Modeling, analysis and simulation of a Pan Tilt Platform based on linear and nonlinear systems.

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***Abstract*—This paper deals with the modeling, analysis and controlled simulation (motion control) of a two degree of freedom pan tilt platform (PTP) for positioning or aiming a device. The PTP has two revolute joints. The PTP, with two degrees of freedom, is a device that makes it possible for the camera to point in a desired direction when mounted under a carrier such as an aircraft. The objective of this research work is to derive a linear and nonlinear control system of the PTP to point in a desired direction within allowable specifications. This paper covers the feedback control system with PD controllers for position control of pan and tilt mechanism. These PD controllers derived from linear system model are also used in nonlinear control system simulation. The nonlinear control system developed in this paper is nearest to the physical system. To perform the analysis and simulation, all the parameters involved in the system dynamics were identified. The practicality of this research work can be extended to a broad range of applications. The most apparent use is in security, where there is a strong emphasis on reliably neutralizing threats without risking human life. The unmanned aerial systems (i.e. the PTP on unmanned aerial vehicle) can provide significant reductions in manpower and risk to humans for critical security roles. The role of PTP has been increased significantly in the last few years in the security and defense applications. Such applications include target acquisition, intelligence, surveillance and reconnaissance, border patrol, search and rescue, law enforcement, entertainment and environmental monitoring.**

I. Introduction

THE schematic of the PTP has two degrees of freedom as shown in fig. 1. The PTP includes a base, a rotatable pan mechanism and a rotatable tilt structure for supporting and orienting the device in a desired direction. It can continuously revolve about the pan axis and 90 degrees of motion range in the tilt axis. The pan mechanism (base) is rotatable about a pan axis, and the tilt mechanism is rotatable about a tilt axis supported on the base, that is perpendicular to the pan axis. A pan motor and a tilt motor drive the PTP. There is a gear on the shaft of the motor. Through the mechanism of gear, sprocket and belt, the torque is transferred to the structure.

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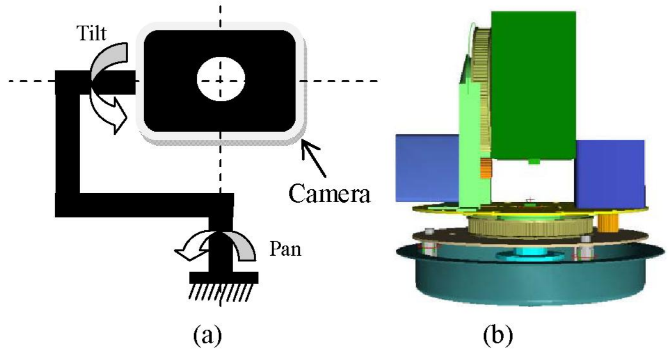


Fig. 1. (a) Schematics and (b) CAD of the PTP

We have built two types of models, a linear model and a non-linear model. In the linear model, the Coulomb friction, the centrifugal and the coriolis forces have been neglected. But in the nonlinear model, viscous friction, the Coulomb friction, mass, inertia, centrifugal and coriolis forces have been used. The parameters (friction, mass, inertia, coriolis forces) were identified from experiments or obtained from computer aided design for analysis and simulation purposes. The linear control system along with proportional-derivative (PD) controllers are analyzed and simulated in Section III. The nonlinear control system is analyzed and simulated with the PD controllers in Section IV. Both the models have been built according to the following allowable specifications

1. The settling time is expected to be within .1 to .5 seconds.
2. The steady state error can be tolerated within ± 2%.
3. The %overshoot is expected to be kept below 22%.

The transmissive optoschmitt sensor consisting of an infrared emitting diode facing an Optoschmitt detector encased in a black thermoplastic housing is used. It checks that %OS does not increase more than the desired value. The encoder attached with motor gives the position of motor shaft. The positioning accuracy of 0.0015 rad was achieved for the Pan Tilt Platform. This was achieved by using the DC-motor having encoder of resolution 4096 pulses/revolution. This ensures the steady state error stays below the desired value (<2%). The results are presented in section V.

