Indian Institute of Space Science and Technology
AV336 - Digital Signal Processing Lab
Department of Avionics

Labsheet 7

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1. Given the following requirements on a filter in continuous time, derive (manually) the desired ideal frequency response $H_d(e^{j\omega})$ in the discrete frequency domain.

   - sampling frequency = 8 kHz, and,
   - pass all signals below 1 kHz with a gain of 1, and,
   - cutoff all signals above 1 kHz.

   Also derive the corresponding impulse response $h_d[n]$.

2. Let $w[n]$ be a rectangular window of length $M+1$. That is

$$w[n] \quad = \quad \begin{cases} 1, & \text{for } n \in \{0, 1, \ldots, M\}, \\ 0, & \text{otherwise.} \end{cases}$$

   (a) Plot the magnitude (in dB) and phase response of the window (i.e., $20log_{10}|W(e^{j\omega})|$ and $\angle W(e^{j\omega})$) for $M = 10, 50$ and 100. Plot the magnitude response such that the gains are normalized by the maximum gain in the passband. What do you observe for different values of M?

   (b) Demonstrate the effect of using rectangular windows with $M = 10, 50$ and 100 on the $h_d[n]$ obtained in Task 1. That is, for each M, obtain $h[n] = h_d[n]w[n]$ and plot the corresponding magnitude and phase plots of the DTFT of $h[n]$, i.e., $H(e^{j\omega})$. (Magnitude plot should be in dB scale). What do you observe? What are the values of the peak overshoots and undershoots for each value of M. Do they change as M changes?

3. Study what the following inbuilt Matlab window functions do:

   (a) bartlett
   (b) hamming
   (c) hanning
   (d) blackman
   (e) kaiser

4. In Task 3 above, you would have found that each function can be used to generate windows $w[n]$ of any needed length. The windows are generated such that they have symmetry around $(M+1)/2$.

   (a) For each window function above, plot the magnitude (in dB) and phase response of the window (i.e., $20log_{10}|W(e^{j\omega})|$ and $\angle W(e^{j\omega})$) for $M = 10, 50$ and 100. Observe and tabulate the maximum sidelobe amplitude and the width of the main lobe for each window and for each M.

(b) For each M and for each window function above, i.e., a $w[n]$, plot the magnitude (in dB) of the filter that would be obtained when using the FIR response $h[n] = w[n]h_d[n]$, where $h_d[n]$ is the desired response derived in Task 1. What is the peak approximation error (in dB) that you obtain for each window and for each M?

5. Suppose one desires to design the following low pass filter (this is a specification of the desired response $H_d(e^{j\omega})$.

$$|H_d(e^{j\omega})| \text{ is } \begin{cases} \in [1-0.01, 1+0.01], & \text{for } 0 \leq |\omega| \leq 0.25\pi, \\ \in [0, \delta], & \text{for } |\omega| > 0.3\pi. \end{cases}$$

   (a) Obtain a complete specification of $H_d(e^{j\omega})$ so that we have a filter with linear phase response

   (b) Design a filter which meets the above specifications using either Hamming, Hanning, or Blackman windows. Use the width of the main lobe and the peak approximation error that you have found in Task 4 above for this. Do this design for $\delta = 0.01$ and $\delta = 0.001$.

   (c) Plot the desired magnitude plot along with the magnitude plot of the filter that you have designed and comment on the differences.

6. Suppose one desires to design a filter using the Kaiser window method. We will use the $H_d(e^{j\omega})$ defined in Task 5 as the desired frequency response. Using the design formulae that we have studied in class design a linear phase low pass filter using the Kaiser window for $\delta = 0.01$ and $\delta = 0.001$.

7. Study what the Matlab inbuilt function "fir1" does. Use "fir1" to obtain a fir filter matching the desired response in Task 6.

8. Generate a signal $x[n] = cos(0.1\pi n) + 2cos(0.5\pi n)$ for $n \in \{0, 1, \ldots, 5 * M^*\}$ where M^* is the maximum length of the filters that you have designed in Tasks 5, 6, 7. For each of the filters that you have designed above in Tasks 5,6, and 7, obtain the signal $y[n]$ which results when $x[n]$ is passed through the filter. Plot $x[n]$ and $y[n]$ for all three cases. Also plot their DTFT magnitudes, i.e., $|X(e^{j\omega})|$ and $|Y(e^{j\omega})|$. What do you observe?