AV314 - Pulse modulation

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Figures from "Communication Systems" by Haykin and "An Intro. to Analog and Digital Commn." by Haykin and Moher

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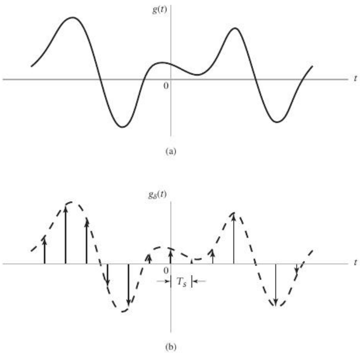
Introduction

- Analog and digital pulse modulation
- Analog pulse modulation: A parameter of a periodic pulse train is varied
  - Amplitude, Width, or Position
- Digital pulse modulation: Coded pulses (e.g., presence or absence)
- In any form of pulse modulation, analog continuous time information is converted into a discrete time signal

Sampling



- Suppose $g(t)$ is a bandlimited finite energy signal
- The sequence $g[n] = g(nT_s)$ is obtained by "sampling" the signal at instants nT_s
- The sampling period is T_s and the sampling rate/frequency f_s is $\frac{1}{T_s}$
- This idealized model of sampling is called instantaneous sampling

Sampled signal - model

- We obtain a sequence $g[n] = g(nT_s)$ after sampling
- To analyze systems which are fed with sampled signals - either discrete time or continuous time analysis
- If continuous time analysis, then we use the following representation for the sampled signal
- $g_\delta(t) = \sum_{n=-\infty}^{\infty} g(nT_s)\delta(t - nT_s)$

Relationships between $g(t)$ and $g_\delta(t)$

- What is the FT $G_\delta(f)$?

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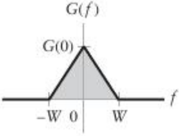
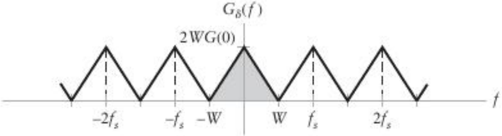
- What is the FT $G_\delta(f)$?
- $G_\delta(f) = f_s \sum_{m=-\infty}^{\infty} G(f - mf_s)$
- Suppose the signal $g(t)$ is bandlimited with bandwidth $2W$
- Suppose the sampling frequency is also $2W$.

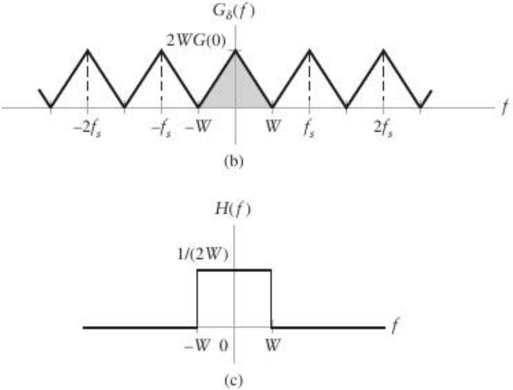


(a)



(b)

Relationships between $g(t)$ and $g_\delta(t)$



- In this case, it is possible to recover $g(t)$ from $g_\delta(t)$ using a reconstruction filter
- The reconstruction filter is non-causal. Read about the time-domain interpolation function from the text.

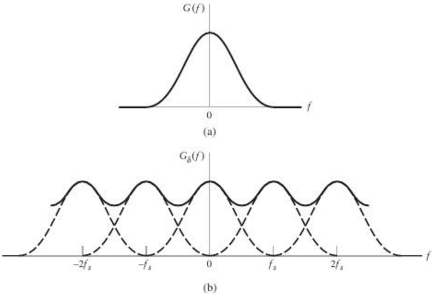
Sampling theorem

Theorem

A bandlimited signal $g(t)$ of finite energy with no energy in frequencies higher than W is completely specified by the samples $g(nT_s)$ where $T_s \leq \frac{1}{2W}$. Furthermore, $g(t)$ maybe recovered from its samples $g(nT_s)$ in this case.

