AV312 - Lecture 16

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Figures from "Communication Systems" by Haykin and "An Intro. to Analog and Digital Commn." by Haykin and Moher

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Review of last classes

- Power spectrum for BPSK, BASK, and BFSK
- Bandwidth requirements for BASK, BPSK, and BFSK

Today's class

- QPSK
- Intersymbol interference
- Nyquist bandwidth and channel
- Today's scribes are Nikhil Mahesh Kumar and Rahul Kumar

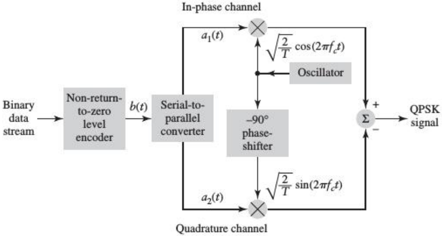
Quadriphase shift keying (QPSK)

- For $kT \leq t < (k+1)T$

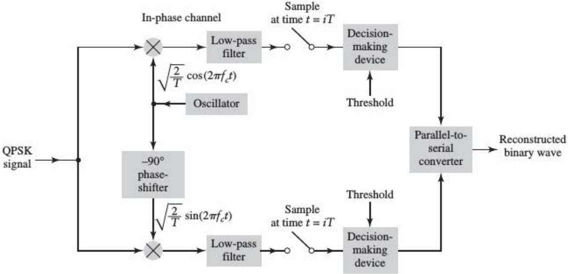
$$s(t) \quad = \quad \sqrt{\frac{2E}{T}} cos\left(2\pi f_c t + (2i-1)\frac{\pi}{4}\right),$$

  where $i \in \{1, 2, 3, 4\}$
- Note that the time interval T is possibly different from T_b
- A generalized version is M-ary PSK
- Recall quadrature carrier multiplexing - two signals multiplexed together using quadrature carriers but no increase in bandwidth
- $s(t) = \sqrt{\frac{2E}{T}} cos\left((2i-1)\frac{\pi}{4}\right) cos(2\pi f_c t) - \sqrt{\frac{2E}{T}} sin\left((2i-1)\frac{\pi}{4}\right) sin(2\pi f_c t)$
  - Combination of two BPSK signals - the bits can be chosen independently
  - A representation using vectors - constellation diagram

QPSK - generation and detection

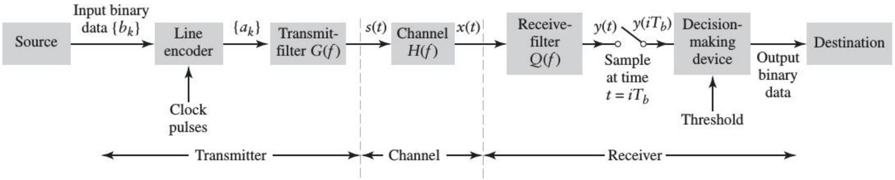


(a)



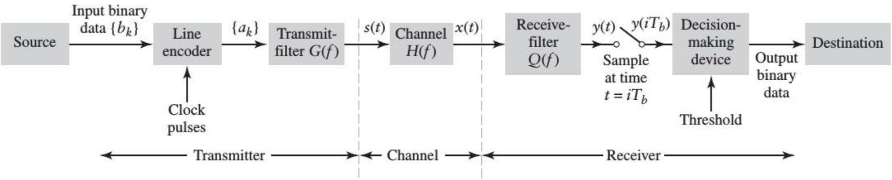
(b)

Recall: Digital transmission system



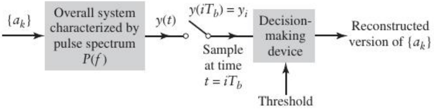
- Source of digital information - characterized by bit duration T_b (or bit rate)
- Converted into a line code whose levels are represented by a_k (say $-A$ and $+A$)
- Further transformation of a_k to "match" the signal to the channel (what if the channel were bandpass?)
- We obtain a continuous time signal $s(t)$ which is transmitted over the channel
- At the receiver need to convert it into a digital signal - so synchronized sampling, usually at rate T_b
- A decision device decides whether 0 or 1 was transmitted
- Let us think about a baseband digital transmission system

An effective pulse shape $p(t)$



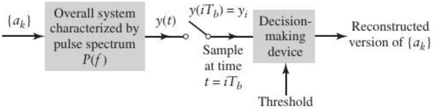
- Note that all filters and the channel are assumed to be LTI
- Let us think about a_k-s as an impulse train
- Then $s(t) = \sum_{k=-\infty}^{\infty} a_k g(t - kT_b)$ since we are transmitting every T_b secs.
- $x(t) = s(t) \star h(t)$
- $y(t) = x(t) \star q(t)$
- $y(t) = \sum_{k=-\infty}^{\infty} a_k p(t - kT_b)$, where $p(t) = g(t) \star h(t) \star q(t)$
- Or $P(f) = G(f)H(f)Q(f)$

An effective pulse shape $p(t)$



- $y(t) = \sum_{k=-\infty}^{\infty} a_k p(t - kT_b)$, where $p(t) = g(t) \star h(t) \star q(t)$

Intersymbol interference problem



- At the sampling instants $y(iT_b)$ we have $y(iT_b) = \sum_{k=-\infty}^{\infty} a_k p((i-k)T_b)$ (notation: pulse is centered at zero)
- Suppose $y_i = y(iT_b)$ and $p_i = p(iT_b)$
- $y_i = \sum_{k=-\infty}^{\infty} a_k p_{i-k}$
- We need $y_i = p_0 a_i$ for all i. Let us say that $p_0 = \sqrt{E}$
- What we have is $y_i = \sqrt{E} a_i + \sum_{k \neq i} a_k p_{i-k}$

