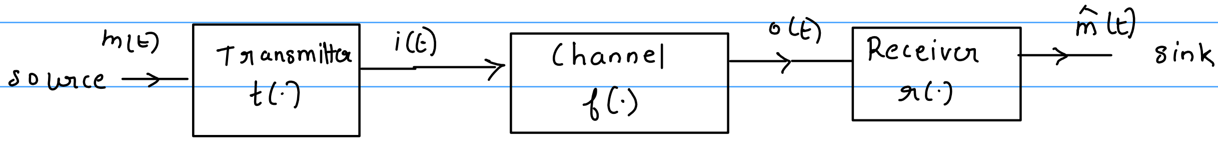
Lecture 4 30/07/2019

Review

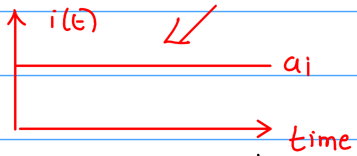


Design $t(\cdot)$ and $r(\cdot)$
such that $e(m(t), \hat{m}(t)) \leq \varepsilon$.
given $f(\cdot)$

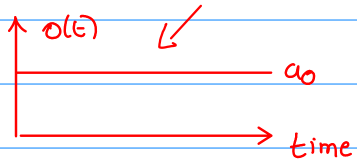
- Let us take an example to understand channel modelling.

Consider a channel with the following $\underline{I}$ and $\underline{O}$ (recall : $i(t) \in I$, $o(t) \in O$).
$I = O =$ set of all DC signals.

So a $i(t)$ would be

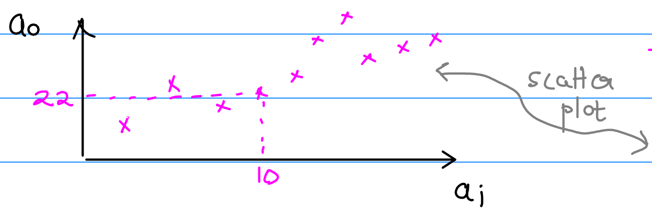


$o(t)$:



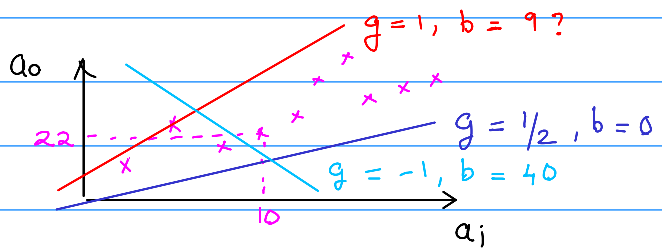
We have a channel that gives $o(t) = f(i(t))$.

Collect data about the channel – i.e., for an a_i what is a_o.

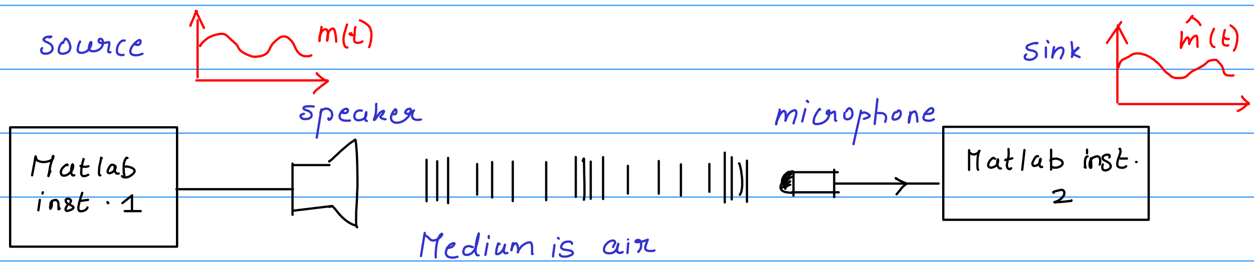


a_i	a_o
5	10.5
⋮	⋮
10	22
⋮	⋮
20	38

What is the relationship $f(\cdot)$ that $\underline{\text{best fits}}$ this data?



- Let us now consider a more complicated example



how to find $f(\cdot)$ for the above channel ? ← compare with earlier approach.

I and O are now not sets of scalars, but they are sets containing bounded functions.

$f(\cdot)$ processes signals rather than scalars.

recall that earlier we had linear/affine

recall that we first need some form or structure for $f(\cdot)$ and then we need to fit $f(\cdot)$ with the specified structure to the data that we collect.

- What are structures / forms for $f(\cdot)$?
- What data should we collect ?
- How do we fit $f(\cdot)$ to the data ?

Structures / Forms for $f(\cdot)$: (Signals and systems review) → review references for signals and systems.

