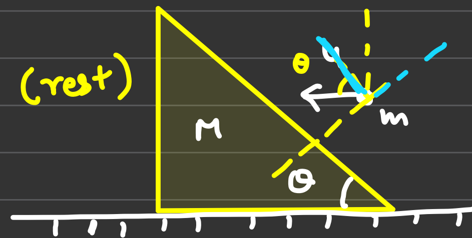
E&M6

VMC Vidyamandir Classes SINCE 1986

IIT JEE | MEDICAL | FOUNDATION

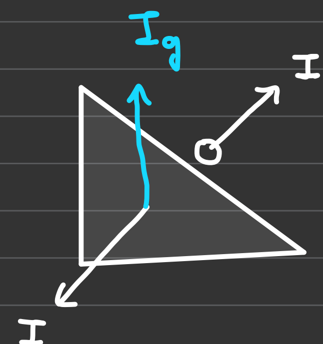
Q)

Steps:

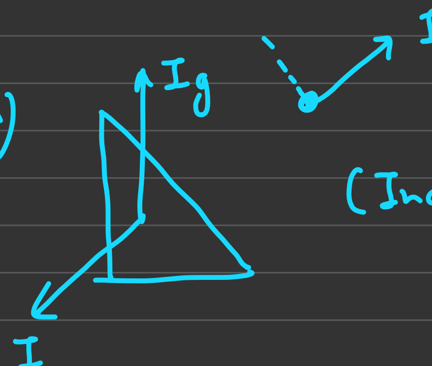


① ✓

② Impulse diagram

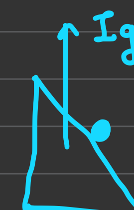


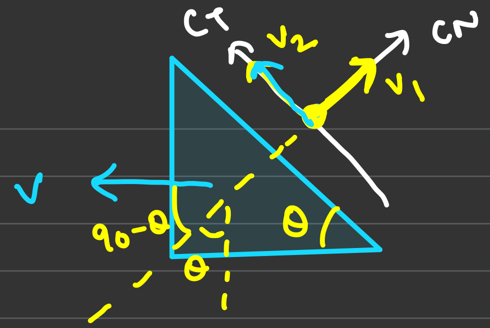
a)



(Individual)

b) System



③ 

'e'

find velocities After Collision

④ Apply $v_{sep} = e\, v_{app}$ "along Common Normal"

$$(v_1 + v\sin\theta) = e \times u\sin\theta \quad \text{— ①}$$

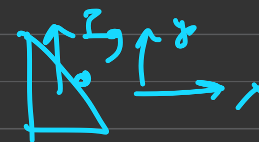
?

⑤ a) { if impulse along any axis on system or object is $=0$ then momentum must not change (momentum is conserved) }

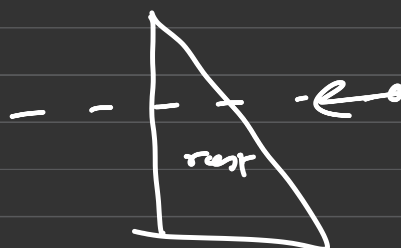
$\underline{mu}\cos\theta = \underline{mv_2}$ { on point $\perp$ to impulse axis }

$v_2 = u\cos\theta$

b) we can apply law of Con. of L.M along x-axis as impulse along that axis on System = 0



before Collision

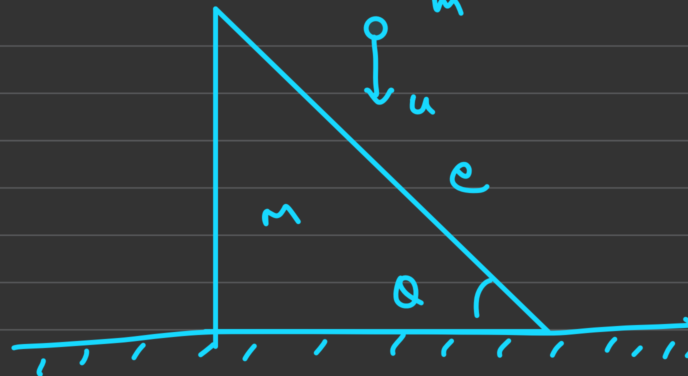


After collision

$0 - \underline{mu} = -\underline{MV} + \underline{m(v_1\sin\theta - v_2\cos\theta)}$

· Solve all three eqn (iii)

Homework:



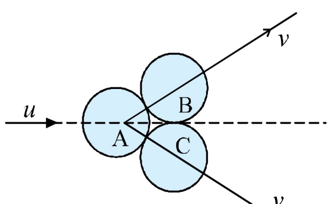
find velocities after collision?

