Artificial Potential Fields

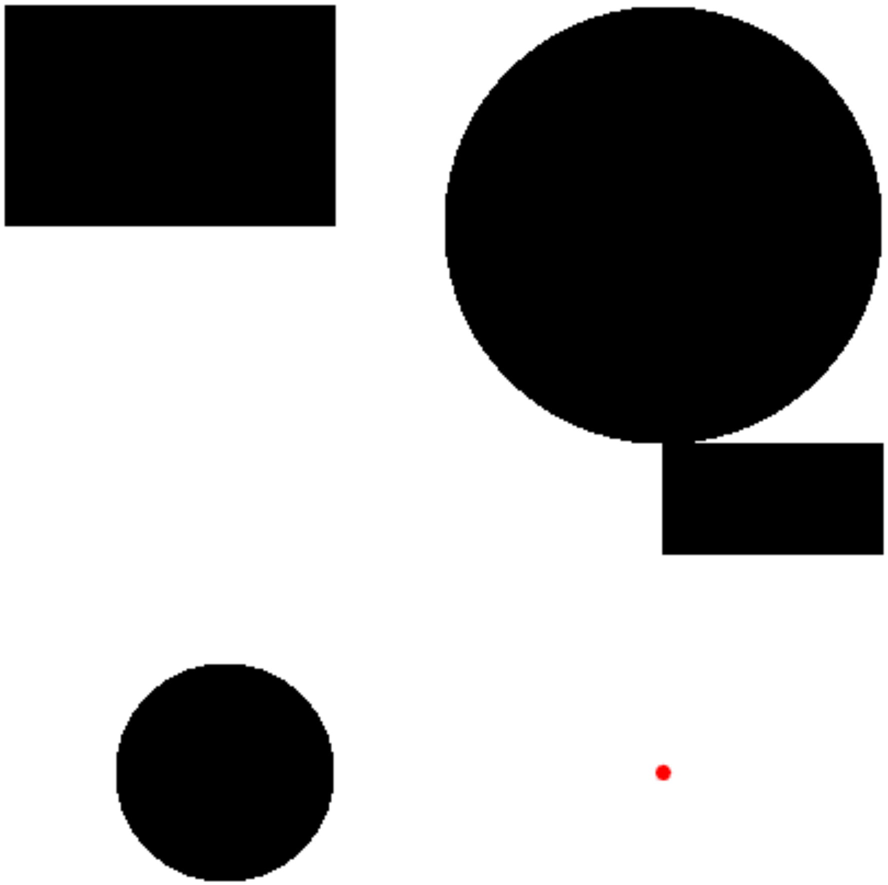
Constructing Artificial Potential Fields

SECTION 4.1

- **The basic idea is to try to construct a smooth function over the extent of the configuration space which has high values when the robot is near to an obstacle and lower values when it is further away.**
- **We also want this function to have it’s lowest value at the desired goal location and it’s value should increase as we move to configurations that are further away.**

- **If we can construct such a function we can use it's gradient to guide the robot to the desired configuration.**
- **As usual it's easiest to start with an example in a 2 dimensional configuration space where we can more easily visualize what is happening.**
- **This figure shows a typical 2 dimensional configuration space where the black regions correspond to configuration space obstacles and the red dot indicates the desired goal configuration.**

Example 2D Configuration Space



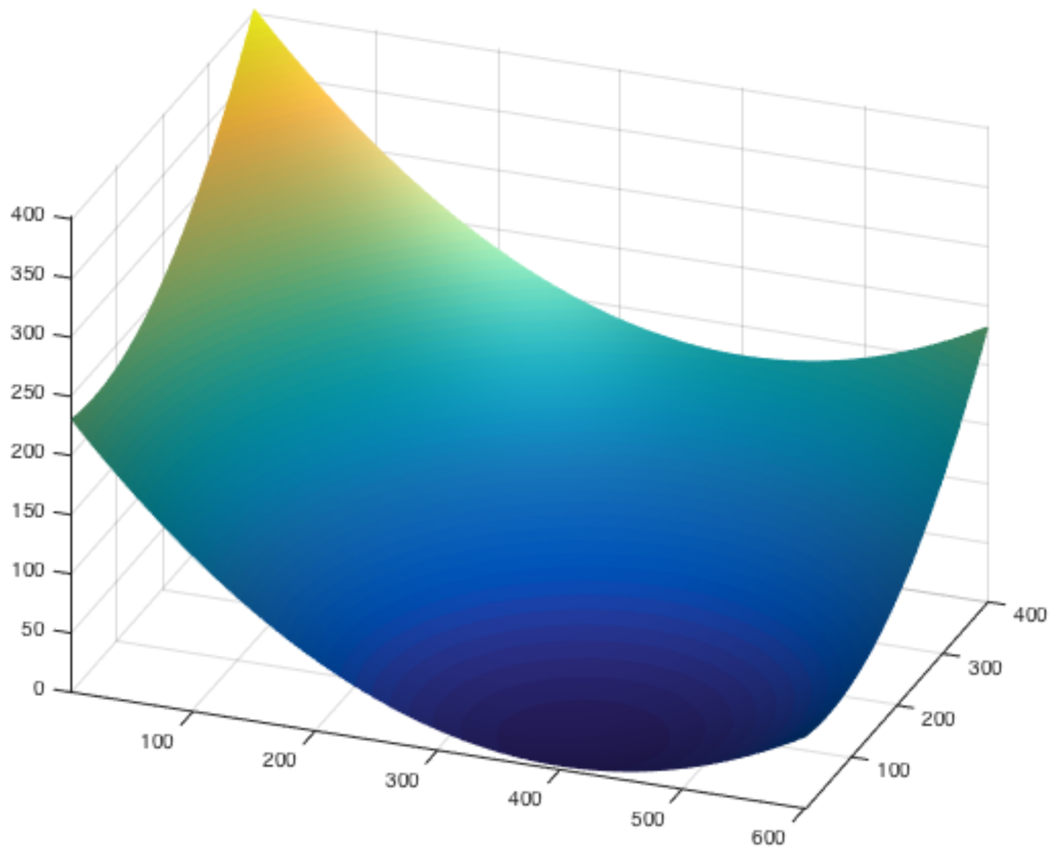
Constructing an Attractive Potential Field

- An attractive potential function, $f_a(\mathbf{x})$, can be constructed by considering the distance between the current position of the robot, $\mathbf{x} = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$, and the desired goal location, $\mathbf{x}_g = \begin{pmatrix} x_1^g \\ x_2^g \end{pmatrix}$, as follows:

$$f_a(\mathbf{x}) = \xi(\|\mathbf{x} - \mathbf{x}_g\|^2)$$

- Here ξ is simply a constant scaling parameter

Visualizing the Attractive Potential Field



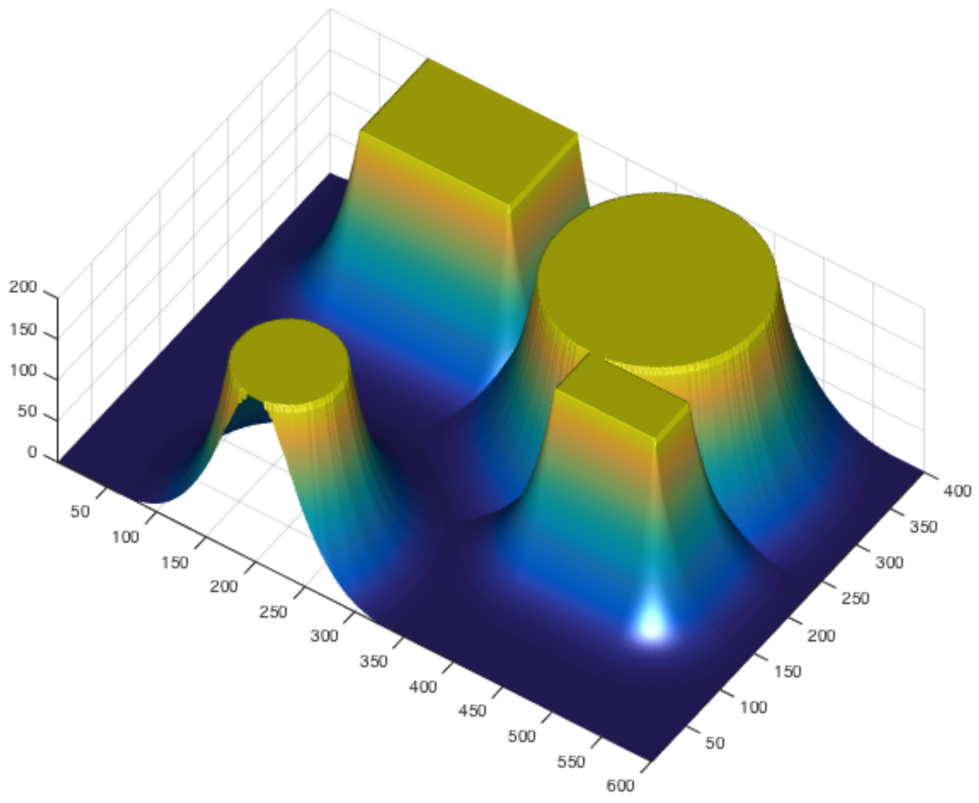
Constructing a Repulsive Potential Field

- A repulsive potential function in the plane, $f_r(\mathbf{x})$, can be constructed based on a function, $\rho(\mathbf{x})$, that returns the distance to the closest obstacle from a given point in configuration space, $\mathbf{x}$.

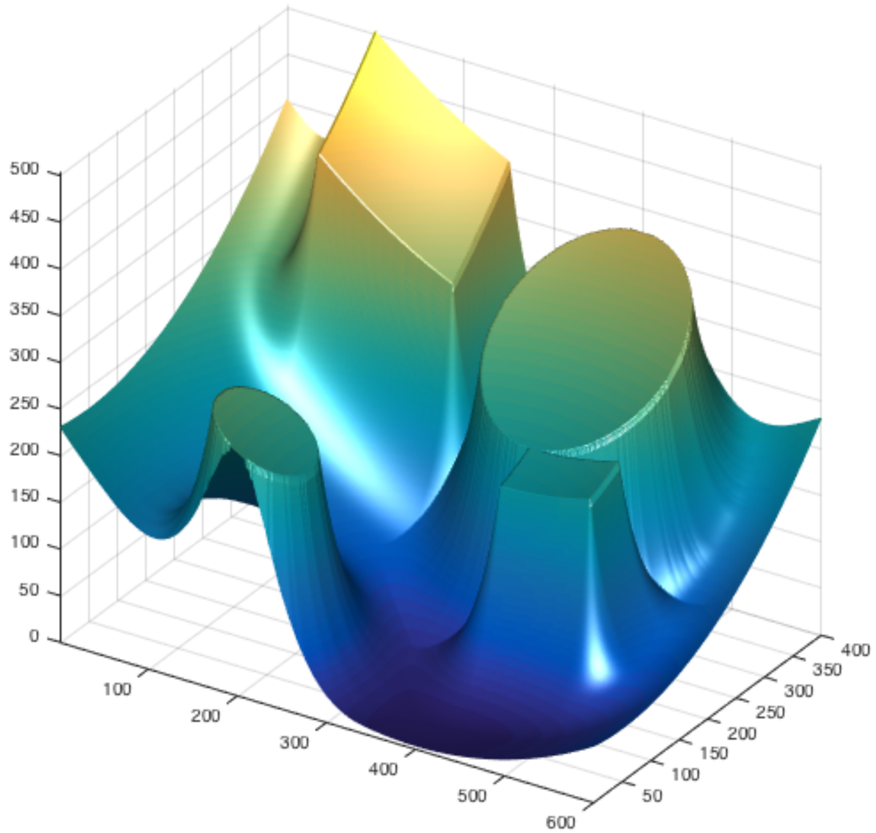
$$f_r(\mathbf{x}) = \begin{cases} \eta(\frac{1}{\rho(\mathbf{x})} - \frac{1}{d_0})^2 & \text{if } \rho(\mathbf{x}) \leq d_0 \\ 0 & \text{if } \rho(\mathbf{x}) > d_0 \end{cases}$$

