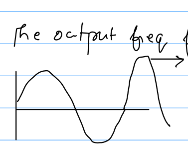
Carson's formula. – Review

The output freq from the VCO is $\underbrace{K_f \max|m(t)|}_{\Delta f_{max}}$



max freq deviation.

The FM signal's B/W $\propto$ $2\Delta f_{max}$
one sided B/W.

$\propto$ one sided B/W of $m(t)$.
$2 f_m$.

$$(2\Delta f_{max} + 2f_m)$$
$$= 2f_m\left(1 + \frac{\Delta f_{max}}{f_m}\right) \rightarrow \beta : \text{FM modulation index.}$$
$$= 2f_m(1+\beta)$$

when $\beta \ll 1$, then Carson's formula says B/W is $2f_m$
$\beta > 1$, B/W is $2\Delta f_{max}$

$\beta \ll 1$: narrowband FM modulation.
$\beta > 1$: wideband FM modulation.

Analysis of FM signal spectrum of wideband FM.

$$m(t) = A_m \cdot \cos(2\pi f_m t)$$

F.M signal is : $A_c \cos\left(2\pi f_c t + \frac{k_f A_m}{f_m}\sin(2\pi f_m t)\right)$
$$= Re\left\{A_c e^{j2\pi f_c t} \cdot e^{j\frac{k_f A_m}{f_m}\sin(2\pi f_m t)}\right\}$$
$$= Re\left\{\underbrace{A_c e^{j\frac{k_f A_m}{f_m}\sin(2\pi f_m t)}} \cdot e^{j2\pi f_c t}\right\}$$

consider $A_c e^{j\frac{k_f A_m}{f_m}\sin(2\pi f_m t)}$
$\| $
$A_c e^{j\frac{k_f A_m}{f_m}\cdot\sin(2\pi f_m(t + n/f_m))}$ } complex periodic signal.
period = $1/f_m$

F.S ? $\underbrace{e^{j\beta\sin(2\pi f_m t)}}_{x_p(t)} = \sum_{k=-\infty}^{\infty} c_k \cdot e^{j2\pi k \cdot f_m \cdot t}$

$$c_k = f_m \int_0^{1/f_m} e^{j\beta\sin(2\pi f_m t)} e^{-j2\pi k f_m t}\, dt$$
$$= \underbrace{f_m \int_0^{1/f_m} e^{-j(2\pi k f_m t - \beta\sin(2\pi f_m t))}\, dt}_{\text{Bessel function}}$$

This class:

$$c_k = \frac{1}{2\pi}\int_0^{2\pi} e^{-j(x\underline{k} - \beta\sin(x))}\, dx$$

$x = 2\pi f_m t$
$dx = 2\pi f_m dt$

Bessel function of the first kind of order $\underline{k}$
$$= J_k(\beta)$$

$$e^{-j(xk - \beta\sin(x))} = \underbrace{\cos(xk - \beta(\sin(x)))}_{\text{even}} - j\underbrace{\sin(xk - \beta\sin(x))}_{\text{odd}}$$

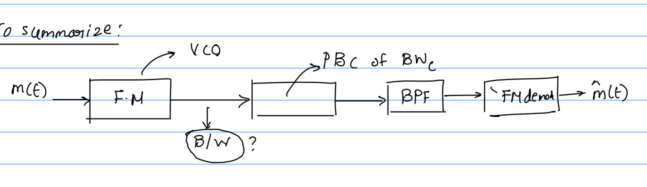
so $J_k(\beta)$ is real!

$$= \sum J_k(\beta) e^{j2\pi k f_m t}$$ → complex BB.

- $J_k(\beta) = (-1)^k \cdot J_{-k}(\beta)$ $(\beta - 1 \quad \beta + 1)$ → FM signal's B/W $= 2f_m(\beta+1)$
$|J_k(\beta)| = |J_{-k}(\beta)|$
- $|J_k(\beta)|$ for $|k| > \underline{\beta + 1}$.

