Aalto University

Distributed Algorithms 2020

2a Graph-theoretic foundations

In any d-regular graph ($d \geq 1$),
a minimum **vertex cover** is
always a d-approximation
of a minimum **dominating set**

degree of a node =
number of neighbors

d-regular graph =
all nodes have degree *d*

set $C \subseteq V$ is a ***vertex cover*** if every edge has at least one endpoint in C

minimum vertex cover = vertex cover with smallest possible number of nodes

set $D \subseteq V$ is a ***dominating set*** if each node is in D or has got at least one neighbor in D

minimum dominating set = dominating set with smallest possible number of nodes

k-approximation of minimum dominating set:

1. a dominating set
2. at most *k* times as large as minimum dominating set

If:

- $d = 1, 2, 3 \ldots$
- G is a d-regular graph
- X is a minimum vertex cover for G

Then:

- X is a d-approximation of a minimum dominating set

If:

- $d = 1, 2, 3 \ldots$
- G is a d-regular graph
- X is a minimum vertex cover for G
- Y is a minimum dominating set for G

Then:

- X is also a dominating set
- $|X| \leq d \cdot |Y|$

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simple undirected graph $G = (V, E)$

set of nodes V

set of edges E

edge $e = \{u, v\}$

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