- Here η is simply a constant scaling parameter and d_0 is a parameter that controls the influence of the repulsive potential

Visualizing the Repulsive Potential Field



Visualizing the Combined Potential field



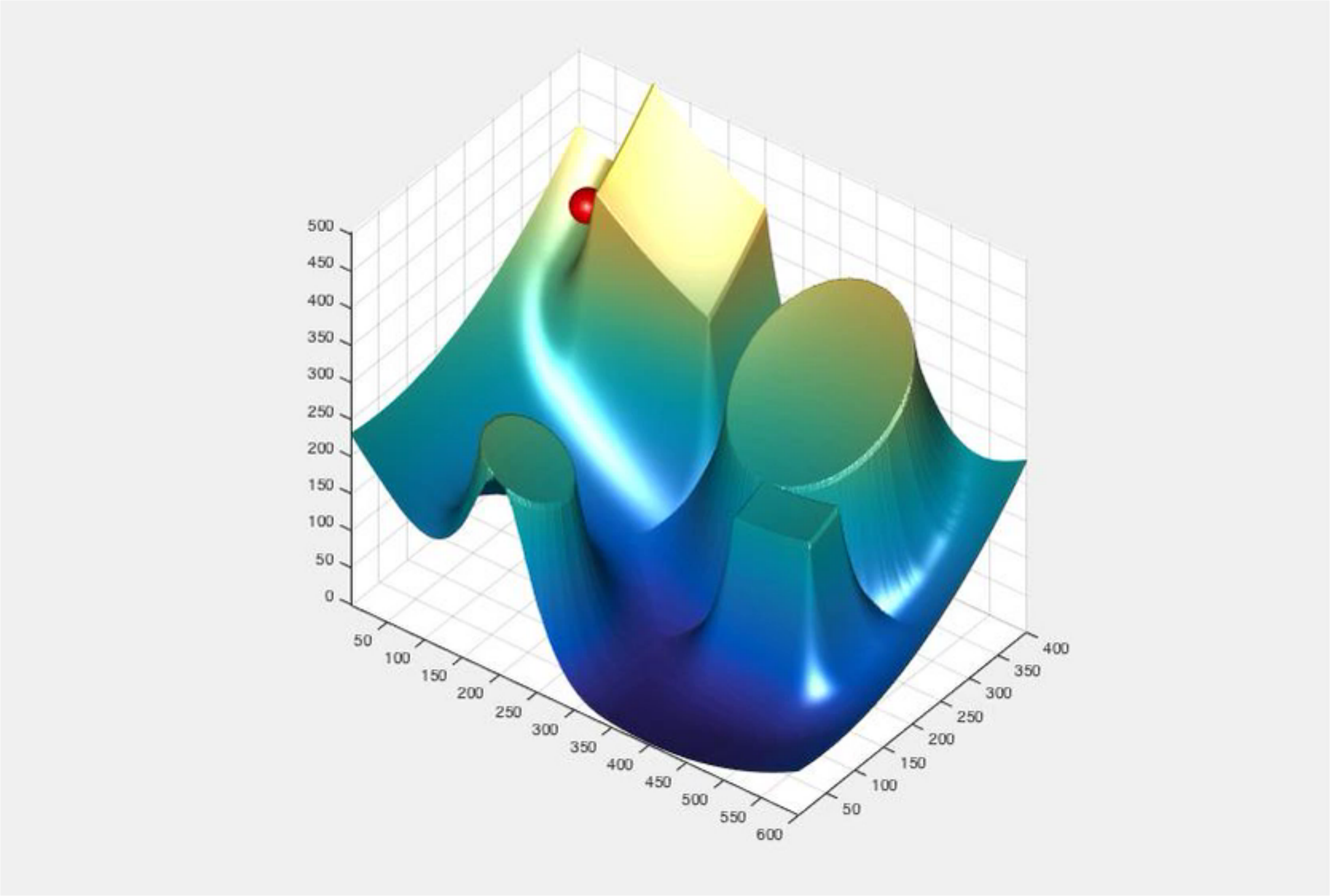
Gradient Based Control Strategy

- While robot position is not close enough to goal
  - Choose direction of robot velocity based on the gradient of the artificial potential field:

$$\mathbf{v} \propto -\nabla f(\mathbf{x}) = -\begin{pmatrix} \frac{\partial f(\mathbf{x})}{\partial x_1} \\ \frac{\partial f(\mathbf{x})}{\partial x_2} \end{pmatrix} \tag{1}$$

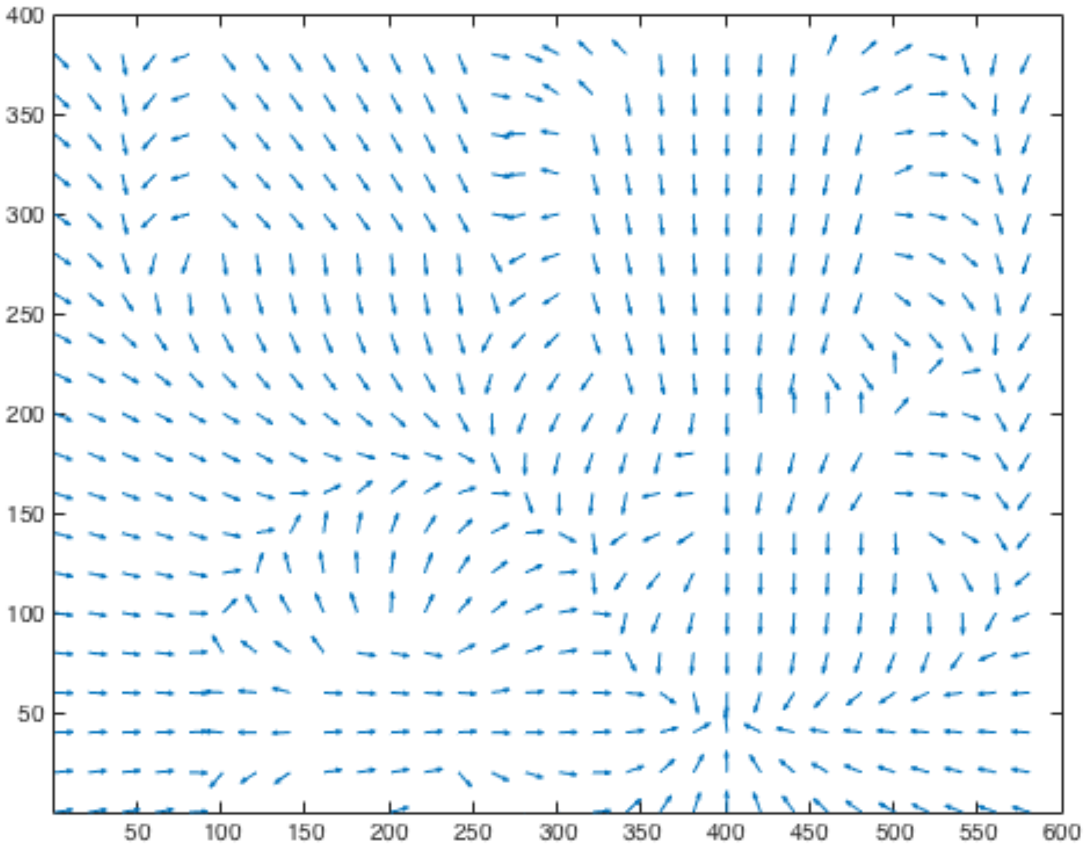
  - Choose an appropriate robot speed, $\|\mathbf{v}\|$

Animation of Gradient Based Control Scheme



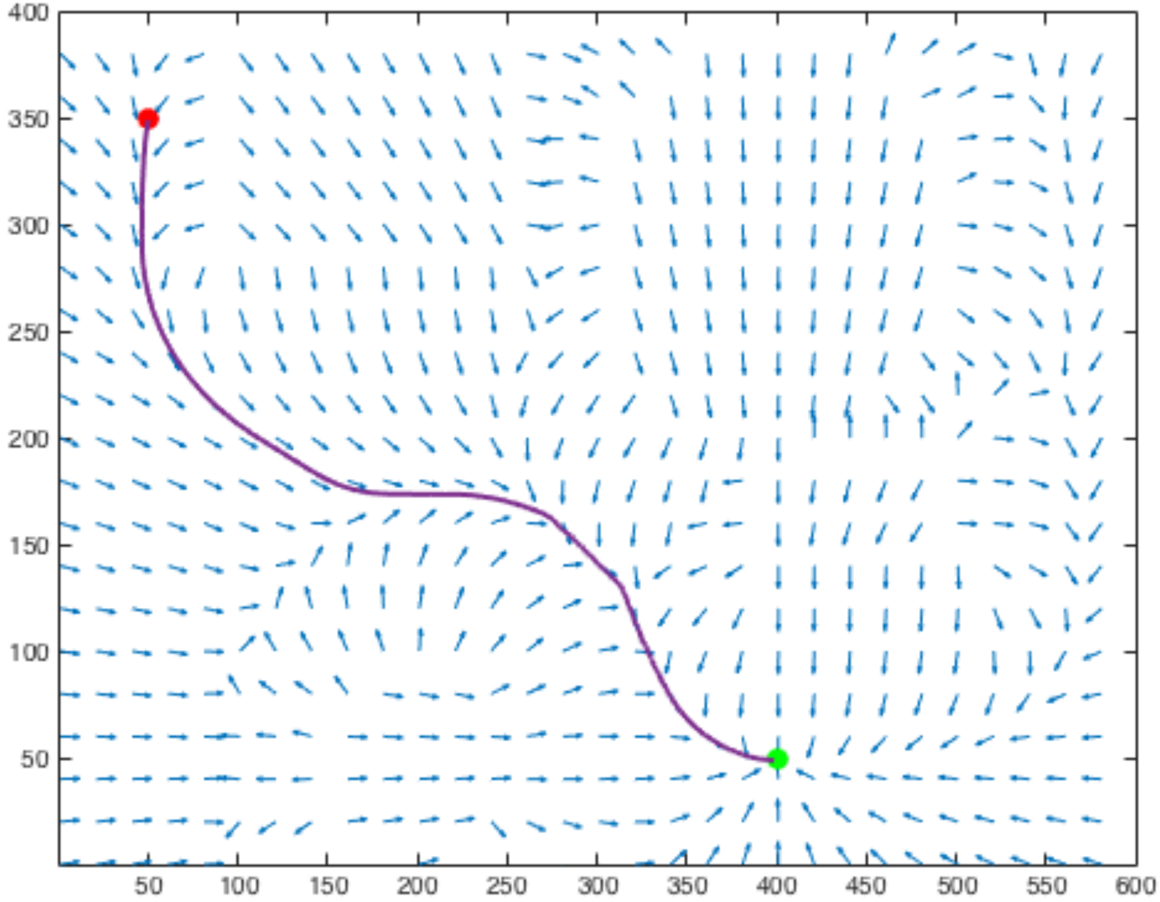
Quiver Plot

- The arrows in this figure denote the direction of the gradient vector at various points in the configuration space.



