Optimal Allocation of Phasor Measurement Units using Practical Constraints in Power Systems

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Abstract

The purpose of the research is to find a strategic placement of phasor measurement units (PMUs) that can maintain the observability of the power system using a binary variant of particle swarm optimisation method (BPSO). In recent years, integer linear programming (ILP) is the common optimisation method used to solve this problem. However, recent studies have shown that deterministic methods such as ILP are still unable to find the most optimal placement of PMUs. Therefore, there is still room for further investigation using heuristic algorithm due to its search-based nature. It is important to determine the most strategic placement of PMUs to ensure that the PMUs are fully utilised due to their expensive price tag. The challenge in using heuristic algorithm lies in its weakness when involving large-sized problem since it is prone to stuck in local optima as shown in earlier studies where most of them applied their proposed approach to small bus systems. To prevent the BPSO method from being stuck in local optima, a mutation strategy is proposed in addition to the V-shaped sigmoid function replacing the S-shaped sigmoid function. The proposed method is designed to intensify the local search of the algorithm around the current best solution to help instigate the particles from being stuck in local optima and consequently finding a better solution. This is shown to be a crucial factor based on the numerical results obtained where it outperforms all methods for all bus systems tested including the IEEE 300-bus system in terms of measurement redundancy and the number of PMUs. This study also considers other problem constraints such as zero-injection bus, single PMU loss and also PMU's channels limit. Most of the existing studies considered PMU to have an unlimited number of channels whereas in practical, PMU does have a finite number of channels that it can use.

Acknowledgement

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List of Symbols

C_1	Acceleration constants
C_2	Acceleration constants
$[A]$	Binary connectivity matrix
$[X]$	Binary decision variable vector
BOI	Bus observability index
L	Channels limit
K	Constriction factor
M	Desired value of measurement redundancy
b	Decision vector variable for single PMU loss
$Gbest$	Global best model
$gbest^t$	Global best position in the swarm at iteration t
ω	Inertia weight
$Lbest$	Local best model
$T_{\max}$	Maximum iteration
$V_{\max}$	Maximum velocity
$\omega_{\max}$	Maximum value of inertia weight
J_1	Measurement redundancy
$[D]$	Measurement redundancy for corresponding bus
$V_{\min}$	Minimum velocity
$\omega_{\min}$	Minimum value of inertia weight
N	Number of bus
N_Z	Number of bus adjacent to ZIB
BI_i	Number of bus incidents to bus i
N_L	Number of bus that not being observed twice by PMUs placement

BR_i	Number of branch combinations for bus i
N_{obs}	Number of observable bus
N_{PMU}	Number of PMUs
$s(p)$	Number of PMUs being placed
BC_i	Number of possible combinations of L out of BI_i
$R(p)$	Number of unobservable buses
$pbest_{ij}^t$	Personal best position for particle i in dimension j discovered so far
x_{ij}^t	Position of particle i in dimension j at iteration t
$[H]$	Possible combination bus for channel limit
R_1	Random numbers that are uniformly distributed between [0,1]
R_2	Random numbers that are uniformly distributed between [0,1]
S	Swarm size
$SORI$	System of redundancy index
G	Total measurement redundancy when considering single PMU loss
v_{ij}^t	Velocity of particle i in dimension j at iteration t
w_1	Weight parameter for the number of bus observed
w_3	Weight parameter for the number of bus that is not being observed twice by PMUs placement
w_4	Weight parameter for single PMU loss measurement redundancy
w_2	Weight parameter for the number of PMUs
C	Weight parameter value for the measurement redundancy

Abbreviations

ABM	Augmented bus merging
BSDP	Binary semi-definite programming
BOI	Bus observability index
BPSO	Binary particle swarm optimisation
CM	Conventional measurement
DE	Differential evolution
ES	Exhaustive search
GA	Genetic algorithm
GPS	Global positioning signal
IGA	Immunity genetic algorithm
ILP	Integer linear programming
KCL	Kirchoff's current law
MICA	Modified imperialist competitive algorithm
NASPI	North American SynchroPhasor Initiative
NSGA	Non-dominated sorting genetic algorithm
OPP	Optimal PMUs placement
PCR	PMU configuration rules
PEV	PMU effective value
PMU	Phasor measurement unit
PSO	Particle swarm optimisation
SA	Simulated annealing
SCADA	Supervisory control and data acquisition
SORI	System observability redundancy index
SQP	Sequential quadratic programming
TRR	Topology re-configuration rules
ZIB	Zero-injection bus

Chapter 1

Introduction

1.1 Motivation of the research

In the current modern era where almost everything has been modernised for a more efficient, better reliability, and independent monitoring, traditional power grids are also in the transitional process to become a modernised power grid, or widely known as the smart grid. The vision for the smart grid is to monitor and manage the power grid as efficiently as possible while providing better reliability and stability. It is a welcome consideration to replace an ageing infrastructure with a smart grid that uses advanced technologies to achieve this vision. One of the advanced technologies used is phasor measurement unit (PMU). PMU is a measurement device that can measure bus voltage phasor at the bus it is installed and the branch current phasor that is adjacent to it, hence, the name. PMU is also equipped with Global Positioning System (GPS), thus, the measurement data provided by the PMU can be calculated real-time due to time-stamping and synchronisation. This knowledge is vital to electric utilities especially to operator engineers where it allows them to identify, anticipate, and correct problems in case when irregular system conditions occur. It aligns with smart grid visions to have a better monitoring of the power grid and more reliable. However, the implementation of PMU has been progressing very slowly due to substantial investment needed for the placement sites.

The PMU placement sites need to have a communication facility for the PMU to operate and the limited number of placement sites that have it hinders its implementation. Furthermore, the cost of the PMU itself is readily expensive although the price is expected to be decreased when there are more demands in future. Research conducted in recent years have shown that by utilising the PMU attributes and the use of the Kirchoff's current law (KCL) and Ohm's law, the number of PMUs required to achieve full observability of a power system can be reduced if the PMUs are strategically placed in a power system.

Many optimisation methods have been used in recent years to determine the strategic placement of PMUs in a power system such as integer linear programming (ILP), simulated annealing (SA), exhaustive search (ES), genetic algorithm (GA), differential evolution (DE), and also binary particle swarm optimisation (BPSO). Among these optimisation methods, the ILP is the most dominant method used in solving this problem. However, recent studies have shown that the PMUs placement obtained using the ILP method is not an optimal solution. Since the optimal PMUs placement problem does not have a unique solution, there is a possibility that meta-heuristic algorithms can be used to deal with the optimal PMUs placement problem. This is because meta-heuristic algorithms use randomisation in their algorithm formulation, hence, different results are expected from these algorithms which might improve the current solution. Compared to other optimisation methods, BPSO algorithm uses the intelligence of the particles to find the quality solution. In addition, its ease of implementation and quick convergence are the two attributes that make it an attractive solution.

Numerous factors were considered in prior studies such as conventional measurement, zero-injection bus, single PMU loss, measurement redundancy, and also PMU's channel limit. Among the factors mentioned earlier, channel limit is the least factor being considered for the optimal PMU placement (OPP) across many optimisation methods especially in the BPSO algorithm. Theoretically, the PMU that has a fewer channel limit is cheaper than the one with more channels because its measuring ability is restricted. Therefore, the motivation of this thesis is to investigate the application of BPSO algorithm to solve the OPP problem while considering normal operation, zero-injection bus, single PMU loss, measurement redundancy, and also PMU's channel limit.

1.2 Aim of the research

The main aim of the research is to determine the minimum number of PMUs required across different IEEE bus systems using the BPSO algorithm. Factors such as zero-injection bus, measurement redundancy, and also PMU's channel limit are considered in this thesis. As other optimisation methods, BPSO algorithm also has drawbacks that may affect its general performance to achieve the primary aim of the research. Problems that have been documented in the existing studies include the lack of diversity during the searching process which may affect the quality solution that the algorithm can find. Various methods have been proposed by the existing studies. However, the effectiveness of the proposed methods from the existing studies is rarely tested on larger systems, where the particle diversity is the most important. Thus, the second aim of this thesis is to propose a method that can overcome issues concerning particle diversity and to ensure that it can be applied in larger systems. The proposed method will then be validated using MATLAB software to illustrate its efficiency compared to the standard BPSO algorithm. The obtained results of the proposed method will also be further evaluated by comparing it with the results of prior studies. The proposed method is expected to improve particle diversity to avoid it from being trapped in local optima and must be able to obtain a high quality solution across all IEEE bus systems tested.

To summarise, the objectives of the research are listed as follows:

- To determine the minimum number of PMUs required across different IEEE bus systems using BPSO algorithm while considering normal operation, zero-injection bus, measurement redundancy, single PMU loss, and PMU's channel limit.
- To propose a method that can overcome the issue concerning lack of diversity which will be able to give quality solution in larger systems.
- To evaluate the proposed method efficiency, the proposed method is validated using MATLAB software and the results obtained are compared with the existing studies to evaluate their superiority.

1.3 Contributions of the research

The main contributions of this thesis lie in the improvements made to the standard BPSO performance, its application in solving the OPP problem across different IEEE bus systems while considering normal operation, zero-injection bus, single PMU loss, measurement redundancy, and also PMU's channel limit. The contributions developed in this thesis are detailed as follows:

- This thesis proposes a mutation strategy that is combined with a constriction factor and a V-shaped sigmoid to the standard BPSO algorithm. In order to evaluate its performance for solving the OPP problem, the proposed method is investigated using MATLAB software and it is applied to the IEEE 14-bus, 24-bus, 30-bus, 39-bus, 57-bus, 118-bus, and 300-bus systems. It appears that the proposed method generates better results compared to the standard BPSO algorithm and also the existing studies that use BPSO algorithm in their approach.
- The improvement made to the standard BPSO algorithm is more prominent when involving larger bus systems where standard BPSO algorithm is rarely tested.
- Furthermore, the proposed method converges faster than the standard BPSO algorithm for each bus system. Although quick convergence is sometimes related to being trapped in local optima, the results obtained from the proposed method indicate otherwise.
- The introduction of mutation strategy understandably added computation complexity to the BPSO algorithm, hence, increases the computation time. To overcome this problem, the proposed method discards duplicate solutions in its mutation strategy to ensure that it is more efficient. A satisfactory computation time is achieved, which is slightly longer than the standard BPSO algorithm. However, it is interesting to note that the computation time is superior when compared with the existing studies' computation time.
- For the case involving a single PMU loss, this thesis formulated the problem constraint as a fitness function. Consequently, it is easier to implement and more convenient.
- This thesis also proposes a topology transformation method by using the ILP method for the case involving zero-injection bus. The topology transformation method eliminates the need for having an observability analysis to pinpoint the accurate bus to place a PMU, which is required for the existing studies that use bus merging method.

1.4 Thesis layout

The organisation of this thesis out of chapter 1 is as follows:

- Chapter 2 elaborates the OPP problem including the PMU's observability rules used to determine the observability of the power system. It also features a concise review of a relevant literature to solve the optimal PMU placement problem. The proposed method for ILP when dealing with ZIB is also discussed in this chapter.
- Chapter 3 presents the conventional particle swarm optimisation method and how it is implemented in general. This chapter also explains the parameters used and other variants of particle swarm optimisation method introduced in recent years to overcome its drawbacks.
- Chapter 4 presents the proposed mutation strategy and the use of the V-shaped sigmoid in order to overcome the BPSO algorithm drawbacks. This chapter also further elaborates the reasons behind the parameters selection and how they contributed in improving the algorithm to achieve the desired result.
- Chapter 5 applies the proposed method into different IEEE bus systems and the simulation results for each case considered are presented. The obtained results are also compared with the existing studies to validate their effectiveness in solving the OPP problem.
- Chapter 6 concludes the thesis by explaining the contributions of the thesis. Future work is also discussed.

1.5 List of publications

1) N. H. A. Rahman and A. F. Zobaa, "Optimal PMU placement using topology transformation method in power systems," *J. Adv. Res.*, vol. 7, no. 5, pp. 625–634, Sep. 2016.
2) N. H. A. Rahman, A. F. Zobaa, and M. Theodoridis, "Improved BPSO for optimal PMU placement," in *Power Engineering Conference (UPEC), 2015 50th International Universities*, 2015, pp. 1–4.
3) N. H. A. Rahman and A. F. Zobaa, "Integrated mutation strategy with modified binary PSO algorithm for optimal PMUs placement," submitted to IEEE Industrial Informatics (under revision).

Chapter 2

Background Study and Literature Review

2.1 Introduction

A new concept of the next-generation electric power system, called smart grid, has been touted as the saviour in the mission of having a modern power grid infrastructure that offers improved efficiency, reliability, and safety through automated control and modern communication technologies, replacing the current electrical grid that has been ageing. It correlates with the increased demand for electricity which has gradually increased over the years as reported by the US Department of Energy report [1]. Current infrastructure's incapability of handling an automated analysis and lack of situational awareness were the two major factors that led to the major blackout in the US history. The blackout that happened in 2003, which lasted for two days, affected 55 million people and caused a massive economic loss which was reported to be approximately between USD4 and USD 10 billion [2]. In today's world, where everything is moving rapidly, the impact would be massive, and thus, it is crucial that preventive measures need to be taken quickly and seriously.

The lack of situational awareness of the current infrastructure is generally due to the way metering data is monitored. The smart grid aims to improve the way data is monitored by incorporating phasor measurement unit (PMU) in its infrastructure. Currently, the supervisory control and data acquisition (SCADA) system collects one data point every 1 to 2 seconds, whereas PMUs collect 30 to 60 data points per second [3]. Additionally, a common time reference is supplied by a global positioning system (GPS) for all acquired data to satisfy the need of a real-time control, thereby, allowing more accurate assessment of the current conditions of the power systems [4]. This benefit has led to the installations of over 1,000 PMUs across North America as reported by the North American SynchroPhasor Initiative (NASPI) as of March 2014 [5]. Figure 2.1 shows the PMU locations in the North American Power Grid. The implementation of PMU has been progressing quite slowly over the years, mainly due to the installation and networking costs involved [6]. Hence, careful planning is a prerequisite for the successful deployment of PMUs.

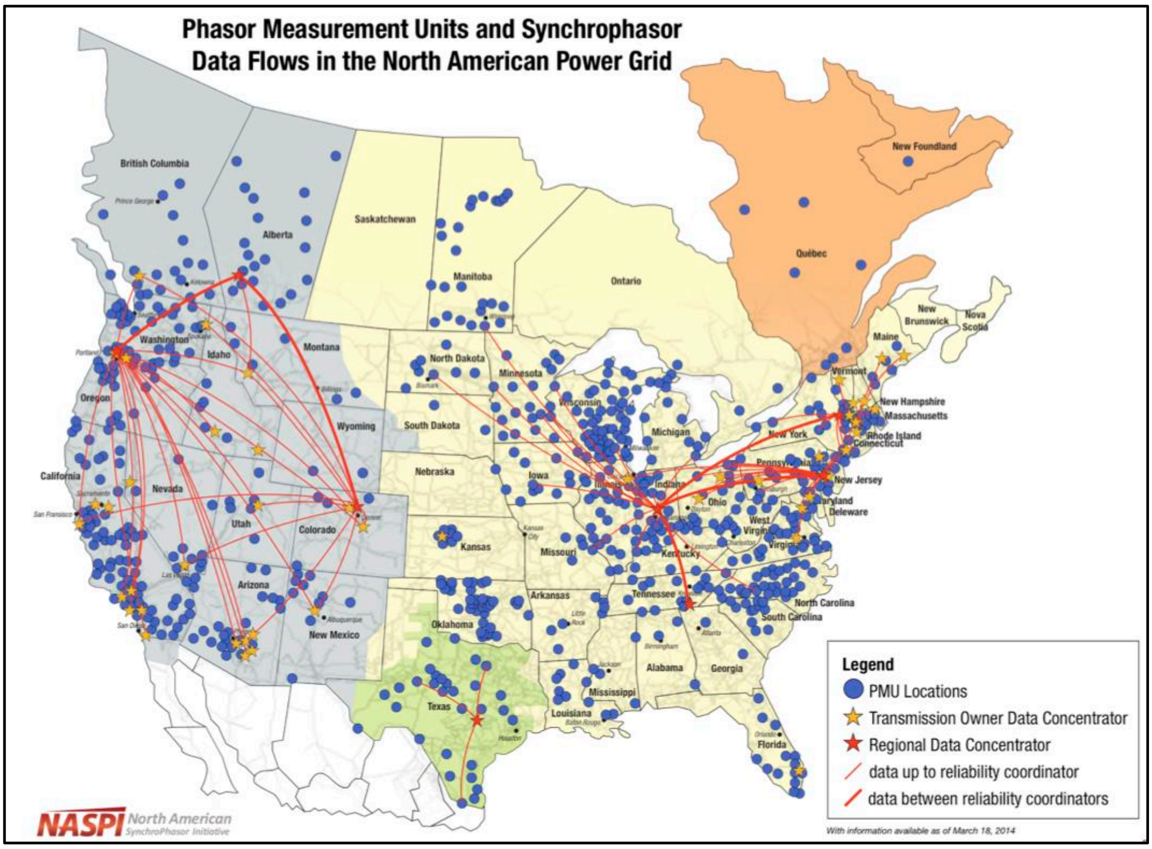

Figure 2.1: Location of PMUs in the North American power grid [5]

2.2 Literature review

Over a long period of time, various techniques have been proposed using different kinds of optimisation methods to solve the OPP problem to make the power system observable. The optimisation methods used can be divided into two main categories: mathematical and heuristic algorithms [7].

There are several mathematical optimisation methods that have been proposed over the years for solving the OPP problem. The integer linear programming (ILP) method is the most common optimisation method used to solve the OPP problem. The ILP method solves the problem in deterministic manner. In ILP, in order to achieve the best solution based on the desired objective, the constraints to solve the OPP problem need to be defined accordingly. Therefore, the constraints formed are very crucial when using the ILP method to solve the OPP problem.

The integer programming formulation to solve the OPP problem was first proposed by Bei and Abur [8] where linear constraints are formed based on network connectivity matrix which makes the assessment of network observability much easier. However, the constraints become nonlinear when the existence of conventional measurements and ZIB are considered in the problem formulations. Later, Gou [9] proposed an ILP method that considered both conventional measurement and ZIB, similar to the research work in [8]. The proposed method managed to eliminate the nonlinear constraints of its problem formulations using a permutation matrix. The proposed method was extended in [10] by considering cases of redundant PMU placement, full observability, and incomplete observability concept which was based on the *depth-of-unobservability* concept introduced in [11].

The depth-of-unobservability concept mentioned above was also discussed in the research by Dua et. al [12] and the authors proposed an ILP method that better optimises the staging of PMUs placement over a given time horizon. The proposed method was also expanded by incorporating a single PMU loss in the problem formulation. In order to evaluate the quality of PMU placement set obtained from the proposed method, bus observability index (BOI) and system observability redundancy index (SORI) were introduced [12].

Abbasy and Ismail [13] proposed a merging method called the augmented bus merging (ABM) which was formulated as a binary ILP. The ABM was modelled for single and multiple PMU loss in addition to power flow and ZIB. The ABM model was proposed to overcome the issue regarding merging method introduced in [14] where it requires topological observability test to determine the accurate location for corresponding PMUs location. This issue occurs because the two linear constraints from the two buses are merged to become one constraint. Therefore, in the event where a PMU is suggested to be placed at one of the two buses, it might indicate that a PMU needs to be placed at one of the bus or both buses to ensure full network observability, thereby requiring a topological observability test to ensure that the PMU is correctly placed. The merging method was also applied in the research work in [15] for a case that considers zero-injection bus and also channel limits. Three strategies reflecting which bus is selected to be merged are considered and the results indicate that bus selection influences the results. More recently, Rahman and Zobaa [16] proposed bus selection rules for merging method for a case considering CM and ZIB, also by using the ILP method. The bus selection rules ensure that the PMU placement sets obtained from the merging method are of the highest quality based on the BOI and SORI index introduced in [12] while at the same time eliminate the need for a topological observability test.

In [17], channels limit was also considered as one of the problem formulations. The proposed model is more relaxed and flexible, which allows more constraints such as line outage and a single PMU loss to be considered. The work in [18] was further enhanced in [15] using mixed ILP, where the effect of network sparsity was investigated in this method. The results indicate that the loss of sparsity leads to a strategic placement of a smaller number of PMUs having a larger number of channels.

The ILP method can only produce a single solution set, even when there are multiple solutions available. This applies to the earlier literatures as well. Often, the solution given by the ILP method is not the optimal solution. Recently, several methodologies have been proposed to address this problem. A sequential quadratic programming (SQP) method was introduced in [19] where it is capable of producing different sets of optimal solutions. The optimal solution set is selected based on the highest SORI value, which indicates the most reliable solution. Another work in [20] introduced a binary semi-definite programming (BSDP) method for solving the OPP problem where it promises to give the optimal solutions. While the results obtained from the SQP and BSDP methods are better than the earlier literatures, there are also results that are not feasible in some cases. For example, the results for the cases of zero-injection bus obtained using the SQP method for the IEEE 30-bus, 57-bus, and 118-bus are not feasible. Meanwhile, for the BSDP method, the results considering zero-injection bus are also not feasible for the IEEE 57-bus system. This is probably due to the mathematical complexity introduced in these methods which make the constraints formed to be inaccurate to solve the OPP problem.

Heuristic by definition is "to discover or learn something by trial and error". Often, it is preferred over other deterministic algorithms such as ILP if there is no indication that the best optimal solution is found. Even though the ILP method is the dominant optimisation technique for solving the OPP problem in recent years, the recent studies elaborated towards the end of the previous section indicated that there is still an opportunity to further investigate the best possible solution.

In contrast to the ILP methods described in the previous section, heuristic algorithms rely on parameters which require fine tuning to ensure that the algorithms are able to find the best possible solutions, instead of a set of constraints. In addition, depending on the number of objectives, each objective may be formed as a component of a fitness function, in which each solution obtained by the algorithms will be evaluated to ensure that it is applicable to the desired objective. However, the time taken by the algorithms may increase as the problem size increases. In addition, it may not converge to an optimal solution for a large-sized problem.

There are a number of studies that adopted heuristic algorithms to solve the OPP problem such as exhaustive search [21], [22] and Tabu search [23]. In recent years, meta-heuristic algorithms have been actively used for solving the OPP problem. In contrast to the simple heuristic algorithms, meta-heuristic algorithms adopted randomisation in the algorithms formulation. The randomisation is supposed to instigate the search on a global scale, thereby, making most of the meta-heuristic algorithms to become suitable for global optimisation. Meta-heuristic algorithms such as simulated annealing, genetic algorithm, differential evolution, and also particle swarm optimisation have been actively used to solve the OPP problem in recent years.

Simulated annealing (SA) was inspired by the formation of crystals in solids during cooling. This concept can be seen during the iron age where blacksmiths discovered that, the slower the cooling, the more perfect the crystals formed [24]. In [25], a dual search technique that uses both modified bisecting search and SA was used. The SA role in the proposed method was to find the ideal PMU placement sets which were gathered from the bisecting search. A graph theoretic procedure, which helps to identify a reasonable starting point, was used to reduce the time taken for the searching process in the bisecting search. Nuqui and Phadke [11] proposed a novel concept of depth-of-unobservability to solve the pragmatic communication-constrained OPP problem using SA. Initially, tree search placement technique was used to jumpstart the SA such that the conditions of depth-of-one unobservability and depth-of-two unobservability are achieved. Then, the SA was used to optimise the PMUs placement considering communication-constrained. This technique ensures a dispersed placement of PMUs that have communication facilities around the system whilst ensuring the distance between the unobserved buses and those observed is not too far, which allows the PMUs phased installation in the future.

Genetic algorithm (GA) is an optimisation based method that imitates the process of natural evolution. For instance, the complicated changing environment forces each species to adapt in order to prolong its survival. Each species is encoded in its chromosomes which will continue to transform when reproduction occurs. All of these changes will improve the species characteristics over time as the improved characteristics are inherited by future generations [24]. This analogy is translated to GA where individuals in a population, also known as chromosomes, represent potential solutions to a given problem, where when they are combined, they will produce a new individual that is close to the optimum. This process will continue until termination conditions are satisfied. In GA, three operators namely crossover, selection, and mutation are responsible for the combination process. These three operators are also responsible for increasing the population diversity in GA.

A non-dominated sorting genetic algorithm (NSGA) was proposed in [26] to solve OPP based on two competing objectives, which are minimisation of PMUs and maximisation of measurement redundancy. The NSGA initially used graph theory procedure and simple GA to estimate individual optimal solutions. Then, the NSGA was used to find the non-dominated solution that signifies the best trade-off between two competing objectives. The research work in [27] proposed the topology based formulated algorithm that uses GA to solve the OPP problem. The topology based formulated algorithm involves the merging process of zero-injection bus with one of its adjacent buses, similar to the one that has been discussed earlier. The results from the proposed method indicate that it can be used to solve OPP.

Aminifar et. al [28] proposed an immunity genetic algorithm (IGA) to determine the minimum number of PMUs required to make the power system observable. The IGA approach was developed to overcome the blindness in action of the crossover and mutation operators by curbing repetitive and useless work that may cause degeneracy from the evolution process. The results from the proposed method indicate that it converged quicker compared to the classic GA approach.

A topologic constraint-based GA to determine an optimal PMU configuration was also presented in [29]. The proposed method also introduced topology re-configuration rules (TRR) and PMU configuration rules (PCR) as well as PMU effective value (PEV) to evaluate the rank of each bus. Buses that have lower priority are removed from the possible PMU placements, hence, reducing infeasible solution space and time taken by the algorithm to finish.

Differential evolution (DE) is also an optimisation method that depends on the initial population generation, selection, cross-over, and mutation, similar to that of GA. However, in DE, a mutation operator does not depend on the predefined probability distribution function. Instead, a new arithmetic operator was introduced where the value is based on the differences between randomly selected pairs of individuals. The value is then added to a targeted third individual to form a new individual which will be evaluated by the fitness function. In the event where the new individual is better than the third individual, it will be replaced with the new individual.

DE algorithm adopted in [30] attempted to solve OPP considering zero-injection bus and measurement redundancy. In addition, the set of buses in a power system where a PMU must be installed and prohibited from being installed are also considered. The proposed method was applied on different IEEE buses system and the results are comparable with other published results.

The DE concept was also proposed by Rajasekhar et. al [31] to determine the required number of PMUs for fault observability of a power system. Zero-injection bus was also considered in this approach and the results of the proposed method were compared with the results obtained from the ILP method. The results showed the proposed method ensures global optimisation, contrary to the ILP method that tends to get trapped in local optima. In [32], a new DE algorithm based on Pareto non-dominated sorting was proposed to solve a multi-objective OPP problem, which in this case is to determine the minimum number of PMUs required and a single PMU losses. It is mentioned that many Pareto-optimal solutions can be found using this approach and its flexibility allows more objectives to be considered for future research.

For particle swarm optimisation (PSO), it is inspired by the movement of a flock of bird or fish schooling. A swarm of particles, where each particle represents a potential solution, is maintained by the PSO algorithm. The particles are flown through the search space, where the position of each particle is adjusted based on its own experience and neighbours. The experience in this case refers to the best position each particle has found with respect to the fitness function. Originally, PSO was developed for real-valued optimisation problem [33]. However, a discrete binary version of PSO (BPSO) was introduced later to solve discrete optimisation problem [34]. The OPP problem is formulated as a discrete problem, hence, BPSO was used in recent studies that aimed to solve the OPP problem. Detailed explanation of the PSO and BPSO algorithms is discussed further in Chapter 3.

Depth-of-unobservability concept introduced by Nuqui and Phadke [11] was adapted in BPSO by Sharma and Tyagi [35] to find the minimum number of PMUs required while maximising measurement redundancy. Rather et. al [36] incorporated constriction factor with BPSO and implemented the correction method to improve particles diversity. The correction method re-assigns PMUs that were initially placed on all buses adjacent to it in order to minimise the possibility of particles getting trapped in local minima. In addition, to reduce computation complexity, identical position arrays are filtered out in each iteration. In a research made by Ahmadi et al. [37], BPSO was used to find the minimum number of PMUs required considering normal operation, measurement redundancy, zero-injection bus and also power flow measurement. An enhanced PSO approach for power system as suggested in [38] was adopted in BPSO for optimal PMU placement in the study by Chakrabarti et. al [39]. The enhanced PSO approach involves the inclusion of additional rules for velocity updates if the particles could not find any feasible solution. The main principle of the method is to ensure that the knowledge of feasible solutions should always be used to drive the swarm to find the best solution. In the case where none of the particles have found a feasible solution, a random search is conducted to enhance the possibility of quickly finding a viable solution.

Similar to [39], Hajian et. al [40] also proposed a new velocity update equation in their approach to solve the OPP problem using BPSO. Besides the velocity update equation, the authors also introduced new observability rules to deal with cases involving zero-injection bus, loss of a single PMU, and also loss of a single branch. Maji and Acharjee [41] introduced a new approach to control inertia weight for BPSO called the exponential BPSO. Contrary to linear inertia weight, the inertia weight will exponentially decrease when the approach is used. The authors claimed it improves the algorithm's searching ability. In addition, mutation operator was also incorporated with the proposed approach. Hybrid BPSO was explored by Wang et. al [42] in dealing with OPP problem by combining simulated annealing and BPSO. The crossover and mutation techniques used in genetic algorithm are also incorporated to encourage particles diversity.

A probability constraint called Metropolis is used to decide if the mutated particle can replace the original particle. Ghaffarzadeh et. al [43] also explore the idea of solving optimal PMU placement using hybrid BPSO by introducing the combination of differential evolution (DE) and BPSO. The program is divided into two stages where during the first stage, the DE is used for a global search while for the second stage, BPSO is used to refine the best solution obtained during the first stage. Overall, the authors claimed it is intended to balance the trade-off between crude global search and intensive local search provided by both DE and BPSO. Besides minimising the number of PMUs, the cost of PMU is also considered as the optimisation problem.

2.3 Phasor Measurement Unit

2.3.1 Phasor

To determine the state of a power system, the values of all state variables, namely voltage magnitudes and phase angles of all system buses, need to be known [44]. Knowing these values allows the calculation of active and reactive power flows through the power system. Due to the distance that separates the system nodes, determining the values of all state variables is a difficult task, in which synchrophasor measurements are capable of solving it with ease.

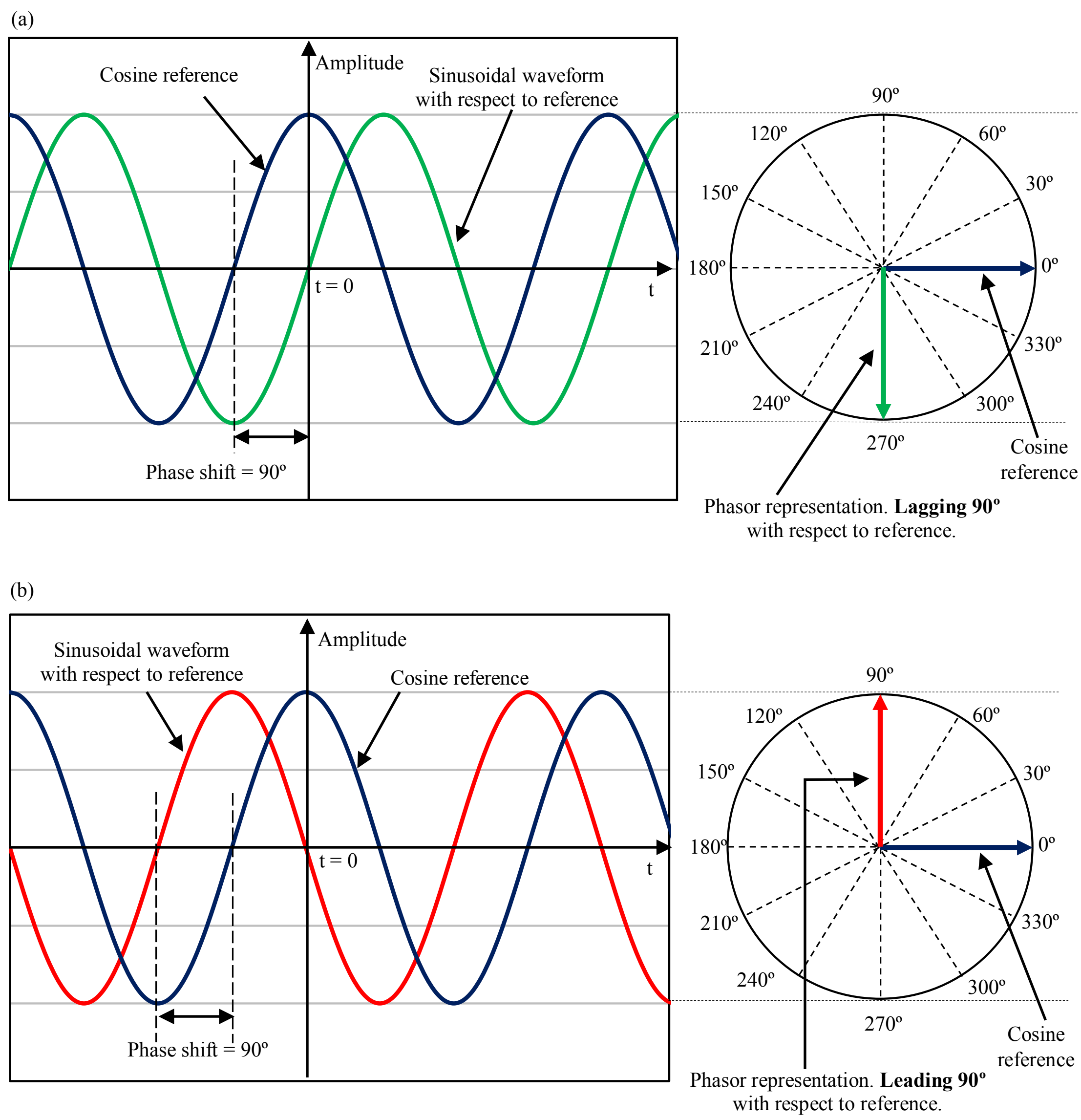

Figure 2.2: Phasors data received from two remotes location, (a) phase angle = -90 degree in respect to cosine reference, (b) phase angle = 90 degree in respect to cosine reference

Synchrophasor measurements refers to the precise time-synchronised measurements of magnitude and phase angle of the sine waves found in electric grid. The magnitude and phase angle of voltage and current sinusoidal waveforms at a certain point in time denote a complex number which is referred to as phasor. The magnitude is determined based on the amplitude of the sinusoidal waveform, while the phase angle is decided with respect to the same time reference as shown in Figure 2.2. In Figure 2.2(a), the sinusoidal waveform is lagging with respect to the cosine reference. According to the IEEE C37.118 communication protocol [45], since it is lagging, the phase angle is negative, whereas in Figure 2.2(b), the sinusoidal waveform is leading with respect to the cosine reference, hence, the phase angle is positive. Since the PMU is equipped with GPS, each phasor measurement is time-stamped, hence, allowing phasor measurements taken by the PMUs in multiple locations to synchronise.

2.3.2 Structure of PMU

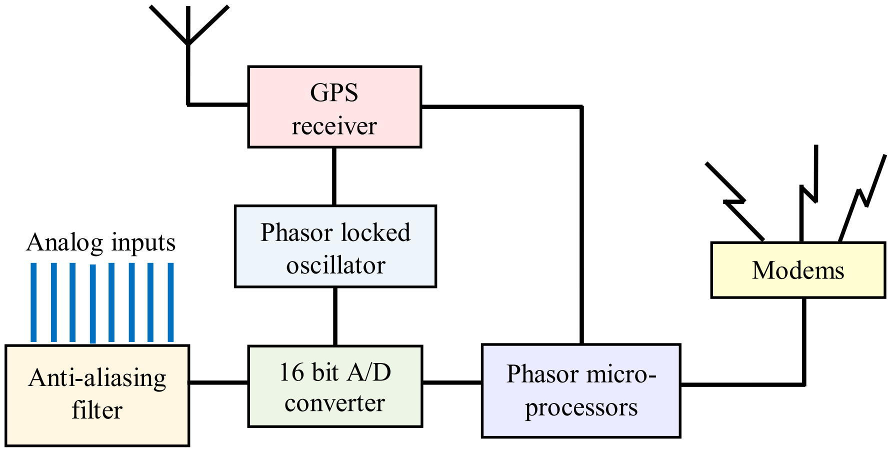

Figure 2.3: Block diagram of PMU

Figure 2.3 shows the block diagram of a PMU. Firstly, the input signals are received in analogue form. The input signals consist of measured currents and voltages from voltage and current transformers. Then, it is processed by the anti-aliasing filter to remove high-frequency components that exceed the Nyquist sampling rate [46]. Hence, the frequency components that exceed the Nyquist sampling rate will be suppressed. The phase locked oscillator is responsible for dividing the GPS 1 pulse per second into the required number of pulse per second with respect to the waveform sampling. Then, by using a 16-bit precision A/D converter, the analogue signal will be digitised at sampling instants defined by the sampling time signals from the phase locked oscillator before it is fed to the phasor microprocessor. The phasor microprocessor is then responsible for re-sampling the signals and calculating the positive sequence from the digitised sample data. The calculated phasor is then time-stamped before it is uploaded to a proper communication channel using modems.