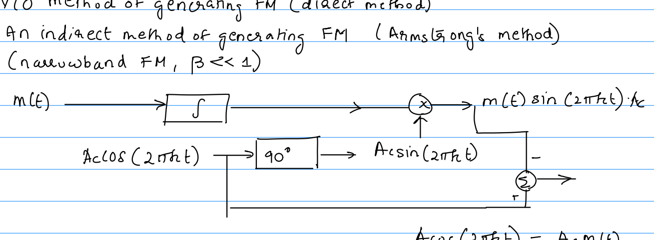
$$\sum_{k=-(\beta+1)}^{(\beta+1)} [J_k(\beta)]^2 = 0.99 \sum (J_k(\beta))^2$$

To summarize:



Frequency modulation

1) VCO method of generating FM (direct method)
2) An indirect method of generating FM (Armstrong's method)
(narrowband FM, $\beta \ll 1$)



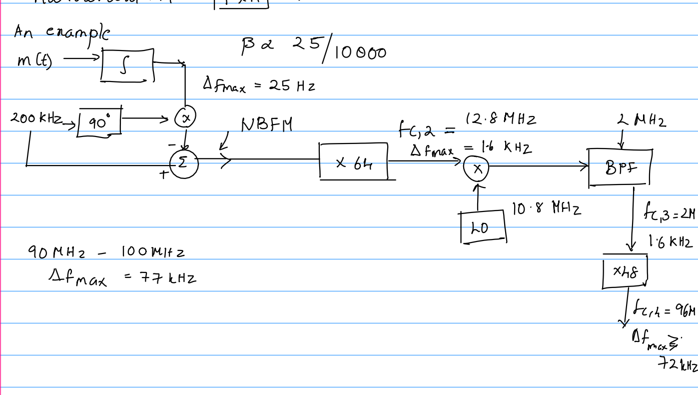
$A_c\cos(2\pi f_c t) - A_c m_1(t)\sin(2\pi f_c t)$

$$A_c \cos\left(2\pi f_c t + 2\pi k_f \int_0^t m(u)\cdot du\right)$$
$$= A_c\cos(2\pi f_c t)\cdot 1 - A_c\sin(2\pi f_c t)\cdot(\int)$$

Wideband FM → Δf_{max}, $f_{c,1}$
narrowband FM → $f \times n$ → $n \times f_{c,1}$, $n\Delta f_{max}$ → β

An example

$\beta \propto 25/10000$



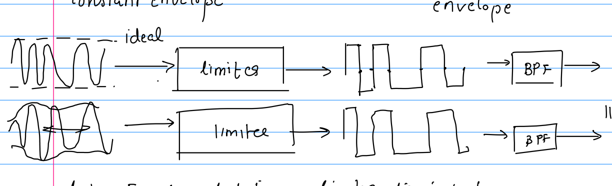
90 MHz – 100 MHz
$\Delta f_{max} = 77$ kHz

Frequency demodulation.

$$A_c\cos\left(2\pi f_c t + \underbrace{2\pi k_f \int_0^t m(u)\cdot du}_{\phi(t)}\right) \rightarrow \boxed{d/dt} \rightarrow ?$$

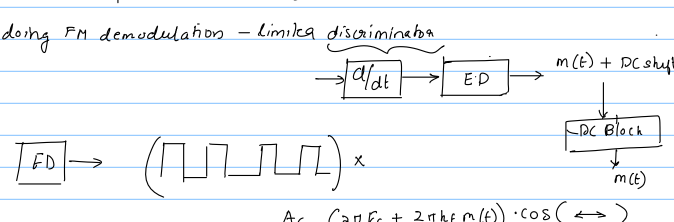
constant envelope

$$\underbrace{A_c(2\pi f_c + 2\pi k_f \cdot m(t))}_{\text{envelope}} \cdot \cos(2\pi f_c t + \underbrace{\phi(t)}_{\text{phase variation}})$$



$A_c\cos(\cdots)$

doing FM demodulation – limiter discriminator



$A_c\,(2\pi f_c + 2\pi k_f m(t))\cdot\cos(\leftrightarrow)$

- Balanced FM demodulator
- complex BB.