AV312 - Lecture 17

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Figures from “Communication Systems” by Haykin and “An Intro. to Analog and Digital Commn.” by Haykin and Moher

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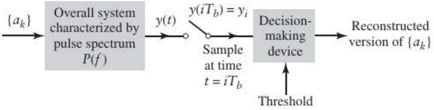
Review of last classes

- Intersymbol interference
- Nyquist bandwidth and channel

Today's class

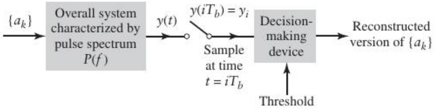
- Raised cosine pulse shaping
- Duobinary signalling
- Today's scribes are Chandu Lal and Ravi Kiran Reddy

An effective pulse shape $p(t)$



- $y(t) = \sum_{k=-\infty}^{\infty} a_k p(t - kT_b)$, where $p(t) = g(t) \star h(t) \star q(t)$

Intersymbol interference problem



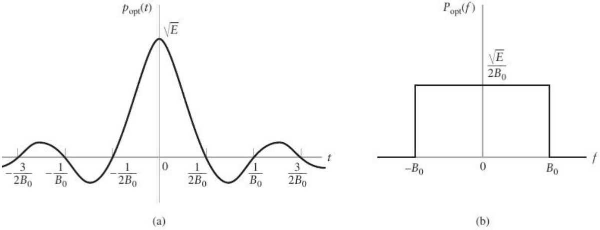
- At the sampling instants $y(iT_b)$ we have $y(iT_b) = \sum_{k=-\infty}^{\infty} a_k p((i-k)T_b)$ (notation: pulse is centered at zero)
- Suppose $y_i = y(iT_b)$ and $p_i = p(iT_b)$
- $y_i = \sum_{k=-\infty}^{\infty} a_k p_{i-k}$
- We need $y_i = p_0 a_i$ for all i. Let us say that $p_0 = \sqrt{E}$
- What we have is $y_i = \sqrt{E} a_i + \sum_{k \neq i} a_k p_{i-k}$

Nyquist channel

- If $y_i = p_0 a_i$ for every i then we require that

$$p_n = \begin{cases} \sqrt{E}, & \text{for } n = 0, \\ 0, & \text{otherwise.} \end{cases}$$

- Note that $p_n = p(nT_b)$
- Is it possible to get $P(f)$? Assuming that $P(f)$ is bandlimited
- Consider the choice of $p(t) = sinc\left(\frac{t}{T_b}\right)$
- With $B_0 = \frac{1}{2T_b}$ we have the following optimal pulse shape $p_{opt}(t)$



- The PAM system with $P_{opt}(f)$ is called the Nyquist channel
- The bandwidth B_0 is called the Nyquist bandwidth

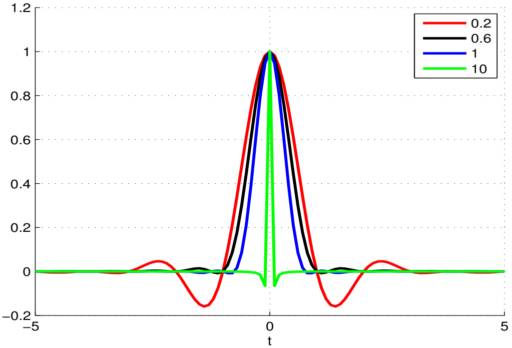
Raised cosine pulse shaping

- The problem with sinc pulses - $\frac{1}{t}$ decay
- How to increase the decay rate?

Raised cosine pulse shaping

- The problem with sinc pulses - $\frac{1}{t}$ decay
- How to increase the decay rate?
- *Damp* the sinc pulse using a window function
- Raised cosine pulse shape (actually damped sinc pulse shape)

$$p(t) = \sqrt{E} sinc(2B_0 t) \frac{cos(2\pi\alpha B_0 t)}{1 - (4\alpha B_0 t)^2}$$

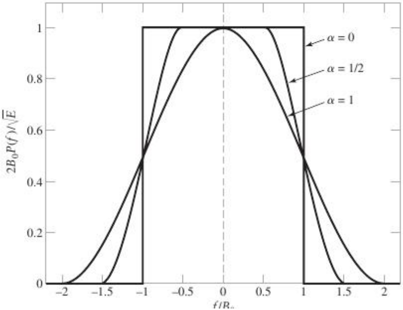


Raised cosine pulse shaping

- The F.T of $p(t)$ is

$$P(f) = \begin{cases} \frac{\sqrt{E}}{2B_0}, \text{ for } |f| \leq f_1, \\ \frac{\sqrt{E}}{4B_0}\left[1 + cos\left\{\frac{\pi(|f|-f_1)}{2(B_0-f_1)}\right\}\right], \text{ for } f_1 < |f| < 2B_0 - f_1, \\ 0, \text{ o/w.} \end{cases}$$

