1) $\int x^4 - 2x\,dx$

$$\int x^4 dx - \int 2x\,dx = \frac{x^{4+1}}{4+1} - \frac{2x}{2} = \frac{x^5}{5} - \frac{2x^2}{2}$$

$$\boxed{\frac{x^5}{5} - x^2 + C}$$ //

2-) $\int \sqrt{x-1}\,dx = \int (x-1)^{\frac{1}{2}}\,dx = \frac{(x-1)^{\frac{1}{2}+1}}{\frac{1}{2}+1} = \frac{(x-1)^{\frac{3}{2}}}{\frac{3}{2}}$

$\sqrt{(x-1)^3} \cdot \frac{2}{3} = \boxed{\frac{2\sqrt{(x-1)^3}}{3} + C}$ //

3-) $\int x(x+1)^2\,dx = \int x(x^2+2x+1)\,dx$

$= \int x^3 + 2x^2 + x\,dx = \int x^3 dx + \int 2x^2 dx + \int x\,dx$

$$\frac{x^{3+1}}{3+1} + \frac{2x^{2+1}}{2+1} + \frac{x^{1+1}}{1+1} = \boxed{\frac{x^4}{4} + \frac{2x^3}{3} + \frac{x^2}{2} + C}$$ //

4-) $\int \operatorname{sen}(3x)\,dx$

$u = 3x \quad du = 3dx$

$dx = \frac{du}{3}$

$\int \operatorname{Sen}(u)\,\frac{du}{3} = \int \operatorname{Sen}(u)\,\frac{1}{3}\,du$

$\int \frac{1}{3} \int \operatorname{Sen}(u)\,du = \frac{1}{3}(-\cos(u)) = \frac{1}{3}(-\cos(3x))$

$$= \boxed{\frac{-\cos(3x)}{3} + C}$$ //

① $\int_0^1 x\sqrt{x}\,dx = \int_0^1 x \cdot x^{\frac{1}{2}}\,dx = \int_0^1 x^{\frac{3}{2}}\,dx$

$= \left[\frac{x^{\frac{3}{2}+1}}{\frac{3}{2}+1}\right]_0^1 \Rightarrow \frac{x^{5/2}}{5/2} = x^{5/2}\cdot\frac{2}{5}\Big]_0^1$

$= \frac{2x^{5/2}}{5} = \frac{2\sqrt{x^5}}{5}\Big]_0^1$

$\frac{2\sqrt{1^5}}{5} - \frac{2\sqrt{0^5}}{5} = \frac{2\sqrt{1}}{5} - \frac{2\sqrt{0}}{5}$

$= \frac{2.1}{5} - \frac{2.0}{5} = \boxed{\frac{2}{5}}$ //

② $\int_0^{\pi} \frac{\operatorname{sen} x\,dx}{2} = \int_0^{\pi} \frac{1}{2}\operatorname{sen}(x)\,dx \int_0^{\pi} \frac{\operatorname{sen}(x)\,dx}{2}$

$\frac{1}{2}\int_0^{\pi} \operatorname{sen}(x)\,dx \Rightarrow \frac{1}{2}\left[-\cos(x)\right]_0^{\pi} \Rightarrow \frac{-1}{2}\cos(x)\Big]_0^{\pi}$

$\frac{-1}{2}\left[\cos(\pi) - \cos(0)\right] = \frac{-1}{2}(-1-1) = \frac{-1}{2}(-2)$

$= \frac{3}{2} \cdot \boxed{1}$

③ $\int_0^{\frac{\pi}{2}} \cos(2x)\,dx$ $\quad u = 2x \quad dx = \frac{du}{2}$

$du = 2dx$

$\int_0^{\pi} \cos(u)\frac{du}{2} = \int_0^{\pi} \cos(u)\frac{1}{2}\,du = \frac{1}{2}\int_0^{\pi}\cos(u)\,du$

