Indian Institute of Space Science and Technology
AV312 - Digital Communication
Department of Avionics

Final Exam for **B.Tech Avionics - Semester V** on **23/11/2015**

Note to the student

1. There are two parts to this question paper. Part-I contains 3 questions for 15 marks and Part-II contains 11 questions for 50 marks. There are 4 pages in total.

2. Note that Part-I will be considered as Quiz-3. Part-II will be considered as the final exam. Answer all questions in each part.

Part-I (for 15 marks)

Question 1 (8 marks) Let $m(t)$ be the signal $cos(2\pi f_m t)$. Assume that $x_1(t)$ is obtained by the SSB-modulation of $m(t)$ using the carrier $cos(2\pi f_c t)$. Also assume that f_m is very small compared to f_c. In order to de-modulate the SSB signal using envelope detection, we transmit the signal $x(t) = x_1(t) + acos(2\pi f_c t)$. Suppose we assume that $x(t)$ is received as it is without any distortion or noise addition. Obtain the output of an ideal envelope detector (which acts like an ideal rectifier followed by an ideal filter) when $x(t)$ is applied to it. Is it possible to recover $m(t)$ back from $x(t)$ using this ideal envelope detector?

Question 2 (3 marks) Suppose $x(t)$ is a real valued signal. Show that $x(t)$ and its Hilbert transform $\mathcal{H}(x(t))$ are orthogonal. Note that two signals $x(t)$ and $y(t)$ are orthogonal if $\int_{-\infty}^{\infty} x(t)y(t)dt = 0$.

Question 3 (4 marks) Assume that the local carrier signal generated while demodulating an SSB signal has a frequency error (offset) of Δ with respect to the carrier frequency f_c used to generate the SSB signal. Write down what the demodulated signal is when the SSB signal consists of the upper sideband only.

Part-II (for 50 marks)

Question 1 (3 marks) Suppose $m(t)$ is a baseband real valued signal bandlimited to $[-f_m, f_m]$ Hz. Let $x(t) = A + m(t)$, where A is a positive real valued constant. Let $x_s(t) = \sum_{n=-\infty}^{\infty} x(nT_s)\delta(t - nT_s)$, where $T_s = \frac{1}{10f_m}$. Assume that $x_s(t)$ is transmitted over an ideal bandpass channel with lower and upper cutoff frequencies $8f_m$ and $12f_m$ (only the positive frequencies are stated here). Let $y(t)$ be the output of the above bandpass channel. Assume that $\hat{m}(t)$ is obtained by applying $y(t)$ to an ideal envelope detector (which acts like an ideal rectifier followed by an ideal filter). What is the relationship between $\hat{m}(t)$ and $m(t)$?

Question 2 (2 marks) Suppose we have a random process $X(t) = Acos(2\pi f_c t)$, where f_c is a constant and A is random and uniformly distributed in $[0, 1]$. Is the process $X(t)$ strictly stationary?

Question 3 (5 marks) Suppose we have a continuous time linear filter with the impulse response

$$h(t) = \begin{cases} ae^{-at} \text{ for } 0 \leq t \leq T, \\ 0, \text{ otherwise.} \end{cases}$$

Find out the power spectral density of the output of the filter for an input signal with power spectral density $S_X(f)$.

Question 4 (3 marks) A signal has a total duration of 100 seconds. It is sampled at a rate of 8kHz and then encoded. The signal to quantization noise ratio is required to be 40 dB. What is the minimum storage capacity needed to store this signal? State all assumptions that you have made.

Question 5 (4 marks) Ten different message signals, each with a bandwidth of 10 kHz, are to be multiplexed and transmitted. Determine the minimum bandwidth required for each method if the multiplexing/modulation method used is: (a) FDM, SSB, (b) TDM, PAM.

