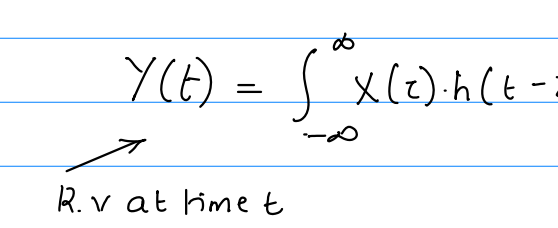


Note that this lecture was given using the board in class. So the actual contents of this lecture note may differ from what was covered in class.

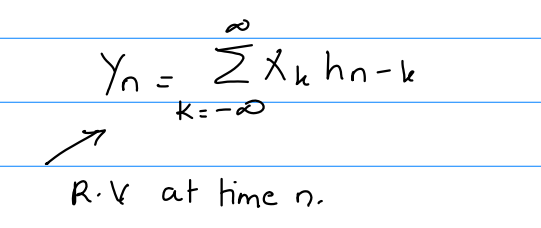
Review:

- CTDP $(x(t)), t \in \mathbb{R}$ - DTRP $(x_n), n \in \mathbb{Z}$



$$y(t) = \int_{-\infty}^{\infty} x(\tau) h(t-\tau) d\tau$$

R.V at time t



$$y_n = \sum_{k=-\infty}^{\infty} x_k h_{n-k}$$

R.V at time n .

Can we characterize FDD of $(y(t))$ as a function of FDD of $(x(t))$ and the impulse response $h(t)$?

example:

$(x(t))$ is IID.

$h(t)$ is $\delta(t) + \delta(t-1)$

$\therefore y(t) = x(t) + x(t-1)$

Is $(y(t))$ IID? Check $y(t)$ and $y(t+1)$'s joint distribution. No!

important to note that the structure of the FDD is not preserved here.

Ⓐ But are there FDD structures which are preserved when LTI filtering is done. We will answer this below.

If not IID can we find the FDD of $(y(t))$?

$Fy(t_0)$ is easy to find: it is the convolution of the pdfs of $x(t)$ and $x(t-1)$ (assuming pdfs can be defined).

important to note that getting the joint distribution is not that easy.

in applications (esp. like computation of SNR) summary statistics are enough.

Ⓐ so we now ask whether it is possible to find out summary statistics

we will answer this below.

Ⓐ FDD structures which are preserved by LTI filtering.

- If $(x(t))$ is strictly stationary then $(y(t))$ is strictly stationary. (also for DT)

- If $(x(t))$ is WSS then $(y(t))$ is WSS.

example: (why should this property hold for SS $(x(t))$?)

Let $(x(t))$ be SS and $h(t) = \delta(t) + \delta(t-1)$

Then $y(t) = x(t) + x(t-1)$

The distribution of $y(t)$ is determined by the joint dist of $x(t), x(t-1)$

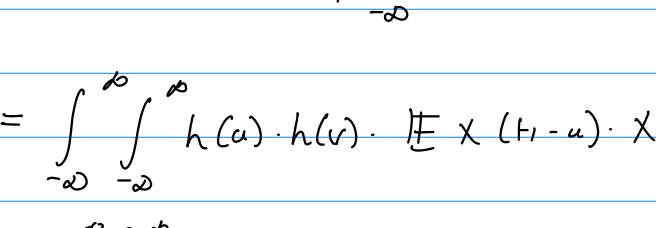
$y(t+z) \rightsquigarrow x(t+z), x(t+z-1)$

since $(x(t))$ is SS so $F_{x(t), x(t-1)} = F_{x(t+z), x(t+z-1)}$

We can think about $(y(t))$'s FDDs in the same manner.

Going into the details of this invariance property for WSS processes will be useful for us.

Invariance ppty for WSS processes:



$(x(t))$ WSS \Rightarrow a) $E x(t) = \mu_x$ (a constant), b) $E x(t_1) x(t_2) = R_x(t_1 - t_2)$

$$E y(t) = ? \quad E y(t) = E \int_{-\infty}^{\infty} x(\tau) h(t-\tau) d\tau = \int_{-\infty}^{\infty} \mu_x h(t-\tau) d\tau = \mu_x \int_{-\infty}^{\infty} h(u) du = \mu_y$$

$$E y(t_1) y(t_2) = E \left[\int_{-\infty}^{\infty} h(u) x(t_1 - u) du \cdot \int_{-\infty}^{\infty} h(v) x(t_2 - v) dv \right]$$

$$= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} h(u) h(v) E x(t_1 - u) x(t_2 - v) du dv$$

$$= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} h(u) h(v) R_x(t_1 - t_2 - u + v) du dv \quad \left\{ \begin{array}{l} \text{from here it can be} \\ \text{seen that } R_x \text{ is a} \\ \text{function of } t_1 - t_2 \end{array} \right.$$

Let $\omega = v - u$

$$= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} h(v - \omega) h(v) R_x(t_1 - t_2 + \omega) dv d\omega$$

$$= \int_{-\infty}^{\infty} R_x(t_1 - t_2 + \omega) \cdot \int_{-\infty}^{\infty} h(v) h(v - \omega) dv d\omega$$

$\omega = -\omega$

$$\text{or } = \int_{-\infty}^{\infty} R_x(t_1 - t_2 - \omega) \cdot \underbrace{\int_{-\infty}^{\infty} h(v) h(v + \omega) dv}_{g(\omega)} d\omega$$

$$= \int_{-\infty}^{\infty} R_x(t_1 - t_2 - \omega) g(\omega) d\omega = (R_x * g)_{t_1 - t_2} = R_y(t_1 - t_2)$$