Trajectory Plot

- Example gradient based trajectory.



Trajectory Plot

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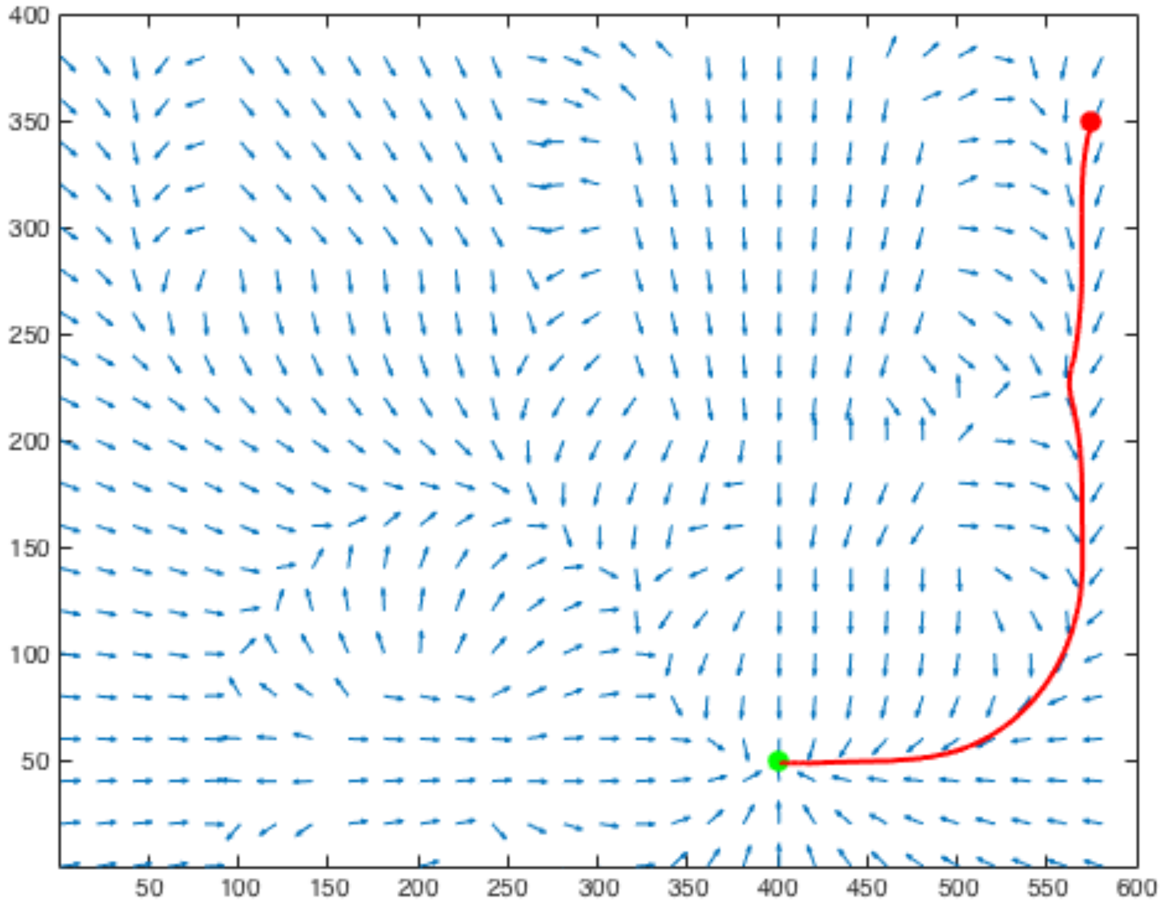

Trajectory Plot

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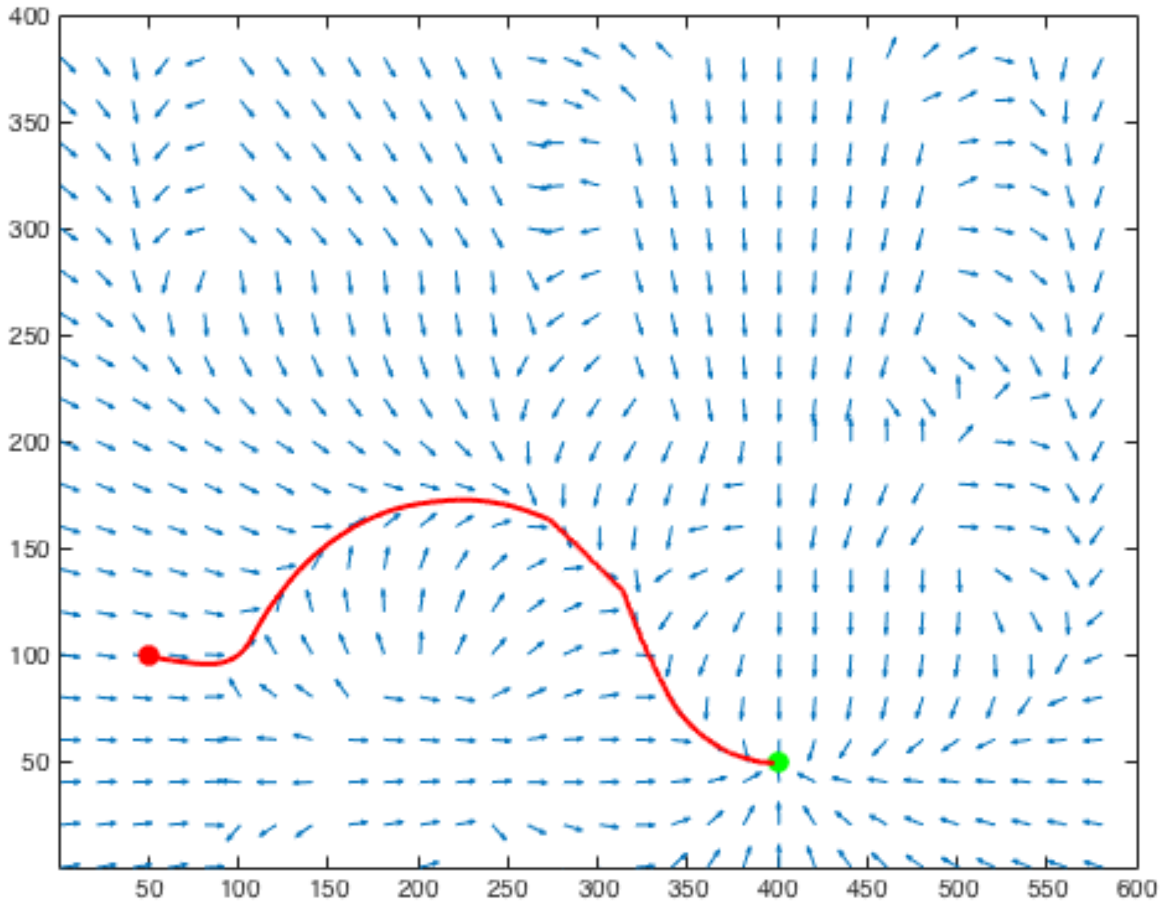
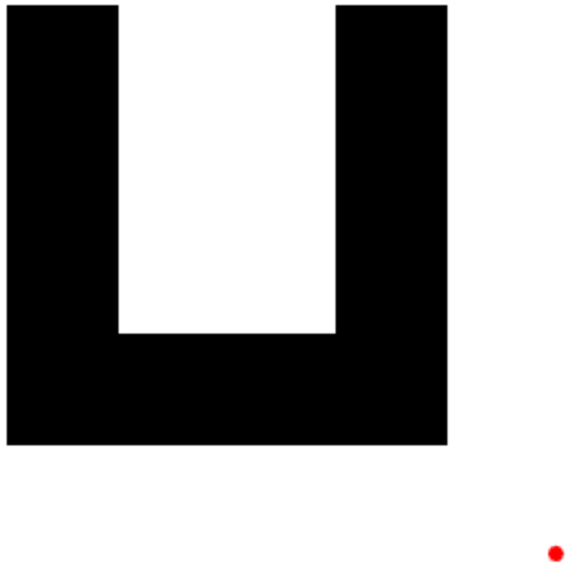


