ECE 580 Spring 2011

# Midterm #2

March 30, (Wednesday)

1. Minimize

$$f = \frac{1}{2}(x_1^2 + 2x_2^2) - x_1 + x_2 + 7$$

using the **rank two correction (DFP)** method. The starting point is $\boldsymbol{x}^{(0)} = \begin{bmatrix} 0 & 0 \end{bmatrix}^\top$.

2. Find the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1,$$

that comes as close as possible to the three data points:

$$(x_1^2, y_1^2) = (2, 1), \quad (x_2^2, y_2^2) = (4, 2), \quad (x_3^2, y_3^2) = (6, 3).$$

3. Consider a series $RLC$ circuit consisting of a resistance $R$, an inductance $L$, and a capacitance $C$. Applying Kirchhoff's voltage law, we obtain the following differential equation modeling the circuit,

$$L\frac{di}{dt} + Ri + V_{out} = V_{in},$$

Our objective is to estimate $L$ and $R$ using available measurements of $i$, $\frac{di}{dt}$, the

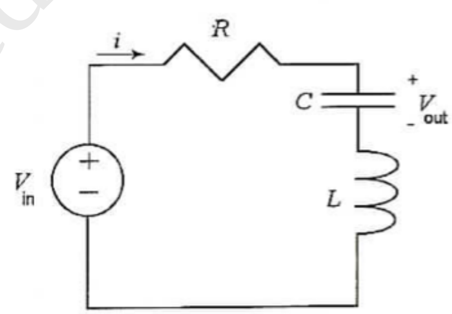


Figure 1: $RLC$ circuit.

output voltage $V_{out}$, and the input voltage $V_{in}$ at three different instances of the circuit operation. The results of measurement experiments are given in the table below. Obtain the least squares estimate of the circuit parameters $L$ and $R$.

Table 1: Measurement data of the RLC circuit.

| Experiment # | $\frac{di}{dt}$ | $i$ | $V_{out}$ | $V_{in}$ |
|---|---|---|---|---|
| 1 | 1 | 0 | -1 | 0 |
| 2 | 0 | 1 | 0 | 2 |
| 3 | 0 | -1 | 0 | 1 |

4.

$$\begin{array}{ll} \text{minimize} & \|\boldsymbol{x}\|_2 \\ \text{subject to} & \begin{bmatrix} a & b \\ a & b \\ a & b \end{bmatrix} \boldsymbol{x} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \end{array}$$

where $a$ and $b$ are non-zero real parameters.

5. Let

$$\boldsymbol{A}_0 = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}, \quad \boldsymbol{b}^{(0)} = \begin{bmatrix} 1 \\ 1 \end{bmatrix},$$

and

$$\boldsymbol{a}_1^\top = \begin{bmatrix} 0 & 1 \end{bmatrix}, \quad b^{(1)} = 1.$$

Use the **recursive least squares** to obtain the least squares solution of the combined system of equations.

6. For the system of linear equations, $\boldsymbol{Ax} = \boldsymbol{b}$, where

$$\boldsymbol{A} = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 0 \\ 0 & -1 & 0 \end{bmatrix}, \quad \boldsymbol{b} = \begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix}.$$

Find the minimum length vector $\boldsymbol{x}^*$ that minimizes $\|\boldsymbol{Ax} - \boldsymbol{b}\|_2^2$.

7. Use the Fundamental Theorem of Linear Programming to solve the problem,

$$
\begin{array}{ll}
\text{minimize} & 3x_1 + x_2 + x_3 \\
\text{subject to} & \overset{2}{x_1} + x_3 = 4 \\
 & \overset{2}{x_2} - \overset{1}{x_3} = 2 \\
 & x_1 \geq 0,\ x_2 \geq 0,\ x_3 \geq 0.
\end{array}
$$