Numerical problem on decision tree

ID3 Algorithm

1. Invented by J. Ross Quinlan
2. Employs a top-down greedy search through the space of possible decision trees.
3. Greedy because there is no backtracking. It picks the highest values first.
4. Select the attribute that is most useful for classifying examples (the attribute that has the highest Information Gain).

Numerical problem on decision tree using GINI INDEX

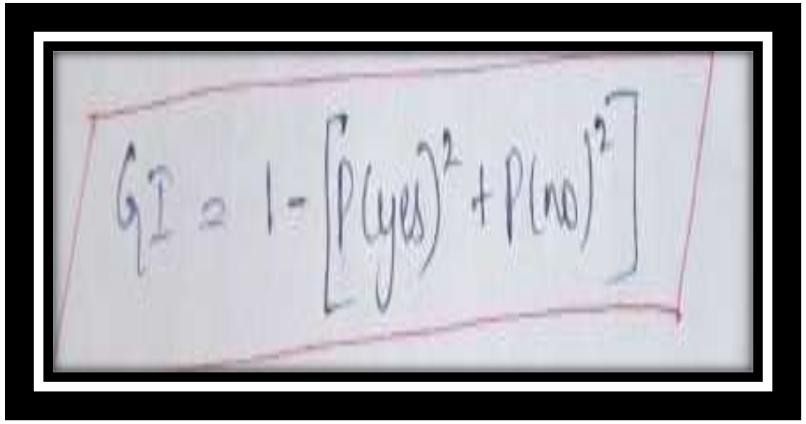
Steps

Find the Gini index for the entire dataset

Find the Gini index for each attribute

Formula for Gini index:

$$\text{Gini} = 1 - \sum_{i=1}^{n} (p_i)^2$$



Weekend	Weather	Parents	Money	Decision
W1	Sunny	Yes	Rich	Cinema
W2	Sunny	No	Rich	Tennis
W3	Windy	Yes	Rich	Cinema
W4	Rainy	Yes	Poor	Cinema
W5	Rainy	No	Rich	Stay In
W6	Rainy	Yes	Poor	Cinema
W7	Windy	No	Poor	Cinema
W8	Windy	No	Rich	Shopping
W9	Windy	Yes	Rich	Cinema
W10	Sunny	No	Rich	Tennis

Decision Tree using Gini Index – Solved Example

Weekend	Weather	Parents	Money	Decision
W1	Sunny	Yes	Rich	Cinema
W2	Sunny	No	Rich	Tennis
W3	Windy	Yes	Rich	Cinema
W4	Rainy	Yes	Poor	Cinema
W5	Rainy	No	Rich	Stay In
W6	Rainy	Yes	Poor	Cinema
W7	Windy	No	Poor	Cinema
W8	Windy	No	Rich	Shopping
W9	Windy	Yes	Rich	Cinema
W10	Sunny	No	Rich	Tennis

- Compute the Gini Index for the overall collection of training examples.
- There are four possible output variables Cinema, Tennis, Stay In and Shopping.
- The data has 6 instances of Cinema, 2 instances of Tennis, 1 instance of Stay In and 1 of shopping.
- $Gini(S) = 1 - [\left(\frac{6}{10}\right)^2 + \left(\frac{2}{10}\right)^2 + \left(\frac{1}{10}\right)^2 + \left(\frac{1}{10}\right)^2] = 0.58$

Decision Tree using Gini Index – Solved Example

Weekend	Weather	Parents	Money	Decision
W1	Sunny	Yes	Rich	Cinema
W2	Sunny	No	Rich	Tennis
W3	Windy	Yes	Rich	Cinema
W4	Rainy	Yes	Poor	Cinema
W5	Rainy	No	Rich	Stay In
W6	Rainy	Yes	Poor	Cinema
W7	Windy	No	Poor	Cinema
W8	Windy	No	Rich	Shopping
W9	Windy	Yes	Rich	Cinema
W10	Sunny	No	Rich	Tennis

