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TEXTURE ANALYSIS – A SURVEY

Harry WECHSLER
Dept. of Electrical Engineering and Computer Science, University of Wisconsin-Milwaukee, Milwaukee, WI 53211, USA

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Abstract. Texture is the term used to characterize the surface of a given object or phenomenon and it is undoubtedly one of the main features used in image processing and pattern recognition. It is commonly agreed that texture analysis plays a fundamental role in classifying objects and outlining the significant regions of a given gray level image. Texture can be seen in images ranging from multispectral remote sensed data to microscopic images. A solution to the texture analysis problem will greatly advance the image processing and pattern recognition fields and it will also bring much benefit to many possible industrial applications.

This survey presents the state-of-the-art in the texture analysis field and it includes also its relation to scientific fields like artificial intelligence and visual perception. The paper outlines the main issues and difficulties and then it presents most of the relevant methods used today in texture analysis. We conclude by outlining directions for future research as they are perceived today in the image processing community.

Zusammenfassung. Die Textur charakterisiert die Oberfläche eines Objektes und ist ein wichtiges Merkmal in der Bildverarbeitung und der Mustererkennung. Sie spielt eine wichtige Rolle in der Klassifizierung von Objekten und in der Erkennung von bestimmten Regionen in Grauwertbildern, und sie kann sowohl in mikroskopischen Bildern, wie auch in jenen der multispektralen Abtastung der Erdoberfläche ermittelt werden.

Eine allgemeine Lösung des Problemes der Texturanalyse wird von grosser Hilfe sowohl für die Bildverarbeitung und die Mustererkennung wie auch für viele industrielle Anwendungen sein.

Dieser Artikel präsentiert den aktuellen Stand der Technik in dem Bereich der Texturanalyse, und beschreibt ihren Zusammenhang mit den Bereichen der künstlichen Intelligenz und der visuellen Wahrnehmung. Die wichtigsten Lösungen und Probleme werden angedeutet und die am meisten benutzte Methoden werden beschrieben. Schliesslich wird es auf die Richtungen hingewiesen, die heutzutage für die zukunftige Forchung wichtig erscheinen.

Résumé. La texture est le terme utilisé pour caractériser la surface d'un objet ou un phénomène donné. Il est sans doute l'une des caracteristiques principales utilisées en traitement d'images et en reconnaissance des formes. Il est généralement admis que l'analyse de texture joue un rôle fondamental dans la classification des objets et dans la détermination des régions significatives d'une image à niveaux de gris donnée. La texture s'observe dans des différents types d'images allant des données multispectrales de télédétection aux images microscopiques. Une solution au problème de l'analyse de texture sera un grand progrès pour le traitement d'images et la reconnaissance des formes et sera bénéfique pour plusieures applications industrielles possibles.

Cette revue présente l'état de l'art de l'analyse de texture et ses relations avec des domaines scientifiques comme l'intelligence artificielle et la perception visuelle. On présente d'abord les points et les difficultés importants dans l'analyse de texture. On décrit ensuite les méthodes les plus importantes utilisées de nos jours. Finalement on indique les directions de recherche pour le futur telles qu'elles sont vues aujourd'hui en traitement d'images.

Keywords. Artificial intelligence, image processing, pattern recognition, texture analysis, visual perception.

1. Introduction

Texture is the term used to characterize the surface of a given object or phenomenon and it is undoubtedly one of the main features used in image processing and pattern recognition. Texture can be seen in images ranging from multispectral remote sensed data to microscopic images. A solution to the texture analysis problem will greatly advance the image processing and pattern recognition fields and it will also bring much benefit to many possible applications in the areas of biomedical image processing (cell analysis), industrial automation (quality control) and remote sensing (crop estimation, ecology studies, etc.). Despite its importance and ubiquity in image data, texture lacks a precise definition. One of the most common definitions describe texture as being generated by one or more basic local patterns that are repeated in a periodic manner over some image region. Pratt *et al.* [53] remarks that such definitions are most applicable to deterministic types of textures such as line arrays, checkerboards, hexagonal tilings, etc., and that the term textured region is usually attributed to image regions devoid of sharp edges. Furthermore, images like the ones identified in an aerial photograph of the earth do not seem to possess an isolatable basic pattern nor a dominant repetition frequency and instead they seem to possess some stochastic structure. Fig. 1 through 6 show textures as reproduced from Brodatz [5] and they cover the range of textures from the deterministic to the stochastic type.

