4 Filter Design Review

For this labsheet, whereever you are asked to plot the magnitude response of filters, you should plot the magnitude normalized with the gain at discrete frequency 0 in dB scale.

1. Given the following requirements on a filter in continuous time, manually derive the desired ideal frequency response $H_d(e^{j\omega})$ in the discrete frequency domain.
   - sampling frequency $= 8kHz$, and,
   - pass all signals below $1kHz$ with a gain of 1, and,
   - cutoff all signals above $1kHz$ (or the cutoff frequency is $\Omega_c = 1kHz$).

   Also derive the corresponding impulse response $h_d[n]$.
   - In general, what is the ideal lowpass filter response $H_d(e^{j\omega})$ that would also have linear phase for a cutoff frequency of ω_c?
   - What is the corresponding impulse response $h_d[n]$?

2. Let $w[n]$ be a rectangular window of length $M+1$. That is

$$w[n] = \begin{cases} 1, & \text{for } n \in \{0, 1, \dots, M\}, \\ 0, & \text{otherwise.} \end{cases}$$

   a) Plot the magnitude (in dB) and phase response of the window (i.e., $20log_{10}|W(e^{j\omega})|$ and $\angle W(e^{j\omega})$) for $M = 10, 50$ and 100. What do you observe for different values of M?
   b) Demonstrate the effect of using rectangular windows with $M = 10, 50$ and 100 on the $h_d[n]$ obtained in Task 1. That is, for each M, obtain $h[n] = h_d[n]w[n]$ and plot the corresponding magnitude and phase plots of the DTFT of $h[n]$, i.e., $H(e^{j\omega})$. (Magnitude plot should be in dB scale). What do you observe? What are the values of the peak overshoots and undershoots for each value of M. Do they change as M changes? What do you need to do to ensure that for each value of M the phase response is linear?

3. Study what the following inbuilt Matlab window functions do:
   a) bartlett
   b) hamming
   c) hanning
   d) blackman
   e) kaiser

4. In Task 3 above, you would have found that each function can be used to generate windows $w[n]$ of any needed length. If length $M+1$ windows are generated, then $w[n]$ has symmetry around $(M+1)/2$.
   a) For each window function above, plot the magnitude (in dB) and phase response of the window (i.e., $20log_{10}|W(e^{j\omega})|$ and $\angle W(e^{j\omega})$) for $M = 10, 50$ and 100. Observe and tabulate the maximum sidelobe amplitude and the width of the main lobe for each window and for each M.

b) For each M and for each window function above, i.e., a $w[n]$, plot the magnitude (in dB) of the filter that would be obtained when using the FIR response $h[n] = w[n]h_d[n]$, where $h_d[n]$ is the desired response derived in Task 1. What is the peak approximation error (in dB) that you obtain for each window and for each M? Again make sure that each filter has a linear phase response.

5. Suppose one desires to design the following low pass filter (this is a specification of the desired response $H_d(e^{j\omega})$).

$$|H_d(e^{j\omega})| \text{ is } \begin{cases} \in [1-0.01, 1+0.01], \text{ for } 0 \leq |\omega| \leq 0.25\pi, \\ \in [0, \delta], \text{ for } |\omega| \geq 0.3\pi. \end{cases}$$

The transition band is $(0.25\pi, 0.3\pi)$.

a) Obtain a complete specification of $H_d(e^{j\omega})$ so that we have a filter with linear phase response.

b) Design a filter which meets the above specifications using either Hamming, Hanning, or Blackman windows separately for the cases $\delta = 0.01$ and $\delta = 0.001$. Use the width of the main lobe and the peak approximation error that you have found in Task 4 above for this (or you can refer to the table containing the main lobe width and peak approximation error that we had discussed in class).

c) For each δ, plot the desired magnitude plot along with the magnitude plot of the filters that you have designed and comment on the differences. Also check whether the designed filters have linear phase responses.

6. Suppose one desires to design a filter using the Kaiser window method. We will use the $H_d(e^{j\omega})$ defined in Task 5 as the desired frequency response.

a) Design a linear phase low pass filter which meets the above specifications using Kaiser window for the cases $\delta = 0.01$ and $\delta = 0.001$. Please use the design formulae (formulae for β and M) that we have studied in class.

b) For each δ, plot the desired magnitude plot along with the magnitude plot of the filters that you have designed and comment on the differences. Also check whether the designed filters have linear phase responses.

7. Study what the Matlab inbuilt function "fir1" does. Use "fir1" to obtain a fir filter matching the desired response in Task 6.

8. Generate a signal $x[n] = cos(0.1\pi n) + 2cos(0.5\pi n)$ for $n \in \{0, 1, \ldots, 5M^*\}$ where M^* is the maximum length of the filters that you have designed in Tasks 5, 6, 7. For each of the filters that you have designed above in Tasks 5,6, and 7, obtain the signal $y[n]$ which results when $x[n]$ is passed through the filter. Plot $x[n]$ and $y[n]$ for all cases. Also plot their DTFT magnitudes, i.e., $|X(e^{j\omega})|$ and $|Y(e^{j\omega})|$. What do you observe?

