Integer Optimization Assignment: Exhaustive Enumeration Method with Nonlinear Constraint

1 Introduction

In this assignment, you will explore the exhaustive enumeration method to solve an integer optimization problem with additional constraints and a nonlinearity. The exhaustive enumeration method involves evaluating all possible solutions to identify the optimal one.

2 Objective

The objective is to model and solve an integer optimization problem with a nonlinear constraint using the exhaustive enumeration method in MATLAB.

3 Problem Statement

We consider a classic integer optimization problem with a relatively large number of combinations and a nonlinear constraint: the project selection problem.

A company has n possible projects to invest in. Each project i has a cost c_i and an expected return r_i. The company has a maximum budget B to invest and can select at most K projects. In addition, there is a nonlinear constraint that limits the total management capacity based on the selected projects.

The objective is to maximize the total return of the selected projects without exceeding the available budget, the maximum number of projects, and the management capacity.

3.1 Nonlinear Constraint

The company's management capacity is limited by the number of projects that can be managed simultaneously. This capacity is modeled by a nonlinear constraint that relates the number of selected projects to a maximum capacity C. Specifically, if x_i is a binary variable indicating whether project i was selected (1) or not (0), the total number of selected projects must satisfy the following condition:

$$\left(\sum_{i=1}^{n} x_i\right)^2 \leq C \tag{1}$$

This formulation reflects that managing a larger number of projects increases management complexity exponentially.

4 Given Data

- **Maximum Budget (B):** 50 million reais.
- **Number of Projects (n):** 20.
- **Maximum Number of Projects (K):** 5.
- **Project Costs (c):**

$$c = [5, 10, 7, 8, 6, 9, 12, 15, 20, 17, 3, 11, 14, 18, 4, 16, 19, 13, 2, 1] \text{ (millions of reais)} \tag{2}$$

- **Expected Returns (r):**

$$r = [10, 15, 9, 14, 11, 13, 20, 25, 30, 28, 8, 17, 22, 26, 12, 24, 27, 19, 7, 5] \text{ (millions of reais)} \tag{3}$$

- **Management Capacity (C):** 16.