

~~Q2~~ Q3 $f_f = 1 \times 10^6 \text{ Hz}$, $f_m = 100 \text{ Hz}$, $f' = 1.1 \times 10^6 \text{ Hz}$

$$x(t) = \cos(2\pi f_f t) \cdot \cos(\beta \sin(2\pi f_m t)) - \sin(2\pi f_f t) \cdot \sin(\beta \sin(2\pi f_m t))$$

$$\approx \cos(2\pi f_f t) - \beta \sin(2\pi f_m t) \cdot \sin(2\pi f_f t)$$

Writing this as complex exponentials:

$$x(t) = \frac{e^{j2\pi f_f t} + e^{-j2\pi f_f t}}{2} + -\beta \sin(2\pi f_m t) \cdot \sin(2\pi f_f t)$$

$$= \frac{e^{j2\pi f_f t} + e^{-j2\pi f_f t}}{2} - \beta \frac{e^{j2\pi f_m t} - e^{-j2\pi f_m t}}{2j} \times \frac{e^{j2\pi f_f t} - e^{-j2\pi f_f t}}{2j}$$

$$= \frac{e^{j2\pi f_f t} + e^{-j2\pi f_f t}}{2} + \frac{\beta}{4} \left\{ e^{j2\pi(f_m + f_f)t} - e^{-j2\pi(f_f - f_m)t} - e^{j2\pi(f_f - f_m)t} + e^{-j2\pi(f_f + f_m)t} \right\}$$

after filtering

$$f(t) = \frac{e^{j2\pi f_f t} + e^{-j2\pi f_f t}}{2} +$$

$$\frac{\beta}{4} \left\{ e^{j2\pi(f_m + f_f)t} + e^{-j2\pi(f_f + f_m)t} \right\}$$

In order to find the complex baseband, consider the time freq. part. Then the spectrum is given by

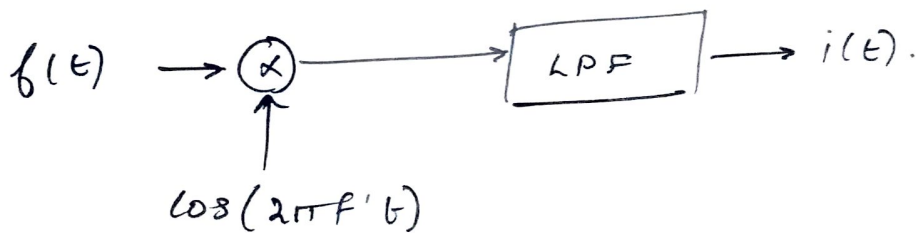
$$\frac{e^{j2\pi f_f t}}{2} + \frac{\beta}{4} e^{j2\pi(f_m + f_f)t}$$

using $f' = 1.1 \text{ MHz}$.

$$e^{j2\pi f' t} \left(\underbrace{\frac{e^{j2\pi (f_c - f') t}}{2} + \frac{\beta}{4} e^{j2\pi (f_m + f_c - f') t}}_{\text{complex baseband.}} \right)$$

$$\frac{1}{2} \cos 2\pi (f_c - f') t + \frac{j}{2} \sin (2\pi (f_c - f') t) + \frac{\beta}{4} \cos (2\pi (f_c + f_m - f') t) + \frac{j\beta}{4} \sin (2\pi (f_c + f_m - f') t)$$

how to calculate $i(t)$.



$$f(t) = \frac{e^{j2\pi f_c t} + e^{-j2\pi f_c t}}{2} + \frac{\beta}{4} \left\{ e^{j2\pi (f_c + f_m) t} + e^{-j2\pi (f_c + f_m) t} \right\}$$

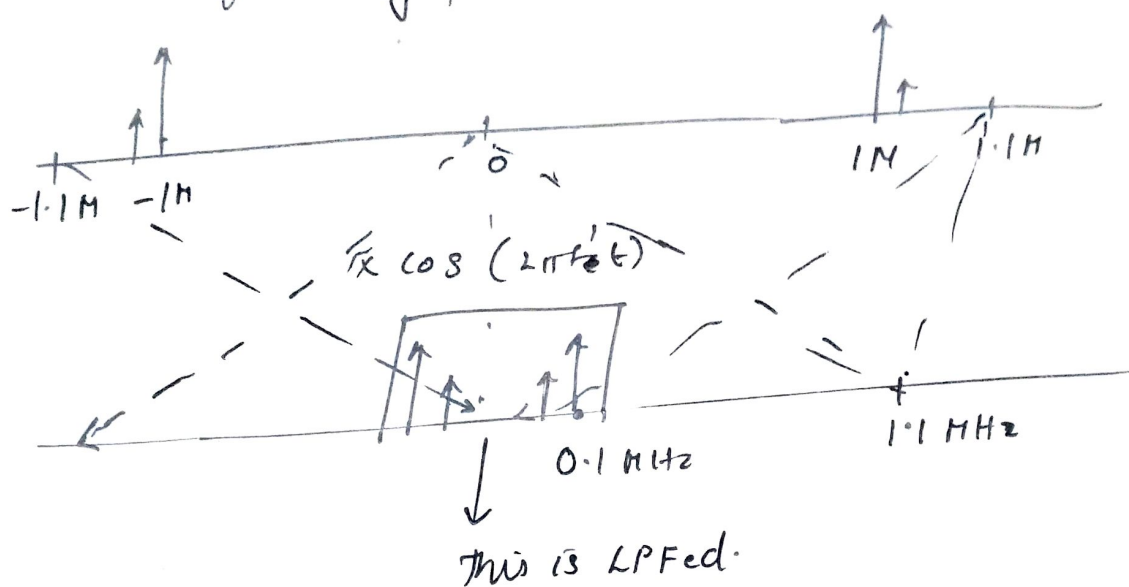
$$f(t) \times \cos(2\pi f' t)$$

$$= \frac{1}{4} \left\{ e^{j2\pi (f_c + f') t} + e^{j2\pi (f_c - f') t} + e^{-j2\pi (f_c + f') t} + e^{-j2\pi (f_c - f') t} \right\}$$

$$+ \frac{\beta}{8} \left\{ e^{j2\pi (f_c + f_m + f') t} + e^{j2\pi (f_c + f_m - f') t} + e^{-j2\pi (f_c + f_m + f') t} + e^{-j2\pi (f_c + f_m - f') t} \right\}$$

Now this gets LPF'd by a filter of cutoff 100 kHz.
 Assume brickwall passes 100 kHz.

Intuition from mag plot.



$$f(t) \times \cos(2\pi f' t) = f(t) \cdot \frac{e^{j2\pi f' t} + e^{-j2\pi f' t}}{2}$$

* $f_f + f'$ terms will get cutoff.

$$f_f - f' = -100\text{ k} \quad \text{and} \quad f_f + f_m - f' = -100\text{ k} + f_m \checkmark$$

$$\begin{aligned} i(t) &= \frac{1}{4} \left(e^{j2\pi(f' - f_f)t} + e^{-j2\pi(f' - f_f)t} \right) \\ &+ \frac{\beta}{8} \left(e^{j2\pi(f' - f_f - f_m)t} + e^{-j2\pi(f' - f_f - f_m)t} \right) \\ &= \frac{1}{2} \cos(2\pi(f' - f_f)t) + \frac{\beta}{4} \cos(2\pi(f' - f_f - f_m)t). \end{aligned}$$

compare with complex BB result obtained earlier.