Aalto University

Distributed Algorithms 2020

4a Fast graph coloring

q = prime number

- $q > 2 \times$ maximum degree

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Color reduction from q^2 to q colors

q = prime number = **7**

- $q > 2 \times$ maximum degree

Color reduction from 49 to 7 colors

49 input colors: (a, b)

- $a = 0, 1, \ldots, 6$
- $b = 0, 1, \ldots, 6$

49 input colors: (a, b)

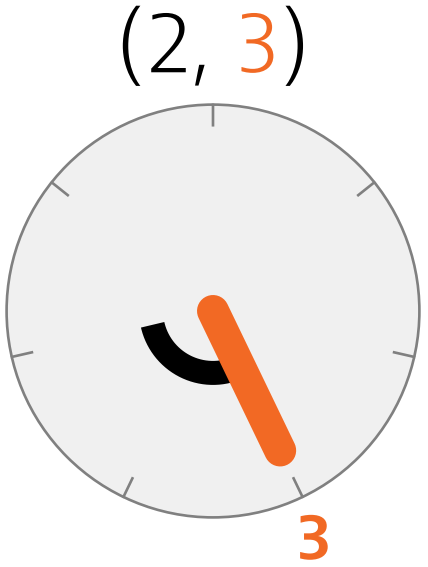
- $a = 0, 1, \ldots, 6$
- $b = 0, 1, \ldots, 6$

7 output colors: $(0, b)$

- $b = 0, 1, \ldots, 6$

$(\boldsymbol{a}, \boldsymbol{b})$ = "clock"

- a = speed
- b = position



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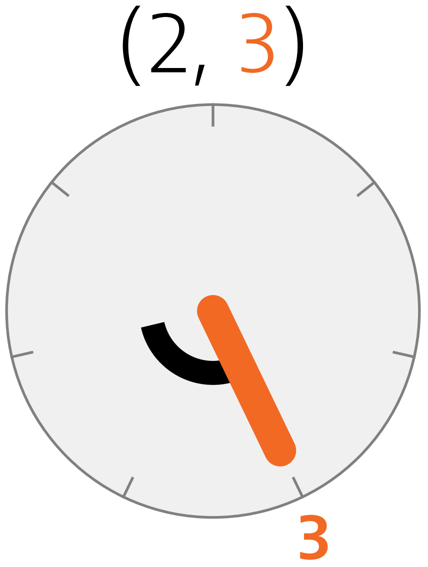
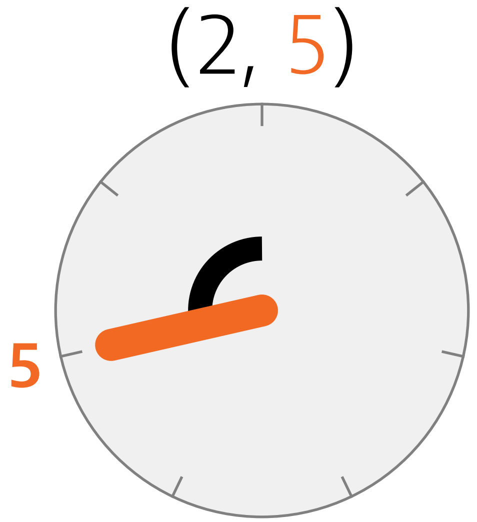


$(\mathbf{a}, \mathbf{b}) \rightarrow$
$(\mathbf{a}, \mathbf{b}+\mathbf{a} \bmod 7)$



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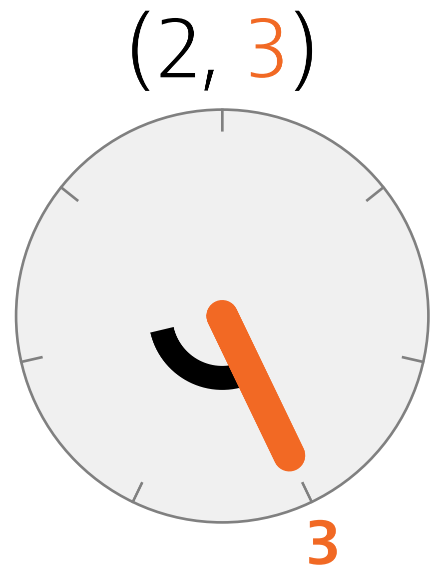
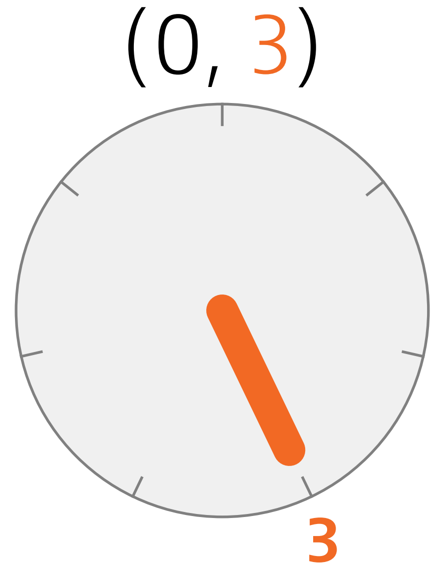


$(\mathbf{a}, \boldsymbol{b}) \rightarrow (\mathbf{0}, \boldsymbol{b})$

if no conflict



Why does it stop in q rounds?

In 7 rounds:

$\leq$ 2 conflicts with one neighbor

In 7 rounds:

≤ 2 conflicts with one neighbor

≤ 1 conflict when it’s running

In 7 rounds:

- $\leq$ 2 conflicts with one neighbor
  - $\leq$ 1 conflict when it’s running
  - $\leq$ 1 conflict when it’s stopped

In 7 rounds:

- $\leq$ 2 conflicts with one neighbor
- $\leq$ 6 conflicts with all 3 neighbors in total

In 7 rounds:

$\leq$ 2 conflicts with one neighbor

$\leq$ 6 conflicts with all 3 neighbors in total

Since 7 > 6, there is a conflict-free round!

Together with other algorithms:

Coloring with Δ+1 colors in $O(\log^* n + \Delta)$ rounds

(starting from unique identifiers)