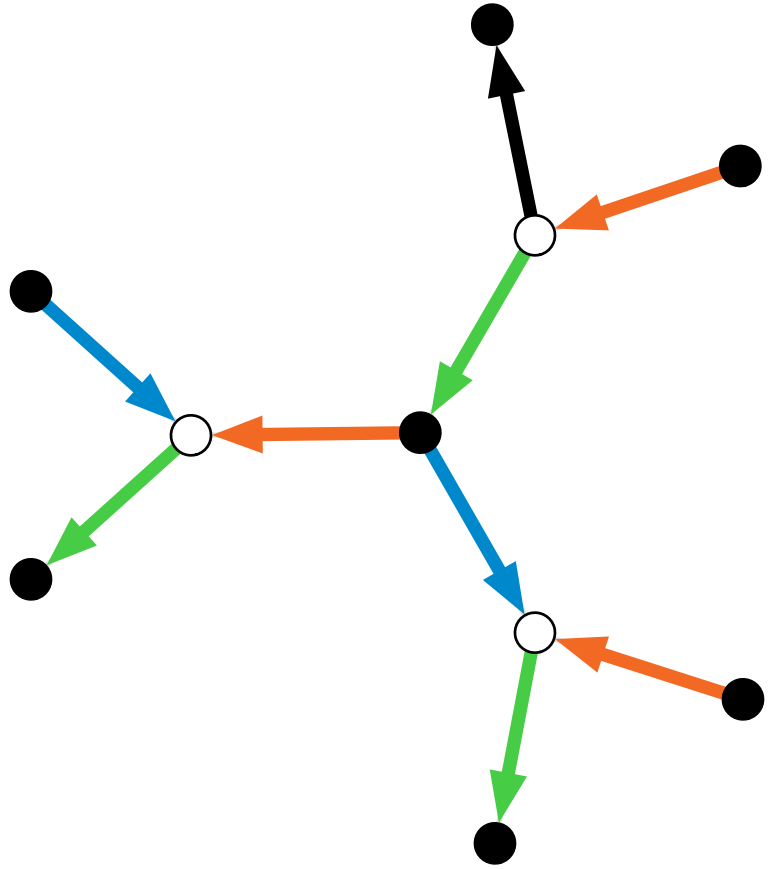

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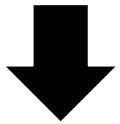

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