Analysis of a Robotic System with Two DOF using Haar Wavelet

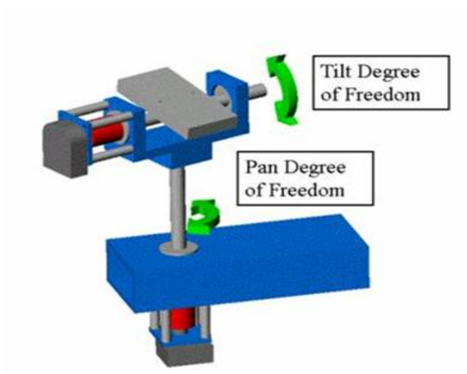
Atul Kumar Pandey, Monika Mittal
Department of Electrical Engineering
National Institute of Technology, Kurukshetra
Haryana, India
atuliec@gmail.com, monika _mittalkkr@rediffmail.com

Abstract— This paper deals with the stability analysis of a robotic system, popularly known as pan tilt platform (PTP), with two degrees of freedom (DOF) using Haar wavelet. It has two revolute joints combining tilt mechanism with pan mechanism. Dynamic model of this robotic system is obtained using Newton-Euler equation. The stability analysis of its dynamic model is carried out using Haar wavelet for the first time. The objective of this paper is to achieve computational savings in the process. The obtained computational savings are tabulated and bar graphed to show comparison with the analytical technique.

Keywords— Two DOF robotic system (Pan and tilt mechanism), linear time invariant system, stability analysis, operational matrices, Haar wavelet

I. INTRODUCTION

Various embodiments have a camera which may be controlled by one or more motors in a base of the camera. Cables and other components may be used to manipulate the camera lens through the side arms of the camera [1]. This arrangement forms a two DOF robotic manipulator called Pan and Tilt mechanism. The tilt mechanism can be rotated about a tilt axis supported on the pan mechanism having one DOF as shown in Fig.1. The tilt mechanism transferred torque to the camera by belt pulley arrangement[2,3].



† Courtesy: http://cats-fs.rpi.edu/~wenj/ECSE446S06/project_overview.html

Fig. 1. Three dimentional view of PTP

Pan arrangement can be also rotated about a pan axis which has one revolute joint (R) as shown in Fig.1. Pan mechanism is used to support the tilt mechanism for achieving desired position and orientation. Pan arrangement has also one DOF with one revolute joint(R). Pan mechanism rotates about a pan axis actuated by a pan motor.

Pan-tilt cameras are often used as components of wide-area surveillance system [2]. A two DOF robotic zoom camera used for surveillance is shown in Fig.2. It is necessary to calibrate these cameras in relation to one another, for obtaining a consistent representation of the entire space. The tilt and pan mechanism is modelled using Newton-Euler equation[3]. The tilt and pan mechanism has been analysed and simulated using lead and PD compensators [2,3].

† Courtesy: www.directindustry.com/ **Pan-tilt**-zoom **camera** / PTZ

Fig. 2. Two DOF robotic zoom camera

Orthogonal functions reduces the computation time for complex as well as simple dynamic system analysis. Some of orthogonal functions such as Block pulse functions [4], Walsh functions [5], Fourier functions [6], Haar functions [9] have been extensively applied for piecewise constant solution to different problems in terms of differential equation, response analysis, optimization and identification of linear and nonlinear systems.The specifications are settling time (Ts) less than 0.5 seconds, steady state error within ±2% and percentage overshoot (%OS) below 22% [3].

The paper is organized as follows: In section II dynamic and state space model of pan and tilt mechanism is discussed. Section III introduces Haar wavelet. Section IV describes analysis of linear time invariant system (LTI) using Haar Wavelet. Section V demonstrates the comparison of solutions using Haar wavelet and analytical methods. Section VI deals with future scope and other applications of this method.

