AVD623: Communication Systems-II

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Lecture 10

Figures are taken from “Communication Systems” by Simon Haykin, “Communication Systems” by Stern and Mahmoud, and “Software receiver design” by Sethares and Johnson.

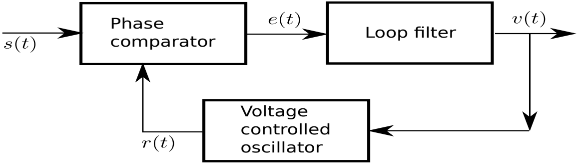
Two issues - Carrier and Timing/Clock Recovery

- A local oscillator needs to produce a replica of the carrier at the receiver
- Replica $\Rightarrow$ match in both frequency and phase
- Difference in frequencies $\Rightarrow$ time varying (linear) difference in phase

$$cos(2\pi f_1 t) \text{ and } cos(2\pi f_2 t)$$

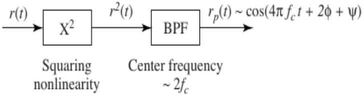
- Phase and frequency estimation using adaptive methods for phase and frequency tracking
  - Phase locked loops
  - Squared difference loop
  - Costas loop
  - Decision directed tracking

Phase and frequency estimation using PLL



- $r(t)$ is the local oscillator's output
- We want $r(t)$ to "match" with $s(t)$ in phase
- We want $e(t)$ to measure the instantaneous phase difference between $s(t)$ and $r(t)$
- Filtered output $v(t)$ controls the VCO output $r(t)$ to match $s(t)$
- $\frac{\Phi_e(s)}{\Phi_1(s)} = \frac{s}{s+2\pi K_o H(s)}$

Squared difference loop



- Recall the FFT based estimation of frequency and phase
  - The carrier was suppressed
  - We got a non-suppressed carrier by using a squaring non-linearity and band pass filtering
- To understand the squared difference loop we will use an assumption that the signal out of the BPF is

$$r_p(t) = cos(4\pi f_c t + 2\phi)$$

- Note that the same method could be applied to the case where the carrier is not suppressed - the frequency and phase would be just half of what we have in the above statement
- Let

$$J_{SD}(\theta) = \frac{1}{4} avg\left\{e^2(\theta, k)\right\} = \frac{1}{4} avg(r_p(kT_s) - cos(4\pi f_0 kT_s + 2\theta))^2$$

Interpreting the PLL as a gradient method

- For the PLL let us define

$$J_{PLL}(\theta) = \frac{1}{2} avg \left\{ r_p(kT_s) cos(4\pi f_0 kT_s + 2\theta) \right\}.$$

- Assuming that $f_0 = f_c$ and $r_p(t) = cos(4\pi f_c t + 2\phi)$, we have that

$$\begin{aligned} J_{PLL}(\theta) &= \frac{1}{2} avg \left\{ cos(4\pi f_c t + 2\phi) cos(4\pi f_c t + 2\theta) \right\} \\ &= \frac{1}{4} cos(2\phi - 2\theta). \end{aligned}$$

- Here note that we should maximize $J_{PLL}(\theta)$ - why?

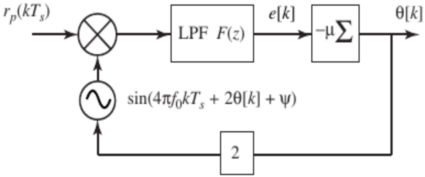
Interpreting the PLL as a gradient method

- The gradient $\frac{dJ_{PLL}(\theta)}{d\theta}$ is

$$-avg\left\{r_p(kT_s)sin(2\pi f_c kT_s + 2\theta)\right\}.$$

- So that update rule becomes

$$\theta[k+1] = \theta[k] - \mu avg\left\{r_p(kT_s)sin(2\pi f_c kT_s + 2\theta)\right\}.$$



Costas loop

- In case of PLL and SD, if the carrier is suppressed, then preprocessing in the form of squaring and BPF is required
- Costas loop does not require this preprocessing
- Consider the following performance function

$$J_c(\theta) = avg\left\{\left[avg\left\{r(kT_s)cos(2\pi f_0 kT_s + \theta)\right\}\right]^2\right\}$$

- What does this performance function capture? Suppose $r(kT_s)$ is $s(kT_s)cos(2\pi f_0 kT_s + \phi)$

$$\begin{aligned}
avg\left\{\left[avg\left\{r(kT_s)cos(2\pi f_0 kT_s + \theta)\right\}\right]^2\right\} &= \\
avg\left\{\left[avg\left\{s(kT_s)cos(2\pi f_0 kT_s + \phi)cos(2\pi f_0 kT_s + \theta)\right\}\right]^2\right\} &= \\
avg\left\{\left[\frac{1}{2}s(kT_s)cos(\phi - \theta)\right]^2\right\} &= \\
\frac{1}{4}s_{avg}cos^2(\phi - \theta).
\end{aligned}$$

