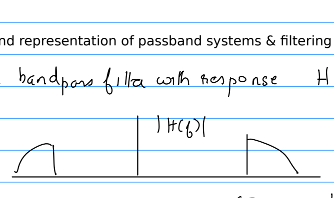
Complex baseband representation of passband systems & filtering

Consider a bandpass filter with response $H(f)$ like



$h(t)$ is a passband signal $\longrightarrow$ CB $\longrightarrow$ $h_b(t)$

$$\begin{array}{ccccc} p(t) & \longrightarrow & \boxed{h(t)} & \longrightarrow & y(t) \\ (P.B) & & (P.B) & & (P.B) \\ \downarrow & & \downarrow & & \downarrow \text{CB} \\ b(t) & & h_b(t) & \longrightarrow & y_b(t) \end{array}$$

S.T $\left(p(t) = 2Re\{ b(t)\cdot e^{j2\pi f_c t}\}\right)$

$$\left(b_r(t) + j b_i(t)\right)\left(\cos(2\pi f_c t) + j\sin(2\pi f_c t)\right)$$

$$y(t) = p(t) * h(t)$$
$$h(t) = 2Re\{ h_b(t) e^{j2\pi f_c t}\}$$

let us try: $\left(b(t)\cdot e^{j2\pi f_c t}\right) * \left(h_b(t) e^{j2\pi f_c t}\right)$

$$= \int_0^\infty h_b(\tau)\cdot e^{j2\pi f_c \tau} \cdot b(t-\tau)\cdot e^{j2\pi f_c(t-\tau)} d\tau$$

$$= e^{j2\pi f_c t}\cdot\left(\int_0^\infty h_b(\tau)\cdot b(t-\tau)\cdot d\tau\right)$$

$$= e^{j2\pi f_c t}\{ b(t) * h_b(t)\}.$$

$$p(t) * h(t) = 2Re\{ b(t) e^{j2\pi f_c t}\} * 2Re\{ h_b(t) e^{j2\pi f_c t}\}.$$

$$= \left(b(t) e^{j2\pi f_c t} + b^*(t) e^{-j2\pi f_c t}\right) * \left(h_b(t) e^{j2\pi f_c t} + h_b^*(t) e^{-j2\pi f_c t}\right)$$

$$\begin{aligned} = \; & e^{j2\pi f_c t}\{ b(t) * h_b(t)\} + \quad \to 1 \\ & b^*(t) e^{-j2\pi f_c t} * h_b(t) e^{j2\pi f_c t} + \\ & b(t) e^{j2\pi f_c t} * h_b^*(t) e^{-j2\pi f_c t} + \quad \to 3,4 \\ & b^*(t) e^{-j2\pi f_c t} * h_b^*(t) e^{-j2\pi f_c t}. \quad \to 2 \end{aligned}$$

$$y(t) = p(t) * h(t) \propto Re\{ (b(t) * h_b(t)) e^{j2\pi f_c t}\}$$

2 $\quad b^*(t) e^{-j2\pi f_c t} * h_b^*(t) e^{-j2\pi f_c t}$

$$= \int b^*(\tau) e^{-j2\pi f_c \tau} \cdot h_b^*(t-\tau)\cdot e^{-j2\pi f_c(t-\tau)} d\tau$$

$$= e^{-j2\pi f_c t} \int b^*(\tau)\cdot h_b^*(t-\tau)\cdot d\tau$$

combine 1 and 2 $= 2Re\{ (b(t) * h_b(t)) e^{j2\pi f_c t}\}$

consider 3 $\quad b^*(t) e^{-j2\pi f_c t} * h_b(t) e^{j2\pi f_c t}$

$$= \int b^*(\tau) e^{-j2\pi f_c \tau}\cdot h_b(t-\tau)\cdot e^{j2\pi f_c(t-\tau)} d\tau$$

$$= e^{j2\pi f_c t} \int b^*(\tau)\cdot e^{-j2\pi(2f_c)\tau} \cdot h_b(t-\tau)\cdot d\tau$$

$$b^*(t) e^{-j2\pi(2f_c)t} \longrightarrow \boxed{h_b(t)} \longrightarrow 0$$

H/W 4

So $y(t) = p(t) * h(t) = 2Re\{ \underbrace{(b(t) * h_b(t))}_{y_b(t)} e^{j2\pi f_c t}\}.$

becoz $y(t) = 2Re\{ y_b(t) e^{j2\pi f_c t}\}.$

Summary.

