Day 5: Normal Distribution I

Normal Distribution

The probability density of normal distribution is:

$$\mathcal{N}(\mu, \sigma^2) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

Here,

- μ is the mean (or expectation) of the distribution. It is also equal to median and mode of the distribution.
- σ^2 is the variance.
- σ is the standard deviation.

Standard Normal Distribution

If $\mu = 0$ and $\sigma = 1$, then the normal distribution is known as standard normal distribution:

$$\phi(x) = \frac{e^{-\frac{x^2}{2}}}{\sqrt{2\pi}}$$

Every normal distribution can be represented as standard normal distribution:

$$\mathcal{N}(\mu, \sigma^2) = \frac{1}{\sigma}\phi(\frac{x-\mu}{\sigma})$$

Cumulative Probability

Consider a real-valued random variable, X. The cumulative distribution function of X (or just the distribution function of X) evaluated at x is the probability that X will take a value less than or equal to x:

$$F_X(x) = P(X \leq x)$$

Also,

$$P(a \leq X \leq b) = P(a < X < b) = F_X(b) - F_X(a)$$

The cumulative distribution function for a function with normal distribution is:

$$\Phi(x) = \frac{1}{2}\left(1 + \mathbf{erf}\left(\frac{x-\mu}{\sigma\sqrt{2}}\right)\right)$$

Where **erf** is the error function:

$$\mathbf{erf}(z) = \frac{2}{\sqrt{\pi}}\int_0^z e^{-x^2}\,dx$$