



## Day 4: Binomial Distribution I ☆

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Terms you'll find helpful in completing today's challenge are outlined below.

### Random Variable

A random variable,  $X$ , is the real-valued function  $X: S \rightarrow \mathbf{R}$  in which there is an event for each interval  $I$  where  $I \subseteq \mathbf{R}$ . You can think of it as the set of probabilities for the possible outcomes of a sample space. For example, if you consider the possible sums for the values rolled by 2 four-sided dice:

- $X = \{2, 3, 4, 5, 6, 7, 8\}$
- $P(X = 2) = P(\{(1, 1)\}) = \frac{1}{16}$
- $P(X = 3) = P(\{(1, 2), (2, 1)\}) = \frac{2}{16}$
- $P(X = 4) = P(\{(1, 3), (2, 2), (3, 1)\}) = \frac{3}{16}$
- $P(X = 5) = P(\{(1, 4), (2, 3), (3, 2), (4, 1)\}) = \frac{4}{16}$
- $P(X = 6) = P(\{(2, 4), (3, 3), (4, 2)\}) = \frac{3}{16}$
- $P(X = 7) = P(\{(3, 4), (4, 3)\}) = \frac{2}{16}$
- $P(X = 8) = P(\{(4, 4)\}) = \frac{1}{16}$

**Note:** When we roll two dice, the value rolled by each die is independent of the other.

### Binomial Experiment

A binomial experiment (or Bernoulli trial) is a statistical experiment that has the following properties:

- The experiment consists of  $n$  repeated trials.
- The trials are independent.
- The outcome of each trial is either success ( $s$ ) or failure ( $f$ ).

## Bernoulli Random Variable and Distribution

The sample space of a binomial experiment only contains two points,  $s$  and  $f$ . We define a Bernoulli random variable to be the random variable defined by  $X(s) = 1$  and  $X(f) = 0$ . If we consider the probability of success to be  $p$  and the probability of failure to be  $q$  (where  $q = 1 - p$ ), then the [probability mass function](#) (PMF) of  $X$  is:

$$p(x) = \begin{cases} 1 - p \equiv q & \text{if } x = 0 \\ p & \text{if } x = 1 \\ 0 & \text{otherwise.} \end{cases}$$

We can also express this as:

$$f(x) = p^x (1 - p)^{1-x}, \text{ for } x \in \{0, 1\}$$

## Binomial Distribution

We define a binomial process to be a binomial experiment meeting the following conditions:

- The number of successes is  $x$ .
- The total number of trials is  $n$ .
- The probability of success of 1 trial is  $p$ .
- The probability of failure of 1 trial  $q$ , where  $q = 1 - p$ .



- $b(x, n, p)$  is the binomial probability, meaning the probability of having exactly  $x$  successes out of  $n$  trials.

The binomial random variable is the number of successes,  $x$ , out of  $n$  trials.

The binomial distribution is the probability distribution for the binomial random variable, given by the following probability mass function:

$$b(x, n, p) = \binom{n}{x} \cdot p^x \cdot q^{(n-x)}$$

**Note:** Recall that  $\binom{n}{x} = \frac{n!}{x!(n-x)!}$ . For further review, see the [Combinations and Permutations Tutorial](#).

## Cumulative Probability

We consider the distribution function for some real-valued random variable,  $X$ , to be  $F_X(x) = P(X \leq x)$ . Because this is a non-decreasing function that accumulates all the probabilities for the values of  $X$  up to (and including)  $x$ , we call it the cumulative distribution function (CDF) of  $X$ . As the CDF expresses a cumulative range of values, we can use the following formula to find the cumulative probabilities for all  $x \in [a, b]$ :

$$P(a < X \leq b) = F_X(b) - F_X(a)$$

## Example

A fair coin is tossed **10** times. Find the following probabilities:

- Getting **5** heads.
- Getting at least **5** heads.
- Getting at most **5** heads.

For this experiment,  $n = 10$ ,  $p = 0.5$ , and  $q = 0.5$ . The respective probabilities for the above three events are as follows:

- The probability of getting **5** heads is:

$$b(x = 5, n, p) = 0.24609375$$

- The probability of getting at least **5** heads is:

$$b(x \geq 5, n, p) = \sum_{r=5}^{10} b(x = r, n, p) = 0.623046875$$

- The probability of getting at most **5** heads is:

$$b(x \leq 5, n, p) = \sum_{r=0}^5 b(x = r, n, p) = 0.623046875$$

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