

# Преобразование тригонометрических выражений

## *Сложность «0»*

В задачах **8.031—8.040** доказать тождество:

$$8.031. \frac{\cos 3\alpha + \cos \alpha}{\sin 3\alpha - \sin \alpha} = \operatorname{ctg} \alpha$$

$$8.032. \frac{1 - \cos 2\alpha}{1 + \cos 2\alpha} = \operatorname{tg}^2 \alpha$$

$$8.033. \frac{\sin 4\alpha - \sin 2\alpha}{\cos^2(3/2)\alpha - \sin^2(3/2)\alpha} = 2 \sin \alpha \quad 8.034. \frac{(\sin \alpha - \cos \alpha)^2}{\sin^2((\pi/4) - \alpha)} = 2$$

$$8.035. \frac{\cos 2\alpha + \sin^2 \alpha}{\sin 2\alpha} = \frac{1}{2} \operatorname{ctg} \alpha$$

$$8.036. \cos 2\alpha + \operatorname{tg}^2 \alpha \cos 2\alpha - 1 = -\operatorname{tg}^2 \alpha$$

$$8.037. \sin 2\alpha \cos \alpha - \cos 2\alpha \sin 3\alpha = -\cos 4\alpha \sin \alpha$$

$$8.038. (\operatorname{tg} 2\alpha - \operatorname{tg} \alpha)(\cos \alpha + \cos 3\alpha) = 2 \sin \alpha$$

$$8.039. \sin 4\alpha \cos 2\alpha - \sin \alpha \cos \alpha = 0,5 \sin 6\alpha$$

$$8.040. \frac{2 \sin \alpha - \sin 2\alpha}{2 \sin \alpha + \sin 2\alpha} = \operatorname{tg}^2 \frac{\alpha}{2}$$

## *Сложность «I»*

В задачах **8.041—8.050** доказать тождество:

$$8.041. \frac{\sin^2 3\alpha - \sin^2 \alpha}{\cos^2 3\alpha - \cos 5\alpha \cos \alpha} = 2 \cos 2\alpha \quad 8.042. \frac{\cos^2 \alpha - \operatorname{ctg}^2 \alpha + 1}{\sin^2 \alpha + \operatorname{tg}^2 \alpha - 1} = \operatorname{ctg}^2 \alpha$$

$$8.043. \frac{\cos^2 2\alpha - 4 \cos^2 \alpha + 3}{\cos^2 2\alpha + 4 \cos^2 \alpha - 1} = \operatorname{tg}^4 \alpha \quad 8.044. \frac{1 + 2 \cos \alpha + \cos 2\alpha}{\cos 2\alpha - 2 \cos \alpha + 1} = -\operatorname{ctg}^2 \frac{\alpha}{2}$$

$$8.045. \frac{\sin \alpha}{\operatorname{tg} \left( \frac{\pi}{4} - \frac{\alpha}{2} \right) (1 + \sin \alpha)} = \operatorname{tg} \alpha$$

$$8.046. \frac{\sin^2 \alpha}{\cos \alpha (1 + \operatorname{ctg} \alpha)} - \frac{\cos^2 \alpha}{\sin \alpha (1 + \operatorname{tg} \alpha)} = \frac{1}{\cos \alpha} - \frac{1}{\sin \alpha}$$

$$8.047. \frac{\cos^4 \alpha - \sin^4 \alpha - \cos^2 \alpha}{2(\cos \alpha - 1)} = \cos^2 \frac{\alpha}{2}$$

$$8.048. \frac{1 + \cos \alpha + \cos 2\alpha + \cos 3\alpha}{\sin 2\alpha + 2 \sin \alpha \cos 2\alpha} = \operatorname{ctg} \alpha$$

$$8.049. \frac{1 + \sin \alpha + \cos \alpha}{1 + \sin \alpha - \cos \alpha} = \operatorname{ctg} \frac{\alpha}{2}$$

$$8.050. 1 + \frac{\cos 4\alpha}{\operatorname{tg} \left( \frac{3}{4} \pi - 2\alpha \right)} = \sin 4\alpha$$

*Сложность «I»*

В задачах **8.051—8.060** вычислить:

**8.051.**  $\sin^2 68^\circ - \sin^2 38^\circ - 0,5\sin 106^\circ + 3$

**8.052.**  $\sin 43^\circ \sin 17^\circ + \sin^2 13^\circ - 2$

**8.053.**  $\cos^2 36^\circ - \cos^2 120^\circ - 0,5\sin 18^\circ - 0,5$

**8.054.**  $\sin 49^\circ \sin 11^\circ + \cos^2 71^\circ + 1$

**8.055.**  $\sin^2 35^\circ + \sin^2 25^\circ + 0,5\cos 10^\circ - 2$

**8.056.**  $\sin^2 44^\circ + \cos^2 46^\circ + \cos 88^\circ + 1,5$

**8.057.**  $\sin^2 16^\circ + \cos 46^\circ \cos 14^\circ + 1$

**8.058.**  $\cos^2 41^\circ + \cos 79^\circ \cos 19^\circ - 1$

**8.059.**  $\sin 67^\circ \sin 7^\circ - \sin^2 37^\circ - 2$

**8.060.**  $\cos^2 84^\circ + \cos 51^\circ \cos 39^\circ + 3$

*Сложность «I»*

В задачах **8.061—8.070** вычислить:

**8.061.**  $\frac{3(\cos 20^\circ - \sin 20^\circ)}{\sqrt{2} \sin 25^\circ}$

**8.062.**  $\frac{(1 + \operatorname{tg} 10^\circ)\cos 10^\circ}{\sqrt{2} \sin 55^\circ}$

**8.063.**  $\frac{\sqrt{2}(\cos 25^\circ - \cos 65^\circ)}{\sin 20^\circ}$

**8.064.**  $\frac{\sin 40^\circ - \cos 40^\circ}{\sqrt{2} \cos 85^\circ}$

**8.065.**  $\frac{1 - \operatorname{ctg}^2 35^\circ}{\cos 70^\circ} \sin^2 35^\circ$

**8.066.**  $\frac{\cos 70^\circ + \sin 70^\circ}{\sqrt{2} \cos 25^\circ}$

**8.067.**  $\frac{1 - 2\cos^2 13^\circ}{\cos 26^\circ}$

**8.068.**  $\sqrt{2} \cdot \frac{\cos 80^\circ + \sin 80^\circ}{\sin 125^\circ}$

**8.069.**  $\frac{1 - 2\sin^2 46^\circ}{8 \cos 92^\circ}$

**8.070.**  $\frac{1 - \operatorname{tg}^2 75^\circ}{\cos 150^\circ} \cdot 2 \cos^2 75^\circ$

*Сложность «I»*

В задачах 8.071—8.080 вычислить:

$$8.071. (\operatorname{tg} 14^\circ + \operatorname{ctg} 28^\circ) \cos 14^\circ \sin 14^\circ \quad 8.072. \frac{\operatorname{tg} 15^\circ - \operatorname{ctg} 15^\circ}{\operatorname{ctg} 30^\circ}$$

$$8.073. \frac{\operatorname{tg} 26^\circ - \operatorname{ctg} 52^\circ}{\cos 78^\circ} \cos 26^\circ \sin 52^\circ \quad 8.074. \sin 24^\circ (\operatorname{tg} 12^\circ + \operatorname{ctg} 12^\circ)$$

$$8.075. \frac{(\operatorname{tg} 15^\circ + \operatorname{tg} 30^\circ) \cos 15^\circ \cos 30^\circ}{\sin 45^\circ} \quad 8.076. \frac{(1 + \operatorname{tg} 75^\circ) \cos 75^\circ}{2\sqrt{2} \sin 120^\circ}$$

$$8.077. (\operatorname{ctg} 27^\circ - \operatorname{ctg} 54^\circ) \sin 54^\circ + 1,5 \quad 8.078. \frac{\operatorname{tg}^2 31^\circ - \sin^2 31^\circ}{4\operatorname{tg}^2 31^\circ \sin^2 31^\circ}$$

$$8.079. \frac{\operatorname{ctg} 31^\circ - \operatorname{tg} 31^\circ}{\operatorname{tg} 31^\circ + \operatorname{ctg} 31^\circ} \cdot \frac{3}{\cos 62^\circ} \quad 8.080. \frac{\operatorname{ctg}^2 34^\circ - \cos^2 34^\circ}{2\operatorname{ctg}^2 34^\circ \cos^2 34^\circ}$$

*Сложность «I»*

В задачах 8.081—8.090 вычислить:

$$8.081. \frac{\sin^2 \frac{\pi}{5} \cos^2 \frac{\pi}{5}}{1 - \cos^4 \frac{2\pi}{5} - \cos^2 \frac{2\pi}{5} \sin^2 \frac{2\pi}{5}} \quad 8.082. \sin \frac{2\pi}{7} \cdot \frac{\left( \operatorname{tg} \frac{\pi}{7} + \operatorname{ctg} \frac{\pi}{7} \right) \cos^2 \frac{\pi}{14}}{1 + \cos \frac{\pi}{7}}$$

$$8.083. \frac{\left( 1 - \operatorname{tg} \frac{\pi}{8} \right)^2 \cos^2 \frac{\pi}{8}}{\sin^2 \frac{\pi}{8}} \quad 8.084. \frac{\left( \sin \frac{\pi}{9} - \cos \frac{\pi}{9} \right)^2 - 1}{\sin \frac{2\pi}{9}}$$

$$8.085. \frac{2 \left( 1 + \sin \frac{2\pi}{11} - \cos \frac{2\pi}{11} \right)}{\sin \frac{\pi}{11} \left( \sin \frac{\pi}{11} + \cos \frac{\pi}{11} \right)} \quad 8.086. \frac{\left( \sin \frac{\pi}{5} + \cos \frac{\pi}{5} \right)^2 - 1}{\sin \frac{2\pi}{5}}$$

$$8.087. \frac{3 \left( \cos \frac{\pi}{7} - \sin \frac{\pi}{7} \right)^2}{\sin^2 \left( \frac{\pi}{4} - \frac{\pi}{7} \right)} \quad 8.088. \frac{\sin^4 \frac{\pi}{9} - \cos^4 \frac{\pi}{9}}{2 \cos \frac{2\pi}{9}}$$

$$8.089. \frac{\left( \operatorname{tg} \frac{\pi}{10} - \operatorname{ctg} \frac{\pi}{10} \right)^2}{\operatorname{ctg}^2 \frac{\pi}{5}} \quad 8.090. \frac{\left( \cos \frac{\pi}{11} + \sin \frac{\pi}{11} \right)^2}{\cos^2 \left( \frac{\pi}{4} - \frac{\pi}{11} \right)}$$