GEOMETRIC PROOF OF LINE INTERSECTION IN 3-SPACE

Given are two lines α and β, and two arbitrary points (which in my ray tracer's case are the origins of parametric representations of these lines) C and D, and the line segment/vector/line $\overline{CD}$, as in the following diagram:

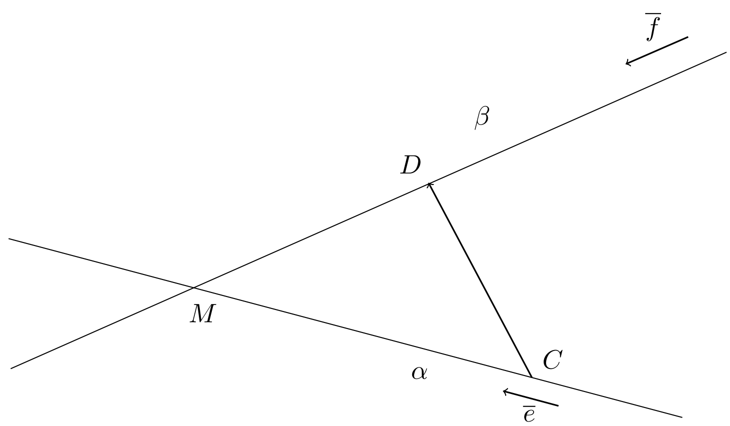

Figure 1.1: intersection diagram

Let e and f be the direction unit vectors of the lines α and β .

Assume that the two points are not either on both lines, since that would be the point of intersection.

If α and β have the same direction they don't intersect, in other words if $\|f \times e\|$ is zero then we know the lines don't intersect.

If the two arbitrary points are on the same line, but not equal, then we know the line won't intersect either, in other words if $\|f \times \overline{CD}\|$ is zero then we know the lines don't intersect.

Otherwise the point of intersection can be found along one of the lines by,

$$M = C \pm \frac{\|f \times \overline{CD}\|}{\|f \times e\|} e$$

Addition when $f \times \overline{CD}$ and $f \times e$ are in the same direction (cross-product of those $= 0$), subtraction otherwise.

Bewijs. herschijf de formule eerst tot,

$$CM = \|C - M\| = \| \pm \frac{\|f \times \overline{CD}\|}{\|f \times e\|} e\| = \frac{\|f \times \overline{CD}\|}{\|f \times e\|}$$

op deze manier wordt duidelijk dat we op zoek zijn naar de afstand van C tot de intersectie. Nu kunnen we geometry gebruiken om te bewijzen dat deze gelijkheid houd.

Laat ϕ de hoek tussen de richtingsvectoren e en f zijn.

Laat ψ de hoek tussen de richtingsvector f en de vector tussen twee punten op α en β $\overline{CD}$ zijn.

In diagrammen,

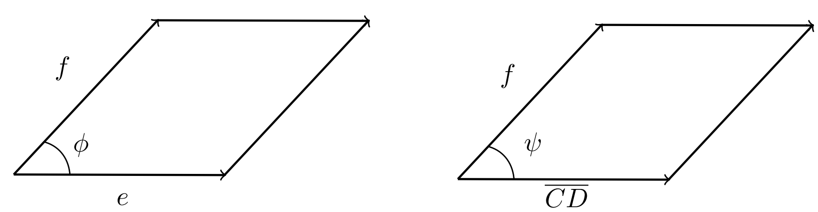

Figure 1.2: kruisproduct diagrammen

beide hoeken zijn aanwezig in het eerste diagram,

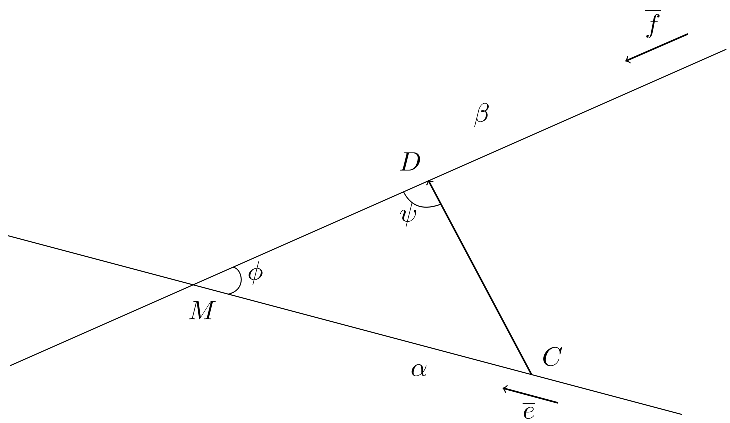

Figure 1.3: diagram met hoeken

Per definitie van het kruisproduct is,

$$CM = \frac{CD\|f\|\sin(\psi)}{\|f\|\|e\|\sin(\phi)}$$

Alle eenheid vectoren vallen nu weg,

$$CM = \frac{CD\sin(\psi)}{\sin(\phi)}$$

Per de wet van sinussen,

$$\frac{\sin(\psi)}{CM} = \frac{\sin(\phi)}{CD}$$
$$\sin(\phi) = \frac{CD\sin(\psi)}{CM}$$

Terug subsitueren en oplossen geeft dan het bewijs,

$$CM = \frac{CD\sin(\psi)}{\frac{CD\sin(\psi)}{CM}}$$

$\square$