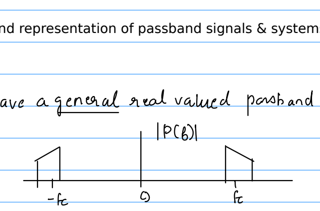
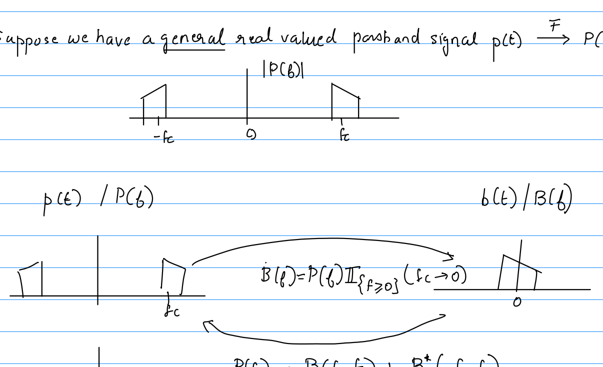
Complex baseband representation of passband signals & systems

Suppose we have a <u>general</u> real valued passband signal $p(t) \xrightarrow{\mathcal{F}} P(f)$



$p(t) \,/\, P(f)$ $\qquad\qquad$ $b(t) \,/\, B(f)$



$$P(f) = B(f-f_c) + B^*(-f-f_c)$$

$$\tfrac{1}{2}\left(p(t) + j\tilde{p}(t)\right) e^{-j2\pi f_c t} = b(t)$$

$$\tfrac{1}{2}\left(p(t) + j\tilde{p}(t)\right)\left(\cos(2\pi f_c t) - j\sin(2\pi f_c t)\right)$$

$$\tfrac{1}{2}\Big(\underbrace{p(t)\cos(2\pi f_c t) + \tilde{p}(t)\sin(2\pi f_c t)}_{\substack{\text{real part in the } b(t) \\ b_r(t)}}\Big) + j\Big(\underbrace{\tilde{p}(t)\cos(2\pi f_c t) - p(t)\sin(2\pi f_c t)}_{\substack{\text{imag part in } b(t) \\ b_i(t)}}\Big)$$

$p(t)$ from $b(t)$?

can we use the relationship $P(f) = B(f-f_c) + B^*(-f-f_c)$?

inverse fourier transform of $P(f)$ gives $p(t)$

inverse " of $\underbrace{B(f-f_c)}_{b(t)\cdot e^{j2\pi f_c t}} + B^*(-f-f_c)$

$$\mathcal{F}^{-1}\left(B^*(-f-f_c)\right) = \underbrace{b^*(t)\, e^{-j2\pi f_c t}}$$

$$= \int b^*(t)\, e^{-j2\pi f_c t} \cdot e^{-j2\pi f t}\, dt$$

$$= \int b^*(t)\, e^{-j2\pi(f+f_c)t}\, dt$$

$$= \left(\int b(t)\, e^{j2\pi(f+f_c)t}\, dt\right)^*$$

$$= B^*(-f-f_c)$$

$$p(t) = b(t)\, e^{j2\pi f_c t} + b^*(t)\, e^{-j2\pi f_c t}$$

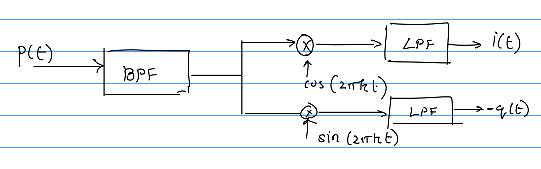
$$= \left(b_r(t) + j\, b_i(t)\right)\left(\cos(2\pi f_c t) + j\sin(2\pi f_c t)\right)$$

$$+ \left(b_r(t) - j\, b_i(t)\right)\left(\cos(2\pi f_c t) - j\sin(2\pi f_c t)\right)$$

$$= 2\left[\left(b_r(t)\cos(2\pi f_c t) - b_i(t)\sin(2\pi f_c t)\right)\right.$$

$$p(t) = 2\left(b_r(t)\cos(2\pi f_c t) - b_i(t)\sin(2\pi f_c t)\right)$$

$$= 2\left(i(t)\cos(2\pi f_c t) - q(t)\sin(2\pi f_c t)\right)$$



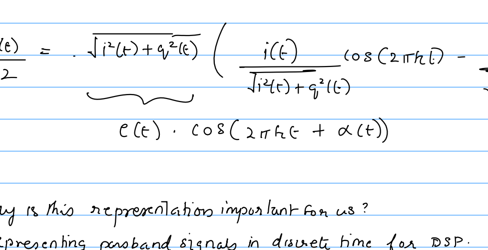
$$\frac{p(t)}{2} = \underbrace{\sqrt{i^2(t)+q^2(t)}}\left(\frac{i(t)}{\sqrt{i^2(t)+q^2(t)}}\cos(2\pi f_c t) - \frac{q(t)}{\sqrt{i^2(t)+q^2(t)}}\sin(2\pi f_c t)\right)$$

$$e(t)\cdot\cos(2\pi f_c t + \alpha(t))$$

→ Why is this representation important for us?

representing passband signals in discrete time for DSP.

for example, suppose we had a passband signal with spectrum as shown.



we know $\dfrac{p(t)}{2} = i(t)\cos(2\pi f_c t) - q(t)\sin(2\pi f_c t)$

if suppose we use our knowledge of Nyquist sampling, then sampling rate $f_s = 1/T_s \geq 2\,\text{GHz}$!!

but $p(t)$ can be obtained from $i(t)$ and $q(t)$ (or from i_n and q_n, where $i_n = i(nT_s')$ and $q_n = q(nT_s')$.)

What rate is required to sample $i(t)$ and $q(t)$?

Suppose f_c is at the center of the passband $i(t)$, and $q(t)$ have two sided B/w = the one sided B/w of $p(t)$ - which is usually $<<$ 1 GHz !

so sampling rate for $i(t) \geq 2 \times \frac{1}{2}$ (one sided B/w of $p(t)$) and similarly for $q(t)$.

But two samples must be taken, resp. of $i(t)$ and $q(t)$ per T_s' to represent $p(t)$ in discrete time domain. (of course, the representation should be s.t. it is possible to recover $p(t)$ from its samples).