

## FunWork #2

Due on February 16

**1.** Many iterative optimization methods use a variable step size. The step size is determined by using a line search that involves locating the minimizer of a function of many variables in a specified direction  $\mathbf{d}$ . This involves the location of an interval in which the minimizer lies and then the interval is reduced. Once the uncertainty interval is determined, one can use the Golden Section search or the Fibonacci method to reduce the uncertainty interval.

We now describe a method that can be used to determine an interval containing the minimizer. We begin by evaluating the given function, say  $f$ , at an initial point  $\mathbf{x}^{(0)}$ . The next step is to evaluate the function at a second point which is a distance  $\varepsilon$  from  $\mathbf{x}^{(0)}$ , where  $\varepsilon$  is the chosen parameter, that is, we evaluate  $f$  at  $\mathbf{x}^{(0)} + \varepsilon\mathbf{d}$ . We then continue to evaluate  $f$  at new points, successively doubling the distance between the points. The process stops when the function increases between two consecutive evaluations. In the one-dimensional example in Figure 1, the function  $f$  increases between  $x_2$  and  $x_3$  and therefore the minimizer is bracketed in the interval  $[x_1, x_3]$ .

Consider now the function

$$f(\mathbf{x}) = \mathbf{x}^\top \begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix} \mathbf{x}$$

with the initial guess  $\mathbf{x}^{(0)} = \begin{bmatrix} 0.8 & -0.25 \end{bmatrix}^\top$ . Assume that the direction of travel is the negative gradient of  $f$ . Take  $\varepsilon = 0.075$ . Bracket the minimizer, that is, use the above described method to find an initial uncertainty region. For more information on bracketing consult Subsection 7.7 on page 123.

**2.** Apply the Golden Section search to the previous problem to reduce the uncertainty region width to 0.2. Organize the results of your computations in table format similar to that of Exercise **7.2** on page 126.

**3.** Repeat Problem **2** using

- (i) the Fibonacci search;
- (ii) the Newton method.

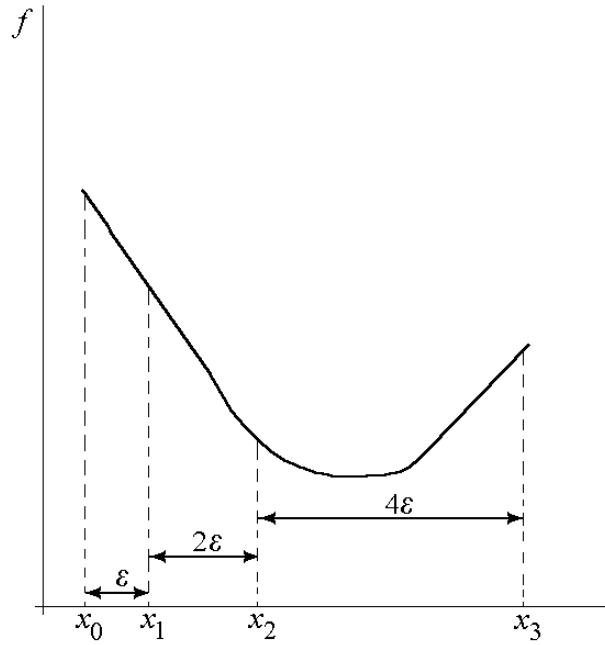


Figure 1: An illustration of the process of bracketing a minimizer.

4. For the MATLAB's "peaks" function given on page 290 in Example 14.3,
  - (a) use MATLAB's commands `meshgrid` and `mesh` to generate its 3D plot. The range of  $x_1$  and  $x_2$  is the same and it should be equal to  $[-3, 3]$ .
  - (b) use the command `contour` to generate 20 contours. Use the same range for  $x_1$  and  $x_2$  as in (a).
5. Minimize the above function using the method of the gradient descent when  $\alpha = 0.02$  and locate these points on the level sets of  $f$ . Connect the successive points with lines or lines with arrows to show clearly the progression of the optimization process. Use two starting points,

$$\mathbf{x}^{(0)} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad \text{and} \quad \mathbf{x}^{(0)} = \begin{bmatrix} -1 \\ 0 \end{bmatrix}.$$

Obtain the sequence of points using the steepest descent method and locate these points on the level sets of  $f$ .