Mitigation of intersymbol interference

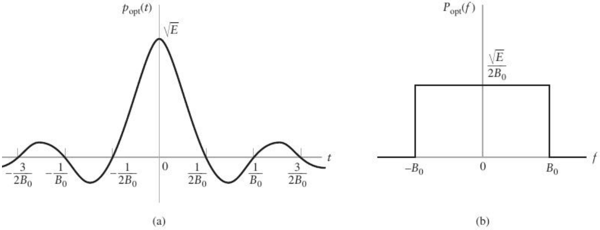
- We have to design $p(t)$ such that $y_i = \sqrt{E} a_i$
- $p(t)$ has to be designed so that $P(f)$ has minimum bandwidth
- Designing $p(t)$ is called pulse shaping

Nyquist channel

- If $y_i = p_0 a_i$ for every i then we require that

$$p_n = \begin{cases} \sqrt{E}, & \text{for } n = 0, \\ 0, & \text{otherwise.} \end{cases}$$

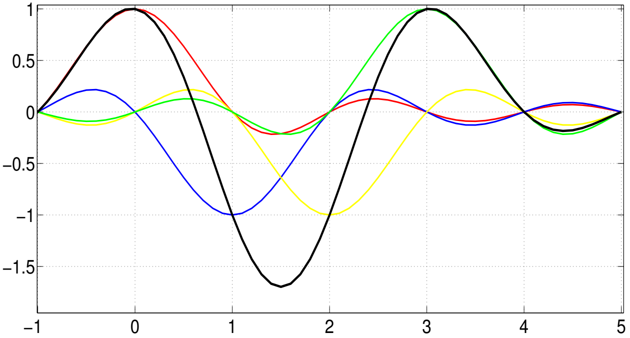
- Note that $p_n = p(nT_b)$
- Is it possible to get $P(f)$? Assuming that $P(f)$ is bandlimited
- Consider the choice of $p(t) = sinc\left(\frac{t}{T_b}\right)$
- With $B_0 = \frac{1}{2T_b}$ we have the following optimal pulse shape $p_{opt}(t)$



- The PAM system with $P_{opt}(f)$ is called the Nyquist channel
- The bandwidth B_0 is called the Nyquist bandwidth

Nyquist channel pulse shaping - issues

- The transfer function $P(f)$ is not realizable
- Issue of timing jitter



- Suppose sampling instants at which decoding is done has a *jitter*. Then is correct decoding possible?

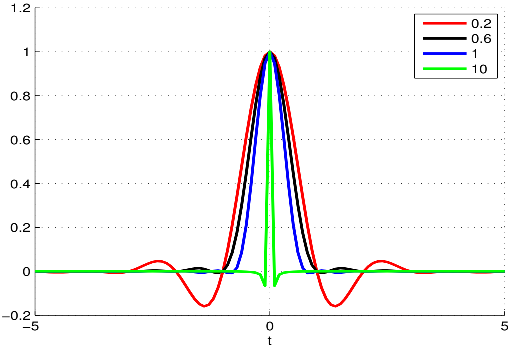
Raised cosine pulse shaping

- The problem with sinc pulses - $\frac{1}{t}$ decay
- How to increase the decay rate?

Raised cosine pulse shaping

- The problem with sinc pulses - $\frac{1}{t}$ decay
- How to increase the decay rate?
- *Damp* the sinc pulse using a window function
- Raised cosine pulse shape (actually damped sinc pulse shape)

$$p(t) = \sqrt{E} sinc(2B_0 t) \frac{cos(2\pi\alpha B_0 t)}{1-(4\alpha B_0 t)^2}$$

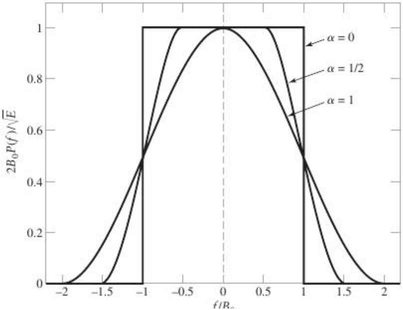


Raised cosine pulse shaping

- The F.T of $p(t)$ is

$$P(f) = \begin{cases} \frac{\sqrt{E}}{2B_0}, \text{ for } |f| \leq f_1, \\ \frac{\sqrt{E}}{4B_0}\left[1 + cos\left\{\frac{\pi(|f|-f_1)}{2(B_0-f_1)}\right\}\right], \text{ for } f_1 < |f| < 2B_0 - f_1, \\ 0, \text{ o/w.} \end{cases}$$

- $\alpha = 1 - \frac{f_1}{B_0}$. α is the roll-off factor.
- Bandwidth of the pulse is $2B_0 - f_1$ or $B_0(1+\alpha)$



Comparison

- Let $r_b = \frac{1}{T_b}$

Scheme	Bandwidth	Power	Rate	Timing Jitter
Rectangular	r_b	95%	r_b	Robust
Sinc	$\frac{r_b}{2}$	100%	r_b	Weak
Raised cosine	$\frac{r_b}{2}(1+\alpha)$	100%	r_b	less than Rect

- Read about square root raised cosing pulse shaping