Homework

Illustration - 19 *Two equal spheres of mass m are in contact on a smooth horizontal table. A third identical sphere impinges symmetrically on them and is reduced to rest. Prove that e = 2/3 and find the loss of KE.*

#

SOLUTION :



Newton's experimental Law :

For an oblique collision, we have to take components along normal i.e., along *AB* for balls *A* and *B*.

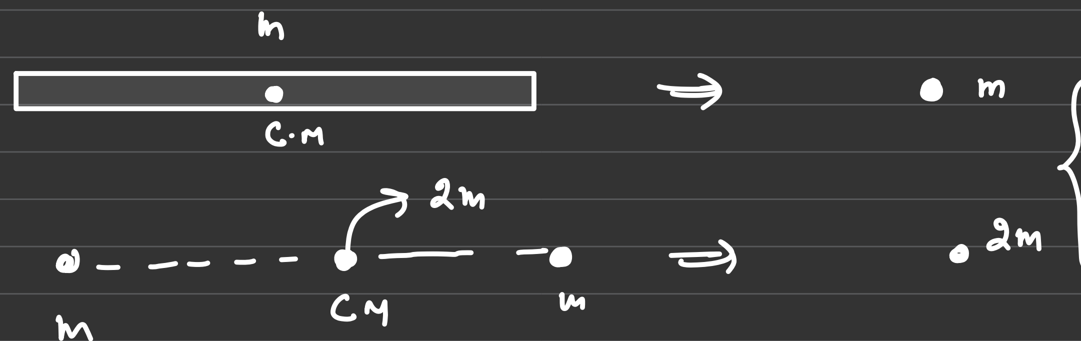
$\Rightarrow \quad v - 0 = e\,(u\cos 30^\circ - 0)$

"Centre of mass"

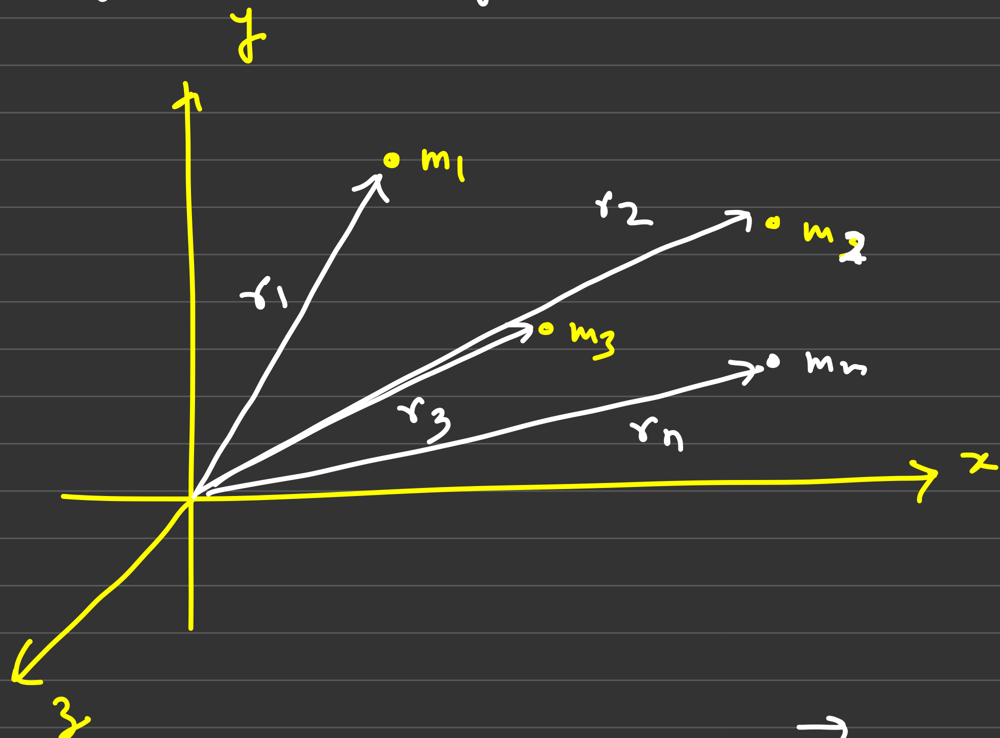
" is a point where we can assume all the mass of the body "

