

# HETEROCEDASTICIDADE CONDICIONAL

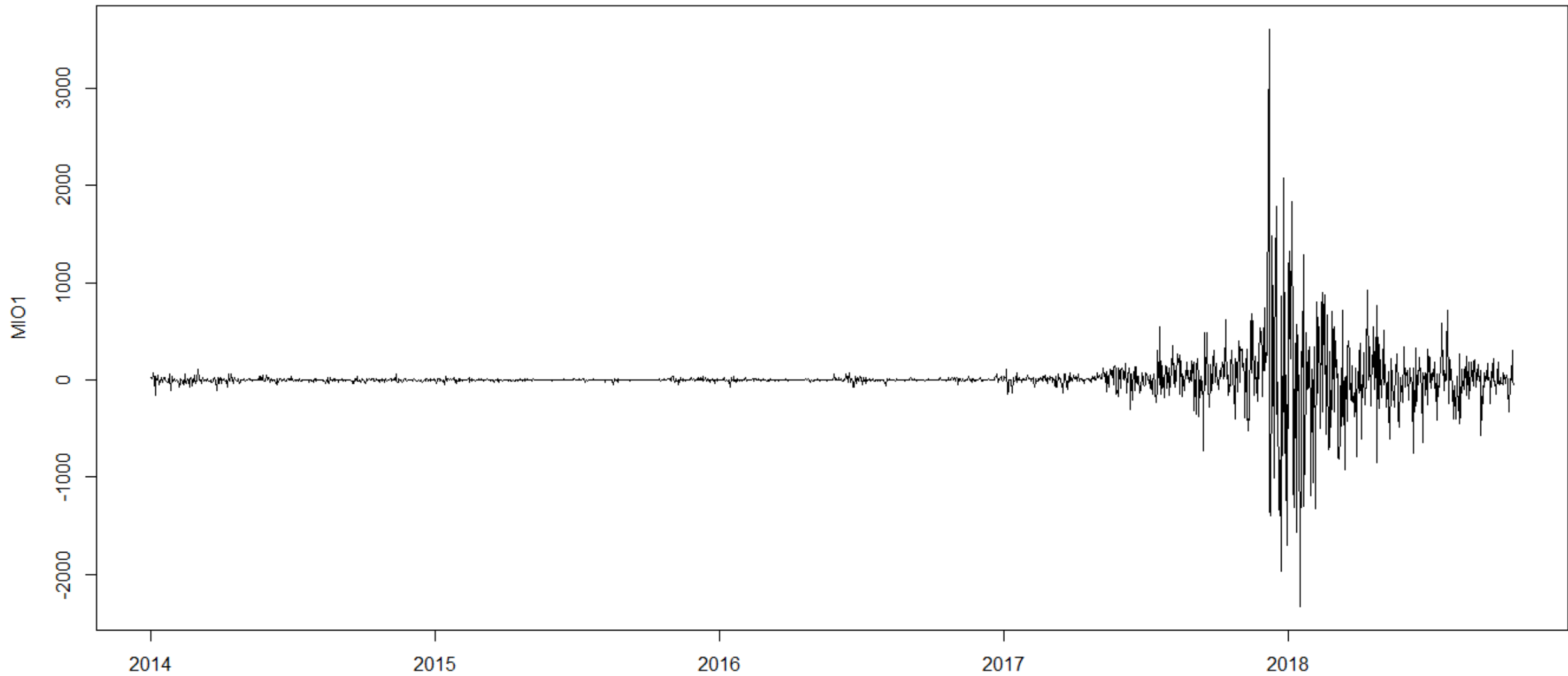
Técnicas de modelagem temporal de variâncias e covariâncias.

Necessidade de se melhorar o modelo CAPM de Sharpe e Lintner que não funcionavam bem empiricamente, Pois as séries financeiras não apresentavam distribuição normal dada a elevada probabilidade de eventos extremos.