II. System Linear Model

Linear system modeling involves the complex reality with the relatively simple models. The result of nonlinear system is closer to realistic system. But exact equivalence of the model and reality is difficult task. The model should match the real system as closely as possible. The linear model follows from the general nonlinear model under certain assumptions. The nonlinear model based on the Lagrange-Euler equation [2] is as follows.

$$u = M(\theta)\ddot{\theta} + C(\theta,\dot{\theta})\dot{\theta} + F(\dot{\theta}) + g(\alpha)$$

Here $u = \tau$ (torque) is control input and θ is feed-back input vector representing the joint orientation, $M(\theta)$ is a symmetric inertia matrix and $C(\theta,\dot{\theta})$ accounts for centrifugal and coriolis forces. The term $F(\dot{\theta})$ accounts for the viscous friction. The term $g(\alpha)$ accounts for gravity forces, here α is angle between the arm length and force (mg). The term $C(\theta,\dot{\theta})$ is effective when the mass centre begins to move away from the centre of rotation. The effect of centrifugal force becomes evident on the distributed mass of the body. Therefore, centre of PTP is located on the rotational axis of the body and term $C(\theta,\dot{\theta})$ is neglected in linear model. Thus the reduced non-linear system equation for single mechanism (joint) is given as

$$\tau = J_{eff}\ddot{\theta} + f_v\dot{\theta} + f_c sgn(\dot{\theta}) \tag{1}$$

Here, τ = Torque, J_{eff} = Effective inertia,
f_v = Viscous friction, f_c = Coulomb friction.

This is a general equation for both the joints and represents the non-linear model of pan and tilt mechanism. The viscous friction is a frictional force that resists objects in motion. The viscous friction is actually a property of the medium in which the motion of the object is occurring. Any fluid medium, such as the grease in the bearings or the air, has an internal resistance to flow, which is represented by the viscous friction. The effective inertial loads of the system are computed from the following relation [1].

$$J_{eff} = \frac{J_a + J_m}{n} + nJ_L \tag{2}$$

J_a = Actuator inertia, J_m = Gear inertia,
J_L = Load inertia, n = Gear ratio

Hence, equations (1) and (2) lead to

$$\tau = \left(\frac{J_a + J_m}{n} + nJ_L\right)\ddot{\theta} + f_v\dot{\theta} + f_c sgn(\dot{\theta}) \tag{3}$$

This nonlinear equation still needs to be linearized in order to develop a controller for the system. The nonlinear term in equation (3) is Coulomb friction. It is a frictional force that exists between two objects that are in contact. The close proximity of their surfaces acts to prevent the motion of the objects. This is a very small force with nonlinearity. Therefore, it is neglected in the linear model of the system. Linearization simplifies the pan/tilt model by focusing on each angle ($\theta_{pan}, \theta_{tilt}$) independently, for the pan and tilt mechanism. Thus, the linear model takes the following form

$$\tau = J_{eff}\ddot{\theta} + f_v\dot{\theta} \tag{4}$$

This second order differential equation can be expressed in state-space form by introducing the state variables: $x_1 = \theta, x_2 = \dot{\theta}$, with the derivatives as $\dot{x}_1 = \dot{\theta}, \dot{x}_2 = \ddot{\theta}$. Thus the state equations are

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = -\frac{f_v}{J_{eff}}x_2 + \frac{u}{J_{eff}} \tag{5}$$

The output equation is

$$y = x_1 \tag{6}$$

Hence the state space model in vector-matrix form is as follows.

