AFRICAN INSTITUTE FOR MATHEMATICAL SCIENCES

(AIMS RWANDA, KIGALI)

Name: Yusuf Brima | Assignment Number: 1
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Question 2

Consider the problem

$$\begin{array}{rc} \text{Minimize} & z = 3x_1 - x_2 + x_3 \\ & x_1 + 2x_2 \leq 4 \\ \text{subject to} & 2x_1 - x_2 + x_3 \geq 1 \\ & x_1, x_2 \geq 0, x_3 \leq 0 \end{array} \tag{1}$$

(a) Is the point $(x_1, x_2, x_3) = (\frac{6}{5}, \frac{7}{5}, 0)$ an optimal solution?

(b) Is the point $(x_1, x_2, x_3) = (\frac{1}{2}, 0, 0)$ an optimal solution?

Question 3

Study the following LP problem using duality concept.

$$\begin{array}{rc} \text{Maximize} & z = z = c_1x_1 + c_2x_2 \\ & -x_1 + x_2 \leq 1 \\ & x_1 + 2x_2 \leq 2 \\ \text{subject to} & 2x_1 + x_2 \geq 0 \\ & 2x_1 - 2x_2 \leq 1 \\ & x_1, x_2 \geq 0 \end{array} \tag{2}$$

(a) Determine the primal solution where the dual variables are given by the vector $v = (0, 1/3, 0, 2/3)^{\mathrm{T}}$
By using Complementary Slackness (3)

$$\bar{W}^{\mathrm{T}}(B - A\bar{X}) = 0 \quad \text{or} \quad \bar{X}^{\mathrm{T}}(A^{\mathrm{T}}\bar{W} - C) = 0 \tag{3}$$

Therefore, $\bar{W}^{\mathrm{T}}(B - A\bar{X}) = 0$ is solved as follows:

$$
\begin{aligned}
0(1-(-x_1+x_2)) &= 0\\
\frac{1}{3}(2-(x_1+2x_2) &= 0\\
0(0-(2x_1+x_2)) &= 0\\
\frac{2}{3}(1-(2x_1-2x_2)) &= 0
\end{aligned}
$$

We therefore, get the following system of simultaneous equations as stated below.

$$
\begin{aligned}
x_1+2x_2 &= 2\\
2x_1-2x_2 &= 1
\end{aligned}
$$

Solving the above simultaneous equation where $x_1 = 1$ and $x_2 = \frac{1}{2}$; the solution for the primal is thus: $\bar{X}^{\mathrm{T}} = (1, \frac{1}{2})^{\mathrm{T}}$

(b) Determine the values of c_1 and c_2 for which the primal solution is optimal.
New $\bar{X}^{\mathrm{T}}(A^{\mathrm{T}}\bar{W} - C) = 0$
To solve for c_1 and c_2, we first determine the dual to the Linear Programming Problem in (2) as follows:

$$
\begin{aligned}
&\text{Minimize} && v = w_1 + 2w_2 + 0w_3 + w4\\
& && -w_1 + w_2 + 2w_3 + 2w_4 \geq c_1\\
&\text{subject to} && w_1 + 2w_2 + w_3 - 2w_4 \geq c_2\\
& && w_1, w_2 \geq 0, w_3 \leq 0, w_4 \geq 0
\end{aligned}
\tag{4}
$$

$\bar{X}^{\mathrm{T}}(A^{\mathrm{T}}W - C) = 0$
From (4), where therefore solve for c_1 and c_2 as follows:

$$
\begin{aligned}
(1)1(-w_1+w_2+3w_3+2w_4-c_1) &= 0\\
(0+\frac{1}{3}+0+\frac{4}{3}-c_1) &= 0\\
c_1 &= \frac{5}{3}
\end{aligned}
$$

$$
\begin{aligned}
(2)\frac{1}{2}(w_1+2w_2+w_3-2w_4-c_2) &= 0\\
(0+\frac{2}{3}+0-\frac{4}{3}-c_2) &= 0\\
c_2 &= \frac{-2}{3}
\end{aligned}
$$