Para o Mercado Financeiro significa modelar a volatilidade e o risco.

Modelos ARCH(p)

Modelos GARCH(p,q)



O teste ARCH-LM serve para testar presença de heterocedasticidade no modelo.

Considere:

$$\hat{\varepsilon}_t = \beta_1 \hat{\varepsilon}_t^2 + \beta_2 \hat{\varepsilon}_{t-1}^2 + \dots + \beta_h \hat{\varepsilon}_{t-h}^2 + \mu_t$$

Teste:

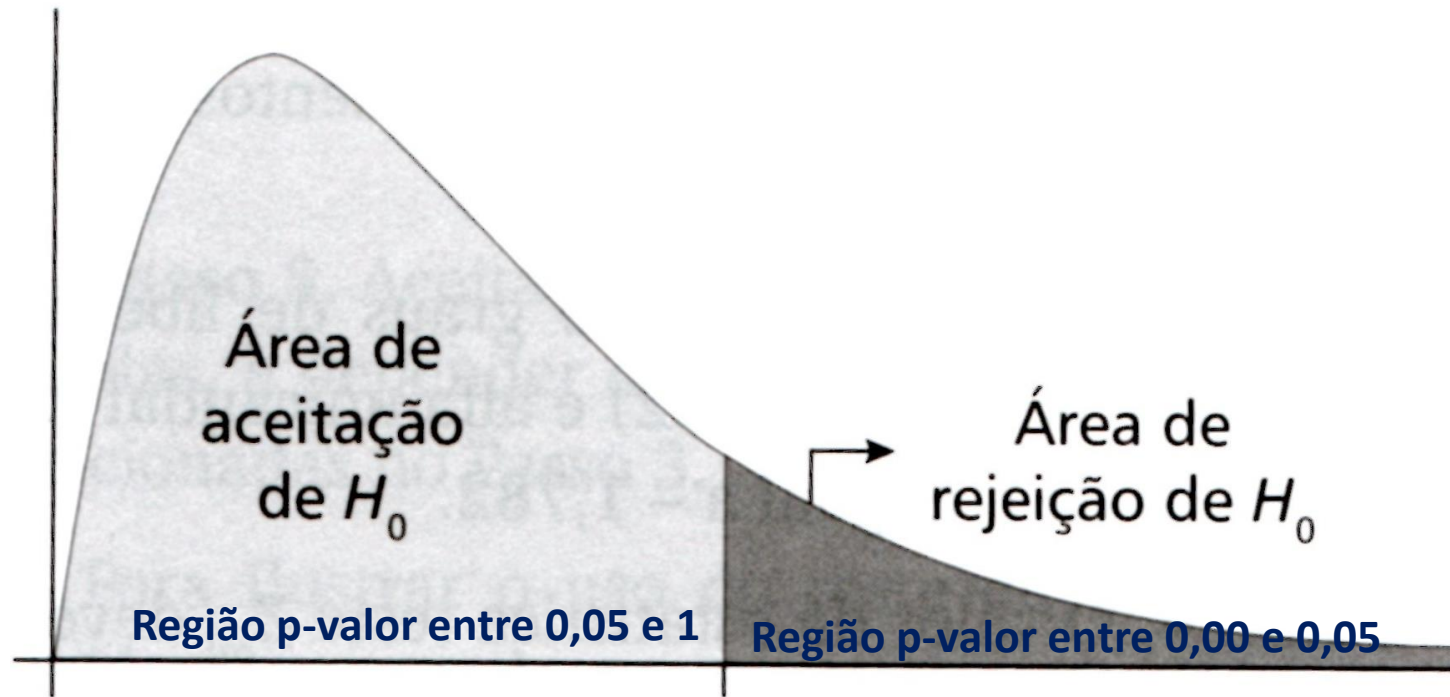
$$H_0: \beta_1 = \beta_2 = \dots = \beta_h = 0$$

