

$$1) \begin{cases} x + y + z = 6 \\ 2x - y + z = 3 \\ x + y - z = 0 \end{cases} \quad \begin{cases} x + y + z = 6 \\ +3y + z = +9 \\ -2z = -6 \end{cases}$$

$$l_2 = -2l_1 + l_2$$

$$\begin{aligned} & \star -2x + (-2y) + (-2z) = -12 \\ & \quad 2x - y + z = 3 \\ & \quad -3y - z = -9^{(-1)} \end{aligned}$$

$$3y + 3 = 9 \rightarrow 3y = 6$$

$$y = 2$$

$$S = \begin{pmatrix} 1 & 2 & 3 \\ x & y & z \end{pmatrix}$$

$$l_3 = -l_1 + l_3$$

$$\begin{aligned} & -x - y - z = -6 \\ & x + y - z = 0 \\ & -2z = -6 \end{aligned}$$

$$z = 3$$

$$x + 2 + 3 = 6$$

$$x = 1$$

$$Q \Rightarrow \begin{cases} x + y + z = 6 \\ 2x - y + z = 3 \\ -x + 2y + 0z = 3 \end{cases} \quad \begin{cases} x + y + z = 6 \\ -3y - 3z = -9 \end{cases}$$

$$l_2 = -2l_1 + l_2 =$$

$$\cancel{-2x} - 2y - 2z = -12$$

$$2x - y + z = 3$$

$$-3y - 3z = -9$$

$$l_3 = l_1 + l_3$$

$$\cancel{x} + y + z = 6$$

$$\cancel{-x} + 2y + 0z = 3$$

$$3y + z = 9$$

$$\cancel{-3y} \rightarrow \cancel{-3 \cdot 0} = -9$$

$$\boxed{y = 3}$$

$$l_3 = l_2 + l_3$$

$$\cancel{-3y} - 3z = -9$$

$$3\cancel{y} + z = 9$$

$$-2z = 0 \quad \boxed{z = 0}$$

$$x + 3\cancel{y} = 6$$

$$\boxed{x = 3}$$

$$S = \begin{pmatrix} 3 \\ 3 \\ 0 \end{pmatrix} \quad \begin{matrix} x \\ y \\ z \end{matrix}$$

$$c) \begin{cases} x + y + z = 6 \\ 2x - y + z = 3 \\ 4x + y + 3z = 1 \end{cases} \rightarrow \begin{cases} x + y + z = 6 \\ 3y + z = 19 \\ 3y + z = 23 \end{cases}$$

$$\begin{array}{l|l} l_2 - 2l_1 + l_3 & l_2 = -4l_1 + l_3, \quad z = K \\ \hline -2x - 2y - 2z = -12 & -4x - 4y - 4z = -24 \\ 2x - y + z = 3 & 4x + y + 3z = 1 \\ -3y - z = -9^{(-1)} & 3y + z = 23^{(-1)} \\ & 3y + z = 23^{(-1)} \end{array}$$

$$\begin{array}{l|l} l_3 \rightarrow l_2 + l_3 & \begin{cases} x + y + K = 6 \\ 3y + K = 9 \end{cases} \\ \hline -3y + z = -9 & \\ 3y + 7z = 23 & \\ 6z = 16 & \end{array} \rightarrow \begin{cases} x + y + K = 6 \\ 3y + K = 9 \end{cases} \rightarrow \boxed{y = \frac{9-K}{3}}$$

$$x + \frac{9-K}{3} + K = 6 \rightarrow x + \frac{9+4K}{3} = 6$$

$$x = 6 - \frac{9+4K}{3} = x = \frac{18-9-4K}{3} = \boxed{\frac{9-4K}{3}}$$

$$S = \left(\frac{9-4K}{3}, \frac{9-K}{3}, K \right) \text{ ou } \emptyset$$

$$2) A = \begin{bmatrix} 5 & 3 \\ 8 & 5 \end{bmatrix} \quad 25 - 24 = 1$$

$$\left| \begin{array}{cc|cc} 5 & 3 & 0 & 1 \\ 8 & 5 & 0 & 0 \end{array} \right| \quad \left| \begin{array}{cc|cc} x & y & 1 & 0 \\ a & b & 0 & 1 \end{array} \right|$$

$$\left| \begin{array}{cc|cc} 5x+3a & 5y+3b & 1 & 0 \\ 8x+5a & 8y+5b & 0 & 1 \end{array} \right|$$

$$\begin{cases} 5x+3a = 1 & \Rightarrow 40x+24a = 8 & (-1) \\ 8x+5a = 0 & \Rightarrow -40x+25a = 0 & (a=-8) \end{cases}$$

$$5x + (3 \cdot -8) = 1 \Rightarrow 5x - 24 = 1 \Rightarrow 5x = 25 \Rightarrow x = 5$$

$$\begin{cases} 5y+3b = 0 & \Rightarrow 40y+24b = 0 \\ 8y+5b = 1 & \Rightarrow -40y+25b = 5 & (-1) \end{cases}$$

$$5y + (3 \cdot 5) = 0 \Rightarrow 5y + 15 = 0 \Rightarrow 5y = -15 \Rightarrow y = -3$$

$$b = 5$$

$$A^{-1} = \begin{bmatrix} 5 & -3 \\ -8 & 5 \end{bmatrix}$$

$$3-a) \left| \begin{array}{cc|cc} 0 & 1 & 4 & 7 \\ 1 & 0 & 2 & 3 \end{array} \right| \xrightarrow{Z} \begin{array}{cc} 0+2 & 0+3 \\ 4+0 & 7+0 \end{array}$$

$$= \left| \begin{array}{cc} 2 & 3 \\ 4 & 7 \end{array} \right|$$

$$b) \left| \begin{array}{ccc|cc} 1 & 5 & 2 & 0 & 1 & -1 \\ -1 & 4 & 7 & 2 & 3 & 0 \end{array} \right| \xrightarrow{Z} \begin{array}{cc} 1+0+(-6) & -1+(5+0) \\ -1+8+(-21) & 1+12+0 \end{array}$$

2x3

$\frac{3}{2} \begin{array}{cc} -3 & 0 \end{array}$

$$= \left| \begin{array}{cc} 5 & 14 \\ -14 & 13 \end{array} \right|$$

$4 = a)$

$$\begin{array}{ccc|ccc} & -2 & & & & 2 \\ & & & & & & & & & \\ 1 & & -2 & & & 3 & & & & \\ & 2 & & & & & & & & \\ & & 1 & & & -1 & & & & \\ & & & & & & & & & \\ -2 & & & & & & & & & \\ & & -1 & & & 2 & & & & \\ & & & & & & & & & \\ 1 & & & & & & & & & 3 \end{array}$$

$$2 + (-4) + (-6) - (-6 + (-8) + 1)$$

$$-8 + 13$$

$$\boxed{\text{Det} = 5}$$

$b)$

$$\begin{array}{ccc|ccc} & & & & & & & & & \\ 1 & & 2 & -1 & 0 & & & & & \\ 3 & & 1 & 0 & 1 & & & & & \\ -2 & & 0 & 2 & 4 & & & & & \\ 0 & & 5 & 1 & -1 & & & & & \end{array}$$

$$\begin{array}{ccc|ccc} & & & & & & & & & \\ & & & & & & & & & \\ -5 & & 3 & & & & & & & \\ 4 & & 0 & & & & & & & \\ 8 & & 1 & & & & & & & \\ -5 & & & & & & & & & \end{array}$$

$0 + 60 + 4 - (0 + (-12) + 20)$

$$64 + 32$$

$$\boxed{\text{Dente} = 96}$$