AFRICAN INSTITUTE FOR MATHEMATICAL SCIENCES

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1 Theoretical exercise

Everyone have to type this document as it is.

Theorem 1. *Let n and m be integers. Then*

i. if n and m are both even,then n+m is even,

ii. if n and m are both odd, then n+m is even,

iii. if one n and m is even and the other is odd, then n+m is odd.

Proof. i. If n and m are even, then there exist integers k and j such that $n = 2k$ and $m = 2j$. Then

$$\begin{aligned} n + m &= 2k + 2j \\ &= 2(k + j). \end{aligned}$$

And since k, $j \in \mathbb{Z}$,$(k + j) \in \mathbb{Z}$. $\therefore n + m$ is even.

□

Theorem 2. *Let $n \in \mathbb{N}$, $n > 1$. Suppose that n is not prime $\Rightarrow 2^n - 1$ is not a prime.*

Proof. Since n is **not** a prime, $\exists a, b \in \mathbb{N}$ such that $n = a \times b$, $1 < a, b < n$. Let $x = 2^b - 1$ and $y = 1 + 2^b + 2^{2b} + \cdots + 2^{(a-1)b}$. Then

$$\begin{aligned} xy &= (2^b - 1)(1 + 2^b + 2^{2b} + \cdots + 2^{(a-1)b}) \\ &= 2^b + 2^{2b} + 2^{3b} + \cdots + 2^{ab} - 1 - 2^b - 2^{2b} - 2^{3b} - \cdots - 2^{(a-1)b} \\ &= 2^{ab} - 1 \\ &= 2^n - 1. \end{aligned}$$

Now notice that since $1 < b < n$, we have that $1 < 2^b - 1 < 2^n - 1$, so $1 < x < 2^n - 1$.
Therefore, x is a positive factor, hence $2^n - 1$ is **not** prime number. □

2 Sub-questions

1. (a) Maxwell's equations:

$$B' = -\nabla \times E, \tag{1a}$$
$$E' = \nabla \times B - 4\pi j, \tag{1b}$$

   (b) To show usage of L'Hôpital's rule:

$$\lim_{x\to 0} \frac{e^x - 1}{2x} \overset{\left[\frac{0}{0}\right]}{\underset{H}{=}} \lim_{x\to 0} \frac{e^x}{2} = \frac{1}{2}$$
$$\mathcal{L}_T(\vec{\lambda}) = \sum_{(x,s)\in\mathcal{T}} \log P(s|x) - \sum_{i=1}^{m} \frac{\lambda^2}{2\sigma^2}$$

   (c) Complex numbers

$$z = \overbrace{\underbrace{x}_{real} + i \underbrace{y}_{imaginary}}^{complex numbers}$$

   (d) To use brackets instead of braces use $\underbrace{\qquad}$ and $\overbrace{\qquad}$ commands

2. (a) Using aligned braces for piecewise functions

$$f(x) = \left\{ \begin{array}{ll} x^2 & : x < 0 \\ x^3 & : x \geq 0 \end{array} \right.$$

   (b) The cases environment allows the writing of piecewise functions

$$u(x) = \begin{cases} \exp x & \text{if } x \geq 0 \\ 1 & \text{if } x < 0 \end{cases}$$

   (c) Matrix and array

$$\begin{matrix} a & b & c \\ d & e & f \\ g & h & i \end{matrix}$$

$$A_{m,n} = \begin{pmatrix} a_{1,1} & a_{1,2} & \cdots & a_{1,n} \\ a_{2,1} & a_{2,2} & \cdots & a_{2,n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m,1} & a_{m,2} & \cdots & a_{m,n} \end{pmatrix}$$

$$M = \begin{bmatrix} \frac{5}{6} & \frac{1}{6} & 0 \\ \frac{5}{6} & 0 & \frac{1}{6} \\ 0 & \frac{5}{6} & \frac{1}{6} \end{bmatrix}$$

$$M = \begin{array}{c c} & \begin{array}{cc} x & y \end{array} \\ \begin{array}{c} A \\ B \end{array} & \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \end{array}$$

(d) Equation columns

$$f(x) = ax^2 + bx + c \qquad\qquad g(x) = dx^3$$
$$f'(x) = 2ax + b \qquad\qquad g'(x) = 3dx^2$$

(e) If you want a brace to continue across a new line, do the following:

$$\begin{aligned} f(x) = \pi \{ & x^4 + 7x^3 + 2x^2 \\ & + 10x + 12\} \end{aligned} \tag{2}$$

