

See discussions, stats, and author profiles for this publication at: <https://www.researchgate.net/publication/37406181>

# Partial Differential Equations for Scientists and Engineers

Article in American Journal of Physics · July 1985

DOI: 10.1119/1.14292 · Source: OAI

CITATIONS  
315

READS  
22,670

1 author:



Stanley Farlow  
University of Maine

21 PUBLICATIONS 1,592 CITATIONS

SEE PROFILE

Lokenath Debnath

**Nonlinear Partial  
Differential Equations  
for Scientists and Engineers**

Birkhäuser  
Boston • Basel • Berlin

# Contents

---

## *Preface*

xiii

<b>1.</b>	<b>Linear Partial Differential Equations</b>	<b>1</b>
1.1	Introduction	1
1.2	Basic Concepts and Definitions	2
1.3	The Linear Superposition Principle	4
1.4	Some Important Classical Linear Model Equations	7
1.5	The Classification of Second-Order Linear Equations and The Method of Characteristics	10
1.6	The Method of Separation of Variables	20
1.7	Fourier Transforms and Initial-Boundary-Value Problems	32
1.8	Applications of Multiple Fourier Transforms to Partial Differential Equations	44
1.9	Laplace Transforms and Initial-Boundary-Value Problems	49
1.10	Hankel Transforms and Initial-Boundary-Value Problems	58
1.11	Green's Functions and Boundary-Value Problems	67
1.12	Exercises	78
<b>2.</b>	<b>Nonlinear Model Equations and Variational Principles</b>	<b>93</b>
2.1	Introduction	93
2.2	Basic Concepts and Definitions	93
2.3	Some Nonlinear Model Equations	94
2.4	Variational Principles and the Euler-Lagrange Equations	99
2.5	The Variational Principle for Nonlinear Klein-Gordon Equations	104
2.6	The Variational Principle for Nonlinear Water Waves	105
2.7	Exercises	107

<b>3.</b>	<b>First-Order, Quasi-Linear Equations and The Method of Characteristics</b>	<b>111</b>
3.1	Introduction	111
3.2	The Classification of First-Order Equations	111
3.3	The Construction of a First-Order Equation	112
3.4	The Geometrical Interpretation of a First-Order Equation	116
3.5	The Method of Characteristics and General Solutions	118
3.6	Exercises	130
<b>4.</b>	<b>First-Order Nonlinear Equations and Their Applications</b>	<b>135</b>
4.1	Introduction	135
4.2	The Generalized Method of Characteristics	135
4.3	Complete Integrals of Certain Special Nonlinear Equations	139
4.4	Examples of Applications to Analytical Dynamics	145
4.5	Applications to Nonlinear Optics	151
4.6	Exercises	156
<b>5.</b>	<b>Conservation Laws and Shock Waves</b>	<b>159</b>
5.1	Introduction	159
5.2	Conservation Laws	159
5.3	Discontinuous Solutions and Shock Waves	172
5.4	Weak or Generalized Solutions	174
5.5	Exercises	181
<b>6.</b>	<b>Kinematic Waves and Specific Real-World Nonlinear Problems</b>	<b>185</b>
6.1	Introduction	185
6.2	Kinematic Waves	185
6.3	Traffic Flow Problems	189
6.4	Flood Waves in Long Rivers	202
6.5	Chromatographic Models and Sediment Transport in Rivers	204
6.6	Glacier Flow	210
6.7	Roll Waves and Their Stability Analysis	213
6.8	Simple Waves and Riemann's Invariants	219
6.9	The Nonlinear Hyperbolic System and Riemann's Invariants	239
6.10	Generalized Simple Waves and Generalized Riemann's Invariants	250
6.11	Exercises	254
<b>7.</b>	<b>Nonlinear Dispersive Waves and Whitham's Equations</b>	<b>263</b>
7.1	Introduction	263
7.2	Linear Dispersive Waves	263
7.3	Initial-Value Problems and Asymptotic Solutions	267

