1) Solve the following optimization problem.

$$\max_{(x_1, x_2)} \quad q_1(x_1) + q_2(x_2)$$

$$\text{if} \quad x_1 + x_2 \leq 5$$

$$x_1, x_2 \in \{0, 1, 2, 3 \dots\}$$

and the functions q_1 and q_2 are defined as:

x	0	1	2	3	4	5
$q_1(x)$	4	7	6	3	8	7
$q_2(x)$	2	5	4	6	5	9

2) A company C has 16 lakh rupees which it needs to allocate to its three departments, Research, Marketing, Development. Each department has submitted four proposals each. For each proposal and each department the cost of implementing the proposal and the revenue are as shown. (cost, revenue)

Proposal	Research	Marketing	Development
1	0, 0	0, 0	0, 0
2	1, 5	2, 8	1, 4
3	2, 6	3, 9	0, 0
4	0, 0	4, 12	0, 0.

Is it possible to formulate the profit maximization problem for this company as a dynamic programming problem? If possible, then write down the corresponding dynamic programming equations.

3) The state space S for a dynamic programming problem is $\{0, 1, 2\}$ and the action space $A = \{a, b\}$. The system evolution function $F(S_t, A_t)$ is given as $F(S, A) = (2S) \bmod 3$ if $A = a$ and $\underset{2}{(S + 2)} \bmod 3$ if $A = b$. How can you solve the problem $\max \sum_{t=0}^{2} R(S_t, A_t)$, where $R(S_t, A_t) = S_t^2$.