#

#



① Centre of mass of system of particles:



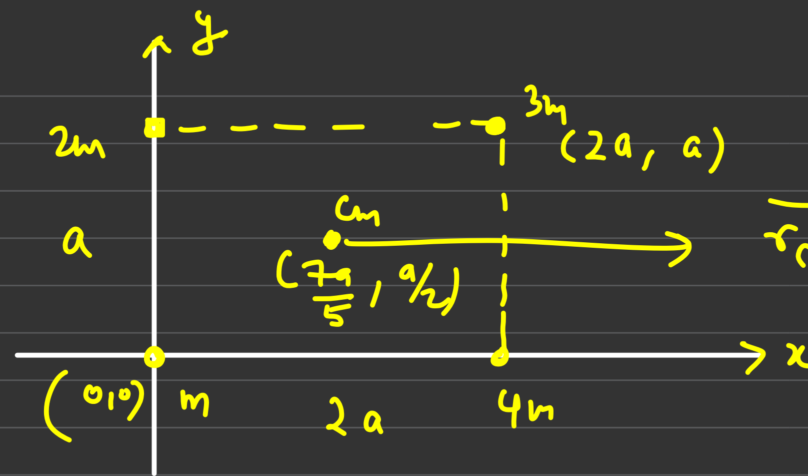
$$\vec{r}_{cm} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + \cdots m_n\vec{r}_n}{m_1 + m_2 + \cdots m_n} = \frac{\Sigma m_i r_i}{\Sigma m_i}$$

$$\begin{cases} x_{cm} = \dfrac{m_1 x_1 + m_2 x_2 \dots m_n x_n}{m_1 + m_2 + \dots m_n} \\[2ex] y_{cm} = \dfrac{m_1 y_1 + m_2 y_2 + \dots m_n y_n}{m_1 + m_2 \dots n} \\[2ex] z_{cm} = \dfrac{m_1 z_1 + m_2 z + \dots m_n z_n}{m_1 + m_2 + \dots m_n} \end{cases}$$

$$\vec{r}_{cm} = x_{cm}\,\hat{i} + y_{cm}\,\hat{j} + z_{cm}\,\hat{k}$$

Q #)

find COM of system of particle ?



$$\vec{r}_{cm} = \frac{7a}{5}\hat{i} + \frac{a}{2}\hat{j}$$

$$x_{cm} = \frac{m \times 0 + 4m \times 2a + 3m \times 2a + 2m \times 0}{10m}$$

$$x_{cm} = \frac{14ma}{10m} = \left(\frac{7}{5}a\right)$$

$$y_{cm} = \frac{m \times 0 + 2m \times a + 3m \times a + 4m \times 0}{10m} = \frac{5ma}{10m}$$

