Activity Sheet:

1. Consider the favorite coin toss experiment. If you toss a biased coin, the probability of obtaining heads is 0.6. If you toss the coin 10 times, what is the probability of getting heads exactly 4 times?

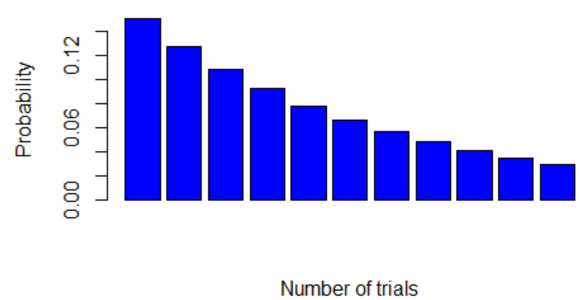
   P (Success) = p = 0.6; P (Failure) = q = 0.4
   Total number of trials, n = 10
   The no. of times we desire to get success (r) or number of favorable events = 4 heads
   $P(X=4) = {}^{10}C_4 \times (0.6)^4 \times (0.4)^6$

   R: dbinom (4, 10, 0.6) or choose (10,4)*(0.6)^4*(0.4)^6

2. You are fond of a particular flavor of ice-cream but it is rarely available in the shop. The probability of getting that ice-cream is only 0.15.
   a. Obtain a distribution table for getting ice-cream in 1st, 2nd,....., 10th visit and generate a plot.

   The success of an event is getting the ice cream in n^{th} trial this case. As per the question, since we are considering number of trials before first success is obtained, this is a Geometric Distribution. Therefore, $P(X=r) = q^{r-1}p$ (where, r-1 are the number of trials before first success is obtained. There can be ten cases for the given question

	Probability P(X= Ice Cream)
1	0.15
2	0.85 * 0.15
3	$(0.85)^2 * 0.15$
4	$(0.85)^3 * 0.15$
5	$(0.85)^4 * 0.15$
6	$(0.85)^5 * 0.15$
7	$(0.85)^6 * 0.15$
8	$(0.85)^7 * 0.15$
9	$(0.85)^8 * 0.15$
10	$(0.85)^9 * 0.15$



   R: geom_distrib = dgeom (x = 0:10,prob = 0.15);
   barplot (geom_distrib ,col = 'blue', xlab = 'Number of trials', ylab = 'Probability')

   b. How many visits on an average are required to get your favorite ice-cream?

Average is nothing but the expected value. E[X] = 1/p for geometric distribution. So, Average No. of visits = 1/p (where p is probability of success) = 1/0.15 = ~7 visits

3. Customers arrive at a bus station at the rate of 5 per minute following Poisson distribution. What is the probability of 3 arrivals in a one-minute interval?

   $P(X = r) = (\lambda^r \times e^{-\lambda}) / r!$
   $\lambda = 5, r = 3$

   $P(X = 3) = (5^3 \times e^{-5}) / 3! = 0.14$
   R: dpois (3, 5, FALSE)

4. Average birth rate = 1.8 per hour. What is the probability that 5 people are born in a 2 hour interval.

   $\lambda = 1.8, r = 5$
   $P(X = 5) = ((\lambda t)^r \times e^{-t\lambda}) / r! = (1.8*2)^5 \times e^{-2*1.8} / 5!$
   R: dpois (5, 1.8*2) = 0.13

5. The time required to repair a machine is an exponential random variable with rate λ = 0.5 jobs/hour.

   What is the probability that a repair time exceeds 2 hours?

   $f(x) = \lambda \times e^{-\lambda x}$
   $P (X > 2) = 1 – P (X \leq 2)$

   R: 1 – pexp (q = 2, 0.5)

6. Compute Z score for the elements in the vector below
   82, 72, 85, 14, 66, 15, 23, 78, 16, 38, 92, 17.

   Compute the mean=∑x/n and SD= $\sum(x\text{-mean})^2/n$ and then for each element x_i compute $(x_i\text{-mean})/SD$

7. If player A gets a goal an average of 70% of the time with SD of 20%. Player B gets a goal an average of 40% of the time with SD of 10%. In a particular game, player A gets the goal 75% of time and player B gets the goal 55% of the time. Which of these two players have done better when compared to their personal track records?

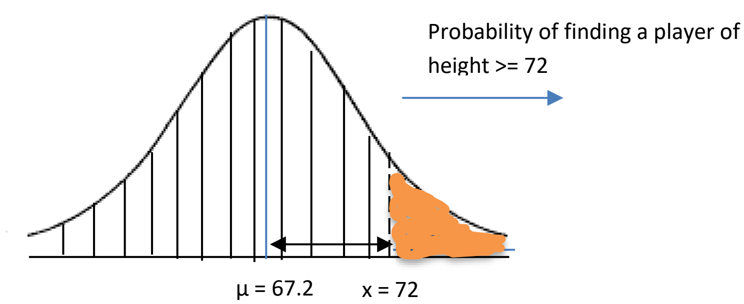
   $\mu_A = 0.7, \sigma_A = 0.2;$
   $\mu_B = 0.4, \sigma_B = 0.1;$

$Z = (x-\mu) / \sigma$
$Z_A = (0.75-0.70) / 0.20 = 0.25$ and $Z_B = (0.55-0.40) / 0.10 = 1.5$

The one with highest Z value has done better against their personal track records. Therefore player B has done better compared to his personal track record.

