Considering absolute deviation
calculating bias/variance wrt to a deviation.

c. Please write down formula for variance ? Briefly explain what the formula represents ? [1+1 points]

Variance is basically how much the result vary when considering collective data set (difference across observation)

$$\text{Variance} = E((X-\mu)^2) = E(X^2) - E(X)^2$$

2

d. Please compute variance using the information for the models MR1 and MR2. [3 points]

variance is $E((X-\mu)^2)$

MR1

$\mu = 1.5$ (average deviation)

$(1-1.5)^2 = 0.25$

$(2-1.5)^2 = 0.25$

$(1-1.5)^2 = 0.25$

$(2-1.5)^2 = 0.25$ $= \frac{0.25 \times 6}{6}$

$(1-1.5)^2 = 0.25$ $\boxed{0.25}$

$(2-1.5)^2 = 0.25$

$(1.5-0.25)^2 = \boxed{1.5625}$ variance from answer

MR2

$\mu = 1.5$ (average deviation)

$(2-1.5)^2 = 0.25$

$(1-1.5)^2 = 0.25$

$(2-1.5)^2 = 0.25$

$(1-1.5)^2 = 0.25$

$(2-1.5)^2 = 0.25$

$(1-1.5)^2 = 0.25$

avg = 0.25

2

$\boxed{1.5625}$ Variance

e. Please compute the MSE (Mean Square Error) for MR1 and MR2. Please write down the formula being used before presenting the computations. [3+1 points]

mean squared error is $\sum |f(x) - y'|^2$ (what the squared no divided)

y' real, $f(x)$ obtained.

0.5

$MR1: |y_1+1-y_1|^2 + |y_2-2-y_2|^2 + |y_3+1-y_3|^2 + |y_4-2-y_4|^2 + |y_5+1-y_5|^2 + |y_6-2-y_6|^2$

$1+4+1+4+1+4 = 15$

MR2

$: |y_1+2-y_1|^2 + |y_2-1-y_2|^2 + |y_3+2-y_3|^2 + |y_4-1-y_4|^2 + |y_5+2-y_5|^2 + |y_6-1-y_6|^2$

$: \boxed{15}$

f. Based on the results above, what would your final model or suggestion be ? Please explain the choice made in detail. [3 points]

So, we would prefer the model having minimum mean squared error this is because we want to have an ideal model that is neither overfitted nor underfitted. by this we want low bias, low variance!!

We would try to have minimum mean squared error.

MSE same so any model.

Refer cont. pages

2. Please answer the following questions regarding POMDPs along with all the steps and computations involved. With the IPL season ongoing, popular site Cricinfo provides the winning chance of the teams (T1, T2) at any point as (z%, 100-z%). This represents the belief (z/100, 1-(z/100)) of winning the match for each team from the current point.

a. You construct your belief vector using Cricinfo and take part in a bet where you bet that T1 would win. Both winning and losing the bet would be same value of 100 rupees (i.e., +100 or -100). Please mention whether the following statement is True or False and present the calculations to support: Using the belief vector, you would expect to receive 60 rupees if z is 60%. [3 points]

Reward

RE. R(100, −100)   if z = 60%. then belief $= \left[\frac{60}{100}, 1-\frac{60}{100}\right]$

$= [0.6, 0.4]$

P(win)   P(lose)

expected utility: $(0.6)(100) + (0.4)(-100)$

$60-40 = 20$

$\boxed{\text{false}}$

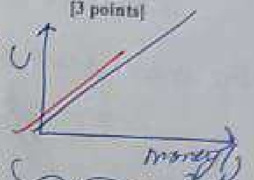
bet

since belief of T1 win > T2 win hence we might win and hence expect ~~0.6×100~~ 100 ruppee !! but actually

b. If you are a risk neutral agent, you would play the bet in (a). Please mention True or False and present a suitable utility function to make the case for your answer along with calculations involved.