978-1-4799-6046-0/14/$31.00 ©2014 IEEE

II. Background and Modeling of PTP

A. Dynamic Model For Tilt Mechanism

The dynamic model of tilt mechanism is obtained by Newton-Euler equation under certain assumptions [2,3].

$$\tau = J_{eff}\ddot{\theta} + f_v\dot{\theta} + f_c\ \mathrm{sgn}(\dot{\theta}) + mgl.\sin(\theta) \tag{1}$$

Where τ=Torque, J_{eff} =Effective inertial load, f_v = Viscous friction, f_c= Coulomb friction, θ = Angle between the force (mg) and arm length l . Now stability analysis of the dynamic equation is done after neglecting sinusoidal and signum function nonlinearities. Transfer function has been obtained using Jacobean matrix in [2] as :

$$G(s) = \frac{31.24954}{s^2 + 0.1578s} \tag{2}$$

The tilt mechanism is marginally stable as one of the system poles lies at the origin. Also, its response does not meet the required performance criteria which is reported to be compensated by a lead compensator as shown in Fig.3. The objective of the lead compensator is to keep settling time less than 0.5 second for the unity feedback system .

Transfer function with compensator is obtained as [2]:

$$T(s) = \frac{829.9s + 8.299}{s^3 + 39.99s^2 + 836.1s + 8.299} \tag{3}$$

The equivalent state space model is obtained for State variables $x_1 = \theta, x_2 = \dot{\theta}, x_3 = \ddot{\theta}$ and $\dot{x}_1 = \dot{\theta}, \dot{x}_2 = \ddot{\theta}, \dot{x}_3 = \dddot{\theta}$.

$$\dot{x} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -8.299 & -836.1 & -39.99 \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u \tag{4}$$

$$y = \begin{bmatrix} 8.2990 & 829.9 & 0 \end{bmatrix} x$$

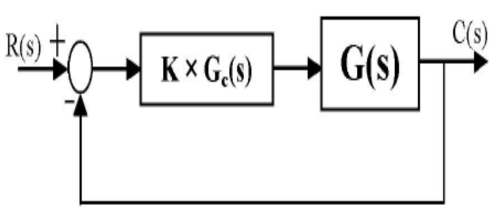


Fig. 3. Closed loop system with compensator

B. Dyanamic Model And Compensator For Pan Mechanism

Similar process can be applied for obtaining transfer function of the pan mechanism in [2,3] as :

$$G(s) = \frac{2.3964}{s^2 + 0.01677s}, \quad T(s) = \frac{848.9s + 8.489}{s^3 + 40.01s^2 + 849.6s + 8.489} \tag{5}$$

State space model for the pan mechanism is also given below using same state variables and procedure .

$$\dot{x} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -8.489 & -849.60 & -40.01 \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u \tag{6}$$

$$y = \begin{bmatrix} 8.489 & 848.9 & 0 \end{bmatrix} x$$

The response analysis for closed loop transfer functions is obtained using Haar wavelet which is described in next section.

III. Brief Review of Haar Wavelet

The orthogonal set of Haar functions is defined as a group of square waves with magnitude ±1 in some intervals and zeros elsewhere [7]. These zeros make the Haar transform faster than other square functions such as Walsh's [4].

$$h_n(t) = h_1(2^j(t - k/2^j)), \tag{7}$$

$$n = 2^j + k,\ j \geq 0, 0 \leq k \leq 2^j,\ n, j, k \in Z$$

First curve $h_0(t)=1$ during the interval $0 \leq t \leq 1$ and is known as scaling function. The second term $h_1(t)$ is the mother wavelet and other curves are generated from it via translations and dilations i.e. $h_1(t)$ is compressed to the half interval (0,1/2) to generate $h_2(t)$. $h_3(t)$ is the same as $h_2(t)$ but delayed from it by 1/2. Similarly, $h_2(t)$ is compressed to quarter interval to generate $h_4(t)$.

Any function y(t) which is square integrable in the interval [0,1) can be expanded in a Haar series [8].

$$y(t) = c_0 h_0(t) + c_1 h_1(t) + \ldots\ldots\ldots\ldots = c_m^T H_m(t) \tag{8}$$

$$c_i = 2^j \int_0^1 y(t) h_i(t) dt \tag{9}$$

For m=4, $H_m(t)$ is calculated by taking the samples of first four Haar functions as follows[12].

$$H_4(t) \overset{\Delta}{=} \begin{bmatrix} h_0(t) & h_1(t) & h_2(t) & h_3(t) \end{bmatrix}^T \tag{10}$$

$$H_4(t) \overset{\Delta}{=} \begin{bmatrix} h_0(t) \\ h_1(t) \\ h_2(t) \\ h_3(t) \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 \end{bmatrix} \tag{11}$$