$$H_1: \beta_1 \neq 0 \text{ ou } \beta_2 \neq 0 \dots \beta_h \neq 0$$

$$ARCH - LM_h = R^2 \xrightarrow{d} \chi^2_h$$

$H_0$ : não há heterocedasticidade

$H_1$ : há heterocedasticidade



Se o valor-p > 0,05 não rejeitaremos a hipótese nula: não há heterocedasticidade

Se o valor-p < 0,05 rejeitaremos a hipótese nula: há heterocedasticidade

## Modelo ARCH (p,q)

A variância segue um processo estocástico condicional ao seu erro anterior t-1, similar a um MA para variância.

$$\sigma^2 = w + \sum_{i=1}^q \alpha_i \varepsilon_{t-1}^2$$

## Modelo Heterocedasticidade Condicional Autorregressivo Generalizado - GARCH(q)

A variância segue um processo estocástico condicional ao seu erro anterior t-1, similar a um MA para variância.

$$\sigma^2 = w + \sum_{i=1}^q \alpha_i \varepsilon_{t-1}^2 + \sum_{i=1}^q \alpha_i \sigma_{t-1}^2$$

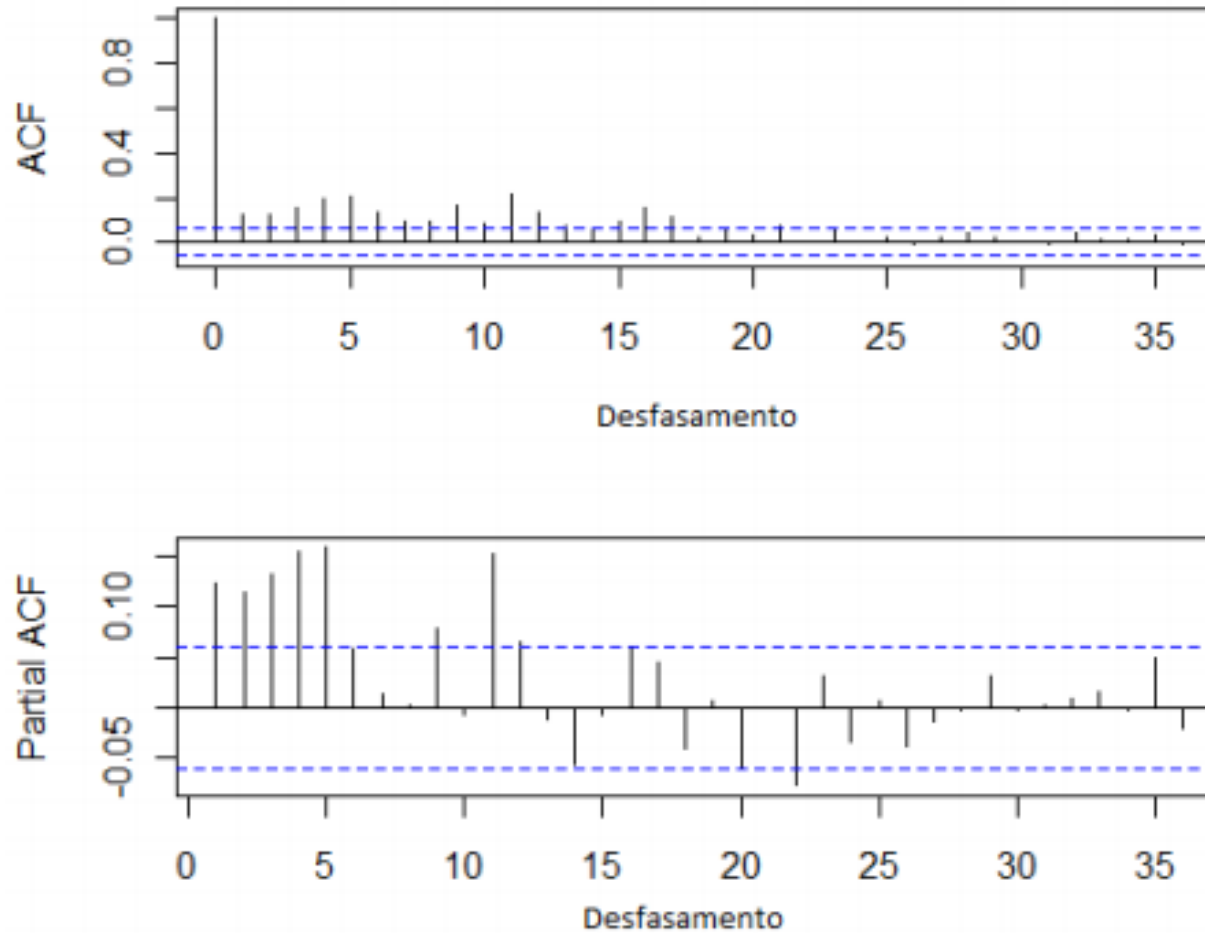
## Função de Autocorrelação –FAC e Função de Autocorrelação Parcial – FAC-P

