

$$1) \begin{vmatrix} 2x & 3y \\ 3 & 4 \end{vmatrix} = \begin{vmatrix} x+1 & 2y \\ 3 & y+4 \end{vmatrix}$$

$$2x \stackrel{A}{=} x+1$$

$$2x - x = 1 \Rightarrow \boxed{x = 1}$$

$$3) y = 2y$$

$$\boxed{y = 0}$$

$$2) \begin{vmatrix} 1 & 5 & 7 \\ 3 & 9 & 11 \end{vmatrix} + \begin{vmatrix} 2 & 4 & 6 \\ 8 & 10 & 12 \end{vmatrix} + \begin{vmatrix} 0 & -1 & -5 \\ 1 & 4 & 7 \end{vmatrix}$$

$$= \begin{vmatrix} 3 & 8 & 8 \\ 12 & 23 & 30 \end{vmatrix} //$$

$$b) \begin{vmatrix} 1 & 5 & 7 \\ 3 & 9 & 11 \end{vmatrix} - \begin{vmatrix} 2 & 4 & 6 \\ 8 & 10 & 12 \end{vmatrix} + \begin{vmatrix} 0 & -1 & -5 \\ 1 & 4 & 7 \end{vmatrix}$$

$$= \begin{vmatrix} -1 & 0 & -4 \\ -4 & 3 & 6 \end{vmatrix} //$$

$$c) \begin{vmatrix} 1 & 5 & 7 \\ 3 & 9 & 11 \end{vmatrix} = \begin{vmatrix} 2 & 4 & 6 \\ 8 & 10 & 12 \end{vmatrix} - \begin{vmatrix} 0 & -1 & -5 \\ 1 & 4 & 7 \end{vmatrix}$$

$$= \begin{vmatrix} -1 & 2 & 6 \\ -6 & -5 & -8 \end{vmatrix}$$

$$d) - \begin{vmatrix} 1 & 5 & 7 \\ 3 & 9 & 11 \end{vmatrix} + \begin{vmatrix} 2 & 4 & 6 \\ 8 & 10 & 12 \end{vmatrix} - \begin{vmatrix} 0 & -1 & -5 \\ 1 & 4 & 7 \end{vmatrix}$$

$$= \begin{vmatrix} 1 & 0 & 4 \\ 4 & -3 & -6 \end{vmatrix}$$

$$3.) a) \begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix} \cdot \begin{vmatrix} 4 & 7 \\ 2 & 3 \end{vmatrix} = \begin{vmatrix} 0+4 & 0+7 \\ 4+0 & 7+0 \end{vmatrix}$$

$$= \begin{pmatrix} 0+2 & 0+3 \\ 4+0 & 7+0 \end{pmatrix} = \begin{vmatrix} 2 & 3 \\ 4 & 7 \end{vmatrix}$$

$$b) \begin{vmatrix} 1 \\ 2 \\ 3 \end{vmatrix} \cdot \begin{vmatrix} 3 & 1 & 1 & 2 \end{vmatrix}$$

$3 \times 1 \quad 3 \times 4$

$$= \begin{vmatrix} 3 & 1 & 1 & 2 \\ 6 & 2 & 2 & 4 \\ 9 & 3 & 3 & 6 \end{vmatrix} \quad 3 \times 4$$

$$c) \begin{vmatrix} 1 & 5 & 2 \\ -1 & 4 & 7 \end{vmatrix} \cdot \begin{vmatrix} 1 & -1 \\ 2 & 3 \\ 3 & 0 \end{vmatrix} = \begin{vmatrix} 1+10+(-6) & -1+15+0 \\ -1+8-21 & 1+12+0 \end{vmatrix}$$

$2 \times 3 \quad 3 \times 2$

$$= \begin{vmatrix} 5 & 14 \\ -14 & 13 \end{vmatrix}$$

$$d) \begin{vmatrix} 1 & -1 & 5 & 0 \\ 2 & 3 & 7 & 1 \end{vmatrix} \cdot \begin{vmatrix} 1 & 1 \\ 2 & 1 \\ 3 & 1 \\ 1 & 1 \end{vmatrix} = \begin{vmatrix} 14 & 5 \\ 30 & 13 \end{vmatrix}$$

$$e) \begin{array}{c|cc|c|cc} 1 & -1 & & 1 & 2 & 3 \\ 2 & 2 & & 4 & -5 & 1 \\ 3 & 4 & & & & \end{array} = \begin{array}{c|cc|c|cc} 1-4 & 2+5 & & 3-1 \\ 2+8 & 4-10 & & 6+2 \\ 3+16 & 6-20 & & 9+4 \end{array}$$

