

Chemical Engineering RAG System

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Question

give a numerical on conduction

Answer

Based on the provided context, a numerical example for steady one-dimensional heat conduction in a plane wall is given in Example 2–11 from "Heat transfer a practical approach" by Yunus A. Cengel and Yunus Cengel.

Problem Statement (Source 7, Page 106): Consider a large plane wall of thickness $L = 0.2$ m, thermal conductivity $k = 1.2$ W/m \cdot $^{\circ}$ C, and surface area $A = 15$ m². The two sides of the wall are maintained at constant temperatures of $T_1 = 120^{\circ}$ C and $T_2 = 50^{\circ}$ C, respectively. Determine: (a) The variation of temperature within the wall and the value of temperature at $x = 0.1$ m. (b) The rate of heat conduction through the wall under steady conditions.

Solution (Source 7, Page 106 & Source 5, Page 109):

Assumptions (Source 7, Page 106-107): 1. Heat conduction is steady (temperature does not vary with time). 2. Heat conduction is one-dimensional (significant in one dimension only, negligible in others). 3. Thermal conductivity is constant. 4. There is no heat generation within the wall.

Properties (Source 7, Page 107): The thermal conductivity is given as $k = 1.2$ W/m \cdot $^{\circ}$ C.

Analysis:

(a) Variation of temperature within the wall:

1. Formulate the differential equation and boundary conditions: For steady one-dimensional heat conduction in a plane wall with constant thermal conductivity and no heat generation, the heat conduction equation in the x -direction (normal to the surface) is (Source 7, Page 107, Eq. 2–17): $d^2T/dx^2 = 0$ The boundary conditions are the specified temperatures at the surfaces (Source 7, Page 107): $T(0) = T_1 = 120^{\circ}$ C (at $x = 0$) $T(L) = T_2 = 50^{\circ}$ C (at $x = L = 0.2$ m)

2. Obtain the general solution of the differential equation: Integrating the differential equation once with respect to x yields (Source 7, Page 107): $dT/dx = C_1$ Integrating a second time gives the general solution (Source 7, Page 107): $T(x) = C_1x + C_2$ where C_1 and C_2 are arbitrary integration constants.

3. Apply boundary conditions to determine constants: * At $x = 0$: $T(0) = T_1 \rightarrow 120^{\circ}$ C = $C_1(0) + C_2 \rightarrow C_2 = 120^{\circ}$ C * At $x = L$: $T(L) = T_2 \rightarrow 50^{\circ}$ C = $C_1(0.2 \text{ m}) + 120^{\circ}$ C $C_1(0.2 \text{ m}) = 50^{\circ}$ C - 120° C = -70° C $C_1 = -70^{\circ}$ C / $0.2 \text{ m} = -350^{\circ}$ C/m

Substituting C_1 and C_2 back into the general solution, the specific solution for the temperature variation within the wall is (Source 5, Page 109): $T(x) = (-350^{\circ}$ C/m) $x + 120^{\circ}$ C

Temperature at $x = 0.1$ m: Substituting $x = 0.1$ m into the temperature relation (Source 5, Page 109): $T(0.1 \text{ m}) = (-350^{\circ}$ C/m)(0.1 m) + 120° C = -35° C + 120° C = 85° C

(b) Rate of heat conduction through the wall under steady conditions:

The rate of heat conduction anywhere in the wall is determined from Fourier's law (Source 5, Page 109): $Q_{\text{wall}} = -kA(dT/dx)$

Since $dT/dx = C1 = -350 \text{ }^{\circ}\text{C/m}$ (from part a), and this is constant for steady one-dimensional conduction with constant k and no heat generation (Source 5, Page 109): $Q_{\text{wall}} = -kA(C1)$ $Q_{\text{wall}} = -(1.2 \text{ W/m} \cdot ^{\circ}\text{C})(15 \text{ m}^2)(-350 \text{ }^{\circ}\text{C/m})$ $Q_{\text{wall}} = 6300 \text{ W}$

The rate of heat conduction through the wall is 6300 W. As noted in the discussion of Example 2-11, under steady conditions, the rate of heat conduction through a plane wall is constant (Source 5, Page 109).

References

1. Heat transfer a practical approach (Yunus A. Cengel, Yunus Cengel) (Z-Library) (Page 131)
2. Heat transfer a practical approach (Yunus A. Cengel, Yunus Cengel) (Z-Library) (Page 291)
3. Heat transfer a practical approach (Yunus A. Cengel, Yunus Cengel) (Z-Library) (Page 314)
4. Heat transfer a practical approach (Yunus A. Cengel, Yunus Cengel) (Z-Library) (Page 81)
5. Heat transfer a practical approach (Yunus A. Cengel, Yunus Cengel) (Z-Library) (Page 109)
6. Heat transfer a practical approach (Yunus A. Cengel, Yunus Cengel) (Z-Library) (Page 42)
7. Heat transfer a practical approach (Yunus A. Cengel, Yunus Cengel) (Z-Library) (Page 106)
8. Heat transfer a practical approach (Yunus A. Cengel, Yunus Cengel) (Z-Library) (Page 141)