AV312 - Lecture 7

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Figures from "Communication Systems" by Haykin and "An Intro. to Analog and Digital Commn." by Haykin and Moher

August 12, 2016

Announcements

- Assignment 2 on the class webpage (deadline August 19th)
- August 15 - holiday
- August 16 - 3rd and 4th hour by Anoop
- August 19 - 1st hour by Anoop
- August 22 - 2nd and 3rd hour by Vineeth
- August 23 - 3rd and 4th hour by Vineeth
- Class test on August 19 shifted to August 22

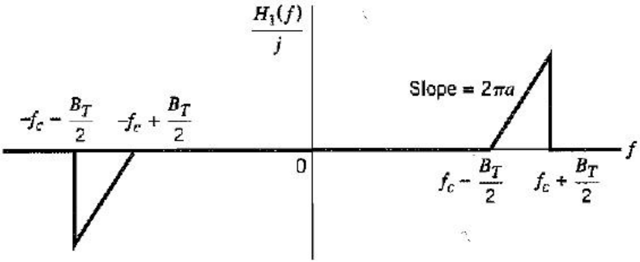
Review of last class

- Frequency modulation
  - FM bandwidth (B_T) and Carson's rule - $2(\Delta f + W)$
  - Frequency modulation systems
  - Effect of channel non-linearities on FM and comparison with AM
  - Frequency demodulation schemes

Today's plan

- Complex baseband representation of passband signals
- Complex baseband representation of passband systems
- Example analysis - FM demodulation
- Today's scribes are Dasara Shirisha and Deepak Kumar

Slope filter/circuit



- Consider the filter with response $H_1(f)$ defined as

$$H_1(f) = \begin{cases} j2\pi a(f - (f_c - \frac{B_T}{2})), & \text{for } f_c - \frac{B_T}{2} \leq f \leq f_c + \frac{B_T}{2}, \\ j2\pi a(f + (f_c - \frac{B_T}{2})), & \text{for } -f_c - \frac{B_T}{2} \leq f \leq -f_c + \frac{B_T}{2}, \\ 0, & \text{otherwise} \end{cases}$$

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- A component of the output is the desired derivative
- What is that component?

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Slope filter output analysis

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- What are the other components?
- Suppose we assume narrow band single tone modulation?
- Analysis of the other component requires some background on "Complex baseband representation of passband signals and systems"

Complex baseband representation of passband signals

- Can every signal at every point in a communication system be represented as a baseband signal?
- If a passband $g(t)$ is real, then $G(-f) = G^*(f)$; do we need both negative and positive frequencies?
- Suppose $g(t)$ (baseband or passband) is real, then the pre-envelope or analytic signal of $g(t)$ is

$$g_+(t) = g(t) + j\hat{g}(t)$$

- The important point here is that

$$G_+(f) = G(f) + sgn(f)G(f)$$

Pre-envelope of $g(t)$

- $G_+(f) = G(f) + sgn(f)G(f)$

$$G_+(f) = \begin{cases} 2G(f), f > 0 \\ G(0), f = 0 \\ 0, f < 0 \end{cases}$$

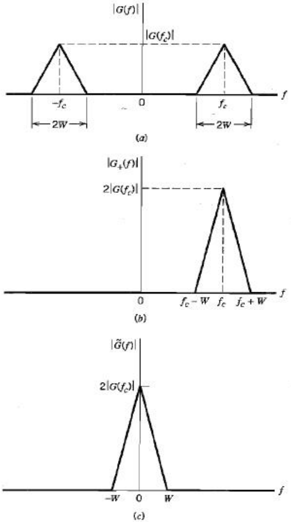
- Can also compute the pre-envelope $g_-(t)$ for negative frequencies

Representation of bandpass $g(t)$

- Assume that $g(t)$ occupies a bandwidth of $2W$ centered at f_c
- Suppose we find a complex signal $\tilde{g}(t)$ such that

$$g_+(t) = \tilde{g}(t)e^{j2\pi f_c t}$$

- Note that $\tilde{g}(t)$ is a low pass signal
- What is $\tilde{G}(f)$?



Representation of bandpass $g(t)$

- The signal $g(t) = Re\left[\tilde{g}(t)e^{j2\pi f_c t}\right]$
- Suppose $\tilde{g}(t) = g_I(t) + jg_q(t)$
- Then $g(t) = g_I(t)cos(2\pi f_c t) - g_Q(t)sin(2\pi f_c t)$
- Can then represent $g(t) = a(t)cos(2\pi f_c t + \phi(t))$. How?
- $a(t)$ is called the natural envelope or envelope of the signal
- $\phi(t)$ is called the phase

Complex baseband representation of passband systems

- Let $h(t)$ be the impulse response FT of a LTI bandpass system
- Let $H(f)$ be the FT of $h(t)$
- Assume that $H(f) = 0$ for $f \notin [f_c - B, f_c + B]$

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- Let $\tilde{h}(t) = h_I(t) + jh_Q(t)$ be the complex baseband impulse response
- Note that $h(t) = Re[\tilde{h}(t)e^{j2\pi f_c t}]$

Complex baseband representation of passband systems

- What is the relationship between $H(f)$ and $\tilde{H}(f)$?

