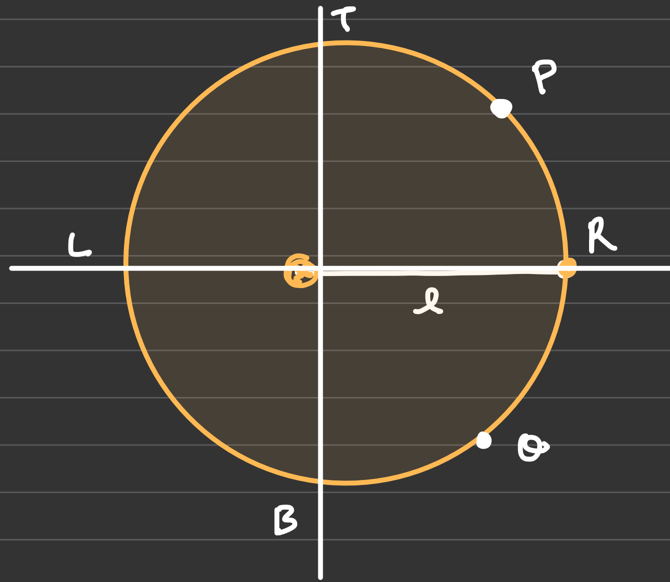
E&M3

VMC Vidyamandir Classes SINCE 1986

IIT JEE | MEDICAL | FOUNDATION

Vertical Circular motion:

①

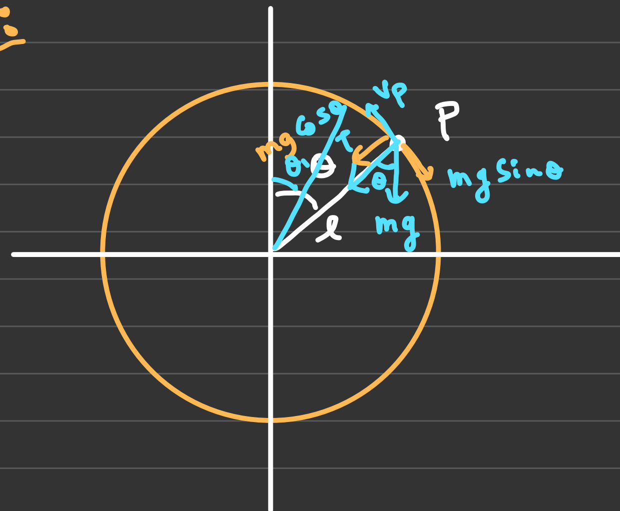


Uniform Circular motion
(Speed const)

Non-Uniform Circular motion
(Speed changing

Analysis:

Case I:



find min velocity at P for which Particle maintain circular motion at this point?

$$mg\cos\theta + T = \frac{mv^2}{l}$$

$$T = \frac{mv^2}{l} - mg\cos\theta \geq 0$$

"for min velocity at this θ to maintain circular path"

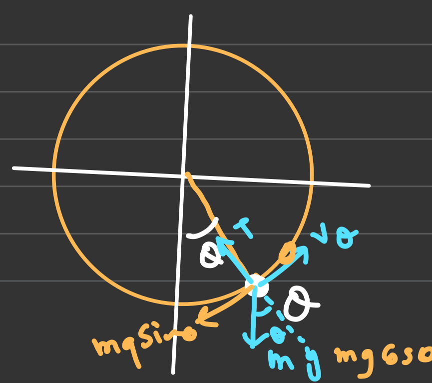
$$\frac{mv_P^2}{l} - mg\cos\theta = 0$$

$$\frac{m v_p^2}{l} = m g \cos\theta$$

$$\boxed{v_p = \sqrt{g l \cos\theta}}$$

"if particle is just maintain at θ then at θ' it will not main circular path as $\underbrace{\theta' < \theta}$ the requirment of speed will increase"

Case II:



