Lecture 2 24/07/2019

Review of lecture 1

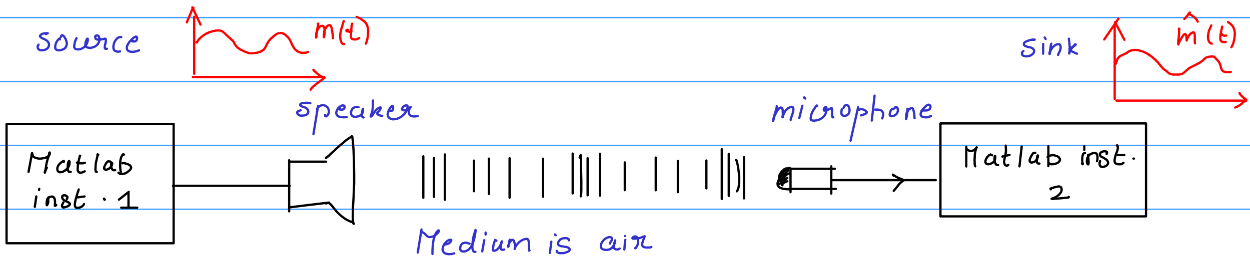
- what is the scope of AV314? What do we study in this course
- Source → has a mechanism / channel → Sink

has information signal $m(t)$ → separated by space / time → needs an exact / approximate reproduction of $m(t)$ called $\hat{m}(t)$

Objectives of this class

- understand channels / mechanisms.
- formal definition of the communication problem.
- start looking at channel modelling.

○ We will use an example of a mechanism or a channel to understand this. The mechanism / channel is an audio channel on a laptop (ideally this should be between two laptops, but for demo. we will only use one laptop)




```
%% sine transmitter
sampling_frequency = 48000;
sampling_period = 1/48000;
total_time = 5;
t = 0:sampling_period:total_time;

frequency = 12500; %Hz
amplitude = 0.1;

x = amplitude * sin(2 * pi * frequency * t);

% Transmit sine
p = audioplayer(x, sampling_frequency);
play(p);
plot(t, x);
% plotspectrum(x, sampling_frequency);
```

```
%% sine receiver
sampling_frequency = 48000;
sampling_period = 1/48000;

total_time = 6;

% Receiver
r = audiorecorder(sampling_frequency, 16, 1);
record(r);
pause(total_time);
stop(r);
rxsounding = double(getaudiodata(r, 'int16'));
plot(rxsounding);
% plotspectrum(rxsounding, sampling_frequency);
```

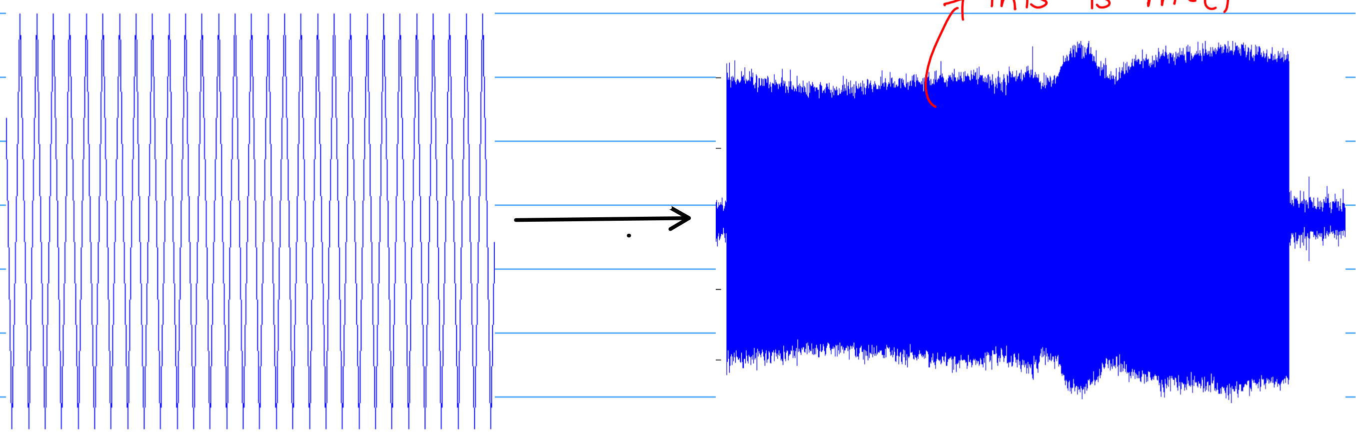
plays a single tone through the speaker. The x array here holds a sampled version of $m(t)$