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ 0 & -\frac{f_v}{J_{eff}} \end{bmatrix} x + \begin{bmatrix} 0 \\ \frac{1}{J_{eff}} \end{bmatrix} u \tag{7}$$

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} x$$

Here

$$x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}, \dot{x} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix}, u = \tau, A = \begin{bmatrix} 0 & 1 \\ 0 & -\frac{f_v}{J_{eff}} \end{bmatrix},$$

$$B = \begin{bmatrix} 0 \\ \frac{1}{J_{eff}} \end{bmatrix}, C = \begin{bmatrix} 1 & 0 \end{bmatrix} \tag{8}$$

These state-space equations can be solved if the coefficients f_v and J_{eff} are known. These coefficients are determined as follows.

A. *Parametric identification*

The viscous friction has been determined experimentally. The experimentally determined values of viscous friction are presented in table I.

Table I
Viscous friction (Nms/rad)

	Positive	**Negative**
***f_v* (tilt)**	0.0013	0.0023
***f_v* (pan)**	0.0055	0.0045

The forward and reverse values of the viscous friction were averaged to one value to work with system/state-space models. To find the effective inertial loads in the system, we have used a combination of calculation and measurements. The inertial load of the pan and tilt mechanism were analyzed in the CAD model. We have determined the inertial load on each axis in CAD software by adding all the components with the proper weights/densities. The gear inertia was determined separately in Pro-E and the actuator inertia was determined from the motor datasheet. The effective load on each axis is calculated by using equation (2), and is shown in table II.

Table II
Inertial loads for pan and tilt mechanism

	J_L $(Kg\,m^2)$	J_m $(Kg\,m^2)$	J_a $(Kg\,m^2)$	n	J_{eff} $(Kg\,m^2)$
Pan mechanism	0.0750	4×10^{-7}	1.6×10^{-6}	$\frac{120}{21}$	**0.4286**
Tilt mechanism	0.0055	4×10^{-7}	1.6×10^{-6}	$\frac{120}{21}$	**0.0314**

B. *Transfer functions*

The transfer function for tilt mechanism is found using (7), (8), tables 1, 2, and $G(s) = C(sI - A)^{-1}B$, and is [9] is given below

$$G(s) = \frac{31.6188}{s^2+0.1581s} \tag{9}$$

The transfer function for pan mechanism is determined similarly, and is given below

$$G(s) = \frac{2.3333}{s^2+0.0042s} \tag{10}$$

III. Linear System with PD Controller

The linear tilt system and linear pan system were controlled by PD controllers as shown in fig. 3.

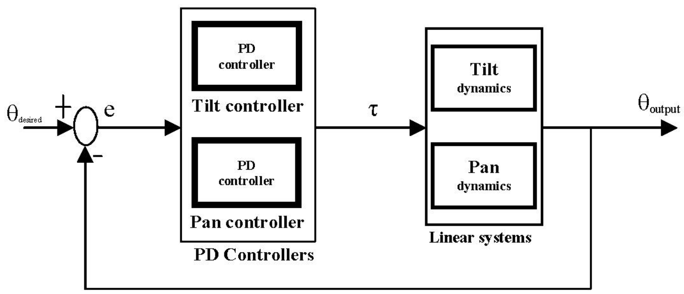


Fig. 3. PTP feedback control system

The tilt system analysis and simulation is shown in section III (A). Similarly, the pan system analysis and simulation is presented in section III (C). The compensated tilt system (tilt system and PD controller) analysis and simulation via the root locus method is presented in section III (B). The compensated pan system (pan system and PD controller) analysis and simulation via the root locus method is presented in section III (D).

A. Tilt system response

The transfer function of tilt system is found in equation (9), section II. According to the root locus technique [9], tilt system has two branches of root locus, symmetrical with respect to the real axis, real-axis segment is [0,-.1581], starting points are the open-loop poles at 0 and -.1581, ending points are the open-loop zeros at ∞ (infinity),∞, real-axis intercept is at -0.079, angle of asymptotes are 90°, 270° and breakaway point is at -0.079. The result of root-locus method, based on simulation performed in Matlab is shown in fig. 5 a. According to the allowable specification; the %OS has to be less than 22%. We have used %OS equal to 5% for which damping ratio (ξ) is equal to 0.6901. The gain (K) has to be designed as shown schematically in fig. 4.

