

① Base conversions \rightarrow division - remainder \rightarrow bigger exponent to smaller exponent

a) $588_{10} = \underline{210210}_3$

A)
$$\begin{array}{r|l} 3 & 588 & 0 \\ \hline & 196 & 1 \\ 3 & 165 & 2 \\ \hline & 21 & 0 \\ 3 & 7 & 1 \\ \hline & 2 & 2 \\ 3 & 0 & \\ \hline & & \end{array}$$

 210210_3

b) $2254_{10} = \underline{33004}_5$

B)
$$\begin{array}{r|l} 5 & 2254 & 4 \\ \hline & 450 & 0 \\ 5 & 90 & 0 \\ \hline & 18 & 3 \\ 5 & 3 & 3 \\ \hline & 0 & \end{array}$$

 33004_5

② Base conversions \rightarrow division - remainder \rightarrow smaller exponent to bigger exponent

a) $20012_3 = \underline{167}_{10}$

A) $20012_3 = X_{10}$

$$2 \times 3^4 + 0 \times 3^3 + 0 \times 3^2 + 1 \times 3^1 + 2 \times 3^0$$

$$= 2 \times 81 + 0 \times 27 + 0 \times 9 + 1 \times 3 + 2 \times 1$$

$$= 162 + 0 + 0 + 3 + 2$$

$$= 167_{10}$$

b) $4103_5 = \underline{528}_{10}$

B) $4103_5 = X_{10}$

$$4 \times 5^3 + 1 \times 5^2 + 0 \times 5^1 + 3 \times 5^0$$

$$= 4 \times 125 + 1 \times 25 + 0 \times 5 + 3 \times 1$$

$$= 500 + 25 + 0 + 3$$

$$= 528_{10}$$

③ decimal fractions to binary

a) 25.84375

the decimal part the fraction part

2 | 25 1
2 | 12 0
2 | 6 0
2 | 3 1
2 | 1 1
0

bottom to top 11001

0.84375
x 2 integer is 1
1.68750
0.68750
x 2 integer is 1
1.37500
0.37500
x 2 integer is 0
0.75000
0.75000
x 2 integer is 1
1.50000
0.50000
x 2 integer is 1
1.00000
0.00000
x 2 integer is 0
0.00000

top to bottom 110110

11001.110110

d) 84.874023

the decimal part on part

2 | 84 0
2 | 42 0
2 | 21 1
2 | 10 0
2 | 5 1
2 | 2 0
2 | 1 1
0

from bottom to top 1010100

0.874023
x 2 integer is 1
1.748046
0.748046
x 2 integer is 1
1.496092
0.496092
x 2 integer is 0
0.992184
0.992184
x 2 integer is 1
1.984368
0.984368
x 2 integer is 1
1.968736
0.968736
x 2 integer is 1
1.937472

from top to bottom 110111

1010100.110111

④ convert binary fractions to decimal

a) 10111.1101

10a) 10111.1101

- 1) write the numbers spaced out
- 2) write 2^x exponents on top of number
- 3) multiply top with bottom for each column, add the results of all columns
- 4) add all numbers

$$\begin{array}{r}
 2^4 \quad 2^3 \quad 2^2 \quad 2^1 \quad 2^0 \quad 2^{-1} \quad 2^{-2} \quad 2^{-3} \quad 2^{-4} \\
 \times \quad 1 \quad \times \quad 0 \quad \times \quad 1 \quad \times \quad 1 \quad \times \quad 1 \quad \times \quad 1 \quad \times \quad 0 \quad \times \quad 1 \\
 \hline
 2^4 + 0 + 2^2 + 2^1 + 1 + \frac{1}{2} + \frac{1}{2^2} + 0 + \frac{1}{2^4} \\
 = 16 + 0 + 4 + 2 + 1 + 0.5 + 0.25 + 0.0625 \\
 = 23.8125
 \end{array}$$

d) 11000010.111

10d) 11000010.111

$$\begin{array}{r}
 2^7 \quad 2^6 \quad 2^5 \quad 2^4 \quad 2^3 \quad 2^2 \quad 2^1 \quad 2^0 \quad 2^{-1} \quad 2^{-2} \quad 2^{-3} \\
 \times \quad 1 \quad \times \quad 1 \quad \times \quad 0 \quad \times \quad 0 \quad \times \quad 0 \quad \times \quad 0 \quad \times \quad 1 \quad \times \quad 0 \quad \times \quad 1 \quad \times \quad 1 \quad \times \quad 1 \\
 \hline
 2^7 + 2^6 + 0 + 0 + 0 + 0 + 2 + 0 + \frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} \\
 = 194.875
 \end{array}$$

⑤ Hexadecimal letters to binary

DEAD BEEF₁₆ using the chart

D = 1101 B = 1011
E = 1110 F = 1111
A = 1010 E = 1110
D = 1101 F = 1111

$$(DEADBEEF)_{16} = (1101\ 1110\ 1010\ 1101\ 1011\ 1110\ 1110\ 1111)_2$$

⑥ decimal to signed-magnitude, one's complement, two's complement representation

- a) 60
- b) -60
- c) 20
- d) -20

	signed-magnitude	one's complement	two's complement
60	00111100	00111100	00111100
-60	10111100	11000011	11000100
20	00010100	00010100	00010100
-20	10010100	11101011	11010100