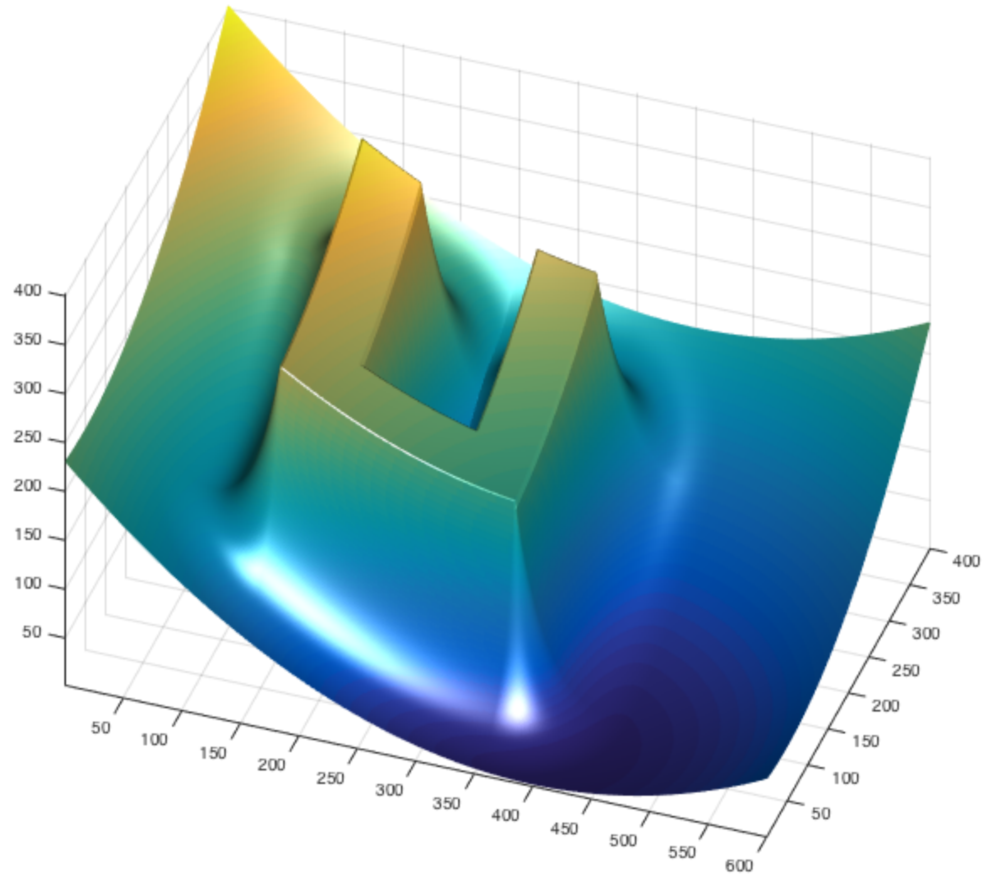
Issues with Local Minima

SECTION 4.2

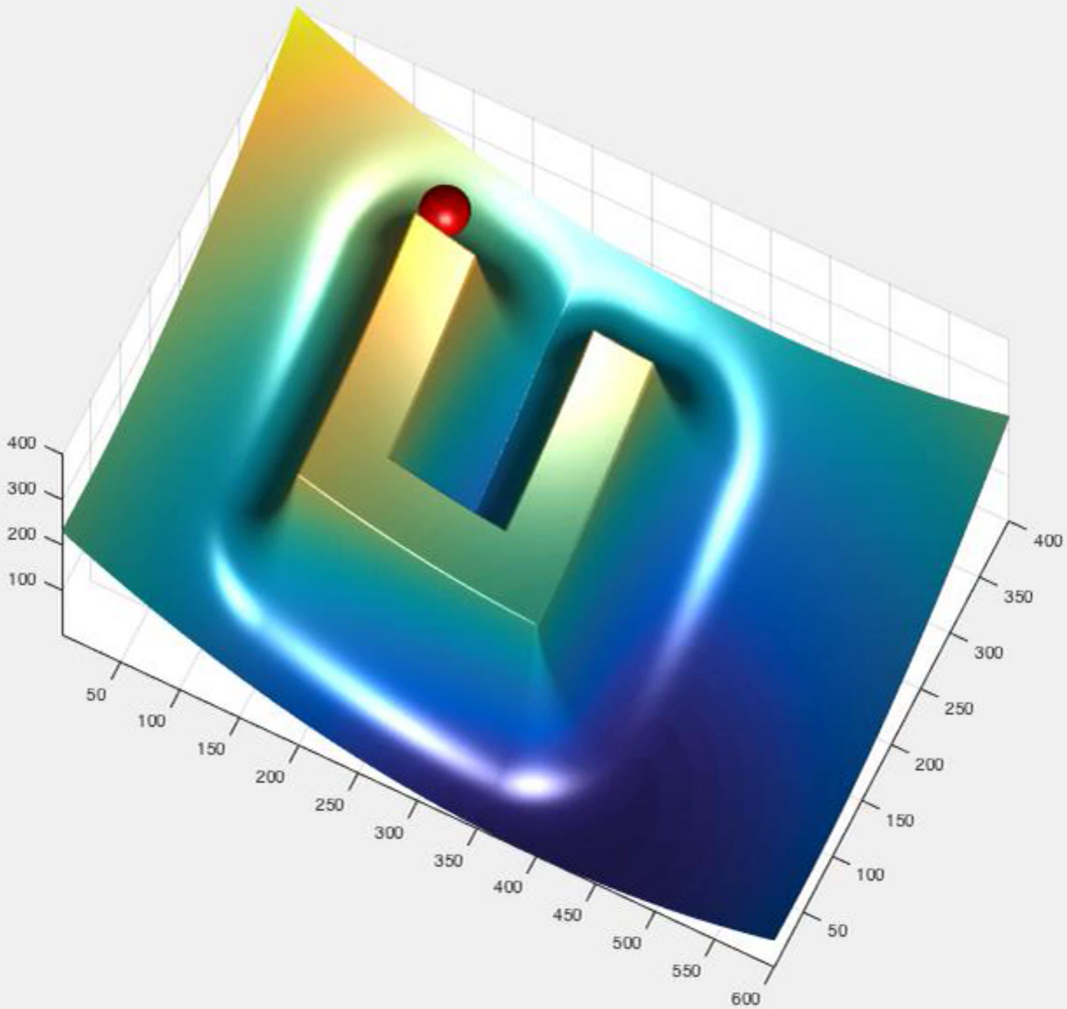
Example Configuration Space



Artificial Potential Field



Animation of Trajectory



Generalizing Potential Fields

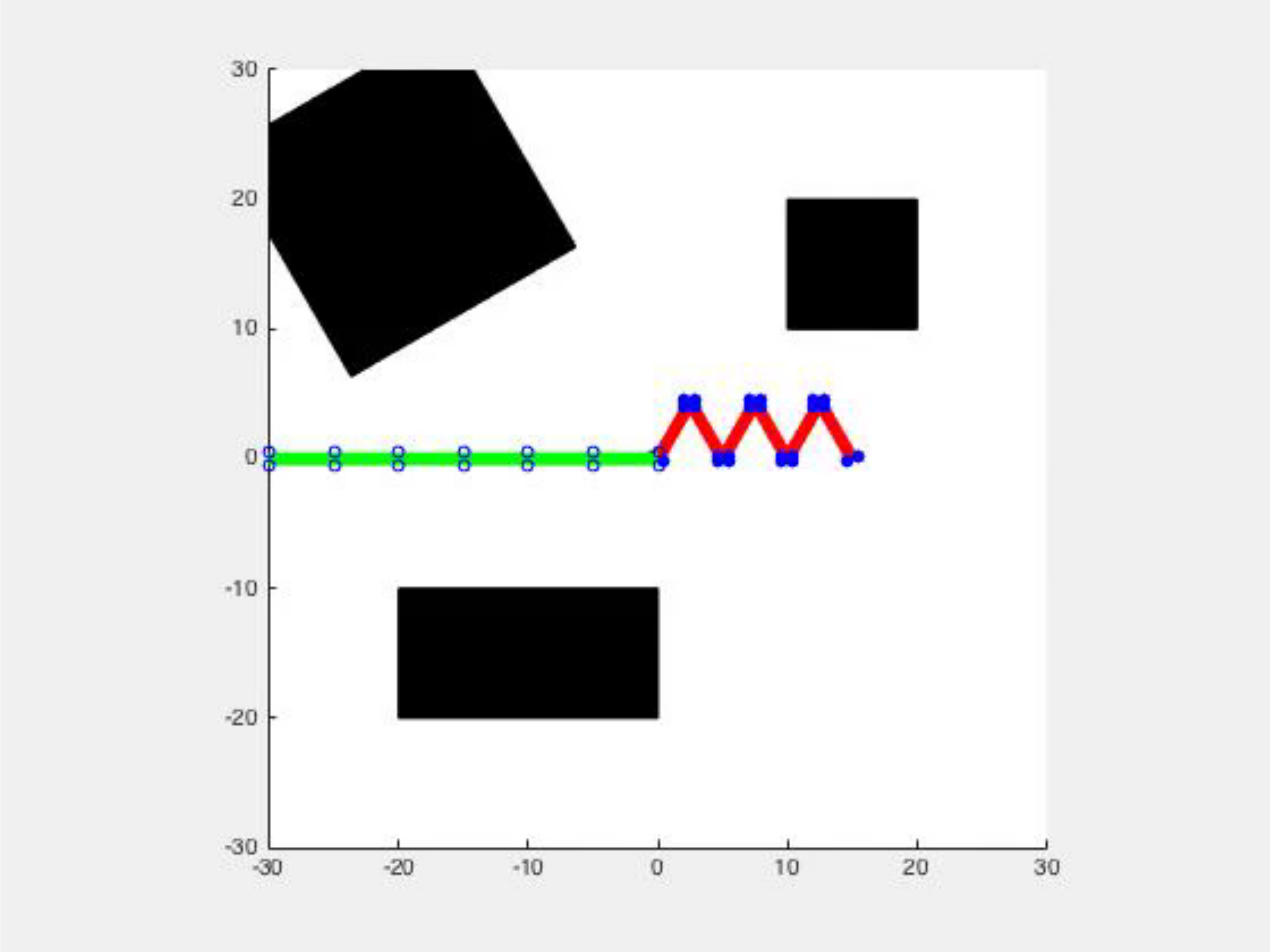
SECTION 4.3

Generalizing Potential Fields

- One approach to generalizing artificial potential fields to more complicated robotic systems which can involve many degrees of freedom is by considering a set of control points distributed over the surface of the robot.
- The position of each of these control points can be computed as a function of the configuration space parameters, $P_i(\mathbf{x})$.
- For each of the control points we can construct an artificial potential field which repels it from obstacles and guides it to its desired location, $f_i(P_i(\mathbf{x}))$
- The final artificial potential function is computed by simply summing over all of the control points: $f(\mathbf{x}) = \sum_i f_i(P_i(\mathbf{x}))$.
- Once again we can construct a control law to move the robot by considering the gradient of the potential field with respect to the configuration space parameters.

$$\mathbf{v} \propto -\nabla f(\mathbf{x}) = -\begin{pmatrix} \frac{f(\mathbf{x})}{\partial x_1} \\ \vdots \\ \frac{f(\mathbf{x})}{\partial x_n} \end{pmatrix} \tag{1}$$

