AV312 - Lecture 9

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Figures from “Communication Systems” by Haykin and “An Intro. to Analog and Digital Commn.” by Haykin and Moher

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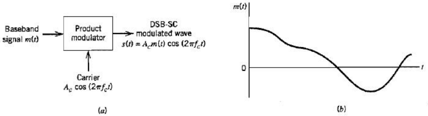
Review of last class

- Frequency modulation
- Signal spectrum and Bandwidth
- Direct and indirect FM generation
- FM demodulation by envelope detection

Today's class

- Phase locked loop (PLL)
  - Carrier recovery using PLL
  - FM demodulation using PLL
  - DSB demodulation using PLL - Costas receiver
- Analysis of PLL
- Today's scribes are Gemi Rachel George and Harshitha Gollamudi

Motivation



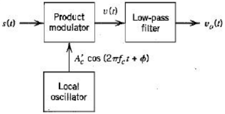
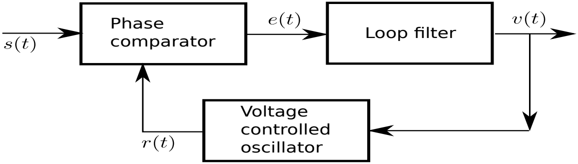


FIGURE 2.7 Coherent detector for demodulating DSB-SC modulated wave.

- A local oscillator needs to produce a replica of the carrier at the receiver
- Replica $\Rightarrow$ match in both frequency and phase
- Difference in frequencies $\Rightarrow$ time varying (linear) difference in phase

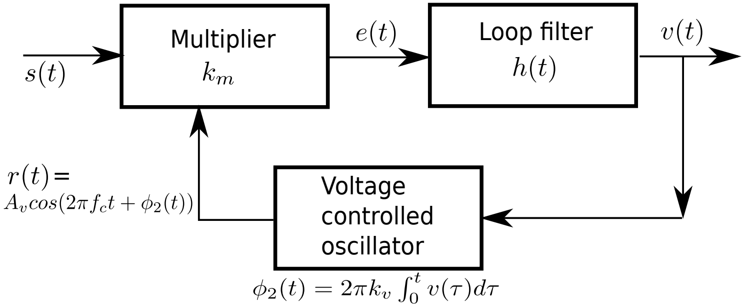
$$cos(2\pi f_1 t) \text{ and } cos(2\pi f_2 t)$$

PLL



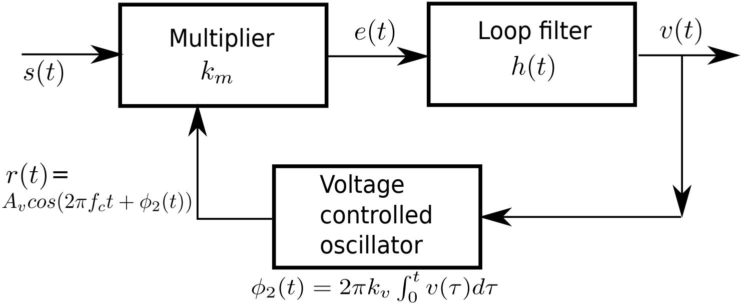
- $r(t)$ is the local oscillator's output
- We want $r(t)$ to "match" with $s(t)$ in phase
- We want $e(t)$ to measure the instantaneous phase difference between $s(t)$ and $r(t)$
- Filtered output $v(t)$ controls the VCO output $r(t)$ to match $s(t)$

PLL - An implementation



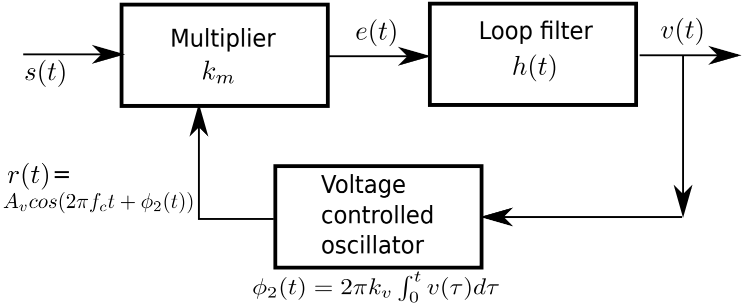
- $h(t)$ is a low pass response
- k_m and k_v are the sensitivities of the multiplier and the VCO respectively

PLL - Applications



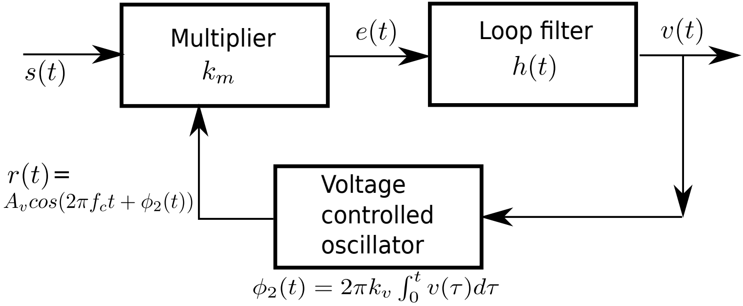
- Assume that phase error $\phi_1(t) - \phi_2(t) \approx 0$
- How does carrier recovery work?