$$= a/2$$

Centre of mass of bodies: / Extended bodies

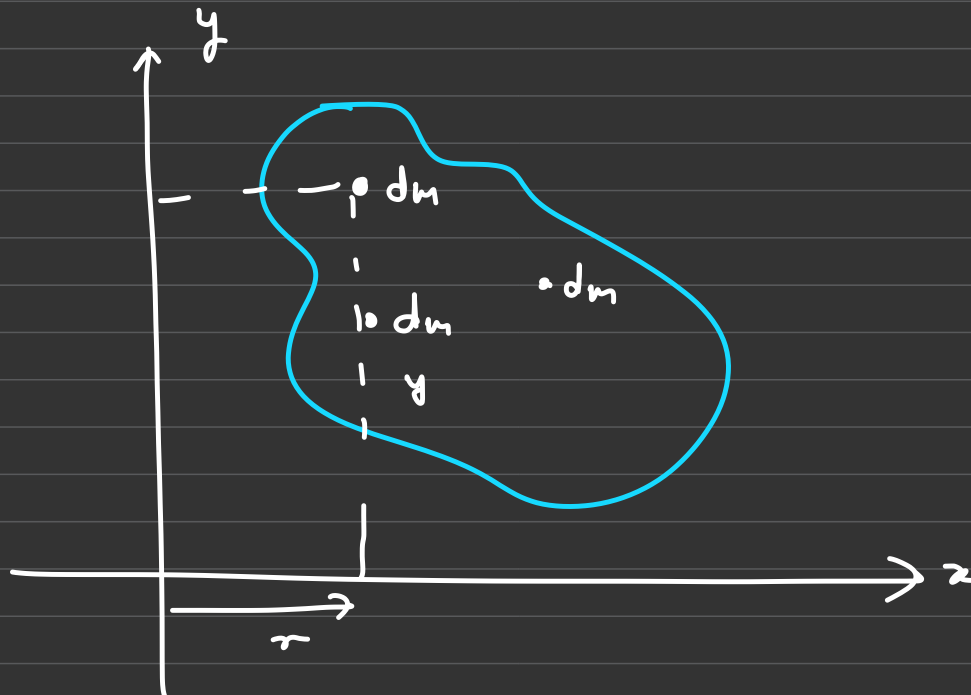
① Ring ② disc ③ Solid sphere

Basic Concept:

$$x_{cm} = \frac{\int dm\, x}{\int dm}$$

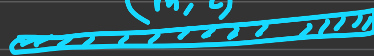
$$y_{cm} = \frac{\int dm\, y}{\int dm}$$

$$z_{cm} = \frac{\int dm\, z}{\int dm}$$



bodies {on basis of mass}

linear body (m, L)



$\lambda = \frac{m}{L}$

mass per unit length

Areal body

m, A

disc

$\sigma = \frac{m}{A}$

Areal mass density

{Hollow sphere / Area body}

volumetric body

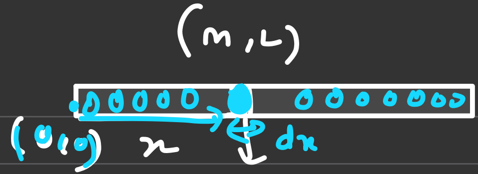
Solid sphere

Volm body

$\rho = \frac{m}{V}$

volumetric mass density

Rod:



dx, dm

"dm is mass dx length rod"

$$L \longrightarrow M$$

$$1 \longrightarrow \frac{M}{L}$$

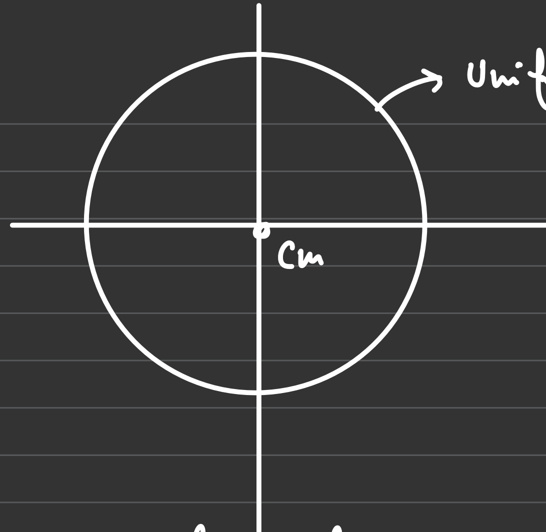
$$dx \longrightarrow \left(\frac{M}{L} \cdot dx\right) = dm$$

$$\Rightarrow \quad x_{cm} = \frac{\int (dm\, x)}{\int dm}$$

$$= \frac{\int \left(\frac{\cancel{M}}{L} \cdot dx\right) \cdot x}{\cancel{M}}$$

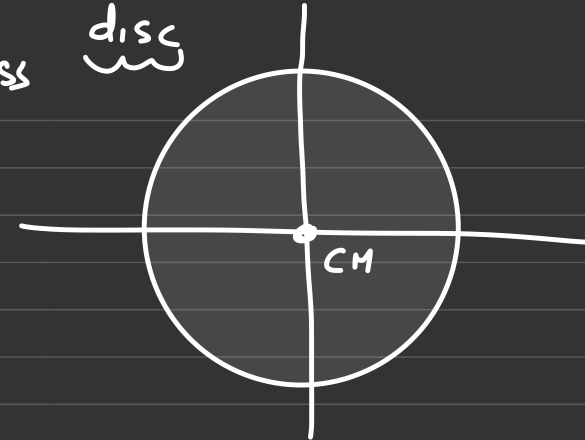
$$= \frac{1}{L} \int_0^L x\, dx = \frac{1}{L}\left[\frac{x^2}{2}\right]_0^L = \left(\frac{L}{2}\right) \underline{\underline{Ans}}$$