Visualizing Control Points on a robot

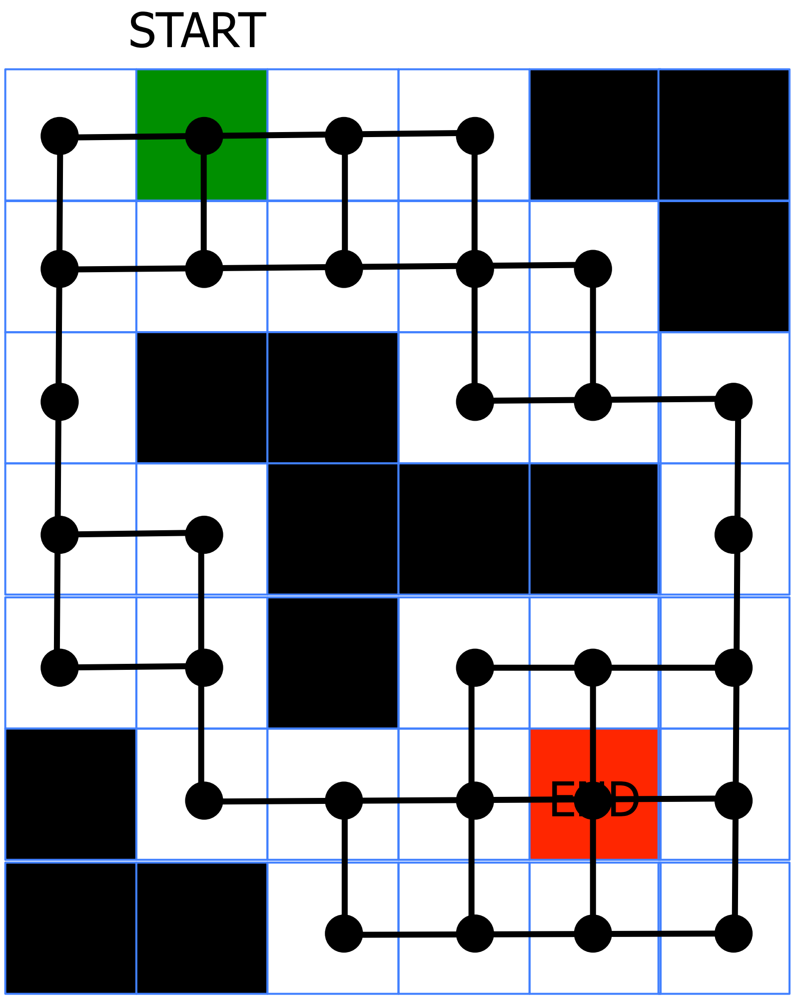


Module Summary

SECTION 4.4

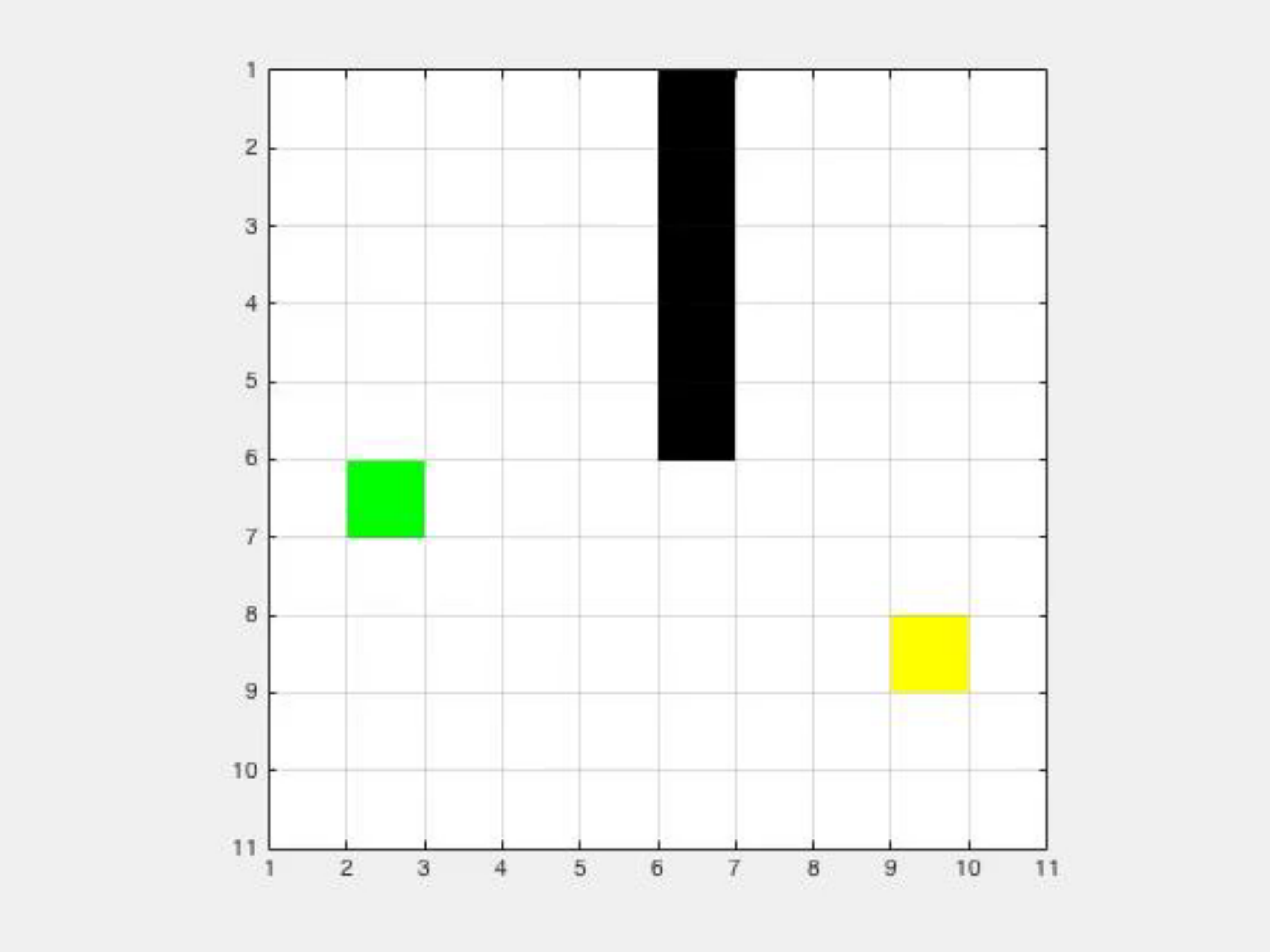
Graph Structure

- We can think of the unoccupied cells as nodes and draw edges between adjacent cells as shown here.
- This set of nodes and edges constitutes a graph.



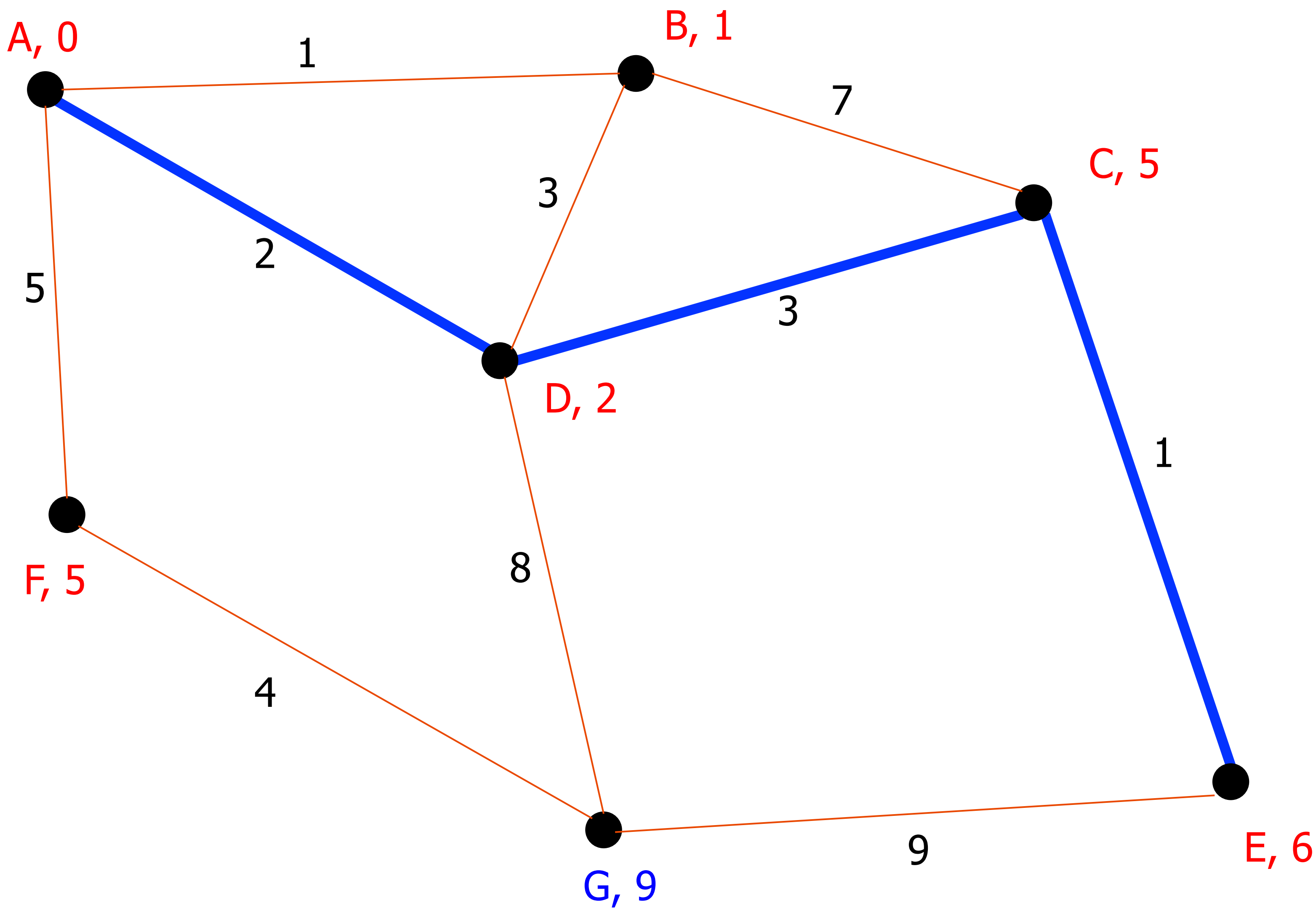
- **For these grid based problems we talked about the breadth first search or grassfire algorithm which searched for a shortest path by exploring outwards from a start location.**

Breadth First Search / Grassfire Algorithm



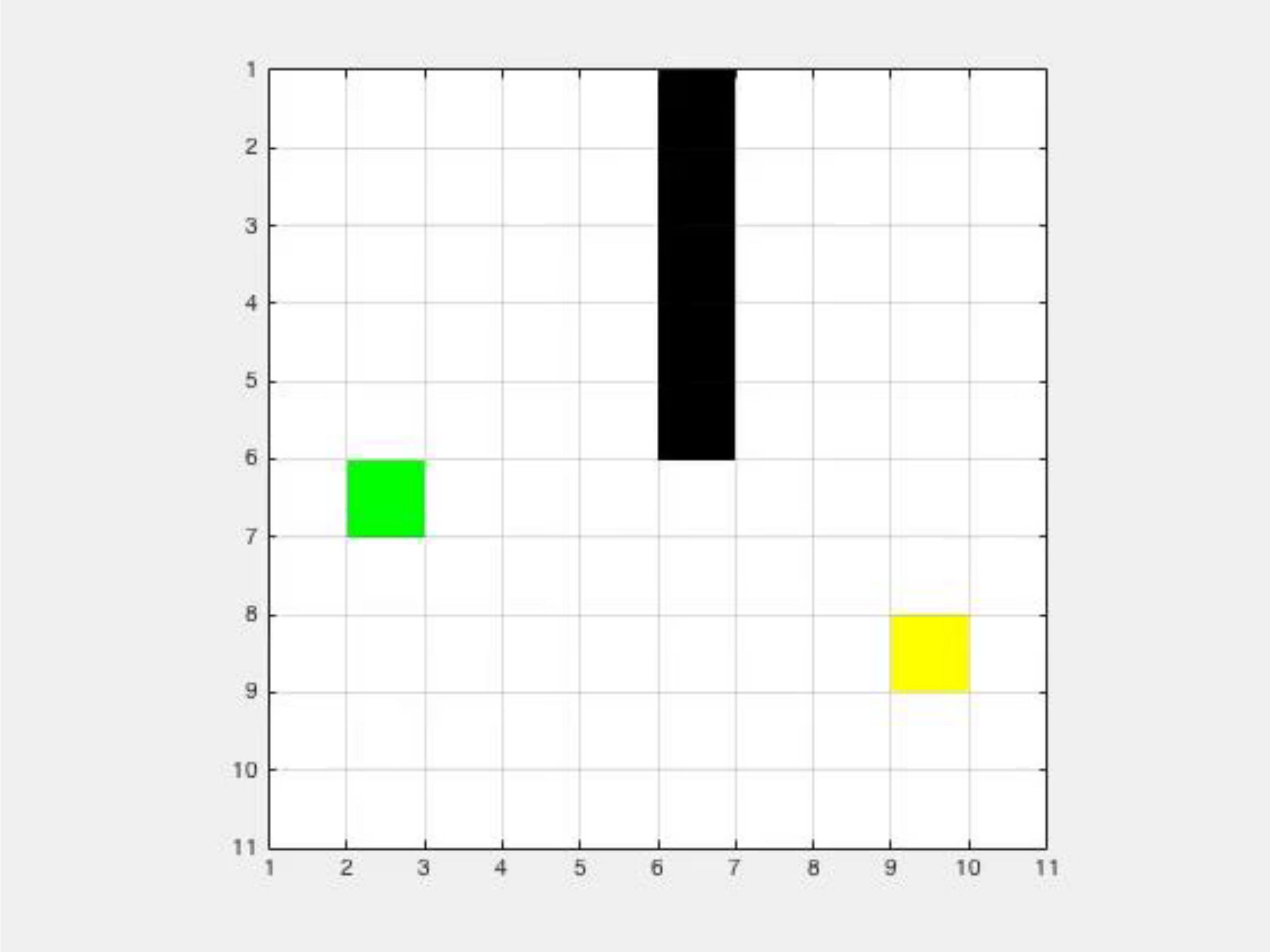
- **We also talked about Dijkstra's algorithm which is a general purpose procedure for planning shortest paths through arbitrary weighted graphs.**

Planning shortest paths – Dijkstra's Algorithm



- **Lastly we described the A* algorithm which is way to speed up the search for a shortest path when you have an informative heuristic to guide you.**

