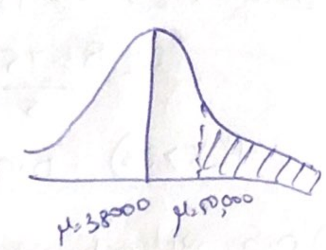
Distribution Assignment

① In pandas assignment

② Given $\mu = 38000$, $\sigma = 10000$



a) $P(X > 50000) = ?$

$$z = \frac{X-\mu}{\sigma} = \frac{50,000-38,000}{10,000} = 1.2$$

$\therefore P(X > 50,000) = 1 - 0.8849 = 0.1151 \approx 0.11$

$\therefore$ No. of firms $= 2000 \times 0.11 = 220$

b) $P(38500 < X < 41000) = ?$

$$z_1 = \frac{38500-38000}{10,000} = 0.05 \qquad z_2 = \frac{41000-38000}{10000} = 0.3$$

$P_1 = 0.5199$ $\qquad$ $P_2 = 0.5120$

$\therefore P(38500 < X < 41000) = P_1 - P_2 = 0.5199 - 0.5120 = 0.0079$

c) $P(30,000 < X < 50000) = ?$

$$z_1 = \frac{30000-38000}{10000} = -0.8 \qquad z_2 = \frac{50000-38000}{10000} = 1.2$$

$P_1 = 0.2119$ $\qquad$ $P_2 = 0.8849$

$\therefore P(30000 < X < 50000) = P_2 - P_1 = 0.8849 - 0.2119 = 0.673$

$\therefore$ No. of firms $= 0.67 \times 2000 = 1340$ firms

③ Given, $n = 20$, options $= 4$

P(exactly 5 wrong answers) $= ?$

Here, $r = 5$ $\quad p = 3/4$ $\quad q = 1/4$

$\therefore P = {}^{20}C_5 (3/4)^5 (1/4)^{15} =$

④ Given, $\lambda = 4$, $r = 0$

$$\text{Probabity} = \frac{e^{-\lambda}\lambda^r}{r!} = \frac{e^{-4}(4)^0}{0!} = \frac{e^{-4}}{1} = e^{-4}$$

(5) a) Given, $\lambda = 3$, $r = 0$

$$\text{probability} = \frac{e^{-\lambda}\lambda^{r}}{r!} = \frac{e^{-3}(3^{0})}{0!} = e^{-3}$$

b) $P(X>=2) = ?$, Here $r=2$

$$= 1 - P(X<2)$$

$$= 1 - e^{-\lambda}\sum_{i=0}^{r}\frac{\lambda^{i}}{i!}$$

$$= 1 - e^{-3}\sum_{i=0}^{2}\frac{(3)^{i}}{i!}$$

$$= 1 - e^{-3}\left[\frac{3^{0}}{0!} + \frac{3^{1}}{1!} + \frac{3^{2}}{2!}\right] = 1 - e^{-3}[1+3+3] = 1 - 7e^{-3}$$

(6) Given, $p = 20\% = 1/5 \Rightarrow q = 4/5$

$$P(X=4) = p^{1}q^{4-1}$$

$$= (1/5)(4/5)^{3}$$

The average no. of inspections to get first defective is mean of Geometric distribution

$$\mu = 1/p = 1/(1/5) = 5$$

(7) Given, $p = 0.3 \Rightarrow q = 0.7$, $n = 5$

$$P(X<=2) = ?$$

$$= \sum_{i=0}^{r} {}^{5}C_{r}\, p^{r} q^{n-r}$$

$$= ({}^{5}C_{0}\, p^{0} q^{5}) + ({}^{5}C_{1}\, p^{1} q^{4}) + ({}^{5}C_{2}\, p^{2} q^{3})$$

$$= {}^{5}C_{0}(0.3)^{0}(0.7)^{5} + {}^{5}C_{1}(0.3)^{1}(0.7)^{4} + {}^{5}C_{2}(0.3)^{2}(0.7)^{3}$$

$$= 0.8369$$

(8) Given, $\mu = 70$, $\sigma^{2} = 200$, $n = 10$, Max = 800 kg

for 10 adults, weight = 700 $\sigma^{2} = 2000 \Rightarrow \sigma = \sqrt{2000}$

$$z = \frac{X - \mu}{\sigma} = \frac{700 - 800}{\sqrt{2000}} = 2.24$$

$$P = 0.98$$

∴ 10 people can safely reach in elevator

If there are twelve adults

weight $= 12 \times 70 = 840$ $\sigma^2 = 2400 \Rightarrow \sigma = \sqrt{2400}$

$$z = \frac{840-800}{\sqrt{2400}} = \frac{-40}{48.98} \approx -0.81$$

$P = 0.20$

It wouldn't be much safe with 12 people

(9) a) Given, $n=50$, $p = 1/2$, $q = 1/2$, $r=20$

$P(X=20) = 50C_{20} (1/2)^{20} (1/2)^{30} =$

b) if $p = 1/4$, $q = 3/4$

$P(X=20) = 50C_{20} (1/4)^{20} (3/4)^{30} =$

(10) Given, $p = 30\% = 0.3 \Rightarrow q = 0.7$, $n=6$

$P(X=2) = 6C_2 (0.3)^2 (0.7)^{6-2} = 0.324$

(11) PDF of exponential distribution is

$$f(x) = \begin{cases} \lambda e^{-\lambda x} & ; x \geq 0 \\ 0 & ; x < 0 \end{cases}$$

Efficiency of typing is $\lambda = 6/77$

Memoryless property of exponential distribution is

$P(T > s+t \mid T > s) = e^{-\lambda t}$

$= P(2 \text{ errors in } 322 \text{ word report})$

$= e^{-(6/77) \times 245}$

where
$s = 77$ words
$t = 322 - 77$
$= 245$ words

(12) Given, $p = 5\% = 0.05 \Rightarrow q = 0.95$, $n=20$

i) $P(X<1) = nC_0 \, p^0 q^{n-0} = 20C_0 (0.5)^0 (0.95)^{20} = 0.35$

ii) $P(X<=1) = nC_0 \, p^0 q^{n-0} + nC_1 p^1 q^{n-1}$

$= 2C_0 (0.5)^0 (2.95)^{20} + 20C_2 (0.5)^1 (0.95)^{19} = 0.735$

iii) $P(X<=2) = \cancel{20} \; P(X<=1) + 20C_2 (0.5)^2 (0.95)^{18} = 0.9246$

(13) Given, $p=0.05 \Rightarrow q=0.95$.

i) $P(X=2, n=5) = 5C_2 (0.05)^2 (0.95)^{5-2} = 0.0214$

ii) $P(X=2, n=2) = 2C_2 (0.05)^2 (0.95)^{2-2} = 0.0025$

iii) $P(X>=1, n=4) = 1 - P(X<1)$

$= 1 - 4C_0 (0.5)^0 (0.95)^{4-0} = 0.1854$

(14) Given, $p=0.2 \Rightarrow q=0.8$, $n=15$

i) $P(X=2) = 15C_2 (0.2)^2 (0.8)^{15-2} = 0.231$

ii) $P(X>=1) = 1 - P(X<1)$

$= 1 - 15C_0 (0.2)^0 (0.8)^{15-0} = 0.965$