$\frac{1}{2}\operatorname{Sen}(u)\,du\Big]_0^{\pi} = \frac{\operatorname{Sen}(u)}{2}\Big]_0^{\pi} \rightarrow \frac{\operatorname{Sen}(\pi)}{2} - \frac{\operatorname{Sen}(0)}{2}$

$\frac{0}{2} - \frac{0}{2} = 0$ //

④ $\int_0^2 (3x-1)^3\,dx \rightarrow u = 3x-1 \quad dx = \frac{du}{3}$

$du = 3dx$

$\int_{-1}^{5} u^3 \frac{du}{3} \rightarrow \frac{1}{3}\frac{u^4}{4}\Big]_{-1}^{5} \rightarrow \frac{u^4}{12}\Big]_{-1}^{5}$

$\frac{5^4}{12} - \frac{(-1)^4}{12} = \frac{625}{12} - \frac{1}{12} = \frac{624}{12} = 52$ //

① $\int x e^{2x}\,dx = x \cdot \frac{e^{2x}}{2} - \int \frac{e^{2x}}{2}\,dx$

$= \frac{x e^{2x}}{2} - \frac{1}{2}\int e^{2x}\,dx$

$\frac{x e^{2x}}{2} - \frac{1}{2}\cdot\frac{e^{2x}}{2} = \frac{x e^{2x}}{2} - \frac{e^{2x}}{4} + C$

$\frac{2x e^{2x} - e^{2x}}{4} = \boxed{\frac{e^{2x}(2x-1)}{4} + C}$ //

② $\int x^3 \operatorname{Sen}(x)\,dx$

$x^3 \cdot (-\operatorname{Cos}(x)) - \int -\operatorname{Cos}(x)\, 3x^2\,dx$

$x^2 \operatorname{Cos}(x) + 3\int \operatorname{Cos}(x)\, x^2\,dx$

$-x \operatorname{Cos}(x) + 3(x)\operatorname{Sen} x - \int \operatorname{Sen}$

$-x^3 \operatorname{Cos}(x) + 3x^2 \operatorname{Sen}(x)$

$-3.2\int x \operatorname{Sen}(x)\,dx = -x^3 \operatorname{Cos}(x) + 3x^2 \operatorname{Sen}(x) - 6$

$\int x(-\operatorname{Cos}(x)) - \int -\operatorname{Cos}(x)\,dx$

$= -x^3 \operatorname{Cos}(x) + 3x^2 \operatorname{Sen}(x) + 6x \operatorname{Cos}(x) - 6\operatorname{Sen}(x)$

$= (-x^3 + 6x)\operatorname{Cos}(x) + (3x^2 - 6)\operatorname{Sen}(x) + C$ //

③ $\int x \cos(2x)\,dx = \frac{x \operatorname{sen}(2x)}{2} - \frac{\int \operatorname{sen}(2x)\,dx}{2}$

$\frac{x \operatorname{sen}(2x)}{2} - \frac{1}{2}\int \operatorname{sen}(2x)\,dx = \frac{x \operatorname{sen}(2x)}{2}$

$-\frac{1}{2}\left(\frac{-\cos(2x)}{2}\right) = \boxed{\frac{x \operatorname{sen}(2x)}{2} + \frac{\cos(2x)}{4} + C}$ //

④ $\int x^3 \ln(x)\,dx = u = \ln(x) \quad du = \frac{1}{x}\,dx$

$dv = x^3\,dx$

$v = \frac{x^4}{4}$

$\int \ln(x)\, x^3\,dx \to \ln(x) \cdot \frac{x^4}{4} - \int \frac{x^4}{4} \cdot \frac{1}{x}\,dx$

$= \frac{x^4 \ln(x)}{4} - \frac{1}{4}\int x^3\,dx = \frac{x^4 \ln(x)}{4} - \frac{1}{4} \cdot \frac{x^4}{4}$

$\frac{x^4 \ln(x)}{4} - \frac{x^4}{16} = \frac{4 \cdot x^4 \ln(x) - x^4}{16}$

$\boxed{\frac{x^4(4\ln(x) - 1)}{16} + C}$ //