The Haar coefficient c_i can be obtained by [3] , however it is more convenient to evaluate it by matrix inversion as:

$$c_m^T = y(t).H_m^{-1} \tag{12}$$

A. Operation Matrix Of Integration

The integration of Haar wavelets can be expanded into Haar series with Haar operation matrix of integration P_m defined in [9] as:

$$\int_0^t H_m(t)dt = P_m . H_m(t) \qquad t \in [0,1) \tag{13}$$

where the square matrix P_m is defined as:

$$P_m = \left[\int_0^t H_m(\tau)d\tau \right] H_m^{-1} \tag{14}$$

Matrix P_m can be also calculated by innovative non-recursive formulation in [9] as:

$$P_m = H_{m.} . Q_{Bm} . H_m^{-1} \tag{15}$$

where Q_{Bm} is the operation matrix of integration [4] for block pulse function at any resolution m.

IV. Analysis of Linear Time Invariant System via Haar Wavelet [7]

A linear system is described by the state equation:

$$\dot{x}(t) = Ax(t) + Bu(t) \text{ , } x(0) = x_0 \tag{16}$$

$$y(t) = Cx(t) + Du(t) \tag{17}$$

Assume that u(t) is a square integral in the interval $0 \le t \le 1$. Haar series expansion of u(t) and $\dot{x}(t)$ can be expressed as :

$$u(t) = G.H(t), \quad \dot{x}(t) = F.H(t) \tag{18}$$

Integration of $\dot{x}(t)$ provides x(t).

$$x(t) = \int_0^t \dot{x}(t) + x_0 = F\int_0^t H(t)dt + x_0 = F.P.H(t) + x_0 \tag{19}$$

Substituting (18) and (19) into (16), we obtain:

$$F.H(t) = A.F.P.H(t) + A.x_0 + B.G.H(t) \tag{20}$$

$$F\text{-}A.P.F = [Ax_0, 0, 0\ldots.., 0] + B.G = G_1 \tag{21}$$

$$\left[f_1, f_2 \ldots\ldots f_{m\text{-}1}\right]^T = \left[I - A \otimes P^T\right]^{-1} . [g_1, g_2, \ldots\ldots g_{m\text{-}1}] \tag{22}$$

where $\otimes$ is Kronecker product defined in [10]. Results obtained in (22) can be used to calculate y(t) from (17) with few matrix operations.

The analysis of the two DOF robotic system using Haar wavelet and analytical methods [11] is discussed in the next section. It is demonstrated that Haar wavelet analysis is computationally more efficient as compared to analytical analysis for a step input.

V. Results

In order to achieve the allowable specifications for the two DOF robotic system, the compensated closed loop transfer function and its state space model for pan and tilt mechanism is described in section II. The model of pan and tilt mechanism gives desired position and orientation of the camera.

The responses of both pan and tilt mechanism are obtained by analytical method using MATLAB for time interval [0, 1]. These are shown in Fig. 4 and Fig. 5, respectively. Same responses are obtained using Haar wavelet algorithm described in Section IV.

Haar wavelet response of tilt mechanism Y_{tilt} is obtained using (17) for m=32 as :

$$Y_{tilt} = [0.1108 \;\; 0.4288 \;\; 0.7814 \ldots\ldots\ldots\ldots\ldots\ldots..]_{1x32}$$

Similarly, Haar wavelet response of pan mechanism Y_{pan} is obtained using (17) for m=32 as :

$$Y_{pan} = [0.1131 \;\; 0.4372 \;\; 0.7947 \ldots\ldots\ldots\ldots\ldots\ldots\ldots]_{1x32}$$

For comparison, obtained Haar responses are superimposed on analytical responses for the respective mechanism in Fig. 4 and Fig. 5.

Design values of steady state error and % OS are tested by using encoder and Optoschmitt sensors in the real time. It is observed from Fig. 4 and Fig. 5 that settling time and steady state error meet the desired performance values. The given system model with controller, gives the desired orientation of camera with a slight difference in the settling time (0.01) and steady state error (0.001). This is due to neglecting nonlinearity terms.