2.4 Application of PMUs in power system

a) **Power system monitoring**

PMU was fundamentally designed to monitor the operation of power systems. The first step of PMUs deployment for wide-area monitoring was during staged tests performed in 1992 where the PMUs were placed in six different places. The PMUs were manually triggered and the measurements data were recorded. The measurements data obtained from the tests proved that the PMUs are dependable in giving accurate phasor measurements [47]. The success of the tests prompts more PMUs installation across multiple places in the US and other countries for further validation tests. In light of the major blackout that happened in the Northeastern US in 2003, PMU measurements were identified to be important in the post-mortem analysis [48]. The final report mentioned that power systems are in need of time synchronised data and consequently recommended all electric utilities to employ PMUs. Currently, the installation of PMUs in power systems is happening around the world where wide area measurement systems (WAMS) are being built around it to ensure various monitoring, protection, and control applications can be developed in central locations [49].

b) **Power system protection**

The benefits provided by the PMU in power system monitoring also extend to power system protection by offering adaptive protection. The synchronised phasor measurement allows relays and protection scheme to adapt to the current system conditions. This is not possible when using the old way of protective power system because it responds on faults based on a predetermined manner [50]. This leads to wrong assumptions to be made and consequently misjudgement of prevailing system condition occurs. In addition to adaptive protection, PMU measurements are also introduced to the fault location technique. Fault location techniques are used in power systems for accurate pinpointing of the faulty position. However, the accuracy of fault location has the tendency to suffer from errors such as variations of source impedance, fault incidence angle, line asymmetry, and loading conditions [51]. The synchronised measurements from the PMUs can rectify this problem [52] and its effectiveness has been verified based on the practical operation in Taiwan power utility [53].

c) **Power system control**

Power system control was primarily local prior to the introduction of real-time phasor measurements. Any control actions taken were made based on local measurements which restrict the ability of the local control of wide area level. The PMU's ability to provide synchronised phasor measurements overcomes this issue since the phasor data will be time tagged so that the control could be based on the actual state of the system [54]. This encourages more development to be done to enhance the robustness of power systems in wide-area monitoring such as wide-area power system stabilisers [55] and wide-area damping controller [56].

2.5 Optimal PMUs placement

PMU is a crucial component in smart grid that requires some considerations if one decides to implement it in a power system. Cost is one of the factors that need to be put into consideration when deciding PMUs implementation. For that reason, it is of the highest priority for electric utilities to plan properly PMUs installation where reliability is maintained while minimising the cost involved. The investigation to find the minimum number of PMUs required while maintaining the complete observability of a power system is referring to the optimal PMUs placement (OPP) problem. The OPP problem mainly focuses on finding a way to determine the minimum number of PMUs required in a power system without compromising power system observability.

The two factors contradict each other where the highest reliability of a power system is achievable with more PMUs installed but the cost involved in the installation will be massive for electric utilities [25]. However, due to the PMU's ability to directly measure branch currents that are adjacent to a PMU installed bus, buses that are adjacent to the PMU installed bus can be made observable when applying the Ohm's law, thereby, avoiding the need for placing PMUs at every bus in a power system [7], [15]. Therefore, numerous studies conducted in the recent years have tried to find a strategic placement of PMUs that can guarantee the power system is observable despite using a small number of

PMUs. Generally, the minimum number of PMUs required to ensure a power system to be observable is within 20-30% of the number of system buses [25].

2.6 Factors considered in solving OPP

In order to solve the OPP problem, studies conducted over the years have introduced several problem constraints to find the best PMUs placement set for different working conditions. The factors are either to take advantage of the power system characteristic to further minimise the number of PMUs required or to ensure the power system remains observable should any contingencies occurred. The following is a brief description of the factors introduced as the problem constraints:

a) **Normal operation**

In normal operation, the OPP problem will be solved by ignoring all factors mentioned below.

b) **Zero-injection bus**

Zero-injection bus (ZIB) refers to the bus in a power system that has no generator or load injected into it. Hence, by considering the presence of ZIB in a power system, fewer PMUs can be used to maintain the power system observability.

c) **Single PMU loss**

As most electrical devices, PMU is also no exception for failure. If one of the PMUs malfunction, the bus that was monitored by the PMU will become unobservable if the bus has no other PMUs monitoring it. Hence, to address this issue, every bus in a power system must be observed by at least two PMUs. That way, if one of the PMUs becomes defective, the observability of the power system will remain.

d) **Channels limit**

According to [15], a PMU from different manufactures does have a finite number of channels. Hence, the PMU will be limited to a certain number of branch currents and bus voltages that it can monitor at one time. Therefore, the PMUs placement set will be different for certain number of channels.

2.7 Observability rules for PMUs

Before explaining about observability, it is important to note the PMU placement model adopted in this thesis. The PMU placement model usually consists of three component types – buses, branches, and injection. Buses in this placement model are referring to substations which are capable to have PMU installed on it. These buses must also be equipped with communication facilities in order for the PMU to operate properly. Meanwhile, branches signify the passages between two neighboring buses where the impedance are assumed to be known. Finally, injection is a variable load or source so that injected current is supplied to the bus [57].

The above descriptions do not take into consideration factors such as physical locations, component states, or the number of transformers in a substation. This is to ensure a simplified topology of a real electric system that can be modelled as a platform for algorithms to easily and quickly used to find the minimal number of PMUs for a complete observability. Observability analysis of a power system can be categorised into two categories — numerical analysis and topological analysis. Numerical analysis is based on whether the measurement gain or Jacobian rank is of full rank, whereas topological analysis relies on whether a spanning tree of full rank can be constructed [23]. In this thesis, topological observability is adopted, in which information concerning network connectivity, measurement type, and their locations are required. In addition, the analysis made is strictly based on logical operations and will not consider any real parameters of the network elements [58].

Firstly, in order to identify a power system as observable, the voltages for all of its buses must be known. The voltage and branch current of a bus can be measured directly by a PMU if it is installed at the bus or indirectly measured using other known parameters such as other bus voltage and branch current. For indirect measurements, using the Ohm's law, buses that are neighbours to the PMU installed bus can have their voltages and branch currents known through calculation. Therefore, every bus can be made observable if the PMUs are strategically placed in a power system. The following describes the observability rules used to determine the network observability of a power system:

Rule 1: A PMU installed bus will have its voltage phasor and all branch currents phasors adjacent to it directly measured by the PMU.

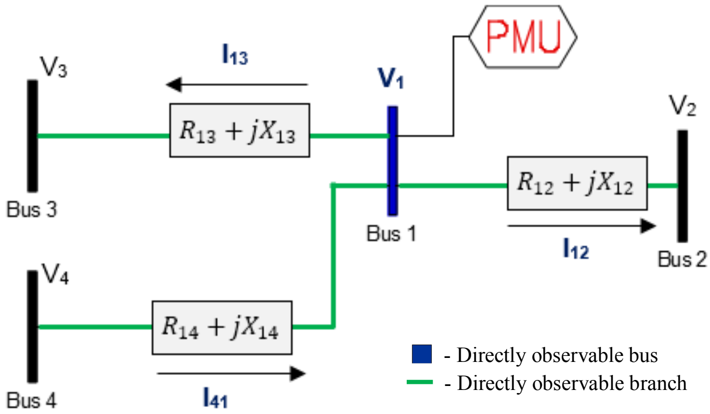

Figure 2.4: Modelling observability for rule 1

Consider Figure 2.4, where bus 1 has a PMU installed on it. Therefore, the values of V_1, I_{12}, I_{13}, and I_{41} can be measured directly by the PMU.

Rule 2: If the voltage at one end and its branch current are known, the voltage at the other end can be calculated.

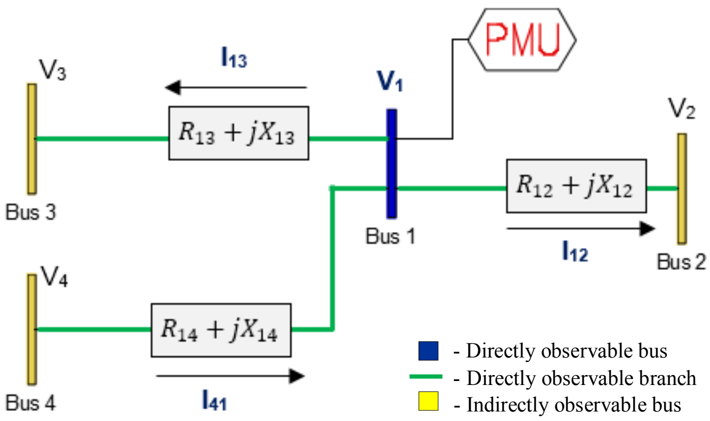

Figure 2.5: Modelling observability for rule 2

Since the value of I_{12}, I_{13}, and I_{41} are known, the voltages at bus 2, 3, and 4 can be calculated using the Ohm's law. The values for V_2 and V_3 are the consequences of V_1 minus the voltage drop caused by the current travelling through the branch current. The following shows how the value of V_2, V_3 and V_4 are calculated.

$$I_{12} = \frac{V_1 - V_2}{R_{12} + jX_{12}}$$
$$V_2 = V_1 - I_{12}(R_{12} + jX_{12}) \tag{2.1}$$

$$I_{13} = \frac{V_1 - V_3}{R_{13} + jX_{13}}$$
$$V_3 = V_1 - I_{13}(R_{13} + jX_{13}) \tag{2.2}$$

$$I_{41} = \frac{V_4 - V_1}{R_{14} + jX_{14}}$$
$$V_4 = V_1 + I_{41}(R_{14} + jX_{14}) \tag{2.3}$$

Rule 3: In the situation where the voltages at both ends are known, the branch currents between the two buses can be calculated according to the Ohm's law.

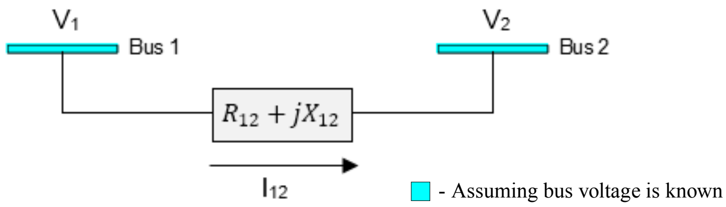

Figure 2.6: Modelling observability for rule 3

Consider Figure 2.6 above where the value of V_1 and V_2 are known and the branch current I_{12} is unknown. Assuming that the current flows from bus 1 to bus 2, by applying the Ohm's law, the value of I_{12} can be calculated as follows:

$$I_{12} = \frac{V_1 - V_2}{R_{12} + jX_{12}} \tag{2.4}$$

Rule 4: In a situation where power flow measurement is present at certain branches, if one end of the branch has its voltage known, the voltage at the other end of the branch will also be known.

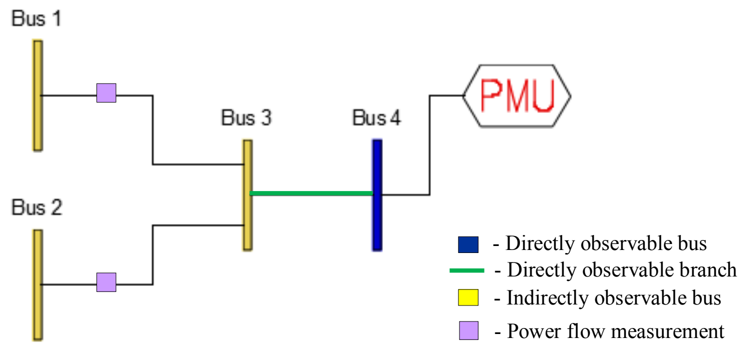

Figure 2.7: Modelling observability rule when considering power flow measurement

To explain the observability rules when considering power flow measurement, consider Figure 2.7 where the power flow measurement is located at branch 1-3 and 2-3 and a PMU is placed at bus 4. The presence of power flow measurement at branch 1-3 and 2-3 means if the voltage at bus 3 is known, the voltage at bus 1 and bus 2 can be calculated through indirect measurement since the branch currents for 1-3 and 2-3 are known. In this case, the voltage at bus 3 is known due to the placement of PMU at bus 4, therefore, ensuring that bus 1 and 2 are observable.

2.7.1 Observability rules when considering ZIB

Zero-injection bus (ZIB) is referring to a bus that has no generator nor load injected into it. Therefore, the sum of branch currents applied at ZIB is zero according to the KCL. If ZIB, including its neighbours have N_Z members, observing $N_Z - 1$ buses is enough to make the unobservable bus become observable. Hence, when considering ZIB, the number of buses to observe is reduced by one for each ZIB available in a power system, subsequently reducing the number of PMUs needed for a complete observability.

In order to assess the topological observability when considering ZIB, the following observability rules are applied:

Rule 5: If all buses adjacent to an *observable ZIB* all are observable except one, the unobservable bus can be deemed observable. For example, if bus 1, 2, and 3 are observable (their voltages are known), V_4 can be calculated when the KCL is applied at bus 2 (ZIB).

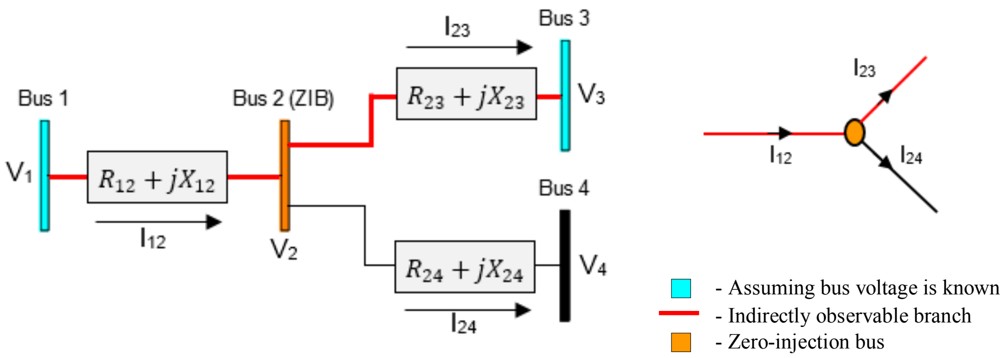

Figure 2.8: Modelling observability for rule 5 when considering ZIB

Consider Figure 2.8 above where bus 2 is an *observable ZIB*. Assuming the value of V_1, V_2, and V_3 are known, the value of I_{12} and I_{23} can be obtained using the Rule 3 earlier. Thefore, by applying the KCL at bus 2 and assuming the current flows as depicted in Figure 2.8, the value of I_{12} is $I_{12} = I_{23} + I_{24}$. Hence, to obtain the value of I_{24} and consequently V_4, both are calculated as follows:

$$I_{24} = \frac{V_2 - V_4}{R_{24} + jX_{24}} \tag{2.5}$$

$$\begin{aligned} V_2 - V_4 &= I_{24}(R_{24} + jX_{24}) \\ V_4 &= V_2 - I_{24}(R_{24} + jX_{24}) \end{aligned} \tag{2.6}$$

Rule 6: If buses adjacent to an *unobservable ZIB* are all observable, the ZIB can be deemed observable.

In this instance, if all buses adjacent (bus 1, 3, and 4) to an *unobservable* ZIB are all observable, the voltage for the *unobservable ZIB* can be determined through calculations.

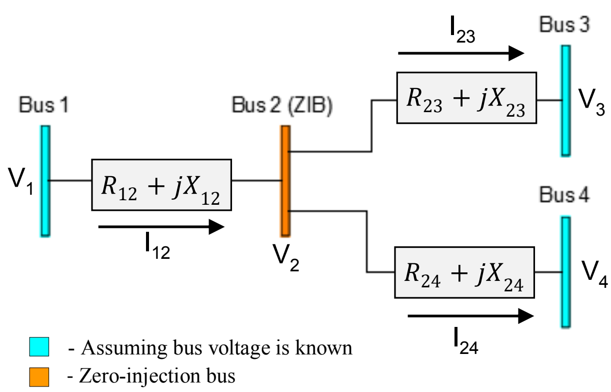

Figure 2.9: Modelling observability for rule 6 when considering ZIB

$$\begin{aligned} V_2 &= V_1 - I_{12}(R_{12} + jX_{12}) \\ V_2 &= V_3 + I_{23}(R_{23} + jX_{23}) \\ V_2 &= V_4 + I_{24}(R_{24} + jX_{24}) \\ 0 &= I_{12} - I_{23} - I_{24} \end{aligned} \tag{2.7}$$

2.8 OPP formulation

Generally, the main objective in the OPP problem is to find the minimum number of PMUs required including its placement to achieve a complete observability of the power system. Therefore, the objective function for OPP problem can be formulated as follows [59]:

$$J = \min\{s(p)\} \tag{2.8}$$

$$\text{subject to } R(p) = 0$$

Where $s(p)$ is the number of PMUs being placed and $R(p)$ is the number of unobservable buses. As can be seen from above, the number of unobservable bus must be zero to indicate the power system is completely observable by the PMUs placement set.

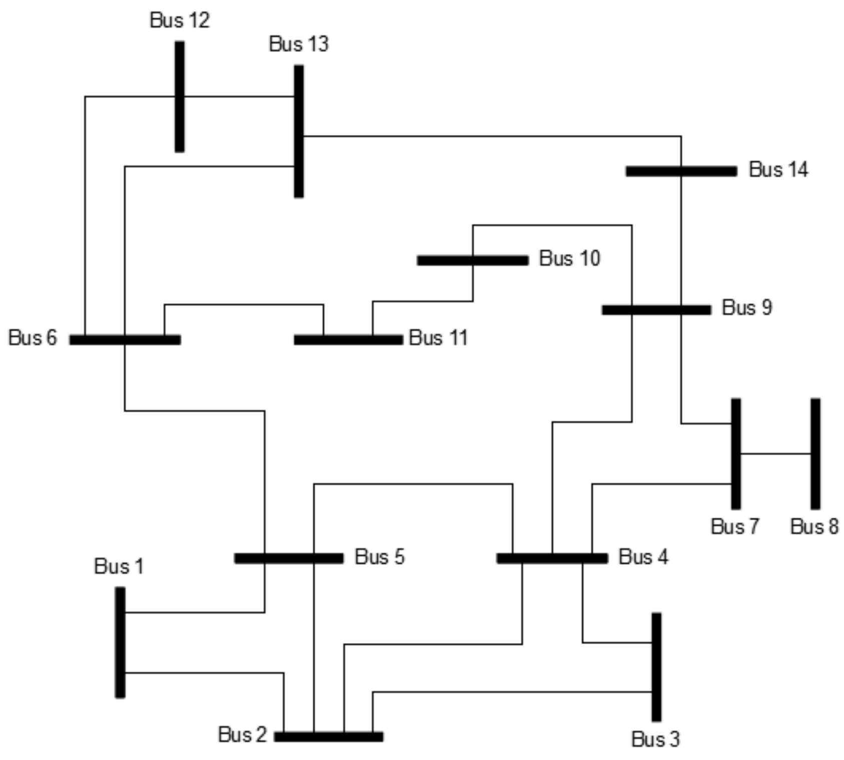

Figure 2.10: IEEE 14-bus system diagram

Since the problem is formulated as binary, a binary representation of the connection between buses in a power system need to be defined. A binary connectivity matrix $[A]$ that indicates connection between buses in a power system can be formed into a simple format in matrix to make ease for the task for topological analysis and solving OPP problem in this thesis using equation (2.9) as follows:

$$A_{ij} = \begin{cases} 1 & \text{if } i = j \\ 1 & \text{if } i \text{ and } j \text{ are connected} \\ 0 & \text{otherwise} \end{cases} \tag{2.9}$$

Based on equation (2.9), the entry is 1 if bus i and bus j are connected, and zero otherwise. In addition, the diagonal entries will also be set as 1. The binary connectivity matrix $[A]$ for the IEEE 14-bus system as depicted in Figure 2.10 is given as follows:

$$[A] = \begin{bmatrix} 1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 1 \end{bmatrix} \tag{2.10}$$

From the binary connectivity matrix in equation (2.10), the constraints can be formulated as follows:

$$f(x) = \begin{array}{ll} f_1 = x_1 + x_2 + x_5 & \geq 1 \\ f_2 = x_1 + x_2 + x_3 + x_4 + x_5 & \geq 1 \\ f_3 = x_2 + x_3 + x_4 & \geq 1 \\ f_4 = x_2 + x_3 + x_4 + x_5 + x_7 + x_9 & \geq 1 \\ f_5 = x_1 + x_2 + x_4 + x_5 + x_6 & \geq 1 \\ f_6 = x_5 + x_6 + x_{11} + x_{12} + x_{13} & \geq 1 \\ f_7 = x_4 + x_7 + x_8 + x_9 & \geq 1 \\ f_8 = x_7 + x_8 & \geq 1 \\ f_9 = x_4 + x_7 + x_9 + x_{10} + x_{14} & \geq 1 \\ f_{10} = x_9 + x_{10} + x_{11} & \geq 1 \\ f_{11} = x_6 + x_{10} + x_{11} & \geq 1 \\ f_{12} = x_6 + x_{12} + x_{13} & \geq 1 \\ f_{13} = x_6 + x_{12} + x_{13} + x_{14} & \geq 1 \\ f_{14} = x_9 + x_{13} + x_{14} & \geq 1 \end{array} \tag{2.11}$$

The operator "+" in equation (2.11) signifies logical "OR" and the value of 1 in the right hand side of the constraints indicates that at least one of the parameters for each constraint must be non-zero. For instance, consider the constraint for f_8 above. The constraint means in order to make sure bus 8 is observable, at least one PMU must be placed at either bus 7 or 8 (or both).

The placement of PMUs can be defined as [X], which acts as a binary decision variable vector where it is formulated as follows:

$$[X] = [x_1 \quad x_2 \quad x_3 \quad \cdots \quad x_N]^T, \text{ where } x_i \in \{0,1\} \tag{2.12}$$

$$x_i = \begin{cases} 1 & \text{if a PMU is installed at bus } i \\ 0 & \text{otherwise} \end{cases} \tag{2.13}$$

2.8.1 Radial bus

A strategic placement of PMUs allows a power system to be made observable with a small number of PMUs. Assuming that a PMU has an unlimited channel, it is evident that placing the PMU at a bus that has many adjacent buses will provide a better network coverage compared to a bus that has very few buses adjacent to it, especially radial bus. Radial bus refers to a bus that only has one neighbour or a bus adjacent to it. Therefore, if a PMU is placed at a radial bus, the maximum number of bus that a PMU can observe is restricted to two buses only – the radial bus and its neighbour. In contrast, placing a PMU at the bus adjacent to radial bus extends the coverage to more bus in addition to the radial bus. Hence, radial bus can be excluded from the PMU candidate placement while at the same time have a PMU placed at the bus adjacent to the radial bus.

2.8.2 Modelling of ZIB

As mentioned earlier, the presence of ZIB can help in further reducing the number of PMUs needed to achieve a complete observability of a power system compared to the normal operation. There are many methods that have been proposed in prior studies in order to deal with ZIB. One of the methods proposed to deal with the existence of ZIB in a power system is the *bus merging* method. The bus merging method involves a merging process between ZIB and one of its neighbours. Hence, in the merging process, the constraints for the two buses will be merged into a single constraint, thereby, reducing the number of constraints that need to be satisfied to ensure every bus is observable by the PMUs placement set. From the observability rules, it is explained that when considering ZIB, if all buses are connected to the ZIB are observable except for one, the unobservable bus can be identified as observable. Therefore, the merged bus implies that if it is observable, the bus that was selected to merge will be observable as well.

In order to explain the bus merging method, consider the IEEE 14-bus system as depicted in Figure 2.10 where bus 7 is a ZIB and it is connected to bus 4, 8, and 9. Generally, there are three strategies that can be used in selecting a candidate bus to merge with ZIB. The three strategies are as follows:

a) *Merge the ZIB randomly with one of its neighbours:*
   In this case, bus 7 is merged with one of its neighbours. For example, bus 7 is merged with bus 9.

b) *Merge the ZIB with its neighbour that has the lowest number of bus adjacent to it:*
   With this strategy, bus 7 is merged with bus 8, which only has one bus adjacent to it.

c) *Merge the ZIB with one of its neighbours that has the highest number of bus adjacent to it:*
   Contrary to the earlier strategies, bus 7 is merged with bus 4, which has five buses associated with it, compared to bus 9 who has four buses connected to it.

Although a bus merging method can be used to find the minimum number of PMUs required when dealing with the existence of ZIB, there are shortcomings that need to be highlighted:

i) If a PMU is required to be placed at the merged bus, it could possibly mean a PMU need to be placed at ZIB, or at the bus selected to merge with ZIB, or at both buses. This situation requires an additional observability test to verify which of the two buses that must be installed with the PMU.
ii) Everytime a merging process takes place, the topology of the system changed. Hence, it could make the topology become complex when dealing with larger system.

2.8.3 Proposed topology transformation method using ZIB for ILP

The work in [13] investigated the effects of the three strategies mentioned in the previous section towards the final PMU placement set obtained from the merging process. It appears that the final PMUs placement set correlates with the bus selected to merge with ZIB. Therefore, the bus selected to merge with ZIB is important when using the bus merging method.

A new bus selection rules for the bus merging method is proposed in this thesis using the ILP method where it gives the accurate PMUs location [16]. Furthermore, the new bus selection rules proposed in this thesis produce a better quality solution compared to the prior proposed bus merging method.

There are four selection rules where each candidate bus needs to be evaluated. Here, the candidate bus means buses that are neighbours to ZIB. The selection rules act as the steps where each candidate bus need to be evaluated.

Rule 1: Candidate bus must not have been merged previously.

Rule 2: Merge the ZIB with radial bus if radial bus is adjacent to ZIB.

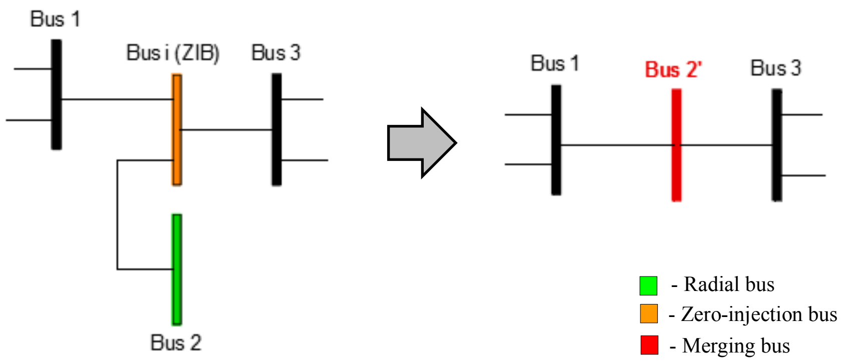

Figure 2.11: Modelling ZIB merging for rule 2, bus *i* is merged with bus 2

Rule 3: If rule 2 is not satisfied, merge the ZIB with the adjacent bus that has the most number of branches and one of its neighbours *must* be connected to the same ZIB and *must not* be a ZIB itself. If there is more than one bus that can apply this rule, randomly pick either bus.

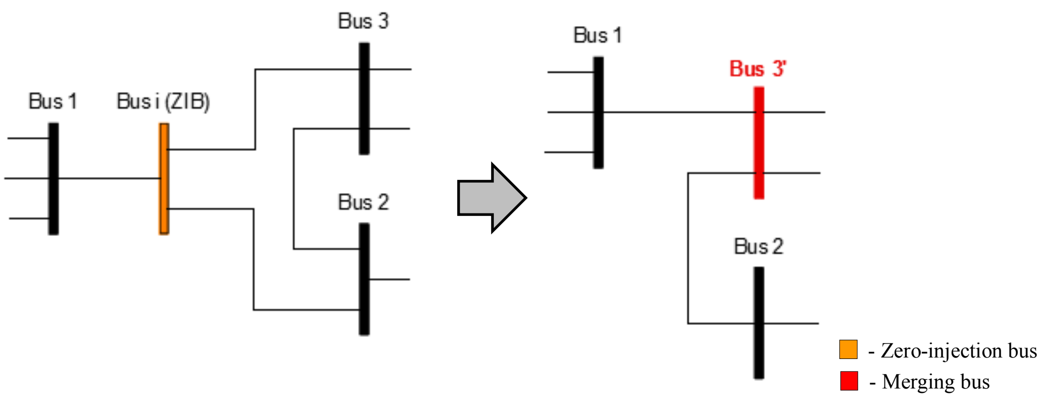

Figure 2.12: Modelling ZIB merging for rule 3, bus *i* is merged with bus 3

Rule 4: Lastly, if the previous rules are not satisfied, merge ZIB with the adjacent bus that has the most number of branches connected to it.

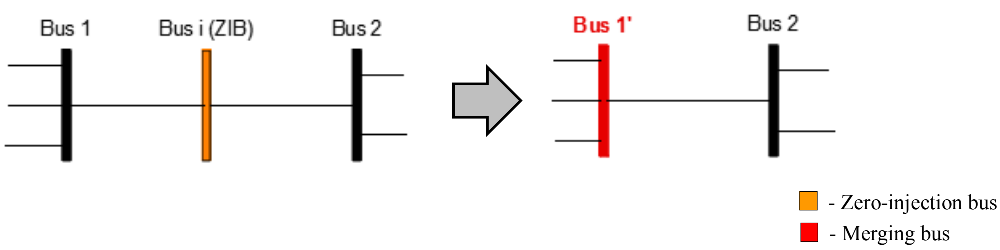

Figure 2.13: Modelling ZIB merging for rule 4, bus *i* is merged with bus 1

It is important to note that a bus can only merged once. Once a bus is merged, it will be excluded from the future merging process.

2.8.3.1 Illustration of the proposed topology transformation method

In order to demonstrate the proposed topology transformation method, the IEEE 30-bus system is used and illustrated in Figure 2.14.

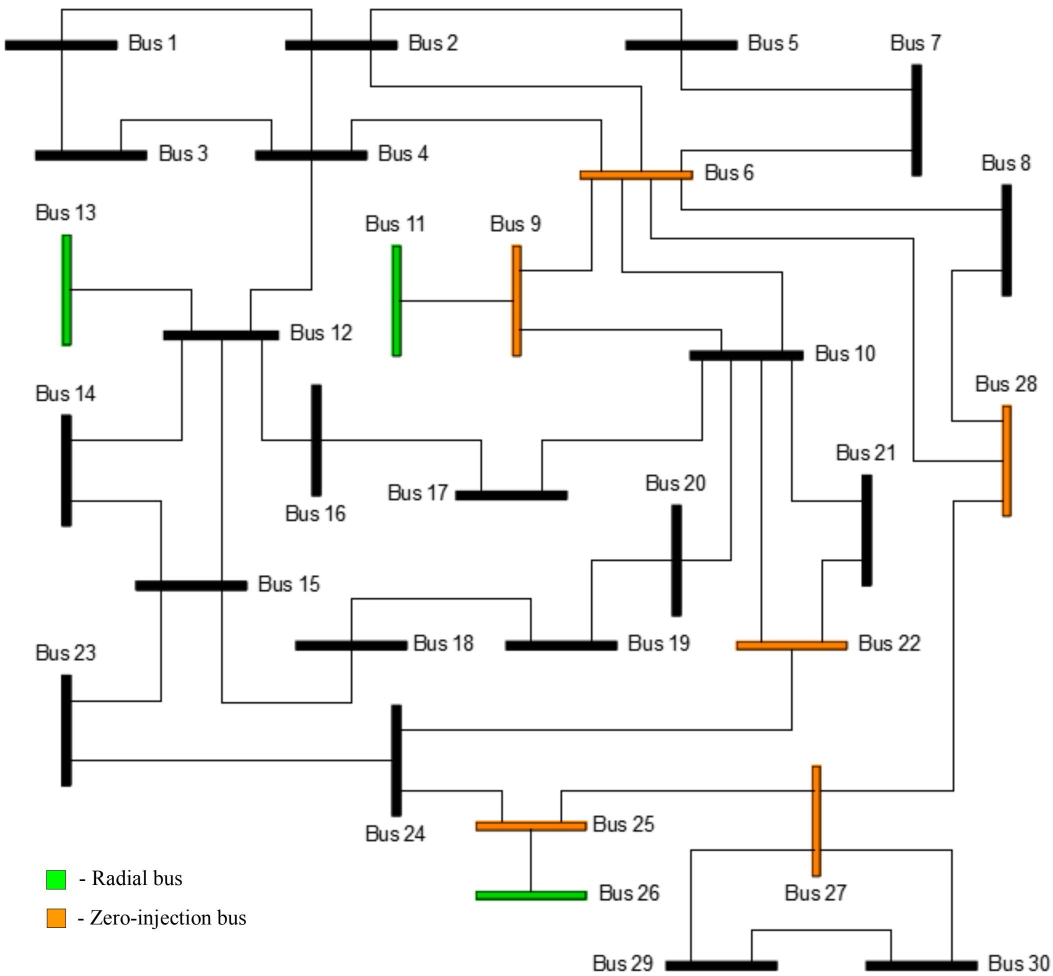

Figure 2.14: IEEE 30-bus system before topology transformation method

There are six ZIB which are bus {6, 9, 22, 25, 27, 28} and three radial bus {11, 13, 26}. The topology transformation process can be explained as follows for each ZIB.

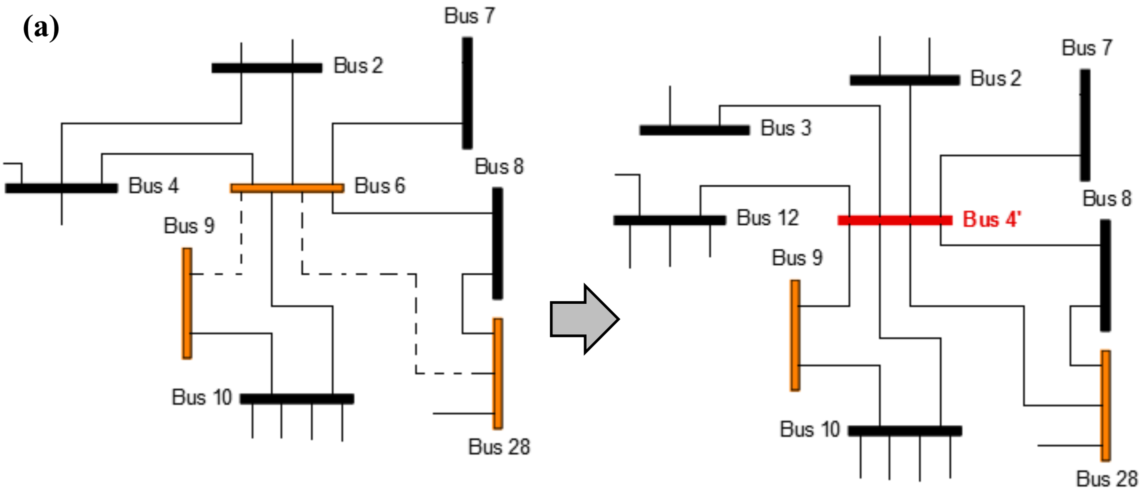

Figure 2.15: The bus merging process for bus 6

The first ZIB is bus 6. From Figure 2.15, it is evident that bus 10 has the most bus connected to it and one of its neighbours is connected to the same ZIB. However, its neighbour that is connected to the same ZIB, which is bus 9 is a ZIB itself, therefore it is invalid for rule 3. This allows bus 2 and bus 4 to be evaluated as the candidate bus to be merged with the ZIB. Since both bus 2 and bus 4 have the same number of branches, bus 4 is randomly picked to merge with ZIB in this case.