[3 points]

thing is if we are risk neutral then we know that



Also if we are risk neutral agent then we would go ahead with bet

because the utility of not playing would be 0 (risk neutral)

but $(0.6)(100) - (0.4)(100) = 20$ if playing

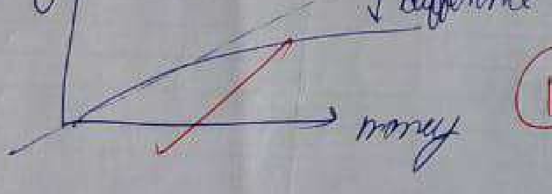
3

Hence you will value this more in plea that you might get 100 with 60% possibility

c. If you are a risk averse agent, you would definitely not play the bet in (a). Please mention True or False and present a suitable utility function to make the case for your answer along with calculations involved.

[3 points]

we know that risk averse agent is one whose graph for utility is concave



1.5

because according to graph above, as well your risk of losing money has more utility than you gaining money

$$\underbrace{U(x) - U(x-c)}_{\text{losing}} > \underbrace{U(x) + U(x+c)}_{\text{winning}}$$

final answer?

~~hence you prefer not play~~

4. If any ... given 'F' grade for the course ...

Best of Luck

d. If the current status from Cricinfo is (z%, 100-z%) and if action Watch will help you understand the game better before taking a bet, please compute the new belief b' if post Watch action you realize that T2 has increased its chances of winning by 15%. [3 points]

since you found via watch, hence you update your belief state based upon what you observe.

initially T_1: Z%

T_2 = 100 − Z%

$(100-Z)\% + \frac{15}{100}(100-Z)\% = \underbrace{\frac{115}{100}(100-Z)\%}_{b}$

$\underbrace{Z\% - \frac{15}{100}(100-Z)\%}_{a}$

0.5

[a, b]

oder check

3. Consider a robot that is moving in an environment. The goal of the robot is to move from an initial point to a destination point as fast as possible. However, the robot has the limitation that if it moves fast, its engine can overheat and stop the robot from moving. The robot can move with two different speeds: Slow and Fast. If the robot moves Fast, it gets a (immediate) reward of 10 and if it moves Slow, it gets a (immediate) reward of 4. We can model this problem as an MDP by having three states: Cool, Warm, and Off. The transitions are shown as below. Assume that the discount factor is 0.9 and also assume that when the robot reaches the (terminal) state Off, it will remain there without getting any reward.

s	A	s'	P(s'|s,a)
Cool	Slow	Cool	1
Cool	Fast	Cool	1/4
Cool	Fast	Warm	3/4
Warm	Slow	Cool	1/2
Warm	Slow	Warm	1/2
Warm	Fast	Warm	1/4
Warm	Fast	Off	3/4

a) Consider the conservative policy J when the robot always moves Slow. Assume that the robot starts at state Cool. What is the value of J(cool) i.e., expected discounted sum of rewards for state Cool. Please show steps of computation. [4 points]

X 3

now, expected discounted sum of reward for state cool

$\gamma\left(\sum P(cool \mid cool, slow) \cdot R(cool)\right)$

$(0.9)(1)(4) = \underline{\underline{3.6}}$

(note that since start state is cool and agent can only move slow it can't reach warm by any mean)

a) Expected discounted of sum of rewards can be seen as

$$\gamma \max \sum P(s'|s,a) \cdot U(s') \quad (1)$$

discount factor ← γ

$U(s')$ → value of that state

$P(s'|s,a)$ → probability of reaching s' via state s by taking action a.

Cont: from cool there are possible fast, fast, slow but it will calculate utility using (1) for each and select the max one.

