Review:

Invariance ppty for WSS processes:

$(X(t)) \longrightarrow \boxed{h(t)} \longrightarrow (Y(t))$

WSS - - - - - - - ?

$(X(t))$ WSS $\Rightarrow$ a) $\mathbb{E}\, X(t) = \mu_X$ (a constant), b) $\mathbb{E}\, X(t_1) X(t_2) = R_X(t_1 - t_2)$

$\mathbb{E}\, Y(t) = ?$

$$\mathbb{E}\, Y(t) = \mathbb{E} \int X(\tau) \cdot h(t-\tau) \cdot d\tau = \int_{-\infty}^{\infty} \mu_X\, h(t-\tau) \cdot d\tau = \mu_X \int_{-\infty}^{\infty} h(u) \cdot du = \mu_Y.$$

$$\mathbb{E}\, Y(t_1) Y(t_2) = \mathbb{E}\left[\int_{-\infty}^{\infty} h(u) \cdot X(t_1 - u)\, du \cdot \int_{-\infty}^{\infty} h(v) \cdot X(t_2 - v) \cdot dv\right]$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h(u) \cdot h(v) \cdot \mathbb{E}\, X(t_1 - u) \cdot X(t_2 - v)\, du\, dv.$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h(u) \cdot h(v) \cdot R_X(t_1 - t_2 - u + v) \cdot du\, dv$$

{ from here it can be seen that R_Y is a function of $t_1 - t_2$

let $\omega = v - u$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h(v-\omega) \cdot h(v) \cdot R_X(t_1 - t_2 + \omega) \cdot d\omega\, dv$$

$$= \int_{-\infty}^{\infty} R_X(t_1 - t_2 + \omega) \cdot \int_{-\infty}^{\infty} h(v) \cdot h(v-\omega) \cdot dv \cdot d\omega$$

$\omega = -\omega$

or $$= \int_{-\infty}^{\infty} R_X(t_1 - t_2 - \omega) \cdot \underbrace{\int_{-\infty}^{\infty} h(v) \cdot h(v+\omega) \cdot dv}_{g(\omega)}\, d\omega$$

$$= \int_{-\infty}^{\infty} R_X(t_1 - t_2 - \omega) \cdot g(\omega)\, d\omega = (R_X * g)_{t_1 - t_2} = R_Y(t_1 - t_2)$$

so $(Y(t))$ is a WSS process.

H/W : Try this out for DTRPs also.

This class:

$R_Y(\tau) = (R_X * g)_\tau$

$R_X(\tau) \longrightarrow \boxed{g(\tau)} \longrightarrow R_Y(\tau)$

$$g(\omega) = \int_{-\infty}^{\infty} h(v) \cdot h(v+\omega) \cdot dv$$

H/W $g(\omega) = (h(v) * h(-v))_\omega$

$R_X(\tau) \xrightarrow{\mathcal{F}} S_X(f)$ $\qquad S_X(f) = \int_{-\infty} R_X(\tau) e^{-j2\pi f\tau} d\tau$

$g(\tau) \xrightarrow{\mathcal{F}} G(f)$

$S_X(f) \cdot G(f) = S_Y(f)$ where $R_Y(\tau) \xrightarrow{\mathcal{F}} S_Y(f)$

a) $R_X(\tau) = R_X(-\tau)$

$S_X(f)$ is real.

b) $G(f) = H(f) H^*(f) = |H(f)|^2$

$\Rightarrow S_Y(f) = |H(f)|^2 S_X(f)$

Instantaneous power.

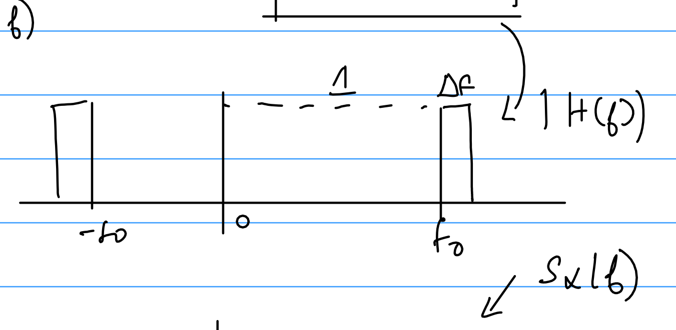
$(X(t))$ } $\rightarrow$ $X^2(t)$

avg is $\mathbb{E} X^2(t) = R_X(0)$

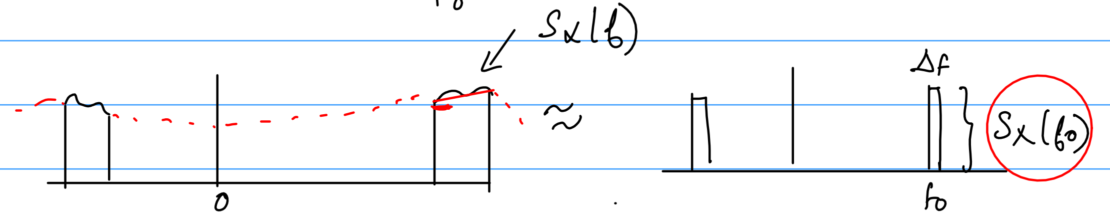
$R_X(\tau) = \int S_X(f) e^{j2\pi f\tau} df \Rightarrow R_X(0) = \int_{-\infty}^{\infty} S_X(f) \cdot df$

What exactly is $S_X(f)$?

$(X(t))$, $S_X(f)$ $\longrightarrow \boxed{\text{BPF}}$ ($H(f)$) $\longrightarrow$ $(Y(t))$, $S_Y(f)$



$S_Y(f)$?



$$\int_{-\infty}^{\infty} S_Y(f) \cdot df = 2\Delta f\, S_X(f_0) = \mathbb{E}\, Y(t)^2$$

power spectral density (PSD).

* PSD(o/p) = PSD(i/p) $\cdot |H(f)|^2$

Example:

$\longrightarrow$ $x(t) + N(t)$ $\boxed{\text{BPF}} \longrightarrow$ $x_0(t) + N_0(t)$