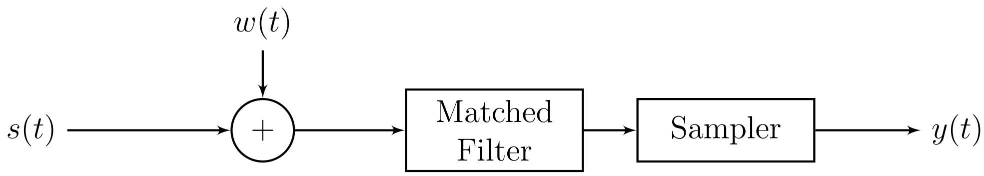
Indian Institute of Space Science and Technology
AV312 - Digital Communication
Department of Avionics

Quiz 2 for **Semester V** on **07/10/2015**

Note to the student

1. There are **5 questions** in this question paper on **2 pages**, for a total of **15 marks**.
2. Answer **all** questions.

Question 1 (5 marks): Consider the following system:



Here $s(t)$ is a time-limited signal in the time interval $[0, T_b]$ and $w(t)$ is a zero mean white Gaussian noise process with power spectral density $\frac{N_0}{2}$. The sampler block samples the output of the matched filter at T_b. Please derive the response of the matched filter which will maximise the SNR of $y(t)$ at the sampling instant T_b.

Question 2 (2 marks): A stationary Gaussian process $(X(t))$ has zero mean and power spectral density $S_{xx}(f)$. Please determine the probability density function of the random variable $X(t_k)$ where t_k is an arbitrary time instant.

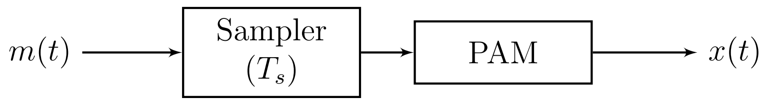
Question 3 (3 marks): Suppose X is a random variable denoting the number on the face when an unfair dice is tossed. The probability of X being the number f is given as $\frac{k}{f^2}$. A discrete time random process $(X[n])$ is constructed by defining $X[n] = \sqrt{n} + X$. Please note that the underlying sample space Ω is $\{1, 2, 3, 4, 5, 6\}$, representing the numbers on the faces of the dice.

1. Draw the sample function of the random process $(X[n])$ for the sample point $\omega = 3$. With what probability does this sample function occur?
2. Find the expectation of $X[n]$, i.e., $\mathbb{E}X[n]$.
3. Is the process $X[n]$ stationary? Please justify your answer.

Question 4 (2 marks): Suppose $(X[n])$ defined for $-\infty < n < \infty$ is an independently and identically distributed (IID) process. The random process $(X[n])$ is filtered by an LTI system with impulse response $h[n] = \delta[n] + \delta[n-1]$. Let $(Y[n])$, again defined for $-\infty < n < \infty$

be the output random process from this filter. Is $(Y[n])$ IID? Please justify your answer. Is $(Y[n])$ identically distributed? Please justify your answer.

Question 5 (3 marks): Consider the following PAM system:



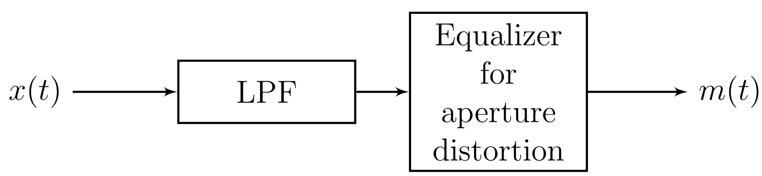
The pulse amplitude modulator puts out the signal $x(t)$ which is obtained by sampling the bandlimited input signal at an appropriate Nyquist rate of $\frac{1}{T_s}$. Here

$$x(t) = \sum_{k=-\infty}^{\infty} m(kT_s)h(t - kT_s),$$

where $h(t)$ is defined as follows:

$$h(t) = \begin{cases} t, \text{ for } t \in [0, \frac{T_s}{2}), \\ T_s - t, \text{ for } t \in [\frac{T_s}{2}, T_s] \\ 0, \text{ otherwise.} \end{cases}$$

For demodulating $x(t)$ (we assume that we receive $x(t)$ itself) we use the following system:



The LPF is assumed to be ideal with the cutoff frequency at $\frac{1}{2T_s}$. The equalizer is used to compensate for the aperture distortion in the PAM modulator. Derive the frequency response of the equalizer which will compensate for the aperture distortion and recover $m(t)$.

Best of luck!