CSCI 2033 - Assignment #1 Key

1- 1.1.1.b) Solve via reduction to uppertriangle form and backward substitution

$$\begin{matrix} 6u+v=5 \\ 3u-2v=5 \end{matrix} \Rightarrow \begin{matrix} 6u+v=5 \\ -\frac{5}{2}v=\frac{+5}{2} \end{matrix} \Rightarrow \underline{v=-1} \Rightarrow 6u+(-1)=5 \Rightarrow \underline{u=1}$$

Double Check

$6(1)+(-1)=5$ ✓

$3(1)-2(-1)=5$ ✓

2- 1.2.20) True or false! $A^2-B^2=(A+B)(A-B)$

$(A+B)(A-B)=A^2+BA-AB-B^2$ BA is not always equal to AB, so $\underline{\text{FALSE}}$

3- 1.3.1.e) Solve with Gaussian Elimination: $\begin{pmatrix} 1 & 1 & -1 \\ 2 & -1 & 3 \\ -1 & -1 & 3 \end{pmatrix}\begin{pmatrix} p \\ q \\ r \end{pmatrix}=\begin{pmatrix} 0 \\ 3 \\ 5 \end{pmatrix}$

$$\left(\begin{array}{ccc|c} 1 & 1 & -1 & 0 \\ 2 & -1 & 3 & 3 \\ -1 & -1 & 3 & 5 \end{array}\right) \Rightarrow \left(\begin{array}{ccc|c} 1 & 1 & -1 & 0 \\ 0 & -3 & 5 & 3 \\ 0 & 0 & 2 & 5 \end{array}\right) \Rightarrow r=\frac{5}{2} \Rightarrow -3q+\overset{25/2}{5\left(\frac{5}{2}\right)}=\cancel{3}\,{}^{6}/_{2} \Rightarrow p+\left(\frac{19}{6}\right)-\left(\frac{\overset{15}{\cancel{5}}}{\cancel{2}\,6}\right)=0$$

$$q=\left(\frac{+19}{2}\right)/_{+3}=\frac{19}{6} \qquad P=\frac{-4}{6}=\frac{-2}{3}$$

$$\boxed{(p,q,r)^T=(-2/3,\ 19/6,\ 5/2)^T}$$

4- 1.3.21.d) Find LU Factorization for $\begin{pmatrix} 2 & 0 & 3 \\ 1 & 3 & 1 \\ 0 & 1 & 1 \end{pmatrix}$

$$\begin{pmatrix} 2 & 0 & 3 \\ 1 & 3 & 1 \\ 0 & 1 & 1 \end{pmatrix}=\begin{pmatrix} 1 & 0 & 0 \\ 1/2 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 2 & 0 & 3 \\ 0 & 3 & -1/2 \\ 0 & 1 & 1 \end{pmatrix}=\begin{pmatrix} 1 & 0 & 0 \\ 1/2 & 1 & 0 \\ 0 & 1/3 & 1 \end{pmatrix}\begin{pmatrix} 2 & 0 & 3 \\ 0 & 3 & -1/2 \\ 0 & 0 & 7/6 \end{pmatrix}$$

$$L=\begin{pmatrix} 1 & 0 & 0 \\ 1/2 & 1 & 0 \\ 0 & 1/3 & 1 \end{pmatrix} \qquad U=\begin{pmatrix} 2 & 0 & 3 \\ 0 & 3 & -1/2 \\ 0 & 0 & 7/6 \end{pmatrix}$$

5- 1.3.33.a) Use LU Factorization to solve $Ax=b_j$ for: $A=\begin{pmatrix} -1 & 3 \\ 3 & 2 \end{pmatrix}$, $b_1=\begin{pmatrix} 1 \\ -1 \end{pmatrix}$, $b_2=\begin{pmatrix} 2 \\ 5 \end{pmatrix}$, $b_3=\begin{pmatrix} 0 \\ 3 \end{pmatrix}$

$$A=\begin{pmatrix} -1 & 3 \\ 3 & 2 \end{pmatrix}=\begin{pmatrix} 1 & 0 \\ -3 & 1 \end{pmatrix}\begin{pmatrix} -1 & 3 \\ 0 & 11 \end{pmatrix}=LU$$

$$Lc_1=b_1 \Rightarrow c_1=\begin{pmatrix} 1 \\ -1-(-3)(1) \end{pmatrix}=\begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

$$Lc_2=b_2 = c_2=\begin{pmatrix} 2 \\ 5-(-3)(2) \end{pmatrix}=\begin{pmatrix} 2 \\ 11 \end{pmatrix}$$

$$Lc_3=b_3 \Rightarrow c_3=\begin{pmatrix} 0 \\ 3-(-3)(0) \end{pmatrix}=\begin{pmatrix} 0 \\ 3 \end{pmatrix}$$

$$Ux_1=c_1 \Rightarrow x_1=\begin{pmatrix} -(1-3\cdot\frac{2}{11}) \\ 2/11 \end{pmatrix}=\begin{pmatrix} -5/11 \\ 2/11 \end{pmatrix}$$

$$Ux_2=c_2 \Rightarrow x_2=\begin{pmatrix} -(2-3\cdot 1) \\ 11/11 \end{pmatrix}=\begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$Ux_3=c_3 \Rightarrow x_3=\begin{pmatrix} -(0-3\cdot\frac{3}{11}) \\ 3/11 \end{pmatrix}=\begin{pmatrix} 9/11 \\ 3/11 \end{pmatrix}$$

