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Vistas in Advanced Computing

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Project Report

My project involved me solving the burgers equation and visualizing the solution. I first tried understanding what burgers equation was representing. I learned that the equation was proposed by J.M Burgers as a simple model for the study of turbulence where the fixed parameter 𝜈 is analogous to the viscosity of a fluid.

The burgers equation looks very similar to the diffusion but instead of D there is 𝜈 and there is another term involved in the equation, . Having done the diffusion equation before, I understand that you initialize a set of values at time = 0 and run a method that approximates the values at the next time step.

To approximate the value at the next time step, I needed a du (the slopes for my initial equations). I rearranged the formula to isolate du by itself which gave me,

I now must decide initial values. I went from 0 to 3 on the x axis with values of 0. My viscosity term is 0.01. I used derivative approximations to figure out the values for the second derivative and the first derivative. For the second derivative I used and for the first derivative I used to approximate. With these equations I wrote a C++ code, stored values into a txt file and plot it in MATLAB over time.

With initial condition of zeros, I see that my plot remains 0 the entire time. I decided that a new initial condition must be used to find a solution I’m looking for. I went through many different initializations which I have provided in my MATLAB code for you to visualize and see. One example I used was the sin function as my initial conditions from 0 to pi. The graph that displayed was a wave function oscillating, but the amplitude was decreasing as time increased. I realized that is due to the viscosity term. When I increase the viscosity term, the wave moves slower and levels out faster while decreasing the viscosity term caused the wave to move faster and levels out slower.

I tried increasing my delta x to have more points and hopefully produced an even more accurate graph. The result I got instead was that the graph blows up to infinity and disappears. Eventually I realized that another condition must be satisfied which is

If this condition is true, the graph remains stable over time. There were other graphs to the burgers equation that I need to represent also. After talking to Dr. Prosperetti, he gave me new initial conditions to try and wanted me to reproduce a shock. The initial conditions to try was a sin function with a large amplitude from 0 to pi and another sin function with a large amplitude from 0 to 2 \* pi. If you run the program I provided in MATLAB with those initial conditions, you can see the shock appearing which is when the slope is approaching a vertical line. If you let the program run iterations after that, the slope becomes undefined which blows up the graph to infinity and becomes unstable, however if your initial conditions satisfy the condition (shown above), the slope will get close to be a vertical line but it does not which causes it to stabilize. If the shock continues, the equation is not complex enough to calculate what happens to the graph afterwards.