- memoryless / with memory
- linear / non-linear
- causal / non-causal
- time invariant / variant

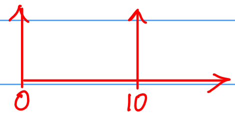
Because we are comfortable with LTI systems, let us assume that the form for $f(\cdot)$ is that it is an LTI system. If $f(\cdot)$ is an LTI system, how do we find $f(\cdot)$?

Recall the following points regarding LTI systems → mathematical models

- Superposition and homogeneity properties
- time invariance
- every LTI system is characterized by its impulse response. (denoted as $h(t)$).

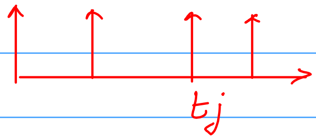
Every signal $i(t)$ can be expressed as a combination of impulse functions

for example



$$i(t) = \delta(t) + \delta(t-10)$$

or



$$i(t) = \sum_i \delta(t - t_i)$$

in general, we therefore have that $i(t) = \int i(\tau)\cdot\delta(t-\tau)\, d\tau$

$h(t)$ is the response to a single impulse

so the response to $i(t)$ or the output $o(t)$ is given by the convolution formula.

- If we know $h(t)$ then we can predict $o(t)$ for any input $i(t)$ as

$$o(t) = \int_{-\infty}^{\infty} i(\tau)\cdot h(t-\tau)\, d\tau$$

- Assume that $h(t)$ is finite energy, i.e., $\int_{-\infty}^{\infty} |h(t)|^2\, dt < \infty$

instead of specifying $h(t)$, we can instead specify $H(f)$ — the spectrum of $h(t)$.

- Assume that $h(t)$'s CTFT (spectrum) exists, i.e.,

$$H(f) = \int_{-\infty}^{\infty} h(t)\, e^{-j2\pi f t}\, dt$$

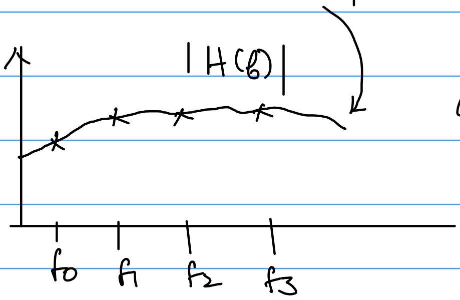
- Then finding $H(f)$ is equivalent to finding the channel model.
- Note that $H(f)$ is complex, and the magnitude and phase spectra need to be specified, in order that the LTI system is completely specified.
- So we need to find $H(f)$ when we consider LTI system models for channels.

How does one find $H(f)$?

- first of all note that $H(f)$ is complex and has a magnitude $|H(f)|$ and an angle $\angle H(f)$.
- using the property that a sinusoidal input to an LTI system produces an output which is a sinusoid with the same frequency, but amplitude and phase modified by $H(f)$ we can devise the following experiment

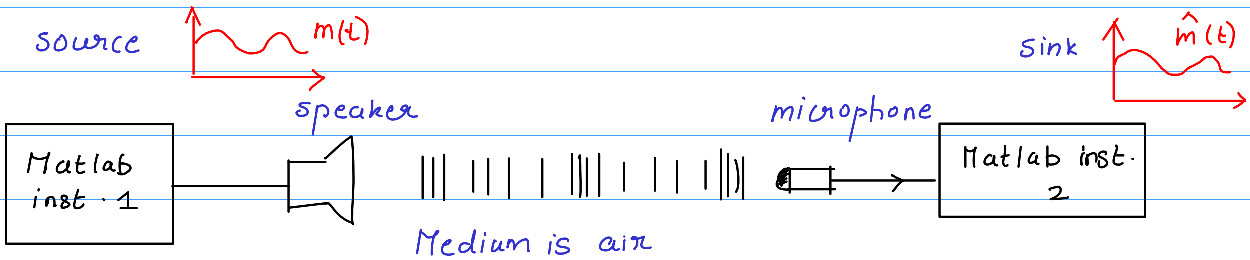
$$s(t) = 1 \cdot \sin(2\pi f_i t) \longrightarrow \boxed{h(t) \text{ or } H(f)} \longrightarrow$$

frequency	$\lvert H(f)\rvert$	$\angle H(f)$
f_1	1	0
f_2	0.5	$-\pi/4$
f_3	0.25	$-\pi/2$
⋮	⋮	⋮



use interpolation, curve fitting to get an approximation $H(f)$.

- Let us go back to the audio channel example.



