AV324 - Communication Systems - II

Tutorial I for Final Exam for **B.Tech Avionics - Semester VI**

---

1) Suppose $m(t)$ is a baseband real valued signal bandlimited to $[-f_m, f_m]$ Hz. Let $x(t) = A + m(t)$, where A is a positive real valued constant. Let $x_s(t) = \sum_{n=-\infty}^{\infty} x(nT_s)\delta(t - nT_s)$, where $T_s = \frac{1}{10f_m}$. Assume that $x_s(t)$ is transmitted over an ideal bandpass channel with lower and upper cutoff frequencies $8f_m$ and $12f_m$ (only the positive frequencies are stated here). Let $y(t)$ be the output of the above bandpass channel. Assume that $\hat{m}(t)$ is obtained by applying $y(t)$ to an ideal envelope detector (which acts like an ideal rectifier followed by an ideal filter). What is the relationship between $\hat{m}(t)$ and $m(t)$?

---

2) Suppose we have a random process $X(t) = Acos(2\pi f_c t)$, where f_c is a constant and A is random and uniformly distributed in $[0, 1]$. Is the process $X(t)$ strictly stationary?

---

3) Suppose we have a continuous time linear filter with the impulse response

$$h(t) = \begin{cases} ae^{-at} \text{ for } 0 \leq t \leq T, \\ 0, \text{ otherwise.} \end{cases}$$

Find out the power spectral density of the output of the filter for an input signal with power spectral density $S_X(f)$.

---

4) Consider a rectangular pulse defined by

$$g(t) = \begin{cases} A, \text{ for } 0 \leq t \leq T, \\ 0, \text{ otherwise.} \end{cases}$$

The matched filter for $g(t)$ is approximated by an ideal low pass filter with bandwidth B. Determine the value of B for which the ideal low pass filter provides the best approximation to the matched filter, considering the fact that maximization of the peak signal to noise ration is the primary objective.

---

5) Assume that an IID bitstream $(B_1, B_2, \dots)$, with each bit B_i chosen uniformly, is transmitted over a channel using BPSK with energy per bit E_b. We assume that the channel adds additive white Gaussian noise (with power spectral density $\frac{N_0}{2}$) to the received signal. The coherent correlation receiver, unfortunately, has been designed as a BASK correlation receiver (at the same carrier frequency as the BPSK signal) to decode BASK signals which are assumed to use an energy per bit of $0.25E_b$. Obtain the average probability of symbol error when this correlation BASK receiver is used to decode the received BPSK signal. Express your answer in terms of the Q or erfc functions.

---

6) Assume that we have a BASK system (transmitter and receiver) which uses an energy per bit E_b and bit duration T_b. The transmission is done over a channel that is assumed not

to introduce ISI, but adds noise which is assumed to be white Gaussian noise with power spectral density $\frac{N_0}{2}$. We assume that one of the components in the transmitter is subjected to power fluctuations. It is assumed that in each bit duration T_b, the transmitter component is either on or off (assume that there are no on-off transients when the component switches on and off), leading to the transmitter being either on or off in each bit duration, independently of the bit that is transmitted. The component on-off process is assumed to be an IID process, with the probability that the component is on in any bit duration being p_{on}. Note that the receiver does not know whether the component is actually on or off in a bit duration. Obtain the average symbol error probability for the above BASK system, if the input bit stream is IID and each bit is uniformly chosen. (Note that this on-off process is an additional source of error in the system.)

---

7) Assume that an IID bitstream $(B_1, B_2, \dots)$, with each bit B_i chosen uniformly, is transmitted over a channel using BASK at carrier frequency f_c, with energy per bit E_b, and bit duration T_b. Obtain the power spectrum of the BASK signal which is transmitted over the channel.

---

8) In wireless communications, regulations limit the amount of spectrum that a transmitter can use. Assume that a transmitter is allocated a bandwidth of $2W$ Hz at a center frequency of f_c. The pulse shaping techniques that we discussed in class are also used to make sure that the transmitted signal energy is "limited" to the region of the spectrum that is allocated to the transmitter. For example, "using" a sinc pulse shape, i.e. $sinc(2Wt)$, would ensure that the transmitted signal is bandlimited to $2W$ Hz. However, such pulse shapes cannot be used in practice. Therefore, consider the case where the pulse shape used is $sinc(2Wt)\times rect(t/T)$, where $rect(x) = 1$ for $|x| \leq 1$. Obtain the fraction of signal energy that is outside the allowed bandwidth of $2W$ Hz.
