PHY 423/623: Computational Techniques & Programming languages

School of Physics, IISER Thiruvananthapuram

Vasanth, 2018

Problem Set 4: Due Apr 03, 2018

Note: Name the programs as 'qnxx_yourname.py' where 'xx' is the question number and 'yourname' is your own name. Put all the files a folder named as your name and upload it in the shared google drive folder.

1. Find the following integral numerically using $n = 4$ Gaussian Quadrature

$$I = \int_{-3}^{3} dx\, e^{-\frac{x^2}{2}}$$

2. Use the composite Simpson's $1/3$ rule using $m = 16$ panels to integrate

$$I = \int_{0}^{1} dx\, \ln(1 + x^2)$$

3. Consider the motion of satellite of mass m around a planet of mass M ($M \gg m$). The planet is fixed at the origin. The force of attraction is

$$F = \frac{GMm}{r^2}$$

Take $G = 1$. Take initial position and velocity of the satellite to be $(0, 2, 0)$ and $(1, 0, 0)$, respectively. Solve the Newton's equation numerically (improved Euler method) to find the trajectory of the satellite during the time interval $[0, 100]$. Take Euler step $h = 0.01$. The trajectory will be confined to the xy-plane. Show the trajectory graphically using `matplotlib`.

4. The equation of motion of a particle falling under gravity and acted by air drag is given by,

$$\frac{dp}{dt} = mg - kv^2$$

where $m = 0.01$ Kg is the mass of the particle, $k = 0.0001$ Kg/m is the drag coefficient. Momentum $p = mv$. The particle is released from rest at $t = 0$. Using the fourth order Runge-Kutta method, find the velocity of the particle as the function of time for $0 < t < 10$ secs. Make a plot of the $v(t)$ data.

5. Consider the general heat flow equation in a 2D metal plate,

$$\frac{\partial^2 T(x,y)}{\partial x^2} + \frac{\partial^2 T(x,y)}{\partial y^2} = \frac{1}{c^2}\frac{\partial T(x,y,t)}{\partial t}$$

We are given the Neumann boundary conditions where the *normal* derivative of the function at all points on the boundary are known.

Work out the formulation for numerical solution of the steady state heat equation. Apply the formulation to a problem where the for a square metal plate the normal derivate of temperature T_{grad} are given as,

$$
\begin{aligned}
T_{grad} &= +400\ C/m \quad \text{(left boundary)} \\
T_{grad} &= -100\ C/m \quad \text{(right boundary)} \\
T_{grad} &= -700\ C/m \quad \text{(top boundary)} \\
T_{grad} &= -200\ C/m \quad \text{(bottom boundary)}
\end{aligned}
$$

6. Consider the wave equation in 1D,

$$\frac{\partial^2 u(x,t)}{\partial t^2} = c^2 \frac{\partial^2 u(x,t)}{\partial x^2}$$

Solve the equation with forward difference method subject to the following initial conditions,

$$
\begin{aligned}
u(x,0) =& f(x) = \sin \pi a, \quad \leq x \leq b \\
u'(x,0) =& g(x) = 0, \quad a \leq x \leq b \\
u(a,t) =& l(t) = 0, \quad t \geq 0 \\
u(b,t) =& r(t) = 0, \quad t \geq 0
\end{aligned}
$$

Take $c = 2$, $a = 0$, $b = 1$, $t \in [0,1]$.

—○—