It is commonly agreed that texture analysis plays a fundamental role in classifying objects and outlining the significant regions of a given gray level image. We concur with Ehrich and Foith [16] about the main issues of concern in texture analysis:

(1) Given a textured region, to which of a finite number of classes does the sample belong?;

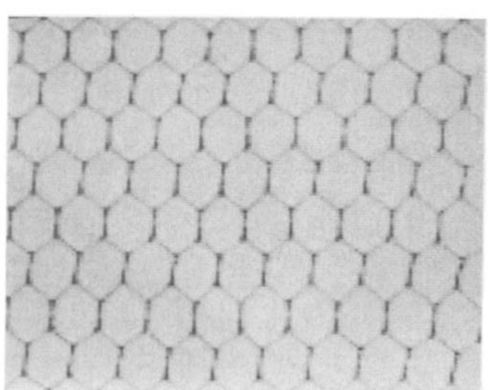

Fig. 1. Netting.

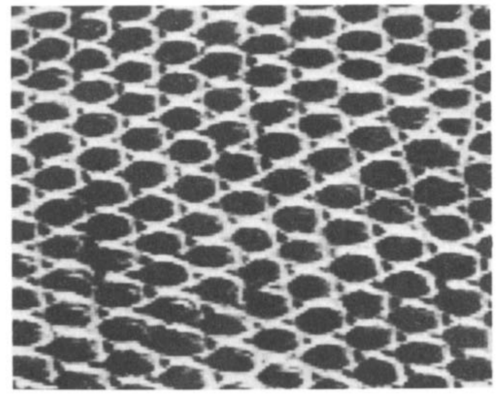

Fig. 2. Reptile skin.

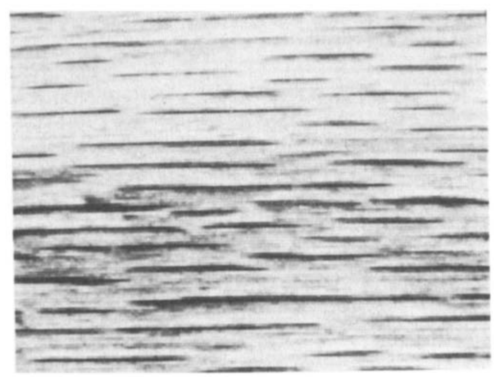

Fig. 3. Wood grain.

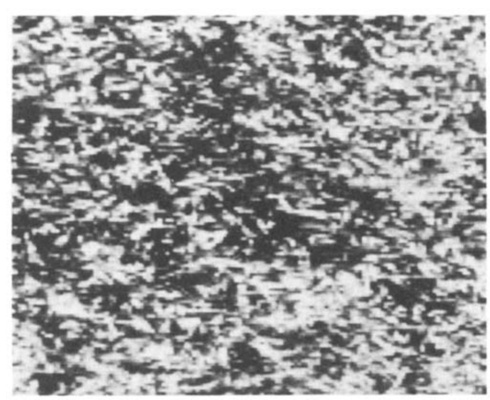

Fig. 4. Grass lawn.

(2) Given a textured region, how can it be described?; and

(3) Given a scene, how can the boundaries between the major textured regions be established?,

Fig. 5. Water.

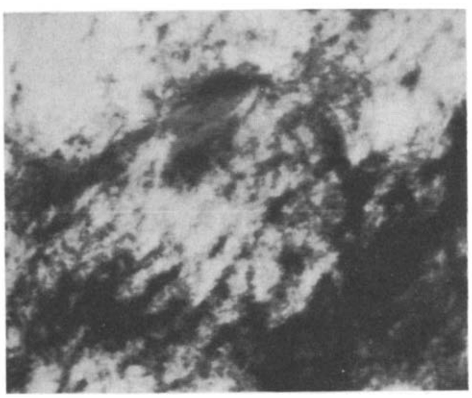

Fig. 6. Clouds.

and that these three problems are listed in order of increased difficulty. The first problem, also known as the texture discrimination (classification) problem, is usually approached by using some statistical method for extracting the characteristic parameters for each one of the given classes. The parameters are then the input features for classification using the well known techniques of statistical pattern recognition. The second problem is more difficult because one could easily find two perceptually different textures and it could be still extremely difficult to describe the differences. The third problem, which is also known as the segmentation problem, is mainly difficult because it is usually unknown at which level of complexity a texture is completed up to its physical boundary, and also because it is not well understood what kind of grouping mechanism is needed for outlining regions of uniform texture. Clearly, an adequate solution to the segmentation problem could be used as well for the first two problems. It should be mentioned that most of the research in the texture analysis field has been done in the context of the first problem mentioned above.