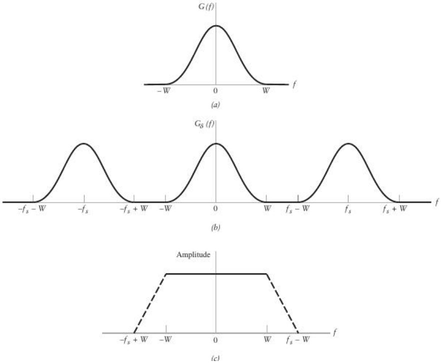
- $T_s \leq \frac{1}{2W}$ or $f_s \geq 2W$.
- The Nyquist sampling rate is $2W$

Aliasing



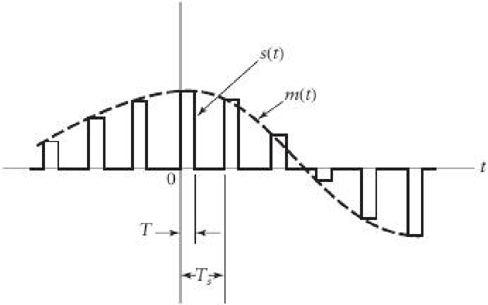
- Aliasing: Either $g(t)$ is not bandlimited or we “undersample” the signal $g(t)$

Aliasing



- Prefiltering using an anti-aliasing filter to bandlimit the signal that is actually sampled
- Sampling may be done at a $f_s > 2W$

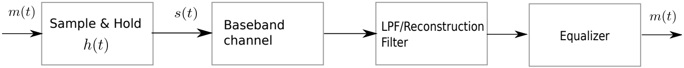
Pulse amplitude modulation



- Message signal $m(t)$ is finite energy and bandlimited
- Sampling frequency is f_s which is greater than or equal to the Nyquist rate
- PAM signal $s(t) = \sum_{n=-\infty}^{\infty} m(nT_s)h(t - nT_s)$
- Note that this is different from $m(t) \times \sum_{n=-\infty}^{\infty} h(t - nT_s)$

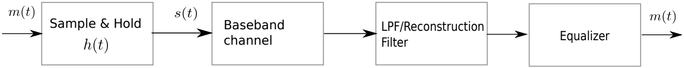
Pulse amplitude modulation and demodulation system

Pulse amplitude modulation and demodulation system



- $s(t) = \sum_{n=-\infty}^{\infty} m(nT_s)h(t - nT_s)$
- What is $m_\delta(t) \star h(t)$?

Pulse amplitude modulation and demodulation system



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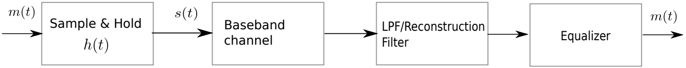
- So ... $m(t) \rightarrow$ Inst. sampling $\rightarrow m_\delta(t) \rightarrow h(t) \rightarrow s(t)$

Pulse amplitude modulation and demodulation system



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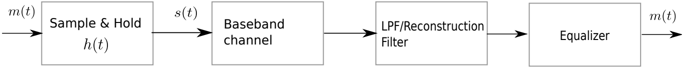
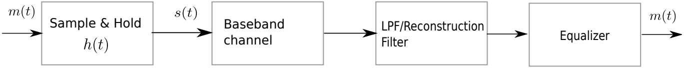
- So ... $m(t) \rightarrow$ Inst. sampling $\rightarrow m_\delta(t) \rightarrow h(t) \rightarrow s(t)$
- The equalizer has to compensate for $h(t)$
- The effect due to the $h(t)$ block is called Aperture effect

Pulse amplitude modulation and demodulation system



- $s(t) = \sum_{n=-\infty}^{\infty} m(nT_s)h(t - nT_s)$
- We can represent $s(t)$ in an alternate way
- Consider $m_\delta(t) \star h(t)$?