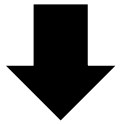

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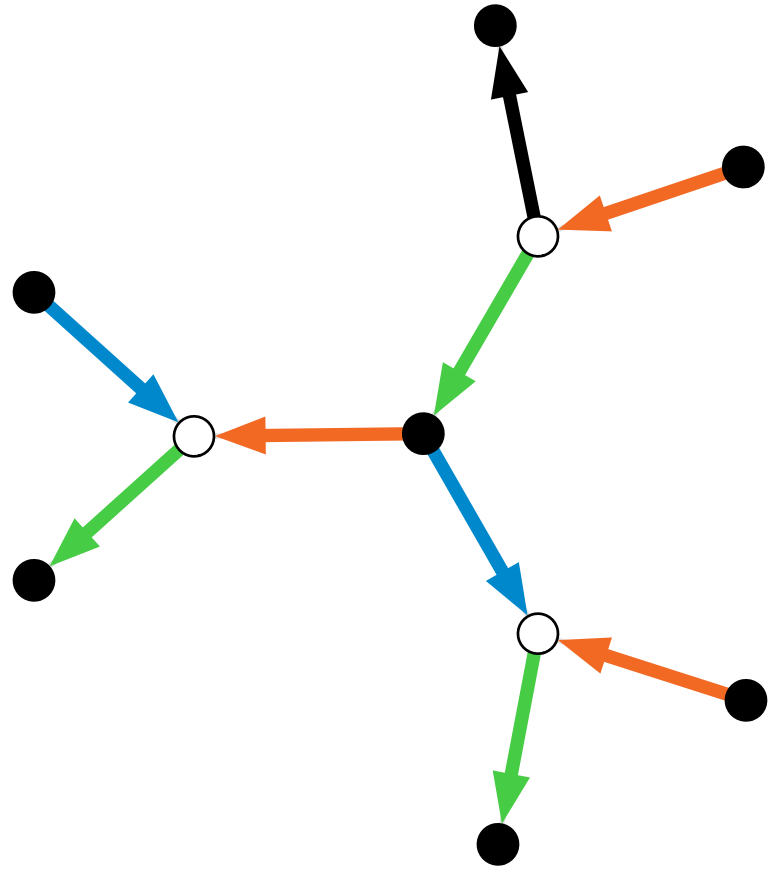