There are two basic philosophies for image segmentation. One philosophy uses the semantic approach, where a priori knowledge about the scene being analyzed is used for the actual segmentation [20]. Yet, there is ample evidence that the eye can perform at least initial segmentation without any a priori knowledge. Such an approach is called a low-level vision approach. Marr [44] and Zobrist and Thompson [74] concur in the idea that any general purpose vision system should start the segmentation task by using the low-level approach. Furthermore, an image segmentation system developed by Ohlander *et al.* [50] provides experimental evidence that primitive cues, texture one of them, could be used for initial segmentation. We concur in the use of the low-level approach as a starting procedure for the texture analysis problem even that a priori knowledge could be used in resolving ambiguities or performing a more efficient goal oriented texture analysis task. Semantic information can help and indeed if introduced it might improve the final result in segmentation of real images [64]. Such issues as described above are intimately related to the artificial intelligence field which tries to endow the computers with human intelligence and therefore enhance their performance.

2. Review

The most formal definitions to characterize texture have been given in the introduction. One definition characterizes textures as visual images which possess some stochastic structure. The other definition describes a texture as being generated by one or more basic local patterns that are repeated in a periodic manner over some image region. As a result of this dichotomy in definition it should be expected that two different approaches would have evolved over the years for the texture analysis problem. The two approaches which indeed crystalized over the years are called the statistical and the structural approach and they analyze the stochastic or repetitive structure of a texture, respectively. Pickett [52] categorizes the two approaches for texture analysis as impressionistic and deliberate. The impressionistic ("statistical") analysis yields an immediate characterization of a texture as being coarse or fine, grainy and so on. The deliberate ("structural") approach seems to be more complicated and it involves arrangements. Pickett also remarks that the taxonomy of textures based on variations in element size or shape is easier to perform than the discrimination of textures based on variations in density and/or arrangement of elements. Julesz [37] would characterize the statistical and structural approaches as perceptual and cognitive, respectively. Therefore, it should be expected that the cognitive task is more difficult than the perceptual one.

2.1. Human vision

Much research has been done in the area of analyzing the human visual system and also in trying to simulate human vision as long as machine vision is involved. (Vision is used here to mean both perception and cognition.) Pickett [52] mentions that some of the basic properties of the optical data which the visual system may measure include various characteristics of elements (size, shape, color, orientation), number or density of elements, and characteristic patterning or arrangement of elements. Hawkins [33] mentions important properties of the visual data like spatial frequency content, gray level content, local shape content and higher order measures comparing the results of the lower order gross measures as the parameters are varied. More recently, Tamura *et al.* [63] performed an important research on how to define textural features corresponding to visual perception. The visual features approximated in computational form were coarseness, contrast, directionality, line-likeness, regularity and roughness.

Perceptual grouping is one of the basic components of human visual organization. Experiments showed that grouping is more primitive and more fundamental than recognition (cognition). Gestalt grouping based on similarity and proximity led Zobrist and Thompson [74] to perform a series of experiments in order to implement some primitive distance functions as might be required for image segmentation, and texture was one of the components used for evaluating the similarity between two groups. Furthermore, Thompson [65] attempted to segment an image built out of four textures following the above ideas. It should be pointed out that the orientation is a very important cue in similarity judgements and aggregate grouping as demonstrated by Beck type experiments. Gestalt grouping has been successfully used by Zahn [73] for taxonomy purposes and the grouping was implemented through MST (minimal-spanning trees).