Pulse amplitude modulation and demodulation system



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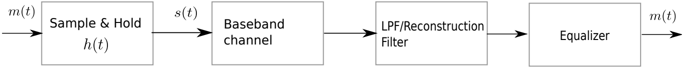

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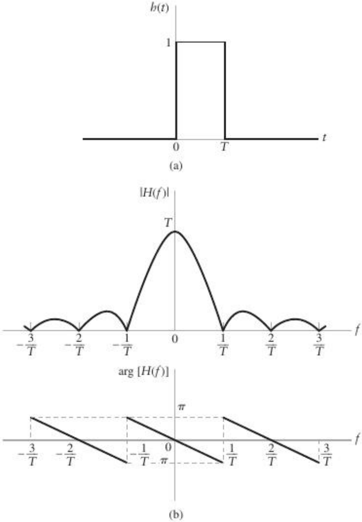
- So $s(t)$ is obtained by
  $m(t) \rightarrow$ Inst. sampling $\rightarrow m_\delta(t) \rightarrow h(t) \rightarrow s(t)$
- The equalizer has to compensate for $h(t)$
- The effect due to the $h(t)$ block is called aperture effect
- If channel has a known impulse response $c(t)$, then equalizer has to compensate for that too

Analysis of PAM modulation and demodulation

- Suppose

$$h(t) = \begin{cases} 1, 0 < t < T, \\ 1/2, t = 0 \text{ or } t = T, \\ 0, \text{ otherwise.} \end{cases}$$

- What is $H(f)$?
  $H(f) = Tsinc(fT)e^{-j\pi fT}$

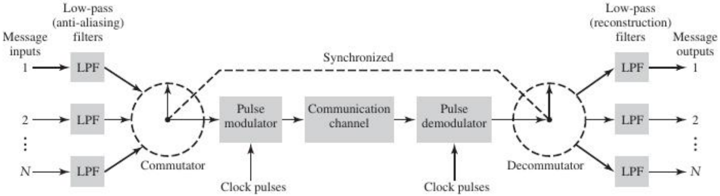


PAM demodulation

- To compensate for the effect of $H(f)$ the equalizer should be $\frac{1}{H(f)}$.
- However, if the equalizer is placed after the LPF, we only need to compensate using $\frac{1}{H(f)}$ for $f \in [-W, W]$
- If $C(f)$ is known, then the equalizer should be designed to be $\frac{1}{C(f)H(f)}$ within the band of interest

Why is PAM used?

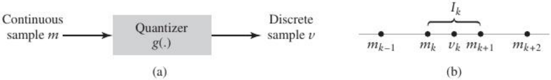
- Using a discrete time signal allows for time division multiplexing
- Read Sections 3.4 (PWM, PPM) and 3.9 (TDM) from the textbook "Communication Systems"



Quantization

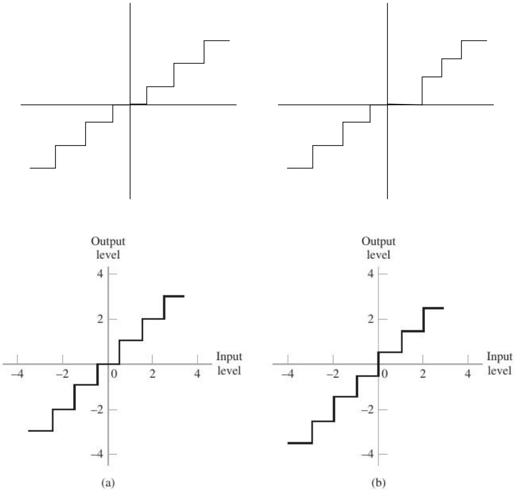
- We are now moving to digital transmission of analog signals - time has been discretized by sampling, amplitude is discretized using quantization.
- $m(t)$ be a finite energy bandlimited signal and $m(nT_s)$ denotes its samples
- Quantization is a mapping $g(.)$; $v(nT_s) = g(m(nT_s))$
- The value of sample is mapped by $g(.)$ to a discrete set
- For brevity, let us drop the time index nT_s