8. A college basketball team has a shortage of one team member and a coach wants to recruit a player. To be selected for training the minimum height recruitment is 72 inches. The average height of the students is 67.2 inches with a variance of 29.34. What is the probability that the coach finds a player from that college?

$\mu = 67.2$; $\sigma^2 = 29.34$, $\sigma = 5.41$, $x = 72$
$Z = (72-67.2)/5.41 = 0.88$
$P(X<=72) = P(Z = 0.88) = 0.811$
$P(X>72) = 1-0.811 = 0.19$



R: 1-pnorm(72,67.2, 5.41) OR 1-pnorm(z-score) i.e. 1-pnorm(0.8856)

9. A certain type of light bulb has an average life of 500 hours, with a standard deviation of 100 hours. The life of the bulb can be closely approximated by a normal curve. An amusement park buys and installs 10,000 such bulbs. Find the total number that can be expected to last for each period of time.

   a. At least 500 hours
   b. Less than 500 hours
   c. Between 350 and 550 hours
   d. More than 750 hours

$\mu = 500$ hrs; $\sigma = 100$ hrs

a. $P(X\geq500)$
   $Z = (500-500)/100 = 0$; $P(Z=0) = 0.5$; R: pnorm (0,0,1) =0.5

P (X≥500) = 1-P (X<500) = 1- P (Z=0) = 1 - 0.5 = 0.5
0.5 x 10,000 = 5,000 bulbs

b. P(X<500)
Z = (500-500)/100 = 0; P (Z=0) = 0.5; R: pnorm (0,0,1) =0.5
P (X<500) = P (Z=0) = 0.5
0.5 x 10,000 = 5,000

c. P(350≤X≤550)
Z = (350-500)/100 = -1.5; P (X=350) = P (Z=-1.5) = pnorm (-1.5, 0, 1) = 0.066 [or pnorm (350,500,100)]
Z = (550-500)/100 = +0.5; P (X=550) = P (Z=+0.5) = pnorm (0.5, 0, 1) = 0.691 [or pnorm (550,500,100)]

Therefore the total no. of bulbs that can be expected to last between 350 hrs and 550 hrs is 69.1% - 6.6% = 62.5% x 10,000 = 6,250 bulbs.

d. P(X>750)
Z = (750-500)/100 = 2.5, P (Z=2.5) = pnorm (2.5, 0, 1) = 0.993
P (X>750) = 1-P (X<750) = 1-P (Z=2.5) = 0.0062; 1-pnorm (2.5, 0, 1) = 0.0062

10. Twelve volunteers were chosen for a blind-fold test to taste 2 soft-drinks A & B. What is the probability that 3 of them were able to correctly identify the drink that they had?

Binomial distribution with n = 12 and p = 0.5 and q = (1-p) = 0.5
$P(X=r) = {}^{n}C_{r} \times p^{r} \times q^{(n-r)}$
Hence $P(X = 3) = {}^{12}C_{3} \times (0.5)^{3} \times (0.5)^{9}$ * =0.05371
R: dbinom (3, 12, 0.5) = 0.053

11. Find the expected value from the given probability distribution table.

1	2	3
0.8	0.6	0.2

1*.8 + 2*.6+3*.2 = 2.4

12. There are four balls in a bag, red, black, green and blue. There is equal probability of getting any colored ball. What is the expected value of getting a green ball out of 20 experiments with replacement?

Expected Value = Number of Experiments × ProbabilityNumber of Experiments × Probability

E(X) = 20 × 0.25 = 5

Exercises:

1. If height of four women is given to be 150 cm, 165 cm, 135 cm and 170 cm, then what will be the expected value of the height of a randomly chosen women?
   The expected value is found to be 155 cm.

2. If the scores of an IQ test are normally distributed with mean of 100 and standard deviation of 10, then what is the probability a person who takes the test will score between 90 and 100.
   $P(90 < X < 110) = P(X < 110) - P(X < 90) = 0.84 - 0.16 = 0.68$

3. The probability that a student is accepted to a prestigious college is 0.3. If 5 students from the same school apply, what is the probability that at most 2 are accepted?
   $b(x \leq 2; 5, 0.3) = 0.8369$

4. Compute variance from the following distribution

8	12	16	20	24
1/8	1/6	3/8	1/4	1/12

$$V(X) = E(X^2) - [E(X)]^2 = 276 - 16^2 = 20$$

5. Five small radios are packed in identical, unmarked individual sealed boxes. Three boxes are on table X and contain 2 radios made by firm A and one by firm B. Two boxes are on table Y and contain one radio made by firm A and one by firm B. If someone moves a box from table X to table Y and you randomly select a box from table Y, what is the probability that you will select a radio made by firm B?
   http://www.intmath.com/counting-probability/9-mutually-exclusive-events.php