The rxsounding variable / array contains $\hat{m}(t)$

- the medium is part of the channel.

this is $m(t)$ (zoomed in view)

this is $\hat{m}(t)$



- Let us now think about another channel, which is a wire (or a pair of wires). This channel also takes in a signal as input and produces a signal as output.
- From what we know about a radio frequency channel, we can say that such a channel also takes a signal as input and produces a signal as output.

So a reasonable way to think about channels (or model channels) is to say that a channel is a transformation (denoted as $f(\cdot)$), which transforms input signals $i(t)$ to output signals $o(t)$.

$$i(t) \longrightarrow \boxed{\begin{matrix}\text{Channel}\\ f(\cdot)\end{matrix}} \longrightarrow o(t)$$

input-output relationship is $o(t) = f(i(t))$.

Many channels may have restrictions on what kinds of $i(t)$ can be inputs to the channel. So we say that $i(t) \in I$. I is the set of allowed input signals. Similarly, $o(t) \in O$; O is the set of all allowed output signals.

an example: a 'perfect' wire $\quad o(t) = f(i(t))$ where $f(\cdot)$ is the identity map.
so that $o(t) = i(t)$.

audio channel example: $o(t) = f(i(t)) = \left(\tfrac{1}{2}\right) i(t) + \left(\tfrac{1}{4}\right) i(t)^2 + n(t) + N(t)$

interference $\quad$ noise

what are the properties expected out of a good channel?

for example - if $i_1(t)$ and $i_2(t) \in I$, and $i_1(t) \neq i_2(t)$, then
$$f(i_1(t)) \neq f(i_2(t)).$$

- Formal definition of a communication problem:

Suppose we start with the assumption that $I = O$ (for simplicity only!)

Case A: $m(t) \in I$, $\hat{m}(t)$ also then $\in I$

In order to use the given channel $f(\cdot)$ we can adopt a strategy like:

$$i(t) = m(t) \longrightarrow \boxed{\begin{matrix}\text{Channel} \\ f(\cdot)\end{matrix}} \longrightarrow o(t)$$

$o(t)$ may or may not be $\hat{m}(t)$.

Case B: $m(t) \notin I$.

In this case we need to transform $m(t)$ to a form compatible with the channel. And we also need to convert the $o(t)$ to $\hat{m}(t)$. These additional operations are shown below:

$$\text{source} \xrightarrow{m(t)} \boxed{\begin{matrix}\text{Transmitter} \\ t(\cdot)\end{matrix}} \xrightarrow{i(t)} \boxed{\begin{matrix}\text{Channel} \\ f(\cdot)\end{matrix}} \xrightarrow{o(t)} \boxed{\begin{matrix}\text{Receiver} \\ r(\cdot)\end{matrix}} \xrightarrow{\hat{m}(t)} \text{sink}$$

Note that in general $t(\cdot)$ and $r(\cdot)$ are required to

a) make $m(t)$ compatible with channel input and to convert channel output to the proper form.

b) undo effects of the channel which might make $\hat{m}(t) \not\approx m(t)$

- The communication problem

Design $t(\cdot)$ and $r(\cdot)$

such that $m(t) \approx \hat{m}(t)$

given $f(\cdot)$

The notion of $m(t) \approx \hat{m}(t)$ can be formalized by using an error function to measure similarity between $m(t)$ and $\hat{m}(t)$.

$$\text{for example} \quad e(m(t), \hat{m}(t)) = \int_{-\infty}^{\infty} (m(t) - \hat{m}(t))^2 \cdot dt$$

and our problem becomes: Design $t(\cdot)$ and $r(\cdot)$

such that $e(m(t), \hat{m}(t)) \leq \varepsilon$.

given $f(\cdot)$