A* Algorithm

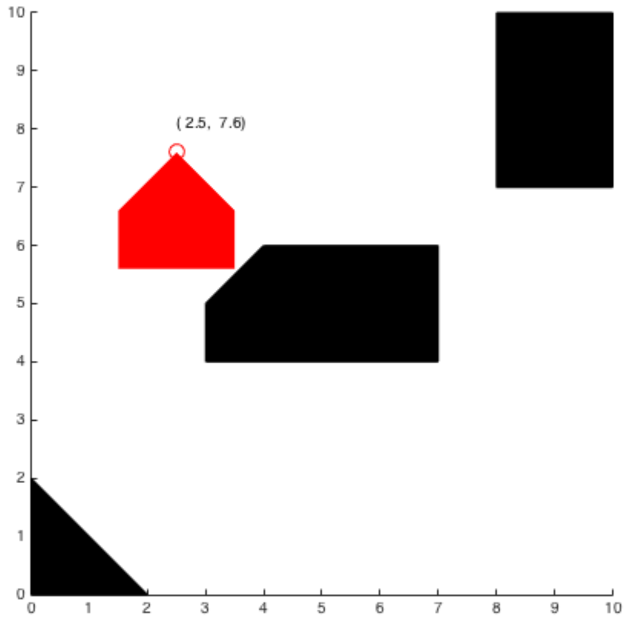


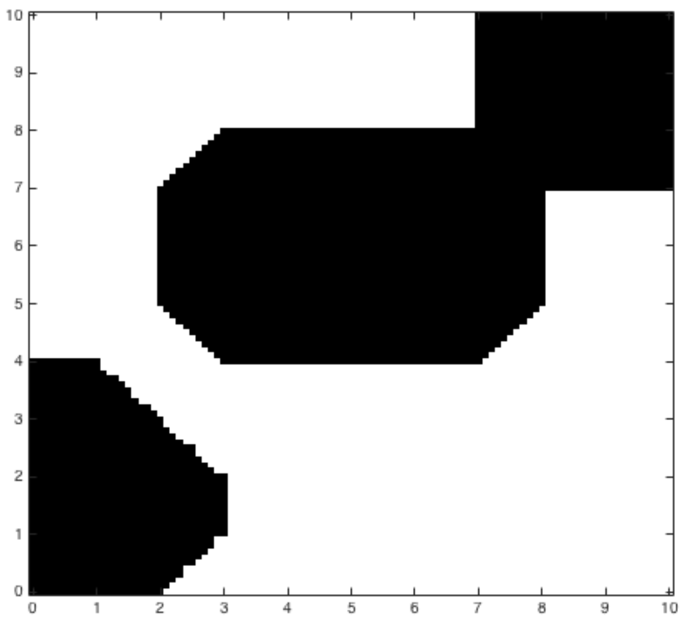
- **These graph based algorithms are important since they serve as the basis for a wide range of path planning procedures.**

- We went on to discuss the concept of configuration space which provided a uniform framework for thinking about a wide range of robotic systems.
- The configuration space of a robot corresponds to the set of all configurations that the robot can take on, the dimensionality and topology of this space depend upon the layout and structure of the system.

- **Given this abstraction we could then talk about configuration space obstacles which correspond to configurations that the robot cannot attain because of obstacles in the workspace.**

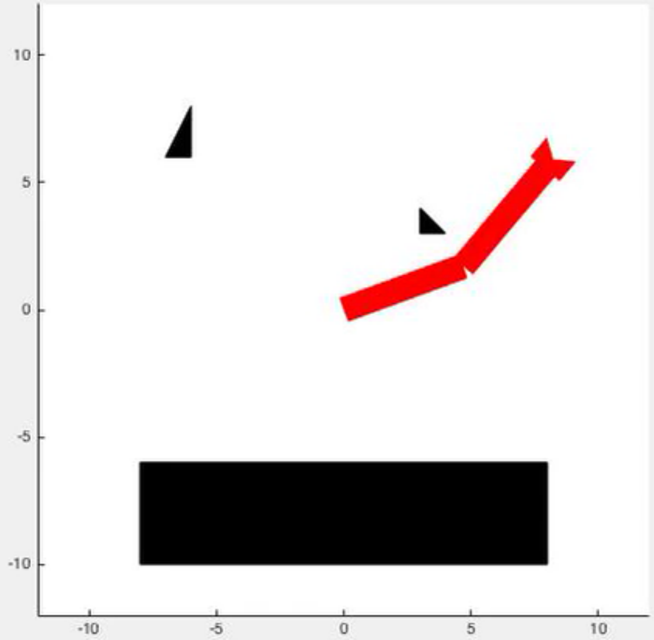
Configuration Space Obstacles

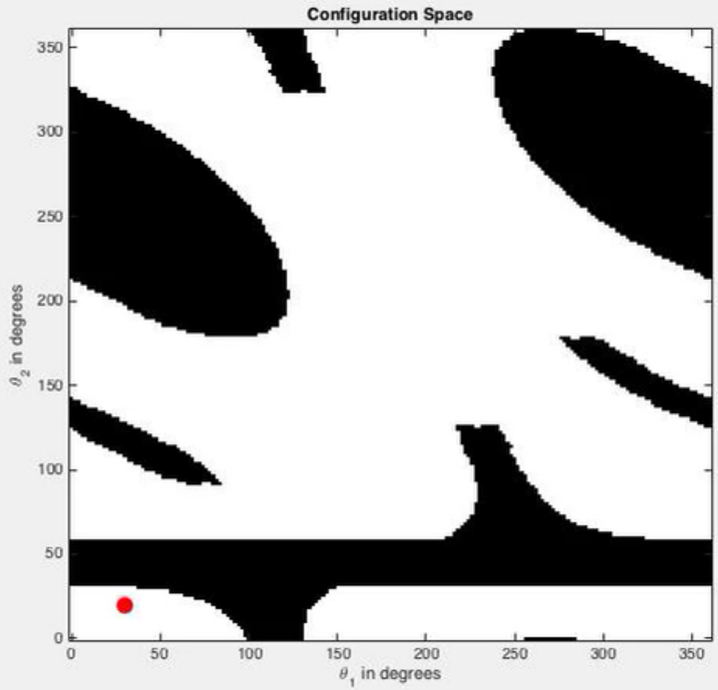




- This notion of configuration space allowed us to think about the motion of the robot in terms of the motion of a point moving through the configuration space while avoiding the configuration space obstacles as shown here