Ring:



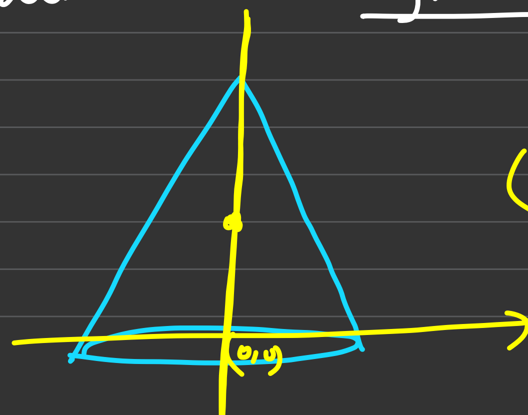
Uniform mass

disc



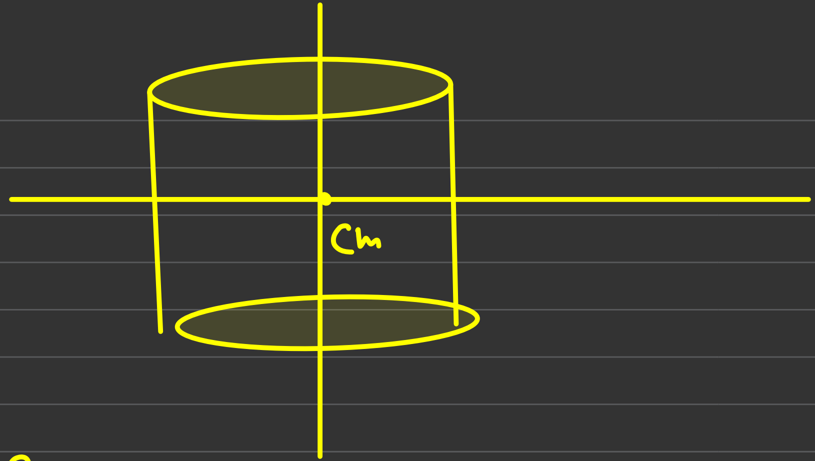
for symmetrical bodies. COM is going to be inter-section of all three "symmetrical axis"

Cone:



$y_{cm} = ?$

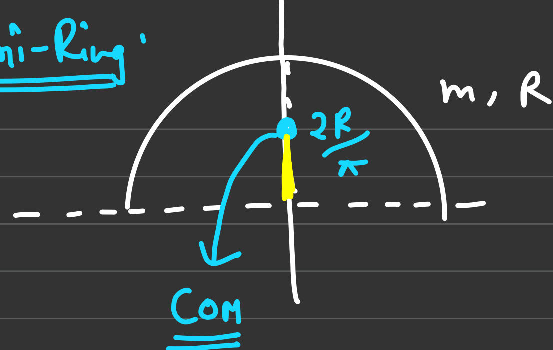
$x_{cm} = 0$



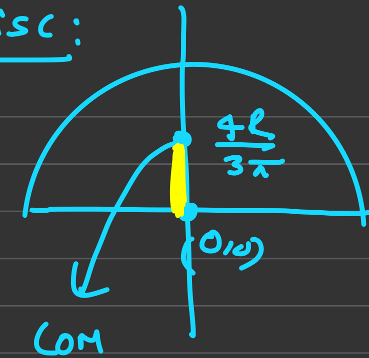
Remember:

1. Semi-Ring:
2. Semi-disc
3. Hemi-Solid sphere
4. Solid cone
5. Hollow Cone
6. Hollow Sphere

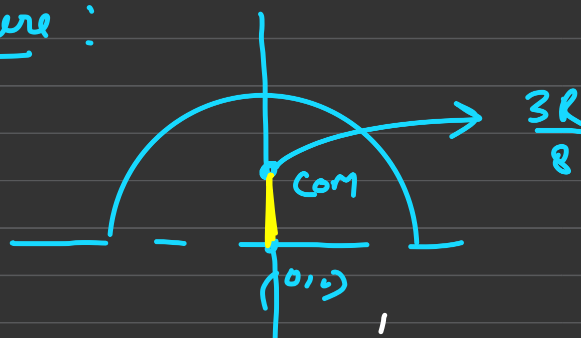
Semi-Ring:



Semi-disc:



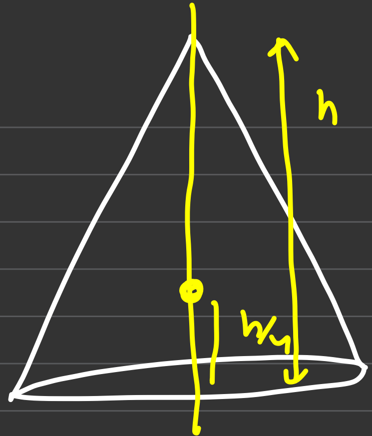
Hemi Solid sphere:



Hemi Hollow sphere:

Solid Cone :

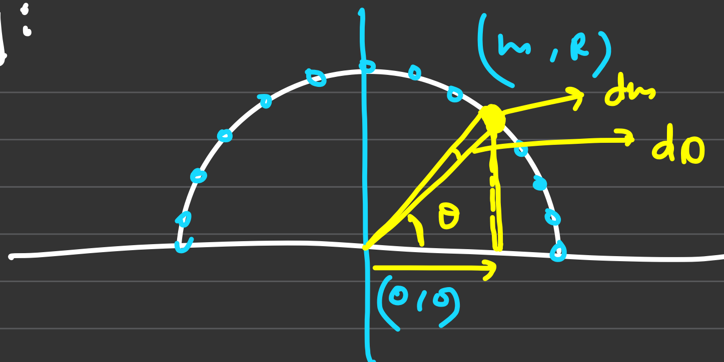
Uniform



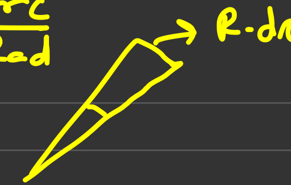
Hollow Cone

H.W

Ring:



$\theta = \dfrac{arc}{Rad}$



$y_{cm} = ?$

$(y = R\sin\theta)$

$$y_{cm} = \frac{\int y \cdot dm}{\int dm} = \frac{\int (R\sin\theta)\cdot dm}{m}$$

$\pi R \longrightarrow m$

$1 \longrightarrow \dfrac{m}{\pi R}$

$R\cdot d\theta \longrightarrow \left(\dfrac{m}{\pi R}\cdot R\,d\theta\right) = dm$

$$= \frac{\int R \sin\theta \cdot \frac{\cancel{M}}{\pi \cancel{R}} \cdot \cancel{R} \cdot d\theta \cdot}{\cancel{m}}$$

$$= \frac{R}{\pi} \int_0^{\pi} \sin\theta \cdot d\theta$$

$$= \frac{R}{\pi} \left[-\cos\theta \right]_0^{\pi}$$

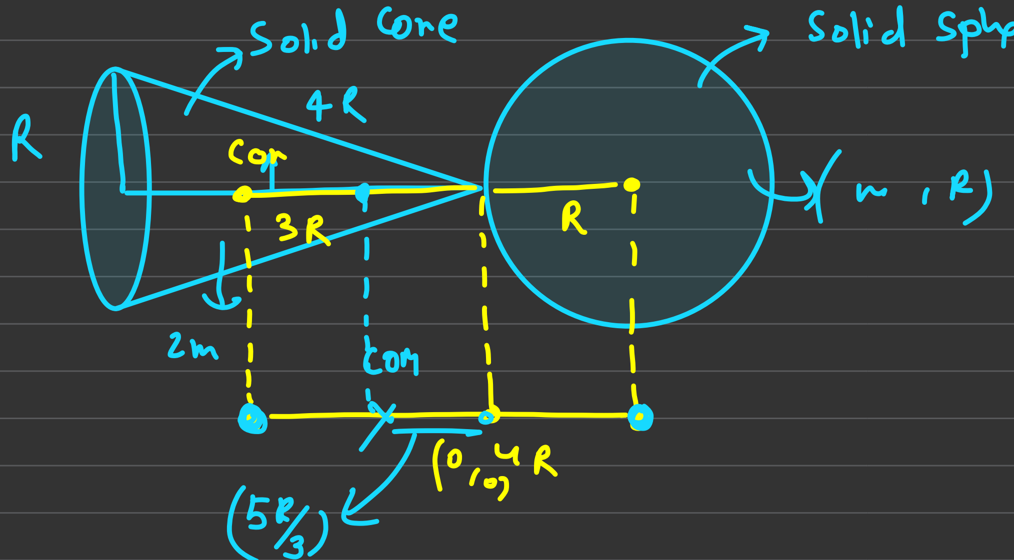
$$= -\frac{R}{\pi} \left[\cos\pi - \cos 0 \right]$$

$$= -\frac{R}{\pi} \left[-1 - 1 \right] = \left(\frac{2R}{\pi} \right) \underline{\underline{Ans}}$$