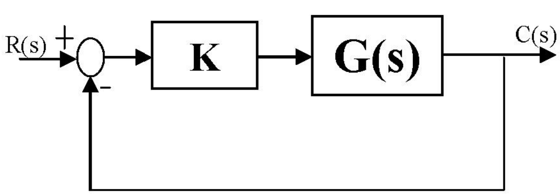


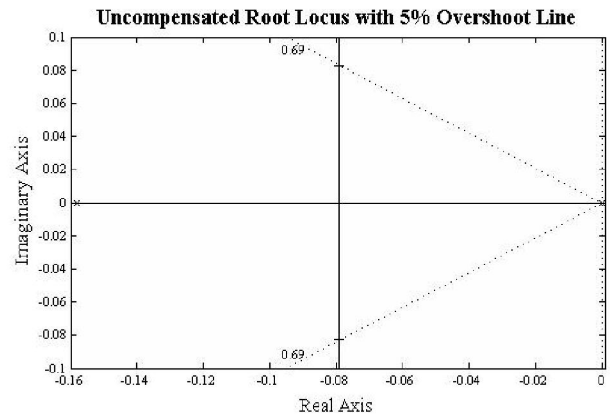
Fig. 4. Closed loop system with gain K

The open-loop transfer function using equation (9) is given as

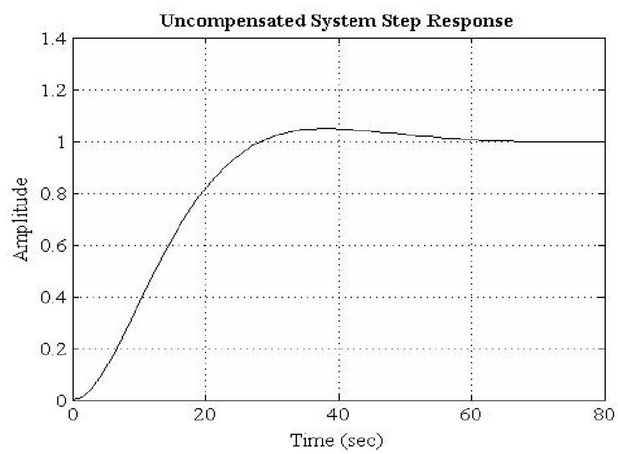
$$KG(s) = \frac{31.6188K}{s^2+.1581s} \tag{11}$$

The root locus for the uncompensated system with a damping ratio of 0.69 is represented by a radial line shown in fig. 5(a). We have found dominant pair of poles at -0.0790 ± 0.0830i along the damping ratio line for a gain K = 0.00041494. The uncompensated system step response is shown in fig. 5(b). The closed loop transfer function (T(s)) based on (11) is as follows

$$T(s) = \frac{0.01314}{s^2+0.1581s+0.01314} \tag{12}$$



(a)



(b)

Fig. 5. Uncompensated system (a) Root locus with radial line (b) step response

As noticed in fig. 5(b), the setting time (50.6 sec) and the steady-state error (12.0342) far exceed the desired performance values. This deficiency has to be compensated by designing the control system.

B. Response of tilt system with PD controller

The objective of a PD controller is to drive the T_s to less than 0.5 sec for the unity feedback system. The compensated system is schematically shown in fig. 6.