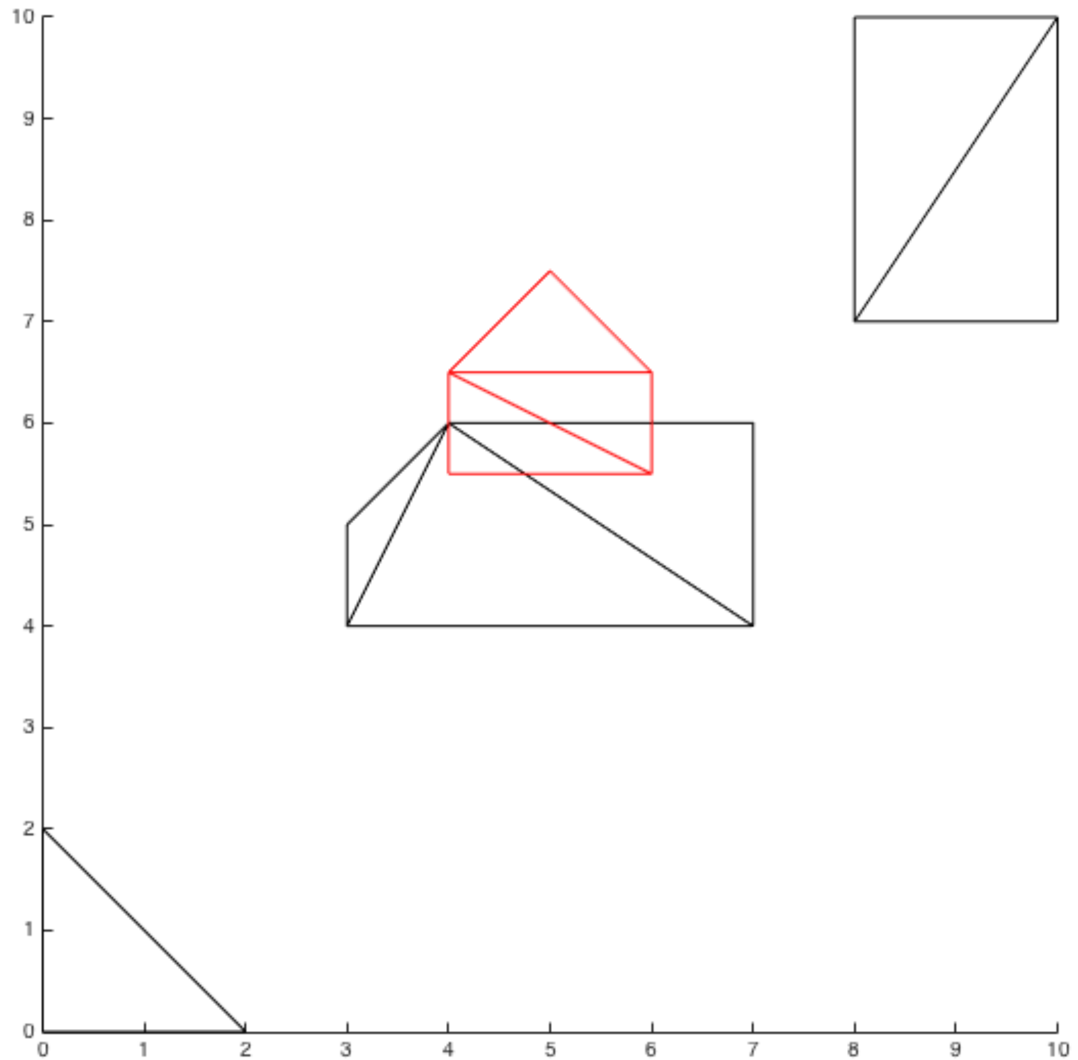
Planning Trajectory for 2 Link Arm





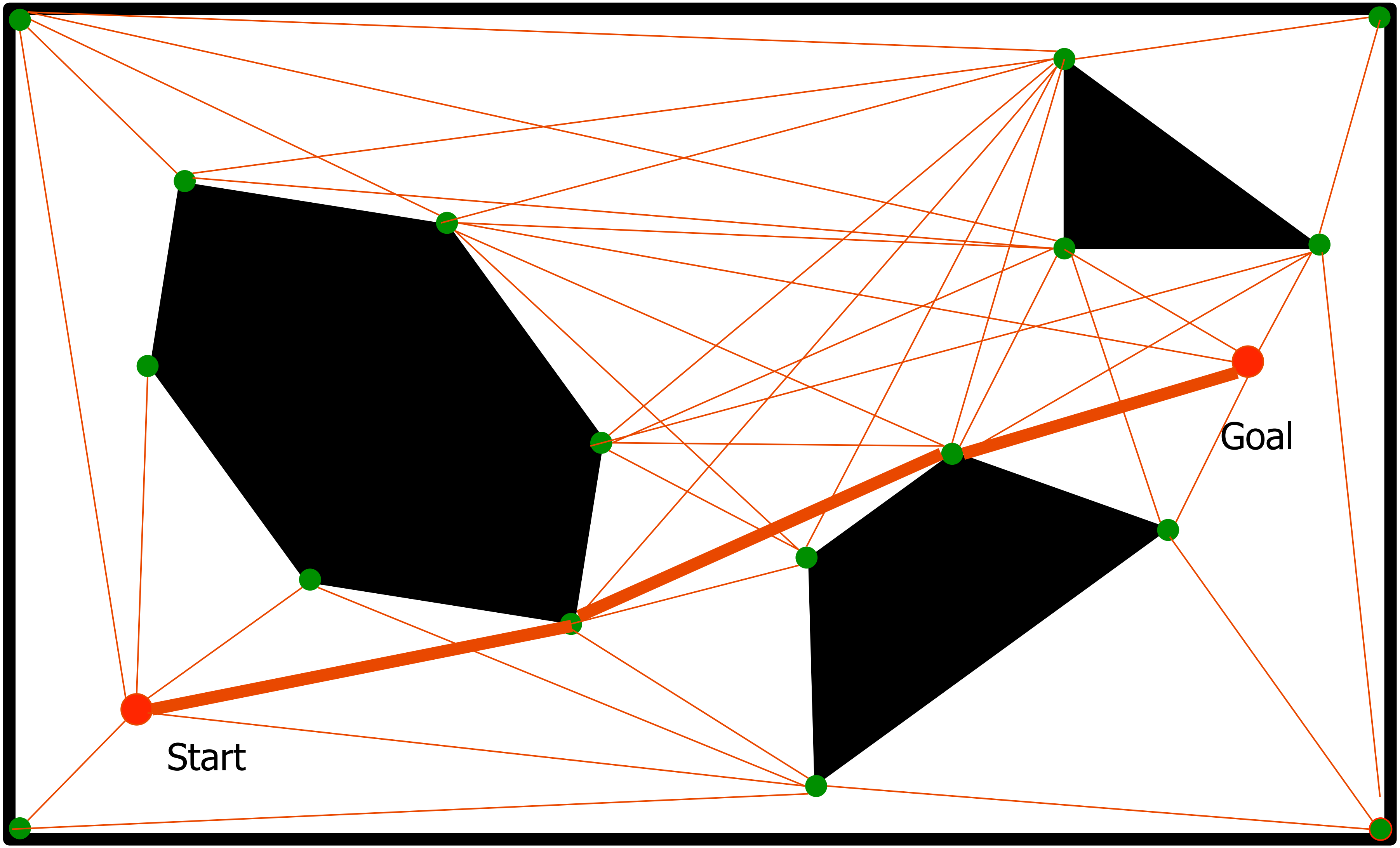
- **In the context of configuration space we talked briefly about CollisionChecking functions that could be used to decide whether or not a give configuration would collide with the workspace obstacles thus providing an implicit description of the configuration space obstacles and the complementary freespace.**

Collision Detection

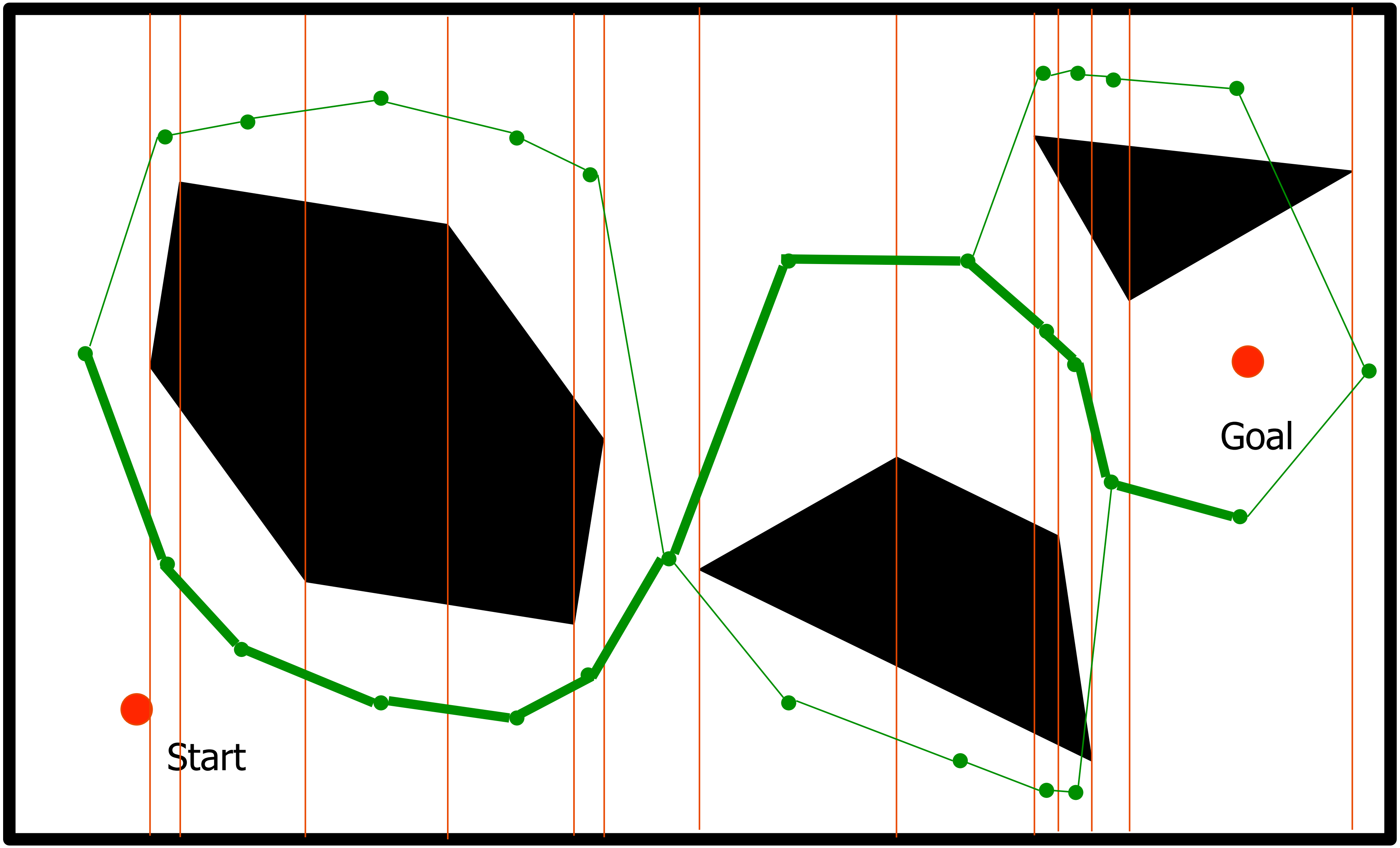


- **We talked about a few algorithms that could be used to plan paths through continuous configuration spaces.**

- **The visibility graph approach,**

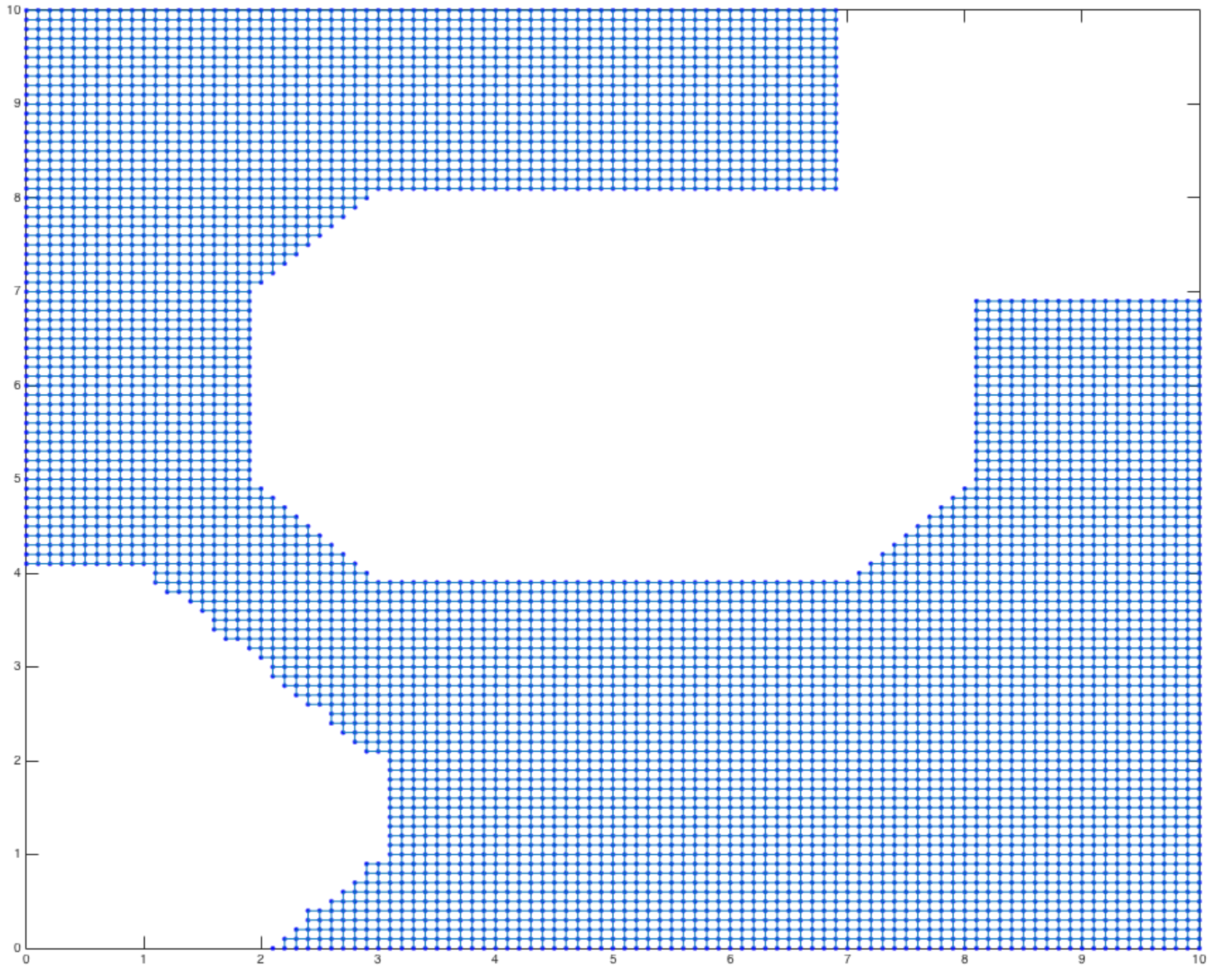

Goal
Start

- **The trapezoidal decomposition approach**


Goal
Start

- **And the grid based approach where we considered a discrete set of evenly spaced points in configuration space.**

