AVD623: Communication Systems-II

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Lecture 8

Figures are taken from "Communication Systems" by Simon Haykin, "Communication Systems" by Stern and Mahmoud, and "Software receiver design" by Sethares and Johnson.

Two issues - Carrier and Timing/Clock Recovery

- A local oscillator needs to produce a replica of the carrier at the receiver
- Replica $\Rightarrow$ match in both frequency and phase
- Difference in frequencies $\Rightarrow$ time varying (linear) difference in phase

$$cos(2\pi f_1 t) \text{ and } cos(2\pi f_2 t)$$

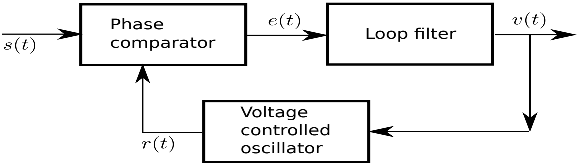
- A clock circuit needs to tick off bit periods and sample the received continuous time waveform at the appropriate times within each bit period

How to do carrier recovery?

Phase and frequency estimation using

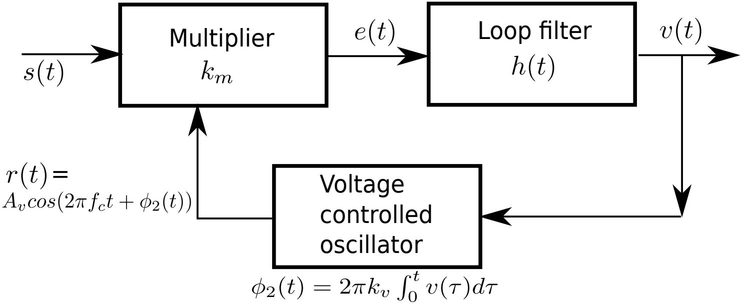
- FFT
- Phase locked loops
- Squared difference loop
- Costas loop
- Decision directed tracking

Phase and frequency estimation using PLL



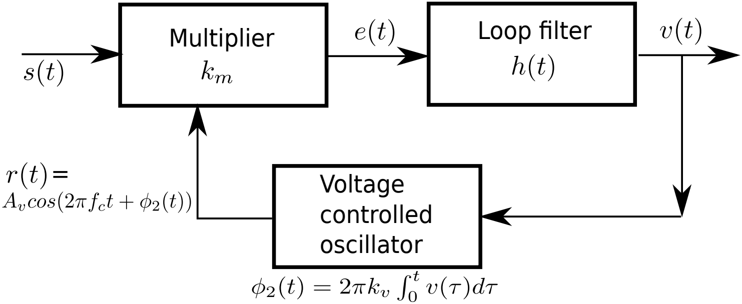
- $r(t)$ is the local oscillator's output
- We want $r(t)$ to "match" with $s(t)$ in phase
- We want $e(t)$ to measure the instantaneous phase difference between $s(t)$ and $r(t)$
- Filtered output $v(t)$ controls the VCO output $r(t)$ to match $s(t)$

PLL - An implementation



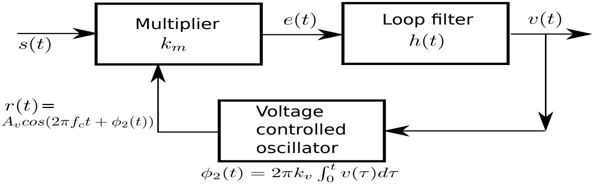
- $h(t)$ is a low pass response
- k_m and k_v are the sensitivities of the multiplier and the VCO respectively

PLL - Applications



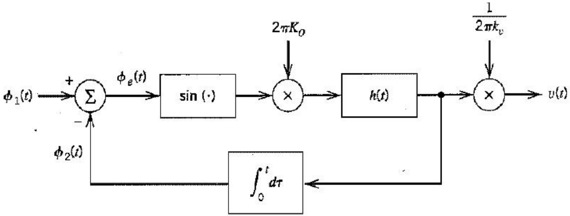
- Assume that phase error $\phi_1(t) - \phi_2(t) \approx 0$
- How does carrier recovery work?