Slow: ~~0.5~~ $(1)(4) = 4$

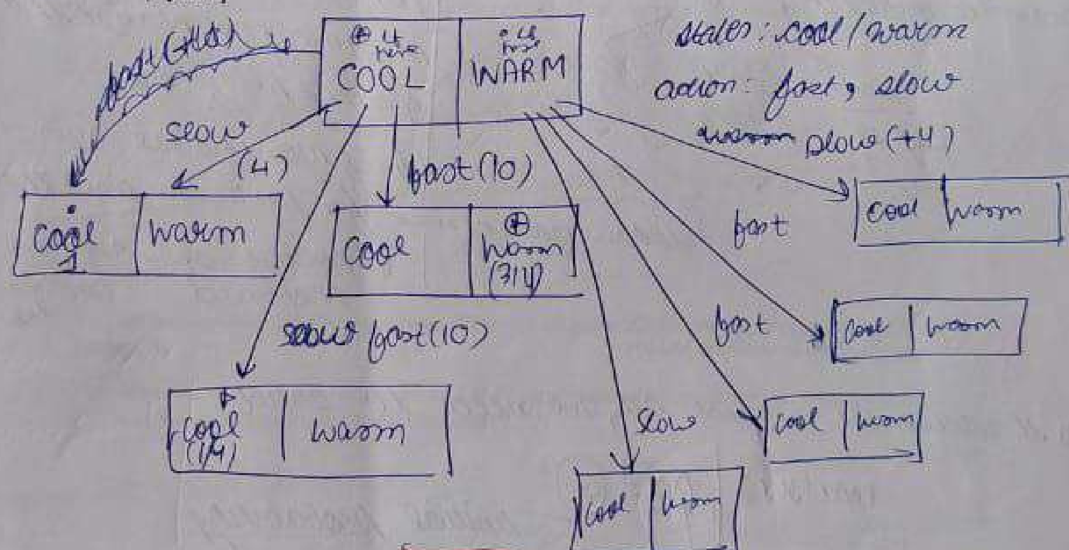
fast: $(\frac{1}{4})(10) = 2.5$ (cool)

fast: $3/4 \times 10 + (2.5)(3) = 7.5$ (warm) Sum??

robot will prefer it max one !!

+3

b) What is the optimal policy for the robot at each state ? Please explain in detail or show computations for how you arrived at the solution. Answer without clear supporting details will receive 0 points. -- **[5 points]** !! by others



states: cool / warm

action: fast, slow

at each state robot will try to choose action that maximizes its utility too. (in above continued).

c) Is it possible to change the discount factor to get a different optimal policy? If yes, please provide the discount factor and the policy it gives and if not, please justify your answer with detailed reasoning. – **[3 points**, mentioning Yes/No without relevant explanation will not receive points]

basically thing is action agent chooses depend upon $P(s'|s,a) \cdot U(s')$ but its not directly dependent on γ — here reward of that state. The overall value would reduce at γ but no policy won't change. $\gamma > 0.1$?

(ii) other is that you choose reward in such a way that equality remains same as say fast = 20 slow = 6

then $(1)(6)$, $(\frac{1}{4})(20)$, $(\frac{3}{4})(25)$ here you notice the the value changed but policy remains the same.

1

d) Is it possible to change the immediate reward function so that J(Cool) [not the policy J but the value provided by J at state Cool] changes but the optimal policy would remain unchanged? If yes, please give such a change and if no justify your answer in a couple of sentences. [3 points, mentioning Yes/No without relevant explanation will not carry points]

basically thing is consider state cool you have three options available out there, cool

slow	fast	fast
$(1)(4)$	$(\frac{1}{4})(10)$	$(\frac{3}{4})(10)$
4	2.5	7.5

the one you choose

Immediate reward function may/may not change the optimal policy e.g., (i) if you choose reward for fast too slow then obviously slow will be preferred

Con't on Nb

2.6

e) If you plan to solve the above MDP using the Linear Programming approach, please present the complete A matrix for this problem. [5 points]

so it would be solved as, we need to satisfy

$\underbrace{\max(\mu x)}$ | $Ax = \alpha$ — initial probability

set of action state

maximising the total reward.

