1 Answer

1.1 Solution 1

The matrix can be decomposed into the following combination:

$$\begin{bmatrix} 1.0 & 2.0 & 8.0 & 34.0 & 1.0 & 11.0 \\ 2.0 & 5.0 & 19.0 & 81.0 & 4.0 & 35.0 \\ 3.0 & 4.0 & 18.0 & 80.0 & 5.0 & 37.0 \end{bmatrix} = \begin{bmatrix} 1.0 \\ 2.0 \\ 3.0 \end{bmatrix} \cdot \begin{bmatrix} 1.0 & 2.0 & 8.0 & 34.0 & 1.0 & 11.0 \end{bmatrix} + \begin{bmatrix} 0.0 \\ 1.0 \\ -2.0 \end{bmatrix} \cdot \begin{bmatrix} 0.0 & 1.0 & 3.0 & 13.0 & 2.0 & 13.0 \end{bmatrix} + \begin{bmatrix} 0.0 \\ 0.0 \\ 4.0 \end{bmatrix} \cdot \begin{bmatrix} 0.0 & 0.0 & 0.0 & 1.0 & 1.5 & 7.5 \end{bmatrix}$$

This means that:

$$A = C_1 R_1 + C_2 R_2 + C_4 R_3$$

Look at the detailed solution to see where R_i and C_j come from.

1.2 Solution 2

The matrix can also be decomposed into two matrices $A = LU$

$$\begin{bmatrix} 1.0 & 2.0 & 8.0 & 34.0 & 1.0 & 11.0 \\ 2.0 & 5.0 & 19.0 & 81.0 & 4.0 & 35.0 \\ 3.0 & 4.0 & 18.0 & 80.0 & 5.0 & 37.0 \end{bmatrix} = \begin{bmatrix} 1.0 & 0.0 & 0.0 \\ 2.0 & 1.0 & 0.0 \\ 3.0 & -2.0 & 4.0 \end{bmatrix} \cdot \begin{bmatrix} 1.0 & 2.0 & 8.0 & 34.0 & 1.0 & 11.0 \\ 0.0 & 1.0 & 3.0 & 13.0 & 2.0 & 13.0 \\ 0.0 & 0.0 & 0.0 & 1.0 & 1.5 & 7.5 \end{bmatrix}$$

2 Detailed Solution

We will now go into the details of $A = \sum C_i R_j$

2.1 Step 0

We prepare ourselves mentally.

2.2 Step 1

We have the matrix

$$A_0 = \begin{bmatrix} 1.0 & 2.0 & 8.0 & 34.0 & 1.0 & 11.0 \\ 2.0 & 5.0 & 19.0 & 81.0 & 4.0 & 35.0 \\ 3.0 & 4.0 & 18.0 & 80.0 & 5.0 & 37.0 \end{bmatrix}$$

We can see that the row R_1 and the column C_1 are both **non-zero vectors**, meaning their multiplication will be a non-zero matrix. Therefore, they are **linearly independent**, and thus we can pick them out.
Also, we have to divide either one of them by the element $e_{11} = 1.0$ to ensure it is not duplicated in the matrix multiplication.
And then, we have

$$\begin{aligned} C_1 \cdot R_1 &= \begin{bmatrix} 1.0 \\ 2.0 \\ 3.0 \end{bmatrix} \cdot \begin{bmatrix} 1.0 & 2.0 & 8.0 & 34.0 & 1.0 & 11.0 \end{bmatrix} \\ &= \begin{bmatrix} 1.0 & 2.0 & 8.0 & 34.0 & 1.0 & 11.0 \\ 2.0 & 4.0 & 16.0 & 68.0 & 2.0 & 22.0 \\ 3.0 & 6.0 & 24.0 & 102.0 & 3.0 & 33.0 \end{bmatrix} \end{aligned}$$

, which when substracted from A_0 will result in a new matrix A_1 with a **lower rank**

$$\begin{aligned} A_0 - C_1 \cdot R_1 &= \begin{bmatrix} 1.0 & 2.0 & 8.0 & 34.0 & 1.0 & 11.0 \\ 2.0 & 5.0 & 19.0 & 81.0 & 4.0 & 35.0 \\ 3.0 & 4.0 & 18.0 & 80.0 & 5.0 & 37.0 \end{bmatrix} - \begin{bmatrix} 1.0 & 2.0 & 8.0 & 34.0 & 1.0 & 11.0 \\ 2.0 & 4.0 & 16.0 & 68.0 & 2.0 & 22.0 \\ 3.0 & 6.0 & 24.0 & 102.0 & 3.0 & 33.0 \end{bmatrix} \\ = A_1 &= \begin{bmatrix} 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 1.0 & 3.0 & 13.0 & 2.0 & 13.0 \\ 0.0 & -2.0 & -6.0 & -22.0 & 2.0 & 4.0 \end{bmatrix} \end{aligned}$$

because R_1 and C_1 have been reduced to 0.
We then increment our row and column indices and continue searching.

