Assignment # 1

Solve the following systems of linear equations by reducing to triangular form and then using Back Substitution.

$$(b)\quad \begin{aligned} 6u + v &= 5, \\ 3u - 2v &= 5; \end{aligned}$$

$V = 5 - 6U$

$3U - 2(5 - 6U) = 5$

$3U - 10 + 12U = 5$

$-10 + 15U = 5$

$15U = 15$

$U = 1 \quad V = -1$

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True or false: If A, B are square matrices of the same size, then

A2 − B2 = (A + B)(A − B).

Gaussian Elimination — regular case

$$(e)\quad \begin{pmatrix} 1 & 1 & -1 \\ 2 & -1 & 3 \\ -1 & -1 & 3 \end{pmatrix} \begin{pmatrix} p \\ q \\ r \end{pmatrix} = \begin{pmatrix} 0 \\ 3 \\ 5 \end{pmatrix},$$

Youtube video:

$$\left(\begin{array}{ccc|c} 1 & 1 & -1 & 0 \\ 2 & -1 & 3 & 3 \\ -1 & -1 & 3 & 5 \end{array}\right)$$

· make the circled numbers to be 1

· make the orange numbers to be 0

$$\left(\begin{array}{ccc|c} 1 & 1 & -1 & 0 \\ 2 & -1 & 3 & 3 \\ -1 & -1 & 3 & 5 \end{array}\right) \rightarrow R_2 + 2(R_3) \left(\begin{array}{ccc|c} 1 & 1 & -1 & 0 \\ 0 & -3 & 9 & 13 \\ -1 & -1 & 3 & 5 \end{array}\right) \rightarrow R_3 = R_1 + R_3 \left(\begin{array}{ccc|c} 1 & 1 & -1 & 0 \\ 0 & -3 & 9 & 13 \\ 0 & 0 & 2 & 5 \end{array}\right)$$

$$\left(\begin{array}{ccc|c} 1 & 1 & -1 & 0 \\ 0 & -3 & 9 & 13 \\ 0 & 0 & 2 & 5 \end{array}\right) \rightarrow R_2 = R_2/-3 \left(\begin{array}{ccc|c} 1 & 1 & -1 & 0 \\ 0 & 1 & -3 & -13/3 \\ 0 & 0 & 2 & 5 \end{array}\right) \rightarrow R_3 = R_3/2 \left(\begin{array}{ccc|c} 1 & 1 & -1 & 0 \\ 0 & 1 & -3 & -13/3 \\ 0 & 0 & 1 & 5/2 \end{array}\right)$$

$$\begin{array}{ccc} p & q & r \end{array}$$
$$\left(\begin{array}{ccc|c} 1 & 1 & -1 & 0 \\ 0 & 1 & -3 & -13/3 \\ 0 & 0 & 1 & 5/2 \end{array}\right) = \begin{aligned} p + q - r &= 0 \\ q - 3r &= -13/3 \\ r &= 5/2 \end{aligned}$$

$\frac{3}{1} \cdot \frac{5}{2}$

$q - \frac{15}{2} = -\frac{13}{3}$

$q = -\frac{13}{3} + \frac{15}{2}$ $\quad 15 \times 3 = 45$

$q = -\frac{26}{6} + \frac{45}{6}$ $\quad 45 - 26 = 19$

$q = \frac{19}{6}$

$p + \frac{19}{6} - \frac{5}{2} = 0$

$p + \frac{19}{6} - \frac{15}{6} = 0$

$\frac{4}{6}$

$p + \frac{2}{3} = 0$

$p = -\frac{2}{3}$

$$\boxed{\begin{aligned} p &= -\tfrac{2}{3} \\ q &= \tfrac{19}{6} \\ r &= \tfrac{5}{2} \end{aligned}}$$

Find the LU factorization of the following matrices:

(d) $\begin{pmatrix} 2 & 0 & 3 \\ 1 & 3 & 1 \\ 0 & 1 & 1 \end{pmatrix}$,

YouTube video:

· U: upper triangle $\begin{matrix} x & x & x \\ 0 & x & x \\ 0 & 0 & x \end{matrix}$

· L: lower triangle $\begin{matrix} 1 & x & x \\ x & 1 & x \\ x & x & 1 \end{matrix}$

· A: original/given matrix

$A = LU$

build L here:

$$L = \begin{bmatrix} 1 & 0 & 0 \\ \frac{1}{2} & 1 & 0 \\ 0 & \frac{1}{3} & 1 \end{bmatrix}$$

$$A = \begin{pmatrix} 2 & 0 & 3 \\ 1 & 3 & 1 \\ 0 & 1 & 1 \end{pmatrix} R_2 = R_2 - \frac{R_1}{2} \quad \begin{pmatrix} 2 & 0 & 3 \\ 0 & 3 & -\frac{1}{2} \\ 0 & 1 & 1 \end{pmatrix} R_3 = R_3 - 0(R_1) \quad \begin{pmatrix} 2 & 0 & 3 \\ 0 & 3 & -\frac{1}{2} \\ 0 & 1 & 0 \end{pmatrix} R_3 = R_3 - \frac{1}{3}(R_2)$$

opposite of the multiplier

opposite of the multiplier

opposite of the multiplier

$$\begin{pmatrix} 2 & 0 & 3 \\ 0 & 3 & -\frac{1}{2} \\ 0 & 0 & 0 \end{pmatrix}$$

$$L = \begin{bmatrix} 1 & 0 & 0 \\ \frac{1}{2} & 1 & 0 \\ 0 & \frac{1}{3} & 1 \end{bmatrix} \begin{bmatrix} 2 & 0 & 3 \\ 0 & 3 & -\frac{1}{2} \\ 0 & 0 & 0 \end{bmatrix}$$

In each of the following problems, find the A = LU factorization of the coefficient matrix, and then use Forward and Back Substitution to solve the corresponding linear systems A x = bj for each of the indicated right-hand sides:

(a) $A = \begin{pmatrix} -1 & 3 \\ 3 & 2 \end{pmatrix}$, $\mathbf{b}_1 = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$, $\mathbf{b}_2 = \begin{pmatrix} 2 \\ 5 \end{pmatrix}$, $\mathbf{b}_3 = \begin{pmatrix} 0 \\ 3 \end{pmatrix}$.

$$LU = \begin{bmatrix} 1 & 0 \\ -3 & 1 \end{bmatrix} \begin{bmatrix} -1 & 3 \\ 0 & 11 \end{bmatrix}$$

$Ly = b_1$

$$\begin{bmatrix} 1 & 0 \\ -3 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$\begin{bmatrix} y_1 \\ -3y_1 + y_2 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$y_1 = 1 \quad y_2 = 2$

$Ux = y$

$$\begin{bmatrix} -1 & 3 \\ 0 & 11 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} -x_1 + 3x_2 \\ 11x_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$x_1 = -\frac{8}{11} \quad x_2 = \frac{2}{11}$

$Ly = b_2$

$$\begin{bmatrix} 1 & 0 \\ -3 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 2 \\ 5 \end{bmatrix}$$

$$\begin{bmatrix} y_1 \\ -3y_1 + y_2 \end{bmatrix} = \begin{bmatrix} 2 \\ 5 \end{bmatrix}$$

$y_1 = 2 \quad y_2 = 11$

$Ux = y$

$$\begin{bmatrix} -1 & 3 \\ 0 & 11 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 2 \\ 11 \end{bmatrix}$$

$$\begin{bmatrix} -x_1 + 3x_2 \\ 11x_2 \end{bmatrix} = \begin{bmatrix} 2 \\ 11 \end{bmatrix}$$

$x_1 = 1 \quad x_2 = 1$

$Ly = b_3$

$$\begin{bmatrix} 1 & 0 \\ -3 & 1 \end{bmatrix}\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 3 \end{bmatrix}$$

$$\begin{bmatrix} y_1 \\ -3y_1 + y_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 3 \end{bmatrix}$$