matrix $(\delta_{ij} - P_{ij}^{a})$ → (probability of reaching state j via action a from i)

1 (when $i=j$)

explained how it will be done on last page — 2

4. Assume that there are 100 students in a class you are part of. Each of the 100 students in the class is provided with a fair coin i.e. P(Heads) = P(Tails) = 0.5. All of you toss your coin simultaneously in each round. Please answer the following questions and present all the relevant reasoning:

a) If the class tossed once, what is the probability that you and your friend both tossed a heads in that round? **[1 points]**

$P(\text{I head}) \cdot P(\text{friend head}) = \left(\frac{1}{2}\right) \cdot \left(\frac{1}{2}\right) = \frac{1}{4} = 0.25$

$\boxed{0.25}$

1

b) If the class plans to toss twice, what is the probability that you would toss a heads in second round conditioned on the fact that you tossed a heads in the first round? **[2 points]**

so A: toss head in second round
B: toss head in first round

$P(A|B) = \frac{P(A \cap B)}{P(B)}$

since A and B are independent event hence $P(A \cap B) = P(A) \cdot P(B)$

$\frac{P(A) \cdot P(B)}{P(B)} = P(A) = \left(\frac{1}{2}\right) = 0.5$

$\boxed{0.5}$

2

c) If the class tossed twice, what is the probability that you obtained heads twice in the two rounds while your friend obtained a tails both times? **[2 points]**

assuming while don't try conditioning here rather simultaneous action

A: you obtain head twice in two round
B: friend obtained tail both time

$P(A \cap B) = P(A) \cdot P(B)$ (since friends toss is independent from your toss)

$\left(\frac{1}{2}\right)\left(\frac{1}{2}\right) \cdot \left(\frac{1}{2}\right)\left(\frac{1}{2}\right) = \frac{1}{16} = \boxed{0.0625}$

2

d) If the class tossed once, what is the probability that you are the only one that tossed a heads while everyone else in the class tossed a tails? **[3 points]**

so A: you tossed heads
B: 99 other tossed tail

$P(A \cap B) = P(A) \cdot P(B)$ (as your toss is independent of their throw)

given 'F' grade for the course and may be debarred ...

Best of Luck

$P(A) = \frac{1}{2}$ (you tossed head)

$P(B) = \underbrace{\left(\frac{1}{2}\right) \cdot \left(\frac{1}{2}\right) \text{------}}_{\text{99 times for each other student for tail}} \left(\frac{1}{2}\right)$

$P(A) \cdot P(B) = \left(\frac{1}{2}\right)^{100} = (0.5)^{100}$

2 3

e) If the class tossed once, what is the probability that both you and your friend tossed a Heads given the information that you are risk averse but your friend is risk seeking? You can define suitable utility functions that model risk averse and risk seeking behavior but please present the inputs modeled.

[3 points]

since you are risk averse: for you since both action have same result be C but $U(x+c) - U(x) < U(x) - U(x-c)$

$\frac{U(x+c) + U(x-c)}{2} < \underbrace{U(x)}_{\text{not playing any more}}$

hence you won't loss!! so p(head) = 0 !!

since friend is risk averge, p(head) = 1/2 hence $P(A \cap B) = 0 \cdot 1/2$

0

$\boxed{0}$

5. Prove the following using truth table method: $(A \to (B \to C)) \Leftrightarrow ((A \land B) \to C)$ **[5 points]**

$X \to Y$ is false only when X is true Y is false

A	B	C	$B \to C$	$A \to (B \to C)$	$A \land B$	$(A \land B) \to C$
T	T	T	T	T	T	T
T	T	F	F	F	T	F
T	F	T	T	T	F	T
T	F	F	T	T	F	T
F	T	T	T	T	F	T
F	T	F	F	T	F	

3. Borrowing writing material or calculators from other students in the examination hall is prohibited.
4. If any student is found indulging in malpractice or copying in the examination hall, the student will be given 'F' grade for the course and may be debarred from writing other examinations.