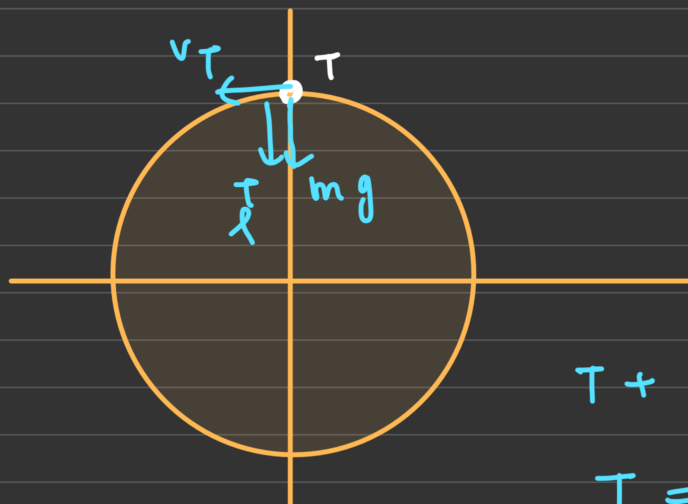
find min velocity at θ for which particle just maintain circular path?

$$T - mg\cos\theta = \frac{m v_\theta^2}{l}$$

$$\Rightarrow \quad T = \frac{m v_\theta^2}{l} + mg\cos\theta \quad \text{——} \ (1)$$

$$\Rightarrow \quad \boxed{v_\theta = 0}$$

Case III:



find min velocity for which particle maintains circular path at 'T'?

$$T + mg = \frac{m v_T^2}{l}$$

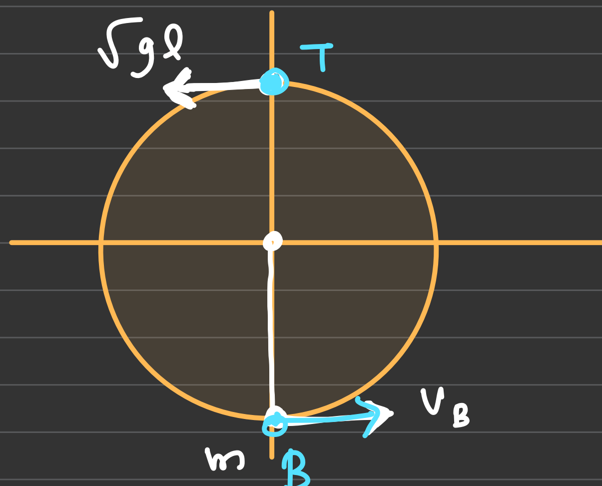
$$T = \frac{m v_T^2}{l} - mg = 0$$

$$\frac{m v_T^2}{l} = mg$$

$$v_T = \sqrt{gl}$$

" if particle is maintaining circular path at topmost point the particle is definitely going to complete circular path "

Case IV :



find **min** velocity at B for which particle is going to complete circular motion

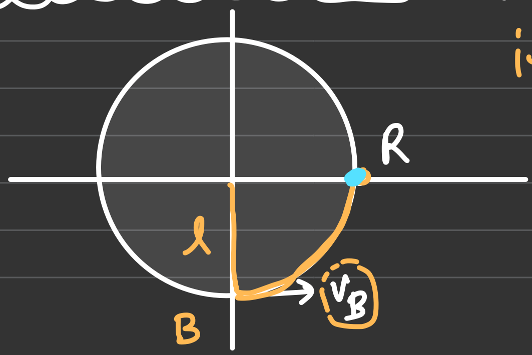
B – T (Law of Conservation Energy)

$$\frac{1}{2}\cancel{m}v_B^2 - \frac{1}{2}(\cancel{m})\left(\sqrt{gl}\right)^2 = \cancel{m}g(2l)$$

$$v_B = \sqrt{5gl}$$

Case V): find min velocity at B for which particle complete circular path till R.

