1- Ache a Integral Indefinidas

a) $\int x\,(2-x^2)^3\,dx$

$(2-x^2)^3 = (2-x^2)(2-x^2)(2-x^2)$

$(4-2x^2-2x^2+x^4)\,(2-x^2)$

$(4-4x^2+x^4)\,(2-x^2)$

$8-4x^2-8x^2+4x^4+2x^4-x^6$

$8-12x^2+6x^4-x^6$

$\int x\,(8-12x^2+6x^4-x^6)\,dx$

$\int 8x-12x^3+6x^5-x^7\,dx$

$\int 8x\,dx - \int 12x^3\,dx + \int 6x^5\,dx - \int x^7\,dx$

$\frac{8x^2}{2} - \frac{12x^4}{4} + \frac{6x^6}{6} - \frac{x^8}{8} + C$

$4x^2 - 3x^4 + x^6 - \frac{x^8}{8} + C$ //

b) $\int e^{2x}\,dx$

$m = 2x$

$dm = 2\,dx$

$dx = \frac{dm}{2}$

$\frac{1}{2}\int e^m\,dm = \frac{1}{2}\cdot e^m = \frac{e^{2x}}{2} + C$ //

c) $\int \cos(8x)\,dx =$

$m = 8x$

$dm = 8\,dx$

$dx = \frac{dm}{8}$

$\frac{1}{8}\;\frac{\operatorname{sen}(m)}{1} = \frac{\operatorname{sen}(8x)}{8} + C$ //

2- a) $\int_{-1}^{2} x(1+x^3)dx = \int_{-1}^{2} x+x^4\,dx$

$\int_{-1}^{2} x\,dx + \int_{-1}^{2} x^4\,dx + \left.\frac{x^2}{2}\right]_{-1}^{2} + \left.\frac{x^5}{5}\right]_{-1}^{2} = \frac{2^2}{2} - \frac{(-1)^2}{2} +$

$\frac{2^5}{5} - \frac{(-1)^5}{5} = \frac{2}{1} - \frac{1}{2} + \frac{32}{5} + \frac{1}{5} = \frac{20-5+64+2}{10} = \frac{81}{10} = 8,1$ //

b) $\int_0^2 \operatorname{sen}(2x)dx$

$m = 2x$

$2.0 = 0$

$dm = 2dx$ $\quad 2.1 = 2$

$dx = \frac{dm}{2}$

$\frac{1}{2}\int_0^2 (m+1)^4 dm = \frac{1}{2}\left.\frac{(m+1)^3}{5}\right]_0^2 \left.\frac{(m+1)}{10}\right]_0^2$

$\frac{(2+1)^3}{10} - \frac{(0+1)^3}{10} = \frac{3^3}{10} - \frac{1^3}{10} = \frac{243}{10} - \frac{1}{10} = \frac{242}{10} = 24,2$

b) $\int_0^2 \operatorname{sen}(2x)dx = m = 2x \quad dx = \frac{dm}{2} \quad x=0 \to m = 2x = 0$

$dm = 2dx \quad x = 2x = m = 2x = 2.2x = 4x$

$\int_0^{4x} \operatorname{sen}(m)\frac{dm}{2} \to \frac{1}{2}\int_0^{2x} \operatorname{sen}(m)\,dm \to \frac{1}{2}\left[-\cos(m)\right]_0^{4x}$

$-\frac{1}{2}\left[\cos 4x - \cos 0\right] = -\frac{1}{2}(1-1) = -\frac{1}{2}.0 = 0$ //

c) $\int_0^4 (\sqrt{2x+1})\,dx = \int_0^8 \sqrt{m+1}\,\frac{dm}{2} = \frac{1}{2}\int_0^8 \sqrt{m+1}\,dm = \frac{2}{2.3}\left[\sqrt{m+1}^3\right.$

$\frac{1}{2}\int_0^8 (m+1)^{\frac{1}{2}}dm \to \frac{1}{2}\left[\frac{(m+1)^{3/2}}{3/2}\right]^2 =$

$\frac{1}{3}\left(\sqrt{(8+1)^3} - \sqrt{0+1}\right) = \frac{1}{3} - (27-1) = \frac{26}{3} = 8,6$ //

d) $\int_0^1 (1-2x)^3 dx$

$\frac{1}{2}\int_0^2 (1-m)^3 dm = \frac{1}{2} \cdot \frac{(1-m)^3}{4}\Big]_0^2$

$\frac{(1-m)^4}{8}\Big]_0^2 = \frac{(1-2)^4}{8} - \frac{(1-0)^4}{8} = \frac{(-1)^4}{8} = \frac{-1}{8}$

$\frac{1}{8} - \frac{1}{8} = 0$ //

f) $\int_0^{\pi/2} 4 \cdot \operatorname{Sen} \frac{x}{2} dx$

$m = \frac{x}{2} = \frac{1}{2}x$ | $\frac{x}{2} = \frac{0}{2} = 0$ $\frac{x}{2} = \frac{\frac{\pi}{2}}{2} = \frac{\pi}{4}$

