Review

(this lecture was given using the board, the transcriptions of this from what was erasing on the board)

- What is a random process?
  - collection of random variables
    $\{X(t)\} = X(t)$, $\forall t \in \mathbb{R}$
  - continuous time and discrete time random processes
- Specification of a random process - by PDF
- PDF structures - IID, IID, Markov, Martingale, GMS
- What is meant by MS RPs? $\mathbb{E}[X(t)]$ = constant, $\mathbb{E}[X(t_1)X(t_2)] = f(t_1 - t_2)$

This class

Our interest is understanding what happens to random signals as processed by systems. Motivation: Run a noisy process a "transformation" of a desired signal (deterministic) and no use (random process).

Let us look back about a simple case - a random signal processed by a memoryless system.

$$\boxed{\{X(t)\}} \longrightarrow \boxed{\quad} \longrightarrow \{Y(t)\}$$

e.g. assume that the system is LTI an amplifier so that $Y(t) = g\,X(t)$.

g gain of the amplifier

- $\{X(t)\}$'s PDF is known. What is $\{Y(t)\}$'s PDF?

(Our interest is to find out what the o/p. RP is and the PDF is the complete specification of the output RP).

Now that $Y(t) = g\,X(t)$ in general $Y(t) = g\,(X(t))$ where g is some transformation function.

Then we have a formula for finding PDF of $Y(t)$. Since we are think about $Y(t_1), Y(t_2)$ $Y(t_1)$ we obtained from $X(t_1)$ $X(t_2)$ as

$$\begin{array}{ccc} X(t_1) & \longrightarrow & Y(t_1) \\ X(t_2) & \longrightarrow & Y(t_2) \end{array}$$

and using the Bivariate method (i.e.e. Jacobian, Radius or Determinant and Tangents for how to do this).

- But finding PDF is difficult!

Our actual interest is not just for memoryless systems but for LTI filters.

$$\{X(t)\} \longrightarrow \boxed{h(t)} \longrightarrow \{Y(t)\} \longrightarrow \text{is } \{Y(t)\} \text{ a random process?}$$

here we can gain insight in characterizing the PDF of $\{Y(t)\}$.

Consider $\{X(t)\}$ - each time we look at this RP we see a time function (which is deterministic) $x(t)$. Assume that the possible time functions are as shown.

A random process with sinusoidal peak.

These are the only functions possible. Each time we obtain the random process we see one of these time functions.

Suppose we look at $\{X(t)\}$ at time t_1 $t < 0$ or > 2. we will always obtain 0 if $t \in [0, 2]$, we will see values $\{0, 1, 2\}$ with $\frac{1}{3}$ probability. This is the space of the random variable $X(t_1)$.

In order to think about what happens when $\{X(t)\} \longrightarrow h(t) \longrightarrow$ we think about how that $\{X(t)\}$ was processed by $h(t)$. In fact of that we know that the o/p is $y(t) = x(t) * h(t)$. Suppose $h(t)$ is

Then we have the following possibilities for the signals that we will obtain on the o/p at time t_1.

cases with the peak each at t o/p y_1 actually $\leq$ RP $Y(t_1)$.

Now consider long that to at the o/p

The values that $Y(t_1)$ takes are marked in the above figure

This each figure $y(t_1) = \int_{-\infty}^{\infty} x(\tau)\, h(t_1 - \tau)\, d\tau$ → value that the RV of RP

Value that the RV at t_1 takes $\quad$ $\{Y(t)\}$'s RPs

$\therefore$ in general $y(t_1) = \int_{-\infty}^{\infty} x(\tau)\, h(t_1 - \tau)\, d\tau$

or $\{Y(t)\} = \left(\int_{-\infty}^{\infty} \{X(\tau)\} * h(\tau) \right)$

The correlational formula holds for random processes also