AV312 - Lecture 10

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Figures from "Communication Systems" by Haykin and "An Intro. to Analog and Digital Commn." by Haykin and Moher

August 23, 2016

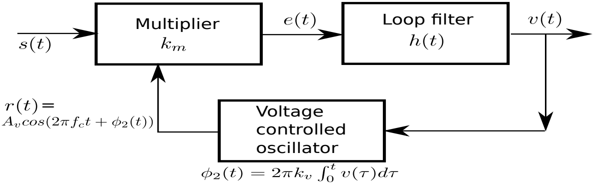
Review of last class

- Phase locked loop (PLL)
  - Carrier recovery using PLL
  - FM demodulation using PLL
  - DSB demodulation using PLL - Costas receiver
- Analysis of PLL

Today's class

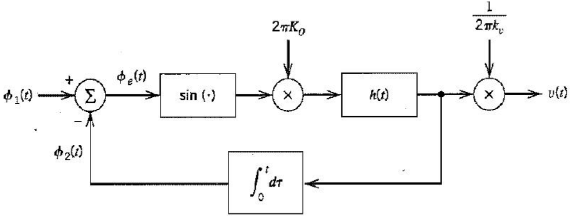
- PLL non-linear model
- Linearized model for PLL
- Analysis of PLL behaviour
- Today's scribes are Husnara Parveen & Sai Bhanumathi

PLL - Model



- Let $s(t) = A_c sin(2\pi f_c t + \phi_1(t))$
- Let $r(t) = A_v cos(2\pi f_c t + \phi_2(t))$
- $e(t) = k_m A_c A_v \left[sin(4\pi f_c t + \phi_1(t) + \phi_2(t)) + sin(\phi_1(t) - \phi_2(t))\right]$
- Since $h(t)$ is a low pass response;
  $v(t) = \int_{-\infty}^{\infty} k_m A_c A_v sin(\phi_1(\tau) - \phi_2(\tau)) h(t - \tau) d\tau$
- $\phi_2(t) = 2\pi k_v \int_0^t v(\tau) d\tau$

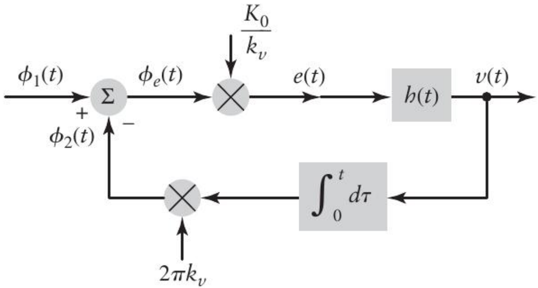
PLL - Model



- $\phi_e(t) = \phi_1(t) - \phi_2(t)$
- Loop gain parameter $K_o = k_v k_m A_c A_v$
- PLL model

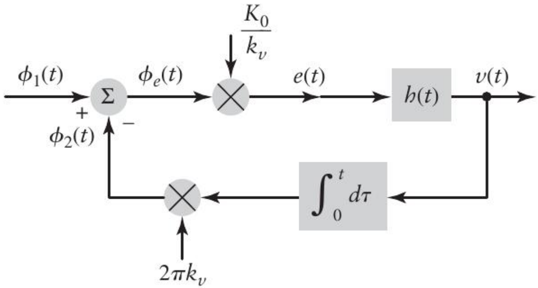
$$\frac{d\phi_e(t)}{dt} = \frac{d\phi_1(t)}{dt} - 2\pi K_o \int_{-\infty}^{\infty} sin(\phi_e(\tau))h(t-\tau)d\tau$$

PLL - Linearized Model



- Assume that $sin(\phi_e(t)) \approx \phi_e(t)$
- We use a Laplace transform domain approach
- $\phi_1(t) \leftrightarrow \Phi_1(s)$, $\phi_2(t) \leftrightarrow \Phi_2(s)$, $v(t) \leftrightarrow V(s)$, $h(t) \leftrightarrow H(s)$

PLL - Linearized Model



- $\frac{K_o}{k_v} H(s) \left(\Phi_1(s) - \Phi_2(s) \right) = V(s)$
- But $\Phi_2(s) = 2\pi k_v \frac{V(s)}{s}$
- $\Phi_1(s) \frac{K_o}{k_v} H(s) = V(s) \left[1 + \frac{2\pi K_o}{s} H(s) \right]$
- $\frac{V(s)}{\Phi_1(s)} = \frac{s(K_o/k_v)H(s)}{s+2\pi K_o H(s)}$ and $\frac{\Phi_e(s)}{\Phi_1(s)} = \frac{s}{s+2\pi K_o H(s)}$

PLL - Behaviour of $\phi_e(t)$

- Assume that the PLL is in "lock" initially
- Assume that the input phase changes, i.e., $\phi_1(t)$ changes
  - $\phi_1(t)$ is a step, ramp
- How does the loop filter affect the $\phi_e(t)$?
  - $H(s) = 1$ or $H(s) = \frac{s+a}{s}$
- The hold-in or lock range of the PLL is the range of frequencies that the PLL can track, once locked
- The pull-in or capture range of the PLL is the range of frequenices that the PLL can lock onto from a free running state
- Refer B.P. Lathi - Modern digital and analog communication systems