Assessing the Assessment-to-Sale Ratio

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Abstract

This paper examines the effect population demographics have on the assessment of homes in Milwaukee, Wisconsin. We examine Milwaukee residential data from 2018 and 2019 joined with American Community Survey 5 Year Estimates. We use hierarchical linear models (HLMs) and multiple mixed logistic regression models to assess the effect population demographics have on the assessment-to-sale ratio. We find that in Milwaukee, Wisconsin, assessment-to-sale ratios in black and hispanic communities, controlling for home characteristics, decrease as their respective populations increase. Further research into the relationship between income, race, and ethnicity could help find a causal link for this difference.

1 Introduction

Municipal assessors have a vested interest in ensuring that property assessments are a fair and equitable reflection of true market value. This paper attempts to examine the effect population demographics have on the assessment of homes in Milwaukee, Wisconsin. Minority communities as well as low income communities could be disproportionately affected, intentionally or not, by choices made in the valuation method of the Assessor's Office. Given past history and present concerns, this was a relevant question to examine. Our project's goal is to determine if Milwaukee property assessments change disproportionately according to demographic factors.

There is a history of local governments assessing properties in predominantly black and/or low-income neighborhoods at a greater overall rate than properties in predominantly white and/or high-income neighborhoods. For example, in Boston, Massachusetts in 1960, the mean assessment-to-sale (ATS) ratio of single-family homes was significantly higher in nonwhite neighborhoods compared to mostly white neighborhoods (0.64 compared to 0.37), and in low income neighborhoods compared to middle and high income neighborhoods (0.53 compared to 0.37 and 0.35 respectively)[1]. Because those being assessed would be paid the same nominal property tax rate, if a person's property is assessed higher than the sales price of their property, then that property is being overtaxed. Alternatively, under assessment can make it difficult to access credit, as banks sometimes rely on county assessments for mortgage applications.

The variable of interest is the ATS Ratio, which consists of the home's assessment value divided by

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[1] David E. Black, "The Nature and Extent of Effective Property Tax Rate Variation Within the City of Boston." National Tax Journal (1972), 206 (Table II)

it's sale price. The mean of the ATS Ratio distribution is not necessarily of concern. If all individuals are equally mis-assessed, no discrimination is occurring. When different properties are over- or under-assessed differently, tax revenue is disproportionately raised. As noted above, taxation is not the only concern; accessing credit could be made increasingly difficult through consistent under-assessment. It is important that particular groups are not being over-assessed or under-assessed. This, in particular, is what we attempt to investigate.

2 Data

The City of Milwaukee Assessor's Office provided data including $8,572$ property sales spanning the years 2018 and 2019. Each observation was an individual home sale, which recorded 61 variables. These variables include home characteristics: sale price, assessed value, land size, census tract, building type, overall quality, kitchen quality, primary wall type, number of bedrooms, etc. Additional census tract level data, namely Median Income, Percent Hispanic, Non-Hispanic White, and Black population, Median Land Size, etc., was retrieved from the American Community Survey 5 Year Estimates[2]. This information was merged by census tract with the data provided by the Assessor's office.

Many unusual sales occur in markets. We are only interested in ordinary residential sales of single housing units -i.e. non-duplex or multifamily residential complexes. Following Makovi (2019)[3], we filtered out any implausible sale price values (below $20,000 and above $10 million). We then follow the recommendation of the International Association of Assessing Officers (IAAO 2013:53) and eliminate all observations whose ATS ratios fell outside the 1.5xIQR range [4]. This process eliminates most of the unusual observations.

The Land Size Units variable was not consistent. Most remaining observations, $5,899$, were measured in front footage, while 894 were measured in square footage. It is impossible to convert these variables, and the Land Size variable is of interest in this study, so we drop the latter group. There were 3 repeat observations, so to keep our data IID we retain the most recent sale.

The data cleaning procedure left us with $5,813$ unique home sale observations.

3 Data Visualization

The raw distribution of the ATS Ratio in our sample is extremely right tailed (Figure A1)[5]. This issue was corrected by the quantile snipping described in the previous section. After removing those observations, the data appears normally distributed (Figure A2). However, according to the Shapiro-Wilk test the data still is not normally distributed. After consultation with our supervising professor, Dr. Banerjee, the cause of this was determined to be the fat tails of the artificially created

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[2]U.S. Census Bureau; American Community Survey, 2019 American Community Survey 5-Year Estimates; generated by Vinnie Palazeti; using api.data.census.gov; (11 December 2020)

[3]Michael Makovi, Is There Discrimination in Property Taxation? Evidence from Atlanta, Georgia, 2010-2016 , (Free Market Institute at Texas Tech University)

[4]We eliminated observations whose ratio is less than the 25th percentile minus 1.5xIQR or more than the 75th percentile plus 1.5xIQR

[5]Figures with "A" prefix are located in the Appendix

distribution. We continue with the analysis notwithstanding these facts, but felt obligated to share these results.