Early experiments performed by Julesz [37] seemed to suggest that differences in the first or second order statistics (brightness and granularity, respectively) allow texture discrimination for a human subject, but that differences in the third or higher order statistics are irrelevant as long as discrimination is concerned. More recently, Julesz [38] presented a new construction of textures, where all third-order statistics are constant, and yet, discrimination is possible. This last result is furthermore substantiated by Gagalowicz [26]. Second-order statistics (i.e., probabilities of the form $P(i, j)$ where $P(i, j)$ represents the likelihood of going from gray level i to gray level j for a given distance d and angle α) have been shown to be very important in the human analysis of textures [37, 53]. As we will see in the next section, computer experiments showed that the above conjecture regarding the importance of second-order statistics holds true for the automated analysis of both artificial and natural textures. Additional experiments performed by Pratt, Faugeras and Gagalowicz [53] showed that the second order statistical measures should be sufficient for texture analysis, but the mean, variance, and autocorrelation function measure, by themselves, although directly or indirectly necessary, are not sufficient. (As it is known, the autocorrelation function is related to the coarseness of texture [34] and as a consequence it should provide a clue about the size of the primitives involved in the structural approach.) The conjecture that the autocorrelation function is not a sufficient measure by itself has been corroborated by Haralick [30] who showed that to the extent that the texture is Markovian the second-order statistics for a given distance will determine the autocorrelation function at many distances. The research on the human visual system, empirical evidence and inventiveness led to different procedures for texture analysis. Two approaches, already mentioned, are the statistical and the structural one, and the features used for analysis are both local and global.

2.2. Statistical methods

The statistical ("impressionistic") approach extracts a set of parameters ("features") from a given image. The parameters are then used as the input features for classification using the well known techniques of statistical pattern recognition [15, 23]. The parameters are derived over the space or frequency domain. Some of the main statistical methods are mentioned next. The gray level difference method [28] estimates the probability density function for differences taken between picture function values. The spatial gray level dependence method [28] estimates the joint gray level distribution for two gray levels located at a distance "d" and an angle "α". The gray-level differences and joint gray level distributions are also known as the first and second order statistics, respectively. The second order statistics are usually tabulated as co-occurrence matrices. Furthermore, first-order statistics are embedded in the second order statistics as marginal density functions. Thus we can not find two pictures with identical second-order statistics and different first-order statistics. Usually a reduction in the number of gray levels via histogram equalization techniques [59] is a necessary preprocessing step for computational efficiency. Common features extracted from the above statistics are the mean, variance (spread of distribution), coarseness, skewness (tiltedness of distribution), kurtosis (sharpness of distribution). The gray level run length method [24] estimates the length of identical runs, where an identical run is defined as a set of connected pixels having the same gray level. One of the characteristic aspects of texture is its spatial granularity ("frequency content") and repeativeness. Therefore, we should expect that transform techniques (mostly digital) are used to extract such information as mentioned above. By far, the most used (orthogonal) transform is the Fourier transform. The Fourier analysis method [41] is a procedure which works in the frequency domain and the features needed for classification like spectral rings or edges are derived from the power spectrum. It is an expensive procedure from a computational viewpoint and an additional problem comes up when we try to evaluate the transform over a non-square region as it is usually the case. Experiments performed by Bajcsy [2] showed that Fourier analysis can provide global information as directionality vs. non-directionality, size information (blob-like), but at the same time, it might lead to spurious results because local information could be jumbled together in the frequency domain. (The same information about directionality could be obtained using local measurements like edge/unit area, histograms corresponding to the direction of the gradient and Hough-type transforms [59].) It is worthwhile to note that Eklundh [18] showed that the textural content of the phase information is low. Haralick *et al.* [28] and Conners and Harlow [8] detail the above methods and their applicability. According to their performance, the procedures mentioned before could be ranked as follows: second-order statistics, first-order statistics, Fourier analysis and gray level run length method [71, 8].

The first and second order statistics (co-occurrence matrices) are by far the most used statistical methods for texture discrimination [31, 40, 62]. One problem with the co-occurrence matrices is related to the need to define the distance "d" and the angle "α" which would fully specify the method. Davis *et al.* [11] rightly point out that a statistical model should be concerned with both the partition of an image into cells and subsequently with the assignment of gray levels to the cells [60]. The first and second order statistics as defined by Haralick [28] are concerned mainly with the assignment aspect. Therefore, Davis *et al.* [11] improve on Haralick's method by defining generalized co-occurrence matrices according to spatial constrained predicates which are related to the partition of a given image into cells. An additional problem related to the nth order statistics in general and the second order statistics in particular is the fact that they depend only on the relative position of the n points, but not on their absolute position. For cloud patterns or blood smears this might be a reasonable assumption, since objects can occur anywhere in the scene. For other types of textures as encountered in chest x-rays or portrait photographs it would not be reasonable to assume position independence [56].

