

1-) $\int_{x=0}^2 \int_{y=0}^{x^2} y \, dy \, dx$
 $x=0 \quad y=0$

$$\int_{y=0}^{x^2} y \, dy = \frac{y^2}{2} \Big|_0^{x^2} = \frac{(x^2)^2}{2} - \frac{0^2}{2}$$

$$\frac{1}{2} \int_0^2 \frac{x^4}{2} = \frac{1}{2} \Big|_0^2 \frac{x^5}{5} = \frac{1}{2} \left(\frac{2^5}{5} - \frac{0^5}{5} \right)$$

$x=0$

$$= \frac{1}{2} \cdot \frac{32}{5} = \frac{32^2}{10^2} = \frac{16}{5}$$

2-) $\int_0^1 \int_0^2 (x+2) \, dy \, dx$
 $x=0 \quad y=0$

$$\int_0^2 (x+2) \, dy \Big|_0^2 (x+2) y \Big|_0^2 = (x+2) y \Big|_0^2$$

$$(x+2)(2-0) = (x+2)2 = 2x+4$$

$$2 \int_0^1 x \, dx + 4 \int_0^1 \frac{x^2}{2} \Big|_0^1 + 4 \cdot x \Big|_0^1 =$$

$$I_0' = 2 \left(\frac{1^2}{2} - \frac{0}{2} \right) + 4(1-0) = 2 \cdot \frac{1}{2} + 4$$

$$= 5 //$$

$$3-) \int_0^1 \left[\int_2^3 x y^2 dy \right] dx$$

$$\int_2^3 x y^2 dy = x \int_2^3 y^2 dy = x \left. \frac{y^3}{3} \right|_2^3 = x \left(\frac{3^3}{3} - \frac{2^3}{3} \right)$$

$$= x \left(\frac{27}{3} - \frac{8}{3} \right) = \frac{19}{3} x$$

$$\int_0^1 \frac{19}{3} x dx = \frac{19}{3} \int_0^1 x dx = \frac{19}{3} \left. \frac{x^2}{2} \right|_0^1$$

$$\frac{19}{3} \cdot \left(\frac{1^2}{2} - \frac{0^2}{2} \right) = \frac{19}{3} \cdot \frac{1}{2} = \boxed{\frac{19}{6}} //$$

$$4.) \int_1^2 \int_1^2 xy \, dy \, dx$$

$$\int_1^2 xy \, dy = x \int_1^2 y \, dy = x \left[\frac{y^2}{2} \right]_1^2$$

$$x \left(\frac{2^2}{2} - \frac{1^2}{2} \right) = x \left(\frac{4}{2} - \frac{1}{2} \right)$$

$$\frac{3x}{2}$$

$$\int_1^2 \frac{3x}{2} \, dx = \frac{3}{2} \int_1^2 x \, dx = \frac{3}{2} \left[\frac{x^2}{2} \right]_1^2$$

$$\frac{3}{2} \cdot \left(\frac{2^2}{2} - \frac{1^2}{2} \right) = \frac{3}{2} \cdot \frac{3}{2} = \frac{9}{4}$$

$$S-1) \int_0^4 \int_0^{\sqrt{x}} (x+y) dy dx$$

$$\begin{aligned} \int_0^{\sqrt{x}} (x+y) dy &= \int_0^{\sqrt{x}} y dy = \left. \frac{y^2}{2} \right|_0^{\sqrt{x}} + \frac{y^2}{2} \\ &= x(\sqrt{x} - 0) + \frac{(\sqrt{x})^2}{2} - \frac{0^2}{2} \\ &= x\sqrt{x} + \frac{x}{2} = x^1 x^{\frac{1}{2}} + \frac{x}{2} = x^{\frac{3}{2}} + \frac{x}{2} \end{aligned}$$

$$\begin{aligned} \int_0^4 \left(x^{\frac{3}{2}} + \frac{x}{2} \right) dx &= \int_0^4 x^{\frac{3}{2}} dx + \frac{1}{2} \int_0^4 x dx \\ &= \left. \frac{2\sqrt{x^4}}{5} \right|_0^4 + \left. \frac{1}{2} \cdot x^2 \right|_0^4 = \frac{2\sqrt{4^4}}{5} - \frac{2\sqrt{0}}{5} \end{aligned}$$

$$+ \frac{1}{2} \left(\frac{4^2}{2} - \frac{0^2}{2} \right) = \frac{2\sqrt{4^4}}{5} + \frac{16}{4} = \frac{64}{5} + 4$$

$$\boxed{= 16,8}$$

$$6 - \int_0^2 \int_0^{x-2} (x+y-1) dy dx$$

$$\int_0^{x-2} (x+y-1) dy = \left[xy + \frac{y^2}{2} - y \right]_0^{x-2} :$$

$$= (x-2-0) + \frac{(x-2)^2}{2} - \frac{0}{2} - (x-2-0)$$

$$= x(x-2) + \frac{(x^2-4x+4)}{2} - (x-2) = \frac{x^2-2x}{1}$$

$$+ \frac{(x^2-4x+4)}{2} - \frac{x-2}{1} =$$

$$= 2x^4 - 4x - x^2 + 4x + 4 = 3x^2 - 10x + 8$$

$$= (x-2)^2 = (x-2)(x-2) = x^2 - 2x - 2x$$

$$+ 4 = x^2 - 4x + 4$$

$$\frac{1}{2} \left[\frac{8x^3}{3} - \frac{16x^2}{2} \right]_0^2 = \frac{1}{2} \left[x^3 - 5x^2 + 8x \right]_0^2$$