Quantization function $g(.)$

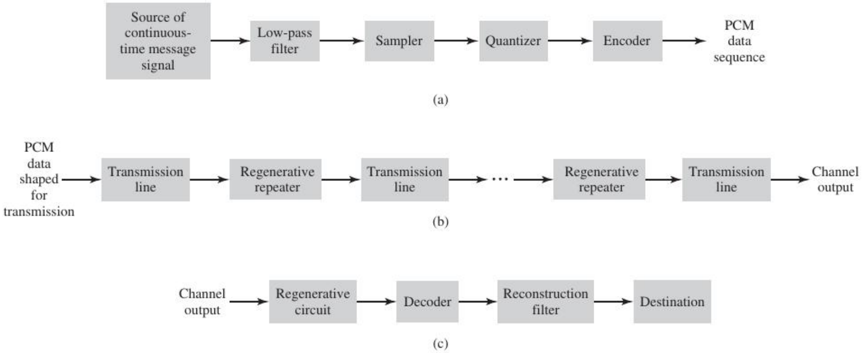


- The mapping $g(.)$ is specified as follows:
  - Let I_k be the interval $(m_k, m_{k+1}]$
  - The amplitude m is represented by the index k if $m_k < m \leq m_{k+1}$
  - The index k is converted to a representation v_k. All $m \in (m_k, m_{k+1}]$ is represented using v_k.
- m_k are called decision levels or thresholds
- v_k are called representation or reconstruction levels
- The spacing between two reconstruction levels, i.e., $v_k - v_{k-1}$ is called quantum or step size
- We are doing scalar quantization - a memoryless transformation

