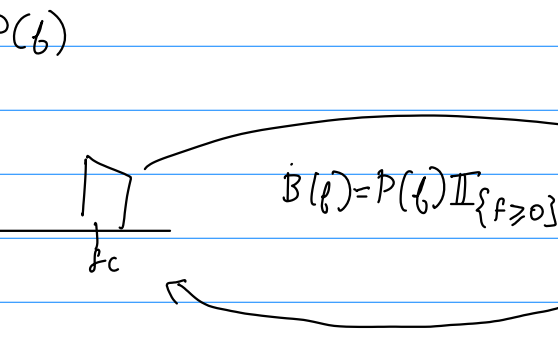


Complex baseband representation of passband signals & systems

Suppose we have a general real valued passband signal  $p(t) \xrightarrow{\mathcal{F}} P(f)$



$$\begin{aligned}
 & \text{Diagram showing the relationship between } p(t) \text{ and } b(t). \\
 & p(t) \text{ is a real signal with spectrum } P(f). \\
 & b(t) \text{ is a complex baseband signal with spectrum } B(f). \\
 & \text{The spectrum } B(f) \text{ is shown as a rectangular pulse from } -f_c \text{ to } -f_c + B \text{ and from } f_c - B \text{ to } f_c. \\
 & \text{The relationship is given by: } B(f) = P(f) \mathbb{I}_{\{f \geq 0\}}(f - f_c) \\
 & \text{and } P(f) = B(f - f_c) + B^*(-f - f_c) \\
 & \text{The complex baseband signal } b(t) \text{ is defined as: } b(t) = \frac{1}{2} (p(t) + j\tilde{p}(t)) e^{-j2\pi f_c t} \\
 & \text{where } \tilde{p}(t) \text{ is the Hilbert transform of } p(t). \\
 & \text{The spectrum of } b(t) \text{ is } B(f) = \frac{1}{2} (P(f - f_c) + \tilde{P}(f - f_c)) \\
 & \text{The real part of } B(f) \text{ is } \frac{1}{2} (P(f - f_c) + \tilde{P}(f - f_c)) \cos(2\pi f_c t) \\
 & \text{The imaginary part of } B(f) \text{ is } \frac{1}{2} (P(f - f_c) - \tilde{P}(f - f_c)) \sin(2\pi f_c t)
 \end{aligned}$$

$p(t)$  from  $b(t)$ ?

can we use the relationship  $P(f) = B(f - f_c) + B^*(-f - f_c)$  ?  
 inverse Fourier transform of  $P(f)$  gives  $p(t)$   
 inverse " of  $B(f - f_c) + B^*(-f - f_c)$

$$\mathcal{F}^{-1} \left( B^*(-f - f_c) \right) = \underbrace{b^*(t)}_{\text{complex conjugate}} e^{-j2\pi f_c t}$$

$$\begin{aligned}
 & = \int b^*(t) e^{-j2\pi f_c t} e^{-j2\pi f t} dt \\
 & = \int b^*(t) e^{-j2\pi (f + f_c) t} dt \\
 & = \left( \int b(t) e^{j2\pi (f + f_c) t} dt \right)^* \\
 & = B^*(-f - f_c)
 \end{aligned}$$

$$\begin{aligned}
 p(t) &= b(t) e^{j2\pi f_c t} + b^*(t) e^{-j2\pi f_c t} \\
 &= (b_a(t) + j b_i(t)) (\cos(2\pi f_c t) + j \sin(2\pi f_c t)) \\
 &\quad + (b_a(t) - j b_i(t)) (\cos(2\pi f_c t) - j \sin(2\pi f_c t)) \\
 &= 2 \left[ b_a(t) \cos(2\pi f_c t) - b_i(t) \sin(2\pi f_c t) \right]
 \end{aligned}$$

$$\begin{aligned}
 p(t) &= 2 \left( b_a(t) \cos(2\pi f_c t) - b_i(t) \sin(2\pi f_c t) \right) \\
 &= 2 \left( i(t) \cos(2\pi f_c t) - q(t) \sin(2\pi f_c t) \right)
 \end{aligned}$$

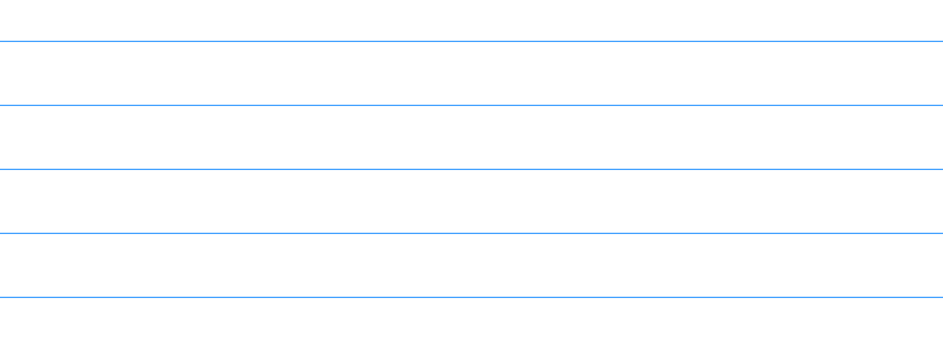


$$\frac{p(t)}{2} = \underbrace{\frac{i^2(t) + q^2(t)}{2}}_{c(t)} \cos(2\pi f_c t + \alpha(t))$$

→ Why is this representation important for us?

Representing passband signals in discrete time for DSP.

for example, suppose we had a passband signal with spectrum as shown.



$$\text{we know } \frac{p(t)}{2} = i(t) \cos(2\pi f_c t) - q(t) \sin(2\pi f_c t)$$

If suppose we use our knowledge of Nyquist sampling, then sampling rate  $f_s = 1/T_s \geq 2 \text{ kHz}!!$

but  $p(t)$  can be obtained from  $i(t)$  and  $q(t)$  (or from  $i_n$  and  $q_n$ , where  $i_n = i(nT_s)$  and  $q_n = q(nT_s)$ )

What rate is required to sample  $i(t)$  and  $q(t)$ ?

Suppose  $f_c$  is at the center of the passband  $i(t)$ , and  $q(t)$  have two sided BW = the one sided BW of  $p(t)$  - which is usually  $< 1 \text{ kHz}$ !

so sampling rate for  $i(t) \geq 2 \times \frac{1}{2}$  (one sided BW of  $p(t)$ ) and similarly for  $q(t)$ .

But two samples must be taken, one of  $i(t)$  and  $q(t)$  per  $T_s$  to represent  $p(t)$  in discrete time domain. (of course, the representation should be such that it is possible to recover  $p(t)$  from its samples).