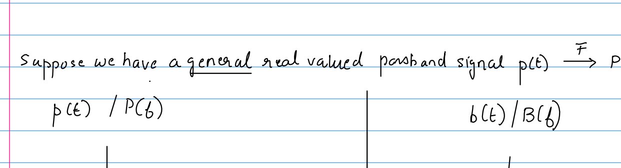
Complex baseband representation of passband signals & systems

Suppose we have a <u>general</u> real valued passband signal $p(t) \xrightarrow{\mathcal{F}} P(f)$



$P(f) = B(f-f_c) + B^*(-f-f_c)$

$p(t) = 2\left(b_r(t)\cos(2\pi f_c t) - b_i(t)\sin(2\pi f_c t)\right)$

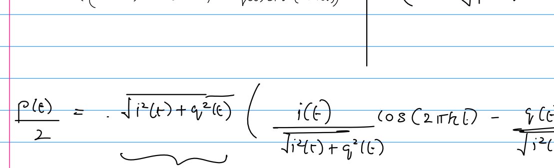
$= 2\left(i(t)\cos(2\pi f_c t) - q(t)\sin(2\pi f_c t)\right)$

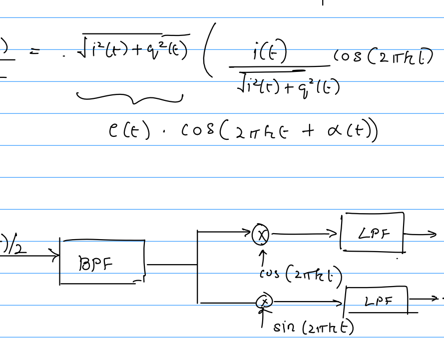
$B(f) = P(f)\,\mathbb{I}_{\{f\geq 0\}}(f_c \rightarrow)$

$\frac{1}{2}\left(p(t) + j\hat{p}(t)\right)e^{-j\pi f_c t} = b(t)$

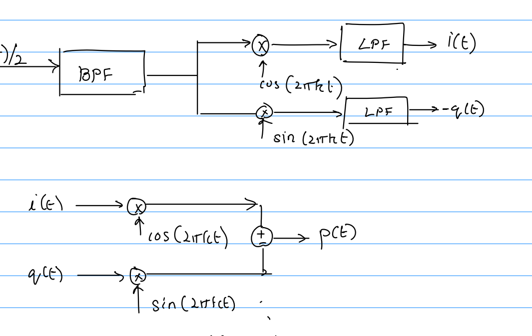
$$\frac{p(t)}{2} = \underbrace{\sqrt{i^2(t)+q^2(t)}}\left(\frac{i(t)}{\sqrt{i^2(t)+q^2(t)}}\cos(2\pi f_c t) - \frac{q(t)}{\sqrt{i^2(t)+q^2(t)}}\sin(2\pi f_c t)\right)$$

$$e(t)\cdot\cos(2\pi f_c t + \alpha(t))$$





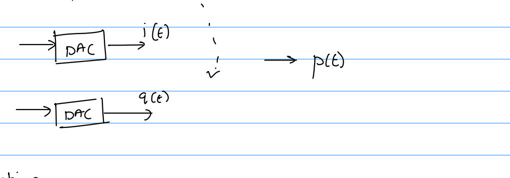
Demonstration



$i(t) \longrightarrow \boxed{ADC} \longrightarrow i_n$

$q(t) \longrightarrow \boxed{ADC} \longrightarrow q_n.$

$p(t) = m(t) \times \cos(2\pi f_c' t)$

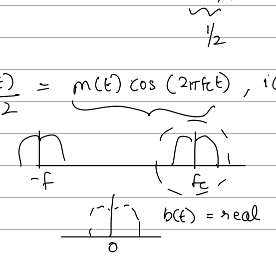


problem : if $p(t) = m(t)\cdot\cos(2\pi f_c' t)$ what will be $i(t)$ and $q(t)$?

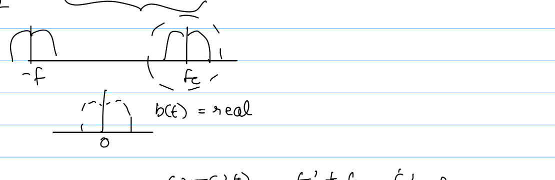
a) $\frac{p(t)}{2} = \frac{\underbrace{\cos(2\pi f_c t)}}{2}$ $i(t)$ and $q(t)$?

