Lista 2

1-) Dados os vetores $u = (2, -3, 2)$ e $v = (-1, 2, 4)$

a) Escreva $w = (7, -11, 2)$ como uma combinação linear de u e v

$$w = a \cdot u + b \cdot v$$

$$(7, -11, 2) = a(2, -3, 2) + b(-1, 2, 4)$$
$$= (2a, -3a, 2a) + (-b, 2b, 4b)$$
$$(7, -11, 2) = (2a - b, -3a + 2b, 2a + 4b)$$

$$\begin{cases} 2a - b = 7 \\ -3a + 2b = -11 \\ 2a + 4b = 2 \end{cases} \qquad \begin{matrix} -l_1 + l_3 = \\ 5b = -5 \\ \boxed{b = -1} \end{matrix}$$

$$2a + 4 \cdot (-1) = 2$$
$$2a - 4 = 2$$
$$2a = 6$$
$$\boxed{a = 3}$$

$$w = 3u + (-1 \cdot v)$$

b) $(-8, 14, k) = a u + b v$

$$= (2a, -3a, 2a) + (-b, 2b, 4b)$$
$$(-8, 14, k) = (2a - b, -3a + 2b, 2a + 4b)$$

$$\begin{cases} 2a - b = -8 \\ -3a + 2b = 14 \\ 2a + 4b = k \end{cases} \qquad \begin{matrix} 2l_1 + l_2 = \\ 4a - 2b = -16 \\ -3a + 2b = 14 \\ \boxed{a = -2} \end{matrix}$$

$$2 \cdot (-2) - b = -8 \qquad \boxed{b = +4}$$

$$2 \cdot (-2) + 4 \cdot (+4)$$
$$-4 + (+16) = 12$$
$$\boxed{K = 12}$$

$u = (-1, 2, 1)$, $v = (1, 0, 2)$, $w = (-2, -1, 0)$

2) a) $a = (-8, 4, 1)$

$(-8, 4, 1) = a u + b v + c w$

$= (-a, 2a, a) + (b, 0, 2b) + (-2c, -c, 0)$

$(-8, 4, 1) = (-a + b - 2c, 2a + 0 - c, a + 2b + 0)$

$$\begin{cases} -a + b - 2c = -8 \\ 2a - c = 4 \\ a + 2b = 1 \end{cases}$$

$2l_1 + l_2$: $-2a + 2b - 4c = -16$, $2a + 0 - c = 4$, $2b - 5c = -12$

$l_1 + l_3$: $-a + b - 2c = -8$, $a + 2b + 0 = 1$, $3b - 2c = -7$

$$\begin{cases} -a + b - 2c = -8 \\ 2b - 5c = -12 \\ 3b - 2c = -7 \end{cases}$$

$-3l_2 + 2l_3$

$-6b + 15 = +36$

$6b - 4c = -14$

$11c = 22$

$c = 2$

$2b - 5 \cdot 2 = -12$

$2b - 10 = -12$

$2b = -2$

$b = -1$

$-a + (-1) - 2 \cdot 2 = -8$

$-a - 1 - 4 = -8$

$-a = -8 + 5$

$-a = -3$

$a = 3$

$A = 3u + (-1 \cdot v) + 2w$

b) $(0,2,3) = au + bv + cw$

$(0,2,3) = (-a,2a,a) + (b,0,2b) + (2c,-c,0)$

$(0,2,3) = (-a+b-2c,\ 2a+0-c,\ a+2b+0)$

$$\begin{cases} -a+b-2c=0 \\ 2a+0-c=2 \\ a+2b+0=3 \end{cases} \qquad \begin{matrix} 2l_1+l_2 \\ -2a+2b-4c=0 \\ 2a+0-c=2 \end{matrix} \quad \Big| \quad l_1+l_3$$

$$\begin{cases} -a+b-2c=0 \\ 2b-5c=2 \\ 3b-2c=3 \end{cases} \qquad \begin{matrix} -3l_2+2l_3 \\ -6b+15c=-6 \\ 6b-4c=6 \end{matrix}$$

$$11c=0$$

$$\boxed{c=0} \qquad 2b-5\cdot 0=2$$

$$2b=2$$

$$\boxed{b=1}$$

$$-a+1=0$$

$$\boxed{a=1}$$

$$\boxed{B = u+v+0\cdot w}$$

3-) a) $\{(1,3)\}$

$(0,0) = a(1,3)$ $\quad \begin{cases} a = 0 \\ 3a = 0 \end{cases}$ $\quad S = \{(0)\}$

$// = (a, 3a)$

L.I

b) $\{(1,3), (2,6)\}$

$1 \times 3 \quad 6 - 6 = \boxed{0}$

$2 \quad 6$

L.D

c) $\{(2,-1), (3,5)\}$

$2 \times -1 \quad 10 - (-3) = \boxed{13}$

$3 \quad 5$

L.I

d) $\{(1,0), (-1,1), (3,5)\}$

$1 \quad 0$

$-1 \quad 1$

$3 \quad 5$

$(0,0) = a(1,0) + b(-1,1) + c(3,5)$

$(0,0) = (a,0) + (-b,b) + (3c, 5c)$

$(0,0) = (a - b + 3c, 0 + b + 5c)$

$\begin{cases} a - b + 3c = 0 \\ 0 + b + 5c = 0 \end{cases}$

$a + 5c + 3c = 0$

$\boxed{b = -5c}$ $\quad \boxed{a = -8c}$

$S = \{(-8c, -5c, c)\}$

L.D

e) $\{(1,-1,1),(-1,1,1)\}$

$(0,0,0) = a(1,-1,1) + b(-1,1,1)$

$(0,0,0) = (a,-a,a) + (-b,b,b)$

$(0,0,0) = (a-b, -a+b, a+b)$

$$\begin{cases} a-b=0 \\ -a+b=0 \\ a+b=0 \end{cases} \qquad \begin{matrix} l_2 + l_3 \\ -a+b=0 \\ a+b=0 \\ 2b=0 \end{matrix} \quad \boxed{b=0}$$

$a+0=0$

$\boxed{a=0}$

$S = \{(0)\}$

L.I

f) $(2,-1,0),(-1,3,0),(3,-4,0)$

3 0

2 -1 0

-1 3 0

3 -4 0

2 0

$0+0+0-(+0+0+0) = \boxed{0}$

L.D

g)

0 1

1 1 1

0 1 1

0 0 1

1 1

$1+0+0-(0+0+0)$

$\boxed{=1}$

L.I

h)

1 -5

1 -1 2

2 3 -3

1 4 -5

1 2

4

$-15+3+16-(6+10+(-12))$

$4-4 = \boxed{0}$

L.D