PLL - Model



- Let $s(t) = A_c sin(2\pi f_c t + \phi_1(t))$
- Let $r(t) = A_v cos(2\pi f_c t + \phi_2(t))$
- $e(t) = k_m A_c A_v \left[sin(4\pi f_c t + \phi_1(t) + \phi_2(t)) + sin(\phi_1(t) - \phi_2(t))\right]$
- Since $h(t)$ is a low pass response; $v(t) = \int_{-\infty}^{\infty} k_m A_c A_v sin(\phi_1(\tau) - \phi_2(\tau))h(t - \tau)d\tau$
- $\phi_2(t) = 2\pi k_v \int_0^t v(\tau)d\tau$

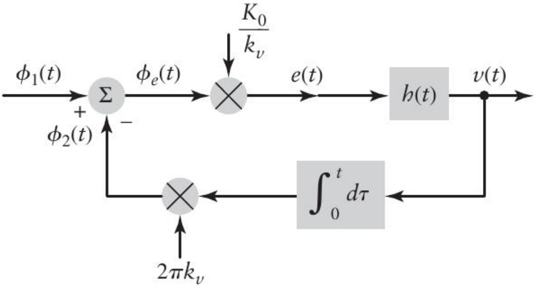
PLL - Model



- $\phi_e(t) = \phi_1(t) - \phi_2(t)$
- Loop gain parameter $K_o = k_v k_m A_c A_v$
- PLL model

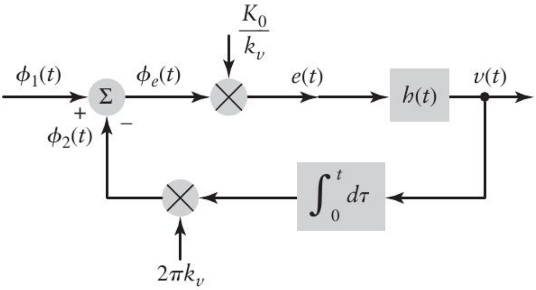
$$\frac{d\phi_e(t)}{dt} = \frac{d\phi_1(t)}{dt} - 2\pi K_o \int_{-\infty}^{\infty} sin(\phi_e(\tau))h(t-\tau)d\tau$$

PLL - Linearized Model



- Assume that $sin(\phi_e(t)) \approx \phi_e(t)$
- We use a Laplace transform domain approach
- $\phi_1(t) \leftrightarrow \Phi_1(s)$, $\phi_2(t) \leftrightarrow \Phi_2(s)$, $v(t) \leftrightarrow V(s)$, $h(t) \leftrightarrow H(s)$

PLL - Linearized Model

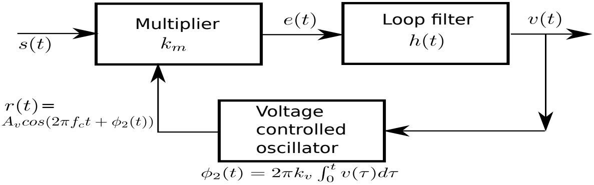


- $\frac{K_o}{k_v} H(s) \left(\Phi_1(s) - \Phi_2(s) \right) = V(s)$
- But $\Phi_2(s) = 2\pi k_v \frac{V(s)}{s}$
- $\Phi_1(s) \frac{K_o}{k_v} H(s) = V(s) \left[1 + \frac{2\pi K_o}{s} H(s) \right]$
- $\frac{V(s)}{\Phi_1(s)} = \frac{s(K_o/k_v)H(s)}{s + 2\pi K_o H(s)}$ and $\frac{\Phi_e(s)}{\Phi_1(s)} = \frac{s}{s + 2\pi K_o H(s)}$

PLL - Behaviour of $v(t)$

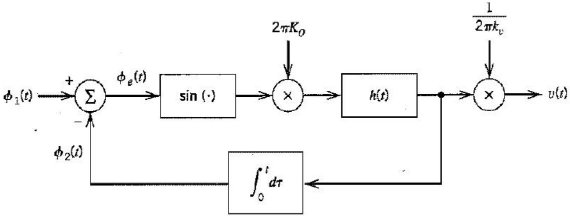
- What if there is a step change in the phase $\phi_1(t)$?
- What if there is a change in the frequency of the input?