Best of Luck

$P(\text{actual}|T_1) = \frac{P(\text{actual} \cap T_1)}{P(T_1)} = \frac{(0.3)}{(0.3)+(0.01)}$

getting positive tested

$\cancel{\frac{(0.3)}{(0.3)+(0.1)+C}}$; $\frac{P(\text{he actually has covid + test})}{P(\text{he gets positive tested})}$

$P(T_1) = P(\text{covid}|T_1) + P(\text{noncovid}|T_1)$

$= (0.3) + (0.01) = \underline{\underline{0.31}}$ on last

23

(b) What is the entropy of the population? ______

we know that Entropy $= \sum -p_i \log p_i$

hence say population: 100, cancer $= \frac{1}{100} \times 100 = 1$, non cancer = 99

$= -\left(\frac{1}{100}\log\left(\frac{1}{100}\right) + \frac{99}{100}\log\left(\frac{99}{100}\right)\right) = -\left(\frac{1}{100}(-2) + \frac{99}{100}\log(.99)\right)$

~3

$= -(-0.02432) = \boxed{0.02432}$

(c) What is the information gain of T1? ______

$\text{Gain}(T_1) = \text{Entropy } S - \sum \frac{|S_v|}{|S|} \cdot \text{Entropy } S_v$ basically

in 100 population

cancer: $\frac{-30}{100}\log\left(\frac{30}{100}\right) = (-0.3)(-0.522) = \underline{\underline{0.1568}}$

noncancer: $\frac{-99}{100}\log\left(\frac{-99}{100}\right) = \underline{\underline{0.98}}$

~1

—continued on last

8. The spam class of a spam dataset has the following probabilities:

P(A|spam) = 0.2, P(B|spam) = 0.3, P(C|spam) = 0.4, P(AB|spam) = 0.15

Given an email with the keywords, {A, B, C}, what will naive bayes compute as the probability that the email is spam? ______ **[5 points]**

now, $P(\text{spam}) = P(A|\text{spam}) + P(B|\text{spam}) + P(C|\text{spam}) - P(AB|\text{spam})$

$= 0.2 + 0.3 + 0.4 = 0.9 - 0.15 = \cancel{0.06)}\ 0.75$

+5

now $P_{\text{naivebayes}} = P(\text{spam}) \cdot [P(A|\text{spam}) \cdot P(B|\text{spam}) \cdot P(C|\text{spam})]$

... the student will be ... grade for the course and may be debarred from writing other examinations.

A	B	C	$B\to C$	$A\to(B\to C)$	$A\wedge B$	$(A\wedge B)\to C$
F	F	T	T	T	F	T
F	F	F	T	T	F	T

clearly in (i) and (ii) for all possible values wherever (i) is true, (ii) is also true hence $\boxed{(A\to(B\to C)) \Longleftrightarrow (A\wedge B)\to C}$ 3

refer back

6. What are all the *infrequent* candidates (after pruning) that would be generated by Apriori if the frequent itemsets are: F = {A, B, C, D, AB, AC, BC, AD, ABC} [Available items are A, B, C, D] **[10 points]**

lets first analyze what all can apriori gen. $^4C_2 = \frac{4\cdot 3}{2}$

A, B, C, D

⟹ AB, BC, CD, AD, BD, AC (CD, BD in this step)

↓ prune (remove CD, BD)

AB, BC, AD, AC +10

↓ these will generate following sets

ABC, ABD, ~~ABC~~, ADC

ABD will get pruned (no BD)

ADC → will get pruned no DC

(ADC, ABD in this step)

↓

ABC left!

(these will be for infrequent ones generated by apriori but will get pruned out!!)

