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7.1

a_1 $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \end{bmatrix}$

$\vec{v_1} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$ $\vec{v_2} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$

$\vec{u_1} = \vec{v_1} \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$, $\vec{e_1} = \dfrac{\vec{u_1}}{|\vec{u_1}|} = \begin{bmatrix} \sqrt{2}/2 \\ 0 \\ \frac{\sqrt{2}}{2} \end{bmatrix}$

$\vec{u_2} = \vec{v_2} - \dfrac{\vec{v_2} \cdot \vec{u_1}}{|u_1|^2} \cdot \vec{u_1}$

$= \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} - \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$, $\vec{e_2} = \dfrac{\vec{u_2}}{|\vec{u_2}|} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$

$\hat{Q} = [\vec{e_1} , \vec{e_2}] = \begin{bmatrix} \sqrt{2}/2 & 0 \\ 0 & 1 \\ \frac{\sqrt{2}}{2} & 0 \end{bmatrix}$

$\longrightarrow Q^* = Q^{-1}$

$\hat{R} = \hat{Q}^*_o A = \begin{bmatrix} \sqrt{2}/2 & 0 \\ 0 & 1 \\ \frac{\sqrt{2}}{2} & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \end{bmatrix} =$

$= \begin{bmatrix} \frac{\sqrt{2}}{2} & 0 & \frac{\sqrt{2}}{2} \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} \sqrt{2} & 0 \\ 0 & 1 \end{bmatrix}$

$A = \hat{Q}\hat{R} = \begin{bmatrix} \frac{\sqrt{2}}{2} & 0 \\ 0 & 1 \\ \frac{\sqrt{2}}{2} & 0 \end{bmatrix} \begin{bmatrix} \sqrt{2} & 0 \\ 0 & 1 \end{bmatrix}$

$*$ tính QR đầy đủ

đưa $q_3 = \begin{bmatrix} q_{13} \\ q_{12} \\ q_{33} \end{bmatrix}$

$$\begin{cases} \langle q_1, q_3 \rangle = \dfrac{1}{\sqrt{2}} q_{13} + 0\, q_{23} + \dfrac{1}{\sqrt{2}} q_{33} = 0 \\[2mm] \langle q_2, q_3 \rangle = 0\, q_{13} + q_{23} + 0\, q_{33} = 0 \end{cases}$$

$(\equiv)$ $\begin{cases} q_{13} = -b \\ q_{23} = a \\ q_{33} = b \end{cases}$, $a, b \in \mathbb{R}$

chọn $a = 0$, $b = 1$

$v_3 = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$ $\Rightarrow$ $\vec{e_3} = \dfrac{\vec{v_3}}{|\vec{v_3}|} = \begin{bmatrix} -\dfrac{1}{\sqrt{2}} \\ 0 \\ \dfrac{1}{\sqrt{2}} \end{bmatrix} = \begin{bmatrix} -\dfrac{\sqrt{2}}{2} \\ 0 \\ \dfrac{\sqrt{2}}{2} \end{bmatrix}$

$A = QR = \begin{bmatrix} \dfrac{\sqrt{2}}{2} & 0 & -\dfrac{\sqrt{2}}{2} \\ 0 & 1 & 0 \\ \dfrac{\sqrt{2}}{2} & 0 & \dfrac{\sqrt{2}}{2} \end{bmatrix} \begin{bmatrix} \sqrt{2} & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$

b, $B = \begin{bmatrix} 1 & 2 \\ 0 & 1 \\ 1 & 0 \end{bmatrix}$

$\vec{v_1} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$, $\vec{v_2} \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}$

$\vec{u_1} = \vec{v_1} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$, $\vec{e_1} = \frac{\vec{u_1}}{|\vec{u_1}|} = \begin{bmatrix} \frac{\sqrt{2}}{2} \\ 0 \\ \frac{\sqrt{2}}{2} \end{bmatrix} = \begin{bmatrix} \frac{1}{\sqrt{2}} \\ 0 \\ \frac{1}{\sqrt{2}} \end{bmatrix}$

$\vec{u_2} = \vec{v_2} - \frac{\vec{u_1} \cdot \vec{v_2}}{|\vec{u_1}|^2} \cdot \vec{u_1} = \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix} - \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$

$\vec{e_2} = \frac{\vec{u_2}}{|\vec{u_2}|} = \begin{bmatrix} \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \\ -\frac{1}{\sqrt{3}} \end{bmatrix}$

$\hat{Q} = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{3}} \\ 0 & \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{3}} \end{bmatrix} \Rightarrow \hat{Q}^* = \begin{bmatrix} \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{-1}{\sqrt{3}} \end{bmatrix}$

$R = \hat{Q}^* B = \begin{bmatrix} \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{-1}{\sqrt{3}} \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 0 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} \sqrt{2} & \sqrt{2} \\ 0 & \sqrt{3} \end{bmatrix}$

* tìm fac QR

đặt $q_3 = \begin{bmatrix} q_{13} \\ q_{23} \\ q_{33} \end{bmatrix}$

$$\begin{cases} \langle q_1, q_3 \rangle = \dfrac{1}{\sqrt{2}} q_{13} + 0 \cdot q_{23} + \dfrac{1}{\sqrt{2}} q_{33} = 0 \\[4mm] \langle q_2, q_3 \rangle = \dfrac{1}{\sqrt{3}} q_{13} + \dfrac{1}{\sqrt{3}} q_{23} - \dfrac{1}{\sqrt{3}} q_{33} = 0 \end{cases}$$

$$\Leftrightarrow \begin{cases} q_{13} + q_{33} = 0 \\ q_{13} + q_{23} + q_{33} = 0 \end{cases}$$

$$\Leftrightarrow \begin{cases} q_1 = t \\ q_2 = -2t \qquad t \in \mathbb{R} \\ q_{33} = -t \end{cases}$$

chọn $t = 1$

$$V_3 = \begin{bmatrix} 1 \\ -2 \\ -1 \end{bmatrix} \Rightarrow \vec{e_3} = \dfrac{\vec{V_3}}{|\vec{V_3}|} = \begin{bmatrix} \frac{1}{\sqrt{6}} \\ -\frac{2}{\sqrt{6}} \\ -\frac{1}{\sqrt{6}} \end{bmatrix}$$

$$B = QR = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{6}} \\ 0 & \frac{1}{\sqrt{3}} & \frac{-2}{\sqrt{6}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{3}} & \frac{-1}{\sqrt{6}} \end{bmatrix} \begin{bmatrix} \sqrt{2} & \sqrt{2} \\ 0 & \sqrt{3} \\ 0 & 0 \end{bmatrix}$$

7.2/
- ma trận $\hat{R}$ sẽ có $r_{ij} = 0$ khi $i > j$ hoặc $j < i$ nếu chúng không thời là chẵn hoặc lẻ
- nếu $i = j$: $r_{ij} \neq 0$
- ~~nếu $i < j$ hoặc $i > j$ hữu~~