7.4	Nonlinear Dispersive Waves and Whitham's Equations	270
7.5	Whitham's Theory of Nonlinear Dispersive Waves	273
7.6	Whitham's Averaged Variational Principle	276
7.7	The Whitham Instability Analysis and Its Applications to Water Waves	278
7.8	Exercises	281
<b>8.</b>	<b>Nonlinear Diffusion-Reaction Phenomena, Burgers' and Fisher's Equations</b>	<b>283</b>
8.1	Introduction	283
8.2	Burgers' Equation and the Plane Wave Solution	283
8.3	Traveling Wave Solutions and Shock-Wave Structure	286
8.4	The Cole-Hopf Transformation and the Exact Solution of the Burgers Equation	289
8.5	The Asymptotic Behavior of the Exact Solution of the Burgers Equation	294
8.6	The N Wave Solution	296
8.7	Burgers' Initial- and Boundary-Value Problem	298
8.8	Fisher's Equation and Diffusion-Reaction Process	301
8.9	Traveling Wave Solutions and Stability Analysis	303
8.10	Perturbation Solutions of the Fisher Boundary-Value Problem	307
8.11	Similarity Methods and Similarity Solutions of Diffusion Equations	309
8.12	Nonlinear Reaction-Diffusion Equations	319
8.13	A Brief Summary of Recent Work with References	324
8.14	Exercises	325
<b>9.</b>	<b>Solitons and The Inverse Scattering Transform</b>	<b>331</b>
9.1	Introduction	331
9.2	The History of the Soliton and Soliton Interactions	331
9.3	The Boussinesq and Korteweg-de Vries (KdV) Equations	336
9.4	Solutions of the KdV Equation, Solitons and Cnoidal Waves	347
9.5	The Lie Group Method and Similarity and Rational Solutions of the KdV Equation	356
9.6	Conservation Laws and Nonlinear Transformations	359
9.7	The Inverse Scattering Transform (IST) Method	363
9.8	Bäcklund Transformations and the Nonlinear Superposition Principle	386
9.9	The Lax Formulation, Its KdV Hierarchy, and the Zakharov and Shabat (ZS) Scheme	391
9.10	The AKNS Method	400
9.11	Exercises	401

<b>10.</b>	<b>The Nonlinear Schrödinger Equation and Solitary Waves</b>	<b>405</b>
10.1	Introduction	405
10.2	The One-Dimensional Linear Schrödinger Equation	405
10.3	The Derivation of the Nonlinear Schrödinger (NLS) Equation and Solitary Waves	407
10.4	Properties of the Solutions of the Nonlinear Schrödinger Equation	412
10.5	Conservation Laws for the NLS Equation	419
10.6	The Inverse Scattering Method for the Nonlinear Schrödinger Equation	422
10.7	Examples of Physical Applications in Fluid Dynamics and Plasma Physics	424
10.8	Applications to Nonlinear Optics	438
10.9	Exercises	449
<b>11.</b>	<b>Nonlinear Klein-Gordon and Sine-Gordon Equations</b>	<b>453</b>
11.1	Introduction	453
11.2	The One-Dimension Linear Klein-Gordon Equation	453
11.3	The Two-Dimensional Linear Klein-Gordon Equation	456
11.4	The Three-Dimensional Linear Klein-Gordon Equation	458
11.5	The Nonlinear Klein-Gordon Equation and Averaging Techniques	459
11.6	The Klein-Gordon Equation and the Whitham Averaged Variational Principle	467
11.7	The Sine-Gordon Equation, Soliton and Anti-Soliton Solutions	470
11.8	The Solution of the Sine-Gordon Equation by Separation of Variables	475
11.9	Bäcklund Transformations for the Sine-Gordon Equation	484
11.10	The Solution of the Sine-Gordon Equation by the Inverse Scattering Method	487
11.11	The Similarity Method for the Sine-Gordon Equation	491
11.12	Nonlinear Optics and the Sine-Gordon Equation	492
11.13	Exercises	496
<b>12.</b>	<b>Asymptotic Methods and Nonlinear Evolution Equations</b>	<b>501</b>
12.1	Introduction	501
12.2	The Reductive Perturbation Method and Quasi-Linear Hyperbolic Systems	502
12.3	Quasi-Linear Dissipative Systems	506
12.4	Weakly Nonlinear Dispersive Systems and the Korteweg-de Vries Equation	508
12.5	Strongly Nonlinear Dispersive Systems and the Nonlinear Schrödinger Equation	521

<i>Contents</i>	<b>xi</b>
12.6 The Perturbation Method of Ostrovsky and Pelinovsky	527
12.7 The Method of Multiple Scales	531
12.8 Method of Multiple Scales for the Case of the Long Wave Approximation	538
<b>Answers and Hints to Selected Exercises</b>	<b>541</b>
<b>Bibliography</b>	<b>559</b>
<b>Index</b>	<b>581</b>