ECE 580 Spring 2018

FunWork #1

Due on February 02

INSTRUCTIONS: The assignment must be typed. Clearly identify the steps you have taken to solve each problem. Your grade depends on the completeness and clarity of your work as well as the resulting answer.

Submit your assignment through the Blackboard. E-mail submissions will not be accepted.

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1. Compute the determinant of the matrix

$$\begin{bmatrix} 1 & 1 & \varepsilon \\ 1 & 1 & \varepsilon^2 \\ \varepsilon^2 & \varepsilon & 1 \end{bmatrix},$$

where

$$\varepsilon = \cos\frac{2\pi}{3} + j\sin\frac{2\pi}{3}.$$

2. Solve the system of linear equations,

$$\left.\begin{array}{rcl} 4bcx_1 + acx_2 - 2abx_3 & = & 0 \\ 5bcx_1 + 3acx_2 - 4abx_3 + abc & = & 0 \\ 3bcx_1 + 2acx_2 - abx_3 - 4abc & = & 0 \end{array}\right\}$$

where $abc \neq 0$.

3. Are the matrices

$$\boldsymbol{A} = \begin{bmatrix} 3 & 2 & -5 \\ 2 & 6 & -10 \\ 1 & 2 & -3 \end{bmatrix} \quad \text{and} \quad \boldsymbol{B} = \begin{bmatrix} 6 & 20 & -34 \\ 6 & 32 & -51 \\ 4 & 20 & -32 \end{bmatrix}$$

similar or not? If they are, then find a similarity transformation, if not, justify why not.

4. Use Theorem 2.2 on page 18 to determine a general solution of the system of linear equations,

$$\left.\begin{array}{rcr} x_1 + x_2 - 2x_3 + 3x_4 &=& 1 \\ x_1 + 2x_2 - x_3 + 2x_4 &=& 3 \\ x_1 - x_2 - 4x_3 + 5x_4 &=& -3 \end{array}\right\}$$

5. Determine a general solution for the system of linear equations,

$$\left.\begin{array}{rcl} 3x_1 + 2x_2 + x_3 &=& 0 \\ 5x_1 + 4x_2 + 3x_3 &=& 0 \\ 4x_1 + 3x_2 + 2x_3 &=& 0 \end{array}\right\}$$

6. Determine a general solution to the matrix equation,

$$\begin{bmatrix} 2 & 1 \\ 2 & 1 \end{bmatrix} \boldsymbol{X} = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}.$$

7.

(a) Find $Df(\boldsymbol{x})$ of

$$f(\boldsymbol{x}) = \boldsymbol{x}^\top \begin{bmatrix} 1 & 5 \\ 3 & 2 \end{bmatrix} \boldsymbol{x} - \boldsymbol{x}^\top \begin{bmatrix} -2 \\ 3 \end{bmatrix} + 5\pi$$

(b) Find the Hessian of

$$f(\boldsymbol{x}) = \frac{1}{2}\boldsymbol{x}^\top \begin{bmatrix} 2 & -3 \\ 5 & 1 \end{bmatrix} \boldsymbol{x} + \boldsymbol{x}^\top \begin{bmatrix} -1 \\ 2 \end{bmatrix} + \log 3$$

8. For the function

$$f = f(x_1, x_2) = 5e^{x_1^3 x_2} + \frac{1}{x_1 x_2^2},$$

(a) find the gradient of f at $\boldsymbol{x} = \begin{bmatrix} 1 & 1 \end{bmatrix}^\top$;

(b) find the rate of increase of f at the point $\boldsymbol{x} = \begin{bmatrix} 1 & 1 \end{bmatrix}^\top$ in the direction $\boldsymbol{d} = \begin{bmatrix} -3 & 4 \end{bmatrix}^\top$;

(c) find the direction of maximum rate of increase at $\boldsymbol{x} = \begin{bmatrix} 1 & 1 \end{bmatrix}^\top$. What is the rate of increase in this direction?

9. For the function

$$f = f(x_1, x_2) = \frac{1}{2}x_1^2 + \frac{1}{2}x_2^2 + 2x_1x_2 + \frac{1}{3}x_2^3 + x_2 + 5,$$

(a) find points that satisfy the first-order necessary conditions for the extremum;

(b) which point is a strict local minimizer? Justify your answer.

10. Represent the following quadratic forms,

(a) $f(x_1, x_2, x_3, x_4) = x_1^2 + x_3^2 + 2x_1x_3 + 4x_1x_4$;

(b) $f(x_1, x_2, x_3) = x_2^2 + x_1x_2 - x_1$;

(c) $f(x_1, x_2, x_3) = x_1^2 + x_2^2 + 6x_1x_2$,

as

$$f = \frac{1}{2}\boldsymbol{x}^\top \boldsymbol{Q}\boldsymbol{x},$$

where $\boldsymbol{Q} = \boldsymbol{Q}^\top$.