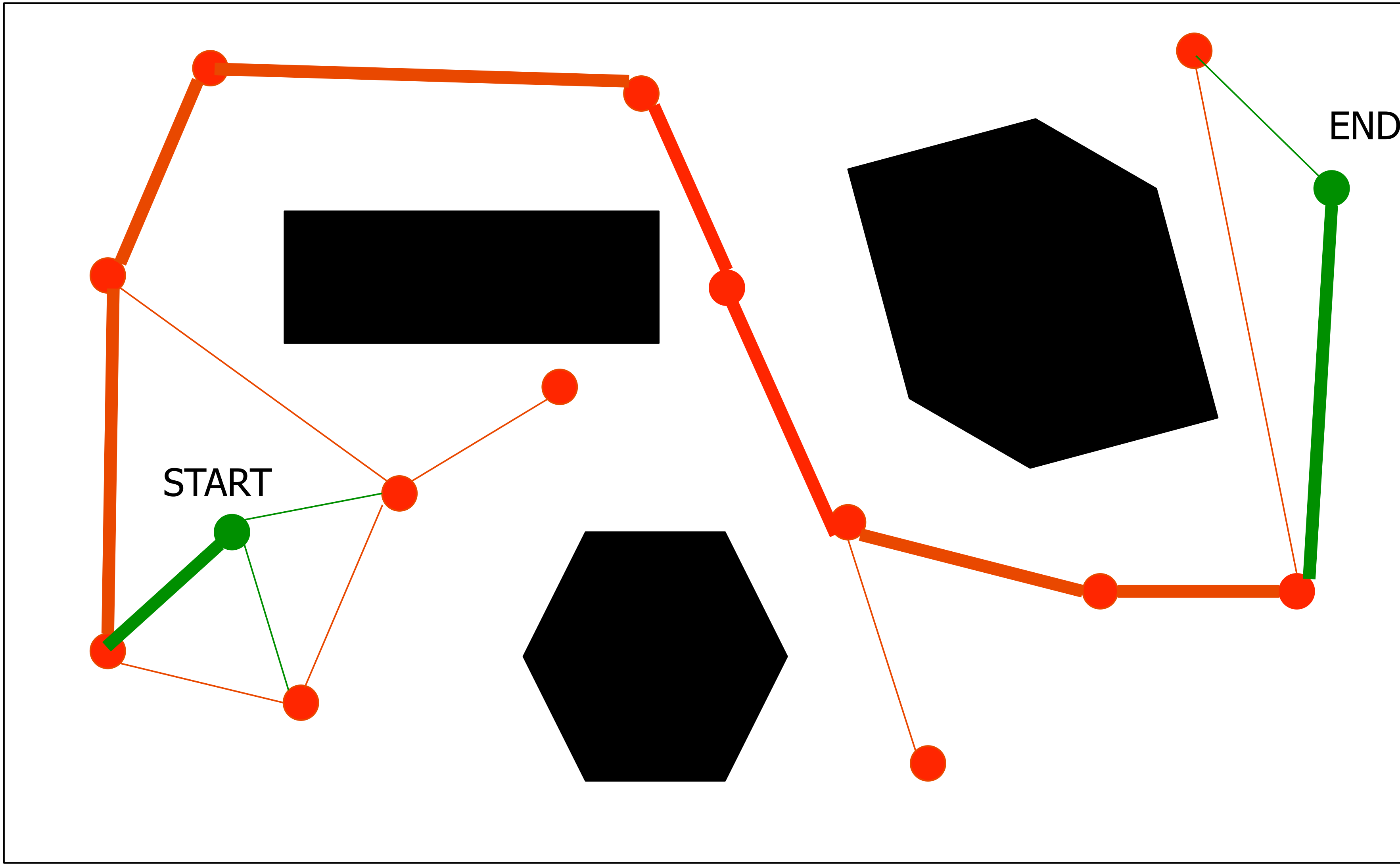
Graph Made from Sampled Points



- **All of these methods represented different approaches to capturing the structure of the continuous configuration space with a discrete graph so that we could apply standard graph based planning algorithms to solve the path planning problem.**

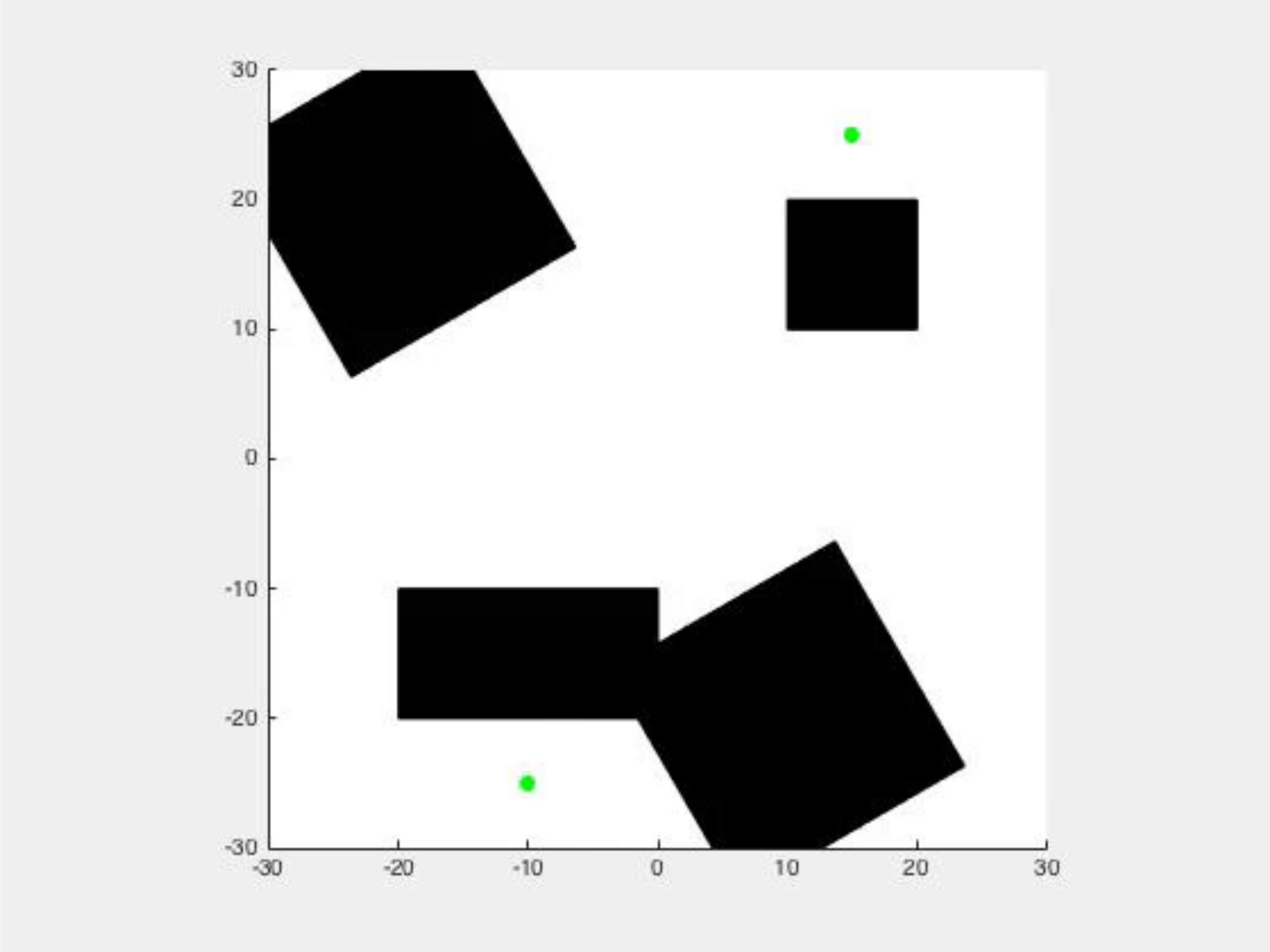
- Another important class of approaches that we touched on was based on the idea of random sampling. Here we construct a graph from randomly chosen samples in the configuration space connected by edges which represent collision free trajectories.
- We described two approaches to utilizing random samples. The probabilistic road map algorithm that sought to construct a roadmap or skeleton of the freespace.

Probablistic Road Map Planning



- **And the Rapidly Exploring Random Tree procedure which constructs ever evolving trees to explore the freespace and forge paths between the start and the goal.**

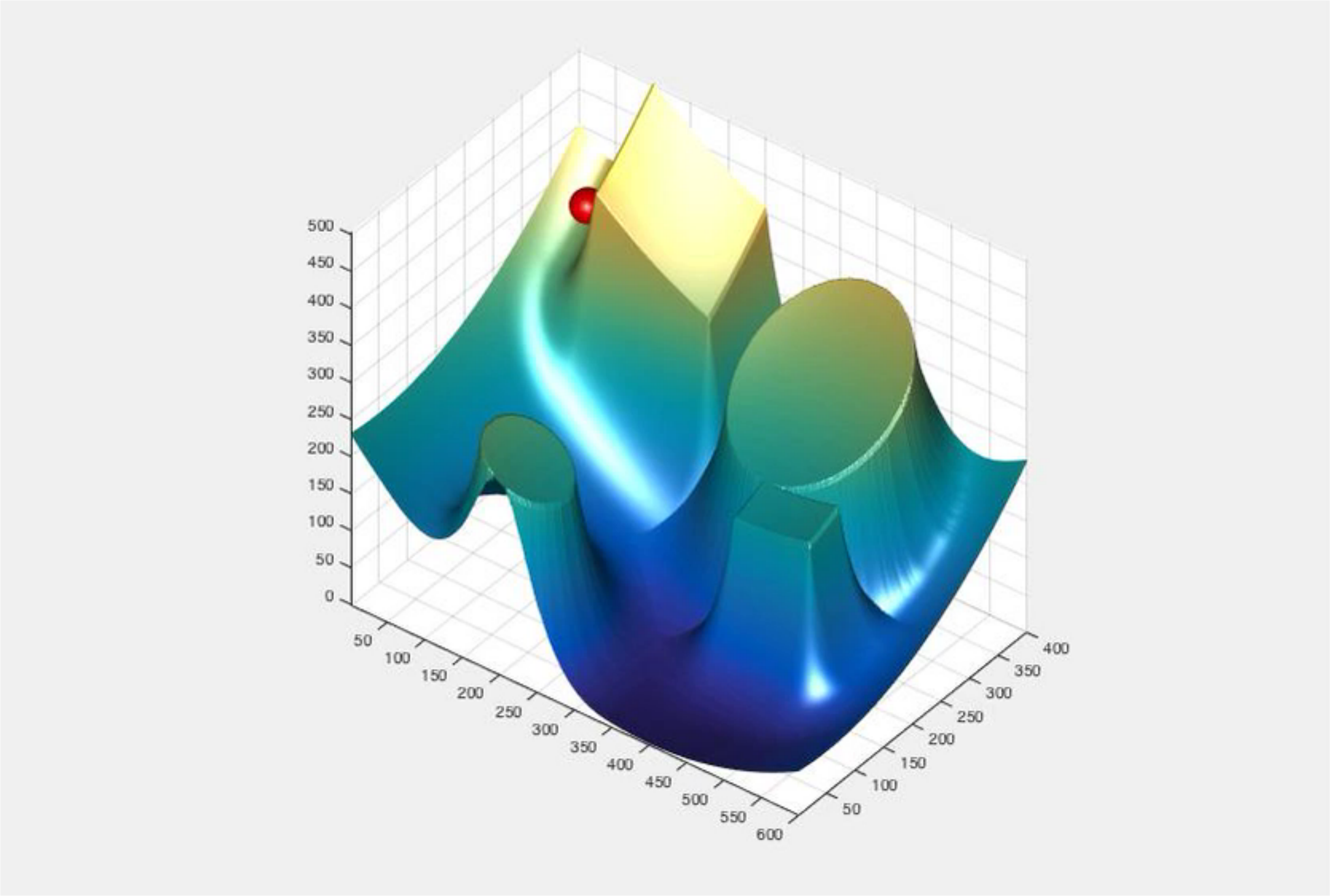
RRT Planning using 2 trees



- **Both algorithms have the pleasing property that they work quite well in practice even on high dimensional configuration spaces. Although when they fail it can be hard to decide whether the failure is due to the lack of any path or a lack of a sufficient set of random samples.**

- Finally we considered approaches based on artificial potential fields which are designed to attract the robot towards the desired configuration and repel it from configuration space obstacles.
- With these approaches we end up steering the robot through configuration space by considering the gradient of the potential function.

Artificial Potential Field Method



- **A strength of these potential field methods is that they are relatively simple to implement and they can often be carried out directly based on sensory input.**

Further Topics In Planning

- Non-holonomic Systems
  - What happens when you can't move freely in all directions?
- Kinodynamic Planning
  - Planning in the face of dynamic constraints
- Planning for Multiple Robots
- Planning with Moving Obstacles.
- Planning in the face of uncertainty