2.3. *Structural methods*

The structural approach assumes that a set of primitive units ("patterns") can be easily identified. It then defines the texture as a combination of such primitives according to different placement rules. There are two major problems with this approach. First, it is not so easy to identify the primitives unless the texture is artificial or not too complex. Secondly, the definition that the patterns are repeated according to some prespecified rules should allow for a stochastic change in the replication process and the same should apply for the patterns themselves. Haralick [30] remarks that tone (primitive) and texture are not independent concepts; when there is little variation of tonal primitives, the dominant property of the image is tone and when there is a wide variation of tonal primitives, the dominant property of that image is texture. The structural approach does not fit the definition of a low-level approach as given in the introduction. It is a cognitive rather than a perceptual approach and it would usually rely on a priori knowledge. All taken into consideration the structural approach is not yet widely used.

An early example of the structural approach is given by Carlucci [6] who defined a texture language for polygon recognition. (For the increasing use of syntactical methods in pattern recognition see Fu [22]; the placement rules might be given in the form of a (stochastic) grammar.) Tomita, Shirai and Tsuji [66] use the structural analysis for classification of textures. A texture element ("primitive") is defined as a connected set of pixels ("area") of "almost" the same gray level. Then the primitives are characterized in terms of the following properties: brightness, area, size directionality and curvature. According to these properties the elements are classified in a number of classes and the above properties are used as textural features. In recognition of an unknown sample, the textural features are evaluated and compared against those of each learned textured class. Lu and Fu [42] define a syntactic model for the generation ("synthesis") and discrimination of structured textures. Tree grammars are used both for synthesis and recognition of basic patterns within fixed-shape windows. Finally, a set of error-correcting tree automata is used as a texture discriminator. Ehrich and Foith [16] present a structural hypothesis about textures which are defined as recursively ("hierarchically") nested intensity regions according to Gestalt criteria. They advocate the use of gray level peaks ("brightness maxima") as textural primitives upon which recursion is based. (The idea of using gray level peaks as textural primitives is also suggested by Mitchell *et al.* [47].) Serra and Matheron suggested the use of mathematical morphology for binary images (a strong restriction on textures) and the idea has been implemented on the Leitz Texture

Analyzer System (TAS) and reported by Mueller and Hunn [49]. With the TAS, the experimenter can select the appropriate (problem-specific) scanning element of a predetermined size or shape which is called a structuring element. Then, the structured element is translated over the whole image and "erodes" all points for which the structuring element centered at that point is totally contained in the given binary image. The "eroded" (binary) picture built of points as described above leads easily to the calculation of textural features like number count, perimeter count, connectivity, chordsize distribution, all according to the structuring element being translated.

2.4. *Texture synthesis and modeling*

The early research in the area of texture analysis was characterized by the use of simple textural properties, some of them motivated by the human visual system. The trend today is toward defining models which could discriminate or characterize a broad range of textures, and this can be seen in the texture analysis and synthesis fields, respectively. (Texture synthesis might be useful for situations where a region of a picture is missing or highly corrupted by noise and synthesized image data could replace it [55].) Rosenfeld and Lipkin [56] synthesize a texture over a given region by constructing or selecting a set of subpatterns and arranging them within the region according to placement rules. The subpatterns are made up of sub-patterns arranged also according to some placement rules, for as many steps as the "resolving power" of the textured display permits. (This is equivalent to a structural approach for texture analysis where the primitives are recursively aggregated [16].) Rosenfeld and Lipkin also point out that the subpatterns which are employed, and the regions over which given sets of subpatterns and given placement rules are used, need not correspond to identifiable perceptual units in the resulting texture. Thus, the above method of texture synthesis makes use of information which cannot always be completely retrieved by analyzing its output, or which, even if available, is not necessarily used by humans when they view the output. The above ideas should lead to caution in the attempt of simulating the human visual system and it would probably be appropriate to say that we should be selective in copying the human visual system. Another point to make is that the search for an underlying generating process of the texture being analyzed should allow for the possibility of different models (generating processes) characterized by variable parameters.