- Computation of Gini Index for Money Attribute
- It has two possible values of Rich (7 examples) and Poor (3 examples).
- For **Money = Poor,** there are 3 examples with "Cinema".
- $Gini(S) = 1 - [\left(\frac{3}{3}\right)^2] = 0$ ✓
- For **Money = Rich,** there are 2 examples with "Tennis", 3 examples with "Cinema" and 1 example with "Stay in", "Shopping" each
- $Gini(S) = 1 - [\left(\frac{2}{7}\right)^2 + \left(\frac{3}{7}\right)^2 + \left(\frac{1}{7}\right)^2 + \left(\frac{1}{7}\right)^2] = 0.694$
- $Weighted\ Average(Money)$

$$= 0 * \left(\frac{3}{10}\right) + 0.694 * \left(\frac{7}{10}\right) = 0.486$$

Decision Tree using Gini Index – Solved Example

Weekend	Weather	Parents	Money	Decision
W1	Sunny	Yes	Rich	Cinema
W2	Sunny	No	Rich	Tennis
W3	Windy	Yes	Rich	Cinema
W4	Rainy	Yes	Poor	Cinema
W5	Rainy	No	Rich	Stay In
W6	Rainy	Yes	Poor	Cinema
W7	Windy	No	Poor	Cinema
W8	Windy	No	Rich	Shopping
W9	Windy	Yes	Rich	Cinema
W10	Sunny	No	Rich	Tennis

- Computation of **Gini Index for Parents** Attribute
- It has two possible values of Yes (5 examples) and No (5 examples).
- For **Parents = Yes,** there are 5 examples, all with "Cinema".
- $Gini(S) = 1 - [\left(\frac{5}{5}\right)^2] = 0$
- For **Parents = No,** there are 2 examples with "Tennis", 1 example with "Stay in", "Shopping" and "Cinema" each
- $Gini(S) = 1 - [\left(\frac{2}{5}\right)^2 + \left(\frac{1}{5}\right)^2 + \left(\frac{1}{5}\right)^2 + \left(\frac{1}{5}\right)^2] = 0.72$
- $\boldsymbol{Weighted\ Average(Parents)}$

$$= 0 * \left(\frac{5}{10}\right) + [0.72 * \left(\frac{5}{10}\right) = 0.36$$

Weekend	Weather	Parents	Money	Decision
W1	Sunny	Yes	Rich	Cinema
W2	Sunny	No	Rich	Tennis
W3	Windy	Yes	Rich	Cinema
W4	Rainy	Yes	Poor	Cinema
W5	Rainy	No	Rich	Stay In
W6	Rainy	Yes	Poor	Cinema
W7	Windy	No	Poor	Cinema
W8	Windy	No	Rich	Shopping
W9	Windy	Yes	Rich	Cinema
W10	Sunny	No	Rich	Tennis

- Computation of Gini Index for Weather Attribute
- It has three possible values of Sunny (3 examples), Rainy (3 examples) and Windy (4 examples).
- For **Weather = Sunny**, there are **2 examples with "Cinema"** and **1 with "Tennis".**
- $Gini(Sunny) = 1 - [\left(\frac{2}{3}\right)^2 + \left(\frac{1}{3}\right)^2] = 0.444$
- For **Weather = Rainy**, there are **2 examples with "Cinema"** and **1 example with "Stay in"**
- $Gini(Rainy) = 1 - [\left(\frac{2}{3}\right)^2 + \left(\frac{1}{3}\right)^2] = 0.444$
- For **Weather = Windy**, there are **3 examples with "Cinema"** and **1 example with "Shopping"**
- $Gini(Windy) = 1 - [\left(\frac{3}{4}\right)^2 + \left(\frac{1}{4}\right)^2] = 0.375$

Weekend	Weather	Parents	Money	Decision
W1	Sunny	Yes	Rich	Cinema
W2	Sunny	No	Rich	Tennis
W3	Windy	Yes	Rich	Cinema
W4	Rainy	Yes	Poor	Cinema
W5	Rainy	No	Rich	Stay In
W6	Rainy	Yes	Poor	Cinema
W7	Windy	No	Poor	Cinema
W8	Windy	No	Rich	Shopping
W9	Windy	Yes	Rich	Cinema
W10	Sunny	No	Rich	Tennis