2.3 Step 2

We have the matrix

$$A_1 = \begin{bmatrix} 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 1.0 & 3.0 & 13.0 & 2.0 & 13.0 \\ 0.0 & -2.0 & -6.0 & -22.0 & 2.0 & 4.0 \end{bmatrix}$$

We can see that the row R_2 and the column C_2 are both **non-zero vectors**, meaning their multiplication will be a non-zero matrix. Therefore, they are **linearly independent**, and thus we can pick them out.
Also, we have to divide either one of them by the element $e_{22} = 1.0$ to ensure it is not duplicated in the matrix multiplication.
And then, we have

$$\begin{aligned} C_2 \cdot R_2 &= \begin{bmatrix} 0.0 \\ 1.0 \\ -2.0 \end{bmatrix} \cdot \begin{bmatrix} 0.0 & 1.0 & 3.0 & 13.0 & 2.0 & 13.0 \end{bmatrix} \\ &= \begin{bmatrix} 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 1.0 & 3.0 & 13.0 & 2.0 & 13.0 \\ 0.0 & -2.0 & -6.0 & -26.0 & -4.0 & -26.0 \end{bmatrix} \end{aligned}$$

, which when substracted from A_1 will result in a new matrix A_2 with a **lower rank**

$$\begin{aligned} A_1 - C_2 \cdot R_2 &= \begin{bmatrix} 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 1.0 & 3.0 & 13.0 & 2.0 & 13.0 \\ 0.0 & -2.0 & -6.0 & -22.0 & 2.0 & 4.0 \end{bmatrix} - \begin{bmatrix} 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 1.0 & 3.0 & 13.0 & 2.0 & 13.0 \\ 0.0 & -2.0 & -6.0 & -26.0 & -4.0 & -26.0 \end{bmatrix} \\ = A_2 &= \begin{bmatrix} 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 4.0 & 6.0 & 30.0 \end{bmatrix} \end{aligned}$$

because R_2 and C_2 have been reduced to 0.
We then increment our row and column indices and continue searching.

2.4 Step 3

We have the matrix

$$A_2 = \begin{bmatrix} 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 4.0 & 6.0 & 30.0 \end{bmatrix}$$

Though either the row R_3 or the column C_3 is a **non-zero vector**, we can see that the element $e_{33} = 0.0$
This means ... and they cannot be picked.
In this case, we increment our column index while keeping the same row until we find a non-zero element. (We will definitely find it because otherwise it will be a zero-vector row case.)

2.5 Step 4

Finally, we have the matrix

$$A_2 = \begin{bmatrix} 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 4.0 & 6.0 & 30.0 \end{bmatrix}$$

We can see that the row R_3 and the column C_4 are both **non-zero vectors**, meaning their multiplication will be a non-zero matrix. Therefore, they are **linearly independent**, and thus we can pick them out.
Also, we have to divide either one of them by the element $e_{34} = 4.0$ to ensure it is not duplicated in the matrix multiplication.
And then, we have

$$\begin{aligned} C_4 \cdot R_3 &= \begin{bmatrix} 0.0 \\ 0.0 \\ 4.0 \end{bmatrix} \cdot \begin{bmatrix} 0.0 & 0.0 & 0.0 & 1.0 & 1.5 & 7.5 \end{bmatrix} \\ &= \begin{bmatrix} 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 4.0 & 6.0 & 30.0 \end{bmatrix} \end{aligned}$$

, which will always be exactly equal to A_2 .
Thus, we are left with the last matrix A_3 with a **rank of zero**, or in other words, a **zero-matrix**.

$$\begin{aligned} A_2 - C_4 \cdot R_3 &= \begin{bmatrix} 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 4.0 & 6.0 & 30.0 \end{bmatrix} - \begin{bmatrix} 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 4.0 & 6.0 & 30.0 \end{bmatrix} \\ = A_3 &= \begin{bmatrix} 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \end{bmatrix} \end{aligned}$$

Bravo! We have successfully decomposed the matrix.