PLL - Applications



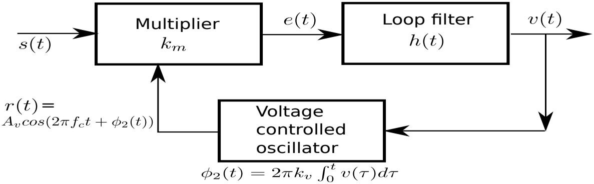
- Assume that phase error $\phi_1(t) - \phi_2(t) \approx 0$
- How does carrier recovery work?
- How does FM demodulation work?

PLL - Applications



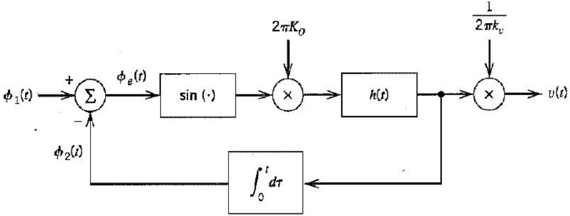
- Assume that phase error $\phi_1(t) - \phi_2(t) \approx 0$
- How does carrier recovery work?
- How does FM demodulation work?
- Read Costa's receiver from the textbook.

PLL - Model



- Let $s(t) = A_c sin(2\pi f_c t + \phi_1(t))$
- Let $r(t) = A_v cos(2\pi f_c t + \phi_2(t))$
- $e(t) = k_m A_c A_v \left[sin(4\pi f_c t + \phi_1(t) + \phi_2(t)) + sin(\phi_1(t) - \phi_2(t))\right]$
- Since $h(t)$ is a low pass response;
  $v(t) = \int_{-\infty}^{\infty} k_m A_c A_v sin(\phi_1(\tau) - \phi_2(\tau)) h(t - \tau) d\tau$
- $\phi_2(t) = 2\pi k_v \int_0^t v(\tau)d\tau$

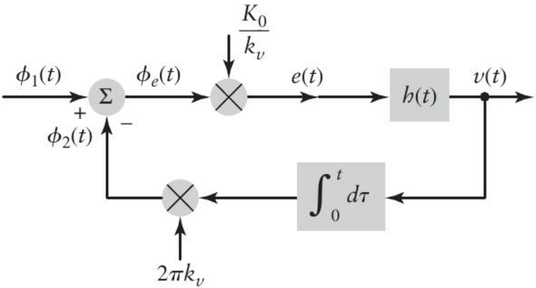
PLL - Model



- $\phi_e(t) = \phi_1(t) - \phi_2(t)$
- Loop gain parameter $K_o = k_v k_m A_c A_v$
- PLL model

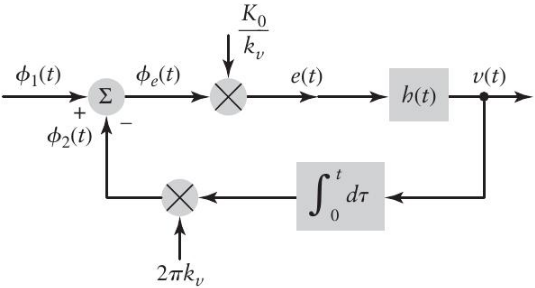
$$\frac{d\phi_e(t)}{dt} = \frac{d\phi_1(t)}{dt} - 2\pi K_o \int_{-\infty}^{\infty} sin(\phi_e(\tau))h(t-\tau)d\tau$$

PLL - Linearized Model



- Assume that $sin(\phi_e(t)) \approx \phi_e(t)$
- We use a Laplace transform domain approach
- $\phi_1(t) \leftrightarrow \Phi_1(s)$, $\phi_2(t) \leftrightarrow \Phi_2(s)$, $v(t) \leftrightarrow V(s)$, $h(t) \leftrightarrow H(s)$

PLL - Linearized Model



- $\frac{K_o}{k_v} H(s)\left(\Phi_1(s) - \Phi_2(s)\right) = V(s)$
- But $\Phi_2(s) = 2\pi k_v \frac{V(s)}{s}$
- $\Phi_1(s) \frac{K_o}{k_v} H(s) = V(s) \left[1 + \frac{2\pi K_o}{s} H(s)\right]$
- $\frac{V(s)}{\Phi_1(s)} = \frac{s(K_o/k_v)H(s)}{s+2\pi K_o H(s)}$ and $\frac{\Phi_e(s)}{\Phi_1(s)} = \frac{s}{s+2\pi K_o H(s)}$

PLL - Behaviour of $v(t)$

- What if there is a step change in the phase $\phi_1(t)$?
- What if there is a change in the frequency of the input?