Weighted Average(Weather)

$$= 0.444 * \left(\frac{3}{10}\right) + 0.444 * \left(\frac{3}{10}\right) + 0.375 * \left(\frac{4}{10}\right)$$

$$= 0.416$$

For Weather - Gini Index: 0.416

For Parents - Gini Index: 0.36

For Money - Gini Index: 0.486

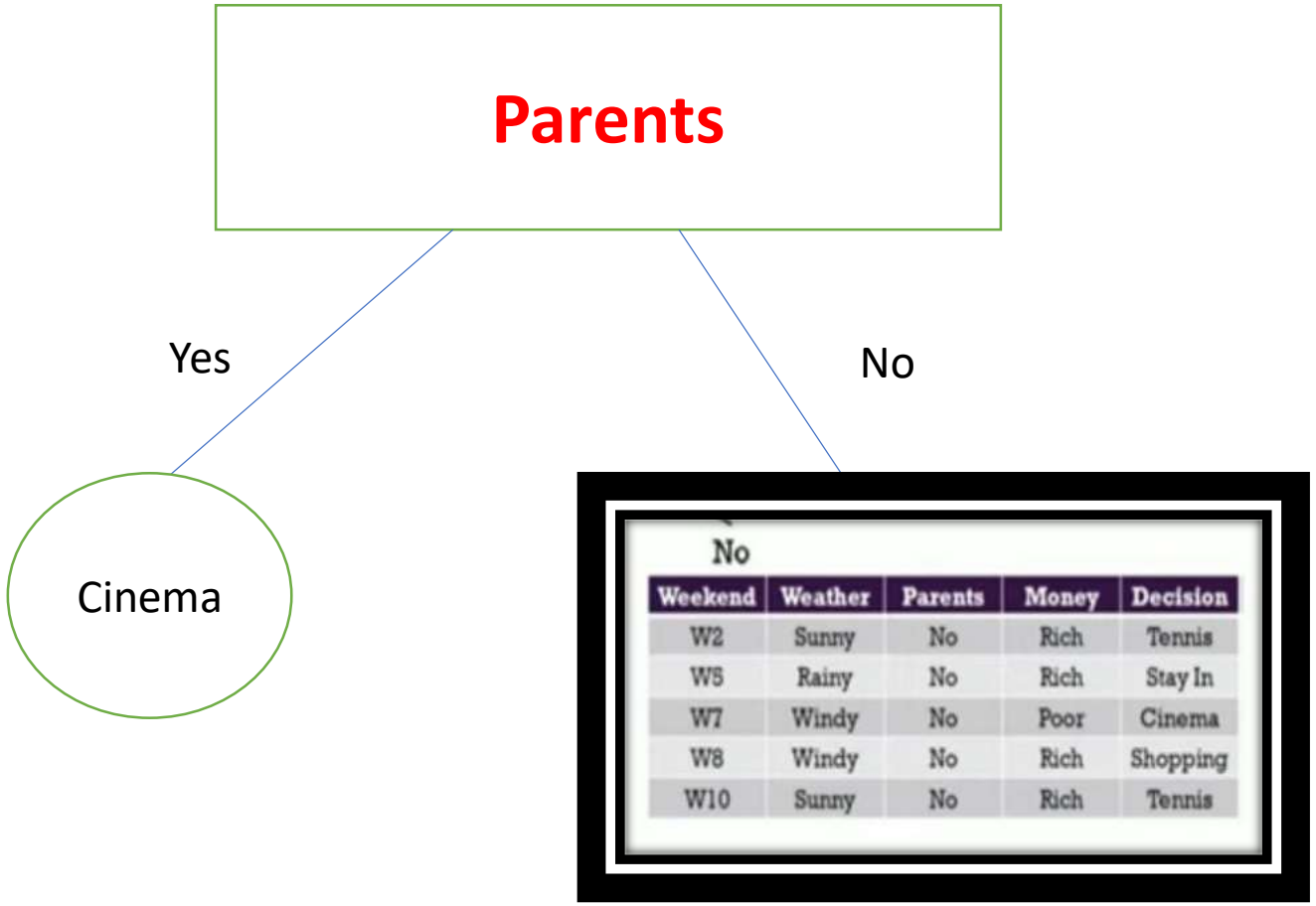
Parents is selected as it has smallest Gini index.