$y_1 = 0 \quad y_2 = 3$

$$\begin{bmatrix} -8/11 \\ 2/11 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 9/11 \\ 3/11 \end{bmatrix}$$

$Ux = y$

$$\begin{bmatrix} -1 & 3 \\ 0 & 11 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 3 \end{bmatrix}$$

$$\begin{bmatrix} -x_1 + 3x_2 \\ 11x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 3 \end{bmatrix}$$

$x_1 = \frac{9}{11} \quad x_2 = \frac{3}{11}$

Write down the inverse of each of the following elementary matrices:

(c) $\begin{pmatrix} 1 & -2 \\ 0 & 1 \end{pmatrix}$,

(d) $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & -3 \\ 0 & 0 & 1 \end{pmatrix}$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$\frac{1}{\det(A)} \quad adj(A)$

$(1 \cdot 1) - (0 \cdot -2)$

$1 - 0$

1

$$\frac{1}{1}\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$$

Add $-3 * R_3$ to R_2

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{pmatrix}$$

Find the inverse of each of the following matrices, if possible, by applying the Gauss–Jordan Method.

(b) $\begin{pmatrix} 1 & 3 \\ 3 & 1 \end{pmatrix}$

$$\frac{1}{(1 \cdot 1) - (3 \cdot 3)}$$

$$\frac{1}{1 - 9}$$

$$-\frac{1}{8}\begin{bmatrix} 1 & -3 \\ -3 & 1 \end{bmatrix}$$

$$\begin{bmatrix} -\frac{1}{8} & \frac{3}{8} \\ \frac{3}{8} & -\frac{1}{8} \end{bmatrix}$$

Which of the following systems has (i) a unique solution? (ii) infinitely many $x - 2y = 1$, solutions? (iii) no solution? In each case, find all solutions:

(b) $$\begin{aligned} 2x + y + 3z &= 1, \\ x + 4y - 2z &= -3. \end{aligned}$$

$$\left(\begin{array}{ccc|c} 2 & 1 & 3 & 1 \\ 1 & 4 & -2 & -3 \end{array}\right)$$

no solutions

Use Gaussian Elimination to find the determinant of the following matrices:

(c) $$\begin{pmatrix} 1 & 2 & 3 \\ 2 & 5 & 8 \\ 3 & 8 & 10 \end{pmatrix}$$

$\frac{2}{1} = 2$

$[1 \ 2 \ 3] \times 2 = [2 \ 4 \ 6]$

$$\begin{array}{r} [2 \ 5 \ 8] \\ - [2 \ 4 \ 6] \\ \hline 0 \ 1 \ 2 \end{array}$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 3 & 8 & 10 \end{bmatrix}$$

$\frac{3}{1} = 3$

$[1 \ 2 \ 3] \times 3 = [3 \ 6 \ 9]$

$[2 \ 8 \ 10]$ $[1 \ 2 \ 3]$

$$
\begin{array}{r}
-\begin{bmatrix} 3 & 6 & 9 \end{bmatrix} \\
\hline
0 \quad 2 \quad 1
\end{array}
\qquad
\begin{bmatrix} 0 & 1 & 2 \end{bmatrix}
$$
$$
\begin{bmatrix} 0 & 2 & 1 \end{bmatrix}
$$

$$\frac{2}{1} = 2$$

$$\begin{bmatrix} 0 & 1 & 2 \end{bmatrix} \times 2 = \begin{bmatrix} 0 & 2 & 4 \end{bmatrix}$$

$$
\begin{array}{rrrr}
 & [0 & 2 & 1 \\
- & 0 & 2 & 4 \\
\hline
 & 0 & 0 & -3
\end{array}
\qquad
\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & -3 \end{bmatrix}
$$

$$1 \times 1 \times -3$$

$$\det(A) = -3$$