$$= \begin{array}{c|cc|c|cc} -3 & 7 & & 2 \\ 10 & -6 & & 8 \\ 19 & -14 & & 13 \end{array}$$

$$f) \begin{array}{c|cc|c|cc} 0 & 1 & 1 & 1 & 4 & 7 \\ 2 & 2 & 0 & 0 & 0 & 1 \\ 0 & 3 & 4 & 1 & 2 & 0 \end{array} = \begin{array}{c|cc|c|cc} 0+0+1 & 0+0+2 & & 0+1+0 \\ 2+0+0 & 8+0+0 & & 14+2+0 \\ 0+0+4 & 0+0+8 & & 0+3+0 \end{array}$$

$$= \begin{array}{c|cc|c|cc} 1 & 2 & & 1 \\ 2 & 8 & & 16 \\ 4 & 8 & & 3 \end{array}$$

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$$4) \text{ c) } \begin{cases} x + 2y + 4z = 1 \\ 3x + 2z = -2 \\ -x + 3y - 2z = 0 \end{cases} \quad \begin{cases} x + 2y + 4z = 1 \\ 7y + 10z = 5 \\ 36z = 18 \end{cases}$$

$$l_2 = -3l_1 + l_2$$

$$+ 23x - 6y - 12z = -3$$

$$3x - y + 2z = -2$$

$$-7y - 10z = -5 \quad (41)$$

$$l_3 = l_1 + l_3$$

$$5y + 7z = 1$$

$$l_3 = 5l_2 + -7l_3$$

$$+ 35y + 50z = 25$$

$$-35y - 11z = -7$$

$$z = \frac{18}{36} = \frac{1}{2}$$

$$7y + 10 \cdot \frac{1}{2} = 5 - 5$$

$$7y = 0 \Rightarrow y = 0$$

$$+ 36z = 18$$

$$x + 2 \cdot 0 + 4 \cdot \frac{1}{2} = 1$$

$$x + 2 = 1 \Rightarrow x = -1$$

$$S = (-1, 0, \frac{1}{2}) //$$

$$b) \begin{cases} x + y = -1 \\ 2x - y - 3z = -2 \\ 5x + 2y - z = -5 \end{cases}$$

$$\begin{cases} x + y + 0z = -1 \\ -3y - 3z = 0 \end{cases}$$

$$l_2 = -2l_1 + l_2$$

$$-2x - 2y = 2$$

$$2x - y - 3z = -2$$

$$-3y - 3z = 0$$

$$l_3 = -5l_1 + l_3$$

$$-5x - 5y = 5$$

$$5x + 2y - z = -5$$

$$-3y - z = 0$$

$$l_3 = -1l_2 + l_3$$

$$3y + 3z = 0$$

$$-3y - z = 0$$

$$z = 0$$

$$y = 3y - 3z = 0$$

$$y = 0/3 = 0$$

$$x + 0 + 0 = -1$$

$$S = (-1, 0, 0)$$

$$c) \begin{cases} 3x + y + 2z = -3 \\ 2x + 2y - z = 1 \\ 4x + 3y + 3z = 3 \end{cases} \rightarrow \begin{cases} 3x + y + 2z = -3 \\ 4y - 7z = 9 \end{cases}$$

$$\begin{array}{l} l_2 \leftarrow 2l_1 + 3l_2 \\ + \quad \begin{array}{l} -6x - 2y - 4z = 6 \\ 6x + 6y - 3z = 3 \end{array} \end{array} \quad \left| \quad \begin{array}{l} l_3 \leftarrow -4l_1 + 3l_3 \\ -12x - 4y - 8z = 12 \\ 12x + 9y + 9z = 9 \end{array} \right.$$

$$\begin{array}{l} \boxed{4y - 7z = 9} \end{array} \quad \begin{array}{l} 5y + z = 21 \end{array}$$

$$\begin{array}{l} l_3 \leftarrow -5l_2 + 4l_3 \\ -20y + 35z = 45 \\ 20x + 4z = 84 \\ 39z = 39 \\ \boxed{z = 1} \end{array} \quad \left| \quad \begin{array}{l} 4y - 7 \cdot 1 = 9 \\ 4y - 7 = 9 \\ 4y = 16 \\ \boxed{y = 4} \end{array} \right.$$

$$\begin{array}{l} 3x + 4 + 2 = -3 \\ 3x = -9 \\ \boxed{x = -3} \end{array}$$