- $\alpha = 1 - \frac{f_1}{B_0}$. α is the roll-off factor.
- Bandwidth of the pulse is $2B_0 - f_1$ or $B_0(1+\alpha)$



Comparison

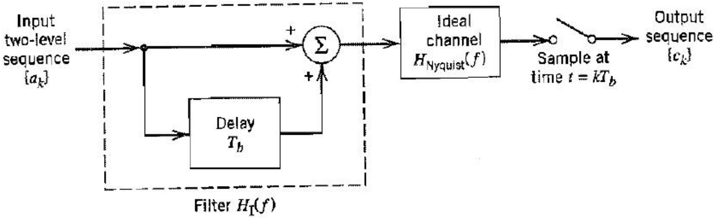
- Let $r_b = \frac{1}{T_b}$

Scheme	Bandwidth	Power	Rate	Timing Jitter
Rectangular	r_b	95%	r_b	Robust
Sinc	$\frac{r_b}{2}$	100%	r_b	Weak
Raised cosine	$\frac{r_b}{2}(1+\alpha)$	100%	r_b	less than Rect

- Read about square root raised cosine pulse shaping

Duobinary signalling

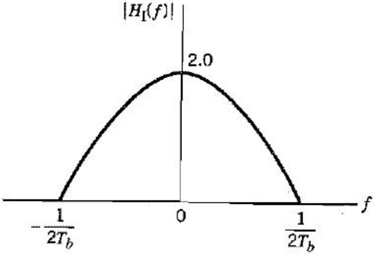
- Let the input bit sequence b_k be converted to a baseband PAM signal $a_k \in \{-1, 1\}$
- Let us think of the sequence a_k as being put into the following system

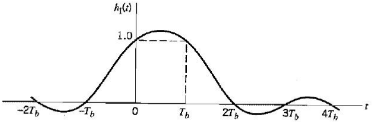


- What is the effective response of the system with a_k as input and c_k as output?

Duobinary response

- The effective response is $H_{nyquist}(f)(1 + e^{-j2\pi fT_b})$
- Or $2H_{nyquist}(f)cos(\pi fT_b)e^{-j\pi fT_b}$
- Note that $H_{nyquist}(f) = 1$ for $|f| \leq \frac{1}{2T_b}$ and 0 otherwise





Duobinary receiver

- $c_k = a_k + a_{k-1}$
- If $\hat{a}_{k-1}$ is the estimate of a_{k-1}, then $a_k = c_k - \hat{a}_{k-1}$
- Prone to error propagation
- Read about the pre-coding method to avoid error propagation from "Communication Systems"
- There are other forms of combining a_k in order to obtain other responses
- Read about the partial response signalling from "Communication Systems"

Zero-forcing equalization

- Recall the digital transmission system block diagram
- A transmit filter $G(f)$, a channel $H(f)$, and a receive filter $Q(f)$
- Let us assume that transmit filtering is not done
- We have $P(f) = H(f)Q(f)$
- We will consider a special form for $Q(f)$ - a linear transversal filter
- The impulse response of $Q(f)$ is $q(t) = \sum_{k=-N}^{N} w_k \delta(t - kT_b)$
- Then $p(t) = h(t) \star q(t)$

Zero-forcing equalization

- $p(t) = h(t) \star q(t)$
- Or $p(t) = \sum_{k=-N}^{N} w_k h(t - kT_b)$
- At the sampling instants $p_n = p(nT_b) = \sum_{k=-N}^{N} w_k h((n-k)T_b)$
- Let $h_n = h(nT_b)$
- Our requirement is

$$p_n = \begin{cases} \sqrt{E}, & \text{for } n = 0, \\ 0, & \text{otherwise.} \end{cases}$$

- Can we adjust w_k to satisfy these requirements?

Zero-forcing equalization

- Our requirement is

$$p_n = \begin{cases} \sqrt{E}, & \text{for } n = 0, \\ 0, & \text{otherwise.} \end{cases}$$

- We can adjust w_k so that

$$p_n = \sum_{k=-N}^{N} w_k h_{n-k} = \begin{cases} \sqrt{E}, & \text{for } n = 0, \\ 0, & \text{for } n = \pm 1, \pm 2, \ldots, \pm N. \end{cases}$$

- The receiver determines h_{n-k} via pilot sequence assisted training