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STAY FIT

COMMUTE SCHEDULE REPORT

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1. INTRODUCTION

Exercise is the ultimate way to increase our metabolic rate and burn fat while improving our health and well-being. Health and fitness have recently become a top priority for many people in the United States. With the increasing acknowledgment of the importance of leading a healthy lifestyle, more and more individuals are setting health and fitness goals for themselves. [1]

The graph below shows the average hours spent per week by different groups of individuals in the United States.

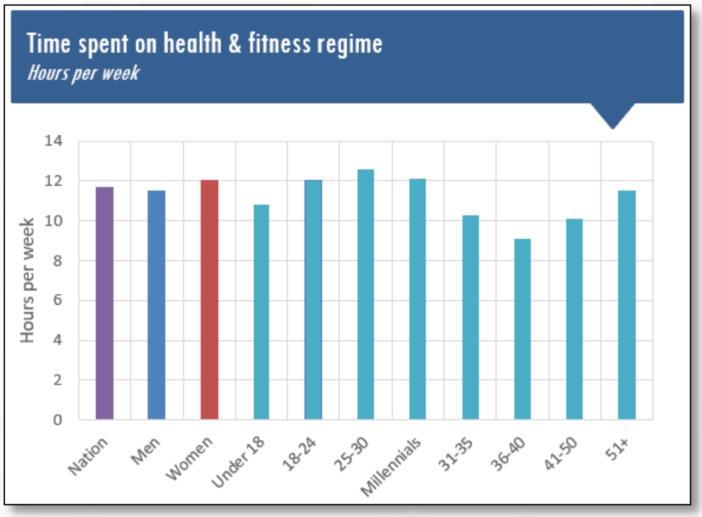

Figure 1: Time spent on health & fitness regime (hours per week) [2]

Results from a recent survey conducted by My Protein, which included

• 1,170 Americans between 18 and 65 show how much time men, women and other age groups spend staying fit and healthy.

• Spend more time exercising than socializing with peers (7.6 vs 4.5 hours per week)

• 42% of millennials will upload a fitness or gym photo to Instagram weekly!

The average American spends 11.7 hours a week, or 25 full days, maintaining their physical condition. [2]

With the increasing number of gyms and fitness facilities, optimization strategies must be used to balance the stressors and significant variables associated with maintaining these facilities. It takes a lot of time and effort for a gym owner to be successful. It requires knowing the many processes involved in running a gym and its purpose of increasing the customer experience while increasing the organization's profitability. StayFit Corporation is one of the organizations that help individuals achieve their health and fitness.

2. PROBLEM STATEMENT

As an ISEN consulting firm, our task is to help the CEO of StayFit Corporation create an effective supervising schedule for managing all of her fitness centers, including old and new sites. We aim to create a strategy that allows the CEO to visit all the centers efficiently and, at the same time, manage her time effectively by maintaining travel time as consistent as possible i.e., minimizing the disparity between longer and shorter trips on any given day is the CEO's top priority.

From the problem description and our understanding, we consider the following important aspects of optimizing the CEO’s supervising/commuting schedule.

- The minimum number of visits for the individual center.
- Different modes of transportation used by the CEO.
- Traveling time, inspection time and others involved in an inspection process.
- Work-life balance of the CEO.
- Minimum and maximum conditions for a distance.