Centre of mass of System of bodies:

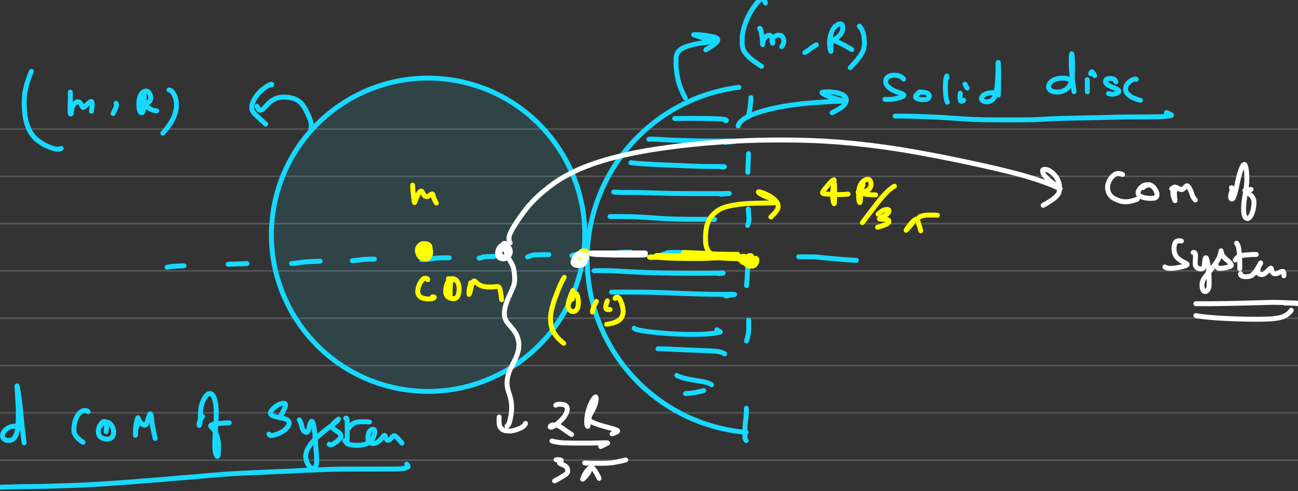
find com?

①



$$x_{cm} = \frac{2\cancel{m}(-3R) + \cancel{m} \times R}{3\cancel{m}} = \left(-\frac{5R}{3}\right)$$

(ii)



find COM of System

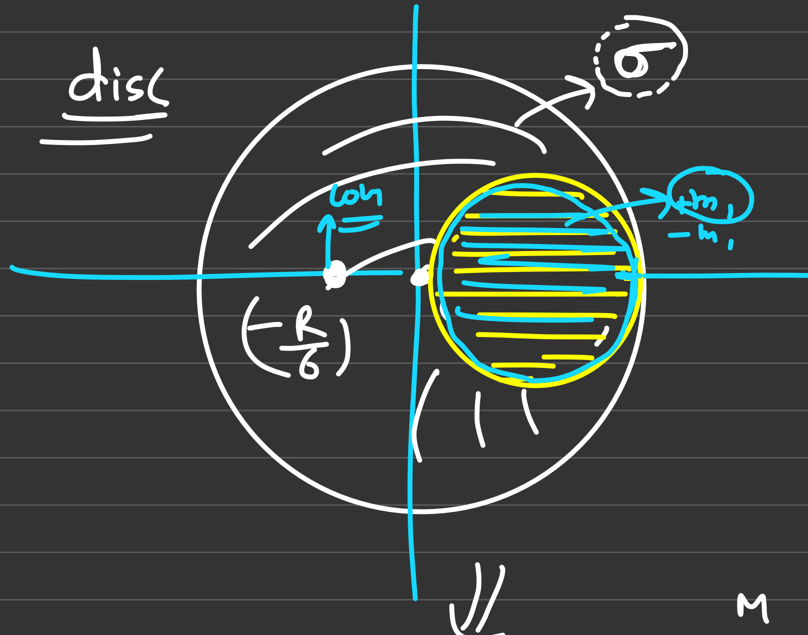
$$x_m = \frac{\cancel{m} \times (-R) + \cancel{m}\left(R - \frac{4R}{3\pi}\right)}{2m}$$