$$= \frac{1}{2} \left[2^3 - 5 \cdot 2^2 + 8 \cdot 2 - 0 + 5 \cdot 0 - 8 \cdot 0 \right]$$

$$= \frac{1}{2} (8 - 20 + 16) = \frac{1}{2} \cdot \frac{4}{1} = \frac{4}{2} = 2$$

$$7) \int_0^3 \int_0^2 (4 - y^2) dy dx = \int_0^3 \frac{16}{3} dx$$

$$= \frac{16}{3} \int_0^3 x^2 dx = \frac{16}{3} \int_0^3 x^0 dx$$

$$\frac{16}{3} \cdot x \Big|_0^3 = \frac{16}{3} (3 - 0) = \frac{16}{3} \cdot 3 = 16 //$$

$$\int_0^2 (4 - y^2) dy = 4y - \frac{y^3}{3} \Big|_0^2 = 4 \cdot 2 - \frac{2^3}{3}$$

$$= (4 \cdot 2 - \frac{8}{3}) - (4 \cdot 0 - \frac{0}{3}) = \frac{8}{1} - \frac{8}{3} = \frac{24 - 8}{3} = \frac{16}{3}$$

$$\begin{aligned}
 0-) \int_0^{\pi} \int_1^2 x \cos(xy) dy dx &= \int_0^{\pi} \\
 & \quad [\sin(2x) - \sin(x)] dx \\
 &= \int_0^{\pi} \sin(2x) dx - \int_0^{\pi} \sin(x) dx
 \end{aligned}$$

$$= \left. \frac{-\cos(2x)}{2} \right|_0^{\pi} - \left. (-\cos(x)) \right|_0^{\pi}$$

$$= \left. \frac{-\cos(2x)}{2} \right|_0^{\pi} + \left. \cos(x) \right|_0^{\pi} = -\frac{\cos(2\pi)}{2}$$

$$- \left(\frac{-\cos(2 \cdot 0)}{2} \right) + \cos \pi - \cos 0$$

$$= \frac{-\cos 2\pi}{2} + \frac{\cos 0}{2} + \cos \pi - \cos 0$$

$$= -\frac{1}{2} + \frac{1}{2} + (-1) - 1 = \boxed{-2}$$

$$\begin{aligned}
 \int_1^2 x \cos(xy) dy &= x \int_1^2 \cos(xy) dy = x \\
 \frac{\sin(xy)}{x} \Big|_1^2 &= \sin(2x) - \sin(x)
 \end{aligned}$$

$$9.) \int_0^1 \int_1^{e^x} dy dx = \int_0^1 \int_1^{e^x} 1 dy dx =$$

$$\int_0^1 \int_1^{e^x} x y dy dx$$

$$= \int_0^1 (e^x - 1) dx = \int_0^1 e^x dx - \int_0^1 1 dx = e^x$$

$$\left| e^x - x \right|_0^1 = e^1 - e^0 - (1 - 0) = e - 1 - 1 =$$

$$\boxed{e-2}$$

$$\int_1^{e^x} x y dy = \int_1^{e^x} y dy = y \Big|_1^{e^x} = e^x - 1$$

$$10.) \int_{-\pi}^{\pi} \int_0^2 (x + \sin(xy) + 1) dx dy =$$

$$= 4 \int_{-\pi}^{\pi} y dy + 2 \int_{-\pi}^{\pi} \sin(xy) dy = 4y \Big|_{-\pi}^{\pi}$$

$$+ 2 [-\cos(xy)]_0^{\pi} = 4y \Big|_{-\pi}^{\pi} - 2 \cos y \Big|_{-\pi}^{\pi}$$

$$= 4\pi - 4(-\pi) - 2 [\cos \pi - \cos(-\pi)] = 4\pi + 4\pi$$

$$- 2 [-1 - (-1)] = 8\pi + 2(-1 + 1) = 8\pi - 20$$

$$= 8\pi$$

$$\textcircled{Q} \cdot \int_0^2 (x + \sin(y) + 1) dx = \int_0^2 x dx + \int_0^2 x \sin(y) dx + \int_0^2 x^2 dx$$

$$= \frac{x^2}{2} \Big|_0^2 + x \sin(y) \Big|_0^2 + \frac{x^3}{3} \Big|_0^2 =$$

$$\left(\frac{2^2}{2} - \frac{0}{2} \right) + (2 \sin(y) - 0 \sin(y))$$

$$+ (2 - 0) = \frac{4}{2} + 2 \sin(y) + 2 =$$

$$2 + 2 \sin(y) + 2 = 4 + 2 \sin(y)$$