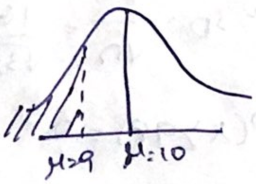
Central Limit Theorem



1) Given $\mu=10$, $\sigma=4$, $n=100$

$P(X<9) = ?$

$\sigma_{SD} = \frac{\sigma_P}{\sqrt{n}} = \frac{4}{\sqrt{100}} = \frac{4}{10} = 0.4$

$z = \frac{X-\mu}{\sigma} = \frac{9-10}{0.4} = \frac{-1}{0.4} = -2.5$

$\therefore P(X<9) = 0.062$

(2) Max = 550 kg, $\mu=50$, $\sigma=15$, $n=10$

for 10 people $\Rightarrow$ $\mu=500$ $\sigma=150$

$\sigma_{SD} = \frac{\sigma_P}{\sqrt{n}} = \frac{150}{\sqrt{10}} = \frac{150}{3.16} = 47.46$

$P(X<550) = ?$

$z = \frac{X-\mu}{\sigma} = \frac{550-500}{47.46} = 1.05$

$\therefore P(X<550) = 0.8531$

$\Rightarrow$ There is a probability of 0.8531 that all 10 students can reach 8th floor safely.

(3) Given $\mu=24$, $\sigma=2.0$, $n=100$

for 100 people $\Rightarrow$ $\mu=240$ $\sigma=200$

$\sigma_{SD} = \frac{\sigma_P}{\sqrt{n}} = \frac{200}{\sqrt{100}} = \frac{200}{10} = 20$

$P(X=250) = ?$

$z = \frac{X-\mu}{\sigma} = \frac{250-240}{20} = \frac{10}{2} = 0.5$

$P(X=250) = 0.6915$

(4) Given, $\mu = 96$, $\sigma = 16$, $n = 35$

$$\sigma_{SD} = \frac{\sigma}{\sqrt{n}} = \frac{16}{\sqrt{35}} = 2.7$$

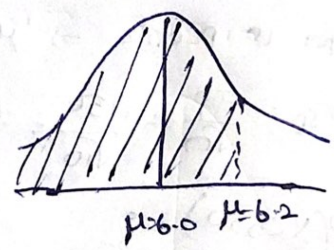
$P(X > 98) = ?$

$$z = \frac{X - \mu}{\sigma} = \frac{98-96}{2.7} = 0.74$$

$$P(X > 98) = 1 - 0.7704$$
$$= 0.2296$$

(5) $\mu = 6.0$, $\sigma = 1.0$, $n = 1$

i) $P(X < 6.2) = ?$



$$\sigma_{SD} = \frac{\sigma}{\sqrt{n}} = \frac{1.0}{\sqrt{1}} = 1$$

$$z = \frac{6.2 - 6.0}{1} = 0.2$$

$\therefore P(X < 6.2) = 0.5793$

ii) $P(X < 6.2) = ?$, $n = 100$

$$\sigma_{SD} = \frac{1.0}{\sqrt{100}} = 0.1$$

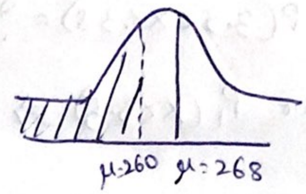
$$z = \frac{X - \mu}{\sigma} = \frac{6.2 - 6.0}{0.1} = 2$$

$\therefore P(X < 6.2) = 0.9772$

(6) This is related to above problem. After making a test with sample of 100 people, probability of men having their head breadth less than 6.2 inch is 0.9772. Almost 97% of men have head breadth less than 6.2 inch. Those helmets would fit almost all men but few. So, the manager reasoned that all helmets should be made for men with head breadths less than 6.2 inch as it satisfies maximum of the condition.

(7) $\mu = 268$ days, $\sigma = 15$ days, $n=25$

$\sigma_{SD} = \frac{15}{\sqrt{25}} = \frac{15}{5} = 3$



$P(X<260) = ?$

$z = \frac{X-\mu}{\sigma} = \frac{260-268}{3} = -2.67$

$\therefore P(X<260) = 0.0038$

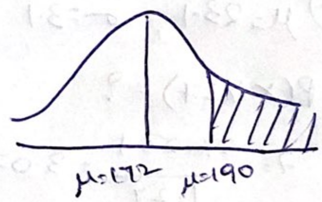
(8) $\mu = 260$ (without diet $\mu = 268$), $n=25$.