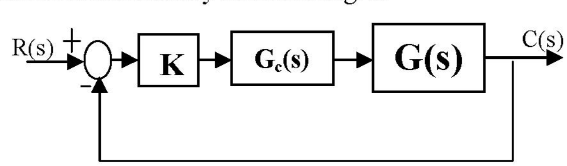


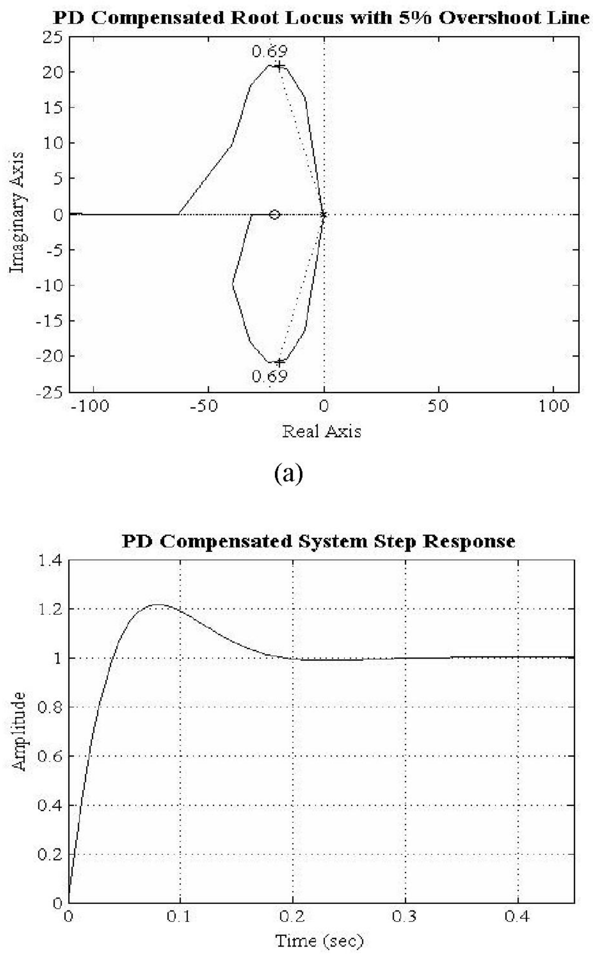
Fig. 6. Closed loop system with compensator

The PD controller is found by using [9]. The PD controller ($G_c(s)$) in Laplace domain is equal to $G_c(s) = s + 21.0808$. The open-loop transfer function resulting for fig. 6 is determined, and is given below

$$KG(s)G_c(s) = K\frac{31.62s+666.6}{s^2+0.1581s} \quad (13)$$

The root locus for the compensated system is shown in fig. 7 (a). A damping ratio of 0.69 is represented by a radial line drawn on the root-locus. We have found dominant pair of poles at -19.3414 ±20.9295i along the damping ratio line for a gain K = 12.184. The compensated system step response is shown in fig. 7 (b). The closed loop transfer function (T(s)) based on (13) is as follows

$$T(s) = \frac{38.21\, s + 805.6}{s^2+38.37\, s + 805.6} \quad (14)$$



(a)

(b)

Fig. 7. Compensated system (a) Root locus with radial line (b) step response

As noticed in fig. 7(b), the setting time (0.2068 sec) and the steady-state error (1.9467×10^{-4}) meet the desired performance values.