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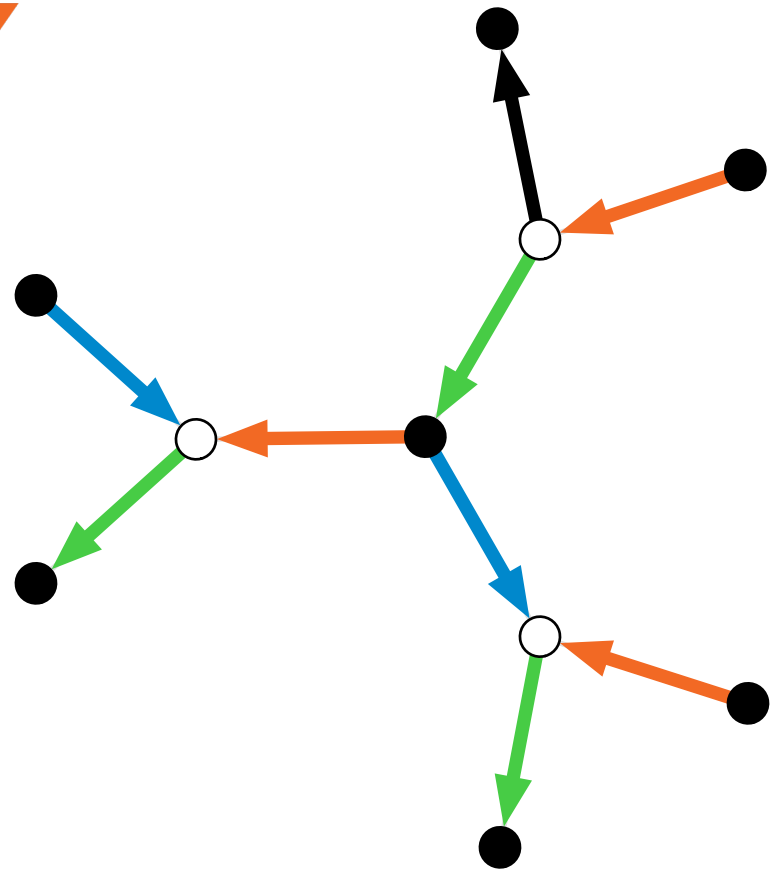

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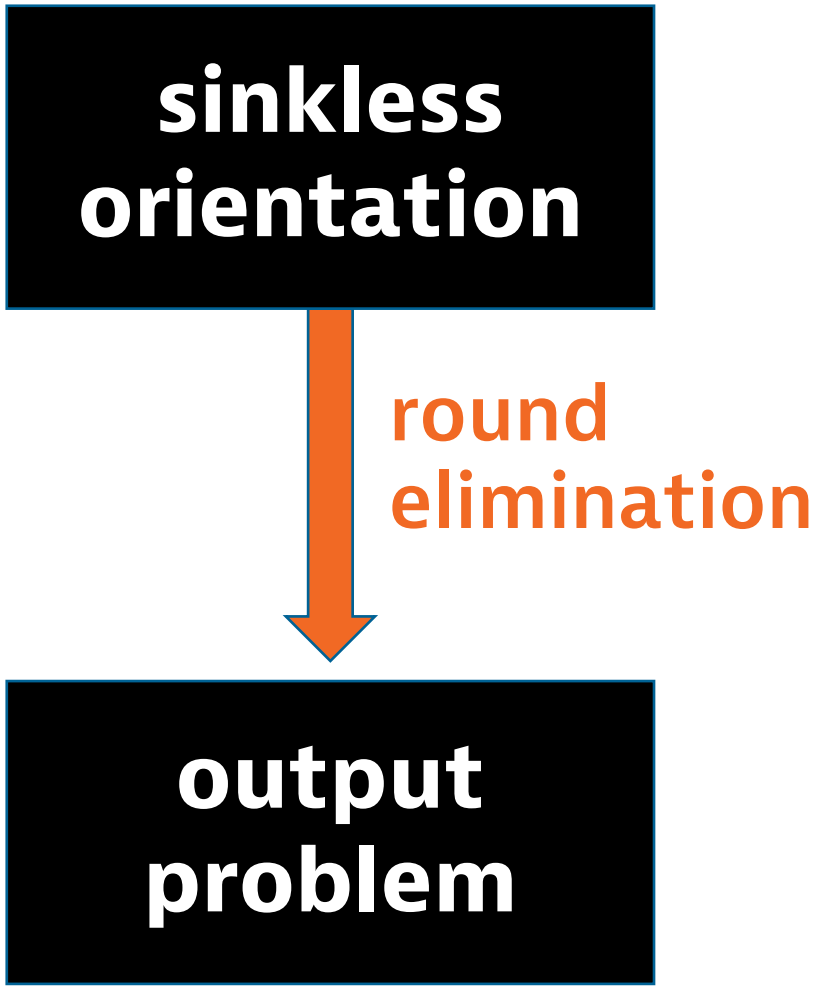