There are seasonal trends in the frequency of ATS Ratio deciles (Figure 1). There is a higher concentration of the 10th decile in 2018, while ATS Ratios in the 1st decile are more common in 2019. We are unsure why this pattern exists. Most purchases occur in the summer, and tail off in the winter.
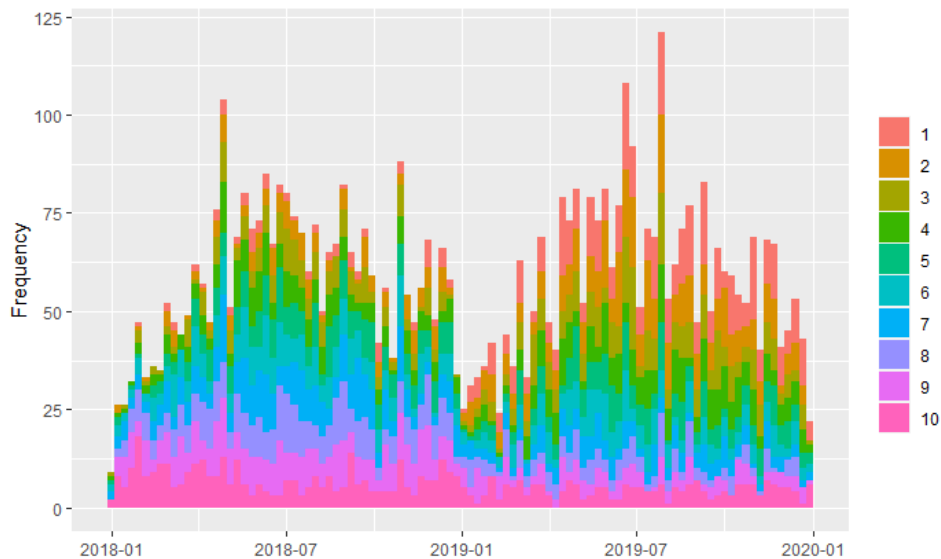

Figure 1: ATS Ratio Decile by Sale Date

The American Community Survey provides population estimates for census tracts. We utilize these estimates, in combination with TIGER/Line Shapefiles, to create the following visualizations. We see, in stark relief, how stratified Milwaukee communities are. These visualizations are near non-overlapping sets. Census tracts that are mostly white are found in the outer sections of the city (Figure 3). Hispanics tend to live towards the city center (Figure 2). African-Americans tend to live in the northern part of the city (Figure 4).
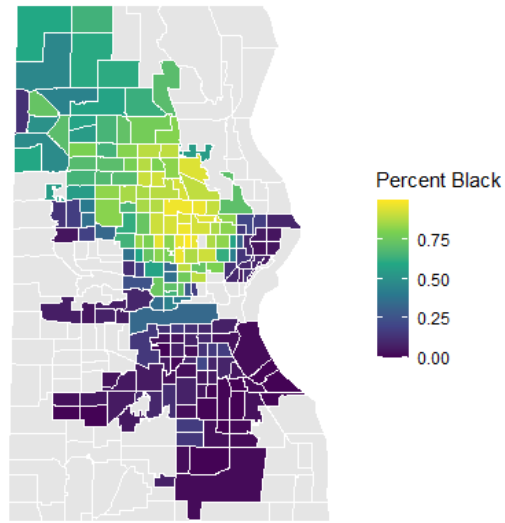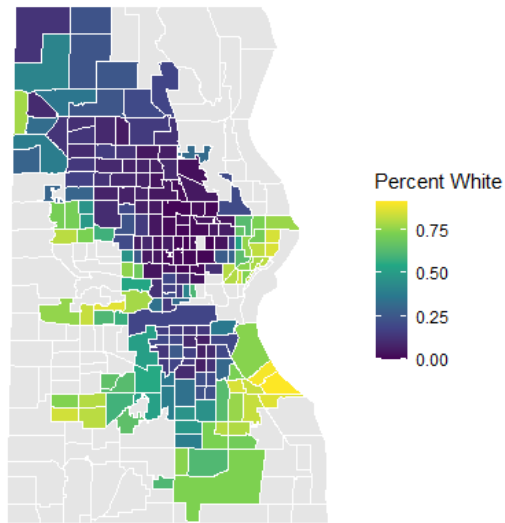
Figure 2: Black Population

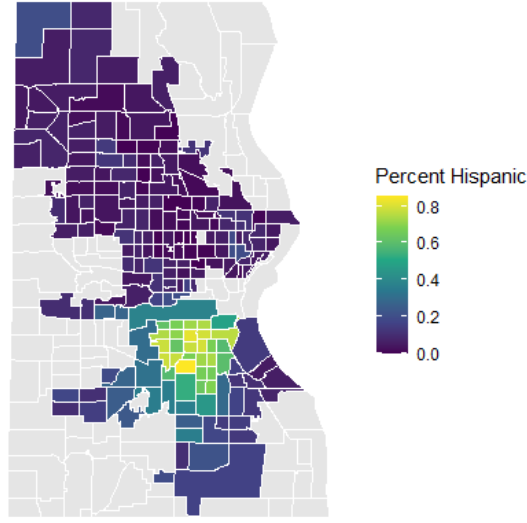Figure 3: Non-Hispanic White Population

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Figure 4: Hispanic Population

Higher income, lower poverty areas tend to be on the outskirts of the city, predominantly to the south (Figure A3). Lower income, higher poverty areas are located closer to the city center and to the north (Figure A4). This is our first introduction to the correlations between racial population demographics and socio-economic status. These correlations will be further explored in the following sections.