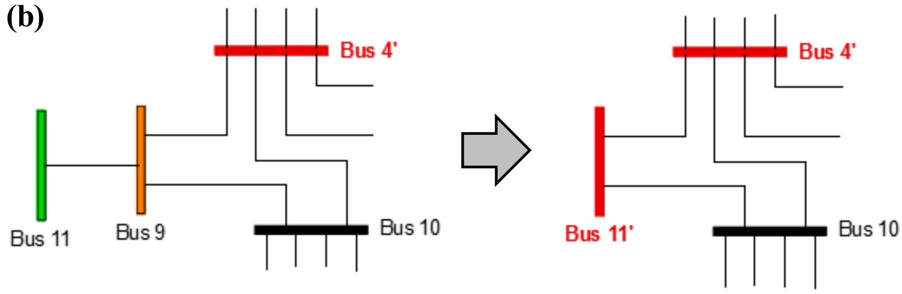

Figure 2.16: The bus merging process for bus 9

The second ZIB is bus 9. In this instance, bus 11 is a radial bus that is connected to bus 9, which is a ZIB. Hence, the radial bus is merged with bus 9 as shown in Figure 2.16.

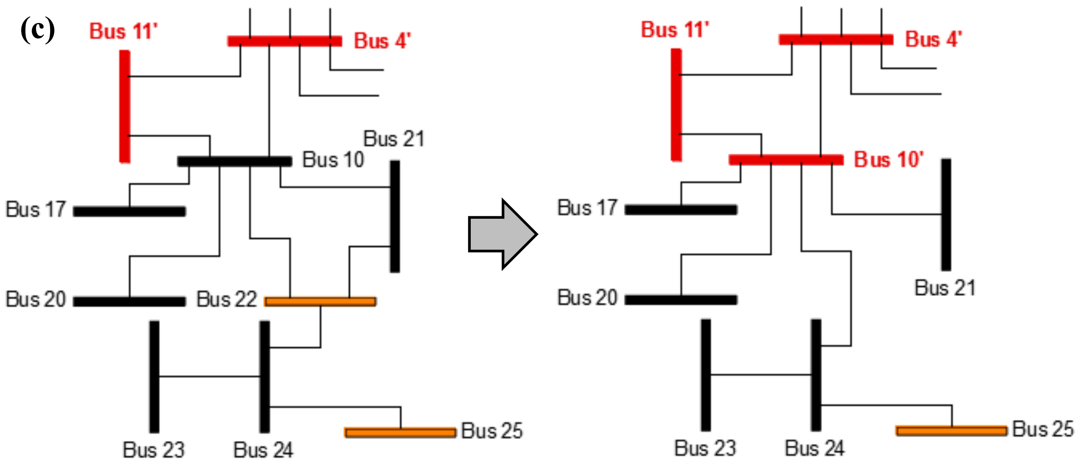

Figure 2.17: The bus merging process for bus 22

For the third ZIB which is bus 22, rule 3 can be applied to bus 10 where it has the most number of branches connected to it and also has a neighbour that is connected to the same ZIB which is bus 21. Hence, bus 10 is chosen as the candidate bus to be merged with bus 22 as shown in Figure 2.17.

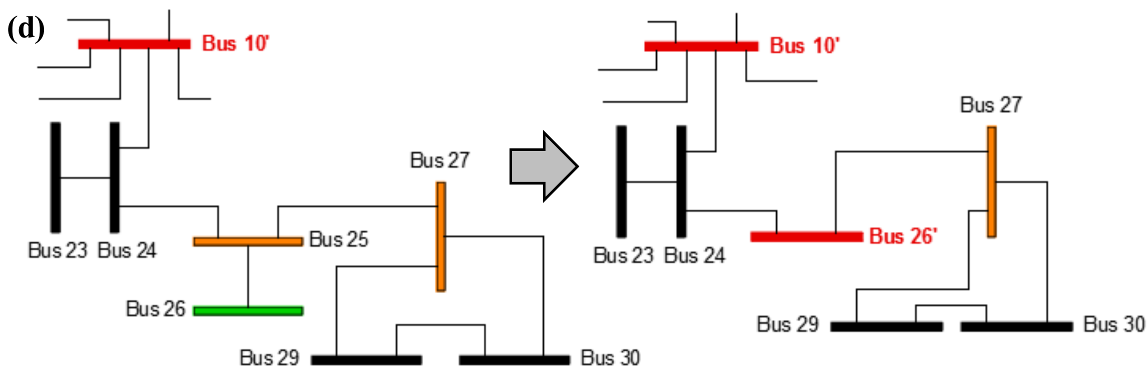

Figure 2.18: The bus merging process for bus 25

The fourth ZIB, which is bus 25, is also similar to that of bus 11 because one of its neighbours (bus 26) is a radial bus. Hence, bus 25 is merged with bus 26 as illustrated in Figure 2.18.

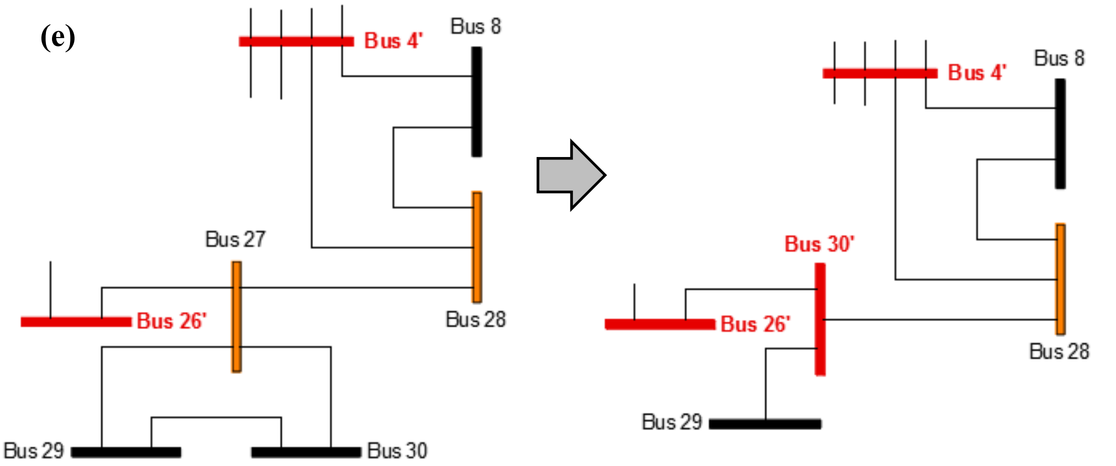

Figure 2.19: The bus merging process for bus 27

In the fifth ZIB, bus 30 is randomly picked to be merged with bus 27 since both bus 29 and bus 30 have the same number of bus connected to them as shown in Figure 2.19.

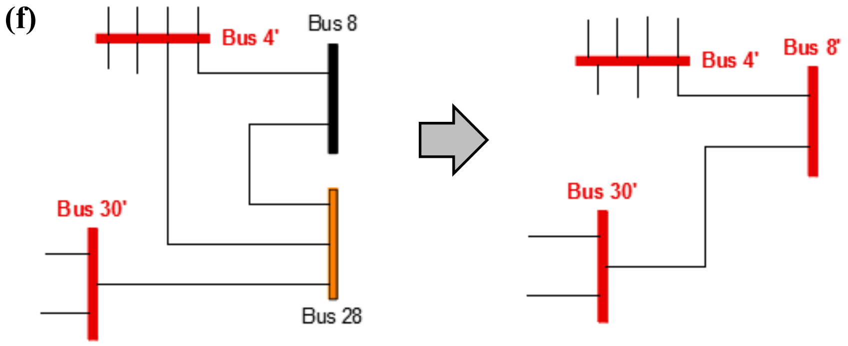

Figure 2.20: The bus merging process for bus 28

For the final ZIB, which is bus 28, since all of its neighbours are already merged, there is only neighbour that is possible for merging which is bus 8. Hence, it is merged with bus 8.

The merging process explained earlier in the IEEE 30-bus is now completed and the consequence of the topology transformation method is shown in Figure 2.21. Notice that the number of bus is reduced from 30 bus to 24 bus due to the bus merging process. As mentioned earlier, for each ZIB present in the power system, one bus is “eliminated” from the power system.

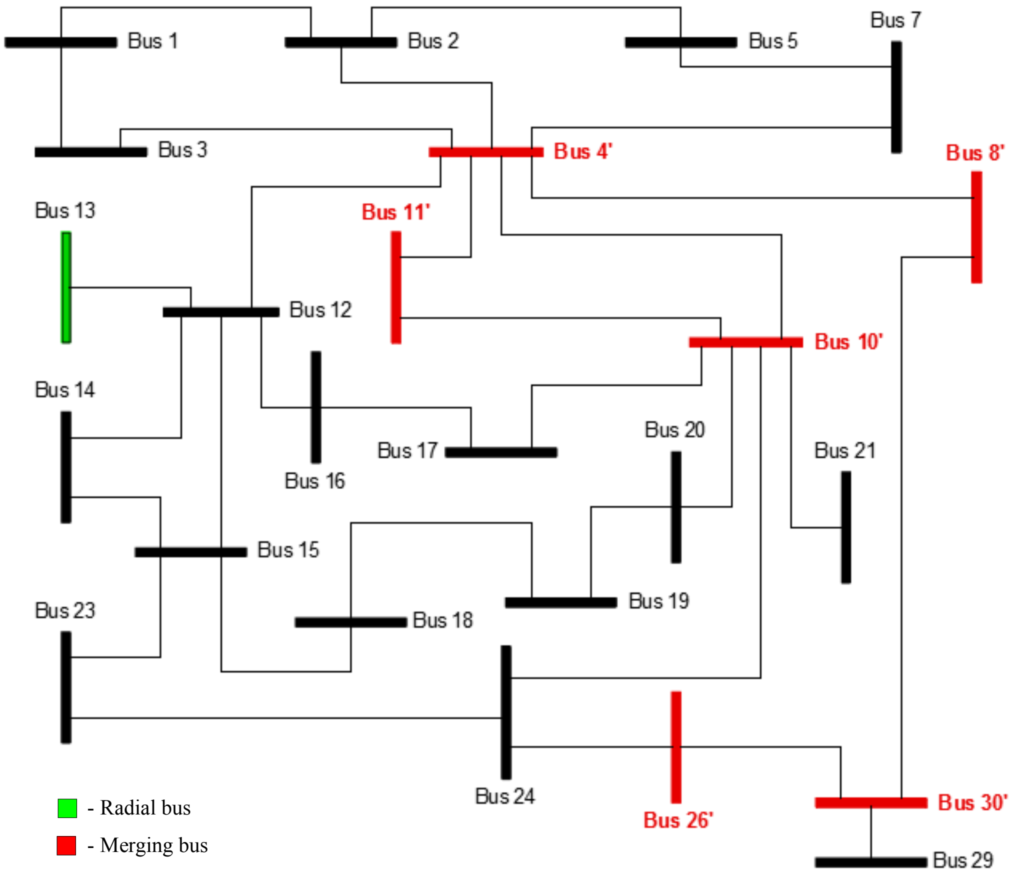

Figure 2.21: IEEE 30-bus system after topology transformation method

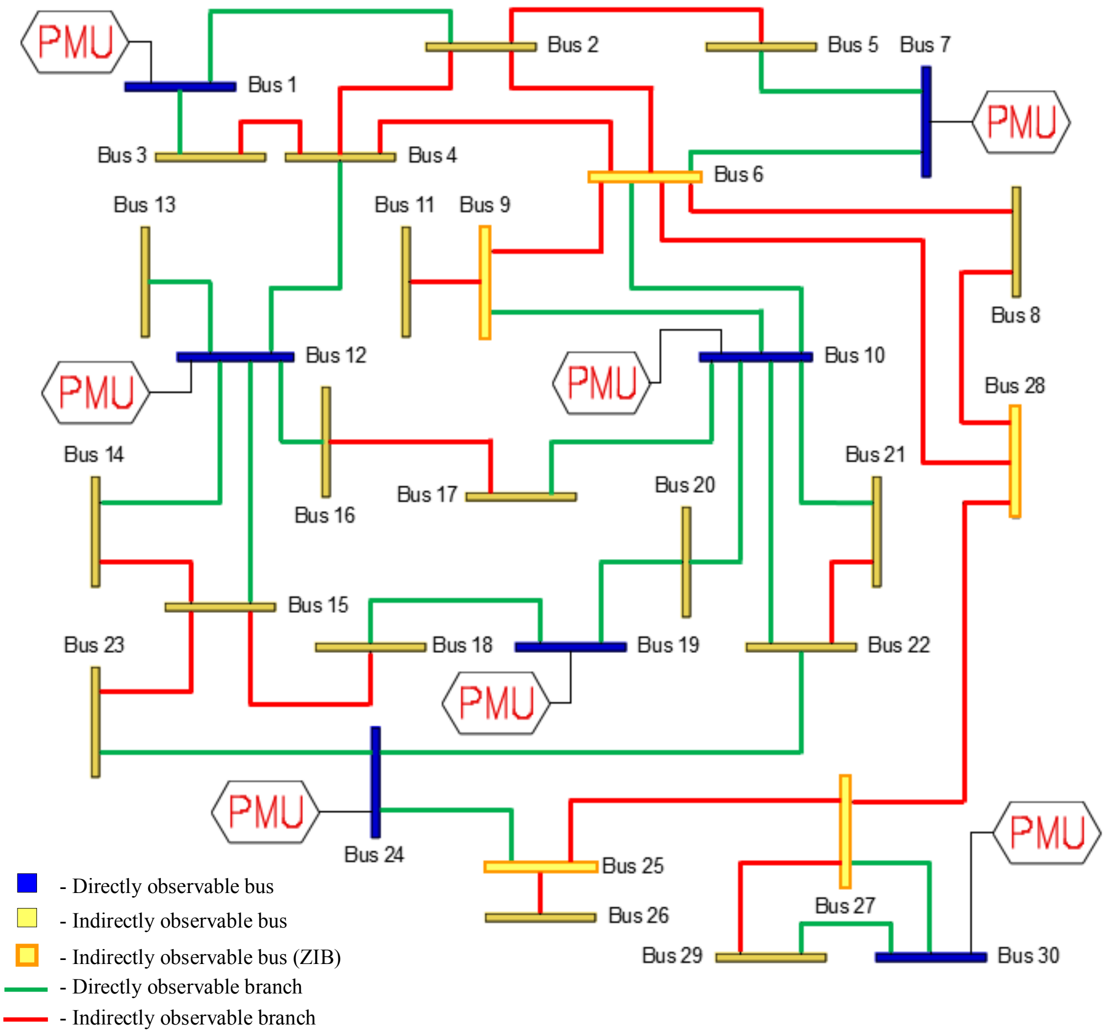

Figure 2.22: Complete observability based on the topology transformation method for IEEE 30-bus system

Figure 2.22 above shows the location of PMUs obtained using the ILP method where there are 7 PMUs being placed at bus 1, 7, 10, 12, 19, 24, and bus 30. In addition, observability analysis of the PMUs placement is also illustrated in Figure 2.22.

2.8.4 Channels limit

Most studies assumed that a PMU has an unlimited number of channels when solving the OPP problem. However, as mentioned earlier, a PMU does have a limited number of channels. This means that in order to ensure a PMU can be used to observe the voltage of the bus it is installed including all branch currents adjacent to it, the PMU needs to have at least the same number of channels.

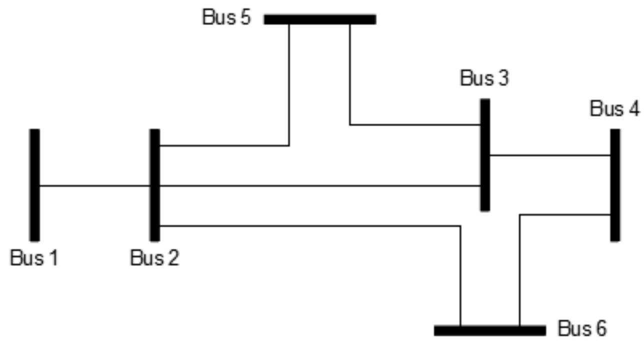

Figure 2.23: 6-bus system

Consider a PMU is installed at bus 2 in a 6-bus system as shown in Figure 2.23. Bus 2 is connected to four buses (bus 1, 3, 5, and 6). In order to measure all buses including itself, a PMU with five channels limit – four channels for branch 2-1, 2-3, 2-5, and 2-6; one channel for bus 2 itself – is needed. However, if the number of channels a PMU has is fewer than the number of branches connected to the PMU installed bus, branch combinations for each bus need to be determined. Consider the following equations [15]:

$$BR_i = \begin{cases} BC_i & \text{if } L \le BI_i \\ 1 & \text{if } BI_i < L \end{cases} \tag{2.14}$$

where L denotes the channels limit, while BI_i is the number of buses incidents to bus i, BR_i is the number of branch combinations for bus i, BC_i is the number of possible combinations of L out of BI_i. Based on equation (2.14), it is clear that if the number of PMU's channel greater than the number of branches which connected to the PMU installed bus, it does not need branch combinations to be identified, hence, $BR_i = 1$, which means a PMU is enough to measure all adjacent buses. In contrast, if the number of PMU's channels is fewer than the number of branches that need to be observed, the number of possible combinations, BC_i can be derived as follows:

$$BC_i = \frac{BI_i!}{(BI_i - (L-1))!(L-1)!} \tag{2.15}$$

Assuming the PMUs channels limit is set as 3, since bus 1, 4, 5, and 6 only have 2 or fewer branches that need to be observed, as indicated in equation (2.14), the number of branch combination is $BR_1 = BR_4 = BR_5 = BR_6 = 1$. Meanwhile, since bus 2 and 3 have 3 or more branches that need to be observed, as indicated in equation (2.14), the number of branch combinations can be determined using equation (2.15).

$$BR_2 \rightarrow BC_2 = \frac{4!}{(4-(3-1))!(3-1)!} = 6 \tag{2.16}$$

$$BR_3 \rightarrow BC_3 = \frac{3!}{(3-(3-1))!(3-1)!} = 3 \tag{2.17}$$

Hence, the summation of branch combinations for the 6-bus system with 3 channels limit is $BR_1 + BR_2 + BR_3 + BR_4 + BR_5 + BR_6 = 13$. Now, since the number of branch combinations for all buses have been determined, a binary connectivity matrix, [*H*] that consists all branch combinations for corresponding bus, where bus *i* will have BR_i rows, can be formed as in equation (2.18). Each row shows the corresponding bus and its neighbours. For instance, consider bus 2 where it has six possible combinations (1-2-3, 1-2-5, 1-2-6, 2-3-5, 2-3-6, 2-5-6). If a PMU is placed at row 3 in equation (2.18), it means that bus 1, 2, and 5 will be observable, while if a PMU is placed at row 5, it means bus 2, 3, and 5 will be observable.

$$[H]=\left[\begin{array}{cccccc}1&1&0&0&0&0\\ \hline 1&1&1&0&0&0\\ 1&1&0&0&1&0\\ 1&1&0&0&0&1\\ 0&1&1&0&1&0\\ 0&1&1&0&0&1\\ 0&1&0&0&1&1\\ \hline 0&1&1&1&0&0\\ 0&1&1&0&1&0\\ 0&0&1&1&1&0\\ \hline 0&0&1&1&0&1\\ \hline 0&1&1&0&1&0\\ \hline 0&1&0&1&0&1\end{array}\right]\begin{array}{ll}\text{Bus 1}&(\text{R1})\\ &(\text{R2})\\ &(\text{R3})\\ \text{Bus 2}&(\text{R4})\\ &(\text{R5})\\ &(\text{R6})\\ &(\text{R7})\\ &(\text{R8})\\ \text{Bus 3}&(\text{R9})\\ &(\text{R10})\\ \text{Bus 4}&(\text{R11})\\ \text{Bus 5}&(\text{R12})\\ \text{Bus 6}&(\text{R13})\end{array}\tag{2.18}$$

Therefore, from equation (2.18), the optimal PMUs placement when the PMUs are restricted to 3 channels for the 6-bus system above is:

$X=$ (2.19)

	R1	*R2*	*R3*	*R4*	*R5*	*R6*	*R7*	*R8*	*R9*	*R10*	*R11*	*R12*	*R13*
$X=$	0	0	**1**	0	0	0	0	0	0	0	**1**	0	0
	Bus 1	*Bus 2*						*Bus 3*			*Bus 4*	*Bus 5*	*Bus 6*

where X is a binary vector that represents the PMUs placement of the branch combinations derived in equation (2.18). Equation (2.19) indicates that two PMUs are required to ensure a complete observability, where one is needed to be placed at bus 2 and one at bus 4. The first PMU placed at bus 2 will measure bus 1, 2, and 5 while the second PMU placed at bus 4 will measure bus 3, 4, and 6 when PMUs channel limit is set to 3.

Theoretically, a PMU that has a fewer number of channel would cost less than the one with more number of channels. Therefore, one of the aim of this thesis is to investigate the number of PMUs required based on different channel limits and consequently the possible ideal number of channels required to achieve the complete observability of the power system.

2.9 Summary

This chapter presents the background study of the OPP problem and how the PMU is able to help in monitoring the power system. Observability rules for the PMUs are explained and explanation on how the problem constraints such as ZIB and channels limit are formulated are also presented in this chapter. The proposed formulation for ZIB by using ILP optimisation method is also explained in this chapter. Literature review of the existing studies using various methods concerning OPP problem is briefly reviewed to improve the understanding of the OPP problem.

Chapter 3

Particle Swarm Optimisation

3.1 Introduction

Particle swarm optimisation (PSO) is a population-based optimisation method developed in 1995 by Kennedy and Eberhart [33]. It was inspired from the studies regarding the movement of organisms in a bird flock. The term swarm in PSO relates to the behaviour of birds, or fishes, generally cruising in the same direction. Meanwhile, the term particle is referring to the element inside the swarm, such as a bird or a fish. Each particle represents a potential solution in PSO. All particles will fly through the search space where their positions are adjusted based on their own experiences and that of their neighbours.

The intelligence of the particles in PSO mentioned above is applicable with five basic principles of swarm intelligence outlined by Millonas [60] as follows:

1. Proximity principle: The population should be able to carry out simple space and time computations.
2. Quality principle: The population should be able to respond to quality factors in the environment.
3. Diverse response principle: The population should not commit its activity along excessively narrow channels.
4. Stability principle: The population should not change its mode of behaviour every time the environment changes.
5. Adaptability principle: The population should be able to change its behaviour mode when it is worth the computational price.

Kennedy and Eberhart argued that PSO conformed the above principles which can be elaborated as follows. Calculations happen in PSO are carried out over time or based on a series of iterations or time steps in n-dimensional space. PSO responds to quality factors possessed by the particle's best position and swarm's best position, which consequently ensures the diversity of response throughout the algorithm. The stability principle is achieved based on the movement of particles that occurs only when the swarm's best position is changed, which in fact also means the population is adaptive to changes.

The intelligence of the swarm in PSO makes it an interesting optimisation method. Its simplicity and ease of implementation have also contributed to its rise in popularity in recent years. Ever since it was found, it has been extensively and successfully implemented in many optimisation problems in various studies [61]–[66]. Furthermore, its ability to converge quickly to acceptable solutions increases the attention it received.

3.2 Conventional PSO

PSO starts by randomly initialising particles in a swarm and uniformly distributed within a defined search space. Each particle i is associated with two vectors, the velocity vector v_{ij} and the position vector x_{ij}, where j is the dimensionality of the problem. The velocity vector indicates particle trajectory from its current position to the next position, while the position vector indicates particle position in the search space. The values for both vectors are updated for every iteration according to the following equations:

$$v_{ij}^{t+1} = \underset{\text{momentum}}{v_{ij}^{t}} + \underbrace{C_1 R_1 (pbest_{ij}^{t} - x_{ij}^{t})}_{\text{cognitive component}} + \underbrace{C_2 R_2 (gbest^{t} - x_{ij}^{t})}_{\text{social component}} \tag{3.1}$$

$$x_{ij}^{t+1} = x_{ij}^{t} + v_{ij}^{t+1} \tag{3.2}$$

where,

- v_{ij}^{t} : The velocity of particle i in dimension j at iteration t
- x_{ij}^{t} : The position of particle i in dimension j at iteration t
- $pbest_{ij}^{t}$: The personal best position for particle i in dimension j discovered so far
- $gbest^{t}$: The global best position in the swarm at iteration t

C_1 and C_2 are acceleration constants used to determine the influence of social and cognitive components, respectively, while R_1 and R_2 are two random numbers that are uniformly distributed between [0,1]. The equation (3.1) shows that the velocity vector consists of three parts which are momentum, cognitive component, and social component. The first part represents a memory of the previous trajectory that acts as a momentum to control particle trajectories from drastic changes from its current position. The second part is called cognitive component which is used to measure the performance of particle i corresponding to its past performances. It also signifies the tendency of the particle to return to its own best experience. The third part is called social component which is used to measure the performance of particle i relative to the entire swarm or its neighbours. Contrary to cognitive component, particles will have the tendency to be attracted to the best position found by the particle's neighbourhood. In short, the value of the velocity vector in equation (3.1) is based on particle i own experience and its neighbours'. The particles movement at each iteration is best illustrated as follows:

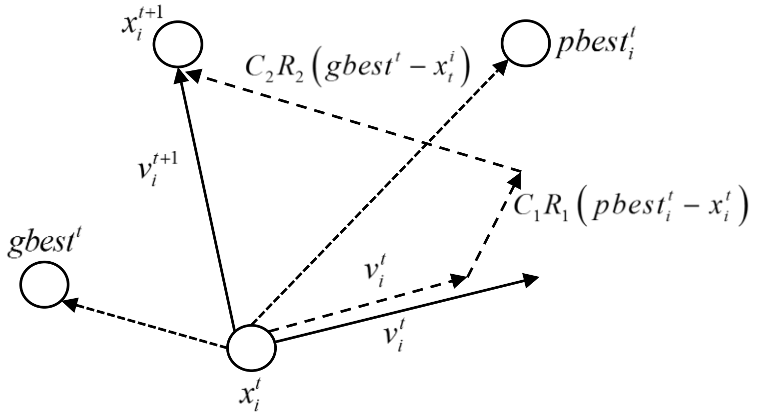

Figure 3.1: PSO position and velocity update

At each iteration, particle i is evaluated using a defined fitness function, f to determine the fitness value. The fitness value is then used to determine the value for *pbest* and also *gbest*. Considering minimisation problem, *pbest* and *gbest* are determined as follows:

$$pbest_{ij}^{t+1} = \begin{cases} x_{ij}^{t+1} & \text{if } f(x_{ij}^{t+1}) < pbest_{ij}^t \\ pbest_{ij}^t & \text{if } f(x_{ij}^{t+1}) \geq pbest_{ij}^t \end{cases} \tag{3.3}$$

$$gbest^t = \min(pbest_i^t, \cdots, pbest_S^t) \tag{3.4}$$

where S is the swarm size, $i \in [1,....,S]$ and $S > 1$

The updated process will be going iteratively until it satisfies the stopping criterion, which is normally defined as the maximum number of iterations allowed, $T_{\max}$. Figure 3.2 illustrates the flowchart of the PSO and the breakdown is described as follows:

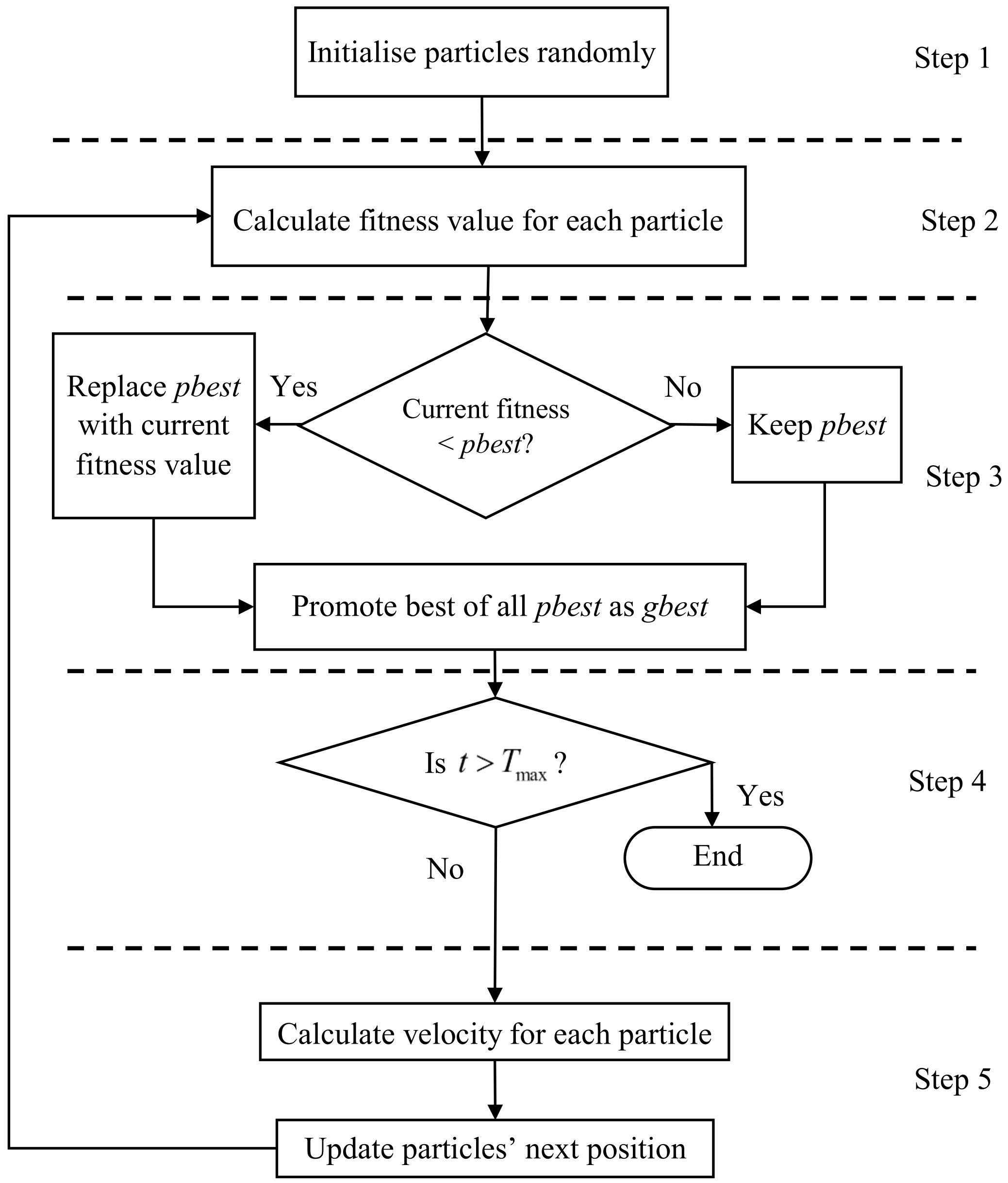

Figure 3.2: General PSO flowchart

Step 1: Particles' position vectors, x and velocity vectors, v are initialised randomly. For v, it is usually defined as $v = 0$ and research work by Engelbrecht [67] indicates that initialising v as zero will produce a better performance. Iteration t is also set as $t = 0$.

Stopping criterion is also defined, which is usually defined as the maximum number of iterations allowed, $T_{\max}$.

Step 2: Each particle is then evaluated using a fitness function defined specifically for the problem under study.

Step 3: When $t = 0$, particles *pbest* do not exist yet. Hence, the initial position vectors, x are used as the starting value for *pbest*. The value for *gbest* is then decided based on the best fitness among all *pbest*. When $t > 0$, the fitness value of each particle calculated in step 2 is compared with the previous fitness to decide the better fitness. If the current fitness is better than the previous fitness, *pbest* is replaced with the current fitness. Otherwise, previous fitness is maintained.

Step 4: The current iteration will then be evaluated once it satisfies the defined stopping criterion. In this case, if $t > T_{\max}$, the PSO algorithm will be terminated. If not, the algorithm will repeat step 2-5 until it satisfies the stopping criterion. The output will be displayed once the PSO algorithm ends.

Step 5: Now that all values are known, particles velocities are calculated using equation (3.1). If velocity clamping technique (explained in section 3.3) is applied, it will be done before updating the particles' next position. The particles' next position is then updated using equation (3.2).

The pseudocode showing PSO algorithm can be derived as follows:

Algorithm 1: Pseudocode of the PSO algorithm

```
for each particle
    Initialise velocity vector, v and position vector, x;
end
for each particle do
    Calculate fitness value;
    if fitness better than pbest then
        Update pbest;
    end
end
Determine gbest among all particles;
for each particle do
    Update velocity using equation (3.1);
    Update particle position using equation (3.2);
end
while stopping criterion is not met
```

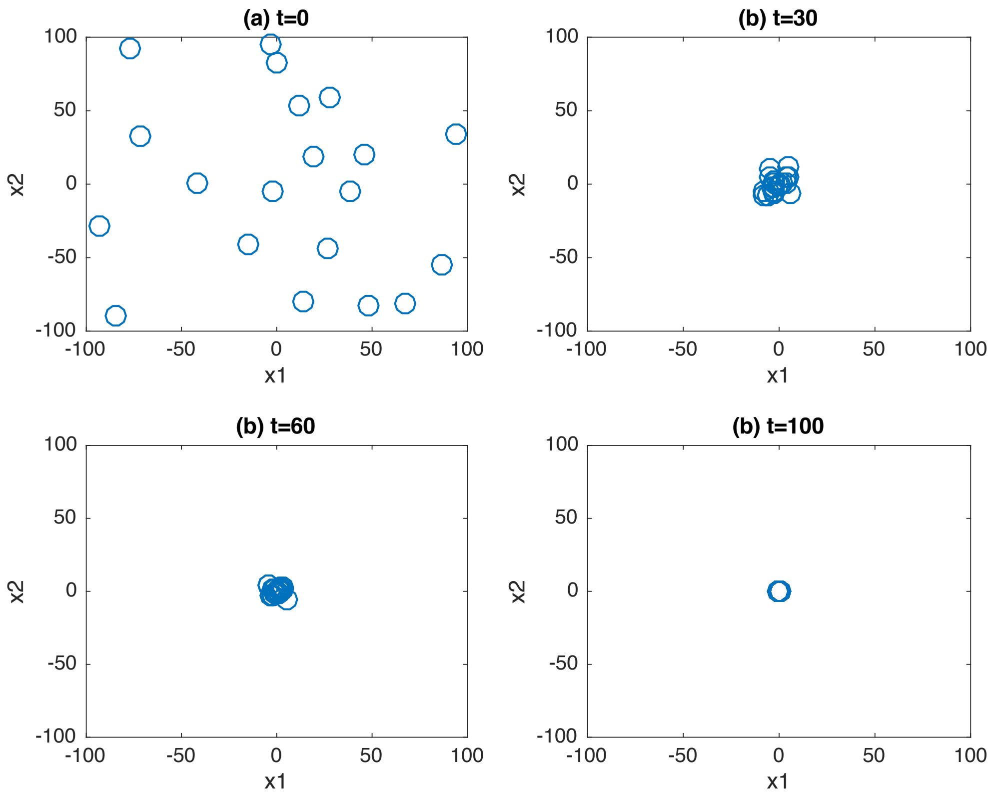

Figure 3.3: Snapshots of particles' movement at specific iteration in PSO (a) t=0, (b) t=30, (c) t=60, (d) t=100

Figure 3.3 shows the snapshots of particles' movements at specific iteration as the particles iterate overtime to minimise the fitness function $f = 3 + x_1^2 + x_2^2$, where the minimum values for this problem are $x_1 = 0.0$ and $x_2 = 0.0$. Figure 3.3(a) indicates the particles' position at $t = 0$, where the particles were initialised randomly within the defined search space, in this case, $-100 \le x \le 100$. As the particles fly through the searching space, which are guided by the best found positions in the search space, at $t = 30$ as per Figure 3.3 (b), the particles seem to have found the global optima and several particles started converging while some still actively searching for other best solutions. At $t = 60$, the particles are no longer capable to update the best positions they have found so far, thus, more particles are now converging to the same position as per Figure 3.3(c). By the time the algorithm reaches the stopping criterion, in this case, the maximum iteration allowed is $t = 100$, all particles have converged to the global optima as per Figure 3.3(d). The intelligence of the swarm in finding the solution to the problem's consideration that can be seen clearly in Figure 3.3 highlights its attractiveness and effectiveness to most researchers.