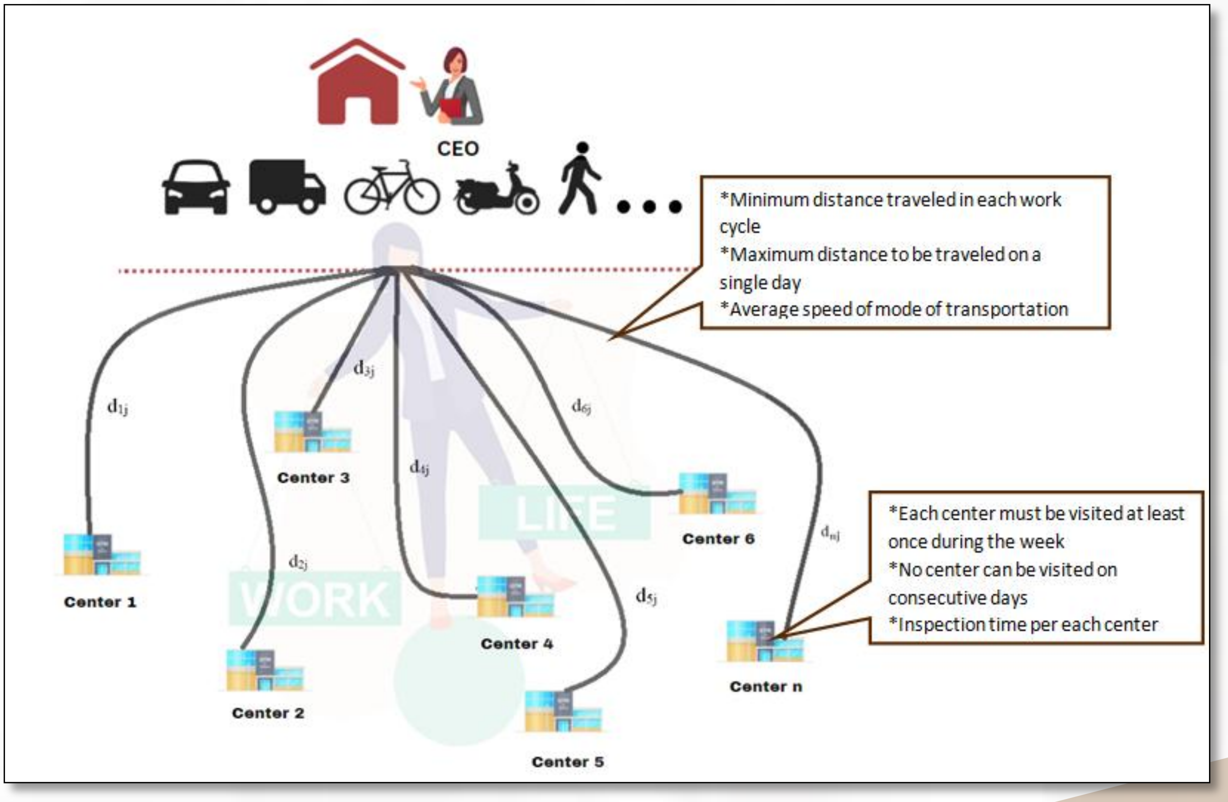

Figure 2: Key Considerations of the Problem Statement [2]

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3. PARAMETERS

1. **<u>Inspection Time parameter</u>:**

Let t_{ik} be the time taken by the CEO to inspect centers "i" on day "k", where i=1,2,…..,n and k=1,2,…..,7;

Here "i" represents the different centers, "k" represents the days in a week and "n" is the number of centers.

<u>For example</u>,

Let us consider,
i: 1= Center 1, 2 = Center 2 … n = Center n
k: 1= Monday, 2 = Tuesday … 7 = Sunday

If the CEO inspects center 1 (i=1) on Monday (k=1), then the parameter is represented as t_{11}.

The below table indicates the time taken by the CEO on days (k) to visit centers (i) where the time is represented as t_{ik} (where $_{ik}$ represents the i^{th} row and k^{th} column).

		day "k"						
		1	2	3	4	5	6	7
center "i"	1							
	2							
	3							
	.							
	n							

2. **<u>Distance parameters</u>:**

- **<u>Fixed Distance</u>:**

Let d_{ij} be the distance traveled by the CEO from her house to the center "i", using transportation mode "j", where i = 1,2,…..,n and j = 1,2,…..,m;

Here "i" represents the different centers, "j" represents the different modes of transportation, "n" is the number of centers and "m" is the number of modes of transportation.

For example,

Let us consider,
i: 1= Center 1, 2 = Center 2 … n = Center n
j: 1 = Car, 2 = Motorcycle, 3 = Walk … m

If the CEO travels from her house to Center 2 using transportation mode 1(car), then the parameter is represented as d_{21}.

- **Minimum Distance:**

Let $dmin_j$ be the minimum distance traveled by each mode of transportation “j” in each work cycle, where j =1,2,…..,m;

Here “j” represents the modes of transportation and “m” is the number of modes of transportation.

