Frequency modulation & phase modulation

Quick review

Spectrum of a FM signal? / B/W of a FM signal?

a) $m(t) = A_m \cos(2\pi f_m t)$; $f_m << f_c$

F.M signal? $= A\cos\left(2\pi f_c t + 2\pi k_f A_m \int_0^t \cos(2\pi f_m u)\,du\right)$

$= A\cos\left(2\pi f_c t + \frac{k_f A_m}{f_m}\sin(2\pi f_m t)\right)$

$= A\left[\cos(2\pi f_c t)\cdot\cos\left(\frac{k_f A_m}{f_m}\sin(2\pi f_m t)\right) - \sin(2\pi f_c t)\cdot\sin\left(\frac{k_f\cdot A_m}{f_m}\sin(2\pi f_m t)\right)\right]$

≈ 1

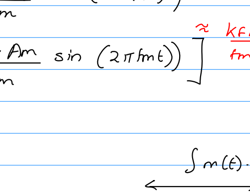
$\approx \frac{k_f A_m}{f_m}\sin(2\pi f_m t)$

if $0 < \frac{k_f\cdot A_m}{f_m} << 1$

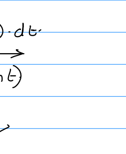
$\int m(t)\cdot dt$

$FM = \underbrace{A\cos(2\pi f_c t)}_{\text{carrier}} - \underbrace{A\sin(2\pi f_c t)\cdot\frac{k_f A_m}{f_m}\sin(2\pi f_m t)}_{DSBSC}$

FM signal's one sided B/W = $(2f_m)$



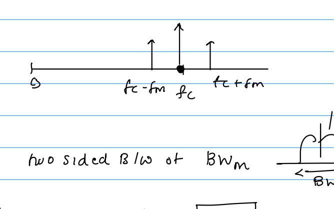
b) $m(t)$ is some baseband signal two sided B/W of BW_m



$\int_0^t m(u)\cdot du = n(t)$ $m(t) \rightarrow \boxed{} \rightarrow n(t)$

FM signal $= A\left[\cos(2\pi f_c t)\cdot\cos(k_f\cdot n(t)) - \sin(2\pi f_c t)\cdot\sin(k_f\cdot n(t))\right]$

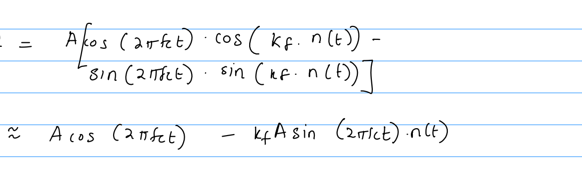
$\approx A\cos(2\pi f_c t) - k_f A\sin(2\pi f_c t)\cdot n(t)$



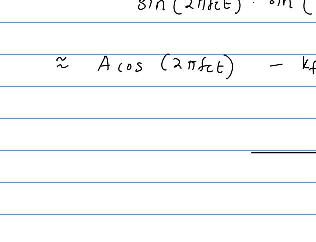
for FM if (k_f is small), then the one sided B/W of the FM signal $\approx$ two sided B/W of the modulating signal $m(t)$.

c) k_f is large.

$m(t)$



FM signal?

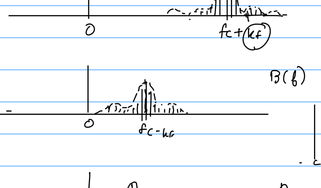


← spectrum for this?

(A) $\cos(2\pi(f_c + k_f))\cdot \times$

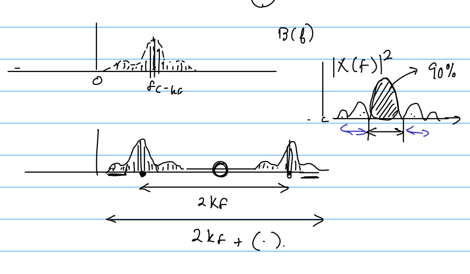
(B) $\cos(2\pi(f_c - k_f)) \times$

(A)'s spectrum?



(B)'s spectrum?

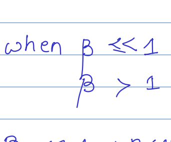
if k_f is large.



Carson's formula $= (\underbrace{2k_f}_{\text{max. freq deviation}} + BW_m) \rightarrow$ refine

Carson's formula

The output freq from the VCO is $\underbrace{K_f \max|m(t)|}_{\Delta f_{max}}$ max freq deviation.



The FM signals B/W $\propto 2\Delta f_{max}$ one sided B/W.

$\propto$ one sided B/W of $m(t)$. $2f_m$

$\left(2\Delta f_{max} + 2f_m\right)$

$= 2f_m\left(1 + \frac{\Delta f_{max}}{f_m}\right)$ → β : FM modulation index.

$= 2f_m(1+\beta)$

when $\beta << 1$, then Carson's formula says B/W is $2f_m$

$\beta > 1$, B/W is $2\Delta f_{max}$

$\beta << 1$: narrowband FM modulation.

$\beta > 1$: wideband FM modulation.

Analysis of FM signal spectrum of wideband FM.

$m(t) = A_m\cdot\cos(2\pi f_m t)$

F.M signal is : $A_c\cos\left(2\pi f_c t + \frac{k_f A_m}{f_m}\sin(2\pi f_m t)\right)$

$= Re\left\{A_c e^{j2\pi f_c t}\cdot e^{j\frac{2k_f A_m}{f_m}\sin(2\pi f_m t)}\right\}$

$= Re\left\{\underbrace{A_c e^{j\frac{2k_f A_m}{f_m}\sin(2\pi f_m t)}}\cdot e^{j2\pi f_c t}\right\}$

consider $A_c e^{j\frac{2k_f A_m}{f_m}\sin(2\pi f_m t)} = A_c e^{j\frac{2k_f A_m}{f_m}\cdot\sin(2\pi f_m(t + n/f_m))}$ } complex periodic signal.

F.S? $\underbrace{e^{j2\beta\sin(2\pi f_m t)}}_{x_p(t)} = \sum_{K=-\infty}^{\infty} c_k\cdot e^{j2\pi k\cdot f_m\cdot t}$

$c_K = f_m\cdot\int_0^{1/f_m} e^{j2\beta\sin(2\pi f_m t)}\, e^{-j2\pi k f_m t}\,dt$

$= \underbrace{f_m\int_0^{1/f_m} e^{-j(2\pi k f_m t - 2\beta\sin(2\pi f_m t))}\,dt}_{\text{Bessel function}}$