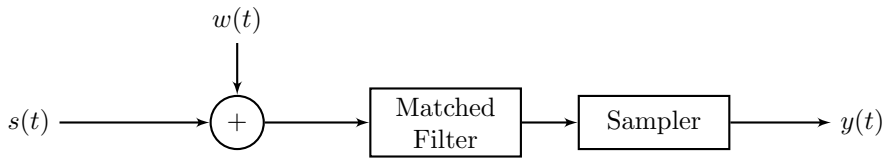
AV324 - Communication Systems II

Tutorial II for Final Exam for **Semester VI**

Question 1 Consider the following system:



Here $s(t)$ is a time-limited signal in the time interval $[0, T_b]$ (i.e. $s(t)$ is zero outside the interval $[0, T_b]$) and $w(t)$ is a zero mean white Gaussian noise process with power spectral density $\frac{N_0}{2}$. The sampler block samples the output of the matched filter at T_b. Derive the response of the matched filter which is used to maximise the SNR of $y(t)$ at the sampling instant T_b. What is the maximum SNR at the sampling instant T_b?

Question 2 Suppose a digital source puts out a sequence of bits $(B_1, B_2, \dots)$. The sequence of bits is modelled by an IID random process where each bit B_i is a Bernoulli random variable taking values 0 and 1 with equal probability.
Draw the block diagram of a BPSK modulator which takes this bit sequence as input and produces a BPSK signal $s(t)$. The modulator uses a carrier frequency of f_c and the energy per bit of the BPSK waveform is E_b.

Question 3 Consider a point to point radio link (such as a satellite link) with link parameters as in Table 1. The receiver of the radio link is as shown in Figure 1. The gains and equivalent noise temperatures of each block are shown either above or below the respective block. Assuming that the link uses BPSK modulation, what is the maximum rate of transmission for a bit error rate of 10^{-6}?

Parameter	Value
EIRP	50 dbW
Frequency of operation	14 GHz
Distance	50000 km

Table 1: Link parameters

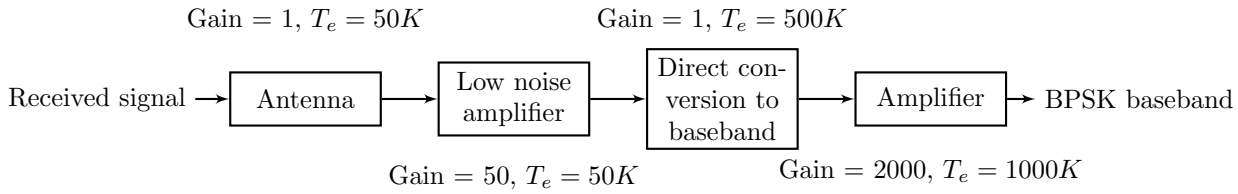


Figure 1: Receiver for the point to point link

Question 4 Suppose we want to transmit the bits that are produced by a digital source over a passband channel using BASK. See Figure 2 for the system diagram. We model the bits which are put out from the source using an IID random process model $(B_1, B_2, \dots)$. We assume that each bit B_i is chosen uniformly

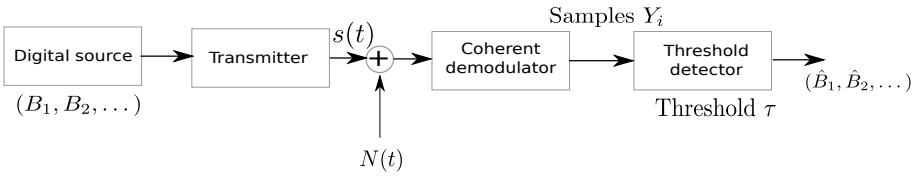


Figure 2: System for Question 8

from $\{0, 1\}$. For transmitting these bits we use BASK which transmits the waveform

$$s(t) = \begin{cases} A_c cos(2\pi f_c t), \text{ for } t \in [(i-1)T, iT] \text{ for } B_i = 1, \\ 0, \text{ for } t \in [(i-1)T, iT] \text{ for } B_i = 0. \end{cases}$$

Note that the bit duration is denoted as T. The BASK signal passes through a passband channel which does not distort the signal but adds white Gaussian noise of PSD $\frac{N_0}{2}$. At the receiver a coherent demodulation scheme is used followed by a threshold detector with a threshold of τ. Note that the coherent demodulator produces samples Y_i at the end of every bit period T. The threshold detector has a threshold of τ and puts out the received bits denoted as $\hat{B}_i$. Find out the probability of error $Pr\left\{\hat{B}_i \neq B_i\right\}$ as a function of τ. What is the optimal value of τ that will minimize the probability of error? Show why this particular value of τ minimizes the probability of error. You can assume that f_c is an integer multiple of $\frac{1}{T}$.