Yes. The diet has an effect on the length of pregnancy

(9) $\mu = 172$, $\sigma = 29$

i) $n=1$, $P(X>190) = ?$

$\sigma_{SD} = \frac{29}{\sqrt{1}} = 29$



$z = \frac{190-172}{29} = 0.620$

$\therefore P(X>190) = 1-0.7324 = 0.2638$

ii) $n=25$, $P(X>190) = ?$

$\sigma_{SD} = \frac{29}{\sqrt{25}} = 5.8$

$z = \frac{190-172}{5.8} = 3.10$

$P(X>190) = 1-0.9990 = 0.001$

iii) Max $= 4750$, $n=25$

[for 25, $\mu = 25 \times 172 = 4300$, $\sigma = 725$]

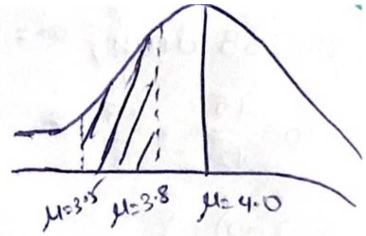
$P(X>4750) = ?$

$z = \frac{4750-4300}{5.8} = 3.103$

$P(X>4750) = 1-0.9990 = 0.001$

(10) $\mu = 4.09 \quad \sigma = 1.59 \quad n = 50$

$P(3.5 < X < 3.8) = ?$



for $P_1(X < 3.5) \Rightarrow z_1 = \dfrac{3.5 - 4.0}{0.24}$

$\sigma_{SD} = \dfrac{1.5}{\sqrt{50}} = 0.24$

$z_1 = -2.08$

for $P_2(X < 3.8) \Rightarrow z_2 = \dfrac{3.8 - 4.0}{0.24}$

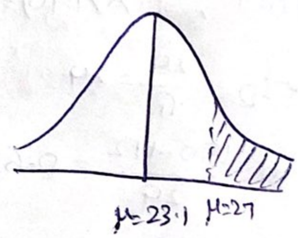
$z_2 = -0.83$

$\therefore P_1 = 0.0188 \quad P_2 = 0.2033$

$P(3.5 < X < 3.8) = P_2 - P_1 = 0.2033 - 0.0188 = 0.1845$

(11) $\mu = 23.1, \quad \sigma = 3.1, \quad n = 6$

$P(X > 27) = ?$



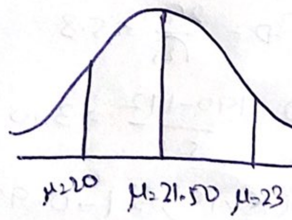
$z = \dfrac{27 - 23.1}{1.27} = 3.07$

$\therefore P(X > 27) = 1 - 0.9989 = 0.0011$

$\sigma_{SD} = \dfrac{3.1}{\sqrt{6}} = 1.27$

(12) $\mu = 21.50 \quad \sigma = 2.22$

$P(20 < X < 23) = ?$



~~σ_{SD}~~ for $P_1(X < 20) \Rightarrow$

$z_1 = \dfrac{20 - 21.50}{2.22}$

$z_1 = -0.68$

for $P_2(X > 23) \Rightarrow z_2 = \dfrac{23 - 21.50}{2.22}$

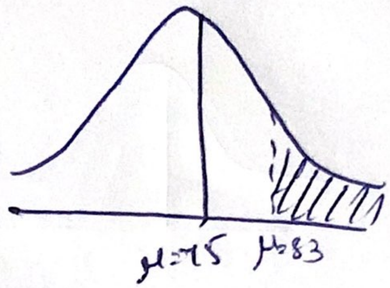
$z_2 = 0.68$

$\therefore P_1 = 0.2483 \qquad P_2 = 0.7517$

$P(20 < X < 23) = P_2 - P_1 = 0.5034$

(13) $\mu = 75$, $\sigma = 5$

i) atleast 83 $\Rightarrow P(X > 83) = ?$



$$z = \frac{83-75}{5} = 1.6$$

$\therefore P(X > 83) = 0.9452$

ii) $n=5$, atleast 83

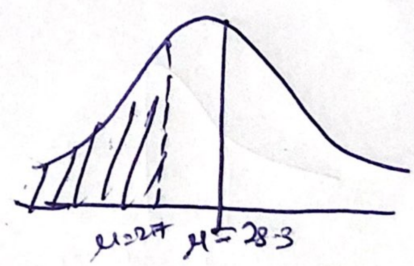
$$\sigma_{SD} = \frac{5}{\sqrt{5}} = \frac{5}{2.24} = 2.23$$

$$P(X > 83) = ? \Rightarrow z = \frac{83-75}{2.23} = 3.59$$

$\therefore P(X > 83) = 0.9998$

(14) $\mu = 28.3$, $\sigma = 2.3$, $n = 10$

$$P(X < 27) \rightarrow ? \Rightarrow z = \frac{27-28.3}{0.72} = -1.80$$



$\therefore P(X < 27) = 0.0359$

$$\sigma_{SD} = \frac{2.3}{\sqrt{10}} = \frac{2.3}{3.16} = 0.72$$