Shape and Size

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Abstract

Humans possess the ability to imagine 3D objects. In this experiment we evaluate how object shape and content size impacts volume assessment. A 2^3 Factorial Randomized Complete Block Design was used. Participants were asked to guess the number of candy in a container. The three container shapes were roughly a sphere, cylinder, and cube. The contents were candies, namely skittles or sprinkles. Our results show that both candy selection and container shape significantly affect the ability to determine the volume of an object.

1 Introduction

Humans possess the ability to imagine 3D objects and mentally rotate said objects. This ability, though, has limitations. A container that is seldom encountered, say a sphere, could hinder imagination. The content of an object could also have an effect. If the size of the contents is very small, people may find it difficult to guess the volume. We designed a 2^3 factorial to test this ability. Three levels of container shape, namely sphere, cylinder, and cube, and two levels, small and large, of candy size were implemented. Participants were asked to guess how many candies were in each of the six objects, i.e. all participants were subjected to all treatments. The absolute difference of the guess and the actual amount in the container was used as the dependent variable. In an attempt to capture undesired variability, we blocked by examiner, experiment location, and participant.

2 Materials

The candies used were skittles and sprinkles purchased from Walmart. A gram of each candy was counted, and each container was then filled and weighed to get an approximate count. The containers, also purchased at Walmart, were a deli meat container (cube), a smart water bottle (cylinder), and the glued tops of two water jugs (sphere). Two of each were purchased, or made, to hold the two candy sizes for a total of six objects. Figure 1 shows the sphere container filled with sprinkles.

Figure 1: Sphere filled with Sprinkles

3 Methods

A 2^3 Factorial Unbalanced Complete Block Design was used. Twenty-seven individuals took part in the study. There were six testing locations, with a varying number of participants at each. Finally, two examiners recorded results. Two main effects, candy size and container shape, and their interaction term were estimated with the ANOVA procedure. Pairwise comparisons were evaluated via Tukey's range test. In order to reduce variability, we nested and blocked certain factors. Participant is nested within location, and location is nested in examiner. We sought to estimate the ability of the true population, not the ability of our participants. Thus, our participant block was a random effect. With the same reasoning, we implemented the examiner and experiment location as random effects. This design is not fully nested, because our two main effects are not nested in any of the blocking factors.

Our final model is:

$$y_{ijklmn} = \mu + \alpha_i + \beta_j + \phi_k + \delta_{l(k)} + \theta_{m(k,l)} + \alpha\beta_{ij} + \epsilon_{n(ijklm)} \qquad (1)$$

with random effects distributed

$$\phi_k \sim \mathcal{N}(0, \sigma_k)$$
$$\delta_{l(k)} \sim \mathcal{N}(0, \sigma_{l(k)})$$
$$\theta_{l(k)} \sim \mathcal{N}(0, \sigma_{m(k,l)})$$

4 Results

The model has an adjusted R^2 value 73.51%. The main effects Candy Type and Container Type are both highly significant. We find that both the size of the candy and the container shape affect the ability to assess volume. The interaction parameter is also highly significant. None of the blocking factors appear significant, though we ought not infer upon them regardless of this fact. Table 1 reports the ANOVA results. This table shows the main effect candy has a much, much higher F Statistic than any other factor.

Table 1: ANOVA table

Source	*df*	F	p
Candy	1	417.11	0.001
Container	2	7.66	0.001
Candy*Container	2	7.47	0.001
Location	4	1.37	0.54
Experimenter	1	0.57	0.54
Participant	21	1.14	0.318

Experiment Results

Next, we assess comparisons via Tukey's range test, commonly referred to as Tukey's Method. In table 2 we test the pairwise comparisons within the Container factor. This test reports that the Cylinder shape has a significantly different mean, while the Sphere and Box shapes do not significantly differ. Using the same method, we then tested the Candy factor. Table 3 shows the means of the two candy sizes differ significantly.

Table 2: Grouping Information using the Tukey Method and 95% Confidence

Container Type	*N*	*Mean*	*Grouping*	
1	54	57997.7	A	
3	54	51812.4	A	
2	54	36208.5		B

Means that do not share a letter are significantly different

Table 3: Grouping Information using the Tukey Method and 95% Confidence

Candy Size	*N*	*Mean*	*Grouping*	
2	81	96463.2	A	
1	81	869.1		B

Means that do not share a letter are significantly different

Finally, we assess the interaction term. Like all good statisticians, we begin by observing the interaction plot. Figure two purports to show a dip in the mean value for the Cylinder shape. There does not seem to be any visual evidence for any other interaction effect.