Law of Conservation Energy:



if it is maintain till R then $v_R = 0$

$v_R = 0$

$$\frac{1}{2} m v_B^2 - \frac{1}{2} m \cancel{v_P^2}_{0} = mgl$$

$$v_B = \sqrt{2gl}$$

$$v_B \leq \sqrt{2gl}$$

{ " then it is going to oscillate in lower half of Circle "

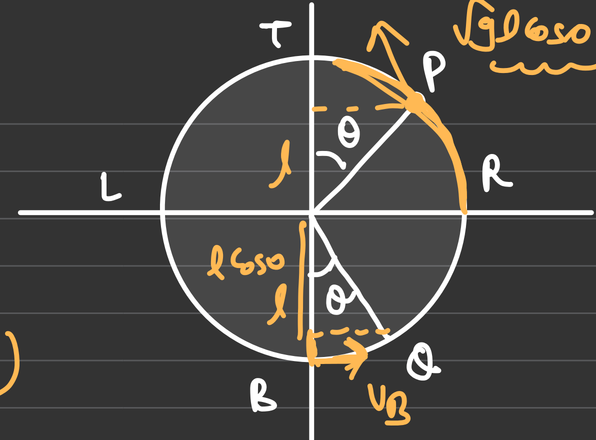
(Case VI)

find min velocity at B for which particle maintains circular path till

a) P

b) Q

a) if it maintains till 'P'



$\frac{1}{2} m \underset{??}{v_B}^2 - \frac{1}{2} m\left(\sqrt{gl\cos\theta}\right)^2 = mg(l + l\cos\theta)$

$v_B =$ Ans.

b) if it maintain circular path till Q the $(v_B)_{min}$?

$v_Q = 0$

$\frac{1}{2} m \underset{??}{v_B}^2 - \frac{1}{2} m(0)^2 = mg(l - l\cos\theta)$

$$\boxed{v_B = \sqrt{2g(l - l\cos\theta)}}$$

Summery:

1. $v_B > \sqrt{5gl}$ it is definately going complete circular path

2. $\sqrt{2gl} < v_B < \sqrt{5gl}$

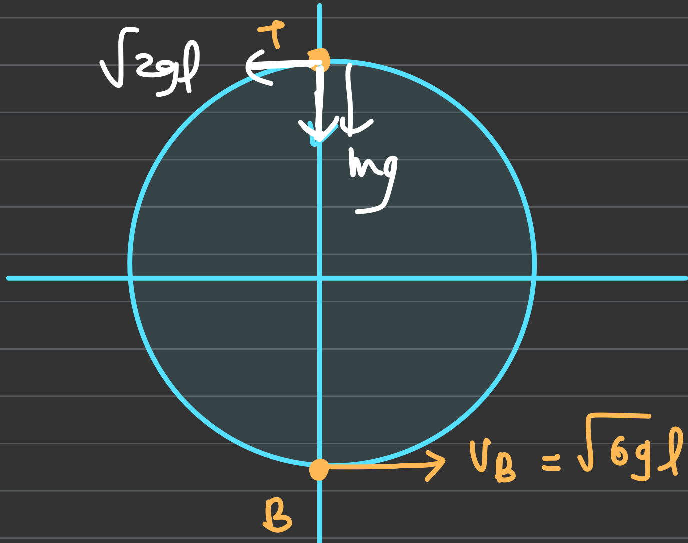
   it is going to slack between R and T

3. $v_B < \sqrt{2gl}$

   it is going to oscillated in lower half or

R to L

Q)



if $V_B = \sqrt{6gl}$ then find velocity and T at topmost point?

① Law of Conservation of Energy from $B \to T$

$$\frac{1}{2} m (\sqrt{6gl})^2 - \frac{1}{2} m (V_T)^2 = mg(2l)$$

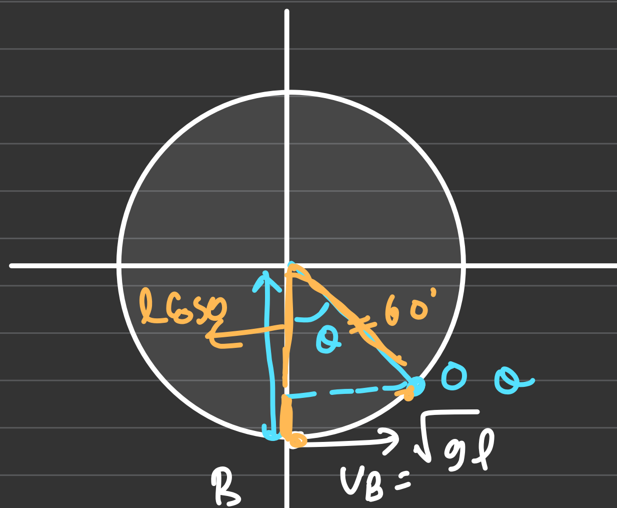
$$\boxed{V_T = \sqrt{2gl}} \quad \underline{Ans}$$

