Lista 3

$u = (x, y)$ $v = (a, b)$

1-) a) $F: \mathbb{R}^2 \to \mathbb{R}^2 : f(x,y) = (x+y, x+2y)$

i) $T(0,0) = T(0+0, 0+2\cdot 0) = (0,0)$ ✓

ii) $T(u+v) = T(u) + T(v)$

x, y

$T(u+v) = T(x+a, y+b)$

$T((x+a)+(y+b), x+a+2(y+b))$

$T(x+y+a+b, 2y+2b+x+a)$

$T(x,y) = T(x+y, x+2y)$

$T(a,b) = T(a+b, a+2b)$ +

$T(x+y+a+b, x+2y+a+2b)$

iii) $T(\alpha u) = \alpha T(u)$

$T(\alpha u) = T(\alpha(x,y))$

$T(\alpha x, \alpha y)$

$T(\alpha x + \alpha y, \alpha x + 2(\alpha y))$

$\alpha T(x,y)$

$\alpha T(x+y, x+2y)$

$T(\alpha x + \alpha y, \alpha x, 2\alpha y)$

T é T.L

//

$u = (x, y)$ $v = (a, b)$

b) $f : \mathbb{R}^2 \to \mathbb{R}^3 : f(x, y) = (x+y, x-y, x+2y)$

i) $\vec{0}$ $T(0,0) = (0+0, 0-0, 0+2 \cdot 0) = (0,0,0)$ OK

II) $T(u+v) = T(u) + T(v)$ OK

$T(x+a, y+b)$

$T(x+a+y+b, x+a-(y+b), x+a+2(y+b))$

$T(x+a+y+b, x+a-y-B, x+a+2y+2b)$

$T(x, y) = T(x+y, x-y, x+2y)$

$T(a, b) = T(a+b, a-b, a+2b)$ †

$T(x+y+a+b, x-y+a-b, x+2y+a+2b)$

III) $T(\alpha u) = \alpha T(u)$

$T(\alpha x, \alpha y)$

$T(\alpha x+\alpha y, \alpha x-\alpha y, \alpha x+2\alpha y)$

$T(x, y) = T(x+y, x-y, x+2y)$

$\alpha(x+y, x-y, x+2y)$

$T(\alpha x+\alpha y, \alpha x-\alpha y, \alpha x+2\alpha y)$

OK

T de B é uma T.L.

$u = (x, y, z)$ $v = (a, b, c)$

c) $f: \mathbb{R}^3 \to \mathbb{R}^2 : f(x, y, z) = (x + y - z, y + z)$

i) $\vec{0}: T(0,0,0) = (0+0-0, 0+0) = (0,0)$

ii) $T(u+v) = T(u) + T(v)$

$u + v = (x+a, y+b, z+c)$

$T((x+a)+(y+b)-(z+c), (y+b)+(z+c))$

$T(x+y+a+b-c-z, y+z+b+c)$

$T_2(x, y, z) = (x+y-z, y+z)$
$T_2(a, b, c) = (a+b-c, b+c)$ +
$T = (x+y+a+b-c-z, y+z+b+c)$

Ok

iii) $T(\alpha u) = \alpha T(u)$

$T(\alpha x, \alpha y, \alpha z)$
$T(\alpha x + \alpha y - \alpha z, \alpha y + \alpha z)$

$\alpha T(x, y, z)$

$\alpha(x+y-z, y+z)$
$T(\alpha x + \alpha y - \alpha z, \alpha y + \alpha z)$

T de C é uma T.L.

