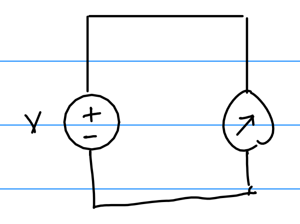
Lecture 27 01/10/2019

Random processes and Noise modelling.

- Signal to noise ratio advantage for FM.

- recall random variables.



n	measurement
1	5.1
2	4.9
3	5.2

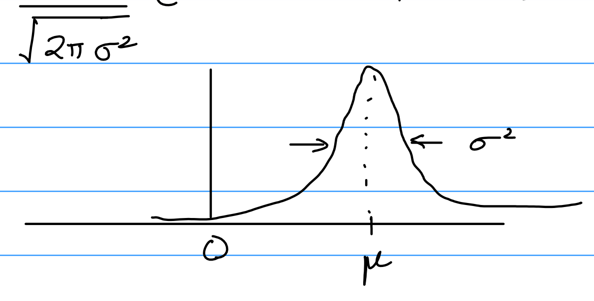
X → S_X : support set (ss sample space)

X → CDF $F_X(x) = \Pr\{X \leq x\}$

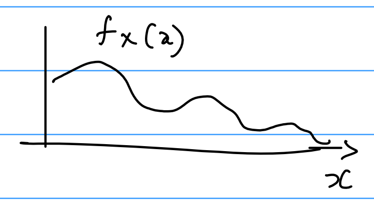
Standard random variables - R.Vs for which PMF / PDF has an analytical formula / standard formula.

Gaussian random variables - PDF?

$$f_X(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-(x-\mu)^2/2\sigma^2} = \mathcal{N}(\mu, \sigma^2)$$



In general



summary statistics

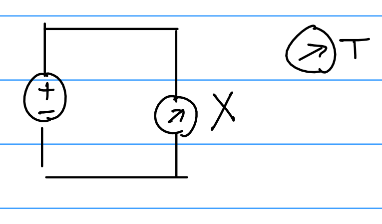
a) mean or expectation

$$\mathbb{E}X = \int x \cdot f_X(x) \cdot dx$$
$$= \sum_1 x \, p_X(x)$$

b) $\mathrm{var}(X) = \mathbb{E}(X - \mathbb{E}X)^2$, $\mathrm{std}(X) = \sqrt{\mathrm{var}(X)}$

c) $\mathbb{E}X^n = \int x^n \cdot f_X(x) \cdot dx \rightarrow n^{th}$ moment

Multiple random variables.

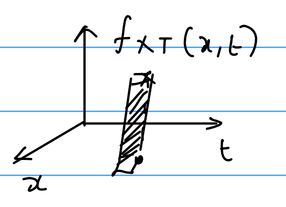


n	X	T
1	5.1	25
2	4.7	27
3	5.2	30
	⋮	⋮

$X - f_X(x)$

$T - f_T(t)$

joint dist $f_{XT}(x, t)$



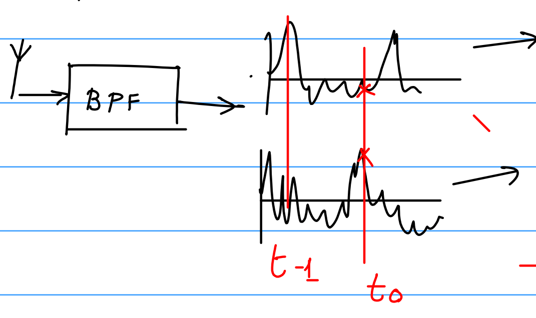
$$f_X(x) = \int_{-\infty}^{\infty} f_{XT}(x, t) \cdot dt$$

covariance $(X, T) = \mathrm{cov}(X, T) = \mathbb{E}[(X - \mathbb{E}X)(T - \mathbb{E}T)]$

correlation $(X, T) = \mathrm{corr}(X, T) = \mathbb{E}[XT]$

can be extended to many random variables.

Random processes.



n	$X(t_0)$
1	-1
2	10
	⋮

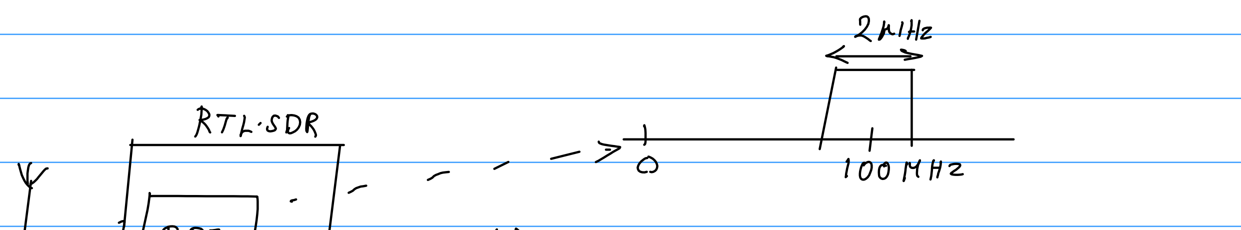
samples taken from a set of functions

sample space

distribution on this set of functions!

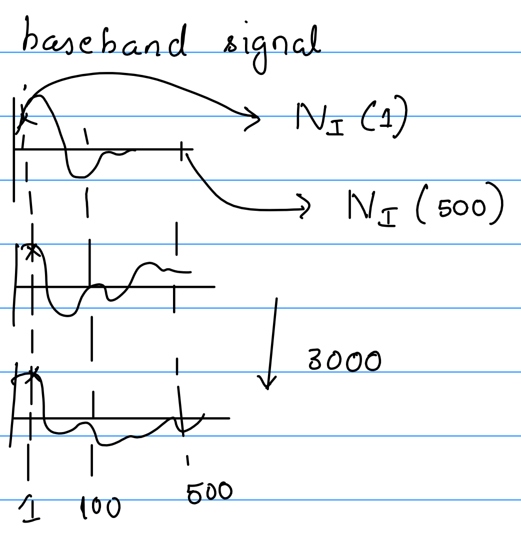
at every time, we have a random variable $X(t)$.

so the collection of such random variables, ie, $(X(t), t \in \mathbb{R})$ is called a random process.



$N(t) = N_I(t)\cos(2\pi f_c t) - N_Q(t)\sin(2\pi f_c t)$

$N_I(t)$ = baseband signal



$N_I(1) \sim \mathcal{N}(\mu, \sigma^2)$

$N_I(100) \sim \mathcal{N}(\mu', \sigma_1^2)$

$N_I(500) \sim \mathcal{N}(\mu^2, \sigma_2^2)$

$N_I(t) \sim \mathcal{N}(0, \sigma^2)$

$(N_I(t), t \in \mathbb{R})$, $F_{N(t)}$