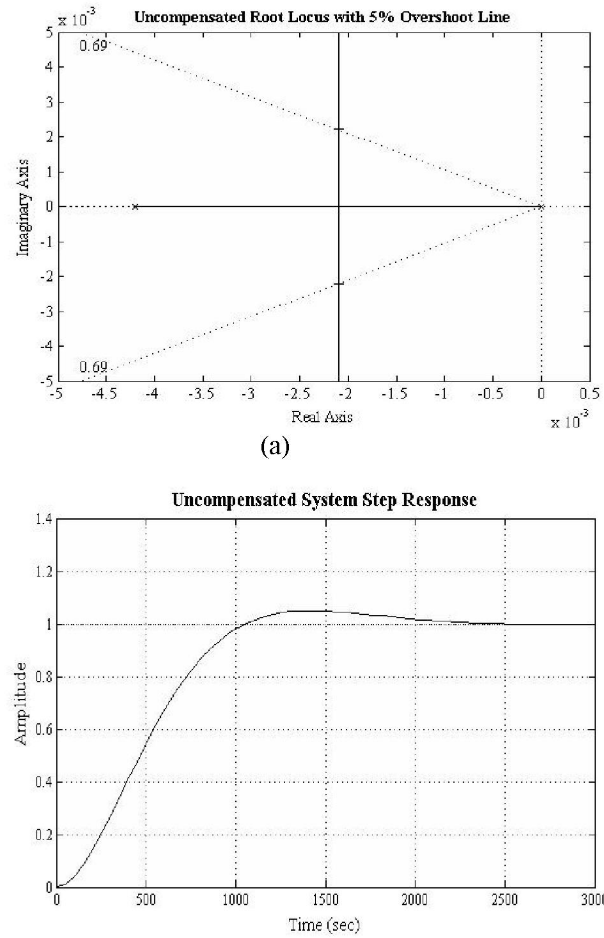
C. *Pan system response*

The transfer function of pan system is found in section II; equation (10). According to the root locus technique, pan system has two branches of root locus, symmetrical with respect to the real axis, real-axis segment [0,-.0042] , starting points of open-loop poles at 0, -.0042, ending points of open-loop zeros at ∞ (infinity), ∞, real-axis intercept at -0.0021, angle of asymptotes equal to $90^{\circ}, 270^{\circ}$ and breakaway point at -0.0021. The result of root-locus method, based on simulation performed in Matlab is shown in fig. 8 (a). We have used %overshoot equal to 5% for which damping ratio (ξ) is equal to 0.6901. Here gain (K) to be designed via root locus technique is as follows.

$$KG(s) = \frac{2.3333K}{s^2+0.0042s} \quad (15)$$

The root locus for the uncompensated system is shown in fig. 8 (a). A damping ratio of 0.69 is represented by a radial line drawn on the root-locus. We have found dominant pair of poles -0.0021± 0.0022i along the damping ratio line for a gain K = 3.9526e-006. The uncompensated system step response is shown in fig. 8 (b). The closed loop transfer function (T(s)) based on (15) is as follows.

$$T(s) = \frac{9.223*10^{-6}}{s^2+0.0042s+9.223*10^{-6}} \quad (16)$$



(a)

(b)

Fig. 8. Uncompensated system (a) root-locus with radial line (b) step response

As noticed in fig. 8 (b), the setting time (1.9048×10^{3} sec) and the steady-state error (455.3986) far exceed the desired performance values. This deficiency has to be compensated by designing the control system.

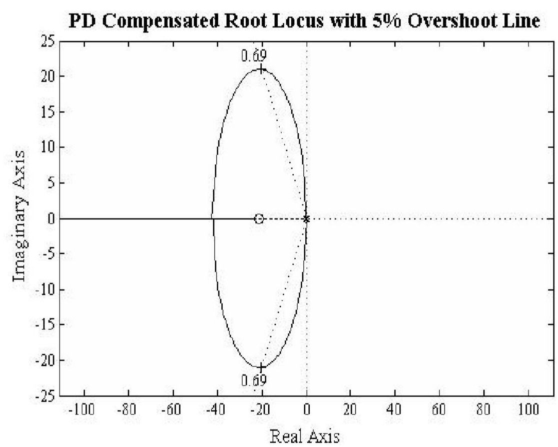
D. Response of Pan system with PD Controller

The objective of a PD controller is to derive the T_s to less than 0.5 sec for the unity feedback system. The compensated system is shown in fig. 6. The PD controller is found by using [9]. The PD controller ($G_c(s)$) in Laplace domain is equal to $s + 21$.The open-loop transfer function for fig. 6 is given as