$dm = \frac{dx}{2}$

$dx = 2 Dm$

$4\int_0^{\frac{\pi}{4}} \operatorname{Sen}(m)\, 2dm \to 4 \cdot 2 \int_0^{\frac{\pi}{4}} \operatorname{Sen}(m)\, dm = 8\left[-\cos(m)\right]_0^{\frac{\pi}{4}}$

$-8\left[\cos(m)\right]_0^{\frac{\pi}{4}} \to -8 \cdot \left(\cos\frac{\pi}{4} - \cos 0\right) = -8 \cdot \left(\frac{\sqrt{2}}{2} - 1\right) =$

$\frac{-8\sqrt{2}}{2} + 8 = -4\sqrt{2} + 8$ //

g) $\int_0^1 \frac{dx}{\sqrt{3x+1}} =$ $m = 3x$ $3x = 3 \cdot 0 = 0$

$Dm = 3dx$ $3x = 3 \cdot 1 = 3$

$dx = \frac{Dm}{3}$

$\int_0^1 \frac{1}{\sqrt{3x+1}} dx = \int_0^1 \frac{1}{(3x+1)^{\frac{1}{2}}} dx \to \int_0^1 (3x+1)^{-\frac{1}{2}} dx \to \int_0^3 (m+1)^{-\frac{1}{2}} \frac{Dm}{3} =$

$\frac{1}{3}\int_0^1 (m+1)^{\frac{1}{2}} dm \to \frac{1}{3}\left[\frac{(m+1)^{\frac{1}{2}}}{2}\right]_0^3 = \frac{2}{3}\left[\sqrt{m+1}\right]_0^3 \to \frac{2}{3}\left[\sqrt{3+1} - \sqrt{0+1}\right]$

$\frac{2}{3}\left[\sqrt{4} - \sqrt{1}\right] = \frac{2}{3}(2-1) \cdot \frac{2}{3} \cdot 1 = \frac{2}{3} = 0{,}666$ ✓

3-) a) $\int x \operatorname{sen}(5x)\,dx =$

$x \cdot \left(\frac{-\cos 5x}{5}\right) - \int \frac{-\cos(5x)}{5}\,dx \;\; -x\,\frac{\cos}{5}(5x) + \frac{1}{5}$

$\int \cos(5x)\,dx \rightarrow -x\,\frac{\cos(5x)}{5} + \frac{1}{5}\,\frac{\operatorname{sen}(5x)}{5}$

$-x\,\frac{(\cos(5x))}{5} + \frac{\operatorname{sen}(5x)}{25} + c$ //

b) $\int x e^{3x}\,dx =$

$\int e^{2x}\,dx = \frac{e^{2x}}{2} \;/\; \int e^{3x}\,dx = \frac{e^{3x}}{3}$

$e^{4x}\,dx = \frac{e^{4x}}{4}$

$\int e^{3x}\,dx = \int e^{u}\frac{du}{3} = \frac{1}{3} = \int e^{u}\,du = \frac{1}{3}e^{u} = \frac{1}{2}e^{2x} = \frac{e^{2x}}{3}$

$\int \underset{u}{x}\, e^{3x}\,\underset{dv}{dx} = x \cdot \frac{e^{3x}}{3} - \int \frac{e^{3x}}{3}\,dx \rightarrow \frac{xe^{3x}}{3} \cdot \frac{-1}{3}$

$\int e^{3x}\,dx = \frac{xe^{3x}}{3} - \frac{1}{3} \cdot \frac{e^{3x}}{3} \rightarrow \frac{xe^{3x}}{3} - \frac{e^{3x}}{9}$

$\frac{e^{3x}}{3}\left(x - \frac{1}{3}\right) + c$ //

c) $\int \ln(2x)\,dx =$

$\ln(2x) \cdot x - \int x \frac{1}{x}\,dx = x\ln(2x) - \int dx$

$x \ln(2x) - x \to x\,(\ln(2x) - 1) + C$

b) $x^2 e^{-x}\,dx$

$x^2 \left(\frac{e^{-x}}{-1}\right) - \int \frac{e^{-x}}{-1}\,dx = x^2 - e^{-x} + \int e^{-x}\,dx = x^2 e^{-x} + e^{-x}$

$e^{-x}(x^2+1) + C$

e) $\int x^2 \cos(3x)\,dx$

$-x^2 \left(\frac{-\operatorname{sen}(3x)}{3}\right) - \int \frac{-\operatorname{sen}(3x)}{3}\,dx = -\frac{x^2 \operatorname{sen}(3x)}{3}$

$\frac{1}{3} \int \frac{\operatorname{sen}(3x)}{3}\,dx + \frac{x^2 \operatorname{sen}(3x)}{3} + \frac{1}{3} \frac{\cos(3x)}{3}$

$-\frac{x \operatorname{sen}(3x)}{3} + \frac{\cos(3x)}{9} + C$