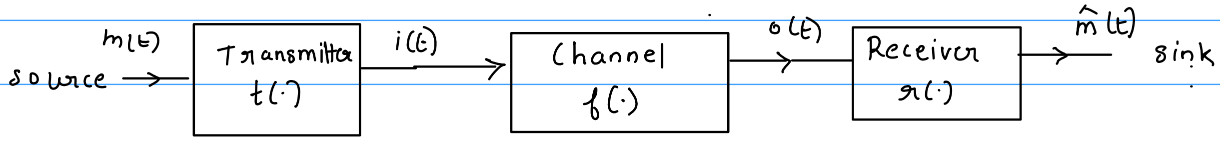
Lecture 7 06/08/2019

Review



Design $t(\cdot)$ and $r(\cdot)$

such that $e(m(t), \hat{m}(t)) \le \varepsilon$.

given $f(\cdot)$ modelled as a LTI filter $(h(t))$

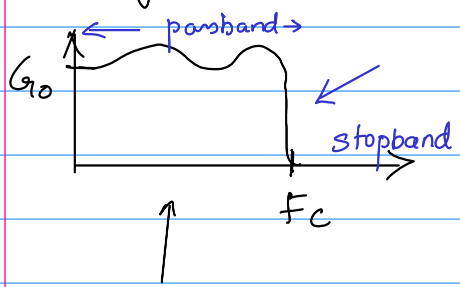
an important idea here is that $t(\cdot)$ and $r(\cdot)$ are used to nullify the effect of the channel (at least in our current understanding).
or counteract

$$r(f(t(m(t)))) \approx m(t)$$

other model - bandpass
passband

Usually wired channels are modelled as low pass filters

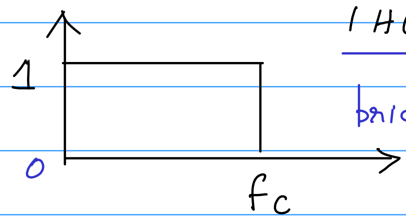
another name for this is : baseband



- other ways in which a LPF is characterized.
- $H(f)$ of the LPF

actual low pass filters

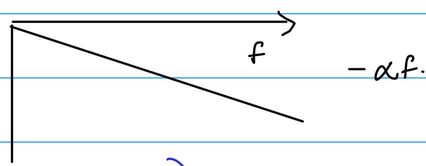
We replace this by an ideal filter (low pass) to study how to design $t(\cdot)$ and $r(\cdot)$ (→ we are studying how to design $t(\cdot)$ and $r(\cdot)$ for baseband channels)

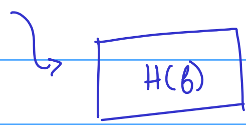


$|H(f)|$

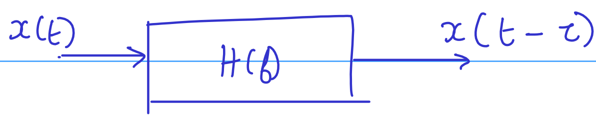
brickwall response

$\angle H(f)$ — linear phase response





Question ? why is linear phase response important ?

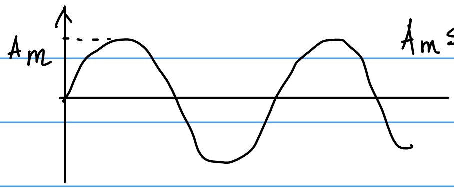


Some examples :

$m(t) \rightarrow \boxed{t(\cdot)} \rightarrow \boxed{LPF} \rightarrow \boxed{r(\cdot)} \rightarrow \hat{m}(t)$

? ?

