AV312 - Lecture 6

Vineeth B. S.

Department of Avionics,
Indian Institute of Space Science and Technology.

Figures from "Communication Systems" by Haykin and "An Intro. to Analog and Digital Commn." by Haykin and Moher

August 9, 2016

Review of last class

- Frequency modulation
  - FM bandwidth (B_T) and Carson's rule - $2(\Delta f + W)$
    - The case of a single tone modulating signal
    - Approximate analysis - DSB signal
    - Complete analysis using the Fourier series representation of $\tilde{s}(t)$

Today's plan

- Frequency modulation systems
- Effect of channel non-linearities on FM and comparison with AM
- Frequency demodulation schemes
- Today's scribes are Daksh Dhiman and Danish Mohammed

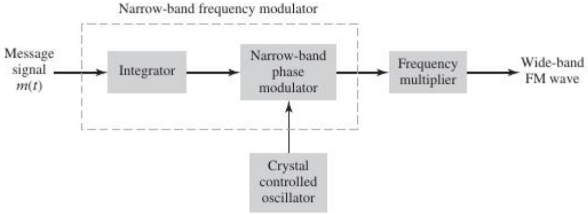
Direct FM generation

- VCO is a system which generates a signal (sinusoid) with instantaneous frequency (ideal)

$$f(t) = f_c + k_f m(t)$$

- The FM signal is generated from a VCO
- The frequency is directly controlled by $m(t)$
- k_f is a parameter of the VCO
- Difficult to obtain wideband FM (why do we need wideband FM?) due to carrier drift

Indirect FM generation (Armstrong modulator)



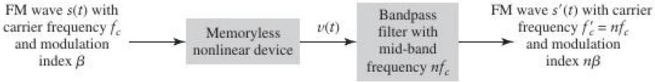
- Generate narrow band FM and then frequency multiply
- The narrow band FM signal is

$$s(t) = A_c cos\left(2\pi f_c t + 2\pi k_f \int_0^t m(u)du\right)$$

- If we frequency multiply by n

$$s'(t) = A_c cos\left(2\pi (nf_c) t + 2\pi (nk_f) \int_0^t m(u)du\right)$$

Frequency multiplication



- The device has IO characteristics given by:

$$v(t) = a_1 s(t) + a_2 s(t)^2 + \cdots + a_n s(t)^n$$

- Would $v(t)$ contain a component centered at nf_c?

Effects of channel non-linearities

- Suppose $v_i(t)$ and $v_o(t)$ are the input and output signals resp.
- The input output relationship is non-linear and an example model is given by

$$v_o(t) = a_1 v_i(t) + a_2 v_i(t)^2 + a_3 v_i(t)^3$$

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- What happens if we send an AM or FM $v_i(t) = s(t)$ through this channel?

Effect on AM signals

- Suppose $v_i(t) = A_c A_m cos(2\pi f_m t) cos(2\pi f_c t)$; $A_c A_m = 1$
- Then we have that

$$\begin{aligned} v_o(t) &= a_1 cos(2\pi f_m t) cos(2\pi f_c t) + a_2 cos^2(2\pi f_m t) cos^2(2\pi f_c t) \\ &\quad + a_3 cos^3(2\pi f_m t) cos^3(2\pi f_c t) \end{aligned}$$

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- What all frequencies would $v_o(t)$ contain?
- What is the desired signal at center frequency of f_c?
- How to extract this desired signal?

Effect on FM signals

- Suppose $v_i(t) = A_c cos(2\pi f_c t + \theta(t))$; $A_c = 1$
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- What all frequencies would $v_o(t)$ contain?
- What is the desired signal at center frequency of f_c?
- How to extract this desired signal?
  - $f_c > 3\Delta f + 2W$

Demodulation of FM signals

- We have $s(t) = A_c cos\left(2\pi f_c t + 2\pi k_f \int_0^t m(u).du\right)$
- We need to extract the instantaneous frequency or phase
- What about $\frac{ds(t)}{dt}$?