For example,

Let us consider,
j: 1 = Car, 2 = Motorcycle, 3 = Walk … m

The minimum distance parameter is represented as:

$dmin_1$ = minimum distance traveled by car

- **Maximum Distance:**

Let $dmax_{jk}$ be the maximum distance traveled by each mode of transportation on day “k”, where “j” =1,2,…..,m and k=1,2,…..,7;

Here “j” represents the different modes of transportation, “k” represents the day in a week and “m” is the number of modes of transportation.

For example,

Let us consider,
j: 1 = Car, 2 = Motorcycle, 3 = Walk … m
k: 1= Monday, 2 = Tuesday … 7 = Sunday

The maximum distance parameter is represented as:

$dmax_{22}$ = maximum distance traveled by motorcycle on Tuesday

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3. **Average Speed parameter**:

Let s_j be the average speed of different modes of transportation "j", where j=1,2,…..,m;

Here "j" represents the mode of transportation and "m" is the number of modes of transportation.

For example,

Let us consider,
j: 1 = Car, 2 = Motorcycle, 3 = Walk … m

The average speed parameter is represented below:

s_1 = average speed by car
s_2 = average speed by motorcycle

4. **Minimum number of visits for center parameter**:

Let min_i be the minimum number of times the CEO visited the centers during a week, where "i" = 1,2,…..,n;

Here "i" represents the different centers and "n" is the number of centers.

For example,

Let us consider,
i: 1= Center 1, 2 = Center 2 … n = Center n

The minimum number of visits for the center parameter is represented as:

min_1 = minimum number of visits for Center 1

5. **Penalty parameter**:

Let p_{jk} be the time penalty when the CEO travels beyond the maximum distance for each mode of transportation on a day in a week, where "j" =1,2,…..,m and k=1,2,…..,7;

Here "j" represents the modes of transportation, "k" represents the day in a week and "m" is the number of modes of transportation.

For example,

Let us consider,
j: 1 = Car, 2 = Motorcycle, 3 = Walk … m
k: 1= Monday, 2 = Tuesday … 7 = Sunday

If the CEO travels more than the maximum distance by car on Wednesday, then the penalty is accrued. So, the parameter is represented as p_{13}.

6. **Total number of modes of transportation:**

Let "Nom_i" be the total number of modes of transportation by which the CEO visits that particular center "i", where "i" = 1,2,…..,n;

Here "i" represents the different centers and "n" is the number of centers.

For example,

Let us consider,
i: 1= Center 1, 2 = Center 2 … n = Center n

The total number of modes of transportation parameter is represented as:

Nom_1 = total number of modes of transportation by which CEO visits center 1

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4. DECISION VARIABLES

1. <u>Travel time:</u>

Let T_{ijk} be the time taken by the CEO to travel from her house to center "i" using transportation mode "j" on a day "k", where i = 1,2,…..,n, j =1,2,…..,m and k=1,2,…..,7;

Here "i" represents the different centers, "j" represents the modes of transportation, "k" represents the day in a week, "n" is the number of centers and "m" is the number of modes of transportation.

2. <u>Binary variables:</u>

- X_{ijk} is the binary variable

  X_{ijk} is 1; if the CEO travels from her house to the center "i" using a transportation mode "j" on a day k

  0; Otherwise

  where i = 1,2,…..,n , j =1,2,…..,m and k=1,2,…..,7;

$$X_{ijk} \begin{cases} 1 & \text{If CEO travels from her house to center i using transportation mode j on day k} \\ 0 & \text{Otherwise} \end{cases}$$

- P_{jk} is the binary variable

  P_{jk} is 1; if CEO exceeds the maximum distance traveled using a different mode of transportation j in a single day k,

  0; Otherwise

  where j = 1,2,…..,m and k = 1,2,…..,7;

$$P_{jk} \begin{cases} 1 & \text{If CEO exceeds the max distance by modes of transportation j on single day k} \\ 0 & \text{Otherwise} \end{cases}$$

5. OBJECTIVE FUNCTION

Minimizing the commute/supervising schedule

$$\text{Minimize } z = \max\left(2 * T_{ijk} + \sum_{i=1}^{n}\sum_{j=1}^{m}(t_{ik} * X_{ijk})\right) - \min\left(2 * T_{ijk} + \sum_{i=1}^{n}\sum_{j=1}^{m}(t_{ik} * X_{ijk})\right)$$

for k = 1,2,…..,7;

The Objective function is to minimize the CEO's supervising hours (which includes traveling & inspection times) i.e., to reduce the difference between the longest and the shortest time scheduled for the CEO in a week.

