EC5.406 – Signal Detection and Estimation Theory – Quiz 1

Date: 29th August, 2025
Instructor: Santosh Nannuru

Maximum marks: 20
Exam duration: 45 minutes

1. [7 marks]
Consider the linear frequency modulated signal observed in presence of white Gaussian noise modeled as,

$$x[n] = \cos(2\pi(f + \alpha n)n) + w[n],\ n = 0, 1, \ldots, N-1,$$

where $w[n]$ are i.i.d. Gaussian random variables with mean 0 and variance σ^2 (known). Assume that the frequency f is known and we are interested in estimating the parameter α.

   (a) Show that the regularity conditions are satisfied for estimation of α.
   (b) Find the CRLB expression for the parameter α.
   (c) Let $f = \frac{1}{2}$ and $N = 2$, plot the CRLB as function of $\alpha \in [-\frac{1}{2}, \frac{1}{2}]$.

2. [7 marks]
You have recently figured out that the stock price $x[n]$ of a company X on the nth day can be accurately modeled using a sinusoidal variation superimposed on a linear trend in presence of some WGN $w[n]$ as,

$$x[n] = An + \sin(\omega n) + w[n],\ n = 0, 1, 2 \ldots N-1,$$

where the parameters A and ω are unknown while the noise variance σ^2 is assumed to be known.

   (a) Find the Fisher information matrix corresponding to the parameters A, ω.
   (b) What are CRLB for the parameters A and ω?
   (c) The following estimator is proposed for the parameter A, $\hat{A} = \alpha \sum_{n=0}^{N-1} x[n]$. Find α such that this estimator is unbiased. You can make the approximation $\sum_{n=0}^{N-1} \sin(\omega n) \approx 0$. What can you say about the efficiency of this estimator?

3. [6 marks]
Solve the following:

   (a) For any arbitrary $p \times q$ matrix M, show that the matrix MM^T is a positive semi-definite matrix.
   (b) Let $\underline{X}$ and $\underline{Y}$ be q and p dimensional random vectors respectively. Their means and covariance matrices are $\mu_{\underline{X}}, \mu_{\underline{Y}}$ and $C_{\underline{X}}, C_{\underline{Y}}$. For a $p \times q$ matrix M, derive the mean and covariance matrix of $\underline{Z} = M\underline{X} + \underline{Y}$.