Objective:

In this session, you will practice and solve some problems that involve identifying certain probability distributions and few applications of central limit theorem.

Key takeaways:

1. Computing distributions in R
2. Identify probability distributions
   - Geometric: For estimating number of attempts before first success
   - Binomial: For estimating number of successes in '*n'* attempts
   - Poisson: For estimating '*n'* number of events in given time window when, on average we see '*m'* events
   - Exponential: Time between events

Distribution	Functions			
Binomial	pbinom	qbinom	dbinom	rbinom
Exponential	pexp	qexp	dexp	rexp
Geometric	pgeom	qgeom	dgeom	rgeom
Normal	pnorm	qnorm	dnorm	rnorm
Poisson	ppois	qpois	dpois	rpois
Uniform	punif	qunif	dunif	runif

'p' – cumulative distribution; (area under the curve) p(x<'a')

'd' – probability density; p(x = 'a') height of the distribution

'q' – inverse of 'p'/ quantile; Value on x-axis corresponding to 'pnorm'

'r' – random number; random number generation for specified probability distribution

Problem statement 1:

1. Consider the favorite coin toss experiment. If you toss a biased coin, the probability of obtaining heads is 0.6. If you toss the coin 10 times, what is the probability of getting heads exactly 4 times?

2. You are fond of a particular flavor of ice-cream but it is rarely available in the shop. The probability of getting that ice-cream is only 0.15.
   a. Obtain a distribution table for getting ice-cream in 1st, 2nd,....., 10th visit and generate a plot.
   b. How many visits on an average are required to get your favorite ice-cream?
3. Customers arrive at a bus station at the rate of 5 per minute following Poisson distribution. What is the probability of 3 arrivals in a one-minute interval?

4. Average birth rate = 1.8 per hour. What is the probability that 5 people are born in a 2 hour interval.
5. The time required to repair a machine is an exponential random variable with rate λ = 0.5 jobs/hour.
   What is the probability that a repair time exceeds 2 hours?
6. Compute Z score for the elements in the vector below
   82, 72, 85, 14, 66, 15, 23, 78, 16, 38, 92, 17.

7. If player A gets a goal an average of 70% of the time with SD of 20%. Player B gets a goal an average of 40% of the time with SD of 10%. In a particular game, player A gets the goal 75% of time and player B gets the goal 55% of the time. Which of these two players have done better when compared to their personal track records?

8. A college basketball team has a shortage of one team member and a coach wants to recruit a player. To be selected for training the minimum height recruitment is 72 inches. The average height of the students is 67.2 inches with a variance of 29.34. What is the probability that the coach finds a player from that college?
9. A certain type of light bulb has an average life of 500 hours, with a standard deviation of 100 hours. The life of the bulb can be closely approximated by a normal curve. An amusement park buys and installs 10,000 such bulbs. Find the total number that can be expected to last for each period of time.

   a. At least 500 hours
   b. Less than 500 hours
   c. Between 350 and 550 hours
   d. More than 750 hours

10. Twelve volunteers were chosen for a blind-fold test to taste 2 soft-drinks A & B. What is the probability that 3 of them were able to correctly identify the drink that they had?
11. Find the expected value from the given probability distribution table.

1	2	3
0.8	0.6	0.2

Compute variance from the following distribution

8	12	16	20	24
1/8	1/6	3/8	1/4	1/12

12. There are four balls in a bag, red, black, green and blue. There is equal probability of getting any colored ball. What is the expected value of getting a green ball out of 20 experiments with replacement?

Exercises:

1. If height of four women is given to be 150 cm, 165 cm, 135 cm and 170 cm, then what will be the expected value of the height of a randomly chosen women?

2. If the scores of an IQ test are normally distributed with mean of 100 and standard deviation of 10, then what is the probability a person who takes the test will score between 90 and 100.
3. The probability that a student is accepted to a prestigious college is 0.3. If 5 students from the same school apply, what is the probability that at most 2 are accepted?
4. Five small radios are packed in identical, unmarked individual sealed boxes. Three boxes are on table X and contain 2 radios made by firm A and one by firm B. Two boxes are on table Y and contain one radio made by firm A and one by firm B. If someone moves a box from table X to table Y and you randomly select a box from table Y, what is the probability that you will select a radio made by firm B?
5. Products produced by a machine has a 3% defective rate. • What is the probability that the first defective occurs in the fifth item inspected?
6. If a production line has a 20% defective rate. What is the average number of inspections to obtain the first defective?
7. Suppose that the amount of time one spends in a bank is exponentially distributed with mean 10 minutes, $\lambda = 1/10$. What is the probability that a customer will spend more than 15 minutes in the bank? What is the probability that a customer will spend more than 15 minutes in the bank given that he is still in the bank after 10 minutes?
8. The probability that you will win a certain game is 0.3. If you play the game 20 times, what is the probability that you will win at least 8 times?

References:

http://onlinestatbook.com/2/probability/binomial.html

http://www-eio.upc.es/teaching/pe/www.computing.dcu.ie/~jhorgan/chapter10slides.pdf

https://onlinecourses.science.psu.edu/stat414/node/138

http://sites.stat.psu.edu/~jiali/course/stat416/notes/chap5.pdf

Bayes theorem

https://onlinecourses.science.psu.edu/stat414/node/43

http://stattrek.com/probability/bayes-theorem.aspx