Indian Institute of Space Science and Technology
AV336 - Digital Signal Processing Lab
Department of Avionics

Labsheet 6

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1. Review the portion on z-transforms as taught in class.

2. For the tasks set out in this labsheet, we will use rational z-transforms for the most part. Recall what rational z-transforms are. We note that rational z-transforms can be represented using their poles and zeros. Think about how you will represent rational z-transforms on a computer. Can rational z-transforms be represented using matlab vectors? Do you need to represent their multiplicities separately? Does there exist a case where separate representation of multiplicities is useful? Does the ordering of poles and zeros (magnitude-wise) in your representation matter? Does there exist a case where an ordered representation is useful?

3. In this task, you will visualize the magnitude $|X(z)|$ of the z-transform $X(z)$ of a sequence $x[n]$. Assume that $X(z)$ is a rational z-transform which is represented using the representation that you have obtained above in (Task 2). Obtain a 3D plot of $|X(z)|$ as a function of the real and imaginary part of z for three choices of rational $X(z)$. You are free to make these three choices. Comment on how $|X(z)|$ behaves near poles and zeros. How does the behaviour near poles and zeros change with the multiplicity of poles and zeros? Comment on the region of convergence of the rational z-transforms you have chosen.

4. In this task, you will obtain the DTFT of a sequence $x[n]$ from its z-transform $X(z)$. Assume that the sequence you are looking at has a rational z-transform $X(z)$ represented using the representation in (Task 2).

   (a) Obtain the magnitude plot of the DTFT from $X(z)$ as discussed in class (hint: use the distance from $e^{j\omega}$ method). Plot the magnitude plot from 0 to 4π radians/sample and comment on the periodicity of the DTFT (use an appropriately sampled ω so that the magnitude plot is smooth).

   (b) Obtain the phase plot of the DTFT from $X(z)$ as discussed in class (hint: use the angle from $e^{j\omega}$ method). Plot the phase plot from 0 to 4π radians/sample and comment on the periodicity of the DTFT (use an appropriately sampled ω so that the phase plot is smooth).

   Obtain the DTFT of two rational z-transforms. You are free to choose $X(z)$.

5. In this task, you will write a function that will "expand out" a rational z-transform as a partial fractions expansion.

   (a) Write a matlab function "partialFractionExpander" that has as input the rational z-transform represented as in Task (2).

   (b) The function should first check if the number of zeros is strictly less than the number of poles.

(c) If it is so, then the function should use the method that we had discussed in class in order to obtain the coefficients of the different terms that arise in the partial fraction expansion. For example, the function should expand rational $X(z)$ as

$$\frac{P(z)}{Q(z)} = \sum_{k=1}^{N} \sum_{s=1}^{s_k} \frac{A_{k,s}}{(z - p_k)^s},$$

where $P(z)$ and $Q(z)$ are polynomials in z, N is the number of poles, p_k represents the k^{th} pole, and s_k is the multiplicity of the k^{th} pole.

(d) The function should put out an output which represents the RHS above. What representation would you choose? Would a list of 3-tuples, with the tuple containing p_k, s_k, and $A_{k,s}$ be sufficient?

(e) Implement this function under the assumption that $s_k \leq 2$ for all k.

(f) Test your function with five different $X(z)$. You are free to choose $X(z)$ (choose $X(z)$ with different number of poles and zeros and multiplicities).

(g) In each case, obtain the partial fractions expansion manually. Compare the expansion that you obtain with the function with the expansion that you obtain manually.

6. Let $x[n]$ be a sequence of length N defined for $n \in \{0, \ldots, N-1\}$.

(a) Write a matlab function "ztransform" which takes as input $x[n]$ and the value of z, say z_0, that we are interested in and computes the $X(z)|_{z=z_0}$.

(b) Modify your function so that it also prints the pole and zero locations, along with their multiplicities.

(c) Suppose we fix $z_0 = re^{-j\frac{2\pi k}{N}}$ for some $r > 0$ and $k \in \{0, \ldots, N-1\}$. Modify your function "ztransform" to compute the z transform of $x[n]$ for z_0 of this form by using FFT. Your modified function should use r and k as input instead of z_0.