Review assignment 2

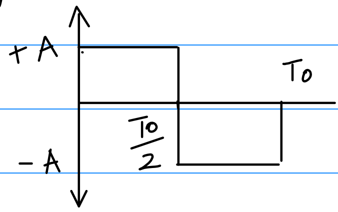
Review of Fourier Series

① Given two periodic signals $x(t)$ and $y(t)$ each with period T_0.
What is the vector representation of $x(t)$ and $y(t)$, when the basis vectors for the vector space are the complex exponentials $\{ e^{+j2\pi k f_0 t}, k \in \mathbb{Z} \}$.
What is the distance between the signals $x(t)$ and $y(t)$?

② Find out the Fourier series / spectrum / vector representation of the following signals

a) $100 \cos(2\pi f_0 t)$

b) The periodic square wave, with one period given by



c) $2\cos(2\pi f_0 t) + 3\sin(2\pi f_1 t)$ (discuss for different cases - $f_0 = f_1$, $f_0 =$ a multiple of f_1, $f_0 \neq$ any multiple of f_1 and vice versa, $HCF(f_1, f_0)$).

③ A signal, that is not realistic / practically realisable, but of immense use in mathematical analysis is the impulse train, defined as

$$i_m(t) = \sum_k i_k \cdot \delta(t - kT_0).$$

- draw an example of an impulse train (ie, choose T_0 and i_k as you wish)
- find out the Fourier series representation of the impulse train $i_m(t)$.

(look at Example 2.4.2 from the textbook)

④ Properties of Fourier series - derive the properties listed below

a) linearity

b) time delay

c) Fourier series of a real valued signal is conjugate symmetric

d) Harmonic structure

e) Differentiation

f) Parseval's identity

(see the section 2.4.1 in the textbook for the definition of all these properties)

(5) Suppose $u(t)$ and $v(t)$ are periodic signals with fundamental frequencies f_u and f_v respectively. ($f_u \neq f_v$). and the following Fourier series exist.

$$u(t) = \sum_{k=-\infty}^{\infty} u_k e^{j2\pi k f_u t} \quad \text{and} \quad v(t) = \sum_{k=-\infty}^{\infty} v_k e^{j2\pi k f_v t}.$$

What can you say about the Fourier series of $u(t) + v(t)$? Can this be expressed in terms of u_k and v_k?

(6) If $u(t)$ and $v(t)$ are two periodic signals with the same fundamental period f_0 and the following Fourier series exist:

$$u(t) = \sum_k u_k e^{j2\pi k f_0 t} \quad \text{and} \quad v(t) = \sum_k v_k e^{j2\pi k f_0 t}$$

a) is the product $u(t)\,v(t)$ periodic? what is the period?

b) Suppose $u(t)$ and $v(t)$ have finite power. What about the product $u(t)\,v(t)$?

c) Can you express the Fourier series of the product $u(t)\,v(t)$ in terms of the terms u_k and v_k:

d) Suppose now that $u(t)$ and $v(t)$ have different fundamental frequencies f_u and f_v respectively with the Fourier series being

$$u(t) = \sum_k u_k e^{j2\pi k f_0 t} \quad \text{and} \quad v(t) = \sum_k v_k e^{j2\pi k f_0 t}$$

Do you think you can get a Fourier series representation for the product $u(t)\,v(t)$ and that too in terms of u_k and v_k. Elaborate ...