Review of last class.

$(X(t))$ - Random variable $X(t)\ \forall t$

Set of all possible joint distributions - FDDs

Getting such a general model - specifying all FDDs - not easy!

Impose structures on the FDDs.

a) Independently distributed RPs.

$$F_{X(t_1)X(t_2)\cdots X(t_m)} = F_{X(t_1)} F_{X(t_2)} \cdots F_{X(t_m)}$$

b) Independently and identically distributed RPs

$$F_{X(t_1)X(t_2)\cdots X(t_m)} = F_X \cdot F_X \cdots F_X.$$

c) Markov RPs.

starts from time 0.

$F_{X(t)}$, for a time $t>0$, $t \in \{0,1,2,3\cdots\}$

$X(0) \sim F_{X(0)}$

$f_{X(1)X(0)} = f_{X(0)} \cdot f_{X(1)|X(0)}$

$f_{X(2)X(1)X(0)} = f_{X(0)} \cdot f_{X(1)|X(0)} \cdot f_{X(2)|X(1)}$ → Markov structure.

$$f_{X(2)X(1)X(0)}(x_2, x_1, x_0) = f_{X(0)}(x_0) \cdot f_{X_1|X_0}(x_1|x_0) \cdot f_{X_2|X_0X_1}(x_2|x_0x_1)$$

Digression:

A, B, C $\quad \mathbb{P}(ABC) = \mathbb{P}(A)\cdot\mathbb{P}(BC/A) = \mathbb{P}(A)\,\mathbb{P}(B/A)\cdot\mathbb{P}(C/AB)$

4) Stationary random processes. (strict stationarity)

$$F_{X(t_1)} = F_{X(t_2)}$$

also $F_{X(t_1)X(t_2)\cdots X(t_m)} = F_{X(t_1+\tau)X(t_2+\tau)\cdots X(t_m+\tau)}$

$\forall m, \forall t_1\cdots t_m, \forall \tau$

5) Weakly stationary random processes (wide sense stationary RPs).

1) $\mathbb{E}X(t) = \text{constant}$

2) $\mathbb{E}X(t_1)X(t_2) = \text{function}(t_1 - t_2) = R_X(\tau)$, $\tau = t_1 - t_2$

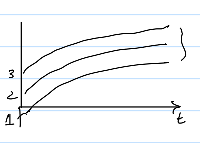
Examples.

1) Is an IID random process strictly stationary?

$F_{X(t_1)\cdots X(t_m)}(x_1,\cdots,x_m) = F_X(x_1)\cdots F_X(x_m)$

$F_{X(t_1+\tau)\cdots X(t_m+\tau)}(x_1,\cdots x_m) = F_X(x_1)\cdots F_X(x_m)$

2) $X(t) = F + \sqrt{t}$



not stationary!

Is it Markov? ✓

IID - No ID - No.

A random process model is given to us.