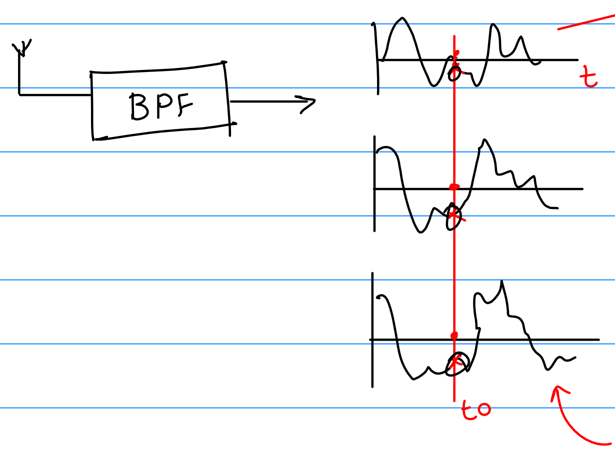
Random processes and Noise modelling. (review of last class)

we are motivated by the need to model noise.



$f(t)$

look at a specific time, say t_0

$f(t_0) \rightarrow$ value of this time function at a specific time can be modelled as a random variable

collection of random variables $(X(t))$

possible to fit a distribution to the random variable at any time t.

for example: $F_{X(t)} = \mathcal{N}(\mu, \sigma^2)$

ordered collection of random variables = Random process

because we have multiple random variables we need joint distributions to describe them.

e.g $X, Y \sim F_{XY}(x, y)$ analogous distribution for $(X(t))$?

$X(t_1), X(t_2) \sim F_{X(t_1)X(t_2)}(x_1, x_2)$

A mathematically "proper" way to describe these distributions:

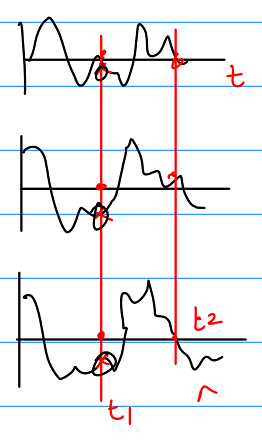
1) fix an integer m (positive)
2) fix $t_1, t_2, t_3 \dots t_m$, $t_1 < t_2 < t_3 < \dots t_m$ $\Big\}$ $\forall m, \forall (t_1 \dots t_m)$
3) $F_{X(t_1)X(t_2)\dots X(t_m)}(x_1, x_2 \dots x_m)$

Finite dimensional distribution (FDD)

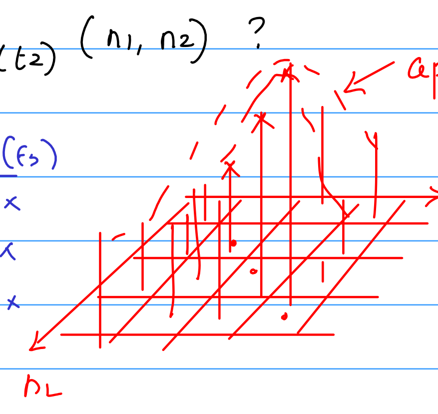
(set of distributions which specify the R.P)

for example: $(N(t))$ $F_{N(t)} \sim \mathcal{N}(0, 1)$

$m = 2$, $t_1 = 10$, $t_2 = 100$ $F_{N(t_1)N(t_2)}(n_1, n_2)$?



	$N(t_1)$	$N(t_2)$	$N(t_3)$
1	-1	0	x
2	-10	1	x
3	-20	0	x



approximate by std dist

repeat this for every possible t_1 and t_2

Getting FDDs is extremely difficult — for building the model
So usually some structure is assumed for the FDD.

a) Suppose the random variables are independent, i.e., $X(t_1)$ and $X(t_2)$, then $X(t_1) \perp X(t_2)$

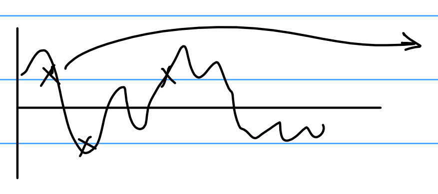
$F_{N(t_1)N(t_2)\dots N(t_m)}(n_1 \dots n_m) = F_{N(t_1)}(n_1) \times F_{N(t_2)}(n_2) \times \dots \times F_{N(t_m)}(n_m)$

called an independently distributed random process.

b) independent and identically distributed random processes. (IID)

e.g: Coin tossing (X_n) N tosses $X_1, X_2, \dots X_N$ $\in \{0, 1\}$

$F_{N(t_1)\dots N(t_m)}(n_1 \dots n_m) = F_N(n_1) \cdot F_N(n_2) \dots F_N(n_m)$



histogram $\approx F_N(n)$

if $F_N(n)$ is Gaussian then white Gaussian noise.

c) Markov random processes.

(x_0, v_0, a_0)

$X_1 = x_0 + v_0 + \frac{1}{2} a_0$
$V_1 = v_0 + a_0$
$A_1 = a_0$

$X_1, X_2, X_3 \dots$ (X_n)
$V_1, V_2, V_3 \dots$
$A_1, A_2, A_3 \dots$