It is also clear from Fig. 4 and Fig. 5 that similar results are obtained using Haar wavelet method which is expected to give more accurate results at higher resolutions.

Computational time for obtaining responses at different resolutions using analytical and Haar wavelet methods are shown in Table I and Table II, respectively for tilt and pan mechanisms. It is observed from Table I and Table II that for all resolutions, computational time in Haar wavelet method is less as compared to those in analytical method for both mechanisms.

Comparison of the computational times in both analytical and Haar wavelet methods are also shown by bar graphs in Fig. 6 and Fig. 7, respectively for tilt and pan mechanisms.

It is evident from Fig. 6 and Fig. 7 that computational time is about 40% less in Haar wavelet method as compared to analytical method for all resolutions. This trend is consistent even at higher resolutions.

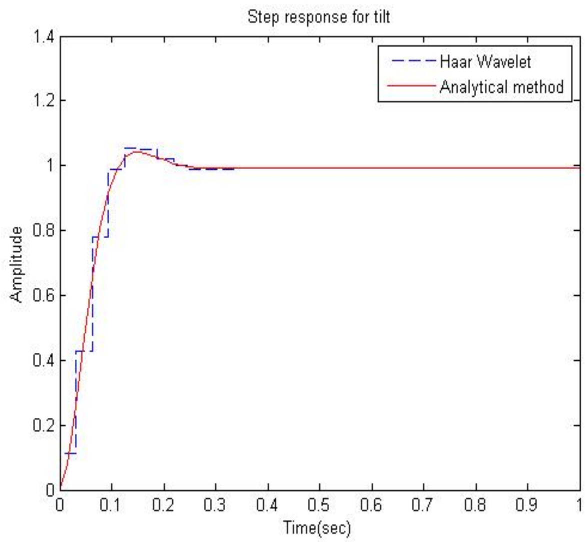


Fig. 4. Step response for tilt mechanism

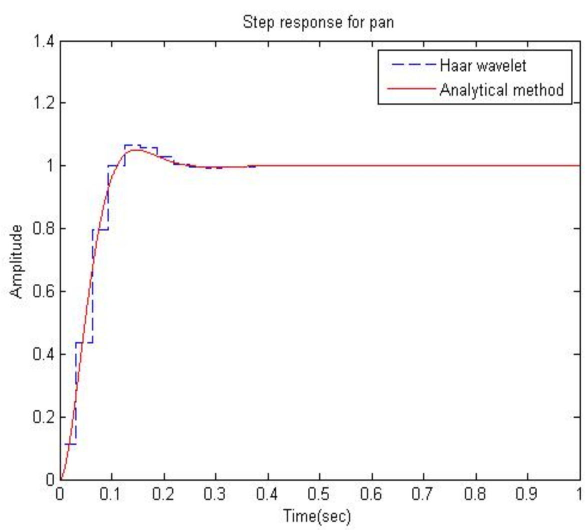


Fig. 5. Step response for pan mechanism

TABLE I. COMPUTATIONAL TIME FOR TILT MECHANISM ANALYSIS

Resolution	m=8	m=16	m=32	m=64	m=128
Computational Time via Haar Wavelet (sec)	0.44045	0.45474	0.46165	0.487760	0.50469
Computational Time via Analytical method(sec)	0.76760	0.78012	0.78673	0.79395	0.80893

TABLE II. COMPUTATIONAL TIME FOR PAN MECHANISM ANALYSIS

Resolution	m=8	m=16	m=32	m=64	m=128
Computational Time via Haar Wavelet(sec)	0.44491	0.45249	0.46024	0.48217	0.50118
Computational Time via Analytical method(sec)	0.75670	0.76793	0.77724	0.78926	0.81258