Complex baseband representation of passband systems

- What is the relationship between $H(f)$ and $\tilde{H}(f)$?
- $2h(t) = \tilde{h}(t) exp(j2\pi f_c t) + \tilde{h}^*(t) exp(-j2\pi f_c t)$
- $2H(f) = \tilde{H}(f - f_c) + \tilde{H}^*(-f - f_c)$
- For $f > 0$, $\tilde{H}(f - f_c) = 2H(f)$

What are we interested in?

- Suppose $x(t)$ is a bandpass signal, with FT $X(f)$
- Let $X(f) = 0$, for $f \notin [f_c - W, f_c + W]$
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- $X(f) \rightarrow H(f)$

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- $y(t) = \int_{-\infty}^{\infty} h(\tau)x(t-\tau)d\tau$
- Is $y(t)$ bandpass?

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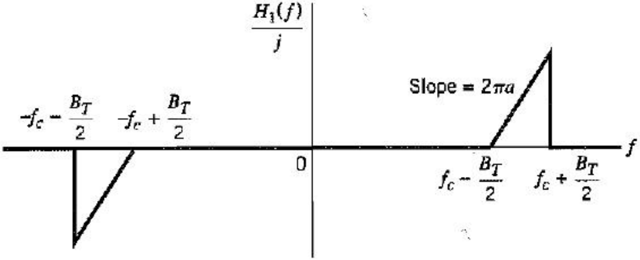
- Can we express $\tilde{y}(t)$ as a function of $\tilde{x}(t)$ and $\tilde{h}(t)$?
- We have that $\tilde{y}(t) = 2\int_{-\infty}^{\infty} \tilde{h}(\tau)\tilde{x}(t-\tau)d\tau$

Slope filter/circuit



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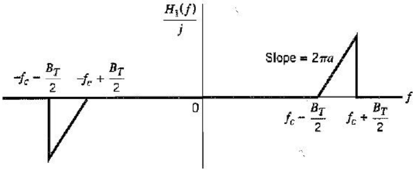

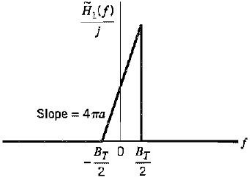

- $\tilde{H}_1(f - f_c) = 2H_1(f), f > 0$

$$\tilde{H}_1(f) = \begin{cases} j4\pi a(f + \frac{B_T}{2}), -\frac{B_T}{2} \leq f \leq \frac{B_T}{2}, \\ 0, \text{ otherwise.} \end{cases}$$

FM $s(t)$ through the slope filter

- $s(t) = A_c cos\left[2\pi f_c t + 2\pi k_f \int_0^t m(u).du\right]$
- $\tilde{s}(t) = A_c e^{\left[j2\pi k_f \int_0^t m(u).du\right]}$
- $\tilde{S}(f) \rightarrow \tilde{H}_1(f) \rightarrow \tilde{S}_1(f)$
- $\tilde{S}_1(f) = \frac{1}{2}\tilde{S}(f)H_1(f)$

$$\tilde{S}_1(f) = \begin{cases} j2\pi a(f + \frac{B_T}{2})\tilde{S}(f), -\frac{B_T}{2} \leq f \leq \frac{B_T}{2} \\ 0, \text{ otherwise.} \end{cases}$$

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- $\tilde{s}(t) = j\pi B_T a A_c \left[1 + \frac{2k_f}{B_T} m(t)\right] e^{j2\pi k_f \int_0^t m(u).du}$
- $s_1(t) = Re\left[\tilde{s}(t)e^{j2\pi f_c t}\right]$
- $s_1(t) = \pi B_T a A_c \left[1 + \frac{2k_f}{B_T} m(t)\right] cos\left[2\pi f_c t + 2\pi k_f \int_0^t m(u).du + \frac{\pi}{2}\right]$
- Use an envelope detector to obtain $\pi B_T a A_c \left[1 + \frac{2k_f}{B_T} m(t)\right]$ if $|\frac{2k_f}{B_T} m(t)| < 1$

Input output relationship

- Why is $\tilde{y}(t) = 2\int_{-\infty}^{\infty} \tilde{h}(\tau)\tilde{x}(t-\tau)d\tau$
- $y(t) = \int_{-\infty}^{\infty} Re\left[h_+(\tau)\right] Re\left[x_+(t-\tau)\right] d\tau$
- An important property of pre-envelopes

$$\int_{-\infty}^{\infty} Re\left[h_+(\tau)\right] Re\left[x_+(\tau)\right] dt = \frac{1}{2} Re\left[\int_{-\infty}^{\infty} h_+(\tau)x_+^*(\tau)d\tau\right]$$

- Then $y(t) = \frac{1}{2} Re\left[e^{j2\pi f_c t} \int_{-\infty}^{\infty} \tilde{h}(\tau)\tilde{x}(t-\tau)d\tau\right]$