4   Pre Regression Analysis

Table 1 provides the Median ATS Ratio by Population Decile. We are interested in how neighborhood population demographics affect assessment, so this is a naturally place to begin. Visually, trends jump out. As the the black population increases, the ATS Ratio trends down. The opposite trend is shown for the White population. This chart purports to show that an increase in White population also increases the median ATS Ratio. There is no visual evidence for a trend within the Hispanic column.

Table 1: Median ATS Ratio by Population Decile

Population Decile	Black	White	Hispanic
1	0.9149	0.8748	0.8847
2	0.8968	0.8695	0.9012
3	0.8892	0.8695	0.8892
4	0.9038	0.8872	0.9016
5	0.8876	0.8948	0.8835
6	0.9069	0.9087	0.8988
7	0.9014	0.9016	0.8914
8	0.8869	0.8999	0.9059
9	0.8713	0.9078	0.9073
10	0.8695	0.9063	0.8748

Table 2 reports the median ATS Ratio within economic strata, namely Median Income, Sales Price, Renter deciles. Similar visual trends jump out. As the decile of median income increases, the ATS Ratio trends toward 1. The trend within sale price decile is opposite and more extreme. The difference within the first and last Sale Price decile is 0.08, compared to 0.02 within the Income deciles 0.009 within the Renter Deciles. There is no visual evidence for a trend within the Renter deciles.

Table 2: Median ATS Ratio by Economic Decile

Decile	Median Income	Sale Price	Renter
1	0.8830	0.9535	0.8914
2	0.8709	0.9487	0.9011
3	0.8730	0.8965	0.9114
4	0.8937	0.9034	0.8992
5	0.8951	0.9092	0.9111
6	0.8935	0.8959	0.8909
7	0.9059	0.8831	0.8828
8	0.9028	0.8756	0.8679
9	0.9077	0.8605	0.8679
10	0.9068	0.8748	0.9006

We use the Mann-Kendall Monotonic Trend test to evaluate this visual evidence. The null hypothesis for this test is that there is no continuously increasing or decreasing (monotonic) trend. The alternative for the two-sided test is there is a monotonic trend. The results of each M-K test are reported in Table 3.

Table 3: Mann-Kendall Monotonic Trend Test

Decile Group	Tau	P Value
Black Population	-0.556	0.0318^*
White Population	0.644	0.0123^*
Hispanic Population	0.156	0.5915
Median Income	0.644	0.0123^*
Sale Price	-0.867	0.0007^{***}
Renter Population	-0.422	0.1074

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

We find significant evidence ($p < 0.05$) for a negative monotonic trend in median ATS Ratio within the Black Population and Sales Price deciles. The p value for Sale Price (0.0007) is much lower than that of Black Population (0.0318). This reflects our intuitive analysis of the large median ATS ratios range within Sale Price deciles. Significant evidence ($p < 0.05$) for a positive monotonic trend is reported for both White Population and Median Income Deciles. We find no significant evidence ($p > 0.05$) for monotonic trends within Renter Population and Hispanic Population deciles.

Figure A5 visualizes the dataset's observation level correlation matrix. Intuitive striations between Kitchen and Full Bath Quality appear in the middle of the correlation plot. Large negative correlations within Kitchen Quality, Full Bath Quality, Condition, and Overall Quality are shown. This is, again, an intuitive result, as these variables are mutually exclusive within their respective sets. Overall, we do not see any odd or extreme correlations.

Figure A6 visualizes the dataset's tract level correlation matrix. There is an extreme negative correlation (-0.84) between Percent Black and Percent White. This reflects the previous visual population stratification reported in the maps of Milwaukee populations. Median Income is positively correlated (0.78) with Percent White and negatively correlated (-0.63) with Percent Black. There is an obvious correlation (-0.80) between Percent Income Poverty and Median Income.

Correlations between variables within the design matrix can lead to multicollinearity. As noted above, the extreme correlation between Percent Black and White populations is due to the stratified population. Black & White Milwaukeeans congregate in different neighborhoods, and are the majority population. This results in their percent populations, nearly always, being reflections. If the population of a tract is 35% Black, the White population is $\sim 60\%$. This holds true for the majority of tracts. Therefore, these variables provide roughly the same information. In regression analysis, either side of an equally divided ratio provides equivalent information. We have encountered this situation in the Milwaukee populations. Thus, we will only include Percent Black and Percent Hispanic in our regression models, leaving Percent White as the reference level.

The correlation between Median Income and Percent Black or White could be troublesome. We will analyze the variance inflation factor of our models to determine if multicollinearity is an issue here. Due to the high negative correlation between Percent Income Poverty and Median Income,

and the fact each is describing the same phenomenon, we will drop the former from the analysis.