3.3 Parameter selection

PSO in nature seems to be sensitive to the tuning of some of its parameters towards the algorithm's performance. Hence, several considerations must be taken into account to ensure that the algorithm does not suffer from poor performance in terms of convergence and efficiency. The detailed explanations are discussed as follows:

1. **Swarm size, *S*** – Swarm size is referring to the number of particles or elements in the swarm. A large swarm increases the potential of covering large parts of the search space. It also may require fewer iterations to achieve a good optimisation result since it has better knowledge of the search space. However, a large swarm also adds more computation complexity, hence, increasing the time it spent to finish the algorithm. Typically, the number of particles suggested is within the range of 20 – 40 but numerous researches have shown that the value should be selected empirically.

2. **Number of iterations, T_{max}** – The number of iterations is referring to the number of times for the algorithm to be executed. Obviously, a large number of iterations increases the time taken for the PSO to end even if the algorithm has already converged. A low number might be the preferred option but it may stop the search process prematurely if the algorithm does not converge. Hence, a well-thought number of iterations to balance the time the PSO takes to finish the algorithm and obtain a good optimisation result are important.

3. **Velocity, *v*** – Equation (3.1) shows that velocity is a stochastic parameter and it defines the particle's trajectory for the next position. It is prone to an uncontrolled trajectory, which enables the particle to go beyond the optimal search region. This phenomenon is called *explosion* and to restrain this phenomenon, the particle's velocity should be clamped into a reasonable range. By clamping velocity, the particles are more controlled in their searching process and it helps the particles to exploit their current position. If the velocity of the particle ever goes beyond the

search space, velocity clamping will make sure it would not go beyond the region of interest. Supposed $V_{\max}$ denotes the maximum allowed velocity, particle velocity is adjusted as follows before the position update:

$$v_{ij}^{t+1} = \begin{cases} v_{ij}^{t+1} & \text{if } v_{ij}^{t+1} < V_{\max} \\ V_{\max} & \text{if } v_{ij}^{t+1} \geq V_{\max} \\ -V_{\max} & \text{if } v_{ij}^{t+1} \leq -V_{\max} \end{cases} \tag{3.5}$$

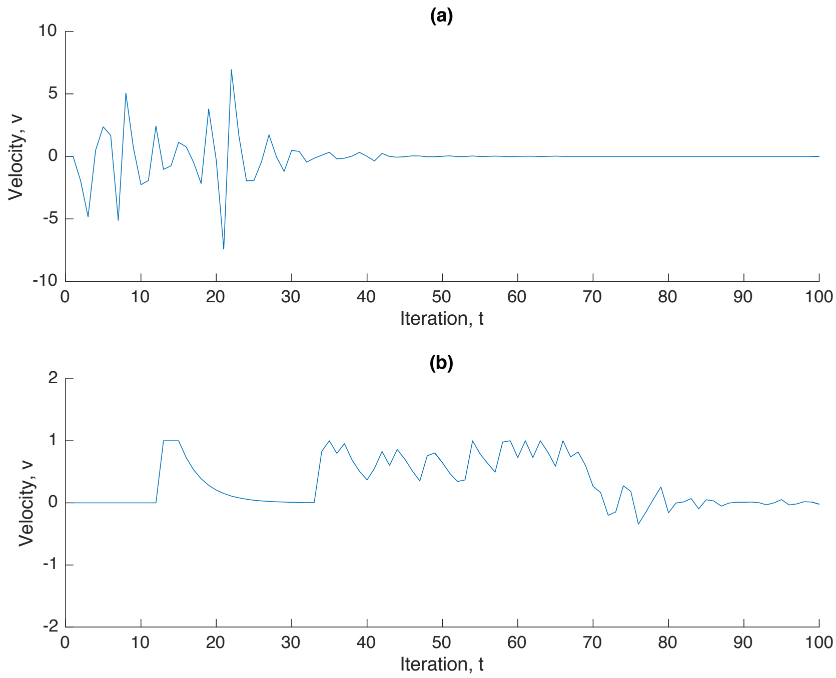

Figure 3.4: Velocity clamping effect (a) Without velocity clamping, (b) With velocity clamping Vmax=1

Equation (3.5) shows how velocity clamping limits the explosion by creating a feasible region for the particles to search. Figure 3.4(a) illustrates the value of velocity vector without velocity clamping where it is not restricted, while Figure 3.4(b) shows the effect of velocity clamping if $V_{\max} = 1.0$, where the value of velocity vector is limited to $-V_{\max} \leq v \leq V_{\max}$. The value set for $V_{\max}$ affects the particles' exploration and exploitation abilities. A large value of $V_{\max}$ encourages global exploration because the particles are allowed to have a large momentum, thus, allowing the particles to reach other regions quickly. Meanwhile, a small value encouraged local exploitation due to a smaller time step, thus, providing a longer time for the particles to refine their current position. However, the particles may be missing a good solution if the value of $V_{\max}$ is set to a very large number since they might move beyond the possible good region due to their faster movement. Whereas, if the value of $V_{\max}$ is set to a very small number, it means the time step needed to reach other regions is increased and the possibility to become trapped in the local optima is higher. It can be said that apart from preventing velocity explosion, velocity clamping also limits the exploration of the particle's search. Normally, the value of $V_{\max}$ is set empirically according to the problem under study. For a start, the value suggested is half the range of possible values for the search space. A uniform velocity for all dimension was suggested by Abido [68] in his research work as follows:

$$V_{\max} = \frac{(x_{\max} - x_{\min})}{N} \tag{3.6}$$

where N is a chosen number of intervals, while $x_{\max}$ and $x_{\min}$ are the maximum and minimum values of positon vector x for a particular iteration.

4. **Inertia weight, ω** – Inertia weight is a new parameter proposed by Shi and Eberhart with the objective to improve the scope of the search, by multiplying the velocity at the previous iteration [69]. It basically controls how much the influence of previous velocity on the new velocity. Equation (3.1) is replaced as follows:

$$v_{ij}^{t+1} = \omega v_{ij}^{t} + C_1 R_1 (pbest_{ij}^{t} - x_{ij}^{t}) + C_2 R_2 (gbest^{t} - x_{ij}^{t}) \tag{3.7}$$

When $\omega \geq 1$, velocities are increased over time and accelerate towards the maximum velocity. When $\omega \leq 1$, particles decelerate until their velocities reach zero. These two behaviours indicate that a better exploration can be achieved when the value of inertia is high while better exploitation is achieved when inertia weight has a small value. Initially, a static value was proposed for inertia weight for the entire search duration. Then, a linear decreasing inertia weight was later proposed and it shows that the PSO performs better when both exploration and exploitation are incorporated in the searching process [70]. This strategy implies that the value of ω is decreasing from a larger value (typically 0.9) to a lower value (typically 0.4) linearly as the number of iteration increases. As the value of ω decreases, it transformed the search model from exploration mode to exploitation mode. It has been successfully used in some applications [38], [71]–[73]. The linear decreasing inertia weight can be defined as follows:

$$\omega^t = (\omega_{\max} - \omega_{\min})\frac{T_{\max} - t}{T_{\max}} + \omega_{\min} \tag{3.8}$$

where,

- $\omega_{\min}$: A minimum value of inertia weight
- $\omega_{\max}$: A maximum value of inertia weight
- $T_{\max}$: A maximum number of iterations allowed

5. **Acceleration constants, C_1, C_2** – The combination of acceleration constants, C_1 and C_2 with random values of R_1 and R_2 ensures that stochastic influence is being maintained for the cognitive and social components in a particle's velocity, respectively. The values of C_1 and C_2 represent how much the particles are influenced by the cognitive and social components in the algorithm. For instance, if the cognitive component has a large value, the movement of the particle will be directed into the direction of the personal best position, which is limiting the influence of the social component. The influence of C_1 and C_2 can be explained further by considering the following scenarios:
   a. If $C_1 = C_2 = 0$, all particles will fly according to their current speed without having any external influences. Therefore, the velocity update equation is calculated as:

   $$v_{ij}^{t+1} = v_{ij}^{t} \tag{3.9}$$

   b. If $C_1 > 0$ and $C_2 = 0$, all particles are flying independently. The velocity update equation then will be:

$$v_{ij}^{t+1} = v_{ij}^{t} + C_1 R_1 \left[pbest_{ij}^{t} - x_{ij}^{t} \right] \tag{3.10}$$

c. In contrast, if $C_2 > 1$ and $C_1 = 0$, particles are attracted to a single point in the entire swarm and the velocity update equation become:

$$v_{ij}^{t+1} = v_{ij}^{t} + C_2 R_2 [gbest^{t} - x_{ij}^{t}] \tag{3.11}$$

d. In the case of $C_1 = C_2$, particles are attracted between the average of *pbest* and *gbest*.

A dynamic value for acceleration constant has also been proposed by Suganthan [74] and Ratnaweera et. al [75] in their research work. The work in [74] explores linear decreasing acceleration constant similar to the linear inertia weight mentioned earlier. Meanwhile, in [75], a cognitive component is reduced while a social component is increased in time. The intention is to ensure the particles move around the search space in the beginning and converge to the global optima in the latter part of the optimisation. This approach can be represented as follows:

$$C_1 = (C_{1f} - C_{1i}) \frac{t}{T_{\max}} + C_{1i} \tag{3.12}$$

$$C_2 = (C_{2f} - C_{2i}) \frac{t}{T_{\max}} + C_{2i} \tag{3.13}$$

where C_{1f}, C_{1i}, C_{2f} and C_{2i} are constant, t is the current iteration, and $T_{\max}$ is the maximum number of iterations. Generally, the values for C_1 and C_2 are empirically selected but the value of 2 for both constants has been suggested as a good starting point [33] [74]. Ozcan and Mohan in their research work shows that if $C_1 + C_2 > 4$, the trajectory of a particle will exponentially grow, causing the danger of a particle moving out of the search space, without returning. Hence, it is in the best interest to maintain $C_1 + C_2 < 4$ to ensure a good performance [76].

6. **Constriction factor, *K*** – It was proposed by Clerc and Kennedy [77] to balance the exploration and exploitation, similar to that of inertia weight and velocity clamping. Constriction factor achieves it by controlling explosion and at the same time allows particles to intensively exploiting their current position. The constriction factor is defined as follows:

$$v_{ij}^{t+1} = K[v_{ij}^{t} + C_1 R_1 (Pbest_{ij}^{t} - x_{ij}^{t}) + C_2 R_2 (Gbest^{t} - x_{ij}^{t})] \tag{3.14}$$

where,

$$K = \frac{2y}{\left|2-\phi-\sqrt{\phi(\phi-4)}\right|}, \text{where } y \in [0,1] \tag{3.15}$$

with

$$\phi = C_1 + C_2, \text{ where } \phi > 4 \tag{3.16}$$

Constriction factor produces a damping effect that decreases the particles trajectories oscillation over time. Thus, convergence is assured over time. From equation (3.16), the damping effect becomes more prominent as the value of ϕ increases, K will get smaller. For instance, if $\phi = 5$, then $K \approx 0.38$. Thus, generally ϕ is set as 4.1, hence, $K = 0.729$, which works fine for most applications. Through constriction factor, any ability to control the change of direction of particles must be done via the constants, C_1 and C_2. The introduction of constriction factor also negates the need to have both inertia weight and velocity clamping because the constriction factor prevents velocity explosion and facilitates convergence.

3.4 Gbest and Lbest models

PSO algorithm can be divided into two models, which are Global best model (*Gbest*) and Local best model (*Lbest*). These two models are distinguished based on the social interaction among particles and also network topology that both models used.

a. **Gbest model** – Each particle in Gbest model is connected with each other and consequently the position of each particle is influenced by the best fit particle in the entire swarm (*gbest*). This means that every particle will be attracted to the *gbest* solution found by a member of the swarm and if the *gbest* is not updated regularly, the swarm may converge prematurely. Since each particle is interconnected with every particle in the swarm, information exchange happens swiftly and each particle can compare the information they have with each other. Based on the description above, the network topology of *Gbest* model can be seen in Figure 3.5(a).

b. **Local best model** – In contrast to the *Gbest* model, each particle in *Lbest* model is only connected to its immediate neighbours. Thus, each particle is only influenced by the best fit particle found by its neighbourhood only. This characteristic means each particle only has limited access to its small neighbourhood, hence, its convergence speed is relatively slow. However, the *Lbest* model compensates it by preventing premature convergence by maintaining particles diversity. Three common network topologies used for *Lbest* model can be seen in Figure 3.5(b)-(d).

The performance comparison made in [78] indicates that the *Gbest* model performs well with respect to success and efficiency while *Lbest* model performs slightly better in terms of consistency. However, the selection of which model to use is usually selected empirically to the characteristics of the problem.

3.5 PSO topology

The social interaction between particles is a primary feature for PSO. The topology reflects the way particles socialise among themselves in the swarm and it controls the exploration versus exploitation tendency [79]. It determines the connection among particles that significantly affect the rate of information flow among the particles. The topology also tells how each particle is influenced by others that are more successful in the neighbourhood. There are many types of neighbourhood in PSO but the following are four topology structures that are most prominently used in various studies.

a) **Star topology** - This topology which is illustrated in Figure 3.5(a) is typically referred to as the *Gbest* model. Each particle is fully connected, thus, making each particle to become neighbours with each other in the swarm. As a result, this means that the experience or information of all particles are known for every particle. The best particle found by any particle of the swarm is shared with every particle, hence, will attract each particle towards it.

b) **Ring topology** – This topology is known as the *Lbest* model. Each particle in this topology is only connected with its immediate neighbour. The convergence is understandably slower since each particle performance is only affected by its immediate neighbour. Figure 3.5(b) illustrates this topology.

c) **Wheel topology** – Each particle in this topology is only connected to one particle that acts as a main particle as shown in Figure 3.5(c). Experience or information gathered by all particles is communicated through the main particle. The main particle compares the performances of all particles and adjusts its position towards the best particle. If the main position is in a better performance than the other particles, it will communicate the improvement to all particles.

d) **Von Neumann** – Particles are connected in a grid structured as shown in Figure 3.5(d). Compared to the ring and wheel topology, the particles in Von Neumann have neighbours above, below, and on each side of the particle.

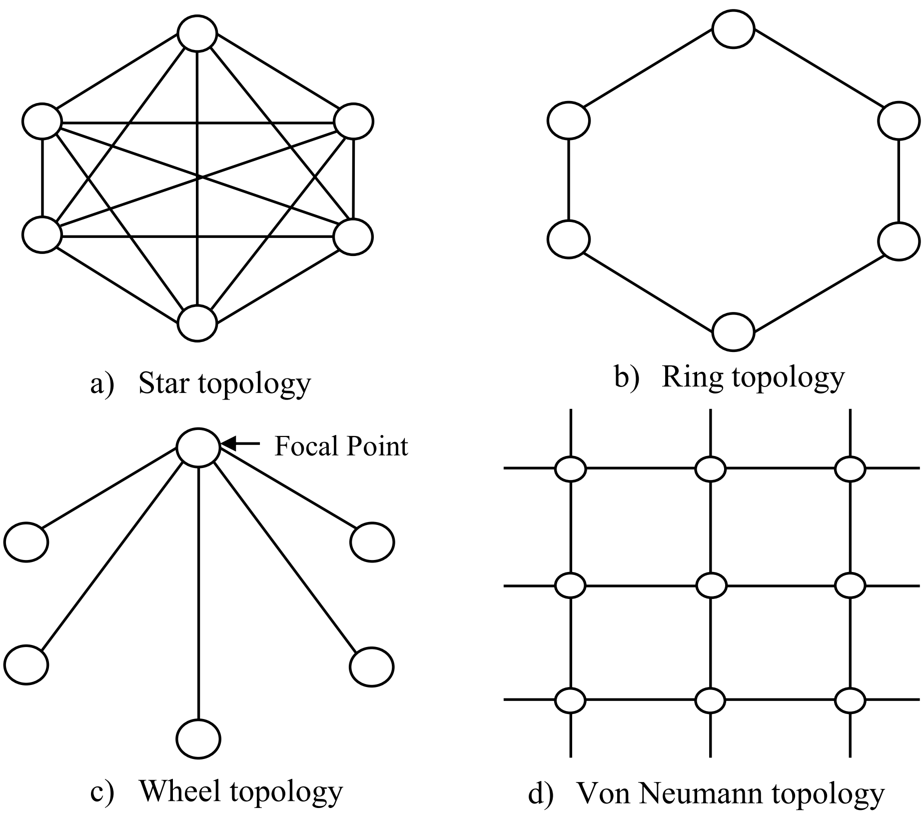

Figure 3.5 PSO topology

Generally, the flow of information in the PSO topology is highly dependent on (i) degree of connectivity among nodes (members of the network), (ii) the amount of clustering (occurs when a node's neighbours are also neighbours to one another, (iii) average shortest distance from one node to another node [80]. Information regarding best particle performance is fast transmitted in the topology where every particle is connected to one another, thus, allows faster convergence. However, it is also exposed to local convergence where the particles have not traveled enough to fully investigate the search space.

3.6 PSO drawbacks

As mentioned in section 3.1, PSO is very quick to obtain a number of high quality solutions and has stable convergence. Furthermore, it is very easy to implement. However, as most optimisation algorithm, PSO is not without any drawbacks. The following are the three main issues that impact PSO performance:

1. **Stagnation** – When a particle is in the situation where its current position is equal to its personal best position and global best position. The particle velocity starts to decrease rapidly to zero, and when it hits zero, it would be very hard for the particle to move from its current position.

2. **Local convergence** – It follows from the stagnation issue mentioned above because of the lack of momentum or impetus available for particles to fly further when they have started converging. This leads to local convergence which prevents the particles to improve their current position. Increasing particles diversity is very helpful to improve the probability for particles to avoid local optima. One way to achieve it is by increasing the number of particles.

3. **Problem scale** – As the dimension is growing, the search space for particles to explore will increase as well. With the lack of particles diversity and stagnation issue mentioned above, it becomes a huge problem that affects the PSO's performance greatly. As the problem scales, increasing the particles is one way to improve the particles' exploration albeit increased computation complexity. Morever, adding a local search method to the algorithm also improves the PSO's performance [81]–[83].

3.7 PSO variants

Over the years, since its inception, numerous PSO variants have been developed mainly to improve its performance in terms of convergence, diversity, and also the accuracy of solutions. Furthermore, it is also developed to ensure that it can operate for various optimisation problems such as dynamic environments problem, multi-objective problem, and also discrete problem. The following are some of the popular variants used recently for various studies:

1. **Fully informed PSO** – Traditionally, a PSO uses one best position as a guide that indicates the best search region other particles should be searching iteratively. In fully informed PSO, each particle is influenced by the successes of all of its neighbours, rather than the performance of only one particle. Thus, the topology that is used for this approach and the swarm size determine how diverse the population will be, which in regard to the searching aspect, determine whether it is more enhanced rather than diluted [84].
2. **Multi-objective PSO** – Many real world optimisation problems demanded simultaneous optimisation of a number of objectives (multi-objectives). The main objective of this variant is to find a set of solutions that optimally balance the trade-offs among the objectives desired. Normally, the task is to find a set of non-dominated solutions by defining an appropriate fitness function to evaluate the quality of the solution in terms of multiple objectives. This variant produces a multiple set of solutions from which the Pareto concept is applied to find the solution that satisfies all objectives.
3. **Adaptive PSO** – In an optimisation problem that dynamically changes over time or based on the environment, adaptive PSO is introduced to detect the environment change and to react accordingly. For instance, the particles may become inactive as they lost the exploration and exploitation abilities when their best positions are almost similar to the best positions achieved by their neighbours. This is the situation where an appropriate action should be taken to avoid the particles from being inactive. The modifications vary from updating inertia weight, reinitialise and increasing particles, and recombination of the current and previous particles best.

4. **Hybrid PSO** – This variant was introduced mainly to overcome the PSO drawbacks by integrating one or more features from other algorithms into the PSO. The main goal is usually to maintain the diversity of the population to prevent local convergence. Commonly, hybrids of GA and PSO are used to increase population diversity where the mutation technique used in GA was integrated into the PSO.

5. **Binary PSO** – The original PSO was introduced to operate in a real-valued space. In order to ensure that it can operate in an optimisation problem that is defined by a discrete value, binary PSO variant is introduced to overcome this problem. This variant uses the same velocity update as in the conventional PSO but the value for the velocity vector is normalised so that it can be used as a probability threshold for particle updating position. This variant is explained in detail in section 3.8

6. **Selecting strategy** – This variant introduces an alternative way of determining *pbest* and *gbest* in the PSO. Normally, *pbest* and *gbest* are determined based on the performance of the swarm. However, this variant uses selection strategies that are designed to improve population diversity, local search ability, and also to improve convergence. Angeline [85] in his research work suggested a tournament selection method used in an evolutionary programming where each solution is sorted according to the performance score based on its fitness value against a randomly selected group of *k* particles. The current position of the top half of the particles will then be duplicated onto the bottom half of the particles. This approach may improve the local search capabilities of the PSO, but it reduces diversity. Padhye [86] in his research work introduced selection strategies for the multi-objective PSO. Among the strategies he proposed was to randomly select *gbest* where one particle dominates many particles. Another strategy is to only update *pbest* if the newest solution is not from the previous searched region. Hence, a better diversity is expected.
7. **Multi-start PSO** – The main objective of this variant is to increase diversity by randomly reinitialise particles positions and/or its velocities. Although many aspects need to be considered such as how it should be randomised and when should randomisation occur. Typically, particles are reinitialised when particles do not improve over time. In the event where only velocities are randomised but

positions are kept constant, particles are forced to search in different random directions, thus, may result in larger parts of the search space explored.

3.8 Binary PSO

PSO was originally developed for continuous-valued spaces. However, there are many optimisation problems that are defined by discrete-valued spaces such as integer programming problems, the travelling salesman problem, scheduling, and routing. Fortunately, the PSO is adaptable to discrete-valued spaces albeit involving a different interpretation of velocity, particle trajectory, velocity clamping, and momentum.

Binary PSO (BPSO) was developed by Kennedy and Eberhart to operate in a discrete search space [34]. Therefore, in BPSO, the position vector x for each particle can only take the binary value of 0 or 1. For example, assuming that particle i has 7 dimensions, the particle will consist of 7 numbers of bit. The particle in BPSO is formed as follows:

$$x = \begin{bmatrix} 1 & 0 & 1 & 0 & 0 & 1 & 1 \end{bmatrix} \tag{3.17}$$

In each iteration, to update each element of position vector x, it is updated based on the following rule:

$$x_{ij}^{t+1} = \begin{cases} 1 & \text{if } rand < sig(v_{ij}^{t+1}) \\ 0 & \text{otherwise} \end{cases} \tag{3.18}$$

Where *rand* denotes the random number in the range of [0,1], while $sig(v_{ij})$ is a sigmoid function which is defined as follows:

$$sig(v_{ij}^{t+1}) = \frac{1}{1 + e^{-v_{ij}^{t+1}}} \tag{3.19}$$

As can be seen from the position update rules in equation (3.18), updating the particle's position in BPSO involves the process of forcing each element of position vector x to take the value of 1 or 0, depending on the random probability against the value of sigmoid. The value of sigmoid is based on the velocity value in BPSO, which is normalised into probability values in the interval of [0,1].

As can be observed by now, the position update rules in equation (3.2) by the conventional PSO is replaced with the position update rules in equation (3.18). It is also important to note that in BPSO, the particle's previous position does not influence the particle's next position like it does for the conventional PSO. It depends solely on the velocity value of each element of the position vector x.

Based on equations (3.18) and (3.19), if $V_{ij} = 2, \therefore sig(V_{ij}) = 0.88$, it indicates the probability of the element of position vector x to take the value of 1 as 88% and 12% probability for the element of position vector x to take the value of 0. Furthermore, if $V_{ij} = 0 \therefore sig(V_{ij}) = 0.5,$ based on the position update rules, the element will be updated randomly since the chance for the element to take the value of 1 or 0 is balanced.

Apart from the sigmoid function and position update rules, the parameters used in the BPSO are similar to that of the conventional PSO. Overall, in BPSO, the velocity update is still the same as the conventional PSO where it still takes a real-valued number. The only change in the binary PSO is the calculation of position vectors.

However, the behaviour and meaning of velocity clamping explained earlier are the opposite of the standard PSO. As mentioned earlier, velocity is interpreted as a probability of change in BPSO. For example, if $V_{\max} = 4,$ then $sig\left(V_{\max}\right) = 0.982$ is the probability of x_{ij} to change to bit 1 and 0.018 the probability to change to bit 0. Hence, it can be said that small values for $V_{\max}$ encourage exploration, even if a good solution has already been found. With large values, new position vectors are unlikely due to low probability to change. This behaviour is contrary to that of the real-valued PSO where small values limit exploration and large values encourage exploration.

3.9 Summary

This chapter elaborates the basic knowledge of the PSO algorithm and how to implement it. In addition, its parameters including the optional parameters are also briefly explained in order to understand the influence of each parameter towards the performance of the algorithm. Lastly, this chapter explains the difference between the conventional PSO and the BPSO, which will be used in this thesis to solve the OPP problem.

Chapter 4

Integrated Mutation Strategy with modified BPSO for OPP

4.1 Introduction

This chapter focuses on the important part of the thesis which is presenting how the BPSO is used for solving the OPP problem. The main objective is to employ BPSO algorithm to determine the minimum number of PMUs for a complete observability while maximising measurement redundancy. Therefore, the algorithm must be able to find the best possible solution that can achieve the objective. To evaluate each possible solution obtained by the particles with respect to the desired objective, the fitness function is used. Hence, the fitness function must be designed so that it can be used to differentiate the quality of each solution presented by the particles accordingly, from which a decision regarding the best solution can be made.

Since the BPSO is a population based optimisation method, it relies heavily on the activity of the particles in the population to find good solutions. Hence, the population needs to be knowledgeable regarding the search space in order to find the best possible solution for the desired objective. However, population based methods tend to experience poor performance when dealing with large-sized problem. It might be the case for most of the studies involving population based methods as discussed in the literature review in section 2.8. Most of the studies applied their proposed approach to the small bus systems such as the IEEE 14-bus, 24-bus, 30-bus, 39-bus, and 57-bus systems. Very few applied their proposed approach to the IEEE 118-bus system and beyond. In addition, none of the studies that used the BPSO to solve the OPP problem have considered the PMU channels limit.

In this chapter, a modified version of the BPSO method is proposed by integrating a mutation strategy and V-shaped sigmoid function to solve the OPP problem. The mutation strategy is proposed such that it will help to intensify the local search of the algorithm to improve the existing solutions. The proposed approach is expected to be able to solve the OPP problem that considers factors such as normal operation, ZIB, single PMU loss, and also PMU channels limit.

4.2 Particles

As explained in section 3.2, each particle carries a possible solution to the problem under consideration. In this thesis, the problem under consideration is to find the minimum number of PMUs that can achieve a complete observability of the power system. Therefore, the structure of each particle is formed to indicate the availability of the PMU at a specific bus.

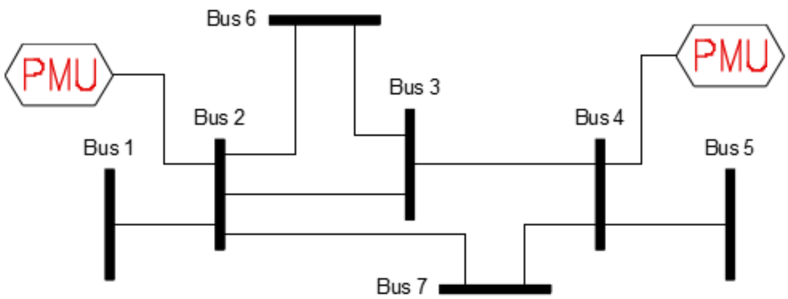

Figure 4.1: 7-bus system

Bus 1	Bus 2	Bus 3	Bus 4	Bus 5	Bus 6	Bus 7
0	1	0	1	0	0	0

Figure 4.2: Particle structure

Figure 4.2 shows how each particle is formed when solving the OPP problem for a 7-bus system as depicted in Figure 4.1. As can be observed from Figure 4.2, each dimension indicates the bus location. Hence, each particle is formed according to the size of the power system. The value 1 at bus 2 indicates a PMU is placed at that bus. If there is no PMU placed at that bus, the value at bus 2 will be 0.

4.3 Measurement redundancy

Since BPSO is a meta-heuristic algorithm, there will be a number of PMU placement sets that contains the same number of the minimum PMUs required for a complete observability. In order to evaluate the quality of each PMU placements set, measurement redundancy concept of bus observability index (BOI) and system of redundancy index (SORI) that are introduced in [12] are used. BOI is defined as the number of times a bus is observed by the PMUs placement set while SORI is referring to the summation of BOI for all buses. The PMUs placement set that has the highest number of SORI indicates the PMUs placement set has a better quality solution and more reliable for possible contingencies compared to the PMUs placement set that has a low number of SORI [19]. Therefore, in this thesis, this concept will be used to evaluate and compare the PMU placement sets obtained from the proposed method and prior studies in terms of measurement redundancy. BOI and SORI are defined as follows:

$$BOI_i = A \times X_i \tag{4.1}$$

$$SORI = \sum_{i=1}^{N} BOI_i \tag{4.2}$$

4.4 Fitness function

Each particle in the BPSO carries possible solutions to the problem under study. The usefulness of each possible solution to the problem under study needs to be evaluated using a fitness function to ensure that the best solution is always selected ahead of other possible solutions. In this thesis, the objective is to find the minimum number of PMUs and maximising measurement redundancy while ensuring a complete observability of the power system. Therefore, the fitness function should be able to evaluate the three important criteria:

i) The power system observability
ii) The number of PMUs to make the power system observable
iii) The measurement redundancy

Based on above criteria, the fitness function for solving the OPP problem can be formulated as follows [37]:

$$Z = \min\{\underbrace{(w_1 \times N_{obs})}_{\text{Observability}} + \underbrace{(w_2 \times N_{PMU})}_{\text{Number of PMUs}} + \underbrace{(J_1 \times C)}_{\substack{\text{Measurement}\\ \text{Redundancy}}}\} \tag{4.3}$$

where

- w_1, w_2, and C are the weight parameter
- N_{obs} is the number of observable bus
- N_{PMU} is the number of PMUs
- J_1 is the measurement redundancy

As can been seen from equation (4.3), the fitness function is formed based on three components—observability, number of PMUs, measurement redundancy—to evaluate the three criteria mentioned earlier. The first component determines how many bus is observed by the PMUs placement set from the particle being evaluated. It has been mentioned in section 2.6 that a power system is deemed observable if all buses in the system are observable by the PMUs. For instance, for a 7-bus system as shown in Figure 4.1, to determine that it is completely observable, the value of N_{obs} must equal to 7, which indicates all seven buses in the power system are observable. The value of N_{obs} can be determined as follows:

$$N_{obs} = |\{x \in BOI \mid x \neq 0\}| \tag{4.4}$$

Since BOI implies how many times each bus is being observed by the PMUs, the number of bus observed by the PMUs placement set can be determined based on how many non-zero elements in vector BOI for the corresponding particle. The second component determines the number of PMUs which can be determined as follows:

$$N_{PMU} = X^T X \tag{4.5}$$

For the third component, the value of J_1 is formulated as follows:

$$J_1 = (M - BOI)^T \times (M - BOI) \tag{4.6}$$

Where M is the desired value of measurement redundancy whose entries are defined as follows:

$$M = \begin{bmatrix} m_1 & m_2 & m_3 & \cdots & m_N \end{bmatrix}^T_{1\times N} \tag{4.7}$$

If the desired value for measurement redundancy is 2, the element of M will be set to 3. The vector $(M - AX)$ calculates the difference between the desired and actual number of times the bus is observed. The maximum value for each element of M is depending on the number of bus adjacent to it. For example, bus 2 in a 7-bus system is connected to four other buses {1, 3, 6, 7}, which means, logically, the maximum measurement redundancy for bus 2 is five (four from PMUs placed at buses incident to it, one from a PMU placed at bus 2). Therefore, the maximum value that can be set for bus 2 is m_2=6.

4.4.1 Fitness function for single PMU loss

In the case of considering a single PMU loss, as mentioned in section 2.5, to ensure that every bus has an alternative PMU observing it in case of a single PMU malfunction, each bus needs to be observed by at least two PMUs. The fitness function in equation (4.3) is modified in this thesis to integrate the single PMU loss as follows:

$$Z = \min\{(w_1 \times N_{obs}) + (w_2 \times N_{PMU}) + \underbrace{(w_3 \times N_L) + (w_4 \times G)}_{\text{Single PMU loss}}\} \tag{4.8}$$

where,

$$N_L = \left|\{x \in D \mid x < 0\}\right| \tag{4.9}$$

$$D = AX - b, \text{where } b = [2 \quad 2 \quad 2 \quad \cdots \quad 2]^T_{1\times N} \tag{4.10}$$

$$G = \sum_{i=1}^{N} D_i \tag{4.11}$$

The parameter N_L denotes the number of bus that is not being observed twice by the PMUs placement and G is the measurement redundancy used when considering a single PMU loss. Therefore, parameters J_1 and C used for the measurement of redundancy in equation (4.3) are replaced with parameter G. In equation (4.10), b denotes vector variables that are equivalent to the number of times a bus needs to be observed. Since each bus, when considering a single PMU loss, has to be observed at least twice by the PMUs placement, the value is set as 2.

4.5 V-shaped sigmoid function

In the previous chapter, it has been explained that the way the BPSO algorithm updates the particles' position is different from the one adopted by the conventional PSO. In the BPSO algorithm, sigmoid function is used to map the velocity value as the probability to update position x. The sigmoid function as in equation (3.19) is illustrated in Figure 4.3

where $-5 < v < 5$. Due to its "S" shape when plotted, the sigmoid function will be known as the S-shaped sigmoid hereafter.

