AV312 - Lecture 8

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Figures from "Communication Systems" by Haykin and "An Intro. to Analog and Digital Commn." by Haykin and Moher

August 22, 2016

Announcements

- Assignment 3 on the class webpage (deadline August 26th)

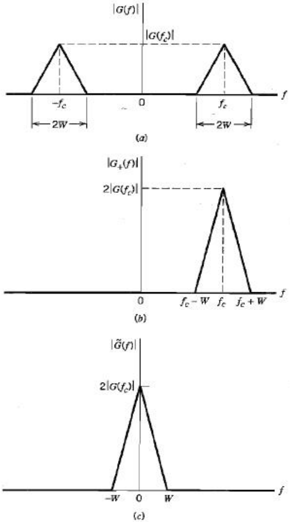
Review of last class

- Complex baseband representation of passband signals
- Complex baseband representation of passband systems
- FM demodulation analysis

Today's class

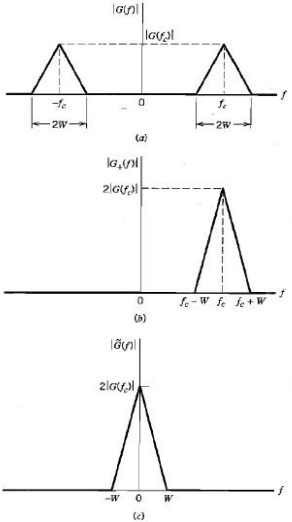
- Review complex baseband representation of passband signals
- Review complex baseband representation of passband systems
- Input output relationship for baseband signals and systems
- Complete FM demodulation analysis
- Today's scribes are Priya Vamshi and Gautam Suresh

Complex baseband representation of passband signals



- Assume that $g(t)$ occupies a bandwidth of $2W$ centered at f_c

Complex baseband representation of passband signals



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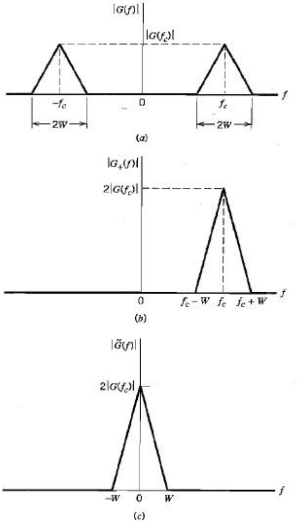

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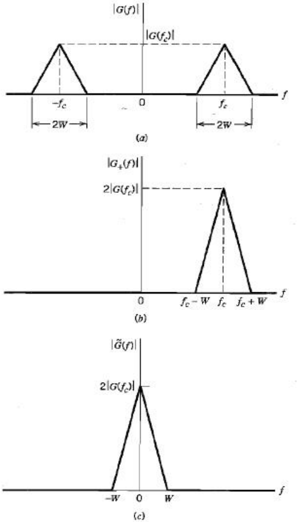
Complex baseband representation of passband signals



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- Complex baseband representation is $\tilde{g}(t)$
- $\tilde{g}(t) = g_I(t) + jg_q(t)$
- $g(t) = a(t)cos(2\pi f_c t + \phi(t))$

Complex baseband representation of passband systems

- Let $h(t)$ be the impulse response FT of a LTI bandpass system
- Let $H(f)$ be the FT of $h(t)$
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- Let $\tilde{h}(t) = h_I(t) + jh_Q(t)$ be the complex baseband impulse response
- Note that $h(t) = Re[\tilde{h}(t)e^{j2\pi f_c t}]$

What are we interested in?

