LIPTE - $\int u \cdot dv = u \cdot v - \int v \cdot dv$

1-) $\int x e^x \, dx = \boxed{e^x (x-1) + C}$

$u = x \rightarrow du = 1\,dx$

$dv = e^x dx \rightarrow v = e^x$

$x \cdot e^x = x \cdot e^x - \int e^x \cdot 1\,dx \rightarrow e^x$

$// = x \cdot e^x - e^x + C$

ou

$= e^x (x-1) + C$ //

2-) $\int x^2 \cos x \, dx = \boxed{x^2 \operatorname{sen} x + 2x \cos x - \operatorname{sen} x + C}$

$u = x^2 \rightarrow du = 2x\,dx$

$dv = \cos x \rightarrow v = \operatorname{sen} x$

$u = x \rightarrow du = 1\,dx$

$dv = \operatorname{sen} x \rightarrow v = -\cos(x)$

$x^2 \cos x = x^2 \cdot \operatorname{sen} x - \int \operatorname{sen} x \cdot 2x\,dx$

$u = x \rightarrow du = 1dx$

$dv = \operatorname{sen} x \rightarrow v = -\cos x$

$// = x^2 \cdot \operatorname{sen} x - 2\int x \cdot \operatorname{sen} x\,dx$

$// = x^2 \cdot \operatorname{sen} x - 2\int x \cdot (-\cos(x))dx - \int -\cos x \cdot 1\,dx$

$// = x^2 \cdot \operatorname{sen} x - 2\int -x\cos(x)dx - \int -\cos x\,dx$

$// = x^2 \cdot \operatorname{sen} x + 2x\cos(x) - \operatorname{sen} x$

$= x^2 \cdot \operatorname{sen} x + 2x \cos(x) - \operatorname{sen} x + C$ //

3-) $\int x \ln(x)\,dx = \dfrac{x^2 \ln(x)}{2} - \dfrac{x^2}{4} + C$

$u = \ln(x) \to Du = \dfrac{1}{x}\,Dx$

$Dv = x \to v = \dfrac{x^2}{2}$

$\int x \cdot \ln(x) = \ln(x) \cdot \dfrac{x^2}{2} - \int \dfrac{\cancel{x}^2}{2} \cdot \dfrac{1}{\cancel{x}}$

$= \dfrac{x^2 \ln(x)}{2} - \int \dfrac{x^2}{2x} = \dfrac{1}{2} \cdot \dfrac{x^{\cancel{2}}}{\cancel{x}}$

$= \dfrac{x^2 \ln(x)}{2} - \dfrac{1}{2} \int x\,dx$

$= \dfrac{x^2 \ln(x)}{2} - \dfrac{1}{2} \cdot \dfrac{x^2}{2}$

$= \dfrac{x^2 \ln(x)}{2} - \dfrac{x^2}{4}$ //

Exercícios

1- Determine os Seguintes Integrais

a) $\int x^2 \, dx$

b) $\int 4x^3 \, dx$

c) $\int 2x^3 + x^2 \, dx$

d) $\int 3x^2 - 5x \, dx$

e) $\int \sqrt{x} \, dx$

f) $\int \sqrt{(x+2)} \, dx$

a) $\int x^2 \, dx = \frac{x^3}{3} + C$

$\frac{x^{2+1}}{2+1} + C = \frac{x^3}{3} + C$

b) $\int 4x^3 \, dx = x^4 + C$

$\frac{\cancel{4}x^{3+1}}{\cancel{4}} = \cancel{x}^4 + C$

c) $\int 2x^3 + x^2 \, dx = \frac{x^4}{2} + \frac{x^3}{3} + C$

$\frac{2x^{3+1}}{3+1} + \frac{x^3}{3} + C = \frac{2x^4}{4} = \frac{x^4}{2}$

d) $\int 3x^2 - 5x \, dx = x^3 - \frac{5x^2}{2} + C$

$\frac{3x^3}{3} - \frac{5x^2}{1+1} + C$

e) $\int \sqrt{x}\, dx = \frac{2\sqrt{x^3}}{3} + C$

$\frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1} = \frac{x^{\frac{3}{2}}}{3/2} = \frac{\sqrt{x^3}}{\left(\frac{3}{2}\right)} = \frac{2\sqrt{x^3}}{3} + C$

f) $\int \sqrt{x+2} = \frac{2\sqrt{(x+2)^3}}{3} + C$

$\frac{(x+2)^{\frac{1}{2}+1}}{\frac{3}{2}} = \frac{(x+2)^{\frac{3}{2}}}{3/2} = \frac{2\sqrt{(x+2)^3}}{3} + C$

2-) a) $\int_1^2 x\, dx = \frac{3}{2}$

$\frac{x^2}{2} = \frac{2^2}{2} - \frac{1^2}{2} = \frac{4}{2} - \frac{1}{2} = \frac{3}{2}$

b) $\int_1^9 \sqrt{x}\, dx$

$\frac{2\sqrt{x^3}}{3} = \frac{2\sqrt{9^3}}{3} - \frac{2\sqrt{1^3}}{3} = \frac{54}{3} - \frac{2}{3} = \frac{52}{3}$

c) $\int_4^9 x^2 \sqrt{x}\, dx = \frac{4118}{7}$

$x^2 \cdot x^{\frac{1}{2}} = x^{\frac{3}{2}}\, dx$

$\left.\frac{x^{\frac{3}{2}+1}}{\frac{3}{2}+1}\right]_4^9 = \frac{x^{\frac{7}{2}}}{\frac{7}{2}} \quad \left.\frac{2\sqrt{x^7}}{7}\right]_4^9 \quad \frac{2\sqrt{9^7}}{7} - \frac{2\sqrt{4^7}}{7}$

$\frac{2 \cdot 3^7}{7} - \frac{2 \cdot 2^7}{7} = \frac{4374 - 256}{7} = \frac{4118}{7}$

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D) $\int_0^{\frac{\pi}{2}} \frac{\text{sen } x}{5}\, dx = \frac{1}{5}$

$\int_0^{\frac{\pi}{2}} \frac{1}{5} \cdot \text{sen } x\, dx \quad \text{sen } x = \frac{1}{5}(-\cos x)\Big]_0^{\frac{\pi}{2}}$

$-\cos\frac{\pi}{2} - (-\cos 0) = 1 \cdot \frac{1}{5} = \frac{1}{5}$ //

e) $\int_{-1}^{2} 6x^4\, dx = \frac{198}{5}$

$6\int x^4\, dx = \frac{6x^5}{5} = \frac{6\, 2^5}{5} - \frac{6 \cdot -1^5}{5} = \frac{192}{5} - \frac{-6}{5} = \frac{198}{5}$

f) $\int_1^2 (5x^{-4} - 8x^{-3})\, dx = \frac{-37}{24}$

$\frac{5x^{-3}}{-3} - \frac{8x^{-2}}{-2}\Big]_1^2 = \frac{5}{-3x^3} - \frac{8}{-2x^2} = -\frac{5}{3 \cdot 2^3} + \frac{4}{2^2}$

$= -\frac{5}{24} + \frac{4}{4} = \frac{-5+24}{24} = \frac{19}{24} - \frac{7}{3} = \frac{19-56}{24} = \frac{-37}{24}$

$= \frac{-5}{3 \cdot 1^3} + \frac{4}{1^2} = \frac{-5}{3} + \frac{4}{1} = \frac{-5+12}{3} \quad \frac{7}{3}$