Schachter *et al.* [60] performed very interesting and important research on random mosaic models for textures. Several models for generating isotropic "cellular" textures would tesselate a region into cells, and then assign gray levels to the cells. (This idea led Davis *et al.* [11] to propose the use of generalized co-occurrence matrices.) Given such a model, it is possible in principle to predict statistical properties of the textures generated by the model, *e.g.*, the expected squared gray level difference between two points of the texture a given distance apart (variogram). These properties can be measured for real textures without having to actually decompose them into cells. Thus, if it is believed that a given texture fits a particular model, the model parameters can be successively adjusted until a good fit between the measured and predicted property value is obtained. The model with these values of its parameters can then be used to provide a description of the texture and the features needed for classification [60]. The above experiments found that Poisson line model fit better textures that appeared relatively random, while the checkerboard model gave better fits for the more regular-appearing textures. Furthermore, such an approach like the one described above is worth mentioning because it links the texture synthesis and analysis processes. (Another attempt to link synthesis and taxonomy was made by Lu and Fu [42], as already mentioned, and by McCormick and Jayaramamurthy [45.) Then, texture models can in principle be used to justify the choice of features for texture classification tasks. One remaining problem, in fact a problem which effects the whole texture analysis field, is that of appropriate resolution or size. Model fitting can yield useful results as long as the cell size distribution for the model is similar to that of the texture.

Several models based upon an underlying generative process have been proposed to date for texture analysis. The parameters which characterize the models can then be used as the features needed for texture classification or segmentation. At the same time such modeling could be used as a data compression method for image transmission. The models are usually one dimensional when it is clear that a textured image should be characterized by a 2-D model rather than a 1-D model. The difficulty in adopting and using a 2-D model is that of choosing a meaningful two-dimensional neighborhood (orientation and size) with respect to the pixel under consideration. McCormick and Jayaramamurthy [45] used an auto-regression model for texture synthesis and the parameters for a given texture were estimated using time series analysis methods [4]. Delp *et al.* [13] also investigated image modeling using an auto-regressive time series model with applications to data compression. A two-dimensional model for the analysis and synthesis of textured images was introduced by Tou *et al.* [68]. When the pixels satisfy a multiplicative process with multiplicative autocorrelation functions, the two-dimensional model may be decomposed into two one-dimensional linear processes. Deguchi and Morishita [12] use a two-dimensional estimation technique for texture characterization (classification) and texture-based image partitioning. Wechsler and Kidode [70] defined a 2-D random walk [21] model and they used it successfully for texture classification. Random walks are performed in a plane domain bounded by an absorbing boundary and the absorption distributions are calculated over a set of windows which are overlapping and cover the whole image. Measurements derived from such distributions are the textural features used for classification. Gagalowicz [25] considers a texture to be the output of a spatial filter excited by white noise (not necessarily Gaussian). The image is then characterized by the texture mean, the histogram of the input white noise, and the transfer function $H(Z)$ of the filter. The mean is easily calculated from the image, the autocorrelation function (second order moments) determines the magnitude of the transfer function $H(Z)$ while higher order moments determine the phase. Once the magnitude and the phase are evaluated inverse filtering will determine the white noise and then the computation of the histogram for the white noise is easily done. It should be pointed out that the white noise probability density and the spatial filter $H(Z)$ do not make up necessarily a complete set of parameters (textural features). Pratt and Faugeras [54] model a stochastic textured array as being generated from an array of independent, identically distributed random variables which passes through a linear or nonlinear spatial operator.

2.5. Texture segmentation

The ultimate goal of texture analysis is to provide for meaningful image segmentation. Limited work has been done in the area of image segmentation using textural features as compared to the effort spent on texture classification. Basically, one could segment a picture by dividing it into squared windows (as small as one pixel) and then, based on the window textural features, assign them to one of a given number of texture classes using a maximum likelihood approach. One of the main difficulties in image segmentation is the intimate relation between segmentation and classification. Namely, in segmentation we must have some a priori knowledge about textural properties of two adjacent regions in order to determine the precise boundary, while in classification (or the process of deriving textural features) we have to know the boundary between the regions in order to calculate "pure" textural features. Following the above reasoning Ando and Doi [1] carry out segmentation and classification simultaneously in an attempt to partition a given textured image by using the histogram and second moments incorporated in a similarity measure.