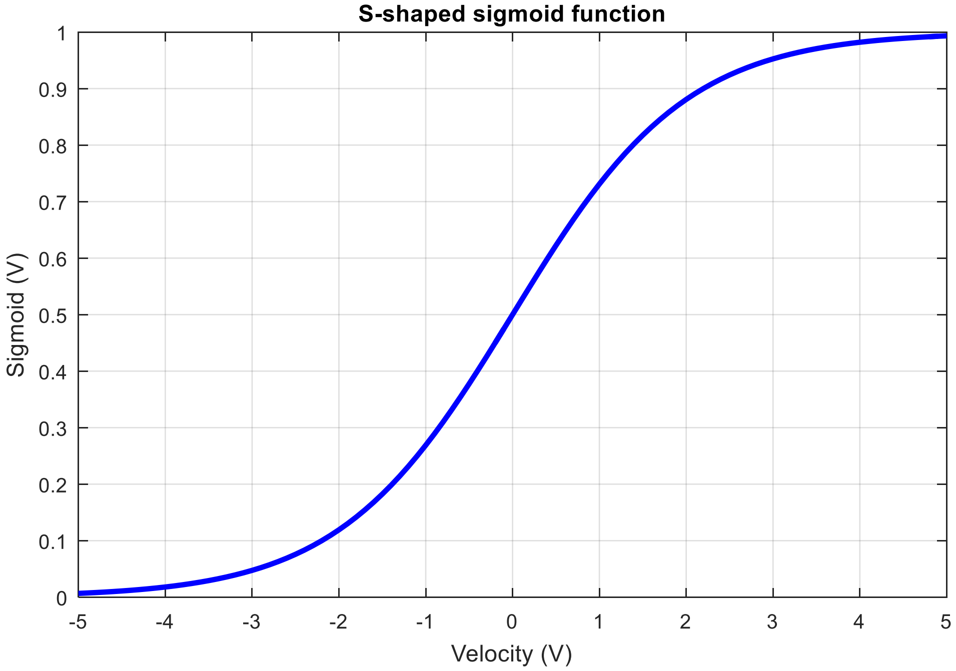

Figure 4.3: S-shaped sigmoid function

From Figure 4.3, based on the position update rules in equation (3.18), a large value for v in the BPSO does not mean a big movement for x similar to that of the conventional PSO. However, it means a large value of v_{ij} increases the probability of x_{ij} to take the value of 1 without considering the previous position. For example, if v=1, the probability of x_{ij} to take the value of 1 is $S(v_{ij}) \sim 0.75$, regardless of its previous position. It is similar if the value of v_{ij} is a large negative value which will increase the probability of x_{ij} to take the value of 0. The velocity is equal to zero when a particle is converged at the *gbest* position. Hence, when v_{ij}=0, the probability for the position vector x_{ij} to take the value 1 or 0 is $S(v_{ij}) = 0.5$, regardless of its previous position. Since in the BPSO algorithm, the particles' previous position does not influence the particles' next position like the conventional PSO does, the particles' next position will be decided randomly since the probability of changing position is now 0.5. This influences the particles diversity with regard to the number of PMUs once the particles have already converged at the best solution found by the algorithm. To demonstrate this behaviour, particle's diversity is measured using the commonly used measurement that is defined as follows [87]:

$$diversity(S) = \frac{1}{S}\sum_{i=1}^{|S|}\sqrt{\sum_{j=1}^{N}(x_{ij} - \bar{x}_j)^2} \tag{4.12}$$

where S is the swarm size, N is the number of bus, x_{ij} is the j^{th} dimension of the i^{th} particle position and $\bar{x}_j$ is the average of the j^{th} dimension over all particles. The diversity graph will show the distribution of the number of PMUs from all particles. Figure 4.4 presents the convergence graph and particles diversity graph for the IEEE 57-bus system:

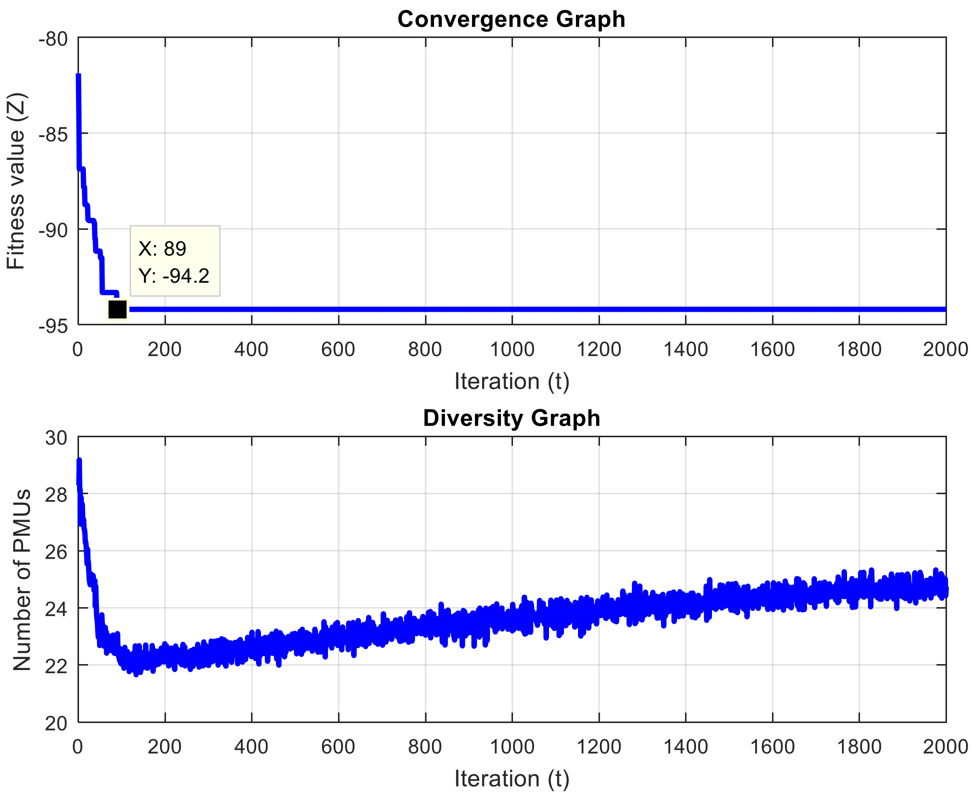

Figure 4.4: Convergence (top) and diversity (bottom) graph for IEEE 57-bus system

As can be seen from Figure 4.4 (top), the particles managed to converge at t=89 and it is unable to improve the fitness value in the remaining iterations. The reason it is no longer able to improve the best solution it has found so far can be explained by considering the diversity graph in Figure 4.4 (bottom). As mentioned earlier, once the particles have converged, the particles' next position will be decided randomly. From the diversity graph, it is evident that the number of PMUs across all particles increases as the algorithm continues its searching process. Subsequently, it will not be able to improve the current best solution because in latter iterations, the particles will often find solutions that use more PMUs. With respect to the IEEE 57-bus system, the minimum number of PMUs that can achieve a complete observability as reported by prior studies is 17 PMUs [36], [37], whereas the algorithm only managed to find best solution that uses 18 PMUs to achieve complete observability of the power system. In this circumstance, the algorithm is unable to find the global optimum solution.

The above description described the weakness concerning the S-shaped sigmoid function and the position update rules in equation (3.18) used for the BPSO algorithm. It also highlights the issue regarding premature convergence that hinders the algorithm from finding better solutions. The following explains in detail the reasons for such behaviour:

i) For the conventional PSO, if the value of v_{ij}=0 it means the particle has already reached the suitable solution so far. Therefore, the corresponding position vector x_{ij} would not be able to instigate any movement anymore since there is no momentum given. However, for BPSO, the particles are forced to take the value of 1 or 0 when their v_{ij} is zero. Hence, once the particles have reached their suitable solutions, the particles tend to consider changing their positions. This prevents the particles from settling and will continue to diverge once they have reached their best solution. This makes the task for exploitation of the search space, which the particles are currently in, becomes harder because the particles will always move to a different position since their previous position is not considered.
ii) The position update rules in equation (3.18) used in the BPSO is largely responsible for the uncontrollable behaviour of the particles. Since the previous position of the position vector x_{ij} is not considered for the next iteration like the conventional PSO, the position update rule becomes more random once it has found its best position. This makes the algorithm hard to reach convergence and suffers from local optima because it will never be able to improve the current position since the particles' position are updated randomly.

In the research work in [88] and [89] , the use of the S-shaped sigmoid function in BPSO are investigated where a new sigmoid function, namely the V-shaped sigmoid function, and new position update rules for position vector x are proposed. The V-shaped sigmoid function and the new position update rules are formulated as follows:

$$sig(v_{ij}^{t+1}) = 2 \times \left| \frac{1}{1+e^{-v_{ij}(t+1)}} - 0.5 \right| \tag{4.13}$$

$$x_{ij}^{t+1} = \begin{cases} (x_{ij}^{t})^{-1} & \text{if } rand < sig(v_{ij}^{t+1}) \\ x_{ij}^{t} & \text{otherwise} \end{cases} \tag{4.14}$$

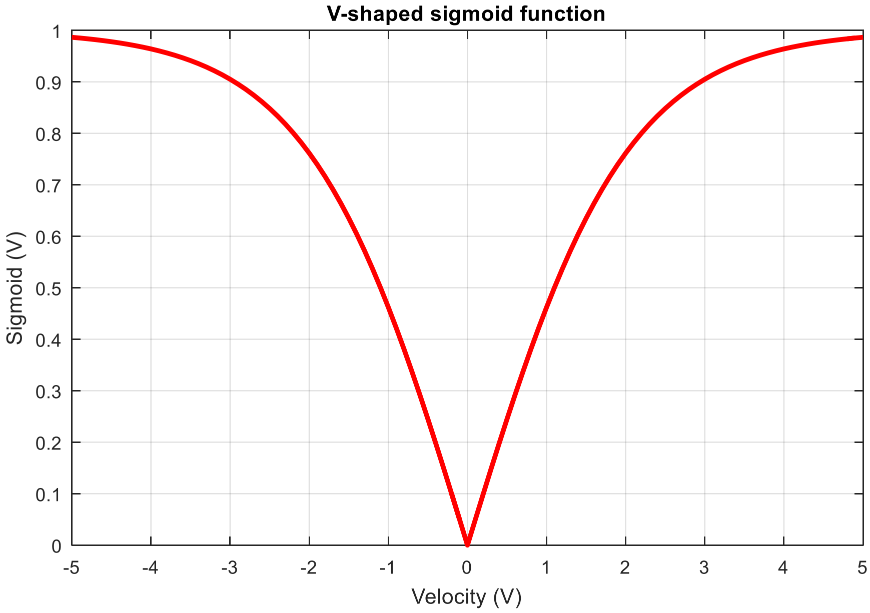

Figure 4.5: V-shaped sigmoid function

As can be seen from Figure 4.5, the new sigmoid function forms "V" when plotted, hence, the name V-shaped sigmoid function. The V-shaped sigmoid function and position update rules in equation (4.13) are formulated such that it can overcome the weakness of the S-shaped sigmoid as follows:

i) The V-shaped sigmoid function interprets a large positive value the same as a large negative value when deciding the probability. Hence, the probability is more consistent regardless of whether the value is positive or negative compared to the S-shaped sigmoid function.

ii) The previous position of vector x_{ij} will be considered when updating the particles' position. It also ensures that the position vector x_{ij} can be in their position when the particles are converged.

According to the research work in [88], the V-shaped sigmoid function outperforms the S-shaped sigmoid in unimodal and multimodal benchmark functions. It also appears that the average mean fitness and average best-so-far value when using the V-shaped sigmoid function improved significantly compared to the S-shaped sigmoid. In order to demonstrate the effects of the V-shaped sigmoid function for solving the OPP problem, it was examined based on two cases. The first case is with the same random seed and the second case is with a different random seed. This is to highlight the performance of both S-shaped sigmoid and V-shaped sigmoid when given identical sets of random numbers and different sets of random numbers. In both case, linear decreasing inertia weight with $\omega_{\max}$ and $\omega_{\min}$ set as 0.9 and 0.4, respectively, were used to examine the performance of the algorithm.

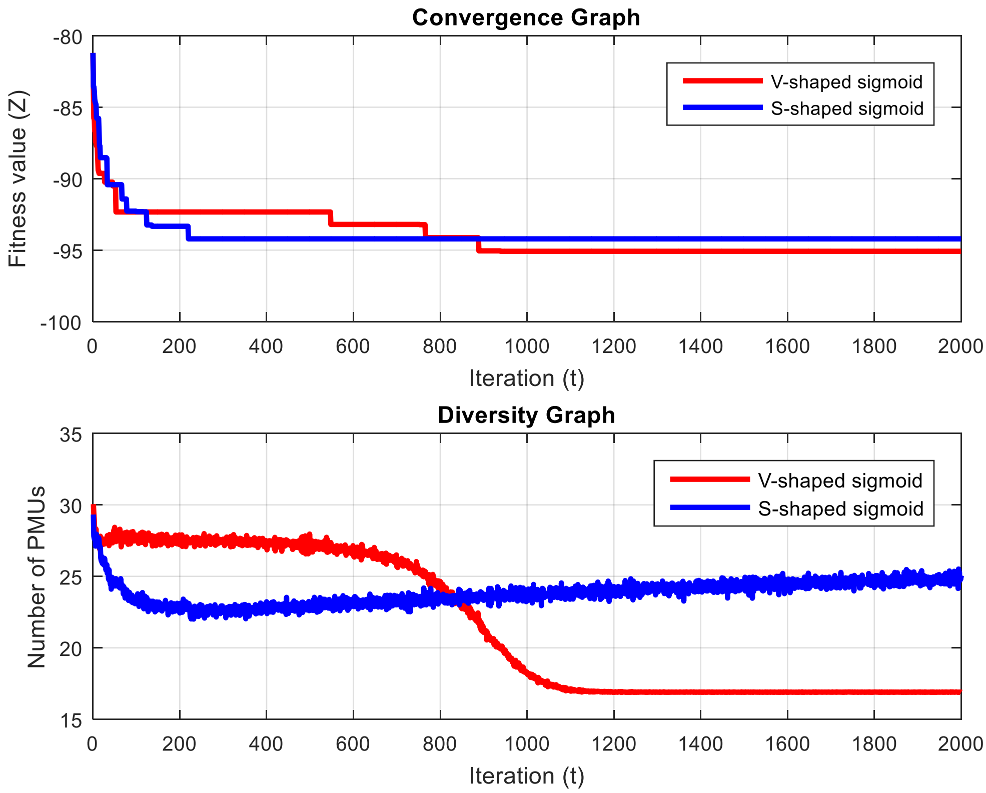

Figure 4.6: Convergence and diversity graph using the same random seed for IEEE 57-bus system

Figure 4.6 shows the performance of the BPSO algorithm when using the V-shaped sigmoid in terms of convergence and diversity if the same random seed is used. The BPSO algorithm that uses the V-shaped sigmoid converged late compared to the S-shaped sigmoid. However, it must be noted that the BPSO algorithm with the V-shaped sigmoid converged to the most minimum fitness value. Since the fitness function in equation (4.3) is to minimise the fitness value, therefore, the solution that managed to converge at the most minimum value is the best solution. It is also important to highlight that the particles diversity is different when using either the S-shaped or V-shaped sigmoid. When using the V-shaped sigmoid, the particles diversity is decreasing as the iteration is counted contrary to when using the S-shaped sigmoid where the particles start with the lowest diversity and ended up with the most diversity.

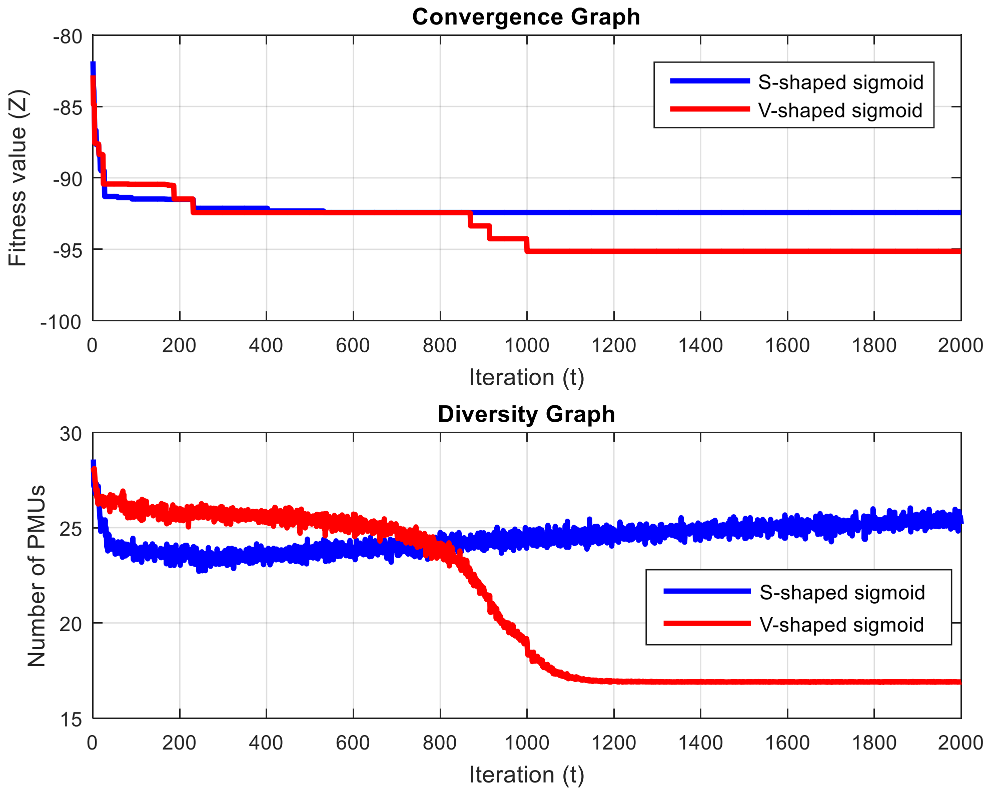

Figure 4.7: Convergence and diversity graph by using the different random seed for IEEE 57-bus system

Meanwhile, the behaviour is also consistent when using different random seeds as depicted in Figure 4.7. It is evident at this point that the use of the V-shaped sigmoid produces completely different behaviours in terms of convergence and particles diversity in particular. This shows the effect of the position update rules in equation (4.14) where it allows particles to be at their current position once it is converging towards the *gbest* where the v_{ij} starts to decrease until v_{ij}=0. The effect of the decreasing linear inertia weight can also be seen as it spends the earlier stages of the algorithm in exploration mode and switches to exploiting mode towards the end of the algorithm when using the V-shaped sigmoid. Therefore, the only downside when using the V-shaped sigmoid is its slow convergence speed albeit the better results from the two test cases.

4.6 Mutation

Mutation technique was often integrated into optimisation methods to make further improvement to the proposed method with regards to population diversity. While mutation operator was generally featured in the PSO, it is seldom applied in the BPSO across various studies. Moreover, mutation technique has only been considered for solving the OPP problem in [90] and [42] where it is triggered similar to that of in [91]. According to the investigation result in [42], the convergence rate of the proposed method over 50 cycles significantly outperforms the basic BPSO, especially in a larger system. However, it is interesting to note that, the additional complexity caused by the integration of SA and mutation technique in the proposed method has increased the computation time. In other research areas, Yudong Zhang et. al [92] adopted the mutation technique in spam detection method, while Menhas et. al [93] investigated the use of both crossover and mutation techniques in the BPSO algorithm. The results obtained from the benchmark functions indicate that they performed better and converged closer to the optimum solution. In addition, it can reach an acceptable solution with fewer iterations.

The improvements made by the mutation technique to the PSO can also be seen in the overview of the following methods:

a) Research work by Jancaukas [94] has shown the integration of mutation and PSO significantly improves the PSO's performance. However, the value for mutation rate needs to be carefully considered to ensure a good result. In the simulation test, it appears that higher values of mutation rate tends to reduce local search performance, hence, preventing the swarm from converging.
b) Liang and Kang [95] adopted mutation approach to update velocity equation where if it satisfies certain criteria, the update velocity equation for each particle will be different for each iteration. The study claims it could form mutation disturbance to avoid from being trapped in local minima. The study also claims particles can reach larger search range.
c) Mortezazadeh et. al [96] applied mutation approach which has two steps. In the first step, mutated particle is generated based on a modified velocity equation if it satisfies the predefined mutation probability parameter. In the second step, the mutated particle is used to replace the current particle if the mutated particle is better than the current particle. Otherwise, the current particle stays.

d) Lu et. al [97] incorporated a mutation operator called the real-valued mutation (RVM), which mimics the sudden change of a gene in natural genetics to enhance the diversity within the swarm, into three variants of PSO algorithms to solve economic dispatch problem with a non-smooth cost function. Particles are randomly selected to mutate in the RVM approach where for each particle selected to mutate, each component will be selected to mutate using a generated randomly binary sequence. The mutated particles will replace their current position for the next iteration regardless of their fitness value. The study indicates the idea is to inject fresh individuals to the swarm in each iteration to increase the chance for the swarm to escape from local optima.

e) Mutation approach in Yuehui et. al [91] will only be triggered if the *gbest* is not updated after certain iterations where particles are selected based on the probability of mutation parameter to mutate. The mutated particles will replace the corresponding particles for the next iteration.

f) While the research works mentioned earlier adapted a random approach in selecting particles to mutate, the swap mutation approach used in [98] selects particles based on their performances. Particles are ranked in each iteration based on the average costs of the generating unit where higher ranked particles will have the highest priority to be dispatched.

From the above literature, it appears that although mutation technique is generally used in the PSO, its usage is very limited when it comes to power system. Hence, there is an opportunity for a research to investigate the effects of mutation in the BPSO, specifically for solving the OPP problem. Therefore, this thesis aims to contribute to this research area by proposing and investigating the application of mutation in the BPSO in order to solve the OPP problem.

In PSO, mutation is adopted in the velocity update in equation (3.1) where the random values of R_1 and R_2 ensure that there is an external influence that may benefit the algorithm in terms of finding the best possible solutions [80]. The influence, however, is decreasing when the particles' *pbest* has the same position as *gbest* as the algorithm iterates. For instance, when *pbest* and *gbest* have the same position, the velocity value for the next position update will be limited to the previous velocity and inertia weight since the cognitive and social component have relatively no influence anymore towards the velocity equation. This leads to premature convergence as the particles are trapped in local optima. This behaviour will be changed if the *gbest* is updated to a new position.

The effect of premature convergence is more profound in the BPSO because the values for the particles' position in the BPSO are limited to 1 and 0 only, unlike the conventional PSO. Hence, particles are very quick to converge to the same position especially if the problem dimension is small. The use of the V-shaped sigmoid also means if the particles have already converged, unless either *pbest* or *gbest* is updated, the particles will not be able to change position since the velocity value will remain zero. Therefore, it is very hard for the particles to escape from their local optima if the particles already converged. Thus, to avoid the particles from being trapped in local optima, an external influence outside of the velocity update equation to help particles to change to new positions may be useful in this circumstance.

In this thesis, a mutation strategy is presented to solve the OPP problem using BPSO. While mutation is usually related to improving particles diversity, in this thesis, the purpose of the mutation strategy is mainly to improve the local search of the algorithm to help instigate particles to escape from the local optima while improving the *pbest* of the corresponding particles. The mutation strategy is designed to exploit the search space obtained from the *pbest* in order to find the better ones. The overall structure of the proposed approach can be seen from Figure 4.8.

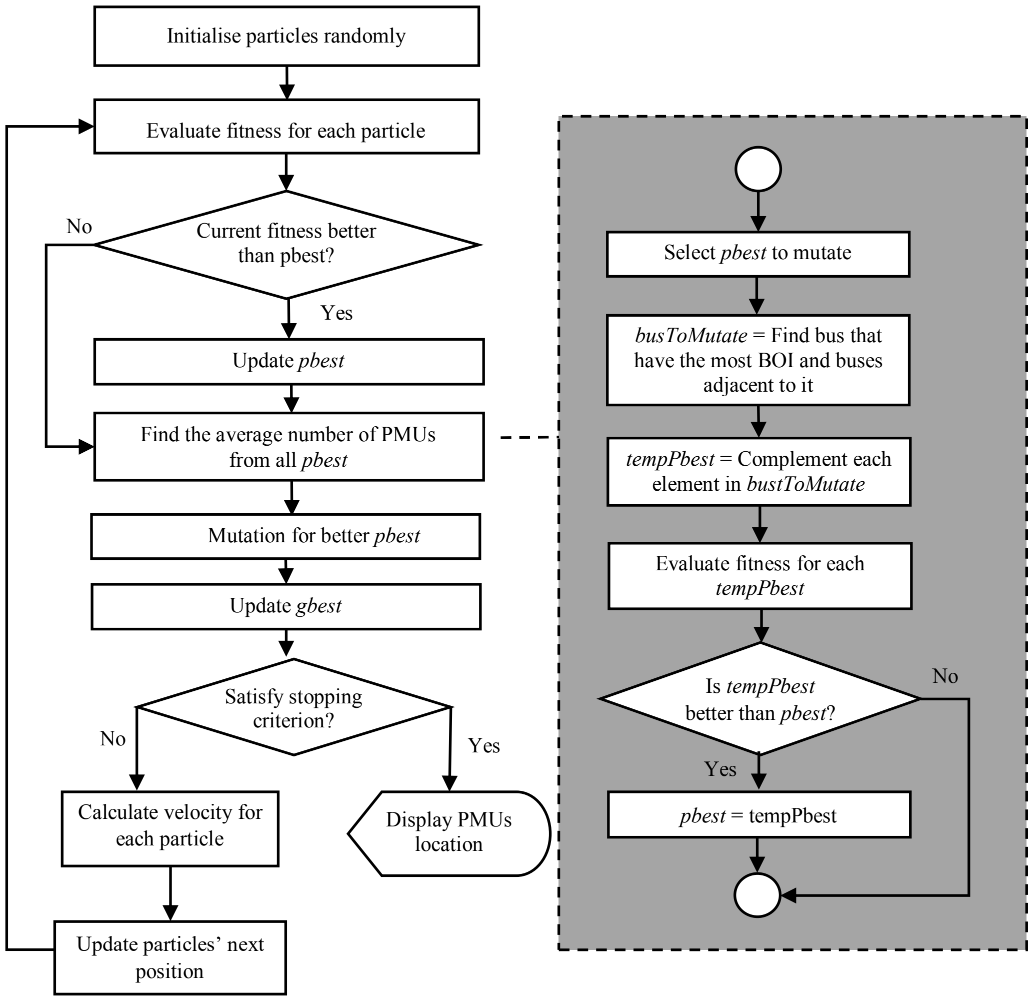

Figure 4.8: The updated flowchart for BPSO algorithm when proposed mutation is adopted

As can be seen in Figure 4.8, after the *pbest* is updated for the current iteration, the mutation process is executed to refine the updated *pbest*. If better solutions are found by the mutation strategy, the better solutions will replace the *pbest*. Otherwise, no change will be made to the *pbest*. Besides that, the mutation strategy also prioritises feasible solution to ensure that the swarm is largely influenced by the feasible solutions. Therefore, *pbest* that are updated using the mutation strategy will always be of feasible solutions. In order to find more feasible solutions through the mutation strategy, a PMU is placed at a bus that is adjacent to a radial bus to confirm the observability of the radial bus. Thus, the placement of other PMUs can be more focused on other regions of the search space. Pre-assigned PMU to a radial bus also helps to increase the convergence speed of the algorithm [99].

Algorithm 2: Pseudocode for the local search

```
Select pbest to mutate based on the average number of PMUs from pbest;
Store it in an array called pbestMutation;
Get the fitness value for all elements in pbestMutation;
for i=1 to pbestMutation do
        Find bus that have the maximum BOI;
        Store in an array called busList;
        for j=1 to busList do
                Find bus that adjacent to busList;
                Store in array called busToMutate including the bus itself;
        end
        for k=1 to busToMutate do
                Complement busToMutate;
                Evaluate the new position, tempPbest;
                Store the fitness value to an array called tempFitness;
        end
        Find the minimum value from tempFitness;
        if tempFitness better than that of pbest then
                Update pbest with the new position;
        else
                Remain pbest
        end
end
```

As mentioned earlier, the purpose of the mutation strategy is to improve the local search of the algorithm. Therefore, the mutation operation will only focus on buses that are adjacent to the bus selected for mutation. Algorithm 2 shows the pseudocode of the mutation strategy proposed in this thesis for solving the OPP problem. It is important to note that, in the proposed approach, only the *pbest* is considered for mutation.

The candidates *pbest* are firstly selected based on the average number of PMUs from the *pbest* of all particles. The candidates *pbest* are stored in an array called the *pbestMutation*. Then, for each element in *pbestMutation*, find the bus that have the maximum BOI and store it in an array called the *busList*. Buses that have the most BOI usually tend to have a cluster of PMUs observing the same bus, thus, the proposed approach will use this as a reason for mutation. For each element in the *busList*, find all buses that are adjacent to the corresponding bus including the bus itself and store them in an array called *busToMutate*. Elements in the *busToMutate* are the buses that will be mutated in this proposed approach. Since the BPSO algorithm uses binary, the state of each element in the *busToMutate* is either “0” (a PMU is not installed at the bus) or “1” (a PMU is installed at the bus). Therefore, the mutation process will basically involve the process of flipping “0” to “1” or “1” to “0”. Figure 4.9 shows the example of the mutation process in this thesis.

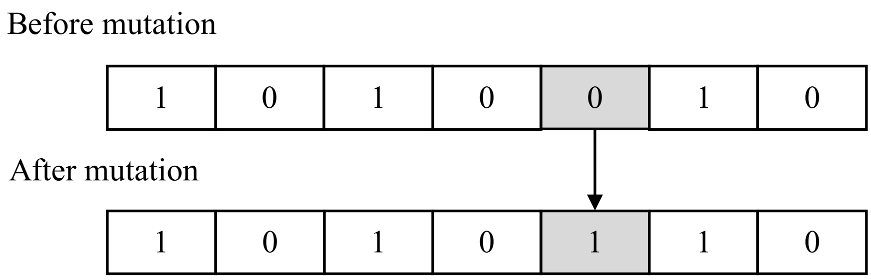

Figure 4.9: The basic mutation process

For each element in the *busToMutate*, only one element is mutated in each iteration as can be seen in Algorithm 2. In each iteration, the new position, *tempPbest*, formed from mutation is evaluated and the fitness value is stored in an array as *tempFitness.* After all elements in the *busToMutate* are finished with the mutation, the minimum value of the *tempFitness* is chosen to be compared with the current *pbest*. If the fitness value of the new position is better than the *pbest*, the *pbest* will then be updated with a new position. Otherwise, no changes will be made and the *pbest* will remain the same.

4.6.1 Illustration of the mutation approach

In order to explain further the process of mutation adopted in the proposed method, Figure 4.10 presents the 6-bus system that will be used to demonstrate the process of mutation in this thesis.

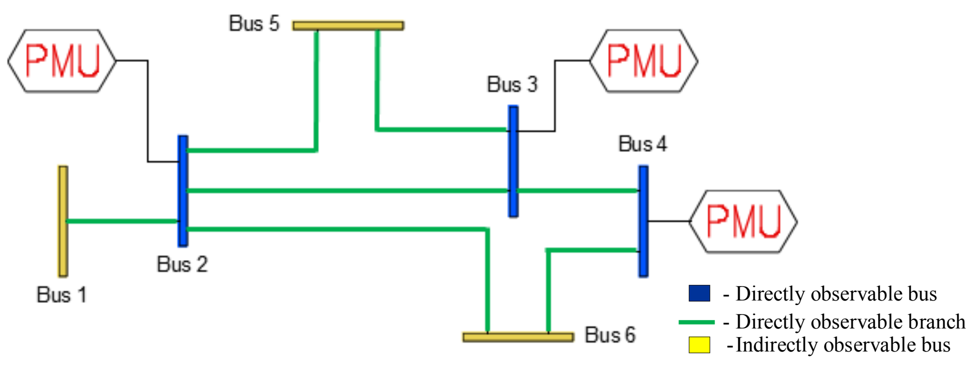

Figure 4.10: Model of 6-bus system used to explain the mutation strategy

Assume that in *t* iteration, the candidate *pbest* has PMUs placed at bus 2, 3, and 4. As can be observed, the system is observable with the current PMUs placement. Table 4.1 shows the location of the PMUs in binary and also the BOI of the 6-bus system based on the PMUs placement.

Table 4.1: Original location of PMUs in 6-bus system

Bus	**1**	**2**	**3**	**4**	**5**	**6**
PMUs Location	0	1	1	1	0	0
BOI when PMU is placed at bus 2	1	1	1	0	1	1
BOI when PMU is placed at bus 3	0	1	1	1	1	0
BOI when PMU is placed at bus 4	0	0	1	1	0	1
BOI_i	1	2	3	2	2	2
$SORI = \sum_{i=1}^{6} BOI_i$	12					

As indicated earlier, the bus that has the most BOI according to the current PMUs placement is chosen as the bus that will be considered for mutation. Since bus 3 has the most BOI, with the value of 3, bus 3 will be stored in an array called the *busList*. Next, the buses that are located next to the *busList* are gathered and stored in an array called the *busToMutate.* In this case, it is the *busToMutate* = {2, 3, 4, 5}, where each element in the *busToMutate* will undergo the mutation process.

Table 4.2: Mutation process for bus 2

Bus	**1**	**2**	**3**	**4**	**5**	**6**
Current *pbest*	0	1	1	1	0	0
tempPbest from the mutation of bus 2	0	0	1	1	0	0
BOI when PMU is placed at bus 3	0	1	1	1	1	0
BOI when PMU is placed at bus 4	0	0	1	1	0	1
BOI_i	0	1	2	2	1	1
$SORI = \sum_{i=1}^{6} BOI_i$	7					

In the first instance, bus 2 will be mutated. As shown in Table 4.2, bus 2 has a PMU placed at the bus in the current *pbest* as indicated by the value of "1". When applying the mutation approach, the value of "1" is flipped to "0". As the result of the mutation, the number of the PMUs is decreased by 1, which consequently influences the value of BOI for each bus. The new position after mutation also made bus 1 unobservable. As mentioned earlier, only feasible solution will be considered when mutating a particle. Therefore, the result of the mutation of bus 2 is invalid and will not be used. Then, the mutation process continues with the second instance, which is to mutate bus 3.

Table 4.3: Mutation process for bus 3

Bus	**1**	**2**	**3**	**4**	**5**	**6**
Current *pbest*	0	1	1	1	0	0
tempPbest from the mutation of bus 3	0	1	0	1	0	0
BOI when PMU is placed at bus 2	1	1	1	0	1	1
BOI when PMU is placed at bus 4	0	0	1	1	0	1
BOI_i	1	1	2	1	1	2
$SORI = \sum_{i=1}^{6} BOI_i$	8					

The result of mutation for bus 3 as indicated in Table 4.3 also reduces the number of PMUs. However, even though the number of PMUs is reduced, the system is still observable. Also, notice that the value of SORI after the mutation of bus 3 is less than the current *pbest* as presented in Table 4.1. This is expected because the measurement redundancy of each bus is influenced by the number of PMUs and their location.

Table 4.4: Mutation process for bus 4

Bus	**1**	**2**	**3**	**4**	**5**	**6**
Current *pbest*	0	1	1	1	0	0
tempPbest from the mutation of bus 4	0	1	1	0	0	0
BOI when PMU is placed at bus 2	1	1	1	0	1	1
BOI when PMU is placed at bus 3	0	1	1	1	1	0
BOI_i	1	2	2	1	2	1
$SORI = \sum_{i=1}^{6} BOI_i$	9					

For the mutation of bus 4, Table 4.4 shows the consequence of the mutation. Similar to the mutation of bus 3 previously, the mutation of bus 4 has reduced the number of PMUs while still maintaining the observability of the system. However, the value of SORI is better than the result obtained from the mutation of bus 3 as given in Table 4.3.

Table 4.5: Mutation process for bus 5

Bus	**1**	**2**	**3**	**4**	**5**	**6**
Current *pbest*	0	1	1	1	0	0
tempPbest from the mutation of bus 5	0	1	1	1	1	0
BOI when PMU is placed at bus 2	1	1	1	0	1	1
BOI when PMU is placed at bus 3	0	1	1	1	1	0
BOI when PMU is placed at bus 4	0	0	1	1	0	1
BOI when PMU is placed at bus 5	0	1	1	0	1	0
BOI_i	1	3	4	2	3	2
$SORI = \sum_{i=1}^{6} BOI_i$	15					

In this last instance, bus 5 is the bus to mutate. In contrast to the mutation of bus 2, 3, and 4 presented earlier, the mutation of bus 5 increases the number of PMUs to four while also increasing the value of SORI to 15 as given in Table 4.5.

When the mutation process is finished, the new position, *tempPbest*, of each mutation is evaluated using the fitness function in equation (4.3). The summary of the mutation process explained earlier is presented in Table 4.6. The mutation of bus 2 as mentioned earlier, is unable to make all buses observable, hence, it is ignored. The mutation process that produces the highest measurement redundancy is the mutation of bus 5. However, it requires more PMUs than the current *pbest,* hence, unable to improve it. The mutation of bus 3 and 4 managed to reduce the number of PMUs compared to current *pbest* without compromising the observability of all buses. Since the objective of solving the OPP problem in this thesis is to minimise the number of PMUs in order to achieve a complete observability of the power system while maximising measurement redundancy, the *tempPbest* that is better than the current *pbest* is from the mutation of bus 4. The mutation of bus 4 ensures that the number of PMUs is fewer and has the most measurement redundancy compared to the *tempPbest* from the mutation of bus 3 and the current *pbest*. The mutation approach demonstrated earlier shows that the approach managed to improve the current *pbest* into a better one.