17a) 60 2 | 60 0
2 | 30 0
positive number 2 | 15 1
2 | 7 1
2 | 3 1
2 | 1 1
0

bottom to top 111100

80, 60 in binary is 111100

So, 60 in normal binary is 00111100 ← flip backwards???

one's complement: 00111100

two's complement: 00111100

signed-magnitude: 00111100

17b) -60 2 | 60 0
negative 2 | 30 0
number 2 | 15 1
2 | 7 1
2 | 3 1
2 | 1 1
0

bottom to top 111100

80, 60 in binary is 111100

So, 60 in normal binary is 00111100 ← flip backwards???

Signed-magnitude: set 1 as the farthest-left bit, since it's a negative number

00111100 becomes 10111100

one's complement: flip 1's to 0's and 0's to 1's

00111100 flips to 11000011

two's complement: add 1 to the above results

$$11000011 + 1 = 11000100$$

$$\begin{array}{r} 11000011 \\ + 1 \\ \hline 11000100 \end{array}$$

17c) 20 2 | 20 0
positive number 2 | 10 0
2 | 5 1
2 | 2 0
2 | 1 1
0

bottom to top 10100

80, 20 in binary is 10100

So, 20 in normal binary is 00101 ← flip backwards???

one's complement: 00010100

two's complement: 00010100

signed-magnitude: 00010100

17d) -20 2 | 20 0
negative 2 | 10 0
number 2 | 5 1
2 | 2 0
2 | 1 1
0

bottom to top 10100

80, 20 in binary is 10100

So, 20 in normal binary is 00010100 ← flip backwards???

Signed-magnitude: set 1 as the farthest-left bit, since it's a negative number

00010100 becomes 10010100

one's complement: flip 1's to 0's and 0's to 1's

00010100 becomes 11101011

two's complement: add 1 to the above results

$$11101011 + 1 = 11101100$$

$$\begin{array}{r} 11101011 \\ + 1 \\ \hline 11101100 \end{array}$$

22. What decimal value does the 8-bit binary number 10110100 have if:

- a) it is interpreted as an unsigned number? **180**
- b) it is on a computer using signed-magnitude representation? **-52**
- c) it is on a computer using one's complement representation? **-75**
- d) it is on a computer using two's complement representation? **-76**
- e) it is on a computer using excess-127 representation? **53**

22a) 10110100

$$\begin{aligned}
 &= 1 \times 2^7 + 0 \times 2^6 + 1 \times 2^5 + 1 \times 2^4 + 0 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 0 \times 2^0 \\
 &= 2^7 + 0 + 2^5 + 2^4 + 0 + 2^2 + 0 + 0 \\
 &= 180
 \end{aligned}$$

22b) 10110100

farthest-left bit is 1, so the number is negative

convert the rest of the number to decimal:

0110100

$$\begin{aligned}
 &= 0 \times 2^6 + 1 \times 2^5 + 1 \times 2^4 + 0 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 0 \times 2^0 \\
 &= 0 + 2^5 + 2^4 + 0 + 2^2 + 0 + 0 \\
 &= 52
 \end{aligned}$$

So, 10110100 from signed-magnitude to decimal is -52

22c) 10110100

farthest-left bit is 1, so the number is negative

① flip all bits, 1's to 0's, 0's to 1's, 10110100 becomes 01001011

② convert result to decimal 1001011

$$\begin{aligned}
 &= 1 \times 2^6 + 0 \times 2^5 + 0 \times 2^4 + 1 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 1 \times 2^0 \\
 &= 2^6 + 0 + 0 + 2^3 + 0 + 2^1 + 1 \\
 &= 75
 \end{aligned}$$

Answer: -75

22d) 10110100

farthest-left bit is 1, so the number is negative

① flip all bits, 1's to 0's, 0's to 1's, 10110100 becomes 01001011

② Add 1 to the above result: 01001011 + 1 = 1001100

$$\begin{array}{r}
 01001011 \\
 + 1 \\
 \hline
 1001100
 \end{array}$$

③ convert result to decimal 1001100

$$\begin{aligned}
 &= 1 \times 2^6 + 0 \times 2^5 + 0 \times 2^4 + 1 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 0 \times 2^0 \\
 &= 2^6 + 0 + 0 + 2^3 + 2^2 + 0 + 0 \\
 &= 76
 \end{aligned}$$

Answer = -76

Note: each shift to the left multiplies the number by 2
each shift to the right divides the number by 2

44. Using arithmetic shifting, perform the following:

- a) double the value 00010101₂ A) shift 1 bit to the left and add 1 zero to the farthest-right: 00010101 → 00101010₂
- b) quadruple the value 01110111₂ B) shift 2 bits to the left and add 2 zero to the farthest-right: 01110111 → 11011100
- c) divide the value 11001010₂ in half C) shift 1 bit to the right and preserve the sign: 11101010

