Physics of the Early Universe (SC1.415)

IIIT-H, Semester Monsoon 24, Assignment 3:

Submission deadline: 21st November 2024

Submit codes also.

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1. Calculate the age of the universe in years in the ΛCDM model using the following density parameters measured by the Planck experiment

$$\Omega_\Lambda^0 = 0.6911\,, \quad \Omega_K = 0\,, \quad \Omega_M^0 = 0.308\,, \quad \Omega_R^0 = 10^{-5}\,.$$

Hint: The ΛCDM model lecture note has been uploaded with some notational changes and updated numbers.

2. The number density of photon between frequency ν and $\nu + d\nu$ in thermal equilibrium at temperature T is given by the Planck's distribution formula

$$n_T(\nu)d\nu = \frac{8\pi\nu^2}{c^3}\frac{d\nu}{\exp(h\nu/k_BT) - 1}\,.$$

Make a plot of energy density between ν and $\nu + d\nu$ as a function of frequency ν for different temperatures T. *Hint: Unlike in the class note, the speed of light c is explicitly kept. The numerical values of different parameters can be found in the notes or the internet.*

3. During the recombination process the ionization fraction is given as

$$X = \frac{-1 + \sqrt{1 + 4S}}{2S}\,, \quad S = n_\gamma \eta \left(\frac{m_e k_B T}{2\pi\hbar^2}\right)^{-3/2} \exp\left(\frac{B_1}{k_B T}\right) \tag{1}$$

where n_γ is the number density of photons at equilibrium temperature T. Make the following two plots: *a)* the isonization fraction X as a function of T, and *b)* the isonization fraction X as a function of T. Calculate the temperature and redshift at which $X = 1/2$.

4. Using the redshift $z_{\rm dec} = 1090$ at the time of decoupling obtain the *i)* horizon distance of light $d_{\rm hor}$ and *i)* the horizon distance of soundwave $d^S_{\rm hor}$. Calculate the angular diameter distance subtended by $d_{\rm hor}$ and $d^S_{\rm hor}$ at the present universe. If two points in the CMB makes an angle θ in the present universe, then show that if the angle $\theta > 1.9^o$ then the two points are not causally connected.