Table 4.6: Summary from the mutation process

Mutation process	PMUs location	SORI	Observability
Current *pbest*	2, 3, 4	12	Yes
tempPbest from the mutation of bus 2	3, 4	7	No
tempPbest from the mutation of bus 3	2, 4	8	Yes
tempPbest from the mutation of bus 4	2, 3	9	Yes
tempPbest from the mutation of bus 5	2, 3, 4, 5	15	Yes

Figure 4.11 shows the new *pbest* obtained from the mutation process, replacing the current *pbest*.

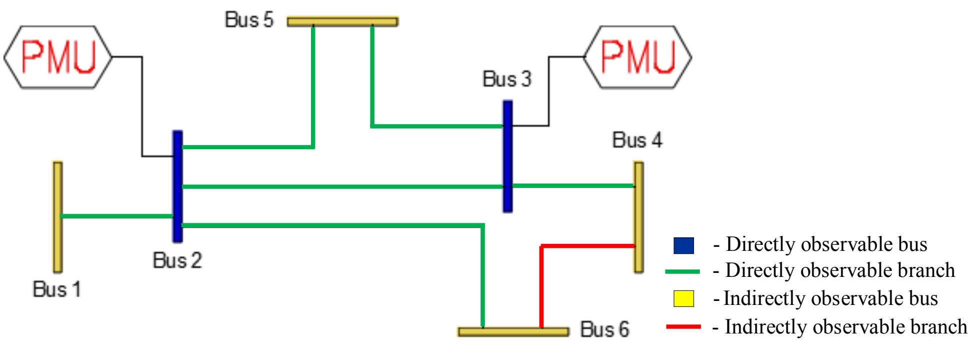

Figure 4.11: New PMUs location after mutation

The mutation strategy is integrated into the BPSO algorithm and applied it to the IEEE 57-bus system to examine the performance. The numerical results as shown in Table 4.7 indicate that the number of PMUs is the same, however, the fitness value is different when the mutation strategy is applied compared to without the mutation strategy. As depicted in Figure 4.12, the performance is significantly better than without integrating the mutation strategy. As can be seen, the convergence speed when using the mutation strategy improved significantly because of the improvement of the local search which allows better solutions to be found earlier. As for the particles diversity, it drops lower than without mutation since the mutation strategy enables the algorithm to find the best optimal solution very early. Therefore, the particles started to converge in the early stage of the algorithm and as indicated in Figure 4.12, the algorithm is no longer able to improve the current result in the latter iterations.

Table 4.7: The PMUs location when the case considering and not considering mutation for IEEE 57-bus system

Method	Number of PMUs, N_{PMU}	PMUs Location	Fitness value
Without mutation	17	1, 6, 9, 15, 19, 22, 25, 27, 32, 36, 38, 41, 47, 51, 52, 55, 57	-95.11
With mutation	17	1, 4, 6, 9, 15, 20, 24, 28, 30, 32, 36, 38, 41, 46, 51, 53, 57	-95.17

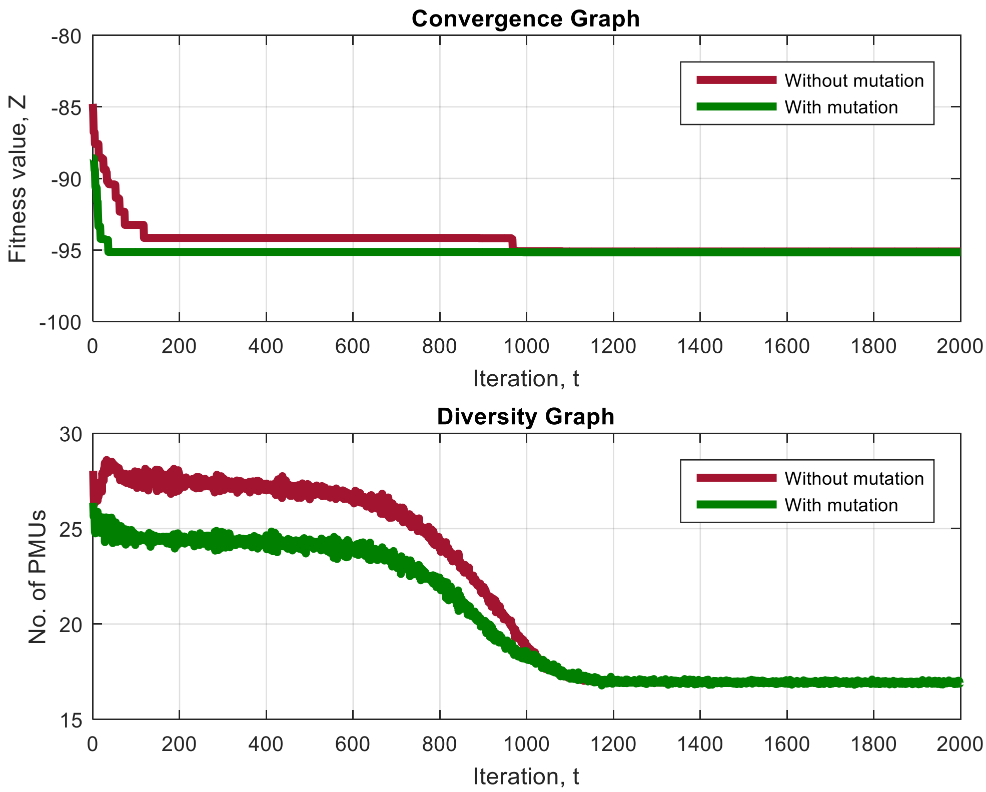

Figure 4.12: Comparing the performance of the algorithm when using mutation and not using mutation for IEEE 57-bus system

4.6.2 Improving computation time

The mutation strategy adopted in the proposed approach as explained in the previous section presents a number of extra fitness evaluations. It consequently increases the computational cost and also the time taken for the algorithm to finish as presented in Table 4.8 for the IEEE 57-bus system.

Table 4.8: Time taken for the proposed method to finish with and without mutation

Method	Time taken	Fitness value	No. of PMUs
Without mutation	16.7171 sec	-95.11	17
With mutation	127.1836 sec	-95.17	17

The time taken when adopting the mutation strategy is about 8 times longer than the approach without the mutation strategy, nevertheless, the mutation strategy managed to improve the result by converging to the most minimum value. Although solving the OPP problem is an offline planning [100], thereby indicating it does not prioritise computation time, it is always good if the optimisation method can achieve the desired result in a short time without affecting the performance of the algorithm.

To address this issue, instead of considering every candidate *pbest*, the mutation approach is designed to only consider a unique solution. The unique solution here means if the candidates *pbest* have the same fitness value, only one of them will be considered for mutation. This is because the *pbest* that has the same fitness value implies that it carries the same PMUs placement set. This means the mutation approach will produce the same result which consequently proves to be a waste of computation cost. To examine the performance of the algorithm, the performance between the mutation approach that considers the unique solution and without considering the unique solution is compared. The experiment was conducted using the same random seed to demonstrate the effects of the approach and the results are presented in Table 4.9. As expected, the time taken by the mutation approach that considers the unique solution is lessened significantly compared to the previous approach because fewer fitness evaluations were executed compared to the mutation strategy that did not consider the unique solution.

Table 4.9: Time taken for the proposed method when considering unique solution and not considering unique solution

Method	Time	Fitness value	No. of PMUs
Mutation without considering unique solution	127.1836 sec	-95.17	17
Mutation considering unique solution	16.4223 sec	-95.17	17

Also, it is worthy to note that the mutation strategy that considers the unique solution does not affect the algorithm's performance at all as depicted in Figure 4.13 since applying mutation to the same PMUs placement set will produce the same result. Although it still needs a longer time to finish the algorithm as given in Table 4.9, the time margin is small compared to the previous approach. Therefore, the proposed method will adopt the unique solution for mutation.

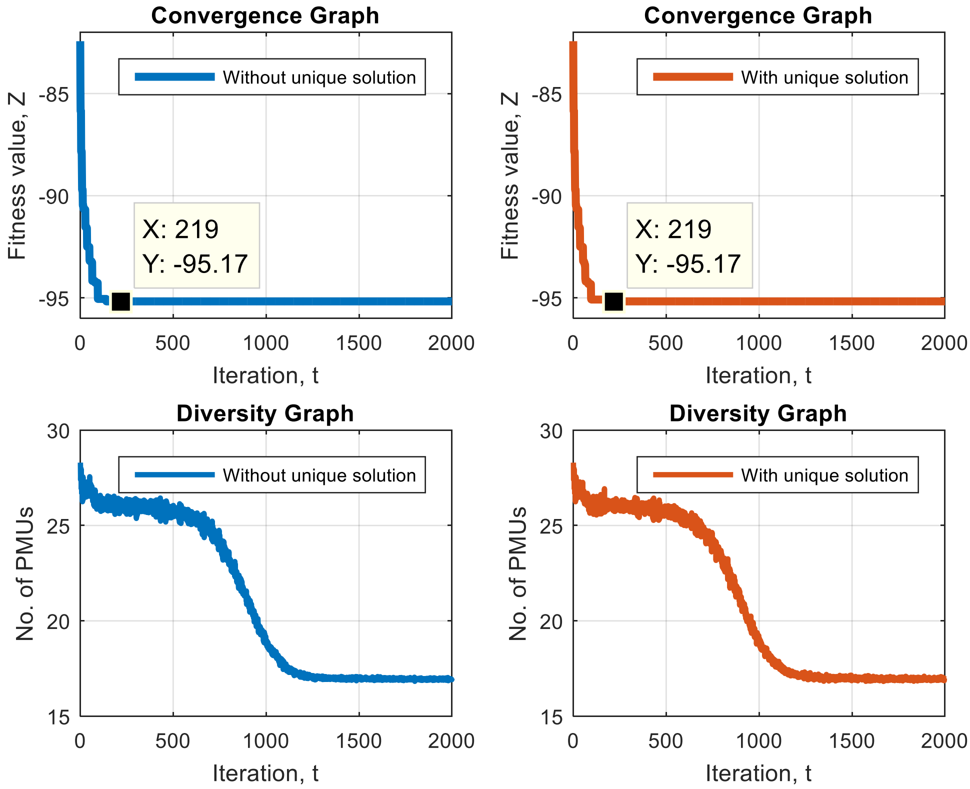

Figure 4.13: Convergence and diversity graph for case considering unique solution and not considering unique solution for IEEE 57-bus system

4.7 Parameter selection

The following sections discuss the selection of parameters used in this thesis to improve its overall performance.

4.7.1 Constriction approach, *K*

Inertia weight and constriction factor as explained in the previous chapter have the same role in mind, which is to control the particles' exploration and exploitation, albeit it is done differently. The conventional PSO does not have any external influences on previous velocity. Therefore, the value of velocity would increase linearly as it iterates. Because of this factor, inertia weight and constriction factor were introduced. The difference between inertia weight and constriction factor can be seen from equations (3.7) and (3.14), respectively where inertia weight applies its influence onto previous velocity, while the constriction factor emphasises its influence onto the three components as a whole.

While inertia weight requires velocity clamping to control the velocity appropriately, constriction factor does not require the use of velocity clamping in its approach. The impact of constriction factor in the proposed approach can be seen in Figure 4.14. As can be observed in the convergence graph, the use of the constriction factor does not significantly affect the convergence speed because both approaches converged very close to each other and at the same fitness value. However, in a larger dimension such as the IEEE 118-bus system, the convergence speed is better when using the constriction factor as can be seen in Figure 4.15. Meanwhile, for particle diversity, in both 57-bus and 118-bus, the use of constriction factor has caused the particle diversity to decrease slowly as the algorithm continues compared to the inertia weight which experienced sudden collapse in particle diversity as soon as it reaches 600 iterations. This behaviour is due to the nature of the constriction factor itself which emphasises on exploiting a focused region of the search space although it is capable of switching to exploratory mode if the swarm discovers a new optimum [101].

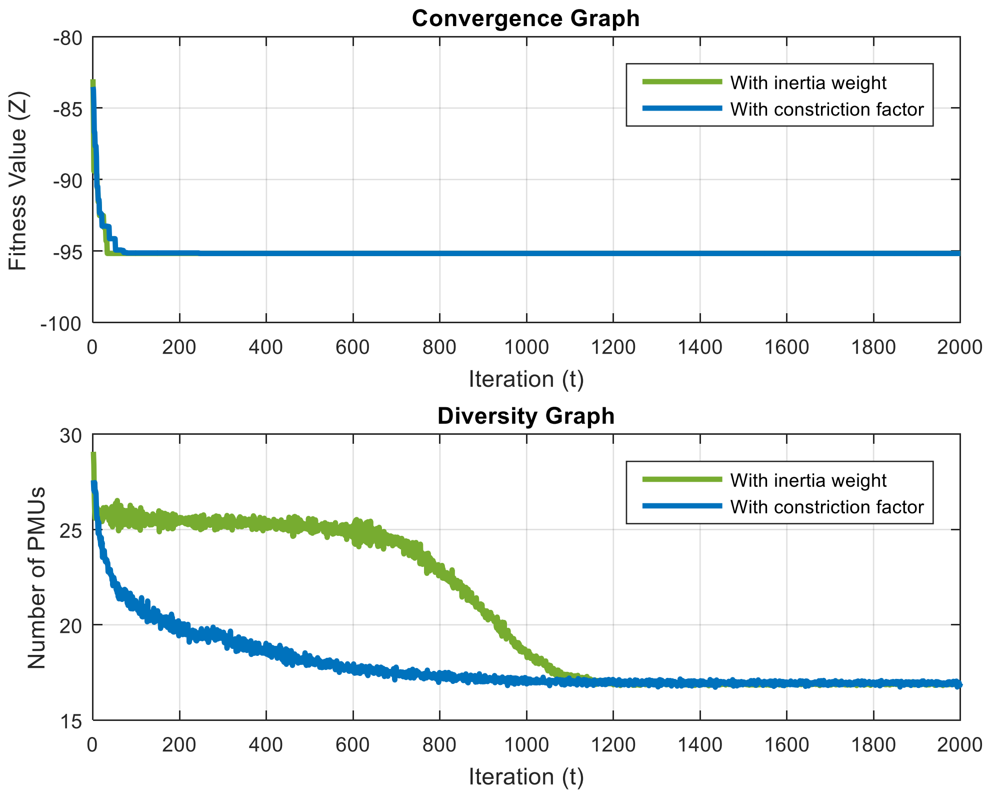

Figure 4.14: Comparison of convergence and diversity graph for IEEE 57-bus system between inertia weight and constriction factor approach when applied to the proposed method

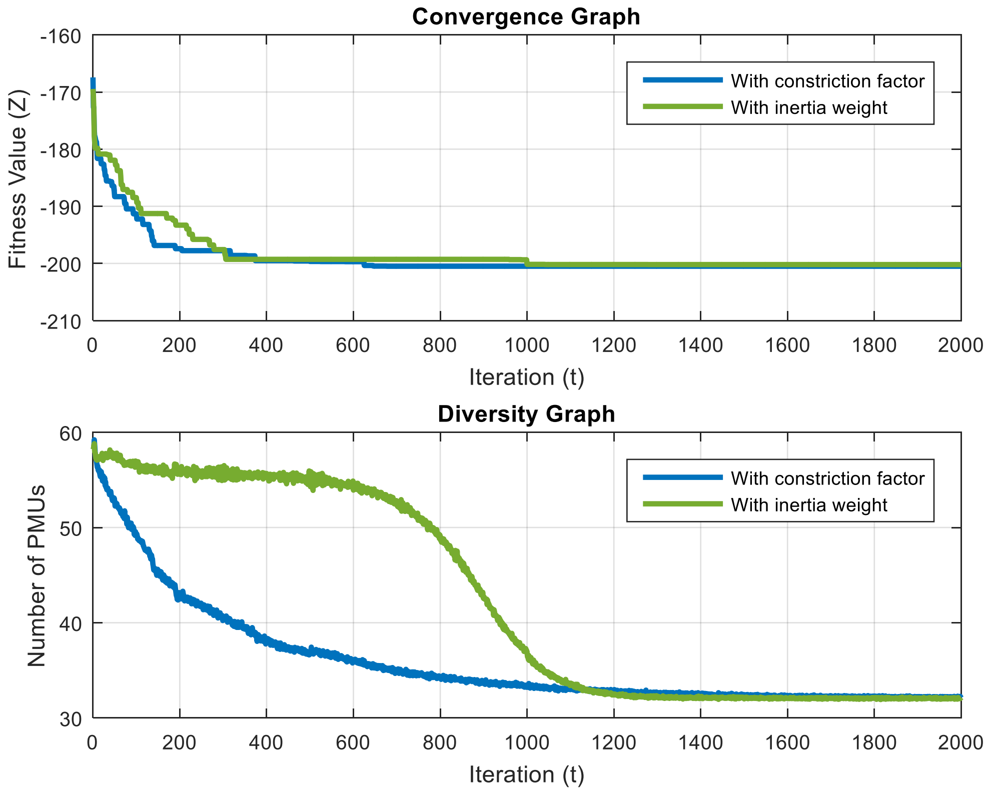

Figure 4.15: Comparison of convergence and diversity graph for IEEE 118-bus system between inertia weight and constriction factor approach when applied to the proposed method

In order to decide which technique is applicable with the proposed method, the convergence rate is used to measure the performance of the two techniques. The convergence rate is referring to the number of times the algorithm manages to converge at the best optimal solution. During the experiment, the algorithm was ran for 30 times for the IEEE 57-bus and 118-bus systems to decide which technique is appropriate for the proposed method.

For the linear decreasing inertia weight, the values used for $\omega_{\max}$ and $\omega_{\min}$ are 0.9 and 0.4, respectively, which are the two common values used and have been reported to produce good performance in the existing studies [38], [41]. For velocity clamping, the value for $V_{\max}$ and $V_{\min}$ were set as 6 and -6, respectively. Meanwhile, for the constriction factor, the value used for C_1 and C_2 is 2.05, which was also the value commonly used in the existing studies when using the constriction factor [36], [102]. Table 4.10 presents the results obtained from the experiment. As can be observed, for the 57-bus system, both approaches managed to find the same minimum number of PMUs required for a complete observability of the power system and also have the same number of convergence rate.

However, for the 118-bus system, although both approaches managed to obtain the same number of PMUs, the convergence rate is more consistent with the use of the constriction factor compared to the linear inertia weight. Therefore, the constriction factor is adopted into the proposed method since it is the most consistent in giving the best possible solution between the two approaches.

Table 4.10: The convergence rate of IEEE 57-bus and 118-bus system using inertia weight and constriction factor

IEEE bus system	Minimum number of PMUs obtained over 30 times	Convergence rate	
		With inertia weight	With constriction factor
57-bus	17	30/30	30/30
118-bus	32	12/30	19/30

4.7.2 Swarm size, *S*

The size of population in the BPSO is also very helpful in improving the particle diversity. It is suggested in [103] that 50 particles are a good starting point. In the experiment, for small systems such as the IEEE 14-bus, 24-bus, 30-bus, and 39-bus systems, 50 particles are good enough to be used as the swarm size. However, for larger systems, the size is not sufficient to achieve the desired consistency. Therefore, in order to decide the swarm size that will be used in the proposed method, a different number of particles was used to evaluate the consistency of the algorithm over the 30 times the algorithm runs. The ideal swarm size is selected based on the average minimum number of PMUs that the algorithm managed to get. Table 4.11 presents the experiment results for different number of particles for the 118-bus system.

As can be observed, six different numbers of particles used in the experimental tests managed to find the same number of PMUs. However, each gives a different convergence rate. It is important to note that increasing the swarm size does increase the possibility of getting the best possible solutions. The convergence rate is increased as the number of swarm increases, with the exception of one. However, it also gives a negative impact to the algorithm as the largest swarm size decreases the convergence rate of the algorithm. This finding is aligned with the studies conducted in [104] which stressed that a large population may give a negative impact to the algorithm's performance. Therefore, the swarm size used in this thesis is $S = N \times 4$.

Table 4.11: Comparing convergence rate and the average number of PMUs based on the size of the swarm for IEEE 118-bus system

Swarm size, *S*	**Maximum SORI**	**Convergence rate**	**Minimum number of PMUs**	**Average minimum number of PMUs**	**Maximum number of PMUs**
50	163	2/30	32	33.3667	35
N	163	7/30	32	33.1000	34
$N \times 2$	164	9/30	32	33	35
$N \times 3$	164	18/30	32	32.4333	34
$N \times 4$	164	23/30	32	32.2667	34
$N \times 5$	164	16/30	32	32.4667	33

4.7.3 Number of Iterations, *t*

The stopping criterion also plays an important role in the BPSO algorithm because it decides when should the algorithm finishes the searching process in order to find the best solution. Usually, the number of iterations is set as the stopping criterion. This means after a certain number of iterations, the algorithm will end regardless of whether it has found the best possible solution or not. A large number of iterations may give more time for the particles to find a better solution but the algorithm might have converged early, thus, making the remaining iterations become pointless. Meanwhile, a low number of iterations may end the algorithm prematurely and the best solution may have not been found.

Therefore, a compromise point needs to be determined to ensure that the algorithm can find the best possible solution in a reasonable time. The ideal number of iterations for the proposed method was investigated and 2000 iterations appears to be the ideal number of iterations. The algorithm managed to find the best possible solution up to the IEEE 300-bus system as presented in Figure 4.16, which is the largest IEEE bus system that this thesis intended to apply in the proposed method.

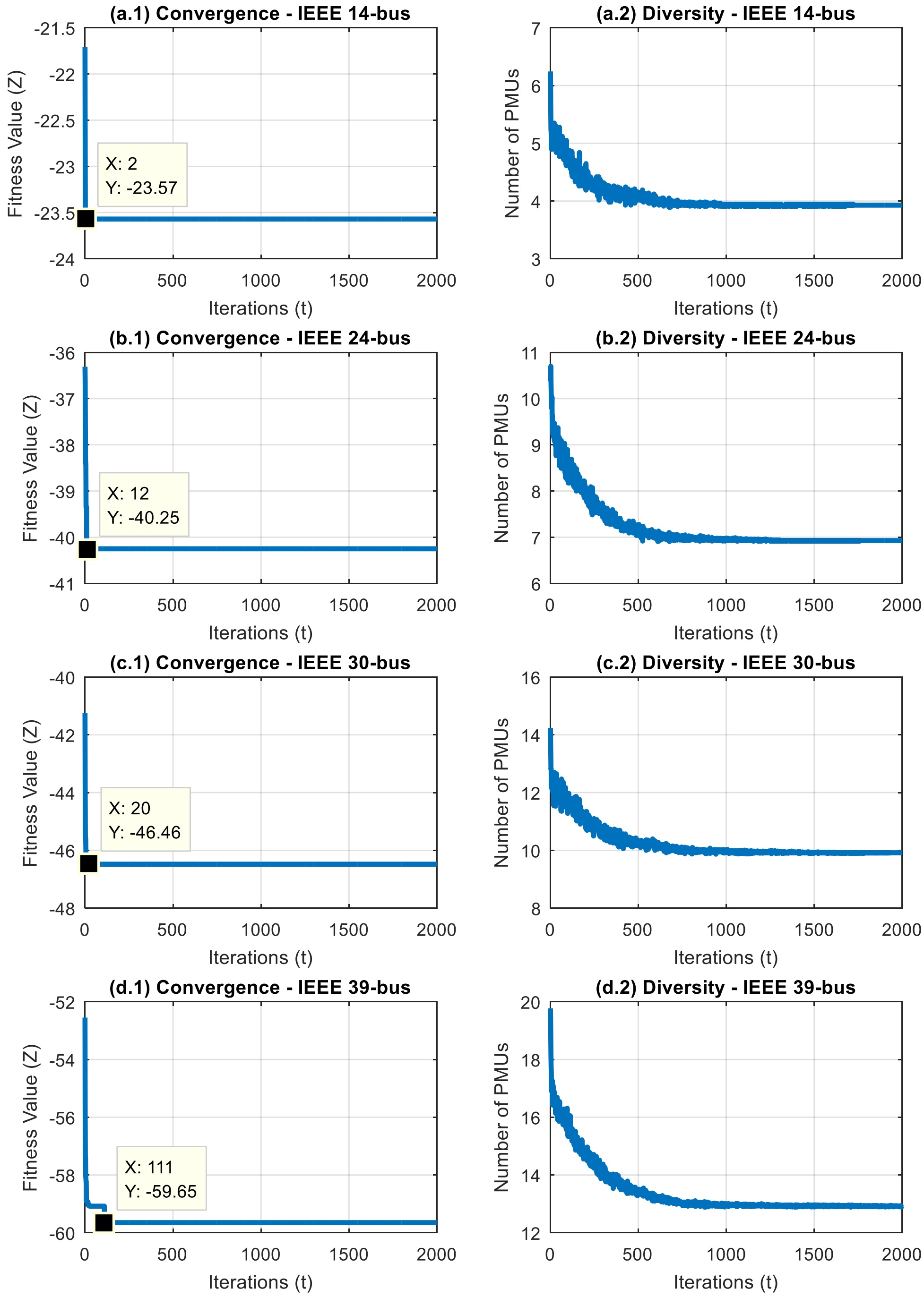

Figure 4.16(a): The convergence graph for all IEEE bus system tested

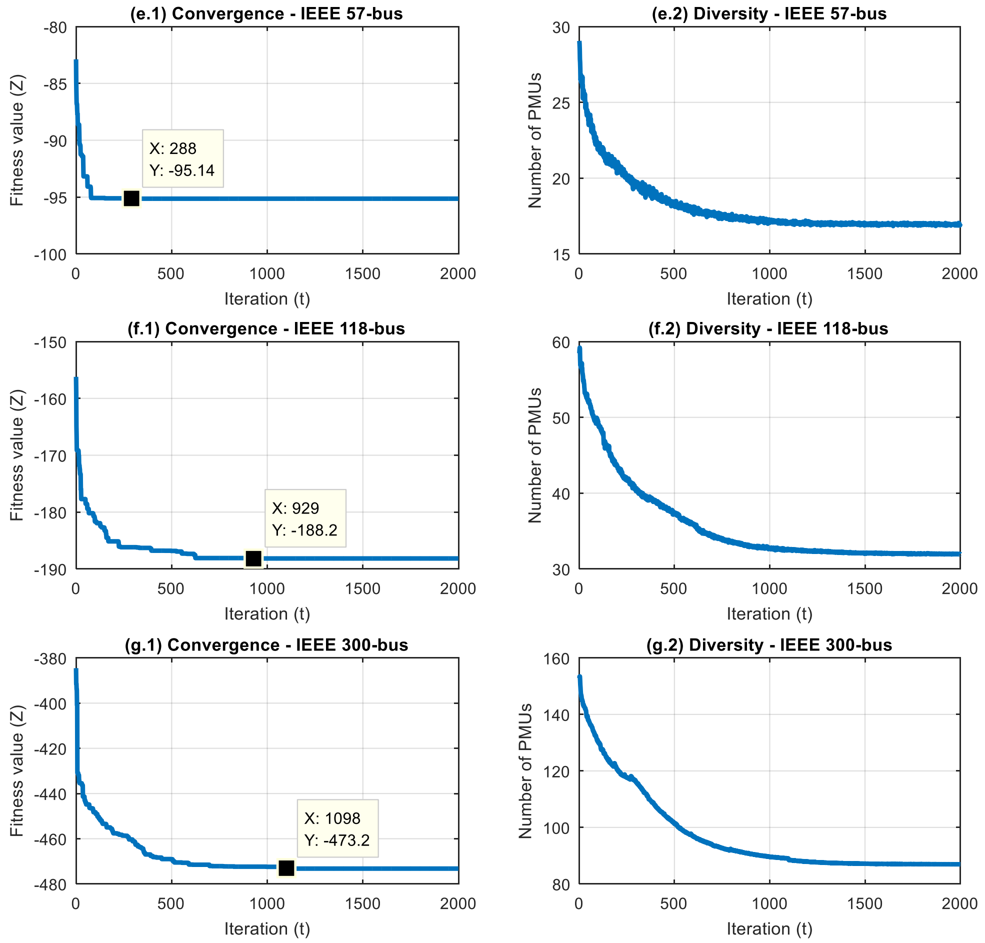

Figure 4.16(b): The convergence graph for all IEEE bus system tested

4.8 Summary

In this chapter, a modified BPSO method is proposed by integrating the mutation strategy and the V-shaped sigmoid function. The fitness function used to evaluate every particle of the desired objective is discussed. In addition to that, the fitness function that incorporates a single PMU loss is also proposed. In order to demonstrate the effectiveness of the proposed method, the influence of both approaches is investigated and analysed using numerical results, convergence and diversity graphs. The parameters selection used by the proposed method is also discussed to evaluate their influences towards the proposed method and the algorithm in particular.

Chapter 5

Results and Discussion

5.1 Introduction

The proposed method has been discussed in details in the previous chapter where the impact of the mutation strategy and V-shaped sigmoid function for solving the OPP problem are highlighted. The simulations are carried out by using MATLAB R2015b software. The technical specification of the computer used to run the simulations is Intel core i5 2.5 GHz with 8 GB of RAM. The value of each parameter used in the equations (4.3) and (4.8) during the simulations is given in Table 5.1.

Table 5.1: The values of each parameter used in simulation

Parameter	Value
Number of particles	$4 \times N$
Number of iterations	2000
Weight value for the number of bus observed, w_1	-2
Weight value for the number of PMUs, w_2	1
Weight value for the number of bus that is not being observed twice by PMUs placement, w_3	2
Weight value for single PMU loss measurement redundancy, w_4	-0.02
Weight value for the measurement redundancy, C	0.01

In this chapter, the proposed method will be applied to various IEEE bus systems ranging from 14-bus until 300-bus systems in order to verify the application of the proposed method in solving the OPP problem. The data for each bus system is obtained from MATPOWER [105] and is shown in Appendix**.**

The objective of the proposed method is to find the minimum number of PMUs that maintains complete observability while maximising measurement redundancy. Since BPSO is a meta-heuristic algorithm, multiple PMUs placement set are expected and to differentiate the quality of each PMU placement set that has the same number of PMUs, the PMUs placement set that has the highest measurement SORI value will be selected as the best optimal result. As mentioned in previous chapter, the PMUs placement set that has the highest measurement redundancy indicate it is better than the PMUs placement set that has low measurement redundancy. In addition, to evaluate the effectiveness of the proposed method in solving the OPP problem, the SORI value for the PMUs placement set obtained from the proposed method and existing studies will be compared. The proposed method is applied to solve the OPP problem by considering several cases. The cases considered for the proposed method are as the following:

i. Normal operation
ii. Case considering ZIB
iii. Case considering single PMU loss
iv. Case considering PMU's channels limit

The algorithm was run for 30 times with random particle initialisation for each case. The result for each case is presented in the following sections. For all test cases, the presence of radial bus is considered to ensure the best possible solution can be obtained. Table 5.2 shows the location of radial bus for each IEEE bus systems.

Table 5.2: The location of radial bus for all tested IEEE bus systems

IEEE bus system	No. of radial bus	Location of radial bus
14-bus	1	8
24-bus	1	7
30-bus	3	11, 13, 26
39-bus	9	30, 31, 32, 33, 34, 35, 36, 37, 38
57-bus	1	33
118-bus	7	10, 73, 87, 111, 112, 116, 117
300-bus	69	84, 171, 185, 213, 222, 227, 230, 233, 236, 239, 241, 250, 281, 319, 320, 322, 323, 324, 526, 528, 531, 552, 562, 609, 664, 1190, 1200, 7001, 7002, 7003, 7011, 7012, 7017, 7023, 7024, 7039, 7044, 7049, 7055, 7057, 7061, 7062, 7071, 7130, 7139, 7166, 9022, 9024, 9025, 9026, 9031, 9032, 9033, 9034, 9035, 9036, 9037, 9038, 9041, 9042, 9043, 9051, 9052, 9054, 9055, 9071, 9072, 9121, 9533

For each radial bus in Table 5.2, the PMU is prevented from being placed at the radial bus. Instead, the PMU is encouraged to be placed at the bus that is adjacent to the radial bus.

5.1.1 Normal operation

In this case, the OPP problem is solved by ignoring ZIB, a single PMU loss and PMU's channels limit. The number of PMUs required for each IEEE bus system tested including their locations and the SORI value for each PMUs placement set is given in Table 5.3.

Table 5.3: PMU locations for normal operation

IEEE bus system	No. of PMUs, N_{PMU}	Locations of PMUs	*SORI*	N_{PMU} / N
14-Bus	4	2, 6, 7, 9	19	0.2857
24-Bus	7	2, 3, 8, 10, 16, 21, 23	31	0.2916
30-Bus	10	2, 4, 6, 9, 10, 12, 15, 19, 25, 27	52	0.3333
39-Bus	13	2, 6, 9, 10, 13, 14, 17, 19, 20, 22, 23, 25, 29	52	0.3333
57-Bus	17	1, 4, 6, 9, 15, 20, 24, 28, 30, 32, 36, 38, 41, 47, 51, 53, 57	72	0.2982
118-Bus	32	3, 5, 9, 12, 15, 17, 21, 25, 28, 34, 37, 40, 45, 49, 52, 56, 62, 64, 68, 70, 71, 76, 79, 85, 86, 89, 92, 96, 100, 105, 110, 114	164	0.2711
300-Bus	87	1, 2, 3, 11, 12, 15, 17, 20, 23, 24, 26, 33, 35, 39, 43, 44, 49, 55, 57, 61, 62, 63, 70, 71, 72, 74, 77, 78, 81, 86, 97, 102, 104, 105, 108, 109, 114, 119, 120, 122, 124, 130, 132, 133, 134, 137, 139, 140, 143, 153, 154, 159, 164, 166, 173, 178, 184, 188, 194, 198, 204, 208, 210, 211, 214, 217, 223, 225, 229, 231, 232, 234, 237, 238, 240, 245, 246, 249, 9002, 9003, 9004, 9005, 9007, 9012, 9021, 9023, 9053	432	0.29

As can be noted in Table 5.3, the number of PMUs required that can achieve complete observability increases as the size of the power system increased. In addition, the number of PMUs is in the range of 20%-30% from the number of bus that needs to be observed as mentioned in the existing study [25].

5.1.2 Case considering ZIB

In this case, the presence of ZIB in power system is considered when solving the OPP problem. The ZIBs locations for every IEEE bus system tested are given in Table 5.4.

Table 5.4: The ZIBs location for every IEEE bus system tested

IEEE bus system	ZIBs location	Number of ZIB
14-bus	7	1
24-bus	11, 12, 17, 24	4
30-bus	6, 9, 22, 25, 27, 28	6
39-bus	1, 2, 5, 6, 9, 10, 11, 13, 14, 17, 19, 22	12
57-bus	4, 7, 11, 21, 22, 24, 26, 34, 36, 37, 39, 40, 45, 46, 48	15
118-bus	5, 9, 30, 37, 38, 63, 64, 68, 71, 81	10
300-bus	4, 7, 12, 16, 19, 24, 34, 35, 36, 39, 42, 45, 46, 60, 62, 64, 69, 74, 78, 81, 85, 86, 87, 88, 100, 115, 116, 117, 128, 129, 130, 131, 132, 133, 134, 144, 150, 151, 158, 160, 164, 165, 166, 168, 169, 174, 193, 194, 195, 210, 212, 219, 226, 237, 240, 244, 1201, 2040, 7049, 9001, 9005, 9006, 9007, 9012, 9023, 9044	66

Meanwhile, the results obtained for each bus system is given in Table 5.5. Notice that the number of PMUs required for all buses are reduced when the presence of ZIB in power system is considered compared to the results obtained for normal operation.

Table 5.5: The PMUs placement for case considering ZIB

IEEE bus system	No. of PMUs, N_{PMU}	Locations of PMUs	*SORI*
14-Bus	3	2, 6, 9	16
24-Bus	6	2, 8, 10, 15, 20, 21	29
30-Bus	7	2, 4, 10, 12, 15, 19, 27	41
39-Bus	8	3, 8, 13, 16, 20, 23, 25, 29	43
57-Bus	11	1, 6, 13, 19, 25, 29, 32, 38, 51, 54, 56	60
118-Bus	28	3, 8, 11, 12, 17, 21, 27, 31, 32, 34, 37, 40, 45, 49, 52, 56, 62, 72, 75, 77, 80, 85, 86, 90, 94, 102, 105, 110	156

300-Bus	69	1, 2, 3, 11, 15, 17, 20, 23, 24, 26, 37, 41, 43, 44, 55, 57, 61, 63, 70, 71, 72, 77, 97, 104, 105, 108, 109, 114, 119, 120, 122, 126, 139, 140, 145, 152, 154, 155, 166, 175, 178, 184, 187, 188, 198, 205, 210, 211, 214, 216, 223, 225, 229, 231, 232, 234, 237, 238, 240, 245, 249, 9002, 9003, 9004, 9005, 9007, 9021, 9023, 9053	393

For example, for the IEEE 39-bus system, in normal operation, the number of PMUs required to achieve complete observability of the power system is 13, whereas, in the case considering ZIB, only 8 PMUs are required. Figure 5.1 illustrates the location of PMUs placement for IEEE 39-bus system and how the PMUs placement set conforms the observability rules that ensure the power system is observable.

