

AK499/AVD871 Tutorial problem. (note change/clarification).

Suppose Alice receives a monthly salary of S_t per month at month t . She spends a amount of money from the total amount of money or capital and invests the rest of the money on which she earns $\theta\%$ interest per month. What should be the investment strategy that will allow her to maximise her spending for H months.

suppose C_t is capital.

Total money in t th month is $C_t + S_t$.

spending is a_t ; $a_t \leq C_t$.

money left is $C_t + S_t - a_t$.
= money invested.

$$\text{so } C_{t+1} = (1 + \theta)(C_t + S_t - a_t)$$

← correction
from what was
written on board.

$$\text{maximise } \sum_{t=0}^{H-1} a_t$$

→ suppose $H = 1$

maximise a_0

$$a_0 \leq C_0 + S_0$$

so take $a_0 = C_0 + S_0$.

→ suppose $H = 2$.

maximise $(a_0 + a_1)$

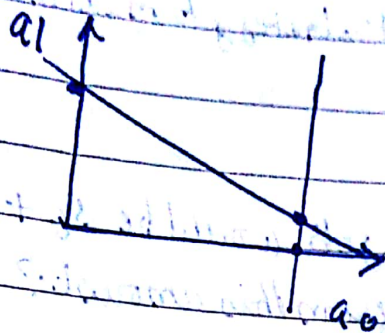
$$a_0 \leq C_0 + S_0, \quad a_1 \leq C_1 + S_1.$$

$$a_0 \leq C_0 + S_0, \quad a_1 \leq ((1 + \theta)(C_0 + S_0 - a_0) + S_1)$$

$$(\max a_0 + a_1)$$

$$a_1 + (1+\theta)a_0 \leq (1+\theta)(c_0 + s_0) + s_1$$

$$a_0 \leq (c_0 + s_0)$$



$$\text{if } a_0 = 0$$

$$a_1 = (1+\theta)(c_0 + s_0) + s_1$$

$$\text{if } a_1 = 0$$

$$a_0 = (c_0 + s_0)$$

$$\text{if } a_0 = (c_0 + s_0)$$

$$a_1 = (s_1)$$

should take $a_0 = 0$

$$\text{and } a_1 = (1+\theta)(c_0 + s_0) + s_1$$

we can define $V_0(c) = c + s_0$

$$V_1(c) = \max_a \{ a + V_0((1+\theta)(c + s_0 - a)) \}$$

$$= \max_a \{ a + (1+\theta)(c + s_0 - a) + s_1 \}$$

$$= \max_a \{ -a\theta + (c + s_0)(1+\theta) + s_1 \}$$

a should be 0.

$$V_1(c) = (c + s_0)(1+\theta) + s_1$$

$$V_2(c) = \max_a \{ a + V_1((1+\theta)(c + s_0 - a)) \}$$

$$= \max_a \{ a + ((1+\theta)(c + s_0 - a) + s_1)(1+\theta) + s_2 \}$$

again $a = 0$.

Save, get a lot of money and then spend it?