Yes

Weekend	Weather	Parents	Money	Decision
W1	Sunny	Yes	Rich	Cinema
W3	Windy	Yes	Rich	Cinema
W4	Rainy	Yes	Poor	Cinema
W6	Rainy	Yes	Poor	Cinema
W9	Windy	Yes	Rich	Cinema

No

Weekend	Weather	Parents	Money	Decision
W2	Sunny	No	Rich	Tennis
W5	Rainy	No	Rich	Stay In
W7	Windy	No	Poor	Cinema
W8	Windy	No	Rich	Shopping
W10	Sunny	No	Rich	Tennis


Parents
Yes
No
Cinema
No
Weekend Weather Parents Money Decision
W2 Sunny No Rich Tennis
W5 Rainy No Rich Stay In
W7 Windy No Poor Cinema
W8 Windy No Rich Shopping
W10 Sunny No Rich Tennis

Weekend	Weather	Parents	Money	Decision
W2	Sunny	No	Rich	Tennis
W5	Rainy	No	Rich	Stay In
W7	Windy	No	Poor	Cinema
W8	Windy	No	Rich	Shopping
W10	Sunny	No	Rich	Tennis

Computation of Gini Index for Parents = No | Weather Attribute

- **Sunny (2 examples)**
- For Parent= No | Weather = Sunny, there are 2 example with "Tennis.
- $Gini(S) = 1 - [\left(\frac{2}{2}\right)^2] = 0$

Decision Tree using Gini Index – Solved Example

Weekend	Weather	Parents	Money	Decision
W2	Sunny	No	Rich	Tennis
W5	Rainy	No	Rich	Stay In
W7	Windy	No	Poor	Cinema
W8	Windy	No	Rich	Shopping
W10	Sunny	No	Rich	Tennis

Computation of Gini Index for Parents = No | Weather Attribute

- **Rainy (1 example).**
- For Parents = No | Weather = Rainy, there is 1 example with "Stay In".
- $Gini(S) = 1 - [\left(\frac{1}{1}\right)^2] = 0$

Decision Tree using Gini Index – Solved Example

Weekend	Weather	Parents	Money	Decision
W2	Sunny	No	Rich	Tennis
W5	Rainy	No	Rich	Stay In
W7	Windy	No	Poor	Cinema
W8	Windy	No	Rich	Shopping
W10	Sunny	No	Rich	Tennis

Computation of Gini Index for Parents = No | Weather Attribute

- **Windy (2 example)**
- For Parents = No | Weather = Windy, there is 1 example with "Cinema" and 1 example with "Shopping".
- $Gini(S) = 1 - \left[\left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2\right] = 0.5$

$$Weighted\ Average(Parents = No\,|\,Weather) = 0 * \left(\frac{2}{5}\right) + 0 * \left(\frac{1}{5}\right) + 0.5 * \left(\frac{2}{5}\right) = 0.2$$

Weekend	Weather	Parents	Money	Decision
W2	Sunny	No	Rich	Tennis
W5	Rainy	No	Rich	Stay In
W7	Windy	No	Poor	Cinema
W8	Windy	No	Rich	Shopping
W10	Sunny	No	Rich	Tennis

Computation of Gini Index for Parents = No | Money Attribute

- Rich (4 examples)
- For Parents = No | Money = Rich, there is 1 example with "stay in" and "Shopping" each and 2 examples of "Tennis".
- $Gini(S) = 1 - [\left(\frac{1}{4}\right)^2 + \left(\frac{1}{4}\right)^2 + \left(\frac{2}{4}\right)^2] = 0.625$

Weekend	Weather	Parents	Money	Decision
W2	Sunny	No	Rich	Tennis
W5	Rainy	No	Rich	Stay In
W7	Windy	No	Poor	Cinema
W8	Windy	No	Rich	Shopping
W10	Sunny	No	Rich	Tennis

For Parents = No | Weather - Gini Index: 0.2

For Parents = No | Money - Gini Index: 0.5

Weather is selected as it has smallest Gini index.

Weekend	Weather	Parents	Money	Decision
W2	Sunny	No	Rich	Tennis
W5	Rainy	No	Rich	Stay In
W7	Windy	No	Poor	Cinema
W8	Windy	No	Rich	Shopping
W10	Sunny	No	Rich	Tennis

Now, for Parent=No & Weather=Sunny, we have all instances as Tennis.