Question 6 (4 marks) Consider a rectangular pulse defined by

$$g(t) = \begin{cases} A, \text{ for } 0 \leq t \leq T, \\ 0, \text{ otherwise.} \end{cases}$$

The matched filter for $g(t)$ is approximated by an ideal low pass filter with bandwidth B. Determine the value of B for which the ideal low pass filter provides the best approximation to the matched filter, considering the fact that maximization of the peak signal to noise ration is the primary objective.

Question 7 (8 marks) Assume that an IID bitstream $(B_1, B_2, \dots)$, with each bit B_i chosen uniformly, is transmitted over a channel using BPSK with energy per bit E_b. We assume that the channel adds additive white Gaussian noise (with power spectral density $\frac{N_0}{2}$) to the received signal. The coherent correlation receiver, unfortunately, has been designed as a BASK correlation receiver (at the same carrier frequency as the BPSK signal) to decode BASK signals which are assumed to use an energy per bit of $0.25E_b$. Obtain the average probability of symbol error when this correlation BASK receiver is used to decode the received BPSK signal. Express your answer in terms of the Q or erfc functions.

Question 8 (5 marks) Consider the BFSK system that we had discussed in class. Let the signals transmitted over the channel for the symbols 0 and 1 be $s_1(t)$ and $s_2(t)$ where

$$s_1(t) = A_c cos\left(2\pi\left(f_c + \frac{\Delta}{2}\right)t\right),$$
$$s_2(t) = A_c cos\left(2\pi\left(f_c - \frac{\Delta}{2}\right)t\right),$$

for $0 \le t \le T_b$. If $f_c > \Delta$ show that the correlation coefficient of the signals $s_1(t)$ and $s_2(t)$ is approximately $sinc(2\Delta T_b)$. Note that T_b is the duration for which the signal corresponding to one bit is transmitted. Determine the minimum value of Δ for which $s_1(t)$ and $s_2(t)$ are orthogonal. Find out the value of Δ that minimizes the average probability of symbol error for this system (state your assumptions on the bit stream that is transmitted).

Question 9 (4 marks) Assume that we have a BASK system (transmitter and receiver) which uses an energy per bit E_b and bit duration T_b. The transmission is done over a channel that is assumed not to introduce ISI, but adds noise which is assumed to be white Gaussian noise with power spectral density $\frac{N_0}{2}$. We assume that one of the components in the transmitter is subjected to power fluctuations. It is assumed that in each bit duration T_b, the transmitter component is either on or off (assume that there are no on-off transients when the component switches on and off), leading to the transmitter being either on or off in each bit duration, independently of the bit that is transmitted. The component on-off process is assumed to be an IID process, with the probability that the component is on in any bit duration being p_{on}. Note that the receiver does not know whether the component is actually on or off in a bit duration. Obtain the average symbol error probability for the above BASK system, if the input bit stream is IID and each bit is uniformly chosen. (Note that this on-off process is an additional source of error in the system.)

Question 10 (4 marks) Assume that an IID bitstream $(B_1, B_2, \dots)$, with each bit B_i chosen uniformly, is transmitted over a channel using BASK at carrier frequency f_c, with energy per bit E_b, and bit duration T_b. Obtain the power spectrum of the BASK signal which is transmitted over the channel.

Question 11 (8 marks) In wireless communications, regulations limit the amount of spectrum that a transmitter can use. Assume that a transmitter is allocated a bandwidth of $2W$ Hz at a center frequency of f_c. The pulse shaping techniques that we discussed in class are also used to make sure that the transmitted signal energy is "limited" to the region of the spectrum that is allocated to the transmitter. For example, "using" a sinc pulse shape, i.e. $sinc(2Wt)$, would ensure that the transmitted signal is bandlimited to $2W$ Hz. However, such pulse shapes cannot be used in practice. Therefore, consider the case where the pulse shape used is $sinc(2Wt) \times rect(t/T)$, where $rect(x) = 1$ for $|x| \leq 1$. Obtain the fraction of signal energy that is outside the allowed bandwidth of $2W$ Hz.

Best of luck!