Gradient approach for Costas loop

- The performance function is

$$J_c(\theta) = avg\left\{\left[avg\left\{r(kT_s)cos(2\pi f_0 kT_s+\theta)\right\}\right]^2\right\}$$

- Using our usual approximation for $\frac{dJ_c(\theta)}{d\theta}$ we have

$$avg\left\{\frac{d\left[avg\left\{r(kT_s)cos(2\pi f_0 kT_s+\theta)\right\}\right]^2}{d\theta}\right\} =$$

$$2avg\left\{\left[avg\left\{r(kT_s)cos(2\pi f_0 kT_s+\theta)\right\}\frac{d\left[avg\left\{r(kT_s)cos(2\pi f_0 kT_s+\theta)\right\}\right]}{d\theta}\right]\right\} =$$

$$-2avg\left\{\left[avg\left\{r(kT_s)cos(2\pi f_0 kT_s+\theta)\right\}\right]\left[avg\left\{r(kT_s)sin(2\pi f_0 kT_s+\theta)\right\}\right]\right\}$$

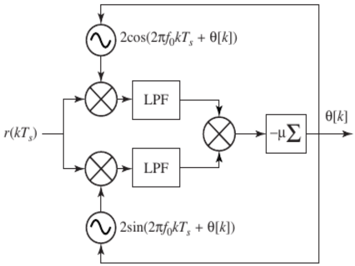
- So the update rule for $\theta[k]$ becomes

$$\begin{aligned}\theta[k+1] &= \theta[k]+\mu\frac{dJ_c(\theta)}{d\theta}\\ &= \theta[k]-\mu avg\Big\{\left[avg\left\{r(kT_s)cos(2\pi f_0 kT_s+\theta)\right\}\right]\times\\ &\qquad \left[avg\left\{r(kT_s)sin(2\pi f_0 kT_s+\theta)\right\}\right]\Big\}\end{aligned}$$

Gradient approach for Costas loop

- So the update rule for $\theta[k]$ becomes

$$\begin{aligned}\theta[k+1] &= \theta[k] + \mu \frac{dJ_c(\theta)}{d\theta} \\ &= \theta[k] - \mu avg\Big\{ [avg\{r(kT_s)cos(2\pi f_0 kT_s + \theta)\}] \times \\ &\quad [avg\{r(kT_s)sin(2\pi f_0 kT_s + \theta)\}] \Big\}\end{aligned}$$



Decision directed phase tracking

- In digital systems, ideally the sample of the received signal should belong to a finite set of possible values. For example, the set of values is $\{1, -1\}$.
- Now if the received value is not 1 or -1 then the error might be due to a phase offset
- This error can be used as a performance function.
- Suppose $s(t)$ is a baseband PAM signal and it is modulated to $s(t)cos(2\pi f_c t + \phi)$
- At the receiver we do $x(t) = avg\left\{s(t)cos(2\pi f_c t + \phi)cos(2\pi f_c t + \theta)\right\}$
- If $\phi = \theta$ then $x(t) = s(t)$ and $x(kT_s) = s(kT_s)$
- Suppose $Q(x)$ is a quantization function that converts x to one of the allowable values
- The performance function for decision directed tracking is

$$J_{DD}(\theta) = \frac{1}{4} avg\left\{\left(Q(x[n]) - x[n]\right)^2\right\}.$$

Decision directed phase tracking

- The performance function for decision directed tracking is

$$J_{DD}(\theta) = \frac{1}{4} avg \left\{ (Q(x[n]) - x[n])^2 \right\}.$$

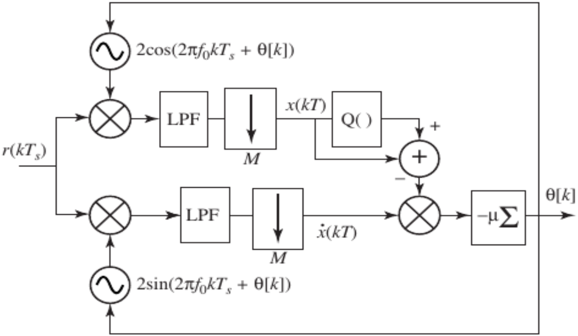
- We will approximate the gradient as follows

$$\begin{aligned} \frac{dJ_{DD}(\theta)}{d\theta} &\approx \frac{1}{4} avg \left\{ \frac{d\left(Q(x[n]) - x[n]\right)^2}{d\theta} \right\} \\ &= -\frac{1}{2} avg \left\{ (Q(x[n]) - x[n]) \frac{dx[n]}{d\theta} \right\} \end{aligned}$$

- Now $x[n] = avg\left\{r(nT)cos(2\pi f_c nT + \theta)\right\}$, so that

$$\frac{dx[n]}{d\theta} \approx -avg\left\{r[nT]sin(2\pi f_c nT + \theta)\right\}$$

Decision directed phase tracking



- The update rule is

$$\theta[n+1] = \theta[n] - \mu\left((Q(x[n]) - x[n])\right) avg\left\{r[nT]sin(2\pi f_c nT + \theta[n])\right\}$$