$= \underbrace{i(t)}_{1/2}\cos(2\pi f_c t) - \underbrace{q(t)}_{0}\sin(2\pi f_c t)$

b) $\frac{p(t)}{2} = \underbrace{m(t)\cos(2\pi f_c t)}$, $i(t)$ and $q(t)$?



c) $\frac{p(t)}{2} = \cos(2\pi f_c' t)$, $f_c' \neq f_c$, $f_c' > f_c$



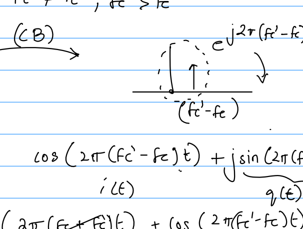
$\underbrace{\cos(2\pi(f_c'-f_c)t)}_{i(t)} + j\underbrace{\sin(2\pi(f_c'-f_c)t)}_{q(t)}$

$2\left(\cos(2\pi f_c' t) \times \cos(2\pi f_c t)\right) = \cancel{\cos(2\pi(f_c+f_c')t)} + \underbrace{\cos(2\pi(f_c'-f_c)t)}$

$2\left(\cos(2\pi f_c' t) \times \sin(2\pi f_c t)\right) = \underbrace{-\sin(2\pi(f_c'-f_c)t)}_{-q(t)}$

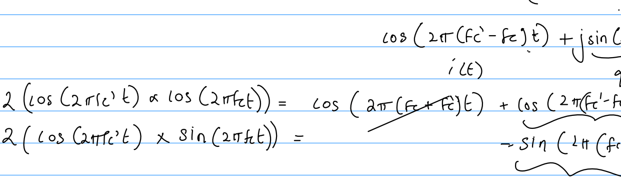
$i(t) = m(t)\cdot\cos(2\pi(f_c'-f_c)t)$

$q(t) = m(t)\sin(2\pi(f_c'-f_c)t).$

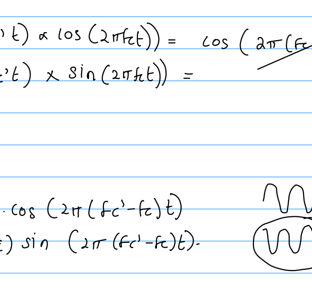


What we have learnt → CB rep for pass band signals

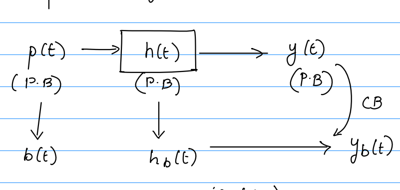
Recall



Consider a bandpass filter with response $H(f)$ like



$h(t)$ is a passband signal $\longrightarrow$ CB $\longrightarrow h_b(t)$



S.T $\left(p(t) = 2\mathrm{Re}\{\,b(t)\,e^{j2\pi f_c t}\}\right)$

$\left(b_r(t) + j\,b_i(t)\right)\left(\cos(2\pi f_c t) + j\sin(2\pi f_c t)\right)$

$y(t) = p(t) * h(t)$

$h(t) = 2\mathrm{Re}\{\,h_b(t)\,e^{j2\pi f_c t}\}$

let us try: $\left(b(t)\cdot e^{j2\pi f_c t}\right) * \left(h_b(t)\,e^{j2\pi f_c t}\right)$

$= \int_0^\infty h_b(\tau)\,e^{j2\pi f_c \tau}\cdot b(t-\tau)\,e^{j2\pi f_c(t-\tau)}\,d\tau$

$= e^{j2\pi f_c t}\cdot\left(\int_0^\infty h_b(\tau)\cdot b(t-\tau)\cdot d\tau\right)$

$= e^{j2\pi f_c t}\left\{\,b(t) * h_b(t)\right\}.$

$p(t) * h(t) = 2\mathrm{Re}\{\,b(t)\,e^{j2\pi f_c t}\} * 2\mathrm{Re}\{\,h_b(t)\,e^{j2\pi f_c t}\}.$

$= \left(b(t)\,e^{j2\pi f_c t} + b^*(t)\,e^{-j2\pi f_c t}\right) * \left(h_b(t)\,e^{j2\pi f_c t} + h_b^*(t)\,e^{-j2\pi f_c t}\right)$

$= e^{j2\pi f_c t}\{b(t) * h_b(t)\} +$
$b^*(t)\,e^{-j2\pi f_c t} * h_b(t)\,e^{j2\pi f_c t} +$
$b(t)\,e^{j2\pi f_c t} * h_b^*(t)\,e^{-j2\pi f_c t} +$
$b^*(t)\,e^{-j2\pi f_c t} * h_b^*(t)\,e^{-j2\pi f_c t}.$

$y(t) = p(t) * h(t) \propto \mathrm{Re}\left\{\left(b(t) * h_b(t)\right)e^{j2\pi f_c t}\right\}$