Modern Complexity Theory (CS1.405)

End Semester Examination (Monsoon 2023)

International Institute of Information Technology, Hyderabad

Time: 3 hours Total Marks: 70

Instructions: Answer <u>*ANY SEVEN*</u> questions from the following questions.

This is a closed book and notes examination.

Regular calculator is allowed.

NO query is allowed in the examination hall.

1. **(a)** Prove that a language L is in the class NP if and only if L has a polynomial-time verifier.

   (b) A coloring of a graph is an assignment of colors to its nodes so that no two adjacent nodes are assigned the same color. Let $kCOLOR = \{\langle G \rangle \mid$ the nodes of G can be colored with k colors such that no two nodes joined by an edge have the same color $\}$. Model this problem mathematically. Show that $kCOLOR$ is in class NP, for all $k \geq 3$.

   [5 + (1 + 4) = 10]

2. **(a)** Suppose a fair dice is thrown. As usual a dice contains six faces 1, 2, 3, 4, 5, and 6, and thus the probability of occurring a face is $1/6$. Let the dice be thrown n times consecutively, where $n \geq 10$. Obviously, the outcome of n throwns is a sequence of n faces. Define the language $nDICETHROWN := \{\langle f_1, f_2, \ldots, f_n \rangle \mid$ the sequence will contain the number of 6-th face ≥ 7, and the face f_i is the outcome of the i-th thrown $\}$. Prove that $nDICETHROWN \in NP$.

   (b) Define the traveling salesperson problem (TSP). Prove that TSP is NP-complete.

   [4 + (1 + 5) = 10]

3. **(a)** Define the space complexity of a non-deterministic Turing machine.

   (b) State the Savitch theorem for any function $f : N \to N$, where $f(n) \geq n$. Deduce the relationship between the classes PSPACE and NPSPACE using this theorem.

   (c) Prove that the true fully quantified Boolean formula (TQBF) problem is PSPACE-complete describing clearly the TQBF problem.

   [1 + (1+2) + 6 = 10]

4. **(a)** Explain the generalized geography game (GG). Prove that GG is PSAPCE-hard.

   (b) Define the following classes: $L^k := SPACE(\log^k n)$, $NL^k := NSPACE(\log^k n)$, and $polyL := \cup_{k \in N} L^k$. Then deduce the relationships between L^k, NL^k, $polyL$, and $PSPACE$.

   [7 + 3 = 10]

5. **(a)** Prove that PATH is NL-complete.

   (b) Show that the language consisting of strings with properly nested parentheses, and (square) brackets is in the class L.

   [5 + 5 = 10]

No mathematical notations
— 2

6. Use *diagonalization* to prove any *two* of the following:

- Power-set of a countably infinite set is uncountable —
- Language $\{\langle M, w\rangle | M \text{ accepts } w\}$ is undecidable —
- SPACE HIERARCHY THEOREM —
- There are functions that are *not* time-constructible
- TIME HIERARCHY THEOREM

[5 + 5 = 10]

7. Give an interactive proof for the language #SAT. Recall that: —

$$\#\text{SAT} = \{\langle \phi, k\rangle | \phi \text{ is a 3CNF Boolean formula that has exactly } k \text{ satisfying assignments}\}$$

[10]

8. Prove or disprove any *four* of the following:

(a) NP $\subset$ ZKP
(b) GNI $\in$ IP —
(c) ZPP = RP $\cap$ co-RP —
(d) IP = co-IP —
(e) BPP $\subset$ BQP —
(f) IP = PSPACE —
(g) Integer Factorization $\in$ BQP

[4 × 2.5 = 10]

9. Suppose $j, r \in N$ are relatively prime and j and r are unknown to us (N is known). Show that if we know the first $2\log_2 N$ bits of j/r, then we can efficiently recover j and r. —

[10]

**************** **End of Question Paper** ********************