7. The probability of cancer in a population is 1%. A test (T1) for cancer identifies cancer patients with a probability of 30% and identifies non-cancer patients with a probability of 99%. **[15 points]**

(a) Given that a patient has tested positive, what is the probability that he actually has cancer? ________

~~Hcam~~ B ~~any population~~ P(cancer) = 0.01

T_1 cancer P(cancer determined by T_1) = 0.3.

P(noncancer by T_1) = 0.99. ~~P(...)~~.

Q9(i)

MS: $\sqrt{(70-60)^2+(70-60)^2} = \sqrt{200} = 14.14$

GM: $\sqrt{(70-60)^2+(70-70)^2} = \sqrt{100+0} = 10$

min (10, 14.14, 11.18) = 10

Single link distance = 10

Q9)

R (65,60)
S (60,60)
M (70,70)

1

G(60,70)

2

Cluster 1

Cluster 2 G — R, S, M

① We will calculate distances between them i.e

Gita – Ram: $\sqrt{(65-60)^2+(60-70)^2} = \sqrt{100+25} = \sqrt{125} = 11.18$

Gita – shyam: $\sqrt{(60-60)^2+(70-60)^2} = \sqrt{0+100} = \sqrt{100} = 10$

Gita – mohan: $\sqrt{(70-60)^2+(70-70)^2} = \sqrt{100+0} = \sqrt{100} = 10$

(a) Single link: minimum distance between two cluster is

single link min (10, 10, 11.18) = $\boxed{10}$

15

(b) Complete link is the maximum distance between two cluster

max (10, 10, 11.18) = $\boxed{11.18}$ Ans

(c) Average link: average distance btween two clusters.

$$\frac{11.18+10+10}{3} = 10.393 \sim \boxed{10.39}$$ Ans

(0.75)(0.2)(0.3)(0.4) = 0.018

Naive bayes basically would try to classify it as Shom as P(Shom)·(P(given condn / Shom))

since all occur simultaneously hence multiplied

9. Data: {(Ram,65,60),(Shyam,60,60),(Gita,60,70),(Mohan,70,70)}. Given that Ram, Shyam and Mohan are in one cluster and Gita is in the other cluster, determine: **[5x3 points]**

(i) Single-link distance between the two clusters: ~~10~~ 10

R(65,60)
S(60,60)
G(60,70) 1

M(70,70) 2

Complete ① on next page ①

Single link is the minimum distance between two clusters.

Calculating all distances as:

RM: $\sqrt{(70-66)^2+(70-60)^2} = \sqrt{25+100} = \sqrt{125} = 11.18$ —cont on next page ①

(ii) Complete-link distance between the two clusters: 14.14

complete link is the maximum distance ~~the~~ between two clusters.

from part ① we have, RM: 11.18, MS: 14.14, GM=10

max(11.18, 14.14, 10) = 14.14

11.18

(iii) Average-link distance between the two clusters: ______

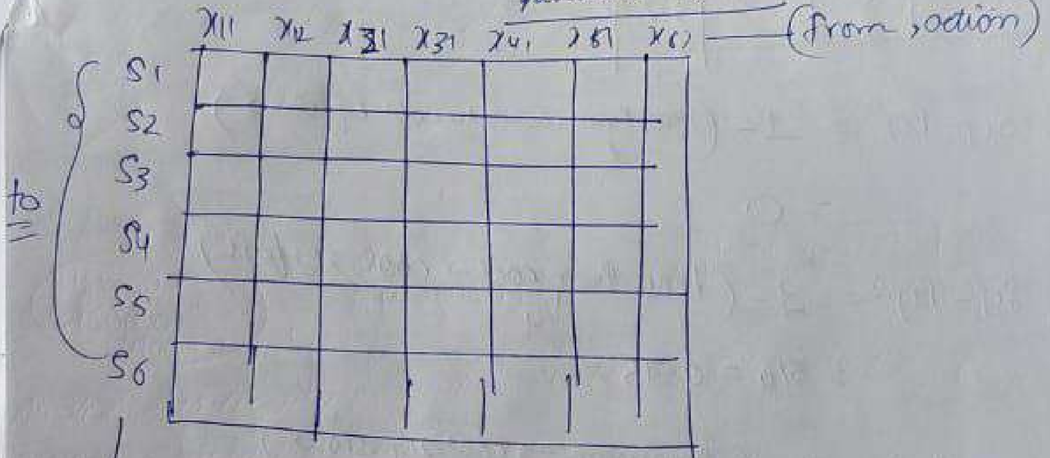
10.39

4. If any student is found indulging in malpractice ... given 'F' grade for the course and may be debarred from writing other examinations.

