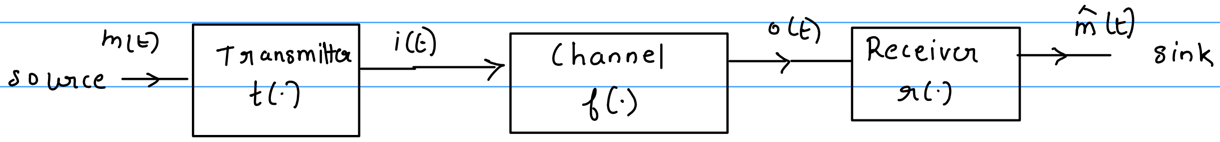
AV314 - Communication Systems I

Lecture 3 29/07/2019

Review of lectures 1 & 2



Design $t(\cdot)$ and $r(\cdot)$
such that $e(m(t), \hat{m}(t)) \leq \varepsilon$.
given $f(\cdot)$

Objectives for lecture 3

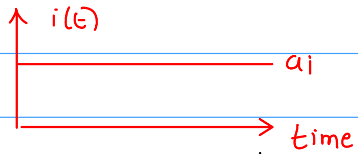
- understand channel modelling – or what is $f(\cdot)$? how to find $f(\cdot)$? etc...
- LTI system models for channels.

o Let us take an example to understand channel modelling.

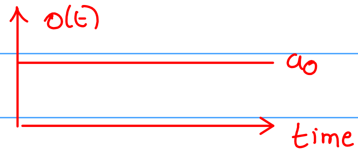
Consider a channel with the following I and O (recall : $i(t) \in I$, $o(t) \in O$).

$I = O =$ set of all DC signals.

So a $i(t)$ would be



$o(t)$:



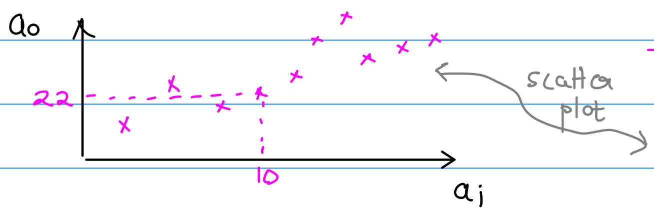
we have a channel that gives $o(t) = f(i(t))$.

* How do we get $f(\cdot)$ for this channel ?

we need to fit a model $f(\cdot)$ to this channel – fit a model to data collected about the channel.

for this channel, we can look at $\xrightarrow{a_i} \boxed{f(\cdot)} \rightarrow a_o$.

collect data about the channel – i.e., for an a_i what is a_o.



a_i	a_o
5	10.5
10	22
20	38

What is the relationship $f(\cdot)$ that best fits this data?

We do not really know what $f(\cdot)$ is / would be correct!

Step 1 So we assume a form/structure for $f(\cdot)$.

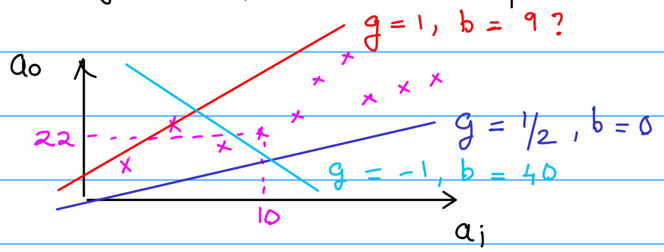
For example, we could have 1) $f(a_i) = g \times a_i$ (g: gain term, a linear model)

2) $f(a_i) = g \times a_i + b$ (b: bias, an affine model)

3) $f(a_i) = g_1 a_i^2 + g_0 a_i + b$.

We will choose a form for $f(\cdot)$ - e.g. $g a_i + b$. (g and b are parameters)

Step 2 Proceed to find for what g and b, we can best explain the data (a_i, a_o)



In order to decide this g and b we will usually solve an optimization problem where we try to minimize the error between what our model $f(\cdot)$ predicts for the a_i-s (which is $f(a_i)$) and the actual data which is a_o.

a_i	a_o
5	10.5
⋮	⋮
10	22
⋮	⋮
20	38

Let the k^{th} data be denoted as $a_{i,k}$ and $a_{o,k}$

Then the error is defined as

$$\sum_k \left(a_{o,k} - f(a_{i,k}) \right)^2.$$

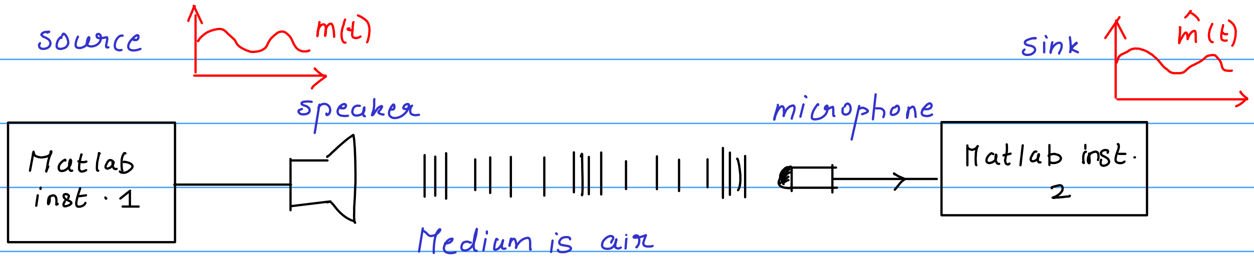
We find that g and b which minimizes $\sum_k (a_{o,k} - f(a_{i,k}))^2$

This is for the example of the channel considered here. But this is a common technique. An important point is that there is no right model for a channel (or anything for that matter, there are only better and better approximations)

We can think of the actual channel as a transformation factual. The model $f(\cdot)$ that we have is only an approximation to this.

Also note that related subjects like RF and μwave will use other models for the channel.

- Let us now consider a more complicated example



how to find $f(\cdot)$ for the above channel ?

I and O are now not sets of scalars, but they are sets containing bounded functions.

$f(\cdot)$ processes signals rather than scalars.

recall that we first need some form or structure for $f(\cdot)$ and then we need to fit $f(\cdot)$ with the specified structure to the data that we collect.

- What are structures / forms for $f(\cdot)$?
- What data should we collect ?
- How do we fit $f(\cdot)$ to the data ?

Structures / Forms for $f(\cdot)$: (Signals and systems review)

- memoryless / with memory
- linear / non-linear
- causal / non-causal
- time invariant / variants

Because we are comfortable with LTI systems, let us assume that the form for $f(\cdot)$ is that it is an LTI system. If $f(\cdot)$ is an LTI system, how do we find $f(\cdot)$?

Recall the following points regarding LTI systems

- Superposition and homogeneity properties
- time invariance
- every LTI system is characterized by its impulse response. (denoted as $h(t)$).

- If we know $h(t)$ then we can predict $o(t)$ for any input $i(t)$ as

$$o(t) = \int_{-\infty}^{\infty} i(\tau) \cdot h(t-\tau)\, d\tau$$

- Assume that $h(t)$ is finite energy, i.e., $\int_{-\infty}^{\infty} |h(t)|^2 dt < \infty$

- Assume that $h(t)$'s CTFT (spectrum) exists, i.e.,

$$H(f) = \int_{-\infty}^{\infty} h(t)\, e^{-j2\pi f t}\, dt$$

- Then finding $H(f)$ is equivalent to finding the channel model.
- Note that $H(f)$ is complex, and the magnitude and phase spectra need to be specified, in order that the LTI system is completely specified.
- So we need to find $H(f)$ when we consider LTI system models for channels.