1) DSB
2) AM
3) SSB

Chap. 3 of UM

4) Complex BB rep. $\longrightarrow$ 2.8 of UM, (exercises)

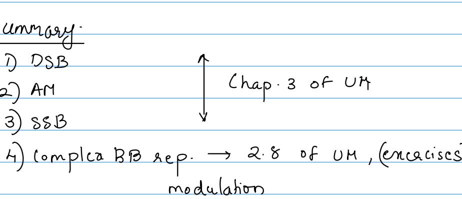
$$m(t) \longrightarrow \underset{\text{modulation}}{\text{PB Signal}} \longrightarrow \text{P.B.C} \longrightarrow \text{demodulate} \longrightarrow \hat{m}(t)$$

Amplitude$(t) \times \cos(2\pi f_c t)$

Reading assgn: Vestigial sideband modulation (TY) (UM).

Classtest on 16th Sept

Frequency modulation & phase modulation



Baseband $m(t) \longrightarrow H(f)$.

$A\cos(2\pi f_c t + 2\pi k_m\cdot m(t))$ – P.M (inst. phase $= k_m\cdot m(t)$)

k_m – P.M sensitivity $(1/V)$

$\underbrace{A\cos\left(2\pi f_c t + 2\pi k_f \int_0^t m(u)\cdot du\right)}$ – F.M $\quad k_f$ – FM sensitivity (Hz/V)

inst freq $= f_c + (k_f\cdot m(t))$

whether FM or PM are equivalent?

$$m(t) \longrightarrow \boxed{F.M} \longrightarrow A\cos\left(2\pi f_c t + 2\pi k_f \int_0^t m(u)\, du\right)$$

$$m(t) \longrightarrow \boxed{PM} \longrightarrow A\cos\left(2\pi f_c t + 2\pi k_m\cdot m(t)\right)$$

can we do FM using PM?

$$m(t) \xrightarrow{\;k_f/k_m\;} \boxed{\int} \xrightarrow{\;k_f/k_m \int_0^t m(u)\cdot du\;} \boxed{PM} \longrightarrow F.M$$

can we do PM with FM?

✓ $\quad m(t) \xrightarrow{\;?\;} \boxed{d/dt} \longrightarrow \boxed{FM} \longrightarrow A\cos(2\pi f_c t + k_m m(t))$

→ assuming that it is possible to recover $m(t)$ from an FM signal from now on, we will look at FM systems

$$m(t) \longrightarrow \boxed{F.M. \text{ modulator}} \longrightarrow A\cos\left(2\pi f_c t + 2\pi k_f \int_0^t m(u)\cdot du\right) \longrightarrow 1$$

Voltage controlled oscillators

$$m(t) \longrightarrow \boxed{VCO} \longrightarrow A\cos\left(\underbrace{2\pi f_c t + \phi(t)}\right)$$

$$f_c + \frac{1}{2\pi}\frac{d}{dt}\phi(t) \propto m(t)$$

$$1 \longrightarrow \boxed{\text{FM demod.}} \longrightarrow \hat{m}(t) \approx m(t)$$

limiter-discriminator

$$A\cos\left(2\pi f_c t + 2\pi k_f \int_0^t m(u)\cdot du\right) \longrightarrow d/dt \longrightarrow$$

$$\underbrace{\boxed{A\, 2\pi f_c\left(1 + \frac{k_f m(t)}{f_c}\right)}}\sin\left(2\pi f_c t + 2\pi k_f \int_0^t m(u)\cdot du + \right)$$

↓ envelope detection

↓ AC coupling

$(2\pi A k_f m(t))$

Recall from DSB: – when can we send $m(t)$ through $H(f)$?
one sided B/W of channel > 2 sided B/W of $m(t)$

$$A\cos\left(2\pi f_c t + 2\pi k_f \int_0^t m(u)\cdot du\right) \xrightarrow{\;\mathcal{F}\;} ?$$

$m(t) \longrightarrow \boxed{FM} \longrightarrow$ (see Matlab demo)