5 Methods

Regression models allows us to move beyond univariate analysis and assess the effect of multiple covariates on the ATS Ratio. Two sets of independent variables were employed, which are referred to as Simple and Specific. Tables 4 and 5 display the variables used in the Simple and Specific models, respectively. The goal of this methodology is to determine how or if increasing the specificity of observation level characteristics impacts the Percent Population coefficients.

Table 4: Simple Model Variables

Tract Level	Observation Level
Percent Black	Log Land Size
Percent Hispanic	Age
Log Median Land Size	Kitchen Quality
Log Median Income	Bathroom Quality
Log Median Sale Price	Overall Condition
Census Tract	Overall Quality
	Home Type
	# of Kitchens
	# of Bedrooms
	# of Full Bathrooms
	# of Half Bathrooms
	Primary Wall
	Sale Month & Year

Table 5: Specific Model Variables

Tract Level	Observation Level
Percent Black	Log Land Size
Percent Hispanic	Age
Log Median Land Size	Kitchen Quality
Percent Land Size Above	Bathroom Quality
Percent Land Size Below	Overall Condition
Log Median Income	Overall Quality
Log Median Sale Price	Home Type
Percent Sale Price Above	# of Kitchens
Percent Sale Price Below	# of Bedrooms
Census Tract	# of Full Bathrooms
	# of Half Bathrooms
	Primary Wall
	Sale Month & Year

Multiple Linear Regression is an insufficient method to assess our research question. The predictor variables in our design matrix vary at hierarchical levels. Specifically, observations within a Census Tract share variance according to their location. Therefore, we employ Hierarchical Linear Modeling (HLM), sometimes referred to as Mixed Effect Modeling. This allows us to use Census Tract as a random effect, and thus capture the variance within and between Tracts.

All numeric variables are scaled and centered prior to modeling. This allows for interpretation on a common scale. Therefore, a one unit change in a dependent variable translates to a one standard deviation in our outcome variable. Because our coefficients are normally less than one, they can be interpreted as, "% of a standard deviation change." We are interested in the specific effect of Percent Black population on the ATS Ratio. The remaining variables in our design matrix are held as constants. The entire regression output is available upon request.

There are interesting implications using a ratio bounded $[0, \infty]$ as the dependent variable. Kronmal (1933)[6] tells us that the interpretation of coefficients is the joint effect, or interaction, of the coefficient and the denominator of the dependent variable. This is a complicating factor. Therefore, we repeated all of our Hierarchical Models using the Log Assessment Value as the dependent variable and included the Log Sales Price as a covariate.

Exploratory analysis revealed the distribution of ATS Ratios for Majority Black neighborhoods was much flatter and left tailed than the total population, Majority White neighborhoods, and Majority Non-Black neighborhoods. This lead us to question if the likelihood of a lower ATS ratio was greater in Black neighborhoods. To answer this question we use Logistic Regression.

6   Results

6.1   Hierarchical Linear Models

We find consistent and significant effects from Percent Black and Percent Hispanic population on the ATS Ratio. Table 6 reports the coefficients of interest, their respective standard error, and the intraclass correlation coefficients for both Simple and Specific models. As noted in the table, all p values are highly significant ($p < 0.001$).

In the Simple model, the Percent Black coefficient and Percent Hispanic coefficient are about -0.12. This coefficient indicates that a one standard deviation increase in the percent Black or Hispanic population is associated with a 0.12 standard deviation decrease in the assessment ratio. This is 12% of a standard deviation. The standard deviation of the assessment ratio, in our data, is about 0.13, so 12% of this is 0.016 or 1.16%. Therefore, observations in a neighborhood with a one standard deviation increase in the percent Black or Hispanic are subject to about a 1.16% decrease in their ATS Ratio.

Next, we interpret the Specific Model results. When the specificity of observation characteristics is increased, we see the coefficients for Percent Black and Percent Hispanic increase. This provides evidence for specific bias relating to the neighborhood demographics. The Percent Black coefficient

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[6]Kronmal, Richard A. "Spurious Correlation and the Fallacy of the Ratio Standard Revisited." Journal of the Royal Statistical Society. Series A (Statistics in Society), vol. 156, no. 3, 1993, pp. 379–392

Table 6: Hierarchical Linear Model

Dependent Variable: ATS Ratio

	Parameter	Simple	Specific
Percent Black	β	−0.1218***	−0.4642***
	Stdev	0.04271	0.0587
Percent Hispanic	β	−0.1198***	−0.1839***
	Stdev	0.02684	0.03498
	ICC	0.0510	0.2037

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

in this model is −0.4642. This coefficient indicates that a one standard deviation increase in the percent Black population is associated with a decrease in 46% of a standard deviation in the assessment ratio. As noted, the standard deviation of the assessment ratio is 0.13. Therefore, 0.46% of this is 0.0614 or 6.14%. Thus, observations in a neighborhood with a one standard deviation increase in the percent Black are subject to a 6.14% decrease in their ATS Ratio.

