Note regarding the Markov property of our system.

19/01/2019

- We are saying that S_{t+1} is specified by S_t for a fixed action a using a conditional pmf.

$$P_{S_{t+1}|s_t}(s_{t+1}).$$

(note that this is intuitively the Markov property. We are using this property in verifying the definition for Markov property below).

The joint distribution of the random variables involved in our system can be written down as

$$(A): P_{S_0 S_1 S_2 \dots S_t}(s_0, s_1 \dots s_t)$$

$$= P_{S_1|s_0}(s_1) \cdot P_{S_2|s_1}(s_2) \cdot P_{S_3|s_2}(s_3) \dots P_{S_t|s_{t-1}}(s_t)$$

for a fixed initial state s_0 (so that $P_{S_0|s_0}(s_0) = 1$).

We will use this property to verify the Markov property.

* Thanks to Raman and Allam Varshith for pointing out the circular like argument here and the mistake in the sequence of steps used in class. Please note that we are starting with a property that we have not yet defined to be Markov (but is from a Markov property).

In order to show that $- P_{S_t|s_{t-1} \dots s_0}(s_t) = P_{S_t|s_{t-1}}(s_t)$

we use $$P_{S_t|s_{t-1} \dots s_0}(s_t) = \frac{P_{S_t, S_{t-1} \dots S_0}(s_t, s_{t-1} \dots s_0)}{P_{S_{t-1}, S_{t-2} \dots S_0}(s_{t-1} \dots s_0)}$$

$$= \frac{P_{S_t|s_{t-1}}(s_t)\, P_{S_{t-1}|s_{t-2}}(s_{t-1}) \cdots P_{S_1|s_0}(s_1)}{P_{S_{t-1}|s_{t-2}}(s_{t-1}) \cdots P_{S_1|s_0}(s_1)}$$

$$= P_{S_t|s_{t-1}}(s_t).$$