9. Suppose one desires to design the following low pass filter (this is a specification of the desired response $H_d(e^{j\omega})$.

$$|H_d(e^{j\omega})| \text{ is } \begin{cases} \in [1-0.01, 1+0.01], \text{ for } 0 \leq |\omega| \leq 0.25\pi, \\ \in [0, \delta], \text{ for } |\omega| > 0.3\pi. \end{cases}$$

a) Obtain a complete specification of $H_d(e^{j\omega})$ so that we have a filter with linear phase response

b) Design filters which meets the above specifications using the frequency sampling method for the cases $\delta = 0.01$ and $\delta = 0.001$.

c) Plot the desired magnitude plot along with the magnitude plot of the filter that you have designed and comment on the differences.

d) For each δ above, plot separate magnitude plots of the filters that you have obtain if you apply circular shifts of $M/4$ and $M/2$ to the $h[n]$. What do you observe?

e) Suppose we need to design a filter with $\delta = 0.001$. Using two frequency samples in a "transition band" is it possible to obtain a $\delta = 0.001$? What should be the values of those two frequency samples? Is there a tradeoff between δ and M?

10. Study what the Matlab inbuilt functions "fir2" and "firls" do. Go through the design examples which are shown in Matlab's help for these two functions.

11. Study what the Matlab inbuilt function "firpm" (or "remez") does. Use firpm to design a linear phase equiripple filter meeting the requirements in Task 1.

12. Matlab also provides filter design tools such as "filterbuilder" and "fdatool". Explore how these tools can be used to design FIR filters.

13. Visualization of the bilinear transform: The bilinear transform is a map between the s-plane and the z-plane and is defined as $z = \frac{1+sT/2}{1-sT/2}$. The inverse map is defined as $s = \frac{2}{T}\frac{1-z^{-1}}{1+z^{-1}}$. In this task you will visualize how an area in the s plane is mapped to z-plane and vice-versa.

    a) Select $N \times M$ points $s_{i,j}$ uniformly in any rectangular region that you wish in the s-plane. (Hint: you can use meshgrid for this purpose). On the z-plane indicate where these points are mapped to under the bilinear transform.

    b) Similarly select $N \times M$ points $z_{i,j}$ uniformly in any rectangular region that you wish in the z-plane. (Hint: you can use meshgrid for this purpose). On the s-plane indicate where these points are mapped to under the bilinear transform.

    c) Verify (using a sufficiently dense set of points) whether the $j\Omega$ axis is mapped to the unit circle under the bilinear transform.

14. Filter design using least squares inverse design: Suppose the desired frequency response $H_d(e^{j\omega})$ can be realized by the causal system with the following z-transform,

$$H_d(z) = \frac{1 + z^{-1}}{1 - 0.5z^{-1}}.$$

    Suppose we approximate $H_d(z)$ using a $H(z)$ of the form

$$\frac{b_0}{1 - a_1 z^{-1} - a_2 z^{-2}}.$$

    Using least squares inverse design, obtain the values of b_0, a_1 and a_2. We note that in least squares inverse design, a set of linear equations have to be written down which constrain the values of b_0, a_1 and a_2. Explore the effect of the number of linear equations that you have on your answer.

15. Filter design using Butterworth analog filter design and impulse invariance: Suppose we have the following desired requirements $H_d(e^{j\omega})$ on the magnitude of digital filter:

    a) Passband edge $= 0.2\pi$

    b) Stopband edge (starting freq) $= 0.4\pi$

    c) Magnitude gain in passband to be $\in [1, 1 - \delta_p]$, where $\delta_p = 0.05$

    d) Magnitude gain in stopband to be $\in [0, \delta_s$, where $\delta_s = 0.001$

    Note that no constraints are being put on the phase response of the filter here. In the design of the filter, explore how you would use the "buttord" inbuilt function in Matlab.

    a) Assuming that there is no aliasing and that $H_d(e^{j\omega})$ has been obtained from sampling of an analog signal $h_a(t)$ uniformly at rate $\frac{1}{T}$, what is $H_a(j\Omega)$ (the CTFT of $h_a(t)$)?.

    b) We note that $H_a(j\Omega)$ can be interpreted as the specification for the design of an analog filter. Obtain a Butterworth filter that is a good approximation to $H_a(j\Omega)$.

    c) Write down the location of the poles of the analog Butterworth filter $H_a(s)$?

d) Under the impulse invariance condition, where are these poles mapped to in the z-plane. Write down the locations of the poles.

e) Plot the frequency response of the filter that you have obtained.

Exploration:

a) Does the design depend on the actual value of T? Is there any change in the frequency response of the realized digital filter if you use different values of T?

b) Repeat the design process but by not compensating for aliasing in the stopband attenuation. How much is stopband attenuation in the final design? Does it meet the given requirements on $H_d(e^{j\omega})$?

c) Using internet resources or Matlab help, find out what the inbuilt function "butter" does. How will you use "butter" for the design problem above?

d) Using internet resources or Matlab help, find out what the inbuilt function "filter" does. Suppose $x[n] = 2cos(0.1\pi n) + 5cos(0.6\pi n)$ for $n \in \{0, \dots, 499\}$. Simulate what happens when the filter that you have designed above is used to filter $x[n]$ in order to obtain $y[n]$. Plot $y[n]$ as well as its DTFT.

16. Repeat the above design using the bilinear transformation. Plot the frequency response of the filter that you have obtained.