② $T + mg = \frac{m v_T^2}{l}$

$$T = \frac{m(\sqrt{2gl})^2}{l} - mg = mg$$

θ) :

find θ_{max} with vertical till it stops



$$\frac{1}{2} m v_B^2 - \frac{1}{2} m \cancel{v_0^2} = mg(l - l\cos\theta)$$

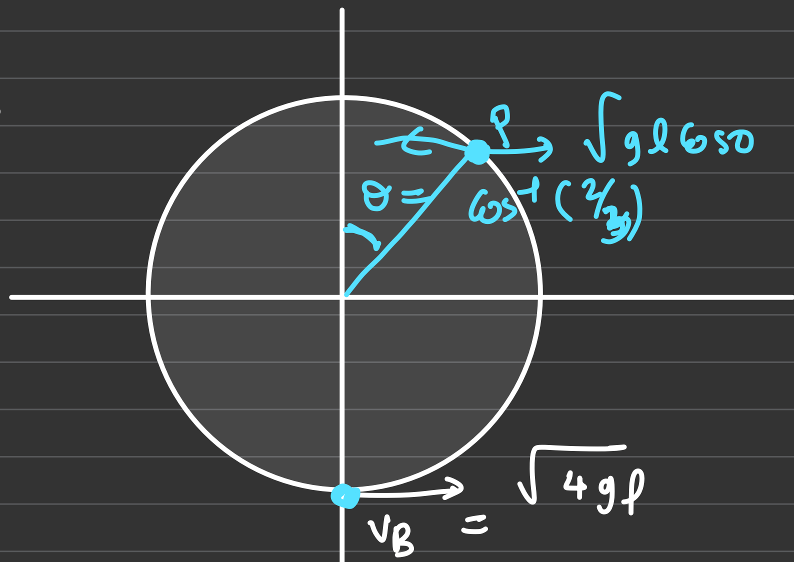
$$\frac{(\sqrt{gl})^2}{2} = g(l - l\cos\theta)$$

$$\frac{gl}{2} = gl\cos\theta$$

$$\cos\theta = \frac{1}{2}$$

$$\theta = 60$$

Q)



a) find θ at which it is leaving circular path?

$$\frac{1}{2}\cancel{m}\left(\sqrt{4gl}\right)^2 - \frac{1}{2}\cancel{m}\left(\sqrt{gl\cos\theta}\right)^2 = \cancel{m}g(l + l\cos\theta)$$

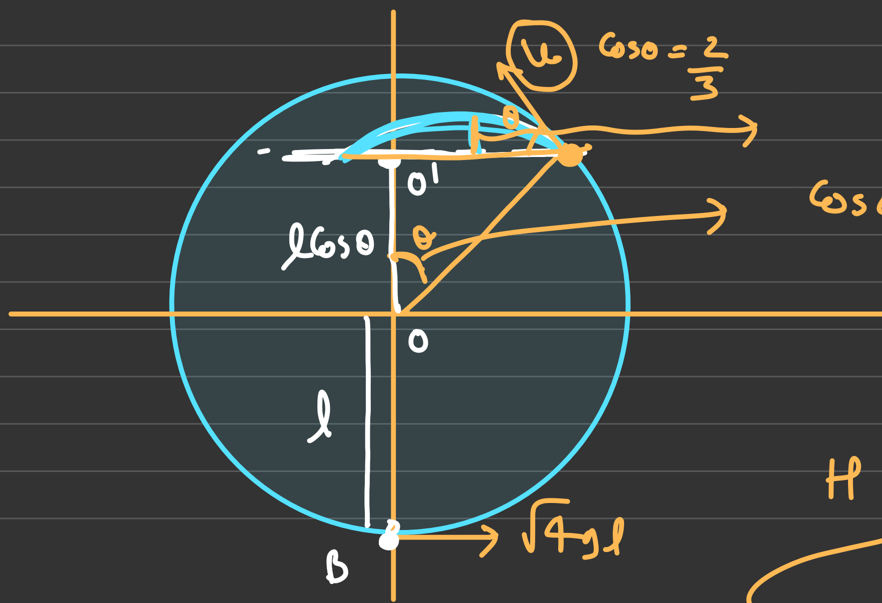
$$2gl - \frac{gl\cos\theta}{2} = gl + gl\cos\theta$$

