AVD623: Communication Systems-II

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Lecture 9

Figures are taken from “Communication Systems” by Simon Haykin, “Communication Systems” by Stern and Mahmoud, and “Software receiver design” by Sethares and Johnson.

Two issues - Carrier and Timing/Clock Recovery

- A local oscillator needs to produce a replica of the carrier at the receiver
- Replica $\Rightarrow$ match in both frequency and phase
- Difference in frequencies $\Rightarrow$ time varying (linear) difference in phase

$$cos(2\pi f_1 t) \text{ and } cos(2\pi f_2 t)$$

- A clock circuit needs to tick off bit periods and sample the received continuous time waveform at the appropriate times within each bit period

How to do carrier recovery?

Phase and frequency estimation using

- FFT
- Phase locked loops
- Squared difference loop
- Costas loop
- Decision directed tracking

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Phase and frequency estimation using

- FFT
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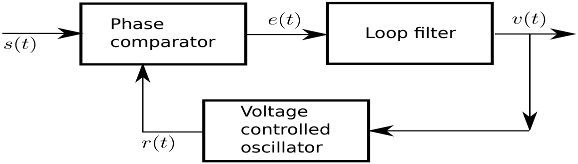
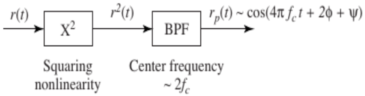
- Squared difference loop
- Costas loop
- Decision directed tracking
- Adaptive methods for phase and frequency tracking

Phase and frequency estimation using PLL



- $r(t)$ is the local oscillator's output
- We want $r(t)$ to "match" with $s(t)$ in phase
- We want $e(t)$ to measure the instantaneous phase difference between $s(t)$ and $r(t)$
- Filtered output $v(t)$ controls the VCO output $r(t)$ to match $s(t)$
- $\frac{\Phi_e(s)}{\Phi_1(s)} = \frac{s}{s+2\pi K_o H(s)}$

Squared difference loop



- Recall the FFT based estimation of frequency and phase
  - The carrier was suppressed
  - We got a non-suppressed carrier by using a squaring non-linearity and band pass filtering
- To understand the squared difference loop we will use an assumption that the signal out of the BPF is

$$r_p(t) = cos(4\pi f_c t + 2\phi)$$

- Note that the same method could be applied to the case where the carrier is not suppressed - the frequency and phase would be just half of what we have in the above statement

Squared difference loop

- Let us assume that we have a VCO that puts out a signal $cos(4\pi f_0 t + 2\theta)$
- We want to adaptively change f_0 and θ to match with f_c and ϕ
- We will first define a performance metric - which is an error which we will try to minimize by adapting f_0 and θ

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- Let
$$J_{SD}(\theta) = avg\left\{e^2(\theta, k)\right\} = \frac{1}{4}avg(r_p(kT_s) - cos(4\pi f_0 kT_s + 2\theta))^2$$
- Suppose $f_0 = f_c$
- Note that if $\theta = \phi$ then $J_{SD}(\theta) = 0$
- We will use a gradient approach to minimize $J_{SD}(\theta)$, the estimate of minimizing θ is obtained from the sequence $\theta[k]$ where
$$\theta[k+1] = \theta[k] - \mu \frac{dJ_{SD}(\theta)}{d\theta}\Big|_{\theta=\theta[k]}$$

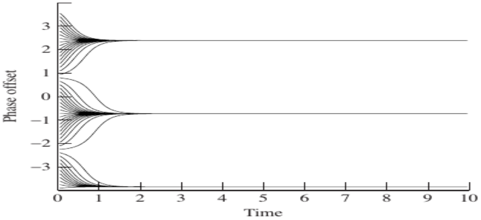
Approximating $\frac{dJ_{SD}(\theta)}{d\theta}$

- We will approximate $\frac{dJ_{SD}(\theta)}{d\theta}$ as follows

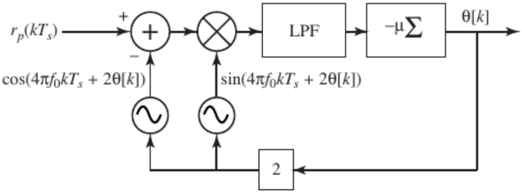
$$\begin{aligned}
\frac{dJ_{SD}(\theta)}{d\theta} &= \frac{davg\left\{e^2(\theta,k)\right\}}{d\theta} \\
&\approx avg\left\{\frac{de^2(\theta,k)}{d\theta}\right\} \\
&= \frac{1}{2}avg\left\{e(\theta,k)\frac{de(\theta,k)}{d\theta}\right\} \\
&= avg\left\{(r_p(kT_s) - cos(4\pi f_0 kT_s + 2\theta[k]))sin(4\pi f_0 kT_s + 2\theta[k])\right\}.
\end{aligned}$$

- Then the recursion becomes

$$\theta[k+1] = \theta[k] - \mu avg\left\{(r_p(kT_s) - cos(4\pi f_0 kT_s + 2\theta[k]))sin(4\pi f_0 kT_s + 2\theta[k])\right\}.$$



Summary of squared difference loop



When is $avg\left\{\frac{d(.)}{d\theta}\right\} = \frac{d(avg\{.\})}{d\theta}$?

- In many of our discussions we have used/will use interchange of differentiation and filtering
- When is this valid? How are the approximations done

Review - Averages and Filters

- Simple average

$$y[N] = avg\left\{x[i]\right\} = \frac{1}{N}\sum_{i=1}^{N} x[i]$$

- Moving average

$$y[k] = avg\left\{x[i]\right\} = \frac{1}{P}\sum_{i=k-(P-1)}^{k} x[i]$$

- Recursive sum

$$y[k] = y[k-1] + \mu x[k]$$

- All of these averages have a low pass characteristic
- When we think about signals at different points in our block diagrams, we can use this equivalence to a LP.

Filters and Derivatives

- Suppose there is a filtering operation which is characterized by a parameter β; $Filtering(\beta)$
- We want to know if $\frac{d}{d\beta} \rightarrow Filtering = Filtering \rightarrow \frac{d}{d\beta}$

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- Case 1: If β is time then this operator commutativity property holds; LTI systems
- Case 2: Suppose β is a parameter of the filter
  - FIR filter with response β_i
  - Output $y[k] = \sum_{i=0}^{P} \beta_i x[k-i]$
  - Then $\frac{d}{d\beta_i} y[k] = x[k-i]$
  - But $\sum_{i=0}^{P} \beta_i \frac{d}{d\beta_i} x[k-i] = 0$
- Case 3: If β is a parameter of the input signal
  - Suppose β is like the phase angle we have seen
  - Again commutativity holds

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- However, we have been looking at parameters which are fixed functions of time in Cases 2 and 3.