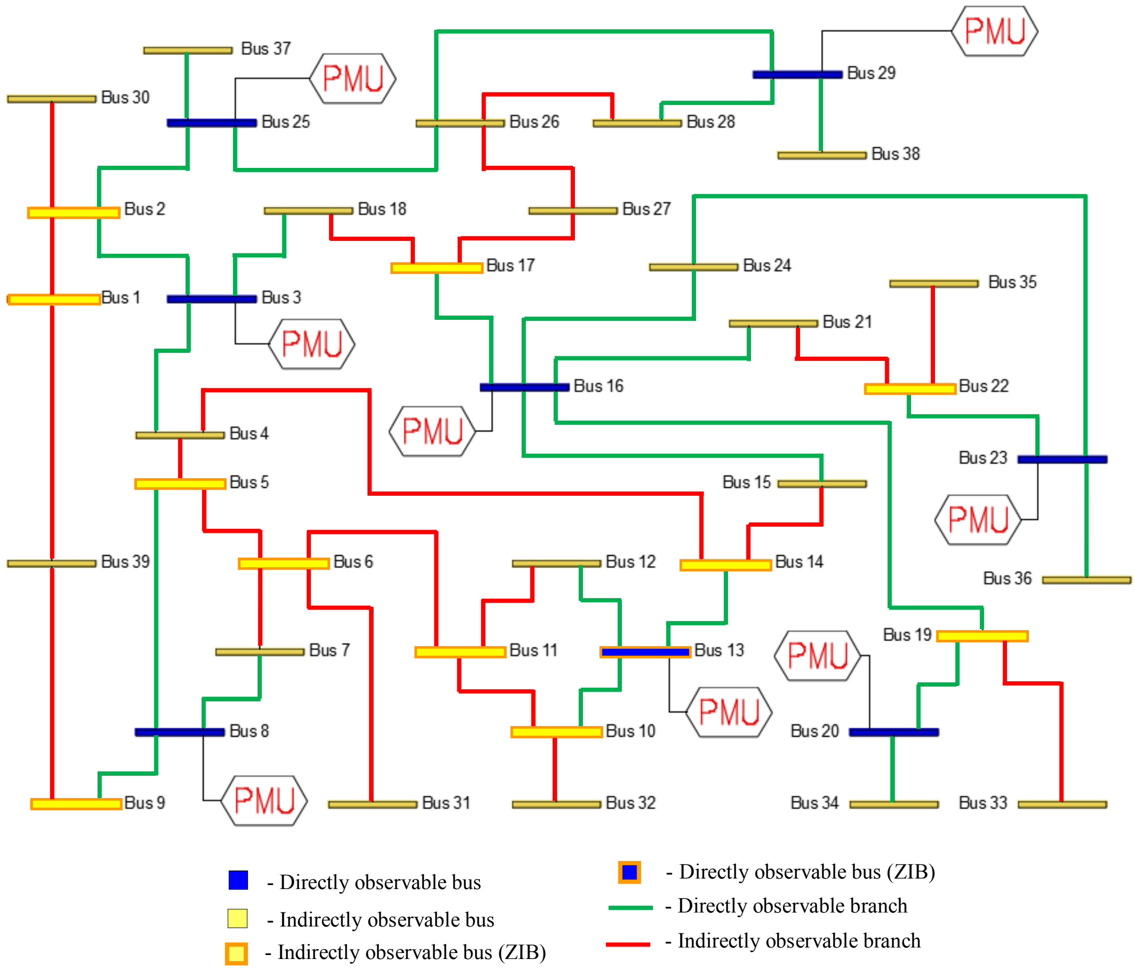

Figure 5.1: The PMUs placement and how it makes IEEE 39-bus system observable

From Figure 5.1, notice that bus 10 and bus 25 are located next to the radial bus, which has been mentioned earlier should have the PMU placed on it. However, since bus 10 is a

ZIB, the PMU is not forced to have it installed at the bus because by applying KCL at bus 10, bus 32 is indirectly observable by the PMUs placement set as shown in Figure 5.1.

5.1.3 Case considering single PMU loss

In this case, each bus in the power system will be monitored by at least two PMUs to ensure if either one of the PMUs responsible in monitoring corresponding bus becomes malfunction, the bus will remain observable through other PMUs. At this point, it is obvious that the number of PMUs required for this case will be increased since each bus needs to be observed by more than one PMU and the results presented in Table 5.6 agree with this notion.

Table 5.6: PMUs placement for case considering single PMU loss

IEEE bus system	No. of PMUs, N_{PMU}	Locations of PMUs	*SORI*
14-Bus	9	2, 4, 5, 6, 7, 8, 9, 10, 13	39
24-Bus	14	1, 2, 3, 7, 8, 9, 10, 11, 15, 16, 17, 20, 21, 23	59
30-Bus	21	2, 3, 4, 6, 7, 9, 10, 11, 12, 13, 15, 16, 18, 20, 22, 24, 25, 26, 27, 28, 30	85
39-Bus	28	2, 3, 6, 8, 9, 10, 11, 13, 14, 16, 17, 19, 20, 22, 23, 25, 26, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39	96
57-Bus	33	1, 3, 4, 6, 9, 11, 12, 15, 19, 20, 22, 24, 25, 26, 28, 29, 30, 32, 33, 34, 36, 37, 38, 41, 45, 46, 47, 50, 51, 53, 54, 56, 57	130
118-Bus	68	2, 3, 5, 6, 9, 10, 11, 12, 15, 17, 19, 21, 22, 24, 25, 27, 29, 30, 31, 32, 34, 35, 37, 40, 42, 43, 45, 46, 49, 51, 52, 54, 56, 57, 59, 61, 62, 64, 66, 68, 70, 71, 73, 75, 76, 77, 79, 80, 83, 85, 86, 87, 89, 90, 92, 94, 96, 100, 101, 105, 106, 108, 110, 111, 112, 114, 116, 117	309

For instance, for the IEEE 57-bus system, the number of PMUs when single PMU loss is taken into account is 33 PMUs. It is more than when it is compared to the normal operation and the case considering ZIB, where the number of PMUs required is 17 PMUs and 11 PMUs, respectively.

5.1.4 Case considering channel limit

For this case, each PMU is restricted to a certain number of channels. Therefore, its capability to measure is limited based on the number of channels set. This case was examined by limiting the number of channels from 2 channels until 5 channels. The number of possible combinations of each IEEE bus system for different number of channels are given in Table 5.7. The number of possible combinations is calculated based on equations (2.14) and (2.15).

Table 5.7: Number of possible combinations of all bus systems using different number of channels

IEEE bus system	Number of branches	Number of channels	Number of possible combinations
14-bus	20	2	40
		3	47
		4	35
		5	18
24-bus	38	2	68
		3	75
		4	51
		5	32
30-bus	41	2	82
		3	103
		4	104
		5	82
39-bus	46	2	92
		3	91
		4	57
		5	43
57-bus	80	2	156
		3	171
		4	143
		5	97

Noted that when considering PMU's channels limit, the dimension of a particle is formed based on the number of possible combinations as shown in equation (2.19). Therefore, the problem dimensions is larger than the number of bus that needs to be observed. Since the population based methods tend to suffer when dealing with large problem dimensions, hence, the importance of population diversity is very crucial to prevent the algorithm from being trapped in local optima.

The PMUs location including the branches where each PMU monitored is presented in Table 5.8-Table 5.12 when considering normal operation. As can be observed from Table 5.8-Table 5.12, the number of PMUs needed to achieve complete observability becomes fewer as the PMU is equipped with more channels.

Table 5.8: The PMUs location for IEEE 14-bus system for the case considering channel limit without considering the existence of ZIB

Channel limit, *L*	No. of PMUs, N_{PMU}	PMU locations with branch assignments
2	7	1{1-2}, 3{3-4}, 5{5-6}, 7{7-8}, 9{9-14}, 10{10-11}, 12{12-13}
3	5	2{2-3,2-5}, 5{5-1,5-6}, 7{7-4,7-8} 10{10-9,10-11}, 13{13-12,13-14}
4	4	2{2-1,2-3,2-5}, 6{6-11,6-12,6-13}, 7{7-4,7-8,7-9}, 9{9-4,9-10,9-14}
5	4	2{2-1,2-3,2-4,2-5}, 6{6-5,6-11,6-12,6-13}, 7{7-4,7-8,7-9}, 9{9-4,9-7,9-10,9-14}

Table 5.9: The PMUs location for IEEE 24-bus system for the case considering channel limit without considering the existence of ZIB

Channel limit, *L*	No. of PMUs, N_{PMU}	PMU locations with branch assignments
2	12	1{1-5}, 2{2-4}, 3{3-9}, 6{6-10}, 7{7-8}, 11{11-14}, 12{12-13}, 15{15-24}, 16{16-19}, 17{17-18}, 20{20-23}, 21{21-22}
3	8	1{1-3,1-5}, 2{2-4,2-6}, 8{8-7,8-10}, 11{11-13,11-14}, 12{12-9,12-23}, 15{15-21,15-24}, 17{17-18,17-22}, 19{19-16,19-20}
4	7	2{2-1,2-4,2-6}, 3{3-1,3-9,3-24}, 8{8-7,8-9,8-10}, 10{10-5,10-11,10-12}, 16{16-14,16-17,16-19}, 21{21-15,21-18,21-22}, 23 {23-12,23-13,23-20}
5	7	2{2-1,2-4,2-6}, 3{3-1,3-9,3-24}, 8{8-7,8-9,8-10}, 10{10-5,10-8,10-11,10-12},16{16-14,16-15,16-17,16-19}, 21{21-15,21-18,21-22}, 23{23-12,23-13,23-20}

Table 5.10: The PMUs location for IEEE 30-bus system for the case considering channel limit without considering the existence of ZIB

Channel limit, *L*	No. of PMUs, N_{PMU}	PMU locations with branch assignments
2	15	1{1-3}, 2{2-4}, 5{5-7}, 6{6-8}, 9{9-11}, 10{10-20}, 12{12-13}, 14{14-15}, 16{16-17}, 18{18-19}, 21{21-22}, 23{23-24}, 25{25-26}, 27{27-28}, 29{29-30}
3	11	3{3-1,3-4}, 5{5-2,5-7}, 9{9-6,9-11}, 10{10-17,10-21}, 10{10-20,10-22}, 12{12-13,12-16}, 15{15-14,15-23}, 18{18-15,18-19}, 25{25-24,25-26}, 27{27-29,27-30}, 28{28-8,28-27}
4	10	2{2-1,2-4,2-5}, 4{4-2,4-3,4-12}, 6{6-7,6-8,6-9}, 9{9-6,9-10,9-11}, 10{10-17,10-20,10-21}, 12{12-13,12-14,12-16}, 18{18-15,18-19}, 24{24-22,24-23,24-25}, 25{25-24,25-26,25-27}, 27{27-28,27-29,27-30}
5	10	1{1-2,1-3}, 2{2-1,2-4,2-5,2-6}, 6{6-4,6-7,6-8,6-28}, 9{9-6,9-10,9-11}, 10{10-17,10-20,10-21,10-22}, 12{12-13,12-14,12-15,12-16}, 15{15-12,15-14,15-18,15-23}, 19{19-18,19-20}, 25{25-24,25-26,25-27}, 27{27-25,27-28,27-29,27-30}

Table 5.11: The PMUs location for IEEE 39-bus system for the case considering channel limit without considering the existence of ZIB

Channel limit, L	No. of PMUs, N_{PMU}	PMU locations with branch assignments
2	21	1{1-39}, 2{2-30}, 3{3-18}, 4{4-5}, 6{6-11}, 6{6-31}, 7{7-8}, 9{9-39}, 10{10-32}, 12{12-13}, 14{14-15}, 16{16-21}, 17{17-27}, 19{19-33}, 20{20-34}, 22{22-35}, 23{23-24}, 23{23-36}, 25{25-37}, 26{26-28}, 29{29-38}
3	14	2{2-3,2-30}, 5{5-4,5-8}, 6{6-7,6-31}, 10{10-11,10-32}, 11{11-10,11-12}, 14{14-13,14-15}, 17{17-18,17-27}, 19{19-16,19-33}, 20{20-19,20-34}, 22{22-21,22-35}, 23{23-24,23-36}, 25{25-26,25-37}, 29{29-28,29-38}, 39{39-1,39-9}
4	13	2{2-1,2-3,2-30}, 6{6-5,6-7,6-31}, 9{9-8,9-39}, 10{10-11,10-13,10-32}, 11{11-6,11-10,11-12}, 14{14-4,14-13,14-15}, 17{17-16,17-18,17-27}, 19{19-16,19-20,19-33}, 20{20-19,20-34}, 22{22-21,22-23,22-35}, 23{23-22,23-24,23-36}, 25{25-2,25-26,25-37}, 29{29-26,29-28,29-38}
5	13	2{2-1,2-3,2-25,2-30}, 6{6-5,6-7,6-11,6-31}, 9{9-8,9-39}, 10{10-11,10-13,10-32}, 13{13-10,13-12,13-14}, 14{14-4,14-13,14-15}, 17{17-16,17-18,17-27}, 19{19-16,19-20,19-33}, 20{20-19,20-34}, 22{22-21,22-23,22-35}, 23{23-22,23-24,23-36}, 25{25-2,25-26,25-37}, 29{29-26,29-28-29-38}

Table 5.12: The PMUs location for IEEE 57-bus system for the case considering channel limit without considering the existence of ZIB

Channel limit, L	No. of PMUs, N_{PMU}	PMU locations with branch assignments
2	29	1{1-17}, 2{2-3}, 4{4-5}, 6{6-7}, 8{8-9}, 10{10-51}, 11{11-43}, 12{12-16}, 13{13-15}, 14{14-46}, 18{18-19}, 20{20-21}, 22{22-23}, 24{24-25}, 26{26-27}, 28{28-29}, 30{30-31}, 32{32-33}, 34{34-35}, 36{36-40}, 37{37-38}, 39{39-57}, 41{41-56}, 42{42-56}, 44{44-45}, 47{47-48}, 49{49-50}, 52{52-53}, 54{54-55}
3	19	1{1-2,1-15}, 4{4-3,4-5}, 8{8-6,8-9}, 11{11-41,11-43}, 12{12-16,12-17}, 14{14-13,14-46}, 19{19-18,19-20}, 22{22-21,22-23}, 25{25-24,25-30}, 27{27-26,27-28}, 29{29-7,29-52}, 32{32-31,32-33}, 35{35-34,35-36}, 39{39-37,39-57}, 44{44-38,44-45}, 48{48-47,48-49}, 51{51-10,51-50}, 54{54-53,54-55}, 56{56-40,56-42}
4	17	1{1-2,1-16,1-17}, 6{6-4,6-5,6-8}, 10{10-9,10-12,10-51}, 15{15-1,15-3,15-45}, 19{19-18,19-20}, 22{22-21,22-23,22-38}, 26{26-24,26-27}, 29{29-7,29-28,29-52}, 30{30-25,30-31}, 32{32-31,32-33,32-34}, 36{36-35,36-37,36-40}, 38{38-37,38-44,38-49}, 41{41-11,41-42,41-43}, 46{46-14,46-47}, 49{49-13,49-48,49-50}, 54{54-53,54-55}, 57{57-39,57-56}
5	17	1{1-2,1-15,1-16,1-17}, 6{6-4,6-5,6-7,6-8}, 9{9-10,9-11,9-12,9-55}, 15{15-3,15-13,15-14,15-45}, 19{19-18,19-20}, 22{22-21,22-23,22-38}, 25{25-24,25-30}, 26{26-24,26-27}, 29{29-7,29-28,29-52}, 32{32-31,32-33,32-34}, 36{36-35,36-37,36-40}, 38{38-37,38-44,38-48,38-49}, 41{41-11,41-42,41-43,41-56}, 47{47-46,47-48}, 51{51-10,51-50}, 53{53-52,53-54}, 57{57-39,57-56}

It is also evident that having PMUs with 4 channels are enough to ensure the power system maintains observable similar to the result when channel limit is ignored as presented in Table 5.3.

Meanwhile, the results obtained for the case considering channels limit and ZIB are presented in Table 5.13-Table 5.17.

Table 5.13: The PMUs location for the IEEE 14-bus system for the case considering channel limit and ZIB

Channel limit, *L*	No. of PMUs, N_{PMU}	PMU locations with branch assignments
2	7	1{1-5}, 2{2-3}, 4{4-7}, 6{6-12}, 7{7-9}, 10{10-11}, 13{13-14}
3	5	2{2-1,2-3}, 6{6-5,6-13}, 6{6-11,6-12}, 7{7-8,7-9}, 9{9-10,9-14}
4	4	2{2-1,2-3,2-4}, 6{6-5,6-11,6-12}, 9{9-7,9-10,9-14}, 13{13-6,13-12,13-14}
5	3	2{2-1,2-3,2-4,2-5}, 6{6-5,6-11,6-12,6-13}, 9{9-4,9-7,9-10,9-14}

Table 5.14: The PMUs location for the IEEE 24-bus system for the case considering channel limit and ZIB

Channel limit, *L*	No. of PMUs, N_{PMU}	PMU locations with branch assignments
2	10	1{1-5}, 2{2-6}, 3{3-24}, 4{4-9}, 7{7-8}, 11{11-14}, 12{12-13}, 17{17-18}, 19{19-20}, 21{21-22}
3	7	1{1-3,1-5}, 2{2-4,2-6}, 8{8-7,8-10}, 13{13-11,13-12}, 16{16-17,16-19}, 21{21-15,21-18}, 23{23-12,23-20}
4	6	2{2-1,2-4,2-6}, 8{8-7,8-9,8-10}, 10{10-5,10-11,10-12}, 15{15-16,15-21,15-24}, 20{20-19,20-23}, 21{21-15,21-18,21-22}
5	6	1{1-2,1-3,1-5}, 2{2-1,2-4,2-6}, 8{8-7,8-9,8-10}, 16{16-14,16-15,16-17,16-19}, 21{21-15,21-18,21-22}, 23{23-12,23-13,23-20}

Table 5.15: The PMUs location for the IEEE 30-bus system for the case considering channel limit and ZIB

Channel limit, *L*	No. of PMUs, N_{PMU}	PMU locations with branch assignments
2	13	1{1-2}, 1{1-3}, 5{5-7}, 8{8-28}, 9{9-11}, 10{10-20}, 12{12-13}, 12{12-14}, 15{15-23}, 16{16-17}, 18{18-19}, 22{22-24}, 29{29-30}
3	8	1{1-2,1-3}, 7{7-5,7-6}, 12{12-4,12-13}, 15{15-14,15-23}, 17{17-10,17-16}, 19{19-18,19-20}, 24{24-22,24-25}, 27{27-28,27-30}
4	7	2{2-1,2-5,2-6}, 4{4-2,4-3,4-6}, 10{10-9,10-17,10-20}, 12{12-13,12-14,12-16}, 18{18-15,18-19}, 24{24-22,24-23,24-25}, 27 {27-25,27-28,27-29}
5	7	2{2-1,2-4,2-5,2-6}, 4{4-2,4-3,4-6,4-12}, 10{10-9,10-17,10-21,10-22}, 12{12-13,12-14,12-15,12-16}, 15{15-12,15-14,15-18,15-23}, 19{19-18,19-20}, 27{27-25,27-28,27-29,27-30}

Table 5.16: The PMUs location for the IEEE 39-bus system for the case considering channel limit and ZIB

Channel limit, *L*	No. of PMUs, N_{PMU}	PMU locations with branch assignments
2	14	1{1-2}, 3{3-4}, 7{7-8}, 10{10-11}, 14{14-15}, 16{16-19}, 16{16-24}, 17{17-18}, 20{20-34}, 21{21-22}, 23{23-36}, 25{25-37}, 26{26-28}, 29{29-38}
3	9	4{4-3,4-14}, 8{8-7,8-9}, 11{11-10,11-12}, 16{16-21,16-24}, 20{20-19,20-34}, 23{23-22,23-36}, 25{25-2,25-37}, 27{27-17,27-26}, 29{29-28,29-38}
4	8	3{3-2,3-4,3-18}, 8{8-5,8-7,8-9}, 10{10-11,10-13,10-32}, 16{16-17,16-21,16-24}, 20{20-19,20-34}, 23{23-22,23-24,23-36}, 25{25-2,25-26,25-37}, 29{29-26,29-28,29-38}
5	8	3{3-2,3-4,3-18}, 8{8-5,8-7,8-9}, 13{13-10,13-12,13-14}, 16{16-17,16-19,16-21,16-24}, 20{20-19,20-34}, 23{23-22,23-24,23-36}, 25{25-2,25-26,25-37}, 29{29-26,29-28,29-38}

Table 5.17: The PMUs location for the IEEE 57-bus system for the case considering channel limit and ZIB

Channel limit, *L*	No. of PMUs, N_{PMU}	PMU locations with branch assignments
2	21	1{1-17}, 2{2-3}, 4{4-6}, 8{8-9}, 10{10-51}, 11{11-43}, 12{12-16}, 14{14-46}, 18{18-19}, 21{21-22}, 24{24-25}, 28{28-29}, 30{30-31}, 32{32-33}, 36{36-40}, 37{37-39}, 41{41-42}, 44{44-45}, 49{49-50}, 52{52-53}, 54{54-55}
3	14	3{3-2,3-4}, 9{9-8,9-55}, 11{11-13,11-43}, 12{12-16,12-17}, 15{15-1,15-45}, 19{19-18,19-20}, 23{23-22,23-24}, 29{29-7,29-28}, 30{30-25,30-31}, 32{32-33,32-34}, 47{47-46,47-48}, 51{51-10,51-50}, 53{53-52,53-54}, 56{56-40,56-42}
4	12	1{1-2,1-16,1-17}, 4{4-3,4-5,4-18}, 10{10-9,10-12,10-51}, 13{13-11,13-12,13-15}, 15{15-3,15-14,15-45}, 20{20-19,20-21}, 25{25-24,25-30}, 29{29-7,29-28,29-52}, 32{32-31,32-33,32-34}, 49{49-38,49-48,49-50}, 54{54-53,54-55}, 56{56-40,56-41,56-42}
5	11	1{1-2,1-15,1-16,1-17}, 6{6-4,6-5,6-7,6-8}, 13{13-9,13-11,13-12,13-14}, 19{19-18,19-20}, 25{25-24,25-30}, 29{29-7,29-28,29-52}, 32{32-31,32-33,32-34}, 38{38-22,38-44,38-48,38-49}, 51{51-10,51-50}, 54{54-53,54-55}, 56{56-40,56-41,56-42,56-57}

Similar to the previous case, based on Table 5.13-Table 5.17 the PMUs with channel limit of 4 are also enough for case considering ZIB except for the IEEE 14-bus and 57-bus systems which needed 5 channels to acquire the same results obtained for case ignoring PMUs channels limit.

5.2 Comparison with previous studies

In order to evaluate the efficiency of the proposed method, the results acquired from the simulations using the proposed method are compared with prior studies. In the first comparison, the number of PMUs and the SORI value are compared against the results obtained from prior studies that used various methods to solve the OPP problem and considering normal operation and ZIB. In the second comparison, the comparison is expanded by including the computation time from prior studies that used BPSO algorithm to solve the OPP problem.

Presented in Table 5.18 is the comparison result between the proposed method with existing studies. All IEEE bus systems tested are compared. It is worth mentioning again that solution that carries the highest SORI is the most quality solution. As evident from the Table 5.18, all studies being compared including proposed method managed to get the same number of PMUs across all IEEE bus systems tested. However, the values of measurement redundancy are different for some bus systems. For the small bus systems, the measurement redundancy obtained by the proposed method is mostly the same as existing studies. However, for large systems such as IEEE 118-bus and 300-bus systems, the difference is more prominent. For example, only 2 out of 11 existing studies being compared with proposed method managed to get the highest measurement redundancy for IEEE 118-bus system which is 164. Meanwhile, for IEEE 300-bus system, the proposed method managed to get the PMUs placement set that has 432 as its measurement redundancy as opposed to the BSDP method which managed to get 423 measurement redundancy. Overall, the proposed method managed to get the highest quality solution for all bus systems.

Table 5.18: The comparison results between proposed method and existing studies on the number of PMUs and measurement redundancy for normal operation

IEEE bus system	**Parameter**	**Proposed Method**	**ES** [21]	**Binary Search** [22]	**DE** [30]	**BPSO** [36]	**BPSO** [37]	**SQP** [19]	**Integer Programming** [106]	**MICA** [107]	**ES** [108]	**Gravitational Search Method** [109]	**BSDP** [20]
14-bus	N_{PMU}	4	4	4	4	-	4	4	4	4	-	4	4
	SORI	19	19	19	19	-	19	19	19	19	-	19	16
24-bus	N_{PMU}	7	7	7	-	7	-	7	-	-	-	-	-
	SORI	31	31	31	-	29	-	31	-	-	-	-	-
30-bus	N_{PMU}	10	10	10	10	10	10	10	10	10	-	10	10
	SORI	52	50	50	52	46	52	48	52	52	-	52	50
39-bus	N_{PMU}	13	13	13	13	13	-	13	13	-	-	-	-
	SORI	52	51	52	52	50	-	52	52	-	-	-	-
57-bus	N_{PMU}	17	17	-	17	17	17	17	-	17	-	-	17
	SORI	72	68	-	72	67	71	71	-	72	-	-	66
118-bus	N_{PMU}	32	32	-	-	32	32	32	32	32	32	32	32
	SORI	164	155	-	-	159	145	163	164	164	156	160	159
300-bus	N_{PMU}	87	-	-	-	-	-	-	-	-	-	-	87
	SORI	432	-	-	-	-	-	-	-	-	-	-	423

Table 5.19: The comparison results between proposed method and existing studies on the number of PMUs and measurement redundancy for case considering ZIB

IEEE bus system	**Parameter**	**Proposed Method**	**ES** [21]	**Binary Search** [22]	**Binary Integer Programming** [110]	**BPSO** [36]	**BPSO** [37]	**ILP** [17]	**BPSO** [40]	**GA** [28]	**ILP** [111]	**BSDP** [20]
14-bus	N_{PMU}	3	3	3	3	-	3	3	3	3	3	3
	SORI	16	16	16	16	-	16	16	16	16	16	16
24-bus	N_{PMU}	6	6	6	-	6	-	-	-	-	-	-
	SORI	29	27	29	-	28	-	-	-	-	-	-
30-bus	N_{PMU}	7	7	7	7	7	7	7	7	7	7	7
	SORI	41	36	39	41	37	34	36	37	33	34	36
39-bus	N_{PMU}	8	8	8	8	8	-	8	8	-	8	-
	SORI	43	43	43	43	40	-	43	43	-	43	-
57-bus	N_{PMU}	11	11	-	11	11	13	11	11	11	11	11
	SORI	60	60	-	59	59	64	60	56	60	59	57
118-bus	N_{PMU}	28	28	-	28	-	29	28	28	28	29	28
	SORI	156	148	-	156	-	155	148	147	148	155	145
300-bus	N_{PMU}	69	-	-	-	-	-	-	-	-	-	70
	SORI	393	-	-	-	-	-	-	-	-	-	378

The same thing can be observed in Table 5.19 for case considering ZIB. The proposed method managed to get the best quality solution including the larger bus systems, namely the IEEE 118-bus and 300-bus systems. For the 300-bus system, the proposed method outperforms the results obtained using the BSDP method by quite significant with 393 measurement redundancy compared to 378 for the BSDP method. Furthermore, it is being observed using fewer PMUs when applying the proposed method compared to the BSDP method, with the former needed 69 PMUs and 70 PMUs for the latter. The results from the two cases that are compared in this section indicate the proposed method managed to get better solution than the existing studies.

For the case considering channels limit for normal operation, the number of PMUs obtained is compared with the existing studies and presented in Table 5.20. The number of PMUs when using different number of channels is consistent across all tested IEEE bus systems.

Table 5.20: Comparison results for case considering channels limit for normal operation

IEEE bus system	Channels limit, L	Number of PMUs, N_{PMU}			
		Proposed Method	Binary Integer [112]	ILP [113]	ILP [15]
14-bus	2	7	7	7	7
	3	5	5	5	5
	4	4	4	4	4
	5	4	4	4	4
24-bus	2	12	-	-	-
	3	8	-	-	-
	4	7	-	-	-
	5	7	-	-	-
30-bus	2	15	15	15	15
	3	11	11	11	11
	4	10	10	10	10
	5	10	10	10	10
39-bus	2	21	-	-	-
	3	14	-	-	-
	4	13	-	-	-
	5	13	-	-	-
57-bus	2	29	29	29	29
	3	19	19	19	19
	4	17	17	17	17
	5	17	17	17	17

Also, from Table 5.20, it can be concluded that the PMUs with 4 number of channels are adequate to achieve a complete observability of the power systems with the minimum number of PMUs as obtained in the existing studies.

Table 5.21 below compares the number of PMUs obtained for case considering PMU's channels limit and ZIB with the existing studies.

Table 5.21: Comparison results for case considering channels limit and ZIB

IEEE bus system	Channels limit, L	Number of PMUs, N_{PMU}					
		Proposed Method	ILP [17]	Firefly [114]	ILP [15]	GA [115]	ILP [113]
14-bus	2	7	7	7	7	7	7
	3	5	5	5	5	5	5
	4	4	4	4	4	4	4
	5	3	3	3	3	3	3
24-bus	2	10	-	-	-	-	-
	3	7	-	-	-	-	-
	4	6	-	-	-	-	-
	5	6	-	-	-	-	-
30-bus	2	13	12	12	13	12	13
	3	8	8	8	9	18	8
	4	7	7	8	7	7	7
	5	7	7	7	7	7	7
39-bus	2	14	14	-	-	-	-
	3	9	9	-	-	-	-
	4	8	8	-	-	-	-
	5	8	8	-	-	-	-
57-bus	2	21	21	21	21	21	21
	3	14	14	14	14	14	14
	4	12	12	13	12	12	12
	5	11	11	12	11	11	11

As can be seen from the Table 5.21, the number of PMUs needed using the proposed method are comparable with the existing studies that used other optimisation methods for all IEEE bus systems. For example, for IEEE 24-bus, 30-bus and 39-bus systems, the PMUs with 4 channels are enough to maintain the observability of the power systems with the most minimum number of PMUs. This results consistent with the results obtained by the existing studies. Therefore, the results proved that the proposed method is capable in solving the OPP problem considering PMU's channels limit using BPSO optimisation method.

In Table 5.22 and 5.23, the computation time from the proposed method is compared with the existing studies that used BPSO algorithm. The computation time is based on the average computation time recorded over 30 times the algorithm was ran by using the proposed method.

Table 5.22: Time comparison for case normal operation

IEEE bus system	**Proposed Method**			**Improved PSO** [36]			**BPSO** [116]		
	No. of PMUs, N_{PMU}	***SORI***	**Avg. Comp. Time (s)**	**No. of PMUs, N_{PMU}**	***SORI***	**Avg. Comp. Time (s)**	**No. of PMUs, N_{PMU}**	***SORI***	**Avg. Comp. Time (s)**
14-Bus	4	19	1.967	-	-	-	4	19	1.5
24-Bus	7	31	3.971	7	29	15.40	-	-	-
30-Bus	10	52	5.565	10	46	82	10	51	6.8
39-Bus	13	52	7.532	13	50	173	-	-	-
57-Bus	17	72	19.08	17	67	350	-	-	-
118-Bus	32	164	87.572	32	159	4980	-	-	-
300-bus	87	432	590.741	-	-	-	-	-	-

Table 5.23: Time comparison for case considering ZIB

IEEE bus system	**Proposed Method**			**Improved PSO** [42]			**Modified BPSO** [40]		
	No. of PMUs, N_{PMU}	***SORI***	**Avg. Comp. Time (s)**	**No. of PMUs, N_{PMU}**	***SORI***	**Avg. Comp. Time (s)**	**No. of PMUs, N_{PMU}**	***SORI***	**Avg. Comp. Time (s)**
14-bus	3	16	5.855	-	-	-	3	16	60
24-bus	6	29	9.835	-	-	-	-	-	-
30-bus	7	41	15.112	6	34	1220.60	7	37	360
39-bus	8	43	18.021	8	43	2778.72	8	43	900
57-bus	11	60	31.858	11	60	3364.36	11	56	2580
118-bus	28	156	135.96	29	152	9627.74	28	147	5100
300-bus	69	393	1068.9	-	-	-	-	-	-

As can be observed, as the number of bus increases, the computation time is also increased. It also evident that the computation time is significantly faster when using the proposed method compared to the existing studies. This shows that in addition of high quality solution obtained by using the proposed method, the computation time is also fast.

5.3 Summary

In this chapter, the proposed method explained in the previous chapter is applied to several IEEE bus systems namely the IEEE 14-bus, 24-bus, 30-bus, 39-bus, 57-bus, 118-bus, and 300-bus. MATLAB software is used to perform the simulations and the results obtained are given in this chapter. To evaluate its effectiveness, the results obtained are compared with existing studies. The comparison results indicate the proposed method managed to outperform existing studies and it can be used to solve the OPP problem for case considering ZIB, single PMU loss and PMU's channels limit.

Chapter 6

Conclusions and Future Work

6.1 Conclusions

In this thesis, optimal PMUs placement set that can guarantee full observability of the power system is investigated based on several practical constraints such as ZIB, single PMU loss, and also PMU's channels limit. It is investigated by using BPSO optimisation method, which uses the intelligence of the swarm to determine the best solution. However, it is widely known that population-based methods such as BPSO has the tendency to have performance issue when dealing with large problem size. As a result, the algorithm tends to converge at local optima, instead of global optima, which gives the optimal solution among all possible solutions. Recent studies in this research area indicated that the balance between exploration and exploitation must be maintained to avoid the particles from being trapped in local optima.

This thesis proposed the use of V-shaped sigmoid function, replacing the S-shaped sigmoid function that is commonly used in the existing studies, to solve the OPP problem to improve the balance between exploration and exploitation of the BPSO algorithm. Based on the experiment, the S-shaped sigmoid sigmoid was not viable to be used when solving the OPP problem for power systems that are larger than the IEEE 57-bus such as the IEEE 118-bus and 300-bus systems since it encourages random positioning of particle, which leads to more exploration and lack of exploitation. The use of V-shaped sigmoid prevents the random positioning of particles without limiting the capability of particles to explore and exploit the position they are in. However, possibility of being trapped in local optima is still exist because the particles no longer carry velocity when they already converged. Therefore, if the particles unable to find the promising region during exploration, it may be trapped in local optima.

A mutation strategy is proposed in this thesis to compliment the V-shaped sigmoid function by refining the local search of the algorithm. The mutation strategy makes use of particle's pbest to find the optimal solution around the pbest, which ensures particles are only driven by the best possible solution to find the promising region. In addition, particle's position can be changed outside of the velocity equation through the mutation strategy. This allows particles to change position even if they are no longer carry velocity, which is very helpful if the particle is trapped in local optima.

To validate the performance of the proposed method, the proposed method was simulated on the IEEE 14-bus, 24-bus, 30-bus, 39-bus, 57-bus, 118-bus, and 300-bus systems by using MATLAB software. Since there is no unique solution for the PMUs placement, in addition of the number of PMUs required to achieve complete observability of the power system, the measurement redundancy, which indicates the reliability of the PMUs placement set, is used. The results indicated that the proposed method is the most reliable method for all IEEE bus systems tested for case considering normal operationg and ZIB. Furthermore, for IEEE 300-bus system, which was never considered in the existing studies that used BPSO algorithm, apart of having the most reliable PMUs placement sets, the proposed method managed to reduce the number of PMUs needed for the case considering ZIB when compared to the deterministic method, namely the BSDP method. This indicated that the proposed method is applicable for large systems.

