

$$1. a) \begin{cases} x + 2y + 4z = 1 \\ 3x - y + 2z = -2 \\ -x + 3y - 2z = 0 \end{cases} \begin{cases} x + 2y + 4z = 1 \\ -7y - 10z = -5 \\ -36z = -18 \end{cases}$$

$$l_2 = \begin{cases} -x - 6y - 12z = -3 \\ 3x - y + 2z = -2 \\ -7y - 10z = -5 \end{cases} \quad \begin{aligned} z &= \frac{-18}{-36} = \frac{1}{2} \\ -7y - 10 \cdot \frac{1}{2} &= -5 \\ -5 &= -5 \end{aligned}$$

$$l_3 = \begin{cases} x + 2y + 4z = 1 \\ -x + 3y - 2z = 0 \\ 5y + 2z = 1 \end{cases} \quad \begin{aligned} 7y &= 0 \\ y &= 0 \end{aligned}$$

$$l_3 = \begin{cases} -35y - 50z = -25 \\ 35y + 14z = 7 \\ -36z = -18 \end{cases} \quad \begin{aligned} x + 2 \cdot 0 + 4 \cdot \frac{1}{2} &= 1 \\ x + 2 &= 1 \\ x &= -1 \end{aligned}$$

$$S = \{(-1, 0, \frac{1}{2})\}$$

$$b) \begin{cases} x + y = -1 \\ 2x - y - 3z = -2 \\ 5x + 2y - z = -5 \end{cases} \begin{cases} x + y^0 = -1 \\ -3y^0 - 3z^0 = 0 \\ 2z = 0 \end{cases}$$

$$l_2 = \begin{array}{l} x + y = -1 \quad (-2) \\ + \quad -2x + 2y + 0z = 2 \\ \hline 2x - y - 3z = -2 \\ -3y - 3z = 0 \end{array} \quad \begin{array}{l} l_3 = -8x - 5y = 5 \\ 8x + 2y - z = -5 \\ \hline -3y - z = 0 \end{array}$$

$$y = 0 \quad z = 0$$

$$x = -1$$

$$S = \{(-1, 0, 0)\}$$

$$l_3 = \begin{array}{l} 3y + 3z = 0 \\ -3y - z = 0 \\ \hline 2z = 0 \\ z = 0 \end{array}$$



$$C) \begin{cases} 3x + y + 2z = -3 \\ 2x + 2y - z = 1 \\ 4x + 3y + 3z = 3 \end{cases} \rightarrow \begin{cases} 3x + y + 2z = -3 \\ -4y + 7z = 9 \end{cases}$$

$$l_2 = \begin{array}{r} 6x + 2y + 4z = -6 \\ -6x - 6y + 3z = -3 \\ \hline -4y + 7z = 9 \end{array}$$

$$l_3 = \begin{array}{r} 12x + 4y + 8z = 12 \\ -12x + (-9y) + (-9z) = -9 \\ \hline -5y - z = -21 \end{array}$$

$$l_3 = \begin{array}{r} -20y + 35z = -45 \\ +20y + 4z = +84 \\ \hline +39z = 39 \end{array}$$

$$z = 1$$

$$y = -\frac{9}{5} - \frac{7}{5} = -4$$

$$x = -(4+2) - \frac{3}{3} = -3$$

$$S = \{(-3, 4, 1)\}$$

$$D) \begin{cases} x + 2y + z + t = 0 \\ x + 3y - z + 2t = 0 \end{cases}$$

$$l_2 = \begin{array}{r} x - 2y - z + t = 0 \\ x + 3y - z + 2t = 0 \\ \hline y - 2t = 0 \end{array}$$

$$\begin{array}{l} x + 2y + t + k_2 = 0 \\ y - 2t + k_2 = 0 \end{array}$$

$$y = +2t - k_2$$

$$x + 2(+2t - k_2) + t + k_2 = 0$$

$$x + 4t - 2k_2 + t + k_2 = 0 \Rightarrow x + 5t - k_2 = 0$$

$$\boxed{x = -5K + K_2}$$

$$S = \{(-5K + K_2, 2K - K_2, K, K_2)\}$$

$$c) \begin{cases} -x + 2y - z = 3 \\ 2x + y + 3z = 5 \\ 3x + 4y + 5z = 13 \end{cases} \quad \begin{cases} -x + 2y - z = 3 \\ 5y + z = 11 \\ 0 + 0 = 0 \end{cases}$$

$$\begin{array}{l|l} l_2: -2x + 4y - 2z = 6 & l_3: -3x + 6y - 3z = 9 \\ \underline{2x + y + 3z = 5} & \underline{3x + 4y + 5z = 13} \\ +5y + z = 11 & \underline{10y + 2z = 22} \\ & \underline{5y + z = 11} \end{array}$$

$$z = K \quad 5y + K = 11$$

$$y = \frac{11 - K}{5}$$

$$-x + 2\left(\frac{11 - K}{5}\right) - K = 3 \Rightarrow -x + 22 - \frac{2K}{5} - K = 3$$

$$-x + \frac{22 - 2K - 5K}{5} = 3 \Rightarrow -x + 22 - 7K/5 = 3$$



$$-X = \frac{3}{1} - \frac{22-7K}{5}$$

$$-X = \frac{15+22-7K}{5} = -X = \frac{-7+7K}{5} \quad (-1)$$

$$X = \frac{7-7K}{5}$$

$$S = \left\{ \frac{7-7K}{5}, \frac{11-K}{5}, K \right\}$$