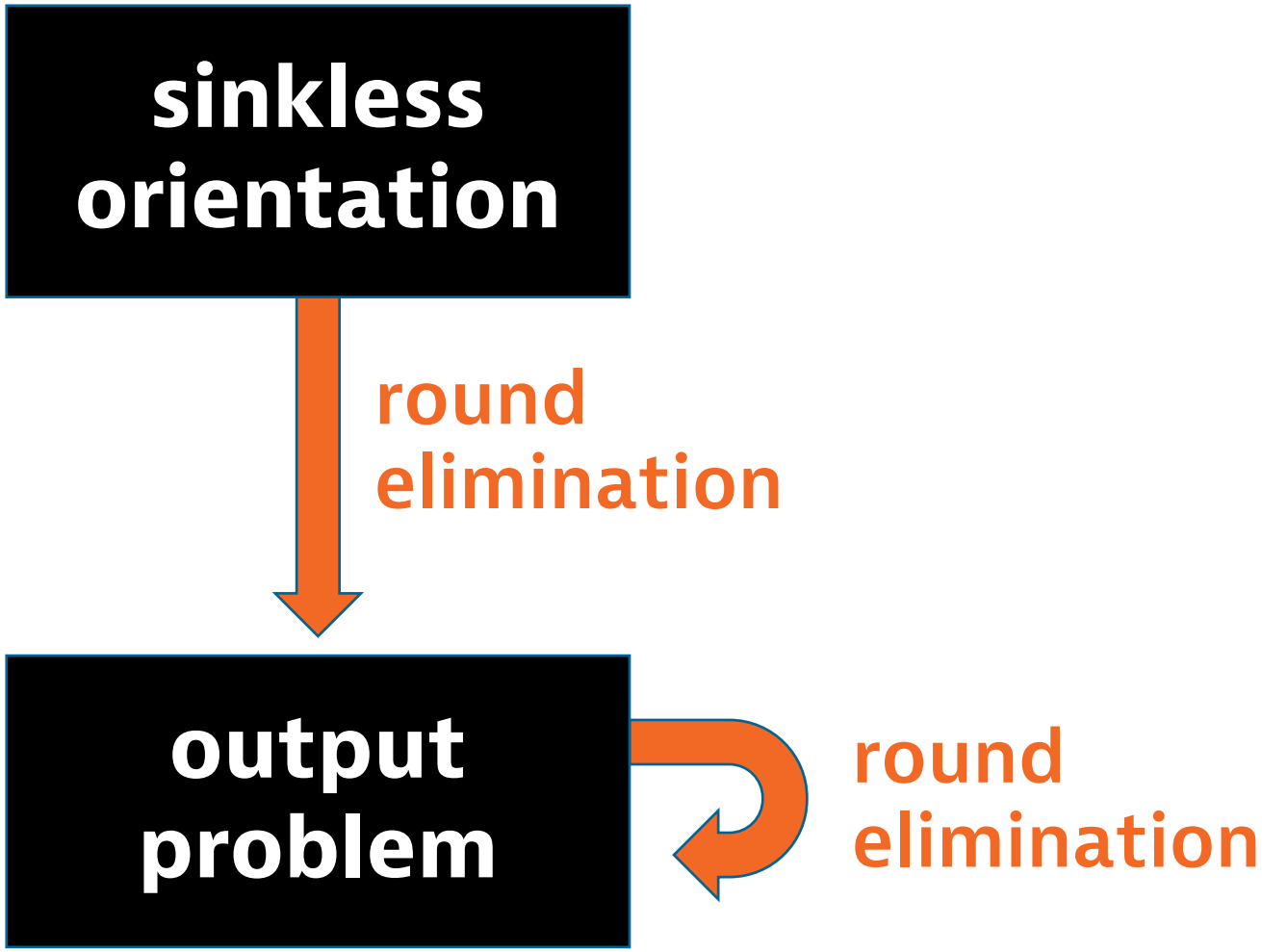

sinkless
orientation
round
elimination
output
problem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