AVD623: Communication Systems-II

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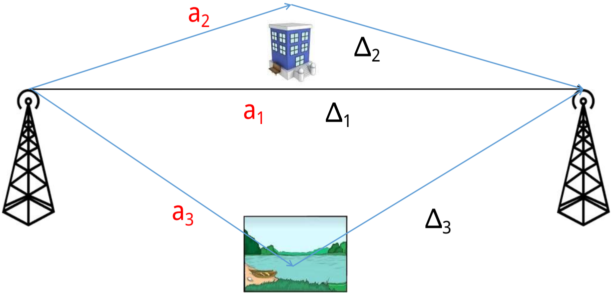
Lecture 5

Figures are taken from “Communication Systems” by Simon Haykin, “Software receiver design” by Sethares and Johnson.

Review

- Intersymbol interference
- Nyquist bandwidth and channel
- Pulse shaping
- Duobinary signalling (Partial response signalling)
- Zero-forcing equalization

Multipath interference



- Suppose there are n paths from the source to the receiver
- The propagation delays on these n paths are $\Delta_1, \Delta_2, \ldots, \Delta_n$
- The corresponding attenuations are $a_1, a_2, \ldots, a_n$
- If $u(t)$ is transmitted we receive $y(t)$ where

$$y(t) = a_1 u(t - \Delta_1) + a_2 u(t - \Delta_2) + \cdots + a_n u(t - \Delta_n).$$

Multipath interference

- If $u(t)$ is transmitted we receive $y(t)$ where

$$y(t) = a_1 u(t - \Delta_1) + a_2 u(t - \Delta_2) + \cdots + a_n u(t - \Delta_n).$$

- The difference $\Delta_n - \Delta_1$ is called the delay spread of the channel
- Recall the digital transmission system block diagram from last class
- Since we sample the received signal we have

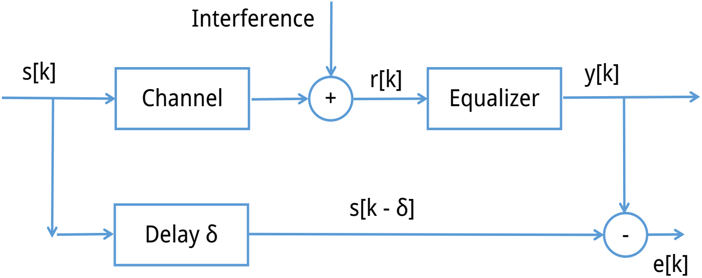
$$y(kT_b) = \alpha_1 u(kT_b) + \alpha_2 u((k-1)T_b) + \cdots + \alpha_N u((k-N)T_b)$$

- In case we have noise or additive interference we have an extra term

$$y(kT_b) = \alpha_1 u(kT_b) + \alpha_2 u((k-1)T_b) + \cdots + \alpha_N u((k-N)T_b) + \eta(kT_b)$$

- Note that in this sampled model, the parameter N should be such that $NT_b \geq \Delta_n$
- Example: Suppose $T_b = 40$ns and $\Delta_n = 40\mu$s, then $N = 100$.

Least squares linear equalization



- Suppose both source and receiver have access to a predetermined sequence of bits
- Then how can we design an equalizer to mitigate ISI

Least squares linear equalization

- Assume that the equalizer is of the linear transversal form
- Let

$$y[k] = \sum_{j=0}^{n} f_j r[k-j]$$

- For example,

$$y[n+1] = [r[n+1], r[n], \ldots, r[1]] \begin{bmatrix} f_0 \\ f_1 \\ \vdots \\ f_n \end{bmatrix}$$

Least squares linear equalization

- We can write

$$\begin{bmatrix} y[n+1] \\ y[n+2] \\ \vdots \\ y[p] \end{bmatrix} = \begin{bmatrix} r[n+1] & r[n] & r[n-1] & \dots & r[1] \\ r[n+2] & r[n+1] & r[n] & \dots & r[2] \\ r[n+3] & r[n+2] & r[n+1] & \dots & r[3] \\ & & \dots & & \\ r[p] & r[p-1] & r[p-2] & \dots & r[p-n] \end{bmatrix} \times \begin{bmatrix} f_0 \\ f_1 \\ \vdots \\ f_n \end{bmatrix}$$

- $Y = RF$
- The matrix R has a special structure - Toepliz

Least squares linear equalization

- The error is $e[k] = s[k - \delta] - y[k]$
- Or $E = S - Y = S - RF$
- We define an error metric $J_{LS} = \sum_{i=n+1}^{p} e[k]^2$
- Our objective is then to choose the f_i such that J_{LS} is minimized

Least squares linear equalization

- The error metric $J_{LS} = \sum_{i=n+1}^{p} e[k]^2$
- Or we have that $J_{LS} = E^T E$
- $J_{LS} = (S - RF)^T (S - RF)$
- Writing this out we have $J_{LS} = S^T S - 2S^T RF + (RF)^T RF$
- We use a mathematical trick here to solve for the optimal F
- Suppose $\Psi = [F - (R^T R)^{-1} R^T S]^T (R^T R)[F - (R^T R)^{-1} R^T S]$
- Then $J_{LS} = \Psi + S^T [I - R(R^T R)^{-1} R^T] S$
- Since only Ψ depends on F we minimize Ψ
- But Ψ is minimized at $F^* = (R^T R)^{-1} R^T S$
- We note that this solution depends on the specification of δ