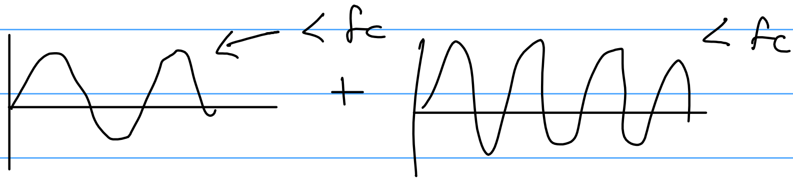
e.g. $m(t)$ is



$A_m \sin(2\pi f t)$, with $f < f_c$

so $t(\cdot)$ and $r(\cdot)$ are identity functions

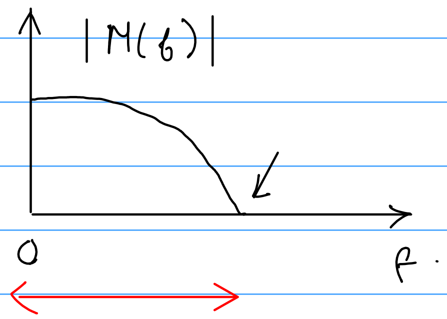
e.g. $m(t)$ is



$A_1 \sin(2\pi f_1 t) +$
$A_2 \sin(2\pi f_2 t)$
$(f_1 < f_c,\ f_2 < f_c)$

$m(t)$ will have some characteristics which are important for designing $t(\cdot)$ and $r(\cdot)$

$$m(t) \ , \ E_m = \int_{-\infty}^{\infty} |m(t)|^2 dt < \infty \ , \ m(t) \xrightarrow{\mathcal{F}} M(f) \checkmark$$

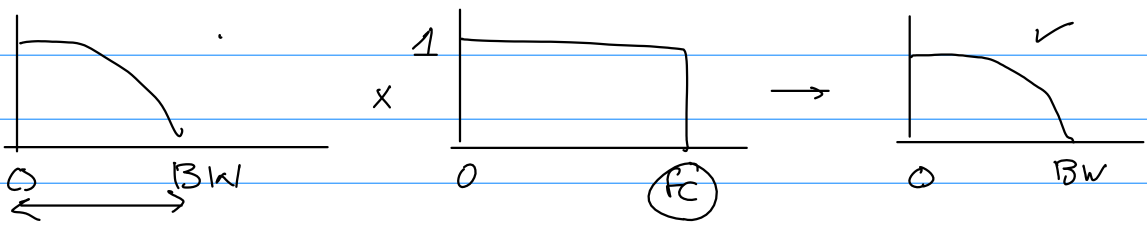


$m(t)$ will a low pass signal

baseband

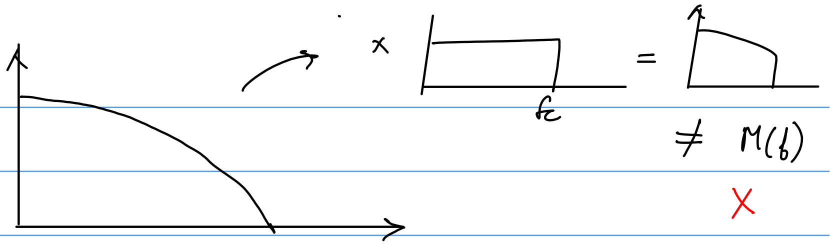
→ more precisely this is one sided bandwidth of a signal.

What is $t(\cdot)$ and $r(\cdot)$ if $f_c >$ Bandwidth ?



If the channel has a $f_c >$ BW of the signal then send $m(t)$ through the channel.

e.g suppose $m(t)$ has a $|M(f)|$ given by



BW $>$ fc

a) try and get another channel (with a larger fc

b) how to reduce BW?

$x(t) \xrightarrow{\mathcal{F}} X(f)$

$x(at) \xrightarrow{\mathcal{F}} X(f/a)$

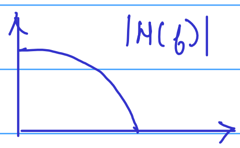
- what is this property of CTFT?
- what is $t(\cdot)$ and $r(\cdot)$ here?

