Indian Institute of Space Science and Technology
AV312 - Digital Communication
Department of Avionics

Quiz 1 for **Semester V** on **21/08/2015**

Note to the student

1. There are **3 questions** in this question paper on **2 pages**, for a total of **15 marks**.
2. Answer **all** questions.

Question 1 (5 marks):

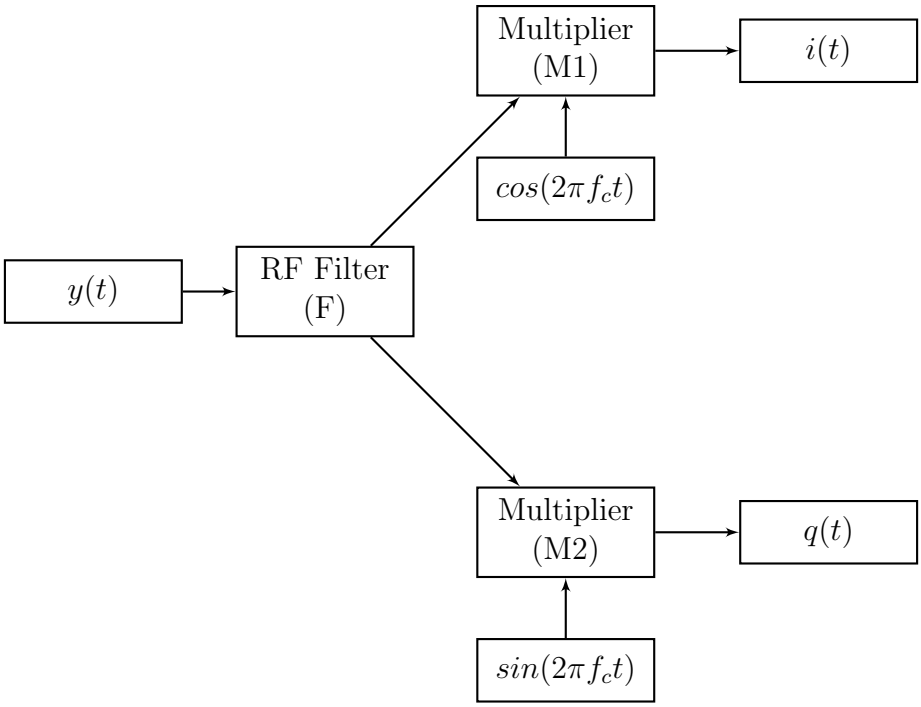
1. Suppose $x(t)$ is a real valued signal for which the Fourier transform $X(f)$ exists. Then prove that $X(f) = X^*(-f)$, where $X^*(f)$ is the complex conjugate of $X(f)$. (1 mark)
2. Suppose $x(t)$ is periodic with period T seconds. What is $X(f)$? (1 mark). (Hint: There is a way of representing periodic signals using δ functions.)
3. Suppose $\mathcal{H}(x(t))$ is the Hilbert transform of $x(t)$, then show that $\mathcal{H}(\mathcal{H}(x(t)))$ is $-x(t)$ (1 mark).
4. Suppose $x(t)$ is a real valued signal. Show that $x(t)$ and its Hilbert transform $\mathcal{H}(x(t))$ are orthogonal (2 marks). Two signals $x(t)$ and $y(t)$ are orthogonal if $\int_{-\infty}^{\infty} x(t)y(t)dt = 0$.

Question 2 (5 marks): Suppose $m(t)$ is a real valued modulating baseband signal, with energy contained in the frequency range $[-B_m, B_m]$. Assume that $x(t)$ is obtained from $m(t)$ by single side band modulation, using upper side band, using a carrier $cos(2\pi f_c t)$ with carrier frequency f_c, where $f_c \gg B_m$. The signal $x(t)$ is transmitted over an ideal channel such that the received signal $y(t)$ is $x(t)$ itself. The signal $y(t)$ is applied to an ideal FM demodulator which recovers the instantaneous frequency $f(t)$ of $y(t)$. Note that the ideal FM demodulator disregards any amplitude variations. Write down an expression for $f(t)$ in terms of $m(t)$.

Question 3 (5 marks): Suppose $m(t)$ is $cos(2\pi 100t)$. A signal $x(t)$ is obtained by narrowband frequency modulation of $m(t)$ with a frequency modulation index $\beta \ll 1$ using a carrier $cos(2\pi f_f t)$, with $f_f = 1$ MHz.

1. Write down an approximate expression for the passband signal $x(t)$, as well as its approximate bandwidth. Justify your approximation. (1 mark)

The signal $x(t)$ is sent through an ideal channel such that the received signal $y(t)$ is $x(t)$ itself. The received signal $y(t)$ is applied to the QAM demodulator shown below:



The RF filter (F) is an ideal bandpass filter with passband from $1MHz$ to $1.2MHz$ (including these frequencies). The output $f(t)$ from the RF filter is applied to two multipliers M1 and M2 as shown. The multipliers multiply the signal $f(t)$ with the locally generated signals $cos(2\pi f_c t)$ and $sin(2\pi f_c t)$ leading to the signals $i(t)$ and $q(t)$. The frequency f_c for the locally generated signals is 1.1 MHz.

2. Write down the complex baseband equivalent (the complex envelope) $f_b(t)$ of the passband signal $f(t)$ centered at 1.1 MHz (2 marks).

3. Describe mathematically the signals $i(t)$ and $q(t)$ (1 mark each).

Best of luck!