In terms of practical constraints considered, based on the literature survey, PMU's channels limit has been considered by other optimisation methods, but never adopted into BPSO algorithm. The challenge is largely revolved around the increase of problem dimension when PMU's channels limit is considered. However, as indicated by the results, the proposed method managed to obtain comparable results with other existing studies.

For the case considering single PMU loss, a new fitness function is proposed where it is integrated into its formulation. This ensures it can be implemented with ease since the minimisation of the fitness function ensures the PMUs placement set is applicable with case considering single PMU loss. In addition, it also helps reduced computational burden since it does not required further observability verification to ensure it meets the desired objective.

This thesis also proposed new selection rules for topology transformation method using ILP when dealing with ZIB. Using the selection rules, bus to merge with ZIB is selected based on certain criteria that will give better measurement redundancy and accurate PMUs placement compared to existing studies that adopted the merging process.

To summarise, for BPSO algorithm that tend to have performance issue when dealing with large problem size, the proposed method proved that it is applicable for larger systems when compared with the existing studies that used different optimisation methods. The consideration of PMU's channels limit further indicated that the proposed method can be used when dealing with large problem size. The use of V-shaped and mutation strategy ensures the quality of PMUs placement sets obtained using the proposed method are the most reliable and globally optimised in this research area.

6.2 Future work

The following describes the future research concerning the application of the proposed method in solving the OPP problem:

- The implementation of the proposed method to the BPSO algorithm encourages the algorithm to find the optimal solution during the beginning of the iterations, especially in small bus systems. Consequently, the remaining iterations proved to be a waste of computational cost since it no longer ables to improve the existing solution. Therefore, a proper stopping criterion that can determine if the algorithm is unable to improve the current solution might help in this regard.
- Implement the proposed method for bus systems larger than IEEE 300-bus system. Although the proposed method performs well for all the IEEE bus systems tested, additional modifications to the proposed method may be needed since each bus system has different topology. For example, the value for the number of iterations may be needed to be increased when dealing with large bus systems since the search space for particles to explore is bigger, hence, it requires more time. Evidently, the proposed method must be validated independently to ensure the obtained result is the optimal solution.

- ❖ Reduce the complexity of the algorithm and the computation time by integrating all practical constraints into a single fitness function. This ensures observability validation to ensure feasibility of each PMUs placement set can be eliminate, which will help reduce the complexity of the algorithm. It also makes ease of implementation for every practical constraints in the algorithm.
- ❖ Investigate the proposed method by using numerical observability. Having the proposed method suitable for both type of observability may improve the flexibility of the proposed method should numerical observability is desired.
- ❖ Recently, due to the expensive cost of a PMU, the implementation of PMUs has been explored to have it incrementally installed. This means, instead of having all PMUs installed at once, the PMUs installation will be done in a given time horizon. Phased installation approach requires a meticulous planning where the initial placement of the PMUs should ensure a uniform distribution of future PMUs installation across the network is achievable. In the meantime, the measurement from the SCADA and the initial PMUs placement should be used to monitor the state of the power system. The depth-of-unobservability concept applied in [11] might be possible to be used here where the PMUs are initially installed such that the initial unobservable bus can be made observable in future PMUs installation without using excessive numbers of PMUs. To integrate the depth-of-unobservability concept into the proposed method, the mutation strategy should be modified to ensure that it is intended to create depth for future PMUs installation, contrary to the current strategy where it is designed to discard any PMUs placement set that is infeasible with respect to the power system's observability.

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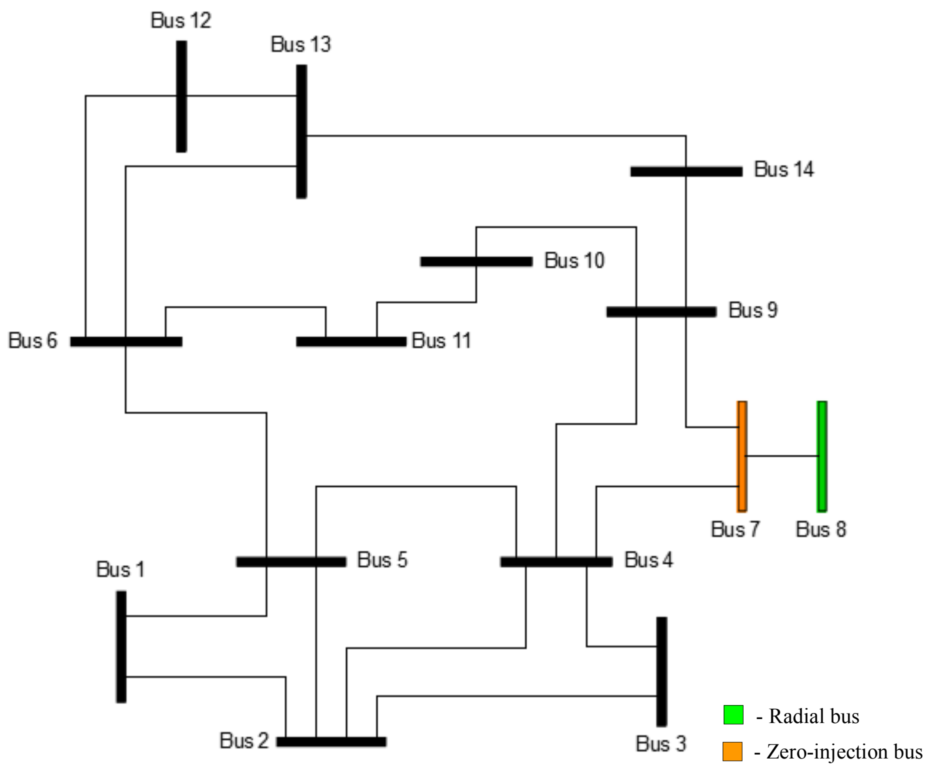
Appendix

IEEE 14-Bus System Data



Branch	***From Bus***	***To Bus***
1	1	2
2	1	5
3	2	3
4	2	4
5	2	5
6	3	4
7	4	5
8	4	7
9	4	9
10	5	6
11	6	11
12	6	12
13	6	13
14	7	8
15	7	9
16	9	10
17	9	14
18	10	11
19	12	13
20	13	14

IEEE 24-Bus System Data

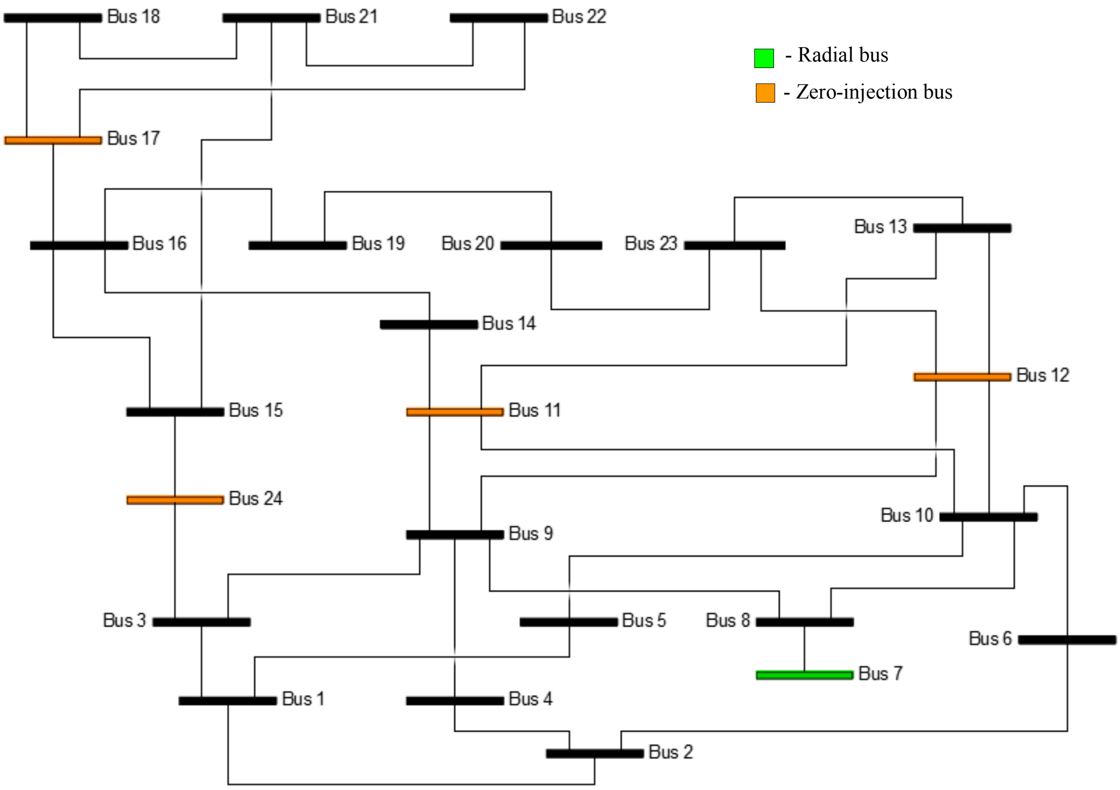
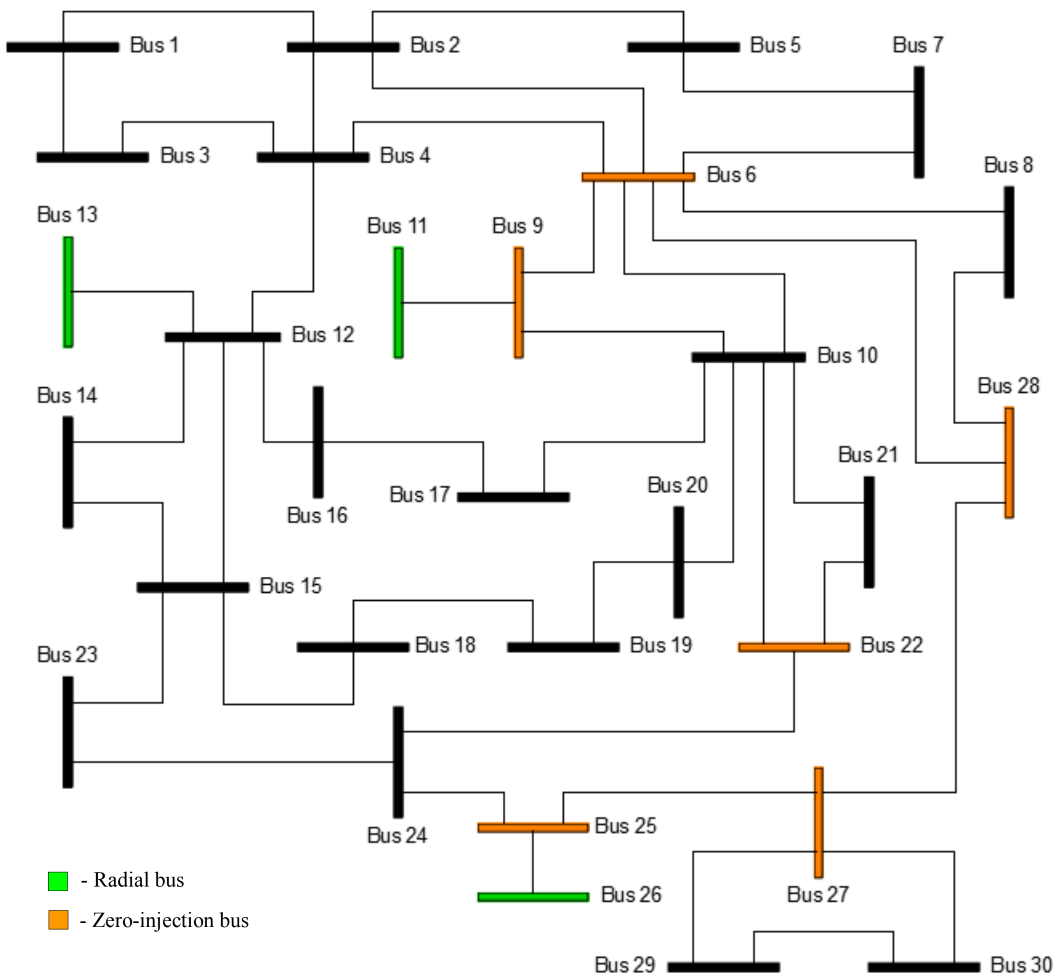


table continued

Branch	*From Bus*	*To Bus*
1	1	2
2	1	3
3	1	5
4	2	4
5	2	6
6	3	9
7	3	24
8	4	9
9	5	10
10	6	10
11	7	8
12	8	9
13	8	10
14	9	11
15	9	12
16	10	11
17	10	12
18	11	13
19	11	14
20	12	13
21	12	23
22	13	23
23	14	16
24	15	16
25	15	21
26	15	24
27	16	17
28	16	19
29	17	18
30	17	22
31	18	21
32	19	20
33	20	23
34	21	22

IEEE 30-Bus System Data



Branch	From Bus	To Bus
1	1	2
2	1	3
3	2	4
4	3	4
5	2	5
6	2	6
7	4	6
8	5	7
9	6	7
10	6	8
11	6	9
12	6	10
13	9	11
14	9	10
15	4	12
16	12	13

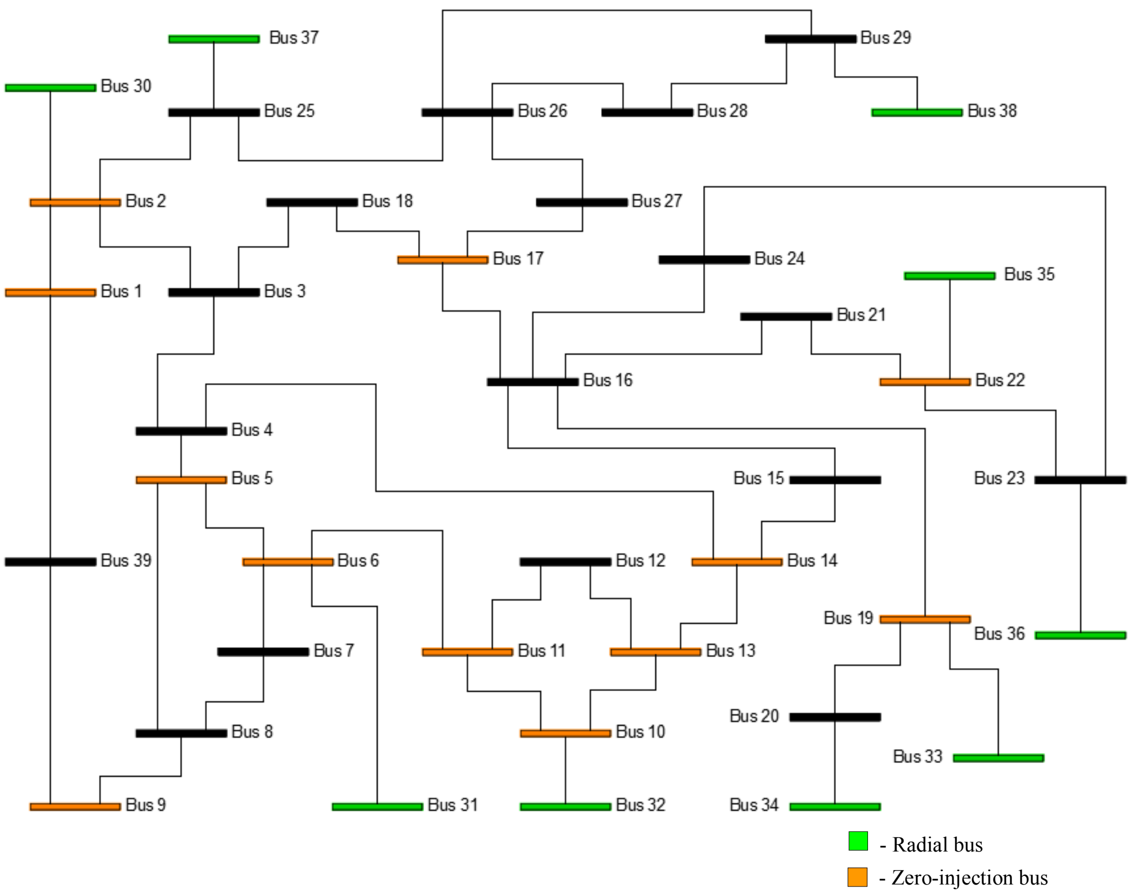
table continued

Branch	From Bus	To Bus
17	12	14
18	12	15
19	12	16
20	14	15
21	16	17
22	15	18
23	18	19
24	19	20
25	10	20
26	10	17
27	10	21
28	10	22
29	21	22
30	15	23
31	22	24
32	23	24

table continued

Branch	From Bus	To Bus
33	24	25
34	25	26
35	25	27
36	28	27
37	27	29
38	27	30
39	29	30
40	8	28
41	6	28

IEEE 39-Bus System Data



Branch	***From Bus***	***To Bus***
1	1	2
2	1	39
3	2	3
4	2	25
5	2	30
6	3	4
7	3	18
8	4	5
9	4	14
10	5	6
11	5	8
12	6	7
13	6	11
14	6	31
15	7	8
16	8	9

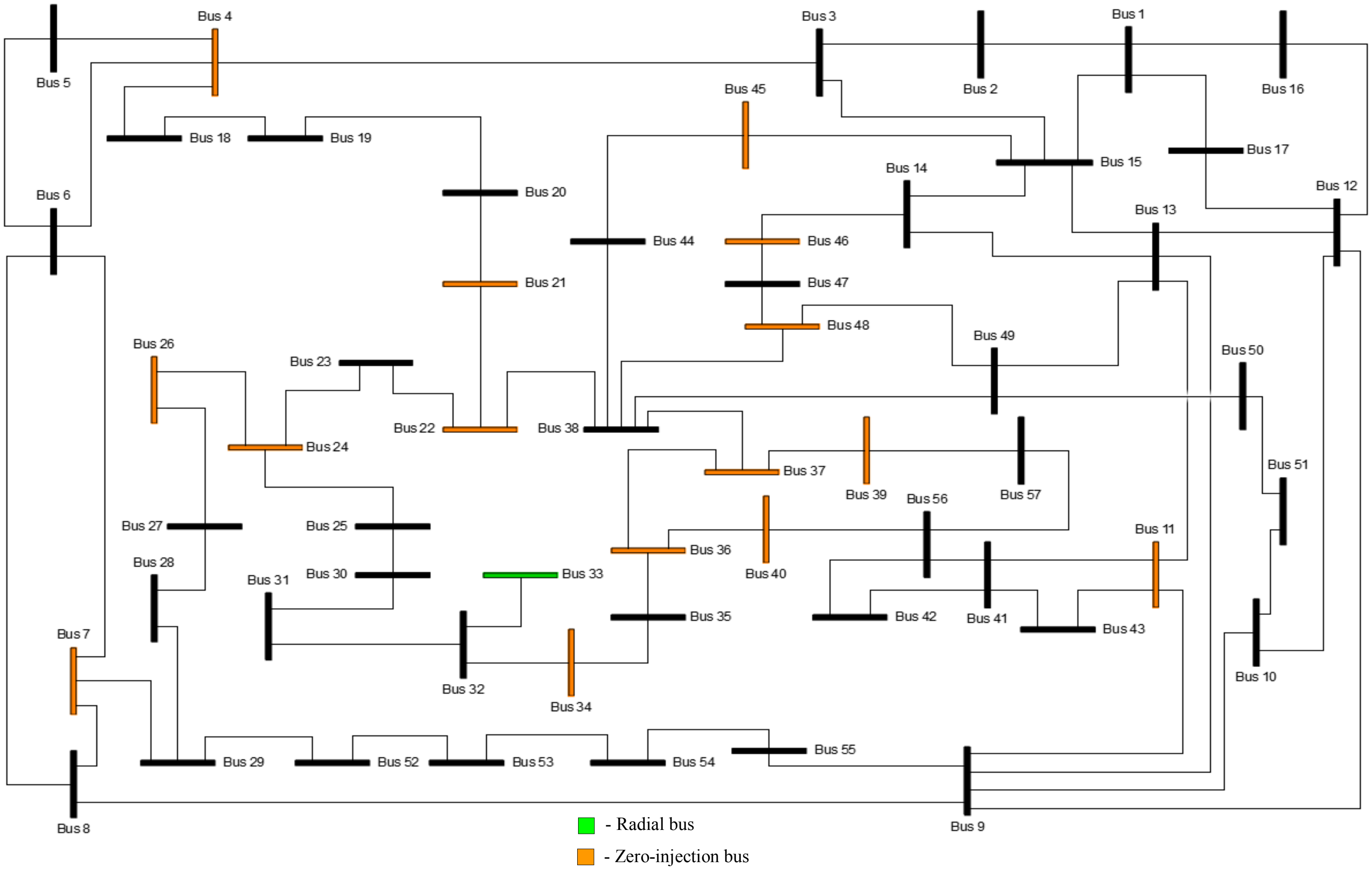
table continued

Branch	***From Bus***	***To Bus***
17	9	39
18	10	11
19	10	13
20	10	32
21	12	11
22	12	13
23	13	14
24	14	15
25	15	16
26	16	17
27	16	19
28	16	21
29	16	24
30	17	18
31	17	27
32	19	20

table continued

Branch	***From Bus***	***To Bus***
33	19	33
34	20	34
35	21	22
36	22	23
37	22	35
38	23	24
39	23	36
40	25	26
41	25	37
42	26	27
43	26	28
44	26	29
45	28	29
46	29	38

IEEE 57-Bus System Data



Branch	***From Bus***	***To Bus***
1	1	2
2	2	3
3	3	4
4	4	5
5	4	6
6	6	7
7	6	8
8	8	9
9	9	10
10	9	11
11	9	12
12	9	13
13	13	14
14	13	15
15	1	15
16	1	16
17	1	17
18	3	15
19	4	18
20	5	6
21	7	8
22	10	12
23	11	13
24	12	13
25	12	16
26	12	17
27	14	15

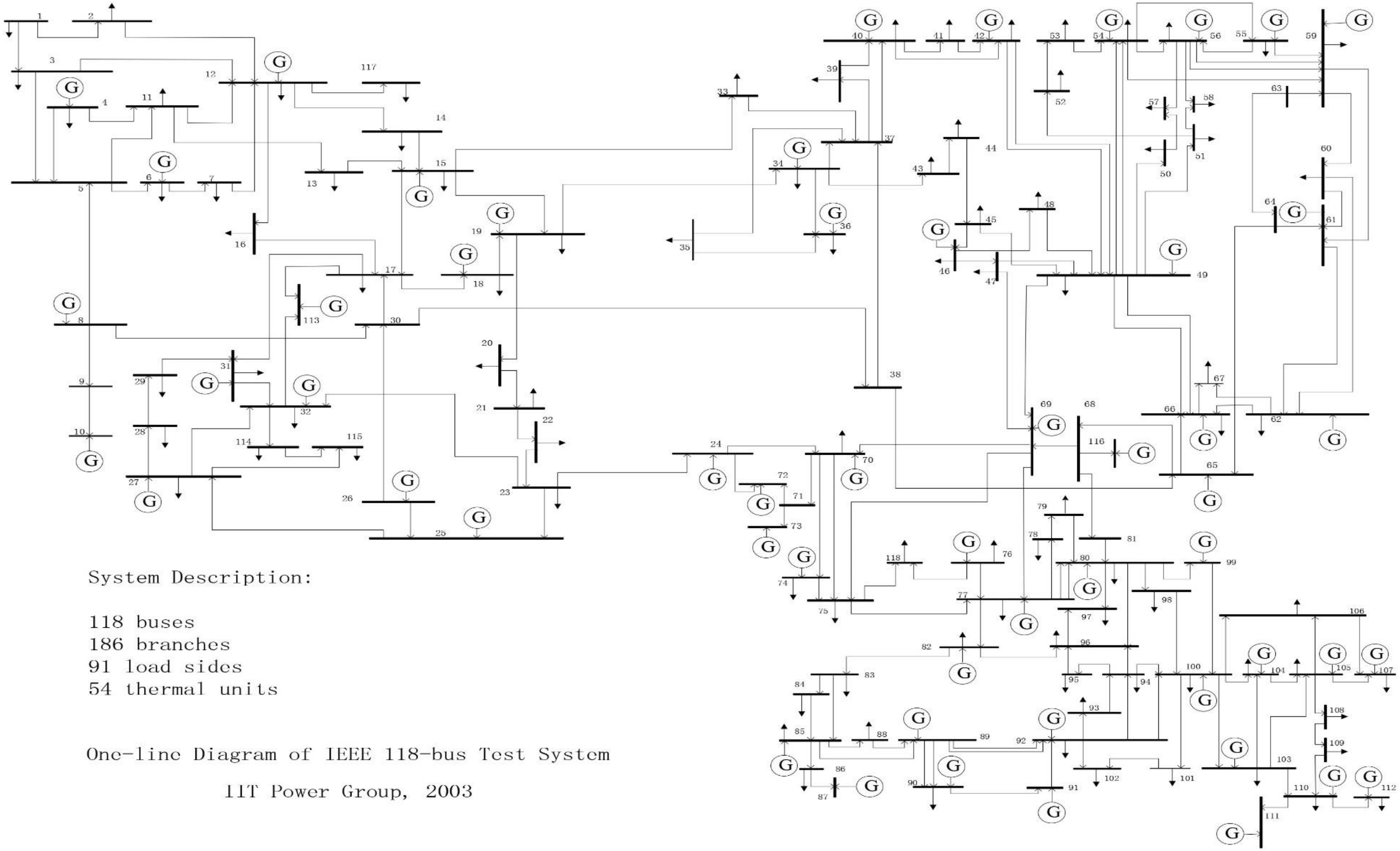
table continued

Branch	***From Bus***	***To Bus***
28	18	19
29	19	20
30	21	20
31	21	22
32	22	23
33	23	24
34	24	25
35	24	26
36	26	27
37	27	28
38	28	29
39	7	29
40	25	30
41	30	31
42	31	32
43	32	33
44	34	32
45	34	35
46	35	36
47	36	37
48	37	38
49	37	39
50	36	40
51	22	38
52	11	41
53	41	42
54	41	43

table continued

Branch	***From Bus***	***To Bus***
55	38	44
56	15	45
57	14	46
58	46	47
59	47	48
60	48	49
61	49	50
62	50	51
63	10	51
64	13	49
65	29	52
66	52	53
67	53	54
68	54	55
69	11	43
70	44	45
71	40	56
72	56	41
73	56	42
74	39	57
75	57	56
76	38	49
77	38	48
78	9	55

IEEE 118-Bus System Data



Branch	From Bus	To Bus
1	1	2
2	1	3
3	4	5
4	3	5
5	5	6
6	6	7
7	8	9
8	8	5
9	9	10
10	4	11
11	5	11
12	11	12
13	2	12
14	3	12
15	7	12
16	11	13
17	12	14
18	13	15
19	14	15
20	12	16
21	15	17
22	16	17
23	17	18
24	18	19
25	19	20
26	15	19
27	20	21
28	21	22
29	22	23
30	23	24
31	23	25
32	26	25
33	25	27
34	27	28
35	28	29
36	30	17
37	8	30
38	26	30
39	17	31
40	29	31
41	23	32

table continued

Branch	From Bus	To Bus
42	31	32
43	27	32
44	15	33
45	19	34
46	35	36
47	35	37
48	33	37
49	34	36
50	34	37
51	38	37
52	37	39
53	37	40
54	30	38
55	39	40
56	40	41
57	40	42
58	41	42
59	43	44
60	34	43
61	44	45
62	45	46
63	46	47
64	46	48
65	47	49
66	42	49
67	45	49
68	48	49
69	49	50
70	49	51
71	51	52
72	52	53
73	53	54
74	49	54
75	54	55
76	54	56
77	55	56
78	56	57
79	50	57
80	56	58
81	51	58
82	54	59

table continued

Branch	From Bus	To Bus
83	56	59
84	55	59
85	59	60
86	59	61
87	60	61
88	60	62
89	61	62
90	63	59
91	63	64
92	64	61
93	38	65
94	64	65
95	49	66
96	62	66
97	62	67
98	65	66
99	66	67
100	65	68
101	47	69
102	49	69
103	68	69
104	69	70
105	24	70
106	70	71
107	24	72
108	71	72
109	71	73
110	70	74
111	70	75
112	69	75
113	74	75
114	76	77
115	69	77
116	75	77
117	77	78
118	78	79
119	77	80
120	79	80
121	68	81
122	81	80
123	77	82

table continued

Branch	**From Bus**	**To Bus**
124	82	83
125	83	84
126	83	85
127	84	85
128	85	86
129	86	87
130	85	88
131	85	89
132	88	89
133	89	90
134	90	91
135	89	92
136	91	92
137	92	93
138	92	94
139	93	94
140	94	95
141	80	96
142	82	96
143	94	96
144	80	97
145	80	98
146	80	99

table continued

Branch	**From Bus**	**To Bus**
147	92	100
148	94	100
149	95	96
150	96	97
151	98	100
152	99	100
153	100	101
154	92	102
155	101	102
156	100	103
157	100	104
158	103	104
159	103	105
160	100	106
161	104	105
162	105	106
163	105	107
164	105	108
165	106	107
166	108	109
167	103	110
168	109	110
169	110	111

table continued

Branch	**From Bus**	**To Bus**
170	110	112
171	17	113
172	32	113
173	32	114
174	27	115
175	114	115
176	68	116
177	12	117
178	75	118
179	76	118

IEEE 300-Bus System Data

Branch	From Bus	To Bus
1	37	9001
2	9001	9005
3	9001	9006
4	9001	9012
5	9005	9051
6	9005	9052
7	9005	9053
8	9005	9054
9	9005	9055
10	9006	9007
11	9006	9003
12	9006	9003
13	9012	9002
14	9012	9002
15	9002	9021
16	9021	9023
17	9021	9022
18	9002	9024
19	9023	9025
20	9023	9026
21	9007	9071
22	9007	9072
23	9007	9003
24	9003	9031
25	9003	9032
26	9003	9033
27	9003	9044
28	9044	9004
29	9004	9041
30	9004	9042
31	9004	9043
32	9003	9034
33	9003	9035
34	9003	9036
35	9003	9037
36	9003	9038
37	9012	9121
38	9053	9533
39	1	5
40	2	6
41	2	8

table continued

Branch	From Bus	To Bus
42	3	7
43	3	19
44	3	150
45	4	16
46	5	9
47	7	12
48	7	131
49	8	11
50	8	14
51	9	11
52	11	13
53	12	21
54	13	20
55	14	15
56	15	37
57	15	89
58	15	90
59	16	42
60	19	21
61	19	87
62	20	22
63	20	27
64	21	24
65	22	23
66	23	25
67	24	319
68	25	26
69	26	27
70	26	320
71	33	34
72	33	38
73	33	40
74	33	41
75	34	42
76	35	72
77	35	76
78	35	77
79	36	88
80	37	38
81	37	40
82	37	41

table continued

Branch	From Bus	To Bus
83	37	49
84	37	89
85	37	90
86	38	41
87	38	43
88	39	42
89	40	48
90	41	42
91	41	49
92	41	51
93	42	46
94	43	44
95	43	48
96	43	53
97	44	47
98	44	54
99	45	60
100	45	74
101	46	81
102	47	73
103	47	113
104	48	107
105	49	51
106	51	52
107	52	55
108	53	54
109	54	55
110	55	57
111	57	58
112	57	63
113	58	59
114	59	61
115	60	62
116	62	64
117	62	144
118	63	526
119	69	211
120	69	79
121	70	71
122	70	528
123	71	72

Branch	From Bus	To Bus
124	71	73
125	72	77
126	72	531
127	73	76
128	73	79
129	74	88
130	74	562
131	76	77
132	77	78
133	77	80
134	77	552
135	77	609
136	78	79
137	78	84
138	79	211
139	80	211
140	81	194
141	81	195
142	85	86
143	86	87
144	86	323
145	89	91
146	90	92
147	91	94
148	91	97
149	92	103
150	92	105
151	94	97
152	97	100
153	97	102
154	97	103
155	98	100
156	98	102
157	99	107
158	99	108
159	99	109
160	99	110
161	100	102
162	102	104
163	103	105
164	104	108
165	104	322
166	105	107

table continued

Branch	From Bus	To Bus
167	105	110
168	108	324
169	109	110
170	109	113
171	109	114
172	110	112
173	112	114
174	115	122
175	116	120
176	117	118
177	118	119
178	118	1201
179	1201	120
180	118	121
181	119	120
182	119	121
183	122	123
184	122	125
185	123	124
186	123	125
187	125	126
188	126	127
189	126	129
190	126	132
191	126	157
192	126	158
193	126	169
194	127	128
195	127	134
196	127	168
197	128	130
198	128	133
199	129	130
200	129	133
201	130	132
202	130	151
203	130	167
204	130	168
205	133	137
206	133	168
207	133	169
208	133	171
209	134	135

table continued

Branch	From Bus	To Bus
210	134	184
211	135	136
212	136	137
213	136	152
214	137	140
215	137	181
216	137	186
217	137	188
218	139	172
219	140	141
220	140	142
221	140	145
222	140	146
223	140	147
224	140	182
225	141	146
226	142	143
227	143	145
228	143	149
229	145	146
230	145	149
231	146	147
232	148	178
233	148	179
234	152	153
235	153	161
236	154	156
237	154	183
238	155	161
239	157	159
240	158	159
241	158	160
242	162	164
243	162	165
244	163	164
245	165	166
246	167	169
247	172	173
248	172	174
249	173	174
250	173	175
251	173	176
252	175	176

Branch	From Bus	To Bus
253	175	179
254	176	177
255	177	178
256	178	179
257	178	180
258	181	138
259	181	187
260	184	185
261	186	188
262	187	188
263	188	138
264	189	208
265	189	209
266	190	231
267	190	240
268	191	192
269	192	225
270	193	205
271	193	208
272	194	219
273	194	664
274	195	219
275	196	197
276	196	210
277	197	198
278	197	211
279	198	202
280	198	203
281	198	210
282	198	211
283	199	200
284	199	210
285	200	210
286	201	204
287	203	211
288	204	205
289	205	206
290	206	207
291	206	208
292	212	215
293	213	214
294	214	215
295	214	242

table continued

Branch	From Bus	To Bus
296	215	216
297	216	217
298	217	218
299	217	219
300	217	220
301	219	237
302	220	218
303	220	221
304	220	238
305	221	223
306	222	237
307	224	225
308	224	226
309	225	191
310	226	231
311	227	231
312	228	229
313	228	231
314	228	234
315	229	190
316	231	232
317	231	237
318	232	233
319	234	235
320	234	237
321	235	238
322	241	237
323	240	281
324	242	245
325	242	247
326	243	244
327	243	245
328	244	246
329	245	246
330	245	247
331	246	247
332	247	248
333	248	249
334	249	250
335	3	1
336	3	2
337	3	4
338	7	5

table continued

Branch	From Bus	To Bus
339	7	6
340	10	11
341	12	10
342	15	17
343	16	15
344	21	20
345	24	23
346	36	35
347	45	44
348	45	46
349	62	61
350	63	64
351	73	74
352	81	88
353	85	99
354	86	102
355	87	94
356	114	207
357	116	124
358	121	115
359	122	157
360	130	131
361	130	150
362	132	170
363	141	174
364	142	175
365	143	144
366	143	148
367	145	180
368	151	170
369	153	183
370	155	156
371	159	117
372	160	124
373	163	137
374	164	155
375	182	139
376	189	210
377	193	196
378	195	212
379	200	248
380	201	69
381	202	211

table continued

Branch	***From Bus***	***To Bus***
382	204	2040
383	209	198
384	211	212
385	218	219
386	223	224
387	229	230
388	234	236
389	238	239
390	196	2040
391	119	1190
392	120	1200
393	7002	2
394	7003	3
395	7061	61
396	7062	62
397	7166	166
398	7024	24
399	7001	1
400	7130	130
401	7011	11
402	7023	23
403	7049	49
404	7139	139
405	7012	12
406	7017	17
407	7039	39
408	7057	57
409	7044	44
410	7055	55
411	7071	71