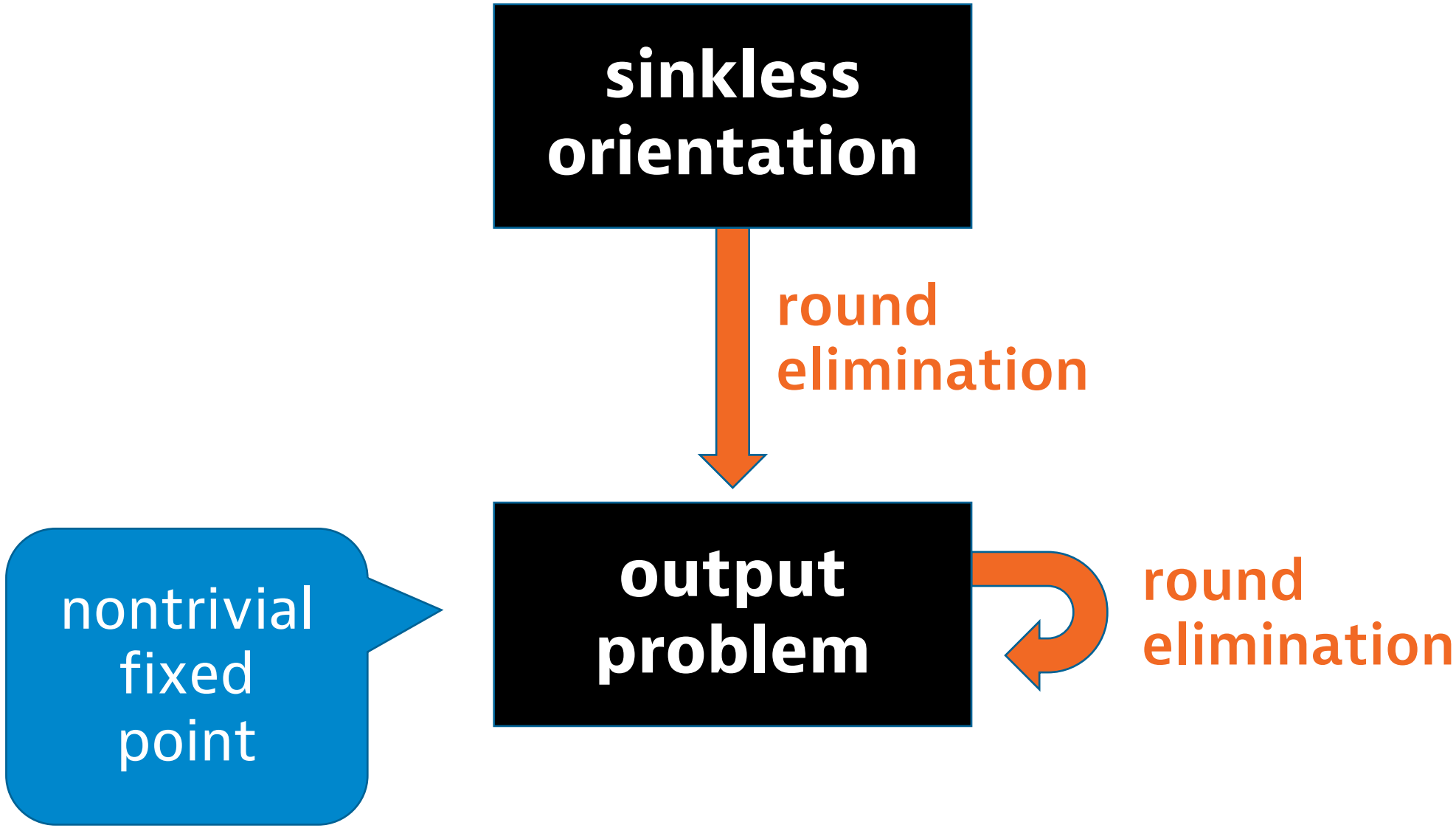
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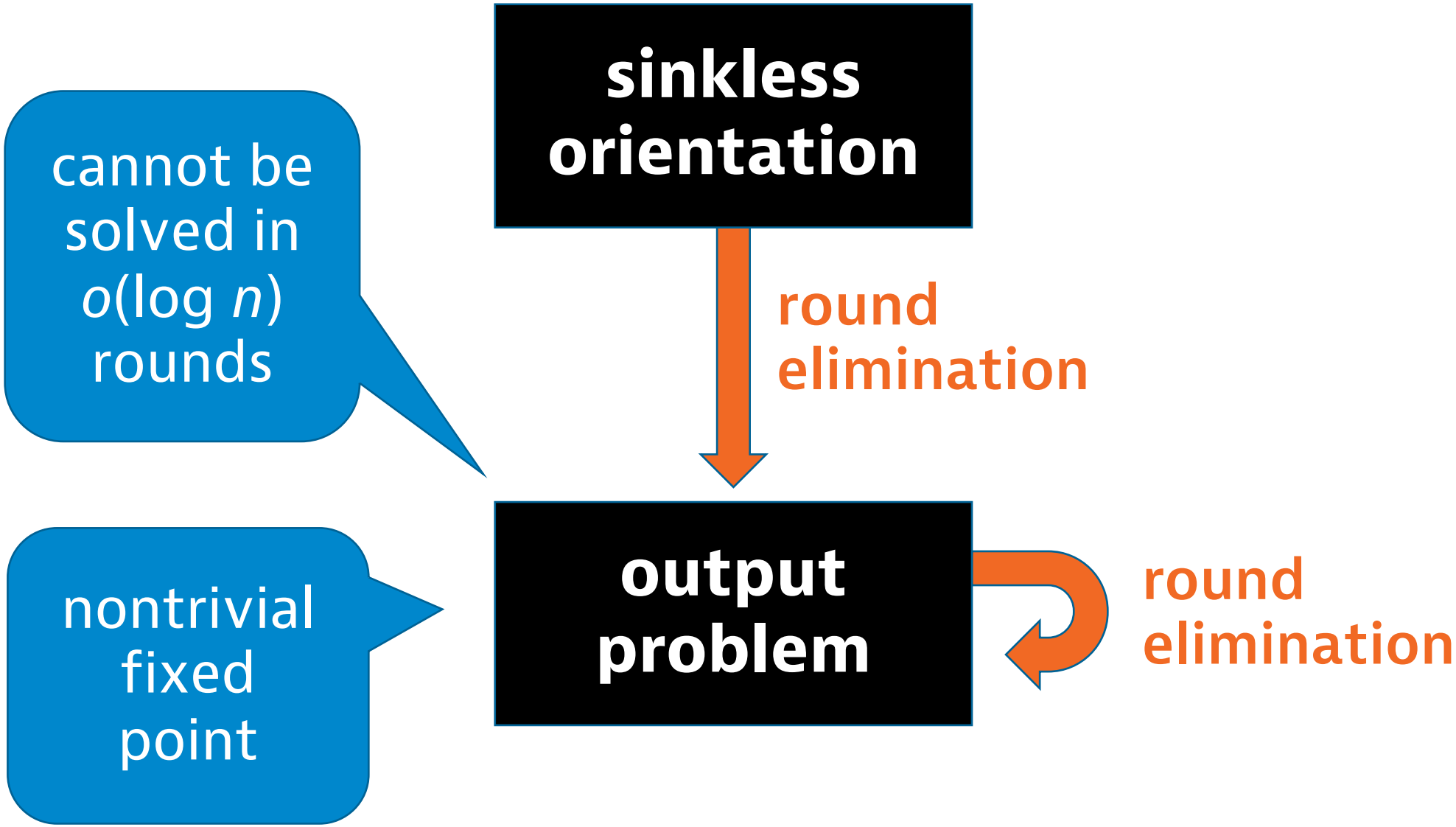
nontrivial fixed point


