Indian Institute of Space Science and Technology
AV336 - Digital Signal Processing Lab
Department of Avionics

Labsheet 9

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1. Visualization of the bilinear transform: The bilinear transform is a map between the s-plane and the z-plane and is defined as $z = \frac{1+sT/2}{1-sT/2}$. The inverse map is defined as $s = \frac{2}{T}\frac{1-z^{-1}}{1+z^{-1}}$. In this task you will visualize how an area in the s plane is mapped to z-plane and vice-versa.
   (a) Select $N \times M$ points $s_{i,j}$ uniformly in any rectangular region that you wish in the s-plane. (Hint: you can use meshgrid for this purpose). On the z-plane indicate where these points are mapped to under the bilinear transform.
   (b) Similarly select $N \times M$ points $z_{i,j}$ uniformly in any rectangular region that you wish in the z-plane. (Hint: you can use meshgrid for this purpose). On the s-plane indicate where these points are mapped to under the bilinear transform.
   (c) Verify (using a sufficiently dense set of points) whether the $j\Omega$ axis is mapped to the unit circle under the bilinear transform.

2. Filter design using least squares inverse design: Suppose the desired frequency response $H_d(e^{j\omega})$ can be realized by the causal system with the following z-transform,
$$H_d(z) = \frac{1+z^{-1}}{1-0.5z^{-1}}.$$
   Suppose we approximate $H_d(z)$ using a $H(z)$ of the form
$$\frac{b_0}{1-a_1z^{-1}-a_2z^{-2}}.$$
   Using least squares inverse design, obtain the values of b_0, a_1 and a_2. We note that in least squares inverse design, a set of linear equations have to be written down which constrain the values of b_0, a_1 and a_2. Explore the effect of the number of linear equations that you have on your answer.

3. Filter design using Butterworth analog filter design and impulse invariance: Suppose we have the following desired requirements $H_d(e^{j\omega})$ on the magnitude of digital filter:
   (a) Passband edge = 0.2π
   (b) Stopband edge (starting freq) = 0.4π
   (c) Magnitude gain in passband to be $\in [1, 1-\delta_p]$, where $\delta_p = 0.05$
   (d) Magnitude gain in stopband to be $\in [0, \delta_s$, where $\delta_s = 0.001$

   Note that no constraints are being put on the phase response of the filter here. In the following, for the impulse invariance based design assume that $T = 1$.

(a) Assuming that there is no aliasing and that $H_d(e^{j\omega})$ has been obtained from sampling of an analog signal $h_a(t)$ uniformly at rate $\frac{1}{T}$, what is $H_a(j\Omega)$ (the CTFT of $h_a(t)$)?.

(b) We note that $H_a(j\Omega)$ can be interpreted as the specification for the design of an analog filter. Obtain a Butterworth filter that is a good approximation to $H_a(j\Omega)$.

(c) Write down the location of the poles of the analog Butterworth filter $H_a(s)$?

(d) Under the impulse invariance condition, where are these poles mapped to in the z-plane. Write down the locations of the poles.

Exploration:

(a) Repeat the design process for $T = \frac{1}{4}$. Is there any change in the frequency response of the realized digital filter.

(b) Repeat the design process but by not compensating for aliasing in the stopband attenuation. How much is stopband attenuation in the final design? Does it meet the given requirements on $H_d(e^{j\omega})$?

(c) Using internet resources or Matlab help, find out what the inbuilt function "butter" does. How will you use "butter" for the design problem above?

(d) Using internet resources or Matlab help, find out what the inbuilt function "filter" does. Suppose $x[n] = 2cos(0.1\pi n) + 5cos(0.6\pi n)$ for $n \in \{0, \ldots, 499\}$. Simulate what happens when the filter that you have designed above is used to filter $x[n]$ in order to obtain $y[n]$. Plot $y[n]$ as well as its DTFT.