$$g\cancel{l} = \frac{3}{2} g\cancel{l} \cos\theta$$

$$\cos\theta = \frac{2}{3}$$

$$\theta = \cos^{-1}(2/3)$$

(b) find maximum Height attained by Particle w.r.t B. ?

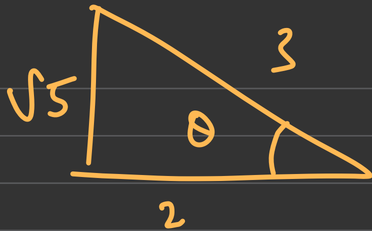


$\cos\theta = 2/3$

$$= BO + OO'$$
$$= l + l \times \frac{2}{3}$$
$$= \left(\frac{5l}{3}\right)$$

$$H = \frac{u^2 \sin^2\theta}{2g}$$

$$u = \sqrt{g l \cos\theta}$$



$$\sin\theta = \frac{\sqrt{5}}{3}$$

$$H = \frac{(\cancel{g} l \cos\theta)\sin^2\theta}{2\cancel{g}}$$

$$H = \frac{l \cos\theta \times \sin^2\theta}{2}$$

$$H_{max} = l \times \frac{\cancel{2}}{3} \times \frac{1}{\cancel{2}} \left[\frac{5}{9}\right] = \frac{5l}{27}$$

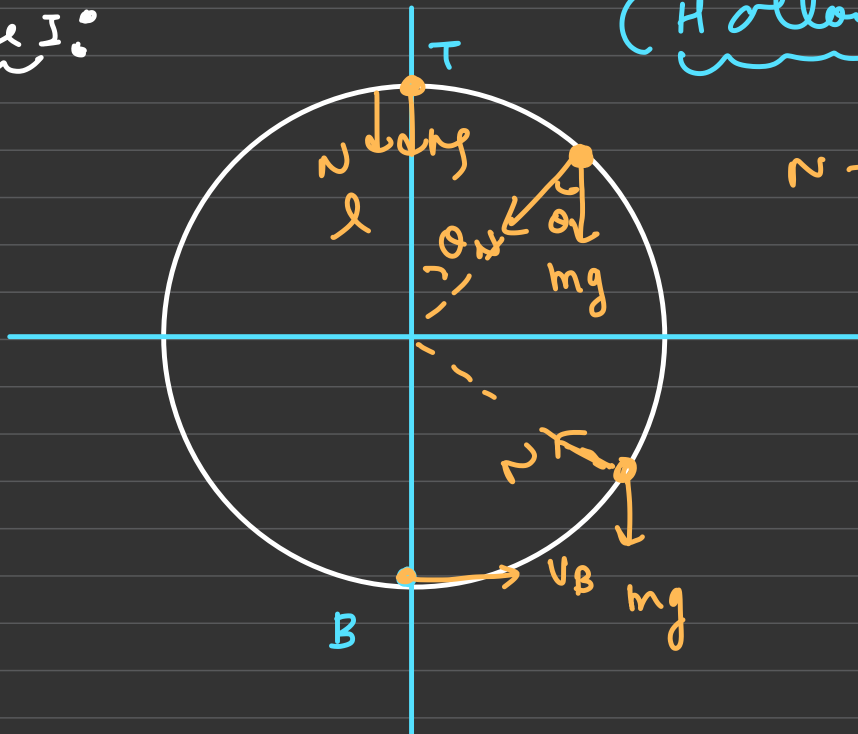
max. Height attained by particle $= \frac{5l}{3} + \frac{5l}{27}$

$$= \frac{50l}{27}$$

Variation in Vertical Circular motion:

(Hollow tube)

Case I:



$$N + mg\cos\theta = \frac{mv_P^2}{l}$$

$$N = 0$$

$$\boxed{v_P = \sqrt{gl\cos\theta}}$$

$$\cancel{N} + mg = \frac{mv_T^2}{l}$$

$$0 > 0$$

$$\sqrt{gl}$$

$$\frac{1}{2}\cancel{m}v_B^2 - \frac{1}{2}\cancel{m}(\sqrt{g0})^2 = \cancel{m}g(2l)$$

$v_B = \sqrt{5g\ell}$

Variation #2

$v_T = 0$

→ massless rod

m → $(v_B)_{min}$ for which this rod completes circular motion?

"In this case, at topmost point particle needs only 0 velocity to pro mean circular path "

law of conservation of Energy to calculate $(v_B)_{min}$ to complete circular path?

$$\frac{1}{2}mv_B^2 - \frac{1}{2}m\cancel{v_T^2} = mg(2l)$$

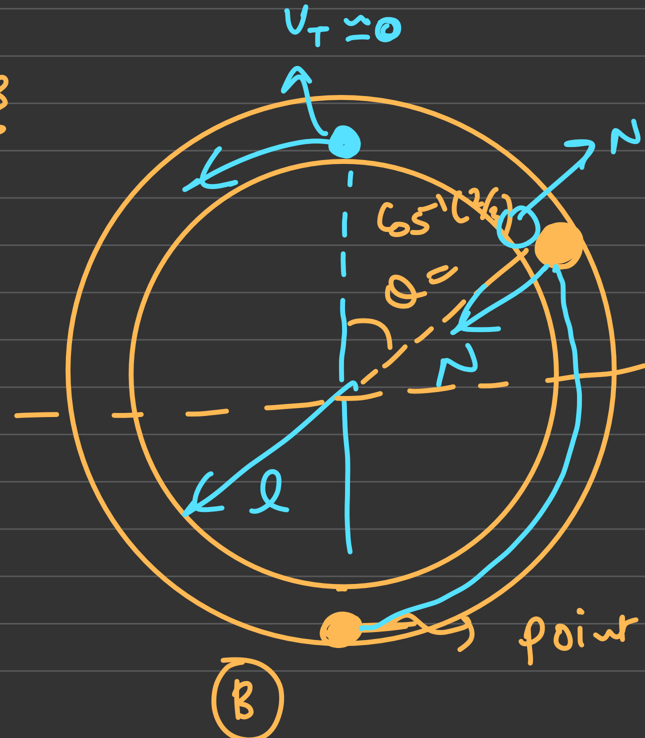
$$0$$

$$(v_B)_{min} = \sqrt{4gl}$$

$$\left\{\begin{array}{ll} v_B > \sqrt{4gl} & \text{then definitely complete circular path} \\ v_B < \sqrt{4gl} & \text{Not going to complete} \end{array}\right.$$

" In this case, due to rigid, particle is never, ever going to leave circular path for any velocity "

Variation #3



" two concentric tubes "

find min velocity at B for which particle is going to complete circular path?

$$\frac{1}{2} m v_B^2 - 0 = mg(2l)$$

$$\boxed{v_B = \sqrt{4gl}}$$ Ans

a) if $\underline{v_B = \sqrt{4gl}}$ then find θ at which particle leaves outer circle and comes in contact with inner circle

$$\left\{ \cos\theta = \frac{2}{3} \right\}$$

b) if in this $v_B = \sqrt{3.5gl}$ then find θ at which it is going to leave out circle and comes in

H.W

Contact with inner circle?

Home work → DTS #2 → Level 1, Level 2

Home work → module → INBE A