International Institute of Information Technology - Hyderabad

EC5.203 - Communication Theory

Final Exam

May 01, 2025

- Number of questions: 8 ; Total points: 40; Time Limit: 180 minutes.
- **Read all instructions and questions carefully.**
- Use of a calculator is permitted.
- This is a closed-book exam.
- Clearly write your assumptions or use of any properties at each step.
- Even if the final answer is correct, only *partial marks* will be given if the approach or methodology is incorrect, not presented clearly, or not legible.
- Even if the final answer is incorrect or incomplete, *partial marks* may be given if the approach or methodology is presented clearly.
- Do not write anything ~~on the question paper~~!, There will be a penalty for marking any answers (including MCQ) on the question paper.
- Follow notations/conventions from class, if not explicitly stated.

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1. State whether the following statements are True or False. Give a reason for your statement in 2-3 sentences. No step marking: both the statement and reason must be right. **(5 points)**

   (a) Digital communication systems are more robust to noise as compared to analog communication systems.

   (b) A mixer cannot be implemented using non-linearity.

   (c) A correlator can be implemented using a matched filter.

   (d) Out of the two 2-ary constellations $[-1\ 1]^T$ and $[1\ 3]^T$, the first constellation has better symbol error probability than the second in the presence of AWGN.

   (e) Using a sinc pulse in the time domain can result in significant intersymbol interference even with far-away symbols in the presence of a timing offset.

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2. Multiple choice questions (1 point each: **5 points**); Negative marking: -0.5 marks; Only one choice is ~~correct.~~

   (a) For the two signals $u_1(t) = e^{-2\pi 10t} I_{[t>0]}(t)$ and $u_2(t) = e^{2\pi 10t} I_{[t<0]}(t)$, single sided bandwidth definition is defined for

   (A) Both $u_1(t)$ and $u_2(t)$ (B) Only for $u_1(t)$

   (C) Only for $u_2(t)$ (D) None of the above

(b) The number of real dimensions for 8-ary biorthogonal modulation scheme is
(A) 1 (B) 2 (C) 4 (D) 8

(c) Consider a signal $p(t)$ sampled at rate $1/T$. Let $P(f)$ denotes the spectrum of $p(t)$ and let $B(f) = \frac{1}{T}\sum_{k=-\infty}^{\infty} P(f + \frac{k}{T})$, then for $p(t)$ to be Nyquist,
(A) $B(f)$ has to be constant (B) $P(f)$ has to be constant
(C) $B(f)$ has to discrete (D) None of the above

(d) For M-ary orthogonal signaling, as M increases,
(A) Power efficiency decreases, and bandwidth efficiency decreases.
(B) Power efficiency increases, and bandwidth efficiency decreases.
(C) Power efficiency decreases, and bandwidth efficiency increases.
(D) Power efficiency increases, and bandwidth efficiency increases.

(e) The optimal detector for M-ary demodulation in the presence of AWGN noise with equal prior probabilities is
(A) MAP (B) ML (C) Minimum distance rule (D) All of the above

3. Answer the following questions:

5 (a) Draw the block diagram of a digital communication system. (2 points)

(b) Prove that the passband waveforms $u_p(t) = u_c(t)\cos(2\pi f_c t)$ and $u_s(t) = u_s(t)\sin(2\pi f_c t)$ are orthogonal assuming that $u_c(t)$ and $u_s(t)$ are real signals. What is the advantage of this result? (3 points)

4. Answer the following questions on analog communication systems

5 (a) Prove that angle modulation is a non-linear operation while amplitude modulation is a linear operation. (2 points)

(b) Prove that FM has infinite bandwidth theoretically. As a special case, derive the narrowband FM expression and its bandwidth. (3 points)

5 5. Applying Gram-Schmidt orthogonalization procedure in the ascending order of index, construct an orthonormal basis set for the signal space spanned by signals $s_1(t), s_2(t), s_3(t)$ and $s_4(t)$ shown in Fig. 1. (5 points)

6. A binary hypothesis problem is specified on an observation y as follows

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$$H_0 : y \sim N(0, \sigma_0^2)$$
$$H_1 : y \sim N(0, \sigma_1^2)$$

where $\sigma_1^2 > \sigma_0^2$. For this problem,

(a) Find the simplified version of MAP rule in terms of y for this problem. (2 points)

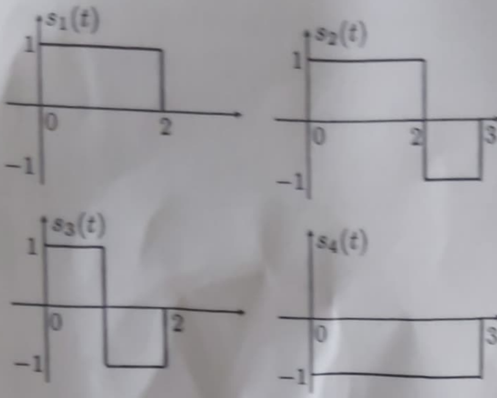

Figure 1: for Q.5

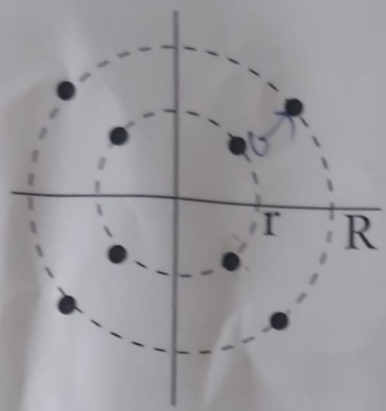

Figure 2: for Q.8

(b) Draw the conditional PDFs of y and the corresponding decision region. **(1 point)**

(c) Derive the conditional error probabilities in terms of the $Q(.)$ function. **(2 points)**

7. Consider a linearly modulated signal $u(t) = \sum_n b[n]p(t - nT)$, where the symbol sequence $\{b[n]\}$ is zero mean and uncorrelated with average symbol energy

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$$\sigma_b^2 = \overline{|b[n]|^2} = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |b[n]|^2,$$

then prove that the PSD is given by

$$S_u(f) = \frac{|P(f)|^2}{T}\sigma_b^2$$

and the power of the modulated signal is

$$P_u = \frac{\sigma_b^2 ||p||^2}{T}$$

where $||p||^2$ denotes the energy of the modulating pulse. **(5 points)**

8. Consider the constellation shown in Fig. 2. For $r = 1$ and $R = 2$, and using $P[Z_i < Z_j | i \text{ sent}] = Q(\frac{d_{ij}}{2\sigma})$, solve the following

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(a) Redraw the constellation and carefully sketch the ML decision regions **(2 points)**

(b) Based on the decision regions in part (a), find the intelligent union bound and the nearest neighbor approximation for the average symbol error probability in terms of E_b/N_0. **(3 points)**