The ATS Ratio is the ratio of the Assessment Value over the Sale Price. In the methods section we provided insights on interpretation from Kronmal (1933). To revisit this, these coefficients are actually the joint effect, or interaction of Sale Price and Percent Black or Hispanic. Depending on ones decisions in the forking paths, this may be either appropriate or not. To alleviate possible objections, we constructed identical models with Assessment Value as the dependent variable and Sale Price as a covariate.

Similar results are found in the models with Log Assessment Value (Table 7) as the dependent variable. Both models report highly significant and negative coefficients for Percent Black and Percent Hispanic. Though, we find mixed results in the transition from the Simple to Specific models. The Percent Black coefficient decreases from −0.1500 to −0.1518, while the Percent Hispanic coefficient increases from −0.5080 to −0.553. Also, the Percent Hispanic coefficient is half as impactful in these models. This provides some evidence for the significance of the interaction between Percent Hispanic and Sale Price.

Table 7: Hierarchical Linear Model

Dependent Variable: Log Assessment Value

	Parameter	Simple	Specific
Percent Black	β	−0.1500***	−0.1518***
	Stdev	0.01871	0.0196
Percent Hispanic	β	−0.0580***	−0.0553***
	Stdev	0.0112	0.0116
	ICC	0.2144	0.2293

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Due to the complexity of these models, we believe interpretation should be done cautiously. Attempting to nail down the specific percentage effect of any single parameter is difficult in high dimensional modeling. Therefore, we opt to take a holistic overview. We observe a constant, negative, and significant impact from Percent Black and Percent Hispanic. We infer from these facts that a negative relationship exists between these parameters and their respective dependent variable.

6.2 Logistic Regression Models

The distribution of ATS Ratios in Majority Black neighborhoods (A6 & A7) is visibly flatter than the overall distribution (A2). This led us to question the likelihood of an ATS Ratio one standard deviation away from the mean. We use Logistic Regression to answer this question.

Following the structure used in the HLM analysis, we sequentially build three models referred to as Simple, Intermediate, and Specific. The variables used in each are listed in Tables A1, A2, and A3, respectively. The purpose of this structure is to determine if increasing the specificity of home characteristics increase or decreases the variable of interest's effect. Each model uses Census Tract as a random effect. Our dependent variable of interest is a binary indicator for an ATS Ratio one standard deviation away from the mean. We build models for both one standard deviation above and one standard deviation below. The predictor of interest is the Percent Black Population.

All numerical variables have been centered and scaled. Therefore, a one unit increase corresponds to a one standard deviation increase in the covariate of interest. The glmer package in R was used to build these models. Unfortunately, with the default nAGQ set to 1, our models would not converge. The nAGQ parameter is the number of points per axis used in evaluating the adaptive Guass-Herminte approximation of the log-likelihood[7]. Values greater than 1 produce greater accuracy, while values lower than one are less accurate. In order for our models to converge we set the value of nAGQ to zero. Therefore, a less exact form of parameter estimation is found. In larger datasets, there is evidence that this setting often makes no difference[8].

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[7] Glmer Function — R Documentation." n.d. RDocumentation.org. Accessed December 11, 2020
[8] Alday, Phillip. "[R-Sig-ME] NAGQ = 0." Stat.Ethz.Ch. September 4, 2017.

Table 8: Multiple Mixed Logistic Regression Models

Dependent Variable: ATS Ratio One Standard Deviation Below

	Parameter	Simple	Intermediate	Specific
Percent Black	β	0.38058***	1.17450***	1.54064***
	Stdev	0.05657	0.11267	0.17196

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

	Parameter	Simple	Intermediate	Specific
Percent Black	Odds Ratio	1.46***	3.24***	4.67***

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

We report significant ($p < 0.001$) coefficients for Percent Black population in all models. Also, as the specificity of observation characteristics increases, the effect of Percent Black population increases. In the Simple model, for a one standard deviation increase in the Percent Black Population, the odds an ATS Ratio one standard deviation below the mean increase by a factor of 1.46. This can also be read as a 46% increase in the odds of an ATS Ratio one standard deviation below the mean. In the Intermediate model, this percentage jumps to a 224% increase in the odds. Finally, in the Specific model, which accounts for the most observation characteristics, we report an odds ratio of 4.67. Therefore, for a one standard deviation increase in the Percent Black Population, the odds of an ATS Ratio one standard deviation below the mean increase by 367%.

The opposite trends are found when the dependent variable is switched to an ATS Ratio one standard deviation above the mean. Table 9 reports the coefficients and odds ratios of interest. As the specificity of home characteristics increase and the Percent Black population increase, the odds of having an ATS Ratio one standard above the mean decrease.

Table 9: Multiple Mixed Logistic Regression Models

Dependent Variable: ATS Ratio One Standard Deviation Above

	Parameter	Simple	Intermediate	Specific
Percent Black	β	0.22***	-1.03***	-1.34***
	Stdev	0.044	0.117	0.181

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

	Parameter	Simple	Intermediate	Specific
Percent Black	Odds Ratio	1.25***	0.36***	0.26***

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

We reiterate the concern about specific interpretation of model coefficients here. Due to the high dimensionality of the models presented, the inference on specific coefficients as "true" measurements, we believe, is not appropriate. We do infer, in an overall manner, that as the Percent Black population increases, accounting for home characteristics, the odds of having an ATS Ratio one standard deviation below the mean increases. Conversely, given the same conditions, the odds of having an ATS Ratio one standard deviation above the mean decreases.