sinkless orientation
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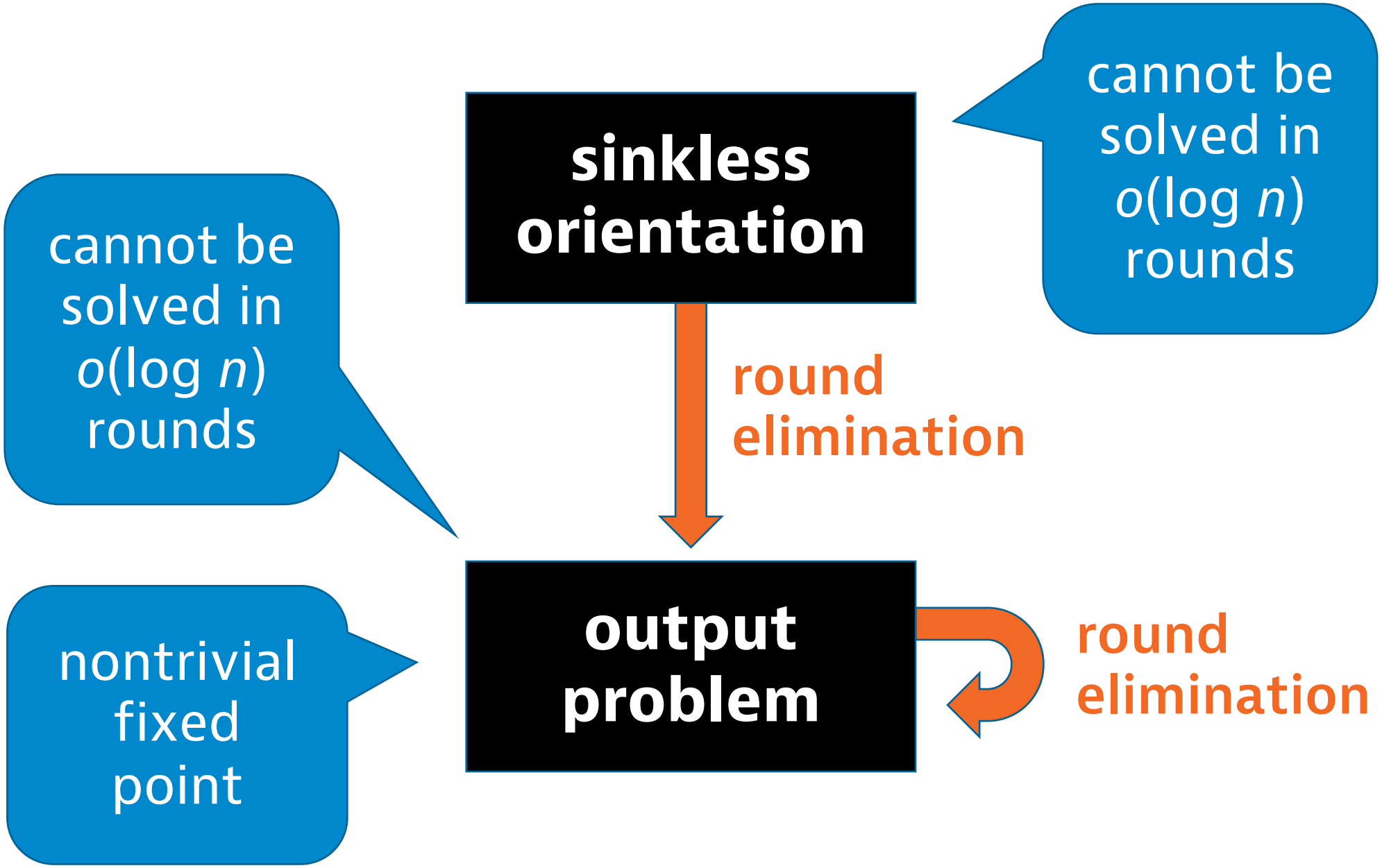
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