We know that the total time (to and fro) traveled by the CEO using transportation mode "j" for the center "i" on day "k" is represented as $2*T_{ijk}$ and t_{ik} is the time taken by the CEO to inspect centers "i" on the day "k".

The travel time for a particular day is calculated as the sum of the round-trip travel time between the CEO's house and the specific center and the inspection time of that center if the CEO visits it.

We will get different values 7 days a week and our objective is to keep the difference minimum so that the CEO can have a consistent supervising schedule.

6. CONSTRAINTS

1. Center visits per week

Each center should be visited at least once a week. Some centers may require additional visits for various reasons i.e., new centers may need more attention and centers with large customer bases or that carry more products must be visited more often.

If the CEO visits the center "i" using a specific mode of transportation "j" on a particular day "k", then the value of the term X_{ijk} will be 1, indicating that the CEO visited the center "i" once.

By calculating the sum of all X_{ijk} values for each center "i" using all transportation modes "j" in 1,2,…..,m on all days from "k" in 1,2,…..,7; should be greater than or equal to 1 so that it satisfies the given condition i.e., each center is visited at least once in a week.

$$\sum_{j=1}^{m}\sum_{k=1}^{7}(X_{ijk}) \geq 1$$

for i= 1,2,..…,n;

2. Minimum number of visits

If the CEO visits the center "i" using a specific mode of transportation "j" on a particular day "k", then the value of the term X_{ijk} will be 1, indicating that the CEO visited the center "i" once.

By calculating the sum of all X_{ijk} values for each center "i" using all transportation modes "j" in 1,2,…..,m on all days from "k" in 1,2,…..,7; should be greater than or equal to min_i so that it satisfies the given condition i.e., each center must be visited for a minimum number of times in each week.

$$\sum_{j=1}^{m}\sum_{k=1}^{7}(X_{ijk}) \geq min_i$$

for i = 1,2,..…,n;

3. No center can be visited during consecutive days

a) From day 1 to 6

If the CEO visits center “i” on day 2, then she is not supposed to visit the same center on day 1 and day 3.

So, from the below constraint, we enforce that the CEO either visits the center “i” on day “k” or day “k+1”, which means either of X_{ijk} or X_{ijk+1} should be 1 but not both i.e the summation of X_{ijk} or X_{ijk+1} should be less than or equal to 1 for each center.

$$\sum_{j=1}^{m}(X_{ijk} + X_{ijk+1}) \leq 1$$

for k = 1,2,.....,6, i = 1,2,…..,n;

b) For day 7

The above constraint is valid only from days 1 to 6, but for day 7 the “k+1” value becomes 8, which violates the number of days a week.

So, we impose the below constraint for day 1 & day 7, which means either of X_{ij7} or X_{ij1} should be 1 but not both.

$$\sum_{j=1}^{m}(X_{ij7} + X_{ij1}) \leq 1$$

for i = 1,2,…..,n;

4. Average speed of different modes of transportation

The CEO uses different modes of transportation to reach centers and each of the transportation modes has its average speed.

We know the average speed of each mode of transportation and the distance between the centers and the CEO’s home.

So, we can calculate the average time taken to travel from home to the center for a specific mode of transportation using the standard time, distance and speed formula.

The value in T_{ijk} is populated only when the CEO visits center "i", which is ensured by X_{ijk} .

$$T_{ijk} = \frac{d_{ij}}{s_j} (X_{ijk})$$

for i = 1,2,…..,n; j = 1,2,…..,m; k = 1,2,….,7;

5. Limitation on transportation mode for each center

Some centers can be visited by a specific mode of transportation due to certain limitations. The data will provide information on the modes of transportation available to reach the center “i”.

Therefore, we can assume that “Nom_i” represents the total number of modes of transportation that can be used to visit that particular center “i”.