7   Conclusion

We find evidence that populations demographics do affect the ATS Ratio. In both of our analyses, Percent Hispanic and/or Percent Black, while controlling for home characteristics and spatially correlated observations via random effects, are significant factors which negatively affect the ATS Ratio. As noted by Makovi, future studies should combine data on county assessments with data on private assessments and loans to determine whether county under-assessment impedes access to credit. We attempted to retrieve this type of information, but it was unavailable. We hope other researchers will continue this work and provide feedback on where we surely made mistakes.

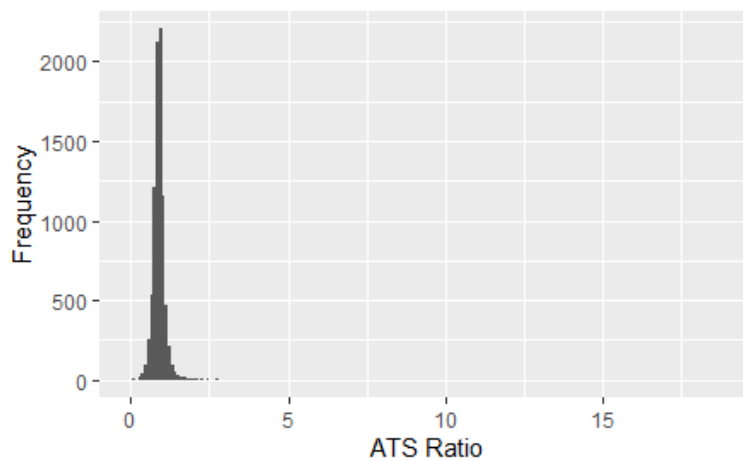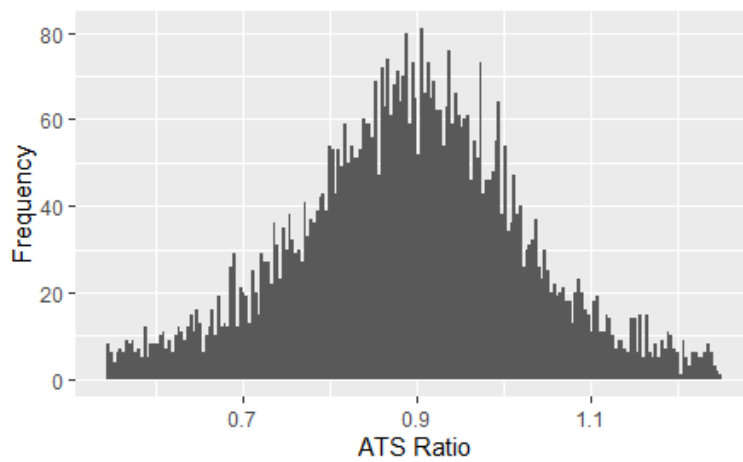# 8 Appendix