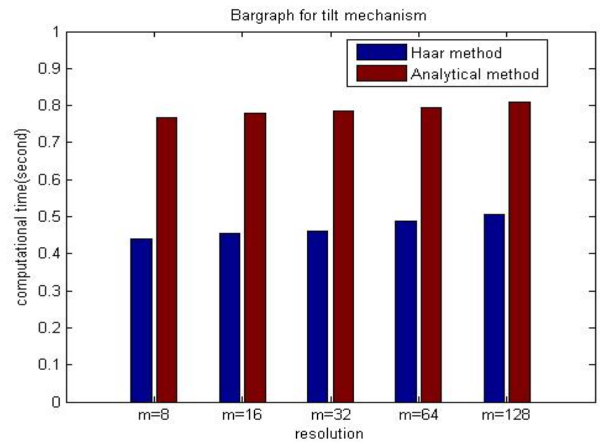


Fig. 6. Bar graph showing computational saving for tilt

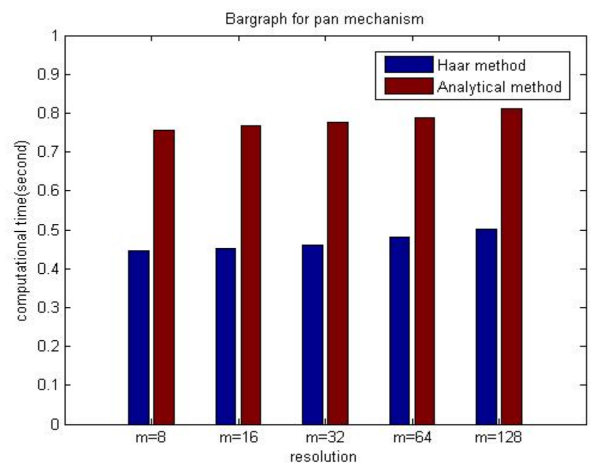


Fig. 7. Bar graph showing computational saving for pan

VI. CONCLUSION

The Haar wavelet method is successfully applied for the analysis of pan and tilt mechanism. This method has been applied to analyze a robotic system with two DOF for the first time as seen from the literature survey. A comparison of computation time in analytical and Haar wavelet is also shown. Lesser computation time is obtained using Haar method as compared to analytical method for both tilt and pan mechanisms at different resolutions. Computational efficiency has been demonstrated using MATLAB R2013a .

In many practical applications, computational savings and control with higher precision, achieved using Haar wavelet method, without the need of excessive computer memory become very important.

REFERENCES

[1] James Davis , Xing Chen ,"Calibrating pan-tilt cameras in wide area surveillance networks" IEEE International Conference on Computer Vision(ICCV2003).

[2] Imran S. Sarwar, Afzaal M. Malik, "Stability analysis and simulation of a two DOF robotic system based on linear control system" 15th International conference on mechatronics and machinevision(2008).

[3] Imran S. Sarwar, Afzaal M. Malik, "Modelling analysis and simulationof a Pan Tilt Platform based on linear and nonlinearsystems",IEEE China , Accepted ,Oct 2008.

[4] K. Maleknejad , M. Shahrezaee , H. Khatami, "Numerical solution of integral equations system of the second kind byBlock–Pulse function" Elsevier Applied Mathematics and Computation 166 (2005) 15–24.

[5] Chen C. F. ,Hsiao C. H., "A state space approach to Walsh series solution of linear system", Int. Znt.JControl,34(1981) 557-584.

[6] Sifuzzaman M., Islam M. R., Ali M. Z. , "Application of Wavelet Transform and its Advantages Compared to Fourier Transform", Journal of Physical Sciences, Vol. 13, (2009) 121-134 .

[7] Chen, C.F., Hsiao, C.H.: Haar wavelet method for solving lumped and distributed-parameter systems.IEEE Proc., Control TheoryAppl.144(1) (1997).

[8] Prof . Akanksha Gupta Prof. Monika Mittal , Dr. Lillie Dewan "Haar Wavelet Based Approach for State Analysis of BiomedicalSystems" IEEE Transaction.

[9] Monika Garg · Lillie Dewan, "Non-recursive Haar Connection Coefficients BasedApproach for Linear Optimal Control"Received: 27 September 2010 / Accepted: 1 December 2011SpringerScience John W. IEEE transaction on circit and system.

[10] John W. Brewer,"Kronecker and matrix calculation in system Theory",IEEE transaction on circit and system .

[11] Norman S. Nise, "Control systems engineering", Wiley student edition, Fourth edition, pp. 515-524, 2004.

[12] Monika garg, Lillie Dewan, "A Generalized Approach for StateAnalysis and Parameter Estimation of Bilinear Systems using Haar Connection Coefficients", World Academy of Science, Engineering andTechnology Vol:5 2011-03-27 .