- Suppose $x(t)$ is a bandpass signal, with FT $X(f)$
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- $X(f) \to H(f)$

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- Can we express $\tilde{y}(t)$ as a function of $\tilde{x}(t)$ and $\tilde{h}(t)$?
- We have that $\tilde{y}(t) = \mathbf{\frac{1}{2}} \int_{-\infty}^{\infty} \tilde{h}(\tau)\tilde{x}(t-\tau)d\tau$

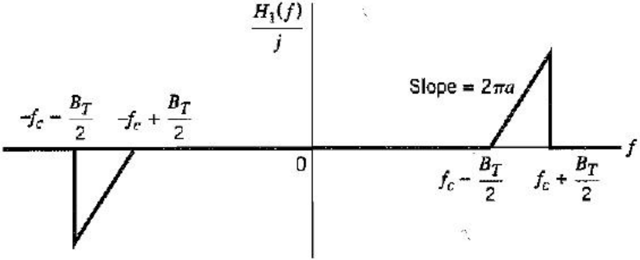
Input output relationship

- Why is $\tilde{y}(t) = \frac{1}{2}\int_{-\infty}^{\infty} \tilde{h}(\tau)\tilde{x}(t-\tau)d\tau$
- $y(t) = \int_{-\infty}^{\infty} Re\left[h_{+}(\tau)\right] Re\left[x_{+}(t-\tau)\right] d\tau$
- An important property of pre-envelopes

$$\int_{-\infty}^{\infty} Re\left[h_{+}(\tau)\right] Re\left[x_{+}(\tau)\right] dt = \frac{1}{2} Re\left[\int_{-\infty}^{\infty} h_{+}(\tau)x_{+}^{*}(\tau)d\tau\right]$$

- Then $y(t) = \frac{1}{2} Re\left[e^{j2\pi f_c t}\int_{-\infty}^{\infty} \tilde{h}(\tau)\tilde{x}(t-\tau)d\tau\right]$

Slope filter/circuit



- Consider the filter with response $H_1(f)$ defined as

$$H_1(f) = \begin{cases} j2\pi a(f - (f_c - \frac{B_T}{2})), \text{ for } f_c - \frac{B_T}{2} \le f \le f_c + \frac{B_T}{2}, \\ j2\pi a(f + (f_c - \frac{B_T}{2})), \text{ for } -f_c - \frac{B_T}{2} \le f \le -f_c + \frac{B_T}{2}, \\ 0, \text{ otherwise} \end{cases}$$

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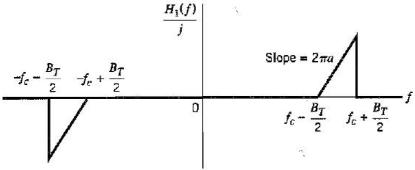

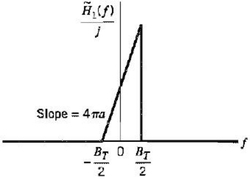

- $\tilde{H}_1(f - f_c) = 2H_1(f), f > 0$

$$\tilde{H}_1(f) = \begin{cases} j4\pi a(f + \frac{B_T}{2}), -\frac{B_T}{2} \leq f \leq \frac{B_T}{2}, \\ 0, \text{ otherwise.} \end{cases}$$

FM $s(t)$ through the slope filter

- $s(t) = A_c cos\left[2\pi f_c t + 2\pi k_f \int_0^t m(u).du\right]$
- $\tilde{s}(t) = A_c e^{\left[j2\pi k_f \int_0^t m(u).du\right]}$
- $\tilde{S}(f) \rightarrow \tilde{H}_1(f) \rightarrow \tilde{S}_1(f)$
- $\tilde{S}_1(f) = \frac{1}{2}\tilde{S}(f)H_1(f)$

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- $\tilde{s}_1(t) = j\pi B_T a A_c \left[1 + \frac{2k_f}{B_T} m(t)\right] e^{j2\pi k_f \int_0^t m(u).du}$
- $s_1(t) = Re\left[\tilde{s}(t)e^{j2\pi f_c t}\right]$
- $s_1(t) = \pi B_T a A_c \left[1 + \frac{2k_f}{B_T} m(t)\right] cos\left[2\pi f_c t + 2\pi k_f \int_0^t m(u).du + \frac{\pi}{2}\right]$
- Use an envelope detector to obtain $\pi B_T a A_c \left[1 + \frac{2k_f}{B_T} m(t)\right]$ if $|\frac{2k_f}{B_T} m(t)| < 1$
- Read text to find out how the bias term in the above expression can be removed