Question 5 Consider the system shown in Figure 3. The input $N(t)$ is a white Gaussian noise process with zero mean and power spectral density $\frac{N_0}{2}$. The processes $N_1(t)$ and $N_2(t)$ are obtained by filtering $N(t)$ using the filters with impulse responses $h_1(t)$ and $h_2(t)$ respectively.

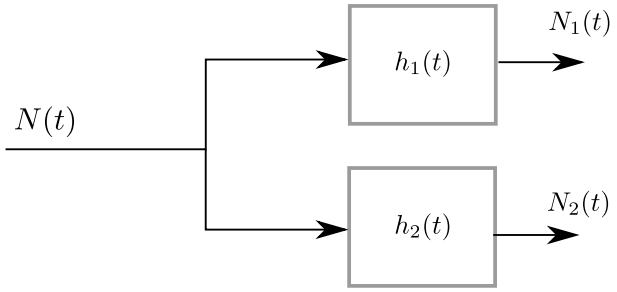


Figure 3: $N_1(t)$ and $N_2(t)$ are obtained via filtering of $N(t)$

Suppose the magnitude spectrums $|H_1(f)|$ and $|H_2(f)|$ for the two filters with impulse responses $h_1(t)$ and $h_2(t)$ are as shown in Figure 4. What is the joint distribution of $N_1(t_1)$ and $N_2(t_2)$ where t_1 and t_2 are two time instants.
Suppose now that the spectrums $H_1(f)$ and $H_2(f)$ are as shown in Figure 5 (note that the Fourier transforms are given, not the magnitude spectrum). In this case, what is the joint distribution of $N_1(t_1)$ and $N_2(t_2)$ where t_1 and t_2 are two time instants.

Question 6 A digital source puts out a bit sequence $(B_1, B_2, B_3, \dots)$ which is assumed to be an IID random process with $B_k \in \{0, 1\}$ and $\Pr\{B_k = 1\} = \frac{1}{2}$. Suppose a coherent BPSK system is used to communicate the above bit sequence. Draw a block diagram showing the different components of the above coherent BPSK system. The reference carrier signal or local oscillator carrier signal at the receiver is in frequency synchronization with the received carrier but out of phase by θ. Derive the bit error probability of the system.

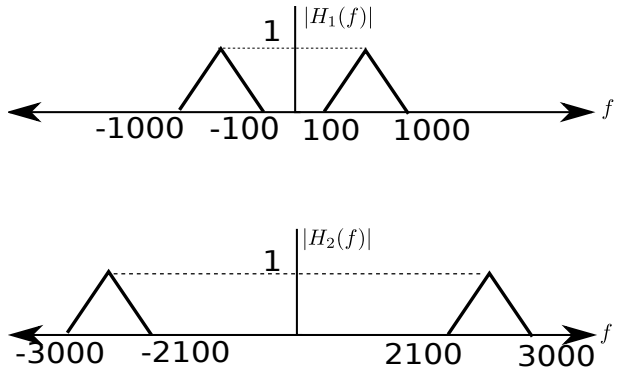


Figure 4: The magnitude spectrums $|H_1(f)|$ and $|H_2(f)|$

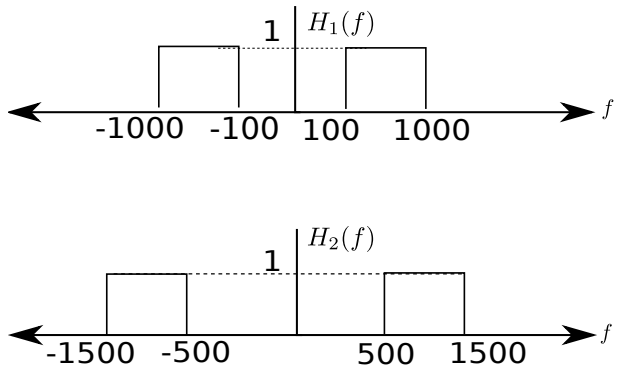


Figure 5: The Fourier transforms $H_1(f)$ and $H_2(f)$

Question 7 A digital source puts out a bit sequence $(B_1, B_2, B_3, \dots)$ which is assumed to be an IID random process with $B_k \in \{0, 1\}$ and $\Pr\{B_k = 1\} = 0.25$. The bit sequence is transmitted using BFSK signalling scheme. Draw a block diagram showing the different components of the above BFSK transmitter. Derive the power spectral density of the BFSK signal obtained, clearly stating any assumptions that you have made.