Summary of last class

- for an ideal baseband channel
  - BW $>$ fc
  - BW $\le$ fc – $t(\cdot)$ and $r(\cdot)$ are identity

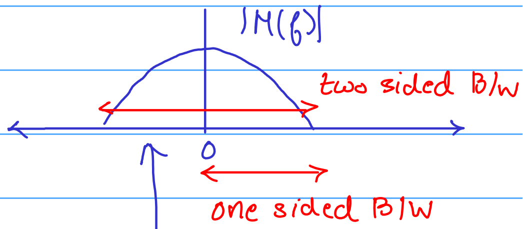
Bandwidth for $m(t)$.

earlier



this is a plot of +ive freq.

$M(f)$ as $\int_{-\infty}^{\infty} m(t) \cdot e^{-j2\pi f t}\, dt$, $f \in (-\infty, \infty)$

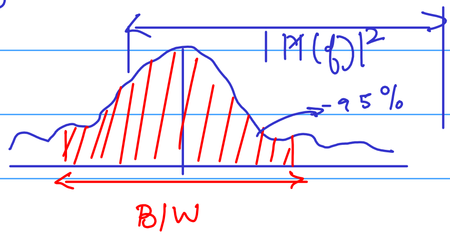


for real valued $m(t)$, "nice" properties of $M(f)$?

$M(-f) = M^*(f)$ ← derived?

Definition of bandwidth: energy containment bandwidth. ← UM

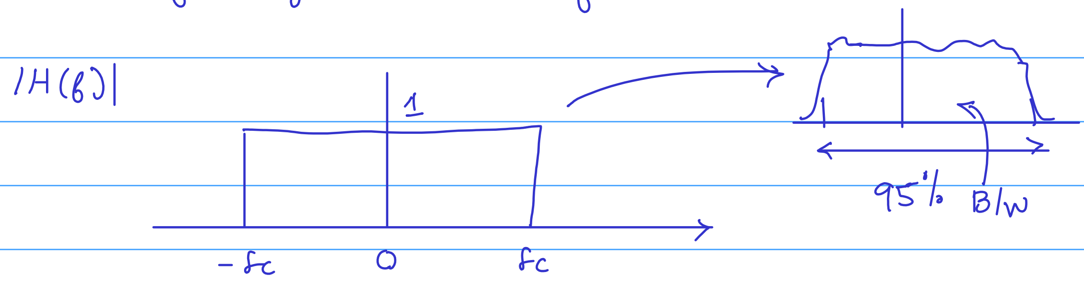
$|M(f)|$ for a real world signal



$e(\cdot)$

Parseval's theorem.

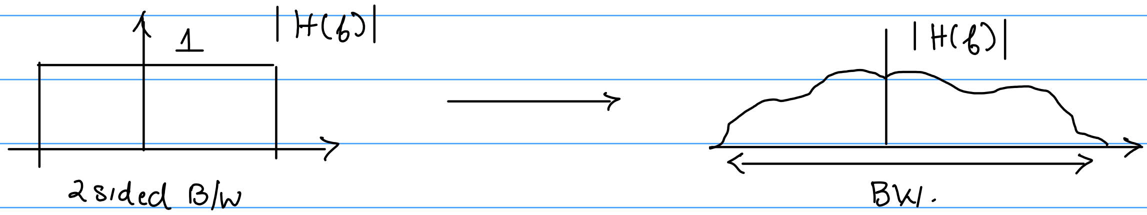
a similar definition for LPF models for channels.



$2f_c$ = 2 sided B/w of our channel.

for baseband channels (which are modelled as ideal LPFs).
t(·) and r(·) are decided just by signal (m(t)) bandwidth
and channel bandwidth.

A slightly more complicated design problem.



suppose m(t) has a 2sided B/w < 2 sided B/w of the channel.

$m(t) \longrightarrow \boxed{H(f)} \longrightarrow$ distorted version of m(t).

so,

equalizer.

$m(t) \longrightarrow \boxed{H(f)} \longrightarrow \boxed{H(f)^{-1}} \longrightarrow m(t-\tau)$

aside:

audio equalizer