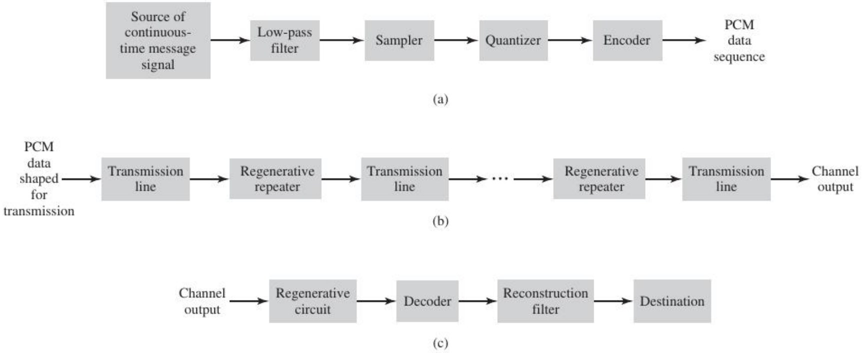
More about Quantization function $g(.)$



Pulse code modulation

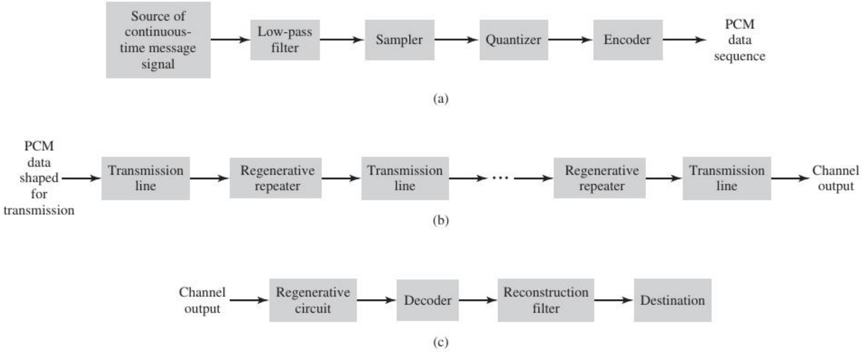


Pulse code modulation - Sampling



- Antialiasing filter + sampling at more than the Nyquist rate

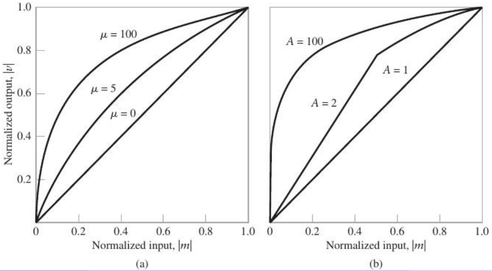
Pulse code modulation - Quantization



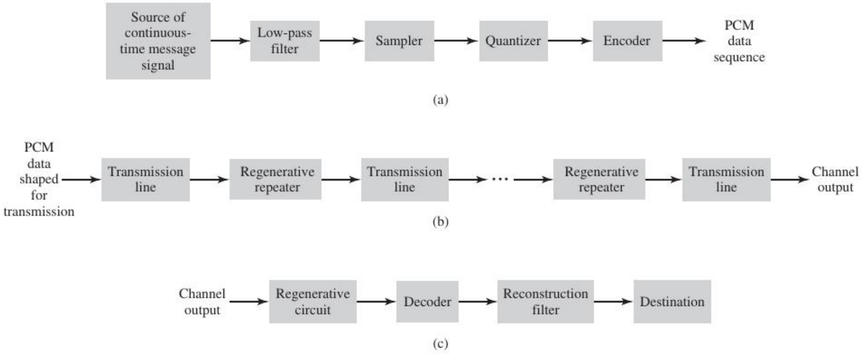
- If the input signal is voice, then non-uniform quantization is usually used

Pulse code modulation - Companding

- Instead of using a non-uniform quantizer, we can transform the signal and then use a uniform quantizer
- If the signal is compressed using a function $c(.)$, i.e., $m_1(t) = c(m(t))$ then it has to be expanded using an inverse function $c^{-1}(.)$
- The combination of compression and expanding is called companding.
- Usually two standard ways of compression (and therefore expansion) are used
- μ-law and A-law (Find out the transformation function from the text)

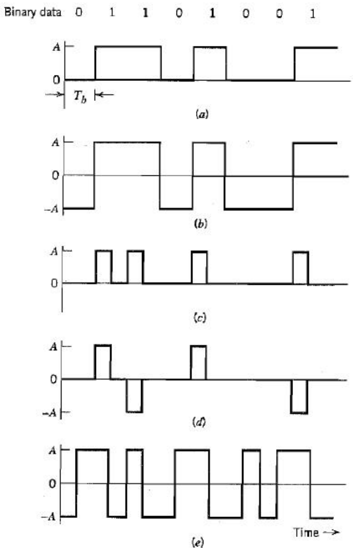


Pulse code modulation - Encoding



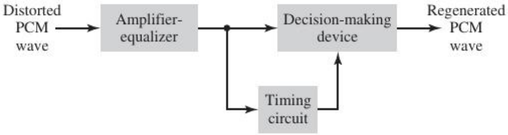
- The quantizer output, i.e., the representation level is encoded using a binary code. Usually this is just a binary representation of the index k that we had seen before.
- Usually a binary code is used because it is easy to distinguish between two levels in noise.

Pulse code modulation - Binary code as signals



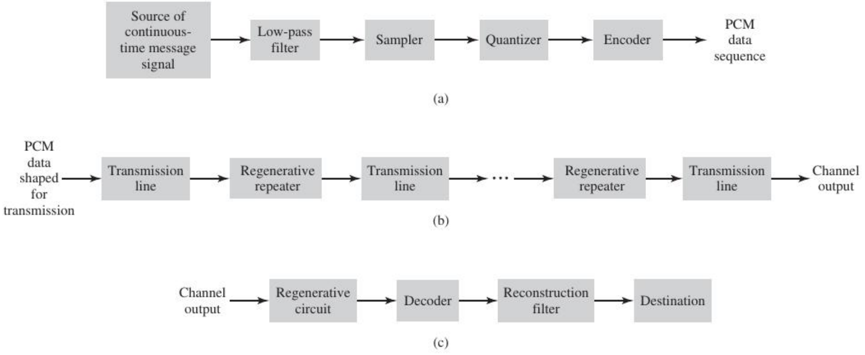
- The binary code is sent over the channel (line) by first converting it into a voltage signal
- There is a mapping from binary $\{0, 1\}$ to pulse shapes. Several possibilities are shown

Pulse code modulation - Regeneration



- We equalize or compensate for the effects of the channel
- The line code is resampled and passed through a decision device to obtain the binary code back
- The binary code is used to regenerate the PCM line code again
- Errors might occur during regeneration.

Pulse code modulation - Receiver



- We obtain the binary code by sampling the line code
- Then the binary code is mapped back to the representative levels v_k and to an impulse train
- Then a reconstruction filter as in the case of PAM is used