

Vektoren Maschinen Prüfung

$$1.) \quad u = (1, 2, 1) \quad \text{u} \quad v = (-1, 0, 2)$$

$$w = (5, 4, -4) = a(1, 2, 1) + b(-1, 0, 2)$$

$$(5, 4, -4) = (a, 2a, a) + (-b, 0, 2b)$$

$$(5, 4, -4) = (a-b, 2a, a+2b)$$

$$\begin{cases} a-b = 5 & a = \frac{5}{2} \quad \boxed{a = 2} \\ 2a + 0 = 4 \\ a + 2b = -4 \end{cases}$$

$2 + 2b = -4 \Rightarrow 2b = -6$
 $\boxed{b = -3}$

$$\boxed{w = 2u + (-3 \cdot v)}$$

$$2.) \quad A = \{(1, -1, 2), (2, 3, -3), (1, 4, -5)\}$$

$$\begin{array}{ccc} 1 & -1 & 2 \\ 2 & 3 & -3 \\ 1 & 4 & -5 \\ 1 & & 2 \end{array}$$

$$\begin{array}{ccc} & 6 & -2 \\ (-15 + 3 + 16) - (6 + 10 - 12) \\ 4 & -4 & = \boxed{0} \end{array}$$

L.D)

$$u = (x, y, z) \quad v = (a, b, c)$$

3-) $\mathbb{R}^3 \rightarrow \mathbb{R}^2 = \langle \text{coordinates} \rangle$

$$(X, y, z) = (x+y, x \cdot z)$$

i $\vec{0}$ $T(0,0,0) = (0+0, 0 \cdot 0) = (0,0)$

ii) $T(u+v) = T(u) + T(v)$
 $u+v = (x+a, y+b, z+c)$

$$T(u+v) = ((x+a)+(y+b), (x+a) \cdot (z+c))$$

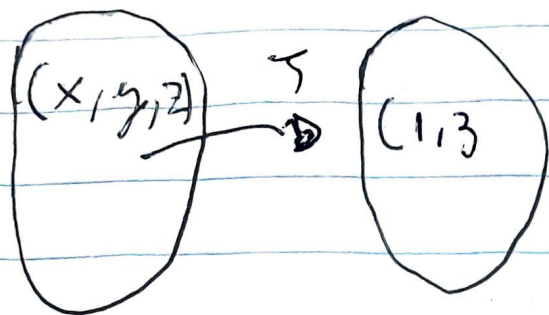
$$T(x+a+y+b, xz+xz+az+ac)$$

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hier e' T.L.

$$\begin{array}{|l} T(u) = (x+y, xz) \\ T(v) = (a+b, ac) \\ T(x+y+a+b, xz+az+ac) \end{array}$$

4-) $T: \mathbb{R}^3 \rightarrow \mathbb{R}^2: T(x, y, z) = (x+z, y+z)$
 $v = (1, 3)$



Suppose $v = (x, y, z)$
 $T(x+z, y+z) = (1, 3)$

$$\begin{cases} x+z = 1 \\ y+z = 3 \end{cases} \quad \begin{cases} x = -z+1 \\ z = 3-y \end{cases} \quad \begin{cases} z = 1-x \\ y = 3-z \end{cases}$$

$$(x, y, z) = (1-z, 3-z, z)$$

$$z=1 \Rightarrow (1-1, 3-1, 1) = (0, 2, 1)$$

$$T(0, 2, 1) = (1, 3)$$

$$v = (1-z, 3-z, z)$$

5-) Determine T.L. $\mathbb{R}^2 \rightarrow \mathbb{R}^3$. $T(1,0) = (2, -1, 0)$
 $T(0,1) = (0, 0, 1)$
 $T_1(1,0) = (2, -1, 0)$
 $T_2(0,1) = (0, 0, 1)$

$$T_1(x,y) = \cancel{T}(x,0) + \cancel{T}(0,y)$$

$$T(x,y) = xT(1,0) + yT(0,1)$$

$$T(x,y) = (2x, -x, 0) + (0, 0, y)$$

$$\boxed{T(x,y) = (2x, -x, y)}$$

$$T(1,0) = (2 \cdot 1, -1, 0) = (2, -1, 0)$$

$$T(0,1) = (2 \cdot 0, -0, 1) = (0, 0, 1)$$

6-) Determine a Imagem do vetor $(4, 2)$ sobre o qual se aplica uma rotação $z = 90^\circ$ no sentido anti-horário. $T(x, y) = (x \cos(z) - y \sin(z), y \cos(z) + x \sin(z))$

$$T(4, 2) = (4 \cdot 0 - 2 \cdot 1, 2 \cdot 0 + 4 \cdot 1)$$

$$T(4, 2) = (0 - 2, 2 \cdot 0 + 4 \cdot 1) = (-2, 4) //$$

$$\begin{vmatrix} 4 \\ 2 \end{vmatrix} \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} -2 \\ 4 \end{pmatrix} //$$

$$7-) A = (1, 2), B = (3, 2), C = (2, 3)$$

a) Reflexão no eixo x ($T(x, y) = (x, -y)$)

$$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{pmatrix} 1 & 3 & 2 \\ 2 & 2 & 3 \end{pmatrix} = \begin{pmatrix} 1 & 3 & 2 \\ -2 & -2 & -3 \end{pmatrix}$$

$$A_1 = (1, -2)$$

$$B_1 = (3, -2)$$

$$C_1 = (2, -3)$$

b) Dilatação de fator $\alpha = 2$ ($T(x, y) = (2x, 2y)$)

$$\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} \begin{pmatrix} 1 & 3 & 2 \\ -2 & -2 & -3 \end{pmatrix} = \begin{pmatrix} 2 & 6 & 4 \\ -4 & -4 & -6 \end{pmatrix}$$

$$A_2 = (2, -4)$$

$$B_2 = (6, -4)$$

$$C_2 = (4, -6)$$