Q3(c) Task: max(rx)

↳ maximising total reward

Such that $Ax = b$ → indicate probabilities of being in particular states



↳ on vertical axis we write the states, on horizontal are the x_{ab} : moving from state a via action b !!

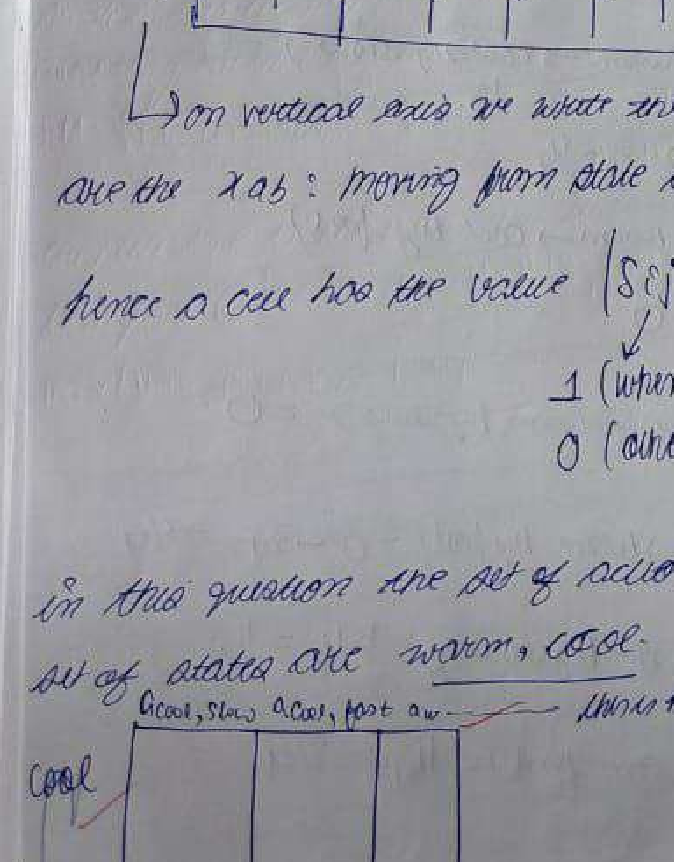
hence a cell has the value $|\delta_{ij} - P_{ij}^{a}|$

δ_{ij} : 1 (when $i=j$), 0 (otherwise)

P_{ij}^{a} ⟶ probability of reaching state j via action a from i.

In this question the set of actions are fast, slow.

set of states are warm, cool.