CSCI 2033 - Assignment #1 Key (cont.)

6 - 1.5.3.c) Find inverse of $\begin{pmatrix} 1 & -2 \\ 0 & 1 \end{pmatrix}$

$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ gives $A^{-1} = \frac{1}{ad-bc}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$ so inverse of $\begin{pmatrix} 1 & -2 \\ 0 & 1 \end{pmatrix}$ is $\frac{1}{1\cdot 1-(-2)0}\begin{pmatrix} 1 & 0 \\ +2 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$

Verify: $\begin{pmatrix} 1 & -2 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ ✓ $\begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & -2 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

#7 - 1.5.3.d) Find inverse of $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & -3 \\ 0 & 0 & 1 \end{pmatrix}$

Note that this looks like the elementary operation that adds a -3 multiple of the 3rd row to the second row. Lets see if the opposite operation is the inverse

$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & -3 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ ✓ $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & -3 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ ✓ So inverse is $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{pmatrix}$

#8 - 1.5.25.b) Find inverse using Gauss-Jordan Method for $\begin{pmatrix} 1 & 3 \\ 3 & 1 \end{pmatrix}$

$\left(\begin{array}{cc|cc} 1 & 3 & 1 & 0 \\ 3 & 1 & 0 & 1 \end{array}\right) \Rightarrow \left(\begin{array}{cc|cc} 1 & 3 & 1 & 0 \\ 0 & -8 & -3 & 1 \end{array}\right) \Rightarrow \left(\begin{array}{cc|cc} 1 & 3 & 1 & 0 \\ 0 & 1 & 3/8 & -1/8 \end{array}\right) \Rightarrow \left(\begin{array}{cc|cc} 1 & 0 & -1/8 & 3/8 \\ 0 & 1 & 3/8 & -1/8 \end{array}\right)$

Verify $\begin{pmatrix} 1 & 3 \\ 3 & 1 \end{pmatrix}\begin{pmatrix} -1/8 & 3/8 \\ 3/8 & -1/8 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ ✓ Inverse: $\begin{pmatrix} -1/8 & 3/8 \\ 3/8 & -1/8 \end{pmatrix}$

#9 - 1.8.1.b) Find a solutions to $\begin{aligned} 2x+y+3z &= 1 \\ x+4y-2z &= -3 \end{aligned}$

$\left(\begin{array}{ccc|c} 2 & 1 & 3 & 1 \\ 1 & 4 & -2 & -3 \end{array}\right) \Rightarrow \left(\begin{array}{ccc|c} 2 & 1 & 3 & 1 \\ 0 & 7/2 & -7/2 & -7/2 \end{array}\right) \Rightarrow \left(\begin{array}{ccc|c} 2 & 1 & 3 & 1 \\ 0 & 1 & -1 & -1 \end{array}\right) \Rightarrow \left(\begin{array}{ccc|c} 2 & 0 & 4 & 2 \\ 0 & 1 & -1 & -1 \end{array}\right) \Rightarrow \left(\begin{array}{ccc|c} 1 & 0 & 2 & 1 \\ 0 & 1 & -1 & -1 \end{array}\right) \Rightarrow$ $\begin{aligned} z &= z \\ y &= -1+z \\ x &= 1-2z \end{aligned}$

Verify: $2(1-2z)+(-1+z)+3z = 1$ ✓ $(1-2z)+4(-1+z)-2z = -3$ ✓ $\boxed{\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -2 \\ 1 \\ 1 \end{pmatrix} z + \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}}$ infinite solutions

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#10 - 1.9.1.c) Use Gaussian Elimination to find determinant of $\begin{pmatrix} 1 & 2 & 3 \\ 2 & 5 & 8 \\ 3 & 8 & 10 \end{pmatrix} = A$

$\begin{pmatrix} 1 & 2 & 3 \\ 2 & 5 & 8 \\ 3 & 8 & 10 \end{pmatrix} \Rightarrow \begin{pmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & 2 & 1 \end{pmatrix} \Rightarrow \begin{pmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & -3 \end{pmatrix} = U$, $\det(A) = \det(U) = 1\cdot 1\cdot -3 = -3$