The basic strategies used for textured image segmentation are the top-down, bottom-up and split-and-merge approaches [51]. (For more on image segmentation see a recent survey by Kanade [39].) The top-down approach is represented among the others by Mitchell and Carlton [48], Ohlander *et al.* [50] and Zucker *et al.* [76]. Mitchell and Carlton use a texture measure that counts the number of local extrema in a window centered at each pixel. Additional features related to the brightness of the image are included in a distance function which performs an initial segmentation using a closest neighbor type approach. The original segmentation is successively refined in a hierarchical way. Ohlander *et al.* use histograms of (textural) properties for segmenting an image which might include heavy ("busy") texture. Zucker *et al.* use local texture operators and then a suppression algorithm thins the output of the local operators. Finally, histogram thresholding yields the segmented image. The bottom-up approach is represented among the others by Ehrich and Lai [17] and Thomson [65]. Chen and Pavlidis [7] use the split-and-merge algorithm [35] for segmenting a textured image. Co-occurrence matrices are first evaluated on a set of regions forming two levels of the quadratic tree ("pyramid") [51]. If the matrices corresponding to a region and its four children in the tree are similar then that region is considered to be of uniform texture. If not it is replaced by its children. Then, the co-occurrence matrices and the adjacent graph are used for additional grouping because regions which are close to each other in the image may lie far apart in the quadratic picture tree. A small region elimination step is also included because texture cannot be defined reliably on very small regions.

2.6. Cell unit

A major stumbling block for the texture analysis problem is that of determining the appropriate size and shape of the area from which the textural features should be extracted. Again we have the same problem we met in the context of the texture segmentation problem. Namely, in order to perform adequate segmentation we have to extract meaningful ("pure") textural features and in order to extract such features we should know the boundaries which surround areas of uniform texture. We call the minimal area of uniform texture in which meaningful measurements can be extracted as *the cell unit.* The question related to the size and shape of the subimage from which to extract textural features does not yet have a satisfactory answer. Haralick [30] points out the following characteristic of a texture. Texture can not be analyzed without a reference frame of tonal primitives being stated or implied. For any smooth gray tone surface, there exists a scale such that when the surface is examined, it has no texture. Then as resolution increases, it takes on a fine texture and then a coarse texture. The above idea amounts to saying that the scale problem is a cognitive rather than a perceptual problem. It should be pointed out that an adequate solution to the cell unit problem should also be able of identifying non-repetitive textures.

Suggested solutions to the cell unit problem include the use of the autocorrelation function [30] and the inertia measure [9]. If the tonal primitives of the image are relatively large, then the autocorrelation function will drop off slowly. If the tonal primitives are small, then the autocorrelation will drop off quickly. To the extent that the tonal primitives are spatially periodic, the autocorrelation function will drop off and rise again in a periodic manner. Hence, the lack of such a periodicity should characterize a non-repetitive texture. The size of the period, small vs. large, could discriminate between fine and coarse textures, respectively. The inertia measure is defined as

$$I(P_\alpha(d)) = \sum_{i=0}^{N-1} \sum_{j=0}^{N-1} (i-j)^2 p_\alpha(i, j/d)$$

where $P_\alpha(d)$ is the co-occurrence matrix for the distance d and angle α, $p_\alpha(i, j/d)$ is an entry in such a matrix and N is the number of gray levels. The inertia measure gives the best "mean square" estimate of the cell unit size. That is, the inertia measure will reach a minimum whenever the intersample spacing distance d is such that two unit patterns match best in the mean square sense.

3. Concluding Remarks

Much research work has been invested in the texture analysis problem but there is still more to come because of the relevance of this problem to image processing and pattern recognition and because of the great number of possible applications.

We believe that the following issues are the ones to dominate the interest of the research community for the next several years. First, most approaches use 1-D modeling rather than 2-D modeling, when it is obvious that cognition and perception of a given image is basically 2-D in its nature. (Furthermore, more sophisticated scene analysis is 3-D and involves depth cues which could be provided by texture gradients.) More work on texture modeling is required and the trend today is to use Markov Random Fields [32]. An additional problem which received little attention is that related to the invariance of the textural analysis procedures to linear and non-linear transformations, like change in orientation and scale. As an example, it is known that the transform method (spatial frequency approach) is not invariant under even a linear translation of gray levels and to compensate for this, probability quantizing should be employed [30]. Last but not least, more experimental work is needed. This should include larger sets of textures on which the procedures are tried and also an attempt to compare the power of different methods on the same set of textures. A beginning in this direction has been made by Weszka *et al.* [71] who concluded that the spatial frequency approaches perform significantly poorer than the first and second order statistics.

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