Physics of the Early Universe (SC1.415)

IIIT-H, Semester Monsoon 24, Assignment 1:

Submission deadline: 31st August 2024

You may take help from any source to solve the problems. But marks will be deducted if your understanding is not reflected in the answers.

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1. Consider two inertial frames S and S' such that their axes are parallel to each other and the S' frame moves with velocity v along the x axis with respect to the S frame. There is an analog clock at rest in the S frame. Show that as seen by an observer from the S' frame, the clock runs slower by a factor $\gamma = 1/\sqrt{1-v^2}$. This is the phenomenon of relativistic time dilation – time passes at different rates to different observer (moving clocks seem to run slower than stationary clocks). *Hint: Proper time is invariant,* i.e., *same for everyone – see notes..*

2. Consider two inertial frames S and S' such that their axes are parallel to each other and the S' frame moves with velocity v along the x axis with respect to the S frame. In the S' frame there is stick of length L_0 lying along the X axis. The length of the same stick when measured from the S frame is ℓ. Using the LT relations between the frames show that $\ell = L_0\sqrt{1-v^2}$. This is the phenomenon of relativistic length contraction. *Hint: In any frame, the act of measurement is simultaneous,* i.e., *the two ends of the stick are measured at the same time. You may assume that speed of light* $c = 1$.

3. Consider two frames S and S' such that their axes are parallel to each other and the S' frame moves with velocity v along the x axis with respect to the S frame. The velocity of an object is u in the S frame and u' in the S' frame. Show that (assuming $c = 1$)

$$u' = \frac{u-v}{1-uv}\,,$$

   This is how velocity addition occurs in relativisty. If v is non-relativistic, *i.e.,* $v << 1$ then $1 - uv \approx 1$, and $u' = u - v$ which is the usual Galilean transformation. *If you do not assume* $c = 1$ *then the formula is* $u' = (u-v)/(1-uv/c^2)$. The above formula can also be derived from the transformation of four velocities (see notes).

4. Using the velocity addition formula derived above, show that the speed of light is constant and is independent of the relative velocity between the source and the observer. *Hint: The object is now light.*

5. Without expanding the sum show that

$$A^\mu B_\mu = A_\nu B^\nu\,. \tag{1}$$

   Hint: Use property of the $\eta_{\mu\nu}$ *given in the class.*

6. Calculate *i)* $\partial_\mu x^\mu$, and *ii)* $\partial_\mu x^\nu$.