	acs	acf	aws	awf	no opp?
C	1	2	3	4	φ
W	5	6	7	8	φ
Opp?	φ	φ	φ	φ	

① $\delta_{ij} - P_{ij}^a = 1 - ($ how from cool → to cool by slow $)$ $=1$

$= 0$

② $\delta_{ij} - P_{ij}^a = 1 - ($ how from cool → cool by fast $)$ $1/4$

$= 3/4 = 0.75$

③ $\delta_{ij} - P_{ij}^a = 0 - ($ from warm → cool by slow $)$ $1/2$

$= 0 - 1/2 = -0.5 = -1/2$

④ $\delta_{ij} - P_{ij}^a = 0 - ($ from warm → cool by fast $)$

$= 0 - (0) = 0$

⑤ $\delta_{ij} - P_{ij}^a = 0 - ($ from cool → warm by slow $)$ 0 $= 0$

⑥ $\delta_{ij} - P_{ij}^a = 0 - ($ from cool → warm by fast $) = 0 - 3/4 = -3/4$

⑦ $\delta_{ij} - P_{ij}^a = 1 - ($ warm → warm by slow $) = 1 - 1/2 = 1/2$

⑧ $\delta_{ij} - P_{ij}^a = 1 - ($ warm → warm fast $) = 1 - 1/4 = 3/4$

---

once you get A do $Ax = \alpha$ ⟶ matrix

Q8) POMDP: partially observable markov decision processes !!

(a) It is false as 0.6 probability means that there are ~~0.60 per~~ 60% percent chances ~~I~~ gonna win In that case win utility amounts to $(0.60 \times 100) = 60$. It can't be the amount of money you get !! It is just utility hence false

(b) So, being risk neutral is when your utility also varies linearly with the amount, ~~clearly risk for losing is more than gaining~~ clearly winning is more than losing for hence,

$\cancel{U + U(x+c) < U - U(x-c)}$ $\qquad U(x+c) - U(x) > U - U(x-c)$

$\cancel{-U(x) + U(x+c) < U - U(x-c)}$

$\cancel{\frac{U(x+c)+U(x-c)}{2} < U(x)}$ $\qquad \underbrace{\frac{U(x+c)+U(x-c)}{2} > U(x)}$

~~hence if you are risk neutral~~ you play the bet!!

you might get 100 with utility 20 hence you shall & utility

(c) yes being risk averse we won't play because here utility of losing > utility of winning hence won't take risk of losing -100 by probability of 40% !!

(d) we know that new belief state is basically the updated probability i.e after considering the evidence confidence you got if it was (Z%, (100-Z)%) and if after watch

is increased by 15% it will be just (taking absolute)

$$\underbrace{((Z-15)\%, (100-Z+15)\%)}$$

new belief state after considering observation

Q3(c) optimal policy of robot is one that maximises reward !!

for the cool state, optimal policy would be to reach warm state by fast action thereby getting reward of (+10) and probability is 3/4. hence relatively higher than what other would provide

for the warm state, optimal policy would be to ~~a~~ off getting reward 10, probability of 3/4 !!

because the net motive is to maximise $\sum P(s'|s,a) \cdot U(s')$

(probability of reaching state s' via state s by taking action a) (U(s') be the value of state it has reached).

3. Borrowing writing ...
4. If any student is found indulging in malpractice ... from writing other examinations ... given 'F' grade for the course and may be debarred

Best of Luck

→ no, optimal policy won't get affected by changing γ since the probability and rewards are still constant

Q7-c

$$(0.02432) - \left(\left(\frac{1}{100}\right)(0.1568) + \left(\frac{99}{100}\right)(0.98)\right)$$

$$= (0.02432) - (0.001568 + 0.9702)$$

$$= -0.947448$$

Q7-a $\frac{P(\text{actually covid} + \text{test})}{P(\text{tested})} = \frac{\frac{30}{100} \times \frac{0.1}{10}}{\left(\frac{30}{100}\right) \times (0.1) + \left(\frac{1}{100}\right)(0.99)}$

$$= \frac{0.03}{0.03 + 0.0099} = \frac{0.03}{0.0399} \approx 75\%$$

— Thanks

Q6) Apriori Algorithm concept + Example

used for pruning out infrequent data set

defined ~~wrt~~ wrt to some threshold

It is based upon fact that a set is frequent if and only if all its subset are frequent!!

~~say that you have AB, BC, CE as frequent ones~~

~~now you can get ABC, ABEC but~~

say you have AB, BC, CD, AD

you formed ABD, BCD, CAD but !! CAD

ABC

will get pruned there's no CA!!

BCD will get pruned there is not BD!!

ABD will get pruned not BD!!

only ABC survives!!

that what question demands returning all the ones that got pruned (infrequent one).