PLL - Model



- Let $s(t) = A_c sin(2\pi f_c t + \phi_1(t))$
- Let $r(t) = A_v cos(2\pi f_c t + \phi_2(t))$
- $e(t) = k_m A_c A_v \left[sin(4\pi f_c t + \phi_1(t) + \phi_2(t)) + sin(\phi_1(t) - \phi_2(t))\right]$
- Since $h(t)$ is a low pass response; $v(t) = \int_{-\infty}^{\infty} k_m A_c A_v sin(\phi_1(\tau) - \phi_2(\tau))h(t-\tau)d\tau$
- $\phi_2(t) = 2\pi k_v \int_0^t v(\tau)d\tau$

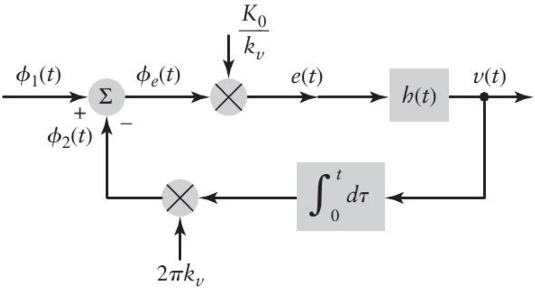
PLL - Model



- $\phi_e(t) = \phi_1(t) - \phi_2(t)$
- Loop gain parameter $K_o = k_v k_m A_c A_v$
- PLL model

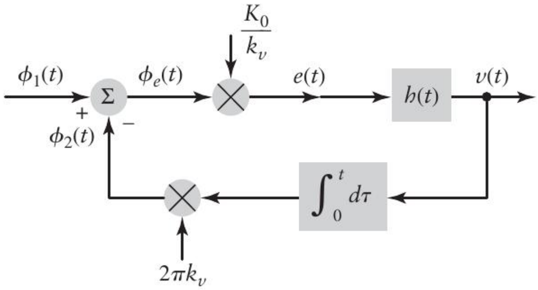
$$\frac{d\phi_e(t)}{dt} = \frac{d\phi_1(t)}{dt} - 2\pi K_o \int_{-\infty}^{\infty} sin(\phi_e(\tau))h(t-\tau)d\tau$$

PLL - Linearized Model



- Assume that $sin(\phi_e(t)) \approx \phi_e(t)$
- We use a Laplace transform domain approach
- $\phi_1(t) \leftrightarrow \Phi_1(s)$, $\phi_2(t) \leftrightarrow \Phi_2(s)$, $v(t) \leftrightarrow V(s)$, $h(t) \leftrightarrow H(s)$

PLL - Linearized Model



- $\frac{K_o}{k_v} H(s) \left(\Phi_1(s) - \Phi_2(s)\right) = V(s)$
- But $\Phi_2(s) = 2\pi k_v \frac{V(s)}{s}$
- $\Phi_1(s) \frac{K_o}{k_v} H(s) = V(s) \left[1 + \frac{2\pi K_o}{s} H(s)\right]$
- $\frac{V(s)}{\Phi_1(s)} = \frac{s(K_o/k_v)H(s)}{s+2\pi K_o H(s)}$ and $\frac{\Phi_e(s)}{\Phi_1(s)} = \frac{s}{s+2\pi K_o H(s)}$

PLL - Behaviour of $\phi_e(t)$

- Assume that the PLL is in "lock" initially
- Assume that the input phase changes, i.e., $\phi_1(t)$ changes
  - $\phi_1(t)$ is a step, ramp
- How does the loop filter affect the $\phi_e(t)$?
  - $H(s) = 1$ or $H(s) = \frac{s+a}{s}$
- The hold-in or lock range of the PLL is the range of frequencies that the PLL can track, once locked
- The pull-in or capture range of the PLL is the range of frequenices that the PLL can lock onto from a free running state
- Refer B.P. Lathi - Modern digital and analog communication systems