Figure A1: Raw ATS Ratio Distribution

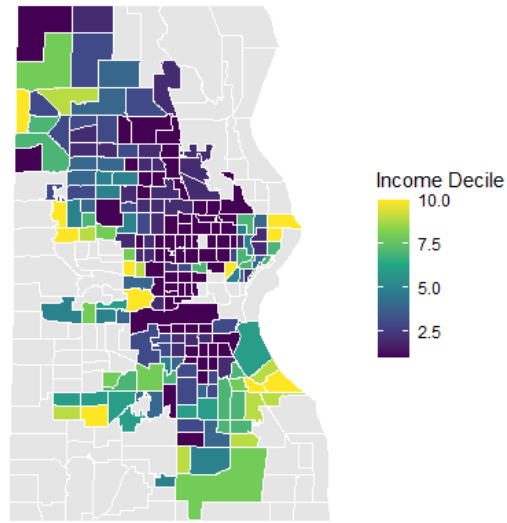Figure A2: Final ATS Ratio Distribution
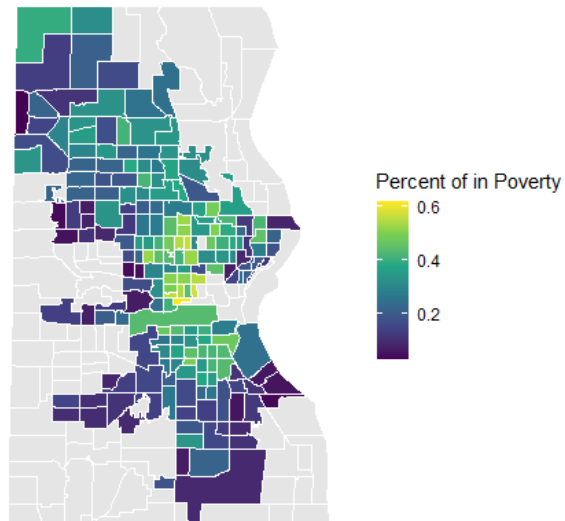
Figure A3: Median Income Estimates
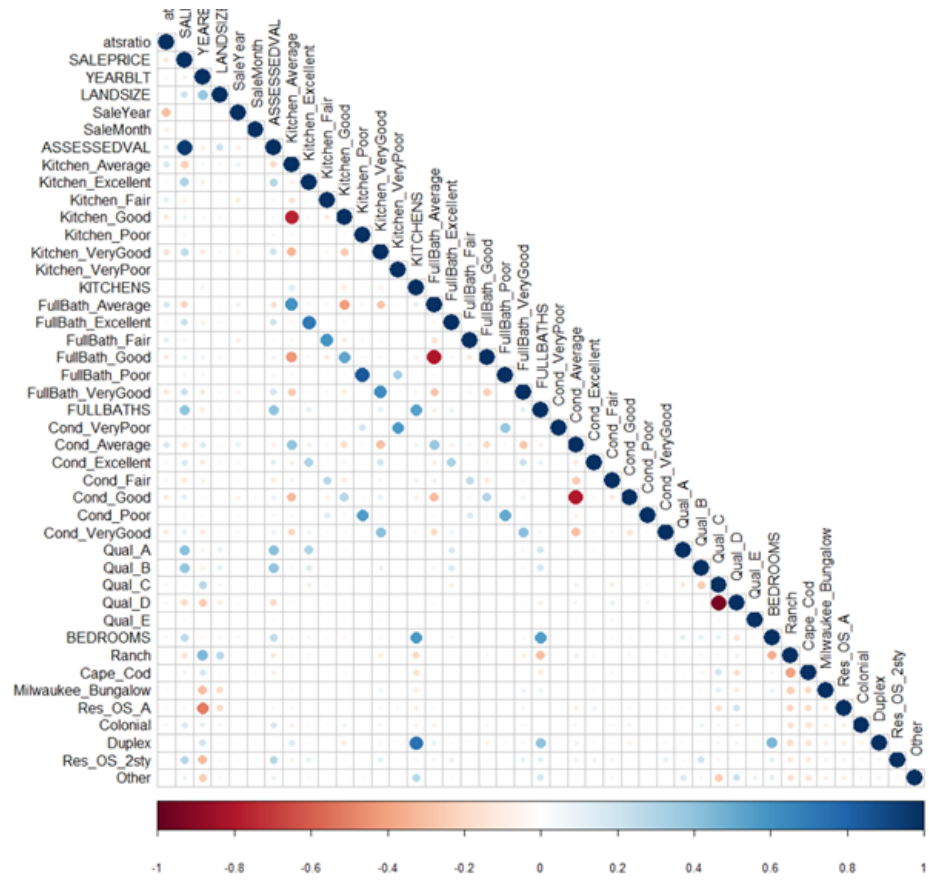