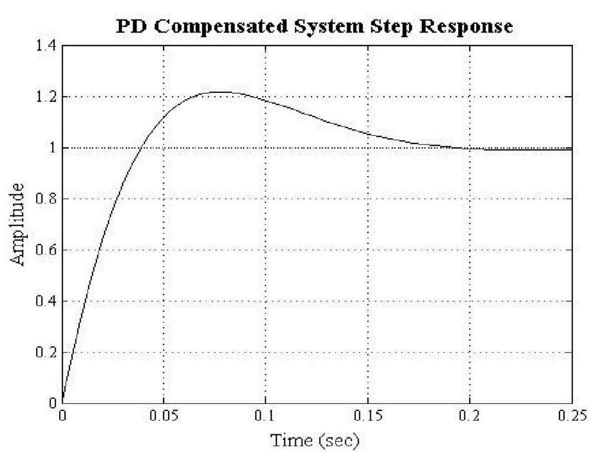
$$KG(s)G_c(s) = K\frac{2.333\,s + 49}{s^2+0.0042s} \tag{17}$$

The root locus for the compensated system is shown in fig. 9 (a). A damping ratio of 0.69 is represented by a radial line drawn on the s-plane. We have found dominant pair of poles -20.0502 ±20.9761i along the damping ratio line for a gain K = 17.1843. The compensated system step response is shown in fig. 9 (b). The closed loop transfer function (T(s)) from (17) is given as

$$T(s) = \frac{40.1\,s + 842}{s^2+ 40.1\,s + 842} \tag{18}$$



(a)



(b)

Fig. 9. Compensated system (a) root-locus with radial line (b) step response

As noticed in fig. 9 (b), the setting time (0.1995 sec) and the steady-state error (1.9467×10^{-4}) meet the desired performance values.

IV. Nonlinear Control System

The nonlinear model [1] of the PTP including both the Coulomb friction and the gravity effect is given below.

$$\tau = \left(\frac{J_a+J_m}{n} + nJ_L\right)\ddot{\theta} + f_v\dot{\theta} + f_c sgn(\dot{\theta}) + mgl\sin\alpha \tag{19}$$

$$\alpha = Angle\ between\ the\ arm\ length\ and\ force(mg)$$

Equation (19) is applied separately for both the mechanisms as presented in fig. 10. The required parameters (mass and arm length) were identified from CAD. The Coulomb fraction was identified experimentally to implement the (19). The mass property (physical design) of PTP is summarized in table III below:

Table III
Mass property data

	Surface Area (m^2)	*m* Mass (Kg)	Volume (m^3)	*l* Length (m)
Pan mechanism	0.2008	1.1452	.000224	0.050
Tilt mechanism	0.0858	0.7395	.000402	0.050
PTP	0.2866	1.8847	.000631	-

The Coulomb friction determined accurately can be accounted for and cancelled in the model. The Coulomb friction has been determined experimentally and is given in table IV.

Table IV
Coulomb friction (Nm)

	Positive	Negative
f_c *(tilt)*	0.0500	-0.0550
f_c*(pan)*	0.0400	-0.0405

The values obtained experimentally and available in CAD are presented in table I, II, III and IV. These values are used in nonlinear model simulation performed in Simulink as explained by Simulink model in fig. 10. The PD controllers were applied as shown in fig. 10. The control law given by [9] is $PD = K_P e + K_D \dot{e}$. We have used values of K, K_D and K_P achieved from section III. For pan and tilt mechanisms, the controller gains determined in section III are given in table V.