By adding the total of all “X_{ijk}” for each center “i” using transportation modes “j” in 1,2,…..,m on days from “k” in 1,2,…..,7 should be greater than or equal to the different modes of transportation which can be used to visit that particular center. i.e, “Nom_i" for “i” in 1,2,…..,n.

$$\sum_{k=1}^{7} \sum_{j=1}^{m} (X_{ijk}) \geq Nom_i$$

for i= 1,2,…..,n;

6. Maximum distance per day

Each mode of transportation has a maximum distance to be traveled in a single day.

We know that for each mode of transportation “j”, there is a maximum total distance “$dmax_{jk}$” to be traveled in a single day and the distance from the CEO’s house to the center “i”, using transportation mode “j” is “d_{ij}”.

The total distance (to and fro) traveled by the CEO using transportation mode “j” for the center “i” can be calculated as $2*d_{ij}$

If the total distance "2*d_{ij}" traveled using each mode of transportation "j" for the center "i" in each day "k" has to be less than or equal to the maximum distance "$dmax_{jk}$" traveled by each mode of transportation "j" on a single day "k".

$$\sum_{i=1}^{n}(2 * d_{ij} * X_{ijk}) \leq dmax_{jk}$$

for j in 1,2,…..,m; k in 1,2,….,7;

7. Time penalty if the transportation mode exceeds the maximum distance

Each mode of transportation has a maximum distance to be traveled in a single day and the transportation mode is used beyond its maximum distance, then a time penalty is incurred for that day.

We know that for each mode of transportation "j", there is a maximum total distance "$dmax_{jk}$" to be in a single day and the distance from the CEO's house to the center "i", using transportation mode "j" is "d_{ij}".

The total distance (to and fro) traveled by the CEO using transportation mode "j" for the center "i" can be calculated as 2*d_{ij}

If the total distance "2*d_{ij}" traveled using each mode of transportation "j" for the center "i" in each day "k" is greater than the maximum distance traveled by each mode of transportation on day "k" then the penalty "P_{jk}" is 1, else 0 [7]

If $\sum_{i=1}^{n}(2 * d_{ij} * X_{ijk}) > dmax_{jk}$, then $P_{jk} = 1$ is enforced; else $P_{jk} = 0$ is enforced.

$$dmax_{k} > \sum_{i=1}^{n}(2 * d_{ij} * X_{ijk}) - M_{1} * (P_{jk})$$

$$\sum_{i=1}^{n}(2 * d_{ij} * X_{ijk}) \leq dmax_{k} + M_{2} * (1 - P_{jk})$$

for j in 1,2,.....,m; k in 1,2,….,7

The above constraints provide the check for 0 and 1 values for P_{jk} and assign the required value to binary variable P_{jk}. If the CEO's total distance exceeds the maximum distance, then only the time penalty is incurred, equivalently P_{jk} takes value as 1. And to find the value of Big M, i.e, M_1 and M_2 we need to know the smallest and largest values it can take. Value of M_1 depends on the lower bound of $dmax_{jk}$ and it can occurs when the CEO uses the transportation mode with lowest average speed and value of M_2 depends on the higher bound of d_{ij} and it occurs when the CEO uses the transportation mode with the highest average speed. [7]

8. Working and Commuting time per day must be limited to 8 hours

a) Time taken to travel from the CEO's home to center "i"
b) Time taken to inspect the center "i"
c) Time taken to travel from center "i" to home (only one center inspection at a time)
d) The CEO must spend 30 minutes at home between two consecutive center inspections.
e) If the transportation mode is used beyond its maximum total distance traveled, then a time penalty is incurred for that day.

We know that the time taken by the CEO to travel from her house to center "i" using transportation mode "j" on a day "k" is T_{ijk}. The total time taken to travel from her house to center "i" and center "i" to her house is "2* T_{ijk}".

The time taken by the CEO to inspect centers "i" on day "k" is "t_{ik}"

The CEO must spend 30 minutes between two consecutive center inspections. Because of work-life balance, the number of times she spend 30 minutes can be represented as the total number of centers visited by different modes of transportation on the day "k" minus one.

