Confidence Interval Assignment

① Given $\bar{X} = 180$, $\sigma = 30$, $n = 1000$, 95% CI = ?

$$\sigma_{SD} = \frac{\sigma}{\sqrt{n}} = \frac{30}{\sqrt{1000}} = 0.95$$

$z = 2$ (for 95% confidence)

$\therefore CI = \bar{X} \pm z\sigma = 180 \pm (2)(0.95)$

② Given, $\sigma = 3.6$, $n = 120$, $\bar{X} = 16.2$, 92% CI = ?

a) $\sigma_{SD} = \frac{\sigma}{\sqrt{n}} = \frac{3.6}{\sqrt{10}} = 0.33$

$z = 1.76$ (for 92% confidence)

$\therefore CI = 16.2 \pm (1.76)(0.33)$

b) Given $\epsilon = 15$ sec ($\cong 0.25$ minutes), $n = ?$ for 92% confidence

we know, $\epsilon = \pm z\frac{\sigma}{\sqrt{n}} \Rightarrow n = \left(\frac{z\sigma}{\epsilon}\right)^2$

$$n = \left(\frac{1.76(3.6)}{0.25}\right)^2 = (25.34)^2 = 642.21$$

$\therefore n \cong 642$

③ Given $\epsilon = 2\% = 0.02$, $n = ?$

a) 90% of CI $\Rightarrow z = 1.645$, Assume $p = 0.5$

$$n = z^2\left(\frac{P(1-P)}{\epsilon^2}\right) = (1.645)^2\left(\frac{0.5(0.5)}{(0.02)^2}\right) = \frac{2.706(0.25)}{0.0004}$$

$n = 1691$

b) $n = 1000$, $P = 400/1000 = 0.40$, 95% CI = ?

$$CI = p \pm z\sqrt{\frac{P(1-p)}{n}} = 0.40 \pm 1.96\sqrt{\frac{0.4(0.6)}{1000}}$$

$CI = 0.40 \pm 0.027$

④ Sample = (0.95, 1.02, 1.01, 0.98)

$n=4$, $\mu_{SD}=0.99$, 95% CI $\Rightarrow z=1.96$, $\sigma=0.032$

$$CI = \mu_{SD} \pm z\frac{\sigma}{\sqrt{n}} = 0.99 \pm 1.96\left(\frac{0.032}{\sqrt{4}}\right)$$

$$CI = 0.99 \pm 1.96(0.016) = 0.99 \pm 0.31$$

∵ Null → $\mu=1$ falls in 95% CI, we accept null. There is no evident at 5% LOS that scale is Inaccurate.

⑤ Given, $\mu=45$, $n=9$, $\mu_{SD}=49.2$, $\sigma=3.5$

Null → $\mu=45$ sec Alternative → $\mu \neq 45$ sec

$$z = \frac{X-\mu}{SE} = \frac{49.2-45}{3.5/\sqrt{9}} = 3.6$$

at 5% significance $\Rightarrow z_{critical}=2$

∵ $z > z_{critical}$, we reject null at 5% LOS.

⑥ Given $\sigma=5$, $n=64$, $\mu=42$, 95% CI = ?

$$CI = S \pm z\frac{\sigma}{\sqrt{n}} = 42 \pm 1.96\left(\frac{5}{\sqrt{64}}\right)$$

$$= 42 \pm 1.225$$

⑦ Given $n=17$, $\Sigma d_i = -3.5$, $\Sigma d_i^2 = 19.13$, 90% CI = ?

$$\bar{X} = \frac{\Sigma d_i}{n} = \frac{-3.5}{17} = -0.21$$

$$\sigma = \sqrt{\frac{\Sigma d_i^2}{n-1}} = \sqrt{\frac{19.13}{17-1}} = \sqrt{1.195} = 1.09$$

$$CI = S \pm z\frac{\sigma}{\sqrt{n}} = -0.21 \pm (1.645)\frac{(1.09)}{\sqrt{17}}$$

$$CI = -0.21 \pm 0.42$$

⑧ Given $\sigma^2=9 \Rightarrow \sigma=3$, $\epsilon=1$, 95% CI, $n=?$

$$n = z^2 \times \frac{\sigma(1-\sigma)}{\epsilon^2} = (1.96)^2 \times \frac{3(1-3)}{1^2}$$

$$n = 23$$

(9) Given, $n=16$, $\mu=141$, $\sigma=4$, 95% CI = ?

$$CI = \bar{x} \pm z\frac{\sigma}{\sqrt{n}} = 141 \pm (1.96)\frac{4}{\sqrt{16}}$$

$$CI = 141 \pm 1.96$$

(10) Given, $n=17096$, Binge drinker $(p) = \frac{3314}{17096} = 0.19$

$$CI = p \pm z\sqrt{\frac{p(1-p)}{n}} = 0.19 \pm 1.645\sqrt{\frac{0.19(0.81)}{17096}}$$

$$CI = 0.19 \pm 0.00492$$

(11) Given, $n=100$, $\mu=49$, $\sigma=4.49$, $z=1.645$

$$CI = \bar{x} \pm z\frac{\sigma}{\sqrt{n}} = 49 \pm (1.645)\frac{4.49}{\sqrt{100}} = 49 \pm 0.739$$

(12) Given, $n=1200$, fraud $(p) = \frac{175}{1200} \approx 0.15$, $z=1.96$

$$CI = p \pm z\sqrt{\frac{p(1-p)}{n}} = 0.15 \pm 1.96\sqrt{\frac{0.15(0.85)}{1200}} = 0.15 \pm 0.196$$

(13) Given, $n=59$, left handed $(p) = \frac{15}{59} = 0.25$, $z=1.96$

$$CI = p \pm z\sqrt{\frac{p(1-p)}{n}} = 0.25 \pm \sqrt{\frac{0.25(0.75)}{59}}(1.96) = 0.25 \pm 0.98$$

(14) Given, $\sigma=475$, $E=100$, $z=1.64$, $n=?$

$$n = \left(\frac{z\sigma}{E}\right)^2 = \left(\frac{1.64(475)}{100}\right)^2 = 60.68$$

$\therefore n \approx 61$

(15) Given, 5 min interval $\Rightarrow$ 68, 42, 51, 57, 56, 80, 45, 39, 36, 79

$n=10$, $\mu=55.3$, $\sigma=15.9$

$$CI = \bar{x} \pm z\frac{\sigma}{\sqrt{n}} = 55.3 \pm (1.96)\frac{15.9}{\sqrt{10}}$$

$$CI = 55.3 \pm 9.86$$