ODE make up quiz

Let us consider the initial value problem

$$\begin{cases} x' = & 3x - 2y \\ y' = & 2x - 2y \end{cases} \quad \text{with } x(0) = 1, \;\; y(0) = 0 \tag{1}$$

The exact solution of the system (1) is given by

$$x(t) = \frac{4}{3}e^{2t} - \frac{1}{3}e^{-t}$$
$$y(t) = \frac{2}{3}e^{2t} - \frac{2}{3}e^{-t}$$

1. Write a python code based on the modified Euler method to compare the exact and approximate solution of (1) on [0.1]. The modified Euler scheme is given by

$$\begin{cases} k_1 = hf(t_n, z_n), \\ k_2 = hf(t_n + h, z_n + k_1), \\ z_{n+1} = z_n + \dfrac{1}{2}(k_1 + k_2). \end{cases} \tag{2}$$

2. Note that the system of ODEs (1) can take the form

$$z' = Az, \tag{3}$$

where $z^T = [x, y]$ and A is a 2×2 matrix. Write a python code based on the exponential time differencing method to compare the exact and approximate solution of (1) on [0.1]. The exponential time differencing scheme is given by

$$z_{n+1} = e^{hA} z_n, \tag{4}$$

Note that to compute the exponential of a matrix you should use the python function *expm* **imported for** *scipy.linalg* **library**

3. Which one of the numerical methods used is more accurate?