(f)

$$\boxed{x^2 + y^2 = z^2} \tag{3}$$

(g)

$$\prod_{\substack{1\le i\le n\\1\le j\le m}} M_{i,j} \tag{4}$$

(h)

$$x = a_0 + \cfrac{1}{a_1 + \frac{1}{a_2 + \frac{1}{a_3 + a_4}}} \tag{5}$$

(i)

$$\cos(2\theta) = \cos^2\theta - \sin^2\theta$$

$$\frac{n!}{k!\,(n-k)!} = \binom{n}{k}$$

$$\frac{\frac{1}{x} + \frac{1}{y}}{y - z}$$

$$\frac{\begin{array}{c}(x_1 x_2)\\ \times (x_1' x_2')\end{array}}{(y_1 y_2 y_3 y_4)} \tag{6}$$

(j)

$$\begin{aligned} &\vdots \\ &= 12 + 7\int_0^2 \left(-\frac{1}{4}\left(e^{-4t_1} + e^{4t_1-8}\right)\right) dt_1 \\ &= 12 - \frac{7}{4}\int_0^2 \left(e^{-4t_1} + e^{4t_1-8}\right) dt_1 \\ &\vdots \end{aligned}$$

(k) Differential equations

$$\frac{\partial u}{\partial t} = h^2 \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right)$$

Table 1: My caption

Name	Bob	
Type	Client	
Parameters	Param1	Value
	Param2	Value
	Param3	Value

3 Equations

$$ax^2 + bx + c = 0, \tag{7}$$

where a, b and c are real numbers, and $a \neq 0$. Equation (7) is the general form of a quadratic equation. Below is a system of equations

$$\int_a^b f(x)dx = (b-a)\left[\frac{f(a)+f(b)}{2}\right] \tag{8}$$

$$\int_a^b f(x)dx = \frac{h}{2}\sum_{k=1}^{N}\left(f(x_{k+1}) + f(x_k)\right). \tag{9}$$

Equation (9) is the general form of equation (8), where the limit of integration is partitioned into N strips of equal intervals given by h.

4 Figures

4.1 One figure

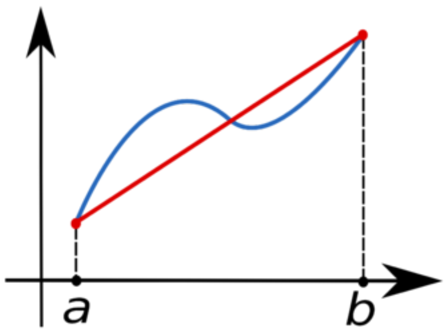
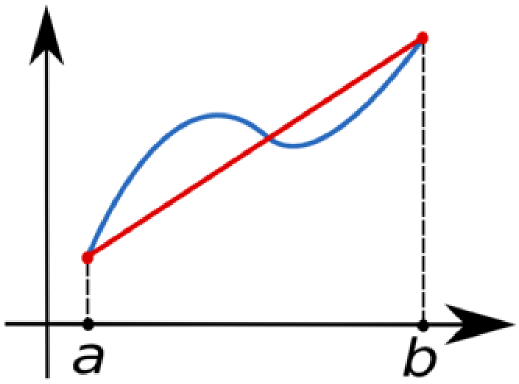


Figure 1: The simplest form of the trapezoidal rule.

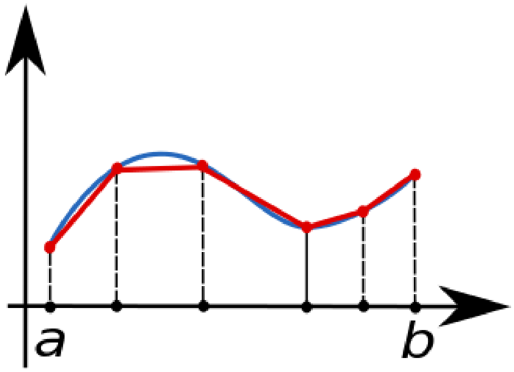
Figure 1 has only one picture. For pictures appearing side by side see section 4.2.

4.2 Figures side by side

This is how you put two pictures side by side. Note that each subfigure has its own caption, and the entire figure has a caption which gives a more general description of the figures. Figure 2a is the same as figure 1. They both correspond to equation (8). Figure 2b corresponds to equation (9). Figure 2 is a pictorial description of equations (8), and (9).



(a) Trapezoidal rule with a single strip.



(b) Trapezoidal rule with N equally spaced strips.

Figure 2: Trapezoidal rules

References

[1] John W. Dower *Readings compiled for History 21.479.* 1991.

[2] E. H. Norman *Japan's emergence as a modern state* 1940: International Secretariat, Institute of Pacific Relations.

[3] Bob Tadashi Wakabayashi *Anti-Foreignism and Western Learning in Early-Modern Japan* 1986: Harvard University Press.