Figure A4: Income Poverty Rate

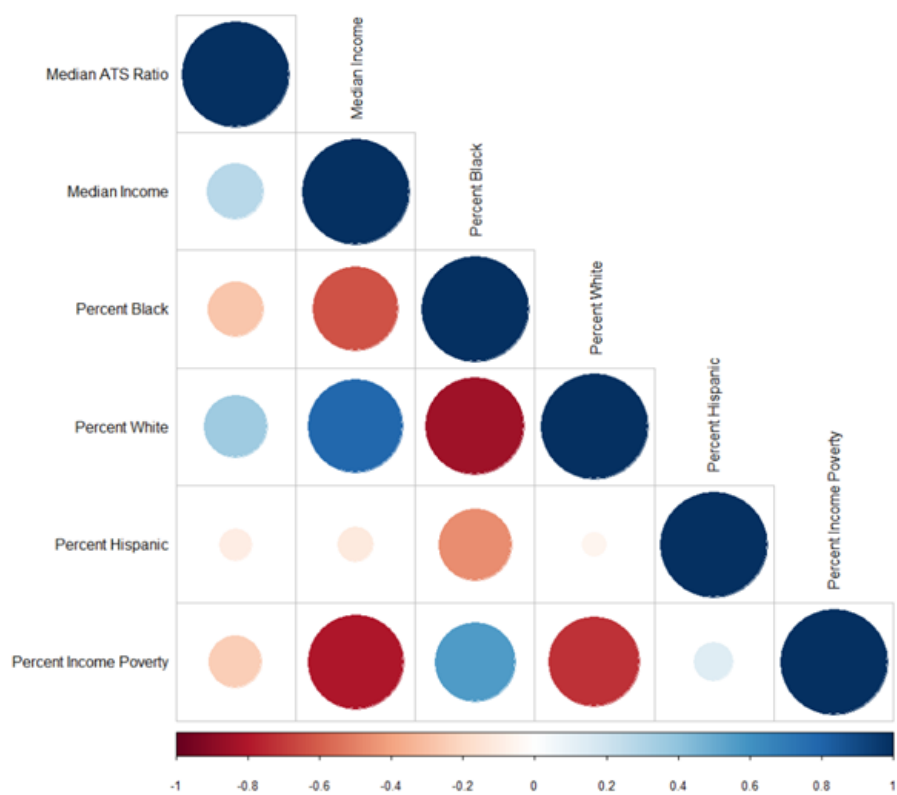Figure A5: Observation Level Correlations
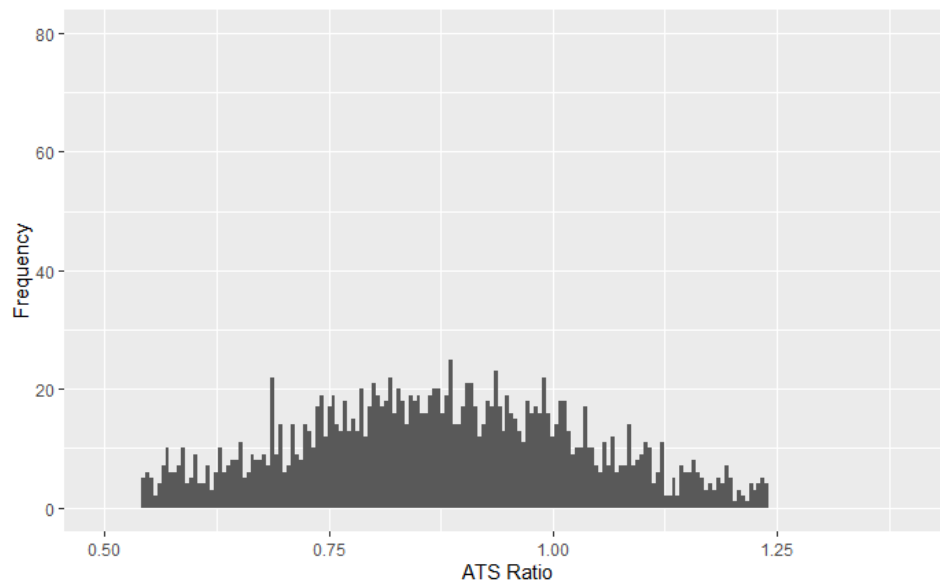
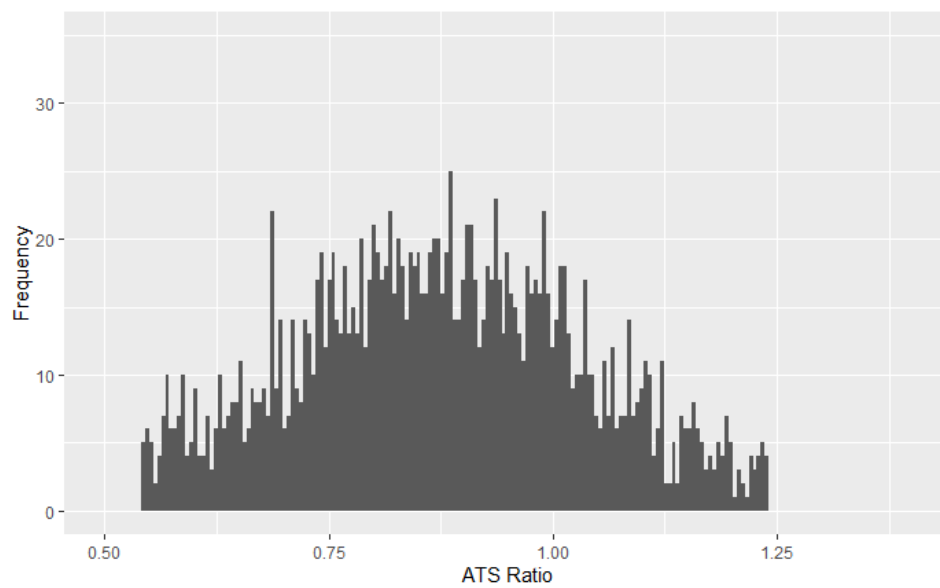Figure A6: Tract Level Correlations

Figure A7: Majority Black Neighborhoods

Figure A8: Majority Black Neighborhoods Rescaled

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Table A1: Logistic Regression: Simple Model Variables

Tract Level	Observation Level
Percent Black	Census Tract ATS Ratio One STDEV Below or Above Mean

Table A2: Logistic Regression: Intermediate Model Variables

Tract Level	Observation Level
Percent Black	Census Tract ATS Ratio One STDEV Below or Above Mean Log Sale Price

Table A3: Logistic Regression: Intermediate Model Variables

Tract Level	Observation Level
Percent Black Log Median Income	Census Tract ATS Ratio One STDEV Below or Above Mean Log Sale Price Log Land Size Home Age of Kitchens of Full Baths of Half Baths of Bedrooms Kitchen Quality Overall Condition Type of Home Primary Wall Sale Month Sale Year

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U.S. Census Bureau; American Community Survey, 2019 American Community Survey 5-Year Estimates, Tables B01003_001E, B02001_002E, B02001_003E,
B03002_003E, B19013_001E, B03003_003E, B17020_001E, B17020_002E,
B17020A_002E, B17020B_002E, C17002_002E, C17002_003E, C17002_004E,
B25003_002E, B25008_001E
; generated by Vinnie Palazeti; using api.data.census.gov; (11 December 2020).