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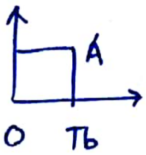
AV324 - Assignment 2 solutions.

1) Channel impulse response is $\delta(t) + \frac{1}{2}\delta(t - T_b)$

$p(t)$ is

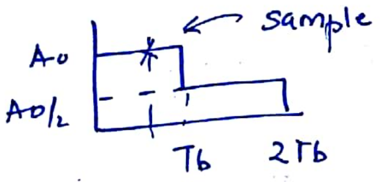


$Y_m = \left(C(X(t)) + N(t)\right)$ sampled every T_b.

note that if $\sum A_k \delta(t - kT_b)$ is sent through the filter with impulse response $p(t)$ and then through the channel with impulse response $\delta(t) + \frac{1}{2}\delta(t - T_b)$, the output of the channel would be

$$\sum A_k \left(p(t - kT_b) + \tfrac{1}{2} p(t - (k+1)T_b)\right)$$

or if a single A_k - say A_0 is sent the o/p is



if we are sampling every T_b, for the first sample time, the $Y_m = Y_0$ is $A_0 + N_0$.

Probability of error or $\Pr\{B_0 \neq \hat{B}_0\} =$

$$= \tfrac{1}{2}\Pr\{Y_0 > 0 \mid B_0 = 0\} + \tfrac{1}{2}\Pr\{Y_0 \leq 0 \mid B_0 = 1\}$$

$$= \tfrac{1}{2}\Pr\{-A + N_0 > 0\} + \tfrac{1}{2}\Pr\{A + N_0 \leq 0\}$$

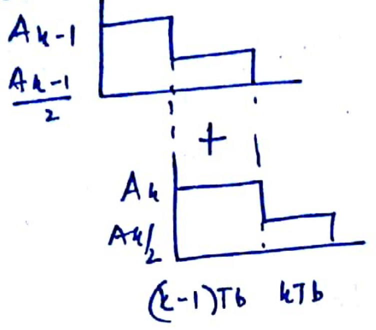
$$= \tfrac{1}{2}\Pr\underbrace{\{N_0 > A\}} + \tfrac{1}{2}\Pr\underbrace{\{N_0 \leq -A\}}.$$

can be computed from the Gaussian Q function.

However for other bit times, there is a small difference.

let us take A_m, $A_m > 0$.

The $r(t)$ that we obtain during the kth bit time consists of the following signal and noise.



happens because of channel being not ideal.

so $Y_m = A_m + A_{m-1}/2 + N_m$.

$$P_r\{B_m \neq \hat{B_m}\} = \sum_{B_{m-1}} P_r\{B_{m-1}, B_m \neq \hat{B_m}\}$$

$$= \sum_{B_{m-1}} \sum_{B_m} P_r\{B_{m-1}, B_m\} \cdot P_r\{B_m \neq \hat{B_m} / B_{m-1}, B_m\}$$

$$= \frac{1}{4}\Big\{ P_r\{B_m \neq \hat{B_m} / 00\} + P_r\{B_m \neq \hat{B_m} / 01\} + P_r\{B_m \neq \hat{B_m} / 10\} + P_r\{B_m \neq \hat{B_m} / 11\}\Big\}.$$

let us take this as an example.

$$P_r\{B_m \neq \hat{B_m} / 10\} = P_r\{-A + A/2 + N_m > 0\}$$

$$= \underbrace{P_r\{N_m > +\tfrac{A}{2}\}}$$

compute from Gaussian Q.

Repeat this for similar terms.

2) $p(t)$ of duration T_b.

MF impulse response is $p(T_b - t)$.

we are interested in the M.F o/p at T_b.

which is $\underbrace{p(t) * \underbrace{p(T_b - t)}_{h(t),\ h(\tau) = p(T_b - \tau)}}$ sampled at T_b.

$$\int_0^{T_b} p(\tau) \cdot h(t-\tau) \cdot d\tau = \int_0^{T_b} p(\tau) \cdot p(t+\tau-T_b) \cdot d\tau$$

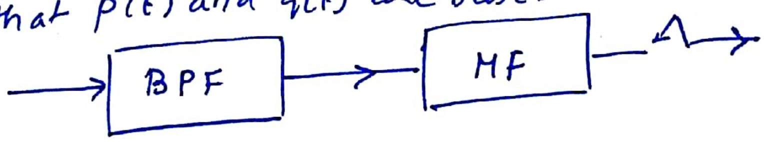
at T_b we therefore have the MF o/p as

$$\int_0^{T_b} p(\tau) \cdot p(\tau) \cdot d\tau$$ → which can be obtained from a correlator.

3) System A - pulse shape $p(t)$, matched filter is $p(T_b - t)$

System B - pulse shape $p(t)$, matched filter? is $q(T_b - t)$

assume that $p(t)$ and $q(t)$ are baseband



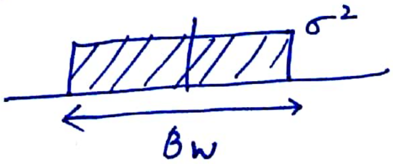
for both systems.

Let us assume that the BPF is such that, and the channel is such that $p(t)$ is transmitted undistorted through the cascade of channel and BPF.

Then the signal at the o/p of the MF for systems A and B are

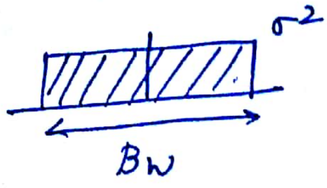
$$\int_0^{T_b} p(t) \cdot p(t) \cdot dt \quad \text{and} \quad \int_0^{T_b} p(t) \cdot q(t) \cdot dt$$

Assuming that the noise PSD is white and some constant σ^2 before the BPF, the PSD at the o/p of the BPF is of the form



Bw is the B/w of the BPF.

The PSD of the noise after MF is



$\times \; |P(f)|^2$ – system A

$\times \; |Q(f)|^2$ – system B.

Since $p(t)$ is passed through BPF w/o distortion, we can assume that effective B/w of $p(t)$ lies within BW.

With a similar assumption for $q(t)$ we can then write the PSD of noise as just $\sigma^2 |P(f)|^2$ and $\sigma^2 |Q(f)|^2$.

noise power is then $\int \sigma^2 \cdot |P(f)|^2 \cdot df$

and $\int \sigma^2 \, |Q(f)|^2 \cdot df$.

so the SNR can be obtained from this (as the ratio).

(see the textbook for a discussion of Cauchy Schwartz inequality which can be used for comparing these SNRs.
– not required for exam).