AV499 - AVD871 - Tutorial Questions.

Formulation of optimization problems as Markov Decision problems.

1) Consider a service counter in your mess. Students queue up in front of the service counter for food service. Consider an observer who records the number of students waiting at the counter for service. The observer sees that every one minute a student may arrive at the counter with probability p and no students arrive with probability $1-p$. The maximum number which arrives in one minute is one student. The observer also sees that a student who is present at the head of the queue gets served with probability q and ~~gets~~ continues to be served with probability $1-q$. Again these observations are made per minute. The queue can hold at most 10 people. The canteen manager can help to control the value of q. by changing the person doing food service. There are two servers - 1st server corresponding to a service probability of q_1 and the second server with a service probability of q_2 with $q_2 > q_1$. Using the 1st server leads to the canteen manager needing to pay c_1 per minute, while the 2nd leads to a payment of c_2 per minute. How should the canteen manager decide which server to use at a time so as to minimize a weighted combination of the average queue length over 1 hour and the payment that he needs to make. Assume that $c_2 > c_1$.

2) At the beginning of each day a certain machine is either working
[R] or broken. If it is broken then the whole day is spent in repairing it and this costs $8c$ in labour and lost production.

If the machine is working, it may be run attended or unattended at costs of c and 0 respectively. In either case there is a chance that the machine will ~~need~~ breakdown and need repair the following day, with probabilities p and p'. Costs are discounted by β, and it is desired to minimize the total expected discounted cost over infinite horizon. Let $F(0)$ and $F(1)$ denote the minimal cost, starting from a morning on which the machine is broken or working resp. Show that it is optimal to run the machine unattended iff $(7p - 8p') \le 1/\beta$.

3) A burglar loots some house every night. His profit from
[R] successive crimes forms a sequence of independent RVs each having exponential distribution with mean $1/\lambda$.
Each night there is a probability q, $0 < q < 1$ of him being caught and forced to return his whole profit. If he has the choice, when should the burgler retire so as to maximize his total expected profit?

4) A gambler has the opportunity to bet on a sequence of N coin
[R] tosses. The probability of heads on the n^{th} toss is p_n, $n \in \{1, \cdots N\}$
For the n^{th} toss he may stake any non-negative amount not exceeding his current capital (which is his initial capital + winnings so far − losses) and call heads or tails. If he calls correctly then he retains his stake and wins an amount equal to it, otherwise he loses his stake. Let $X_0 \geq 0$ denote his initial capital and X_N his capital after the final toss. Determine how the gambler should call and how much he should stake to maximise $E[\log X_N]$.