AV332: Digital Communication Lab

Lab 8
Random variables, Random processes, BER for BASK and BPSK   Date: 13th and 17th October 2016

Please note that this experiment has to be done in MATLAB.

1 Random variables

Prelab assignments

1. Study random variables and common random variable models. Study PMFs, PDFs, CDFs associated with random variables.
2. Study what is meant by a sample from a distribution or a sample/realization of a random variable.
3. Read about pseudo-random numbers from the internet. What is a "seed" associated with a pseudo-random number generator?

In lab tasks

1. Use Matlab to plot the PMFs and CDFs for a Binomial, Poisson, and Geometric distributions. What are the parameters for these distributions? Choose any parameters that you like to plot the PMFs and CDFs.
2. Use Matlab to plot the PDFs and CDFs for Gaussian/Normal, Exponential, and Uniform distributions. What are the parameters for these distributions? Choose any parameters that you like to plot the PMFs and CDFs.
3. Generate 10 independent samples from a Gaussian distribution with mean 1 and variance 2.
4. Let X_i denote the i^{th} independent sample from a Gaussian distribution with mean 1 and variance 2. Let $M_n = \frac{\sum_{i=1}^{n} X_i}{n}$. Plot M_n as a function of n for $n = 10, 50, 100, 200, 500, 1000, 2000$. What do you observe? Write down any reasons for your observations.

2 Random processes

Prelab assignments

1. Study random processes and common random process models. Study FDDs, IID random processes, stationary and non-stationary random processes.
2. Study what is meant by a sample function of a random process or a realization of a random process.
3. Study what is the autocorrelation function of a random process and the power spectral density (PSD) of a random process.
4. Study about LTI filtering of stationary random processes. What is the spectral domain relationship between the input PSD and output PSD of an LTI system with a stationary random process as the input?
5. Read about Ergodic random processes from the textbook.

In lab tasks

1. Write a Matlab function "IIDProcess" to generate a finite length realization of a discrete time IID random process. The inputs to the function "IIDProcess" should be the length of the realization N, the distribution to use, and the parameters of the distribution. The distribution could be either "Gaussian" or "Exponential". Plot 5 realizations of the discrete time IID process.
2. Write a Matlab function "IndProcess" to generate a finite length realization of a discrete time independently distributed process. That is the random variables ($X_i, i \in \{1, 2, 3, \ldots\}$) are mutually independent but not identically distributed. You are free to choose any way to make the random variables non-identically distributed.
3. Write a Matlab function "SineRandomPhase" to generate a sine wave with a random initial phase Φ. Assume that Φ is uniformly distributed in $[0, 2\pi]$. Plot 5 realizations of the sine wave. You are free to assume any value for the amplitude and frequency of the sine wave (Note that you will be implementing a function to generate a discrete time sine wave).

4. Suppose $(X_i, i \in \{1, 2, 3, \ldots\}$ is a stationary random process. Assume that the mean function of the random process is 0 for all i. The autocorrelation function of the random process $R_X(\tau) = \mathbb{E}X_0X_\tau$. Note that computing the autocorrelation function requires one to take an expectation over the joint distribution of X_0 and X_τ for every τ. When a realization of a random process is given, one computes the sample autocorrelation function which is

$$r_X(\tau) = \frac{1}{N-\tau}\sum_{i=1}^{N-\tau} X_i X_{i+\tau}$$

   Write a function "AutoCorrelation" to compute the sample autocorrelation given the sample function of a discrete time random process and the integer valued quantity τ as inputs.
5. Plot the sample autocorrelation function of a discrete time IID process with the distribution being $\mathcal{N}(0, 2)$. What do you observe? What might be the reasons for your observation?
6. Plot the PSD for the discrete time IID process you can generated in the above task. What do you observe? Do you obtain any relationships between the PSD and the moments of the (identical) distribution for the IID process?
7. Suppose you have a discrete time LTI system with impulse response $h_n = a_0\delta(n) + a_1\delta(n-2)$. Suppose $a_0 = a_1 = 1$. Obtain the output from this LTI system if the input is the discrete time process generated in task 5.
8. Obtain the sample autocorrelation function and the PSD of the output obtained in the above task. What do you observe? What happens if you vary a_0 and a_1? Write down your observations and the reasons for your observations.

3 Bit error rate for BASK and BPSK

Prelab assignments

1. Review BASK and BPSK modulation
2. Study what is meant by bit error rate. Study how bit error rate is calculated in practice.
3. Make sure that you have successfully completed all the tasks in lab 7.

In lab tasks

1. Your first task is to implement a BASK modulator and BASK coherent demodulator using the code you have implemented for lab 7.
   (a) Using code from lab 7 implement a function "RandSeqUniform" to generate a random sequence of uniformly distributed bits. The function takes the length of the sequence as input.
   (b) Using code from lab 7 implement a function "BASK" to generate the BASK baseband rectangular pulse sequence for the random sequence of bits obtained above from "RandSeqUniform". Note that the BASK baseband sequence uses rectangular pulses of amplitude A for binary 1 and 0 for binary 0. You can use the T_b value that you have used in lab 7 here.
   (c) Generate the BASK passband waveform corresponding to the above baseband signal. Use a carrier of amplitude 1 and frequency $10/T_b$.
   (d) Implement a coherent demodulator for the above BASK passband waveform and obtain the BASK baseband sequence back.
   (e) Simulate sampling the BASK baseband sequence at the middle of the bit interval T_b. Implement the decision logic (a thresholder) for obtaining the bits from the samples of the BASK baseband sequence.
   (f) Similarly, set up a BPSK modulator and demodulator. Note that the differences from the BASK modulator and demodulator lies in the baseband rectangular pulse sequence (now it is A for binary 1 and $-A$ for binary 0) and the decision logic.
2. Your second task is to simulate additive white Gaussian noise being added to the received signal. For this use the "IIDProcess" function you have implemented in the first section of this lab. The distribution should be Gaussian with mean 0 and variance σ^2. Modify your implementation of the BASK and BPSK systems to add this simulated noise to the BASK and BPSK passband waveforms. Note that the length of the Gaussian noise realization should be the same as that of the signal.

3. Now compute the bit error rate for the bits put out from the decision logic for both BASK and BPSK (Note that this bit error rate has to be computed for a large number of bits. You should generate a random sequence of uniformly distributed bits with the number of bits being 1000).

4. Plot the bit error rate as a function of $\frac{A^2}{\sigma^2}$. What do you observe? What is the significance of $\frac{A^2}{\sigma^2}$? You can keep $\sigma^2 = 1$ and use $A \in \{0.01, 0.1, 1, 2, 5, 10\}$.