SSB modulation / demodulation

$m(t) \to \otimes \to$ BPF $\to$ PBC $\to \otimes \to$ LPF $\to \hat{m}(t)$

$A\cos(2\pi f_c t)$ — DSB modulator

$A\cos(2\pi f_c t)$ ↑ phase matched with received carrier. — same as that of DSB

How to look at SSB signals in time domain?

Hilbert transform

$\to \boxed{\mathbb{H}(\cdot)} \to$

$\mathbb{H}(f) = -j\,\mathrm{sgn}(f)$ — $+j$, 0, $-j$

H/W : UM — wideband $90°$ phase shifter.

$\left(* \frac{1}{\pi t}\right)$ in time domain

how does this equivalence come about?

$\cos(2\pi f_0 t) \to \mathbb{H}(\cdot) \to$

$\frac{1}{2}$ at $-f_0$, $\frac{1}{2}$ at f_0 $= \left(\frac{e^{j2\pi f_0 t} + e^{-j2\pi f_0 t}}{2}\right) \to (\cdot) \to \frac{-je^{j2\pi f_0 t} + je^{-j2\pi f_0 t}}{2}$

$\sin(2\pi f_0 t) \Leftarrow \frac{e^{2j\pi f_0 t} - e^{-j2\pi f_0 t}}{2j} \Leftarrow$

what we obtain is a $90°$ phase shift for all components in a signal?

$m(t)$ – baseband signal — $M(f)$, $\angle M(f)$

$m(t) \to \boxed{\mathbb{H}(\cdot)} \to \tilde{m}(t)$

$\left(m(t) + j\tilde{m}(t)\right)? \xrightarrow{\mathcal{F}} M(f) + j\tilde{M}(f)$

$\tilde{M}(f) = -j\,\mathrm{sgn}(f)\cdot M(f)$

$M(f) + j\tilde{M}(f) = M(f) - j^2\,\mathrm{sgn}(f)M(f)$
$= M(f) + \mathrm{sgn}(f)M(f) =$ $\left(m(t) + j\tilde{m}(t)\right)$ $\mathcal{F}$

$m(t) - j\tilde{m}(t)$?

$m(t) \to \otimes$ with $\cos(2\pi f_c t)$; $m(t) \to \mathbb{H}(\cdot) \to \tilde{m}(t) \to \otimes$ with $\sin(2\pi f_c t)$; both $\to (+) \to i(t)$

is a SSB signal.

$i(t)$ is a lower sideband SSB signal (LSB)

$\mathcal{F}$
$i(t) = m(t)\cos(2\pi f_c t) + \tilde{m}(t)\sin(2\pi f_c t)$.
$I(f) = \frac{1}{2}\left[M(f-f_c) + M(f+f_c)\right] + \frac{1}{2j}\left[\tilde{M}(f-f_c) - \tilde{M}(f+f_c)\right]$

H/W H/W

$\tilde{M}(f) = -j\,\mathrm{sgn}(f)M(f)$

$\frac{1}{2}\left(-j\tilde{M}(f-f_c)\right)$

$\frac{1}{2}\left(-\mathrm{sgn}(f-f_c)\cdot M(f-f_c)\right)$

$=$ $1 \cdot M(f-f_c)$ (only lower sideband) — 0, f_c, F

$M(f-f_c)$ (only lower sideband) — $-f_c$, 0, f_c

Suppose we need to generate a USB signal?
① $m(t)\sin(2\pi f_c t) + \tilde{m}(t)\cos(2\pi f_c t)$ – ?
② $m(t)\cos(2\pi f_c t) - \tilde{m}(t)\sin(2\pi f_c t) \to$ USB signal.

How is the carrier signal modulated in case of SSB?

$m(t)\cos(2\pi f_c t) - \tilde{m}(t)\sin(2\pi f_c t)$

$\underbrace{\sqrt{m(t)^2 + \tilde{m}(t)^2}}_{e(t)}\left(\frac{m(t)}{\sqrt{m^2(t) + \tilde{m}^2(t)}}\cos(2\pi f_c t) - \frac{\tilde{m}(t)}{\sqrt{m^2(t) + \tilde{m}^2(t)}}\sin(2\pi f_c t)\right)$

$e(t)\left(\cos(\alpha(t))\cdot\cos(2\pi f_c t) - \sin(\alpha(t))\sin(2\pi f_c t)\right)$
$= e(t)\cdot\cos(2\pi f_c t + \alpha(t))$

How do we understand SSB demodulation in time domain.

$i(t) \to$ PBC $\xrightarrow{o(t) = i(t - t_p)}$ BPF $\xrightarrow{i(t-t_p')} \otimes \to$ LPF $\to$

$\cos(2\pi f_c(t - t_p'))$

$i(t-t_p') = m(t-t_p')\cdot\cos(2\pi f_c(t-t_p')) - \tilde{m}(t-t_p')\cdot\sin(2\pi f_c(t-t_p'))$
$\times\cos(2\pi f_c(t-t_p'))$ $t = t - t_p'$

$= m(t)\cdot\cos^2(2\pi f_c t) - \tilde{m}(t)\cos(2\pi f_c t)\cdot\sin(2\pi f_c t)$
$= \frac{m(t)}{2} + \underbrace{\frac{m(t)}{2}\cos(2\pi(2f_c)t) - \frac{\tilde{m}(t)}{2}\sin(2\pi(2f_c)t)}_{\text{cutoff by the LPF}}$

Quadrature carrier multiplexing.
$\left(m(t)\cos(2\pi f_c t) + m_2(t)\sin(2\pi f_c t)\right) \times \cos(2\pi f_c t)$
$\times \sin(2\pi f_c t)$