Table V
Controller Gains

Gains	Pan Mechanism	Tilt Mechanism
K	17.1843	12.184
K_D	1	1
K_P	21.00	21.08

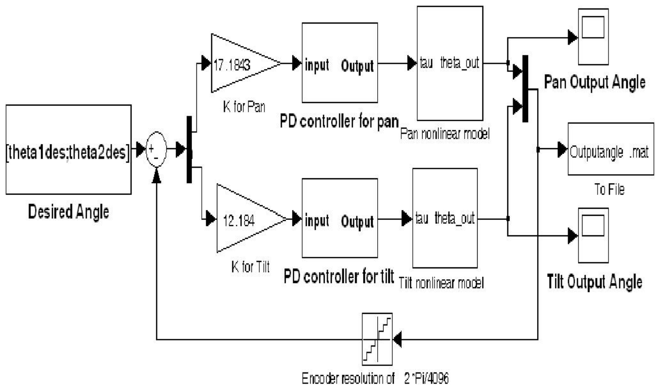
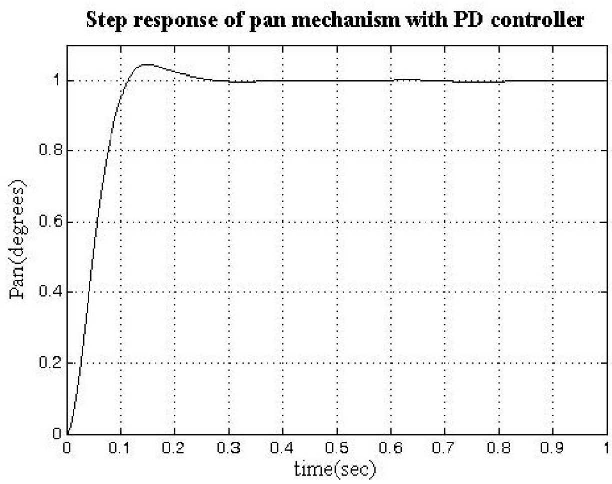
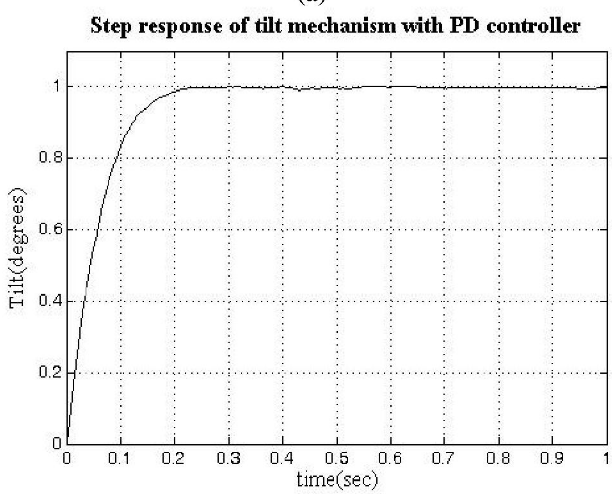


Fig. 10. Pan Tilt Platform Simulation

The PD controllers work with the same gains as found in section III for the PTP in the presence of nonlinearities. The results in fig. 11 verified the results obtained in the section III.



(a)



(b)

Fig. 11. Output of (a) Pan (b) Tilt Platform

V. RESULTS

Linear and nonlinear modeling of the PTP and its simulation based on feedback control system using the PD controllers for both the models leads to the following conclusions.

- The comparison of linear and nonlinear simulation with PD controllers is shown in table VI. The value of T_s achieved was 0.2 seconds, which is less than the design constraint of 0.5 seconds. Our design leads to a steady-state error of 10^{-3}, which is much less than design constraint of ±2%. Thus our design leads to very reliable results.

Table VI
System models with PD controller

	Linear model		Nonlinear model	
	T_S (seconds)	e_{SS}	T_S (seconds)	e_{SS}
Tilt Mechanism	0.2068	1.9467×10^{-4}	0.20	4.4367×10^{-3}
Pan Mechanism	0.1995	4.9881×10^{-6}	0.21	7.9351×10^{-3}

- It is possible to get the desired performance from the controlled system by choosing the appropriate values of parameters of PD controllers in an involved way based on a trail and error method [6-7]. However, we have obtained a better solution by a more effective and reliable technique based on modeling and analysis.
- The linear and non-linear model of system with PD controllers gives us desired orientation of camera with a slight difference in the settling time (10^{-2}) and steady-state error (10^{-3}). This is due to the effect of the Coulomb fraction and gravity.

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