Demodulation of FM signals

- We have $s(t) = A_c cos\left(2\pi f_c t + 2\pi k_f \int_0^t m(u).du\right)$
- We need to extract the instantaneous frequency or phase
- What about $\frac{ds(t)}{dt}$?
- $\frac{ds}{dt} = -2\pi A_c(f_c + 2\pi k_f m(t)) sin\left(2\pi f_c t + 2\pi k_f \int_0^t m(u).du\right)$
- The envelope of the signal contains the message signal
- FM demodulation in principle using a differentiator followed by an envelope detector

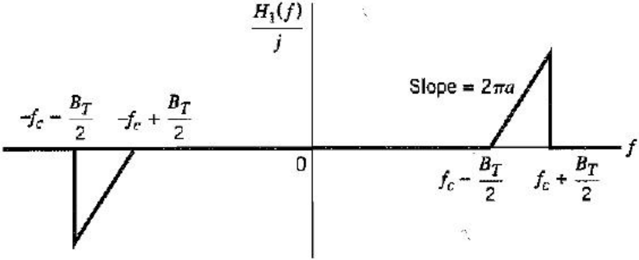
Slope filter/circuit

- Suppose $x(t)$ has a FT $X(f)$, then what is the FT of $\frac{dx(t)}{dt}$?

Slope filter/circuit

- Suppose $x(t)$ has a FT $X(f)$, then what is the FT of $\frac{dx(t)}{dt}$?
- $j2\pi fX(f)$
- So $H(f)$ for a differentiator should be $j2\pi f$
- But we only need to differentiate our signal within the FM transmission bandwidth B_T.

Slope filter/circuit



x

- Consider the filter with response $H_1(f)$ defined as

$$H_1(f) = \begin{cases} j2\pi a(f - (f_c - \frac{B_T}{2})), \text{ for } f_c - \frac{B_T}{2} \leq f \leq f_c + \frac{B_T}{2}, \\ j2\pi a(f + (f_c - \frac{B_T}{2})), \text{ for } -f_c - \frac{B_T}{2} \leq f \leq -f_c + \frac{B_T}{2}, \\ 0, \text{ otherwise} \end{cases}$$

Slope filter output analysis

- A component of the output is the desired derivative
- What is that component?

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Slope filter output analysis

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- $j2\pi afS(f)$
- What are the other components?
- Suppose we assume narrow band single tone modulation?
- Analysis of the other component requires some background on "Complex baseband representation of passband signals and systems"
- Reading: FM stereo multiplexing and superheterodyne receivers

Complex baseband representation of passband signals

- Can every signal at every point in a communication system be represented as a baseband signal?
- If a passband $g(t)$ is real, then $G(-f) = G^*(f)$; do we need both negative and positive frequencies?
- Suppose $g(t)$ (baseband or passband) is real, then the pre-envelope or analytic signal of $g(t)$ is

$$g_+(t) = g(t) + j\hat{g}(t)$$

- The important point here is that

$$G_+(f) = G(f) + sgn(f)G(f)$$

Pre-envelope of $g(t)$

- $G_+(f) = G(f) + sgn(f)G(f)$

$$G_+(f) = \begin{cases} 2G(f), f > 0 \\ G(0), f = 0 \\ 0, f < 0 \end{cases}$$

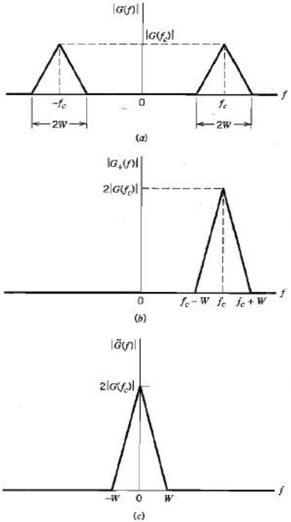
- Can also compute the pre-envelope $g_-(t)$ for negative frequencies

Representation of bandpass $g(t)$

- Assume that $g(t)$ occupies a bandwidth of $2W$ centered at f_c
- Suppose we find a complex signal $\tilde{g}(t)$ such that

$$g_+(t) = \tilde{g}(t)e^{j2\pi f_c t}$$

- Note that $\tilde{g}(t)$ is a low pass signal



Representation of bandpass $g(t)$

- The signal $g(t) = Re\left[\tilde{g}(t)e^{j2\pi f_c t}\right]$
- Suppose $\tilde{g}(t) = g_I(t) + jg_q(t)$
- Then $g(t) = g_I(t)cos(2\pi f_c t) - g_Q(t)sin(2\pi f_c t)$
- Can then represent $g(t) = a(t)cos(2\pi f_c t + \phi(t))$. How?
- $a(t)$ is called the natural envelope or envelope of the signal
- $\phi(t)$ is called the phase