$u = (x, y, z)$ $v(a, b, c)$

d) $f: \mathbb{R}^3 \to \mathbb{R}^3 : f(x, y, z) = (x, xy, z)$

i) $\vec{0}$ $T(0,0,0) = (0, 0.0, 0) = (0,0,0)$

ii) $T(u+v) = T(u) + T(v)$

$u + v = (x+a, y+b, z+c)$

$T(u+v) = (x+a, (x+a)(y+b), z+c)$

$T(x+a, xy + xb + ay + ab, z+c)$

$T(u)\ (x, xy, z) +$
$T(v)\ (a, ab, c)$

$T(x+a, xy+ab, z+c)$

Falha prova ii
Não é uma T.L

e) $f: \mathbb{R}^2 \to \mathbb{R}^2 : f(x, y) = (x + 2y, y)$

i) $\vec{0}$ $T(0,0) = (0+2.0, 0) = T(0,0)$

ii) $T(u+v) = T(u) + T(v)$

$u + v = (x, y) + (a, b)$

$T(x+a, y+b)$

$T(x+a+2(y+b), y+b)$

$T(x+a+2y+2b, y+b)$

$T(x, y) = (x+2y, y)$
$T(a, b) = (a+2b, b)$

$T(x+a+2y+2b, y+b)$

iii) $T(\alpha u) = \alpha T(u)$

$T(\alpha x, \alpha y)$

$T(\alpha x - 2\alpha y, \alpha y)$

$\alpha T(u) = (x+2y, y)$

$T(\alpha x + 2\alpha y, \alpha y)$

$\therefore$ T.L

$u = (x, y)$ $v = (a, b)$

f) $f: \mathbb{R}^2 \to \mathbb{R}^2 : f(x, y) = (x+2, y)$

i) $\vec{0}$ $T(0,0) = (0+2, 0) = (2, 0)$

não é um T.L

g) $f: \mathbb{R}^2 \to \mathbb{R}^2 : f(x, y) = (x, y+2x)$

i) $\vec{0}$ $T(0,0) = (0, 0+2\cdot 0) = (0, 0)$

ii) $T(u+v) = T(u) + T(v)$

$T(x+a, y+b)$	$T(u) = (x, y+2x)$
$T(x+a, y+b+2(x+a))$	$T(v) = (a, b+2a)$ +
$T(x+a, y+b+2x+2a)$	$T(x+a, y+2x+b+2a)$

III) $T(\alpha u) = \alpha T(u)$

$T(\alpha x, \alpha y)$	$T: \alpha(x, y+2x)$
$T(\alpha x, \alpha y + 2\alpha x)$	$T(\alpha x, \alpha y + 2\alpha x)$

Cá T.L //

2) a) $(x+y, x+2y)$

$T(1,0) = (1+0, 1+2\cdot 0) = (1,1)$
$T(0,1) = (0+1, 0+2\cdot 1) = (1,2)$

$\begin{bmatrix} (1,1) \\ (1,2) \end{bmatrix}$

b) $(x+y, x-y, x+2y)$

$T(1,0) = (1+0, 1-0, 1+2\cdot 0) = (1,1,1)$
$T(0,1) = (0+1, 0-1, 0+2\cdot 1) = (1,-1,2)$

$\begin{bmatrix} (1,1,1) \\ (1,-1,2) \end{bmatrix}$

$\begin{bmatrix} 1 & 1 \\ 1 & -1 \\ 1 & 2 \end{bmatrix}$

c) $(x+y-z, y+z)$

$T(1,0,0) = (1+0-0, 0+0)$
$T(0,1,0) = (0+1-0, 0+0)$
$T(0,0,1) = (0+0-1, 0+1)$

$\begin{bmatrix} (1,0) \\ (1,1) \\ (-1,1) \end{bmatrix}^t \begin{bmatrix} 1 & 1 & -1 \\ 0 & 1 & 1 \end{bmatrix}$

e) $(x+2y, y)$

$T(1,0) = (1+2\cdot 0, 0) = (1,0)$
$T(0,1) = (0+2\cdot 1, 1) = (2,1)$

$\begin{bmatrix} (1,0) \\ (2,1) \end{bmatrix}^t$ [illegible] $\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$

f) $(x, y+2x)$

$T(1,0) = (1, 0+2\cdot 1) = (1,2)$
$T(0,1) = (0, 1+2\cdot 0) = (0,1)$

$\begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}^t$