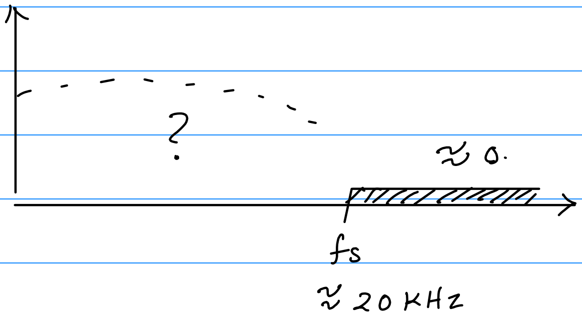
$m(t)$ is stored in an array → $\boxed{h(t) \text{ or } H(f)}$ → $\hat{m}(t)$ is stored in an array

everything in between is considered as the channel.

We have some prior information about $H(f)$ here.

Because we are dealing with audio, the channel would not process signals outside audio range (specially audio range for humans).

so $|H(f)|$

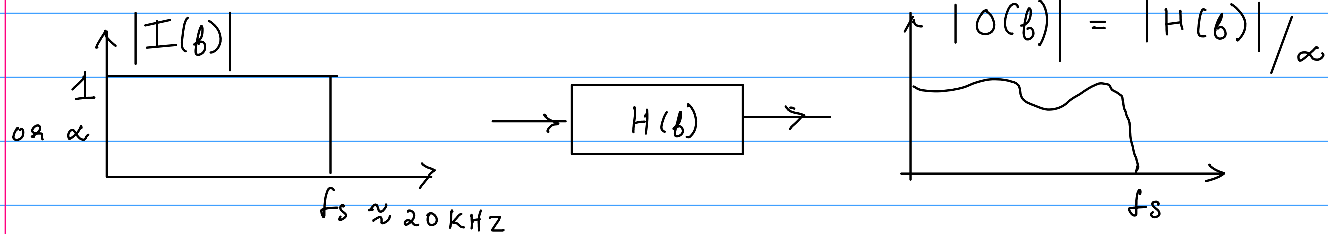


An important property that we have for LTI systems is that

suppose $i(t) \xrightarrow{\mathcal{F}} I(f)$

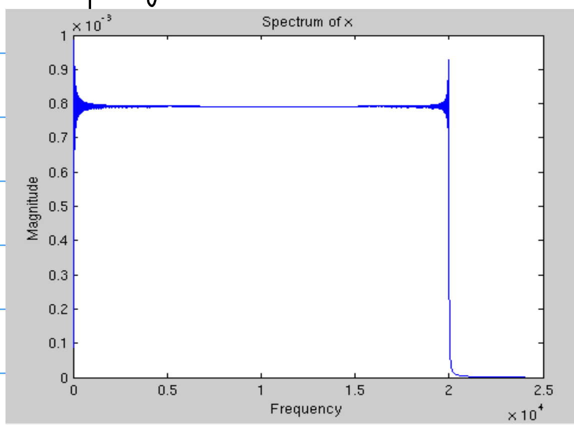
then $o(t) \xrightarrow{\mathcal{F}} O(f)$ and $O(f) = I(f) \cdot H(f)$.

so if we have a special form for $I(f)$, then from measuring $O(f)$ we can find $H(f)$. This is shown below.



Let us conduct an experiment to actually observe this.

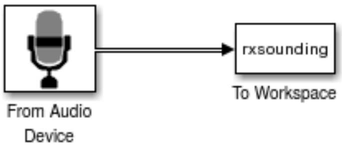
We use a chirp signal as $i(t)$. The chirp's freq is varied from 20 to 20 kHz. The spectrum of the chirp signal turns out to be.



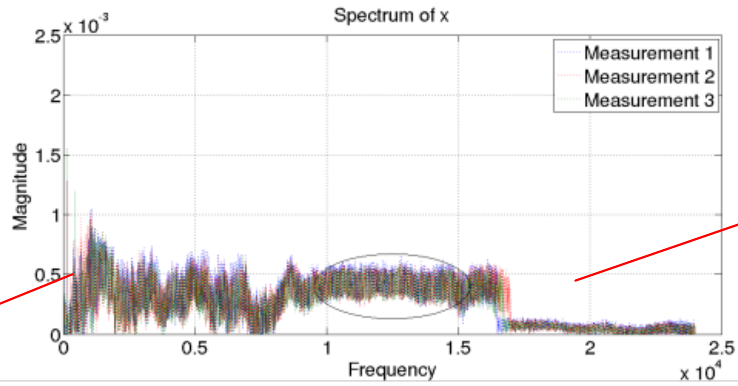
this is an approx. of $I(f)$ that we want.

We send this signal through the audio channel ⇒ we play the signal as an audio signal through the speakers.

The signal is received and the output spectrum is obtained.



Multiple such experiments are done to get multiple output spectra as shown below.



low freq interference could be there.

averaging the received signal for handling noise ?

a channel model which is LTI can be obtained