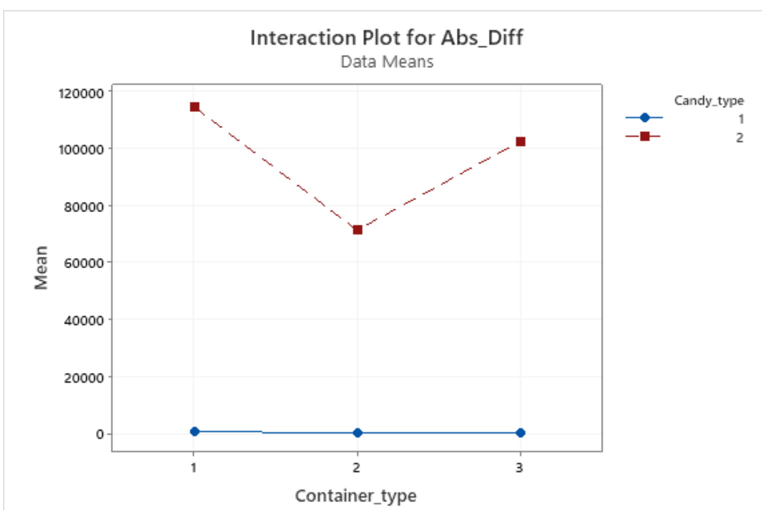

Figure 2: Interaction Plot

To evaluate the evidence suggested in the plot we continue with Tukey's method. Pairwise comparisons for all contrasts were tested. Table 4 confirms the evidence found in the interaction plot. The Cylinder shape combined with the small Candy size does in fact have a mean value significantly different from all other means.

Table 4: Grouping Information using the Tukey Method and 95% Confidence

Candy * Container	*N*	*Mean*	*Grouping*		
2 1	27	114924	A		
2 3	27	102780	A		
2 2	27	71686		B	
1 1	27	1032			C
1 3	27	844			C
1 2	27	731			C

Means that do not share a letter are significantly different

5 Conclusion

Our experiment shows that Container shape and Candy size affect the ability to estimate volume. Specifically, individuals seem to under guess lower values for Cylinders. We also report an interaction effect between Container and Candy, namely within small candy size and the cylinder shape. Future work could more precisely account for the order in which objects are presented. While our location factor does, in some way, capture the variability in presentation, I do not believe it completely accounts for this. There could also be carry over effect. The smaller candy sizes did seem to discourage participants. This could have affected subsequent guesses on large candy sizes.

6 Appendix

Factor Information

Factor	Type	Levels	Values
Location(Experimenter)	Random	6	1(1), 2(1), 3(2), 4(2), 5(2), 6(2)
Experimenter	Random	2	1, 2
Participant(Experimenter, Location)	Random	27	1(1, 1), 2(1, 1), 3(1, 1), 4(1, 1), 5(1, 2), 6(1, 2), 7(1, 2), 8(1, 2), 9(1, 2), 10(1, 2), 11(2, 3), 12(2, 3), 13(2, 3), 14(2, 3), 15(2, 3), 16(2, 4), 17(2, 4), 24(2, 4), 18(2, 5), 19(2, 5), 20(2, 5), 21(2, 5), 22(2, 5), 23(2, 5), 25(2, 6), 26(2, 6), 27(2, 6)
Candy_type	Fixed	2	1, 2
Container_type	Fixed	3	1, 2, 3

Figure 3: Factor Information

Model Summary

S	R-sq	R-sq(adj)	R-sq(pred)
29787.4	78.61%	73.51%	66.79%

Figure 4: Model Summary

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Experimenter	1	793215374	793215374	0.57	0.495 x
Candy_type	1	3.70098E+11	3.70098E+11	417.11	0.000
Container_type	2	13596970171	6798485085	7.66	0.001
Location(Experimenter)	4	5535128797	1383782199	1.37	0.278
Candy_type*Container_type	2	13259302406	6629651203	7.47	0.001
Participant(Experimenter, Location)	21	21196935097	1009377862	1.14	0.318
Error	130	1.15347E+11	887287690		
Total	161	5.39328E+11			

x Not an exact F-test.

Figure 5: Anova Table

Variance Components, using Adjusted SS

Source	Variance	% of Total	StDev	% of Total
Experimenter	-8.49664e+06*	0.00%	0.0	0.00%
Location(Experimenter)	14551609	1.58%	3814.7	12.56%
Participant(Experimenter, Location)	20348362	2.21%	4510.9	14.85%
Error	887287690	96.22%	29787.4	98.09%
Total	922187661		30367.5	

** Value is negative, and is estimated by zero.*

Figure 6: Variance Components