The time elapsed between when she leaves the house for her first visit and when she arrives home after her last trip must be at least 8 hours (*8*60 = 480 min*).

But if the transportation mode is used beyond its maximum distance traveled on that day, then the penalty is incurred. So, CEO's available time for inspections on the day is shortened.

$$\sum_{i=1}^{n}\sum_{j=1}^{m}(2 * T_{ijk}) + \sum_{i=1}^{n}\sum_{j=1}^{m}(t_{ik} * X_{ijk}) + (\sum_{i=1}^{n}\sum_{j=1}^{m}X_{ijk} - 1) * 30 + \sum_{j=1}^{m}(P_{jk} * p_{jk}) \leq 480$$

for k = 1,2,…..,7;

9. Minimum distance in each work cycle

Each mode of transportation has a minimum distance to be traveled in each work cycle.

We know that for each mode of transportation “j”, there is a minimum total distance “$dmin_j$” to be traveled for each work cycle and the distance from the CEO’s house to the center “i”, using transportation mode “j” is “d_{ij}”.

The total distance (to and fro) traveled by the CEO using transportation mode “j” for the center “i” can be calculated as “$2*d_{ij}$”

The total distance “$2*d_{ij}$” traveled using each mode of transportation “j” for the center “i” in each work cycle has to be less than or equal to the minimum distance “$dmin_j$” traveled by each mode of transportation “j”.

$$\sum_{i=1}^{n}\sum_{k=1}^{7}(2 * d_{ij} * X_{ijk}) \geq dmin_j$$

for j in 1,2,…..,m;

10. Non-Negativity

- $t_{ik} \geq 0$
  for i = 1,2,…..,n; k = 1,2,….,7;

- $d_{ij} \geq 0$
  for i = 1,2,…..,n; j = 1,2,…..,m;

- $dmin_j \geq 0$
  for j = 1,2,…..,m;

- $dmax_{jk} \geq 0$
  for j = 1,2,…..,m; k = 1,2,….,7;

- $T_{ijk} \geq 0$
  for i = 1,2,…..,n;   j = 1,2,…..,m; k = 1,2,….,7;

- $s_j \geq 0$
  for $j = 1,2,\ldots,m$;

- $min_i \geq 0$
  for $i = 1,2,\ldots,n$;

- $p_{jk} \geq 0$
  for $j = 1,2,\ldots,m$; $k = 1,2,\ldots,7$;

- “$Nom_i \geq 0$

  for $i = 1,2,\ldots,n$;

11.Binary variables

- $X_{ijk} \begin{cases} 1 & \text{If CEO travels from her house to center i using transportation mode j on day k} \\ 0 & \text{Otherwise} \end{cases}$

  for $i = 1,2,\ldots,n$; $j = 1,2,\ldots,m$; $k = 1,2,\ldots,7$;

- $P_{jk} \begin{cases} 1 & \text{If CEO exceeds the max distance by modes of transportation j on single day k} \\ 0 & \text{Otherwise} \end{cases}$

for $j = 1,2,\ldots,m$; $k = 1,2,\ldots,7$;

7. SUMMARY

The optimization solution for Stay Fit CEO's travel schedule time is provided by this model, specifically minimizing the differences in total commute time and maintaining it as consistent as possible for her work cycle. As an ISEN consultant, we have effectively optimized the CEO's travel time using an integer linear optimization model.

We assumed that the CEO cannot visit the same center more than once in a single day as it sounds more reasonable if the CEO visits a center and gives time to the managers to implement changes from CEO's previous inspection.

The model provides accurate and up-to-date information that closely reflects real-world data for fitness centers. It addresses various constraints that a CEO may encounter, considering parameters such as the maximum distance that can be traveled in a day and the minimum distance that must be traveled in a week using modes of transportation such as biking or walking. It promotes adopting healthy lifestyle habits. Furthermore, the model imposes penalty parameters if the CEO travels beyond the maximum distances using the modes of transportation, also considering the average speed of the modes. The model aims to help the CEO overcome constraints and promote good health habits.

The CEO can use this model to effectively manage her time to visit all health and fitness centers, including those that may open in the future. The proposed result can be achieved efficiently by utilizing different modes of transportation available.

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