O modelo GARCH(p,q) assemelha-se ao modelo tradicional ARMA(p,q).

Assim a FAC e a FAC-P devem sugerir se a série é heterocedástica, Da mesma maneira que dão ideia para as ordens p e q de um modelo ARMA.

A FAC nos sugere a ordem máxima da autorregressão do GARCH para o termo  $\varepsilon_{t-1}^2$

A FACP nos sugere a ordem para o termo  $\sigma_{t-1}^2$





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# Early-warning signals for critical transitions

Marten Scheffer<sup>1</sup>, Jordi Bascompte<sup>2</sup>, William A. Brock<sup>3</sup>, Victor Brovkin<sup>5</sup>, Stephen R. Carpenter<sup>4</sup>, Vasilis Dakos<sup>1</sup>, Hermann Held<sup>6</sup>, Egbert H. van Nes<sup>1</sup>, Max Rietkerk<sup>7</sup> & George Sugihara<sup>8</sup>

**Complex dynamical systems, ranging from ecosystems to financial markets and the climate, can have tipping points at which a sudden shift to a contrasting dynamical regime may occur. Although predicting such critical points before they are reached is extremely difficult, work in different scientific fields is now suggesting the existence of generic early-warning signals that may indicate for a wide class of systems if a critical threshold is approaching.**

It is becoming increasingly clear that many complex systems have critical thresholds—so-called tipping points—at which the system shifts abruptly from one state to another. In medicine, we have spontaneous systemic failures such as asthma attacks<sup>1</sup> or epileptic seizures<sup>2,3</sup>; in global finance, there is concern about systemic market crashes<sup>4,5</sup>; in the Earth system, abrupt shifts in ocean circulation or climate may occur<sup>6</sup>; and catastrophic shifts in rangelands, fish populations or wildlife populations may threaten ecosystem services<sup>7,8</sup>.

It is notably hard to predict such critical transitions, because the state of the system may show little change before the tipping point is reached. Also, models of complex systems are usually not accurate

considered to capture the essence of shifts at tipping points in a wide range of natural systems ranging from cell signalling pathways<sup>14</sup> to ecosystems<sup>7,15</sup> and the climate<sup>6</sup>. At fold bifurcation points ( $F_1$  and  $F_2$ , Box 1), the dominant eigenvalue characterizing the rates of change around the equilibrium becomes zero. This implies that as the system approaches such critical points, it becomes increasingly slow in recovering from small perturbations (Fig. 1). It can be proven that this phenomenon will occur in any continuous model approaching a fold bifurcation<sup>12</sup>. Moreover, analysis of various models shows that such slowing down typically starts far from the bifurcation point, and that recovery rates decrease smoothly to zero as the critical point is

