AV312 - Lecture 4

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Figures from “Communication Systems” by Simon Haykin

August 5, 2016

Announcements

- Assignment 1 has been put up on the webpage
- Deadline is 12th August 2016

Review of last class

- Amplitude modulation and demodulation
  - DSBSC (W_m, no carrier)
  - Plain or large carrier AM (W_m, carrier power)
  - SSB (Hilbert transforms, $W_m/2$, no carrier)
  - VSB ($W_m/2 + f_v$, no carrier, filter property)

Today's plan

- AM: linear modulation, FDM
- Phase and frequency modulation
- Frequency modulation and its bandwidth
- Today's scribes are Aprameyo Roy and Arunima Das

Linear modulation schemes

- All the amplitude modulation schemes that we have studied can be thought of in the following way
- $s(t) = s_I(t)cos(2\pi f_c t) - s_Q(t)sin(2\pi f_c t)$
- The in-phase component $s_I(t)$ and the quadrature component $s_Q(t)$ are derived from the modulating signal $m(t)$
- Usually $s_I(t) = m(t)$ and $s_Q(t)$ is a filtered version of $m(t)$
- For example, $s_Q(t)$ is the Hilbert transform of $m(t)$ for SSB

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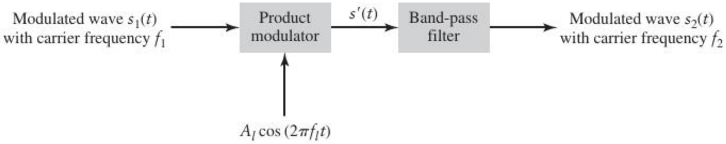

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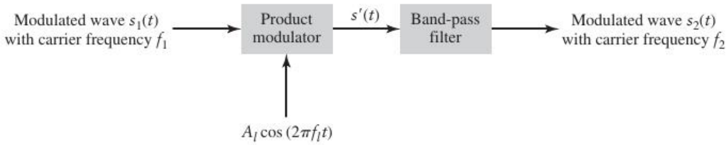
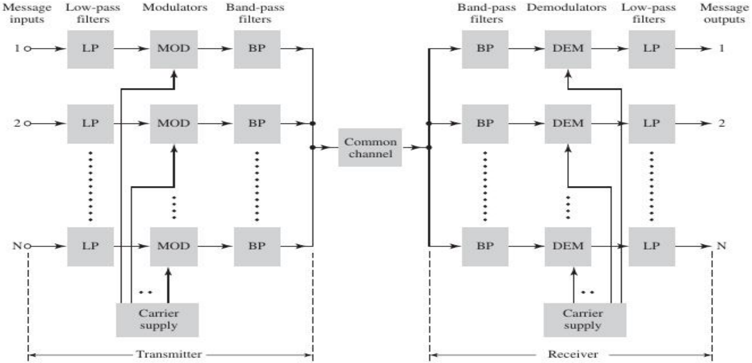


- What is f_l?
- The mixing operation is linear; but it is usually implemented using a non-linearity

Frequency division multiplexing

- A method by which multiple bandlimited baseband signals are combined to a composite signal for transmission
- Uses frequency translation to different carriers



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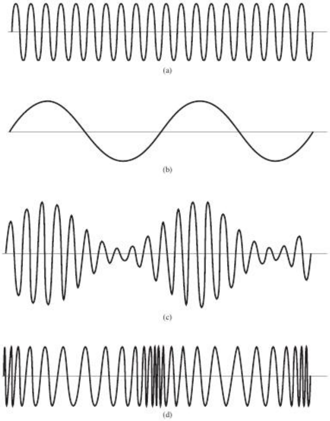

- The modulated signal is $s(t) = A(t)cos(\theta(t))$
- If $A(t)$ is constant then we have constant envelope modulation

Phase and frequency modulation

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- We will consider only FM!

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- Since we did only frequency translation, the bandwidth (one wided) of the modulated signal was equal to the bandwidth (two sided) of the baseband signal
- Is FM a linear transformation?

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- Let $\beta = \frac{\Delta f}{f_m}$ (modulation index, measure of phase deviation)

Bandwidth of FM signals

- The FM signal $s(t)$ for single tone $m(t)$ is therefore

$$s(t) = A_c cos(2\pi f_c t + \beta sin(2\pi f_m t))$$

- Intuitively, as Δf or β increases, the amount by which the frequency changes is large
- Intuitively, then β should affect the bandwidth of the signal
- If $\beta < 1$, we have narrowband modulation
- If $\beta > 1$, we have wideband modulation

Bandwidth of narrowband FM signals

- An approximate approach
- $s(t) = A_c cos(2\pi f_c t + \beta sin(2\pi f_m t))$
- $s(t) = A_c cos(2\pi f_c t) cos[\beta sin(2\pi f_m t)] - A_c sin(2\pi f_c t) sin[\beta sin(2\pi f_m t)]$
- Assume $cos[\beta sin(2\pi f_m(t))] = 1$ and $sin[\beta sin(2\pi f_m t)] = \beta sin(2\pi f_m t)$
- Then
$$s(t) = A_c cos(2\pi f_c t) - A_c \beta sin(2\pi f_c t) sin(2\pi f_m t)$$
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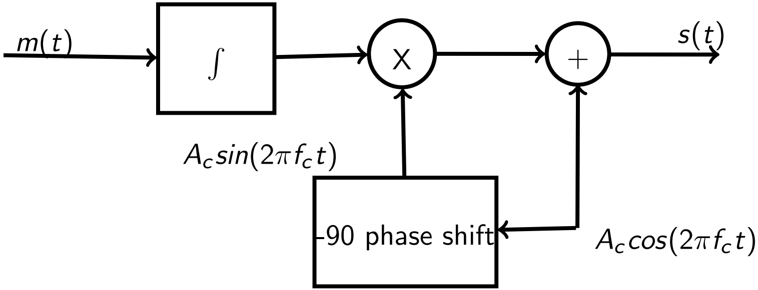
- How to interpret this ?
- The second term is a DSB signal

Narrowband FM modulator



- For general $m(t)$ there will be some distortion in the envelope
- Also higher order harmonics at multiples of f_m would be present

General analysis

- The modulating signal $m(t) = A_m cos(2\pi f_m t)$
- Is $s(t) = A_c cos(2\pi f_c t + \beta sin(2\pi f_m t))$ periodic ?

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- Is $s(t) = A_c cos(2\pi f_c t + \beta sin(2\pi f_m t))$ periodic ?
- $s(t) = Re[\tilde{s}(t)e^{j(2\pi f_c t)}]$
- Here $\tilde{s}(t) = A_c e^{j\beta sin(2\pi f_m t)}$. Is $\tilde{s}(t)$ periodic ?

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- $c_n = f_m \int_{-\frac{1}{2f_m}}^{\frac{1}{2f_m}} \tilde{s}(t)e^{-j2\pi n f_m t} dt$