52. Show how each of the following floating-point values would be stored using IEEE-754 double precision (be sure to indicate the sign bit, the exponent, and the significand fields):

- a) 12.5
- b) -1.5
- c) 0.75
- d) 26.625

⊛ values to sign bit, exponent, significant fields

52a) 12.5

- $12.5 = 1100.1 = 1.1001 \times 2^3$
- exponent field (bias + exponent) = $127 + 3 = 130$
- convert 130 to binary $130 = 10000010$
- positive number, so sign bit = 0

sign	exponent	mantissa
0	10000010	10010000-----0

52b) -1.5

- $1.5 = 1.1 = 1.1 \times 2^0$
- exponent field (bias + exponent) = $127 + 0 = 127$
- convert 127 to binary $127 = 01111111$
- negative number, so sign bit = 1

sign	exponent	mantissa
1	01111111	10000-----0

52c) 0.75

- $0.75 = 0.11 = 1.1 \times 2^{-1}$
- exponent field (bias + exponent) = $127 - 1 = 126$
- convert 126 to binary $126 = 01111110$
- positive number, so sign bit = 0

sign	exponent	mantissa
0	01111110	10000-----0

52d) 26.625

- $26.625 = 11010 = 1.1010 \times 2^4$
- exponent field (bias + exponent) = $127 + 4 = 131$
- convert 131 in binary $131 = 10000011$
- positive number, so sign bit = 0

sign	exponent	mantissa
0	10000011	1010000-----0

⊛ Decode ASCII messages: 1001010 1001111 1001000 1001110 0100000 1000100 1001111 1000101

the following binary ASCII code: 1001010 1101111 1101000

B, B, B, B,		B, B, B,							
		000	001	010	011	100	101	110	1
0000	NULL	DLE	SP	0	@	P	.		p
0001	SOH	DC1	!	1	A	Q	a		q
0010	STX	DC2	"	2	B	R	b		r
0011	ETX	DC3	#	3	C	S	c		s
0100	EOT	DC4	\$	4	D	T	d		t
0101	ENO	NAK	%	5	E	U	e		u
0110	ACK	SYN	&	6	F	V	f		v
0111	BEL	ETB	'	7	G	W	g		w
1000	BS	CAN	(8	H	X	h		x
1001	HT	EM)	9	I	Y	i		y
1010	LF	SUB	*	:	J	Z	j		z
1011	VT	ESC	+	;	K	[k		{
1100	FF	FS	,	<	L	\	l		
1101	CR	GS	=	=	M]	m		}
1110	SO	RS	>	>	N	^	n		~
1111	SI	US	/	?	O	_	o		

John Doe

⊛ unsigned hexadecimal addition:

$$\begin{array}{r} 1 \text{ A } \text{F} \text{ 4} \\ + 3 \text{ 3 } \text{0} \text{ 4} \\ \hline 4 \text{ D } \text{F} \text{ 8} \end{array}$$

$$\begin{array}{r} 3 \text{ D } \text{E} \text{ 7} \\ + \text{D} \text{ 4 } \text{9} \text{ 6} \\ \hline 11 \text{ 2 } \text{7} \text{ D} \end{array}$$

$$\begin{array}{r} \text{E} = 14 \\ + 9 \\ \hline 23 \\ 16 \overline{) 23} \quad 7 \\ 16 \overline{) 1} \quad 1 \\ \hline 17 \\ 16 \overline{) 17} \quad 2 \\ 16 \overline{) 1} \quad 1 \\ \hline 0 \\ 12 \\ 16 \overline{) 17} \quad 1 \\ 16 \overline{) 1} \quad 1 \\ \hline 0 \\ 11 \end{array}$$

Decimal	4-Bit Binary	Hexadecimal
0	0000	0
1	0001	1
2	0010	2
3	0011	3
4	0100	4
5	0101	5
6	0110	6
7	0111	7
8	1000	8
9	1001	9
10	1010	A
11	1011	B
12	1100	C
13	1101	D
14	1110	E
15	1111	F

⊛ unsigned hexadecimal subtraction:

base = 16
borrow = 16

$$\begin{array}{r} 7 \text{ 6} \rightarrow \text{B} \text{ 11} \text{ 6} \\ - 6 \text{ C } \text{12} \text{ 5} \\ \hline 0 \text{ F } \text{1} \end{array}$$

$$\begin{array}{r} 14 \quad 8 \text{ 16} \quad 2 \text{ 16} \quad 24 \quad 8 \\ \text{F} \quad 9 \quad \text{3} \quad \text{8} \\ - 3 \quad 9 \quad 5 \quad \text{D} \\ \hline \text{B} \quad \text{F} \quad \text{D} \quad \text{B} \end{array}$$

46. Draw the combinational circuit that directly implements the following Boolean expression:

$$F(x,y,z) = x + xy + y'z$$

