Hypothesis Testing Assignment

① Given, $n = 256$, $\sigma = 0.65$

i) $H_0 \rightarrow \mu = 2.75$

$H_1 \rightarrow \mu \neq 2.75$

ii) $SE = \frac{\sigma}{\sqrt{n}} = \frac{0.65}{\sqrt{256}} = 0.04$

iii) With $\alpha = 0.05$, the critical regions can be set as 1.96 standard scores above and below mean

iv) $z = \frac{X - \mu}{SE} = \frac{2.85 - 2.75}{0.04} = 2.5$

As $z > z_{critical}$, we reject null hypothesis

② Given, $\sigma = 4.5$, $n = 100$, $X = 52.8$, $\alpha = 0.05$

$z = \frac{X - \mu}{\sigma / \sqrt{n}} = \frac{52.8 - 52}{\sqrt{100}}$

$z = 1.777$

$H_0 \rightarrow \mu = 52$

$H_1 \rightarrow \mu > 52$

At $\alpha = 0.05$, $z_{critical} = 1.65$

As $z > z_{critic}$, null is rejected.

③ Given, $\sigma = 8$, $n = 50$, $X = 32.5$

$H_0 \rightarrow \mu = 34$ $H_1 \rightarrow \mu > 34$ (-ve tail test)

$SE = \frac{\sigma}{\sqrt{n}} = \frac{8}{\sqrt{50}} = 1.131$

$z = \frac{X - \mu}{SE} = \frac{32.5 - 34}{1.131} = -1.32$

At $\alpha = 0.01$, $z_{critical} \cong 2.57$

$\therefore$ Null is accepted.

(4) Given $N_1 = 300$ $X_1 = 120$ $S_1 = 0.53$

$N_2 = 700$ $X_2 = 140$ $S_2 = 0.20$

Null $\rightarrow \mu_1 - \mu_2 < 0.10$ Alternative $\rightarrow \mu_1 - \mu_2 \geq 0.10$

$$z = \frac{(\hat{P}_1 - \hat{P}_2) - D}{\sqrt{\frac{P_1(1-P_1)}{n_1} + \frac{P_2(1-P_2)}{n_2}}} = \frac{(0.53-0.2) - (0.1)}{\sqrt{\frac{(0.53)(0.53+1)}{300} + \frac{(0.2)(1-0.2)}{700}}} = \frac{0.23}{\sqrt{0.00112}}$$

$z = 6.886$

$z_{critical} = 3.09$

So, Null is rejected.

(5) Given, $n = 100 \Rightarrow$ Expected frequencies $= 100/4 = 25$

Actual	Expected	A-E	$(A-E)^2$	$(A-E)^2/E$
41	25	16	256	10.24
19	25	-6	36	1.44
24	25	-1	1	0.04
16	25	-9	81	3.24

$$\chi^2 = \sum \frac{(A-E)^2}{E} = 10.24 + 1.44 + 0.04 + 3.25 = 14.96$$

$\chi^2_{critical}$ (at 0.05 with 3DF) $= 9.82$.

As $\chi^2 > \chi^2_{critical}$, the data suggests that voters do not prefer four candidates equally.

(6) Given,

						total	Mean
A1	80	79	81	70	84	400	80
A2	90	76	88	82	89	425	85
A3	82	68	73	71	81	375	75
							80 $\rightarrow$ Mean of Means

$H_0 \rightarrow \mu_1 = \mu_2 = \mu_3$

$$f \text{ statistic} = \frac{(SSB/m-1)}{(SSW/m(n-1))}$$

$$SSB = 5(80-80)^2 + 5(85-80)^2 + 5(75-80)^2 = 0 + 5(25) + 5(25)$$

$= 250$

$$SSW = (80-80)^2 + (79-80)^2 + (81-80)^2 + (70-80)^2 + (84-80)^2$$
$$+ (90-85)^2 + (76-85)^2 + (88-85)^2 + (82-85)^2 + (89-85)^2$$
$$+ (82-75)^2 + (68-75)^2 + (73-75)^2 + (71-75)^2 + (81-75)^2$$

$$= 0+1+1+100+16+25+81+9+9+16+49+49+4+16+36$$

$$= 412$$

Here $m=3$, $n=5$

$$F = \frac{(250/2)}{(412/12)} = 0.10$$, $F_{critical} = 3.49$ $(\alpha = 0.05)$

As f statistic is low, null is accepted

(7) Given, $\sigma = 20$, $n=200$, $X = 147$, $\alpha = 0.05 \rightarrow z_{critical} = 1.65$

$H_0 \rightarrow \mu \leq 145$ $\quad H_1 \rightarrow \mu > 145$

$$z = \frac{X-\mu}{\sigma/\sqrt{n}} = \frac{147-145}{20/\sqrt{200}} = 1.14$$

As $z < z_{critical}$, Null is accepted with 95% confidence

(8) Given, $\sigma = 100$, $X = 147$

$H_0 \rightarrow \mu \leq 145$ $\quad H_1 \rightarrow \mu > 145$

$$z = \frac{X-\mu}{SE} = \frac{147-145}{100} = 0.02$$

Since, $z < z_\alpha$, we accept null hypothesis.

(9) Given measurements are 70, 69, 73, 68, 71, 69, 71

i) $H_0 \rightarrow \mu = 72$ $\quad H_1 \rightarrow \mu \neq 72$

ii) $\bar{x} = 70.143$, $\sigma = 1.676$, $n=7$

$$z = \frac{\bar{X}-\mu}{\sigma/\sqrt{n}} = \frac{70.143-72}{1.676/\sqrt{7}} = -2.9315$$

iii) Null is rejected at 10%, 5% and 1%.