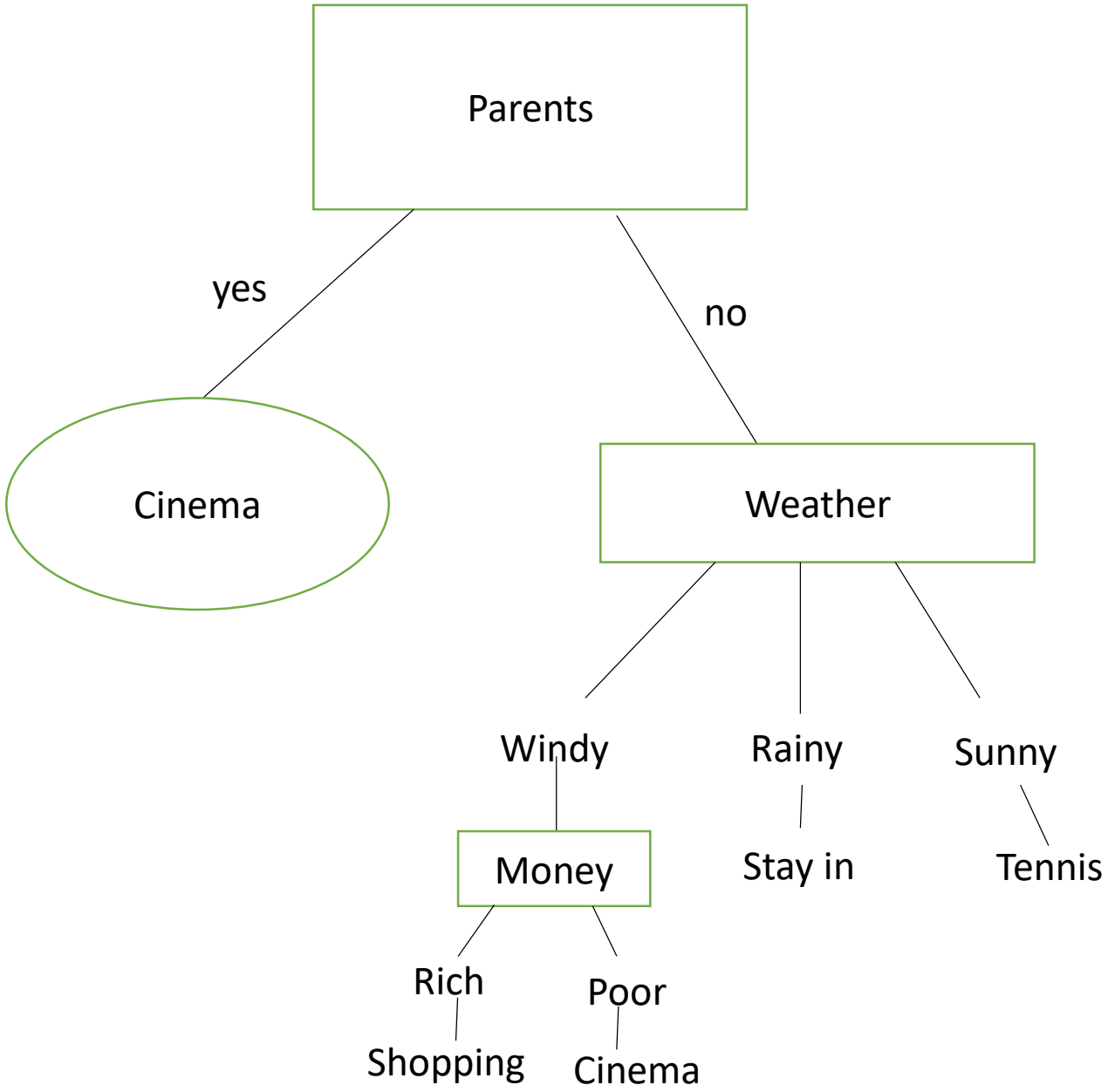
Weekend	Weather	Parents	Money	Decision
W2	Sunny	No	Rich	Tennis
W10	Sunny	No	Rich	Tennis

Now, for Parents=No & Weather=Rainy, we have all instances as Stay In.

Weekend	Weather	Parents	Money	Decision
W5	Rainy	No	Rich	Stay In

Now, for Parent=No & Weather=Windy, we need to split.

Weekend	Weather	Parents	Money	Decision
W7	Windy	No	Poor	Cinema
W8	Windy	No	Rich	Shopping


Parents
yes
no
Cinema
Weather
Windy
Rainy
Sunny
Money
Stay in
Tennis
Rich
Poor
Shopping
Cinema