AV499 - AVD871 – Tutorial - 1

1) Prove whether the following discrete time random processes are Markov or not –

a) $X_1, X_2, X_3 \cdots,$ X_is are IID, each $X_i \sim f_X(x)$. Is (X_i) Markov

b) $X_1, X_2, X_3, \cdots,$ X_is are IID

$Y_1 = X_1$, and

$Y_2 = X_1 + X_2$, and

$Y_n = X_n + X_{n-1} \cdot \forall n.$

Is the process (Y_n) Markov ?

2) Suppose $X_1, X_2, \cdots$ is a discrete time Markov chain which is time homogeneous. The values that X_i can take - called the state space of the Markov chain - are $\{0, 1, 2\}$. The transition probability matrix of the Markov chain is given by

	0	1	2
0	0·7	0·2	0·1
1	0·5	0·4	0·1
2	0·1	0·3	0·6

Given that $X_1 = 1$, what is the joint probability distribution of X_5 and X_3 ?

3) Suppose you observe one run (or sample function) of a random process and see the following –

0 1 0 1 0 0 1 1 1 0 1 0 1 1 0 0 0 1 1.

Suggest an appropriate Markov chain model for this random process based upon this observation.

4) Consider a Markov decision process where the state space is $\{0, 1\}$ and action space is $\{1, 2\}$. (for both states) The transition probability matrices are

$$P^{(1)} = \begin{pmatrix} 0.1 & 0.9 \\ 0.5 & 0.5 \end{pmatrix} \qquad P^{(2)} = \begin{pmatrix} 0.5 & 0.5 \\ 0.5 & 0.5 \end{pmatrix}$$

The reward $R_t(s_t, a_t)$ is Gaussian distributed with a variance of 1 and mean μ_t being a function of s_t and a_t. See table below for what μ_t is

μ_0

s_0	a_0	μ_0
0	1	1
1	1	2
0	2	3
1	2	4

μ_1

s_1	a_1	μ_1
0	1	5
1	1	2
0	2	1
1	2	3

Solve the MDP $\max_{\pi} \mathbb{E}\left[R_0(s_0, A_0) + R_1(s_1, A_1)\right]$

where $s_0 = 1$, π is the set of functions π_0, π_1, $\pi_0(s_0) \rightarrow \{1, 2\}$ and $\pi_1(s_1) \rightarrow \{1, 2\}$.

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5) Suppose Alice receives a monthly salary of s_t in month t. She spends a_t amount of money and adds the remaining amount $s_t - a_t$ to her capital. The capital is invested at an interest rate of θ% per month. What should be the investment strategy that Alice should use to maximise the total consumption (spending) over M months?

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