$$= \frac{-\cancel{R} + \cancel{R} - \frac{4R}{3\pi}}{2}$$

$$= \left(-\frac{4R}{6\pi}\right)\hat{i}$$

COM of Cut | Remove Extended bodies:

\# disc

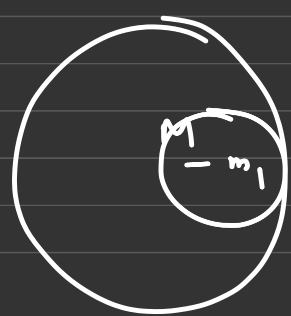


we removed R/2 Part of disc?

find COM of remaining part?

$M_{disc} = \sigma \times \pi R^2 = M$

$m_{disc} = -\sigma(\pi R/2)^2 = \frac{-M}{4}$



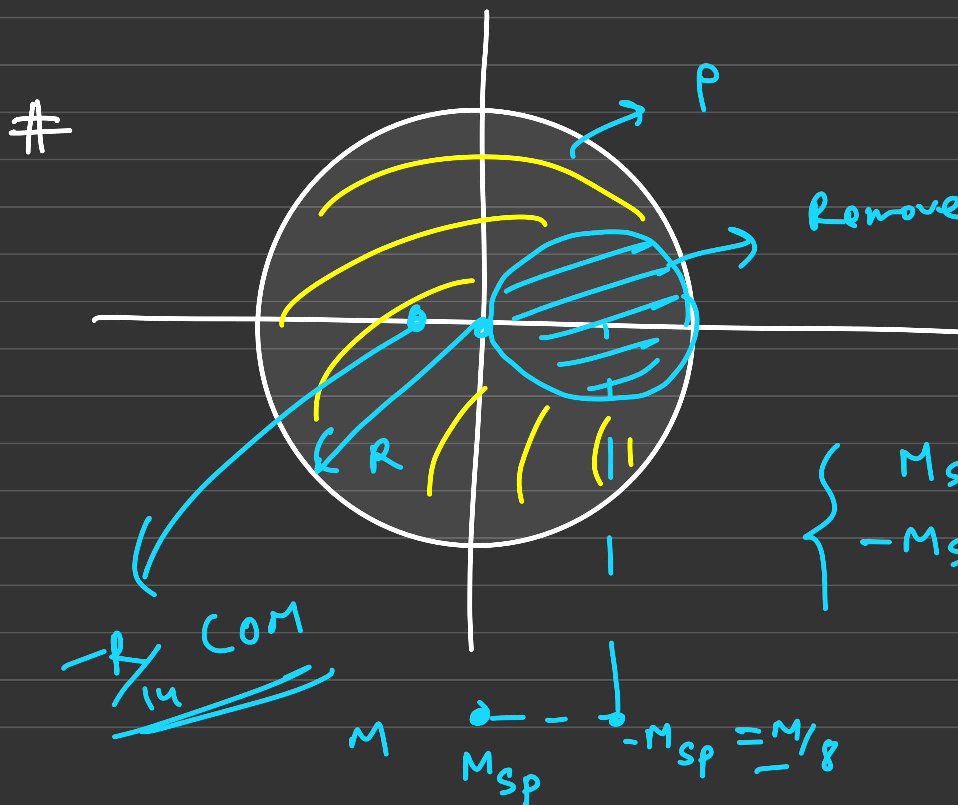
M at $(0, 0)$, $-M/4$ at $R/2$

$$x_{cm} = \frac{M \times 0 - \frac{M}{4} \times R/2}{M - \frac{M}{4}}$$

$$x_{cm} = \frac{-\frac{R}{8}}{3/4}$$

$$= -\frac{R}{8} \times \frac{4}{3} = \left(-\frac{R}{6}\right)$$

#



Remove $R/2$ mass

find COM of remaining part?

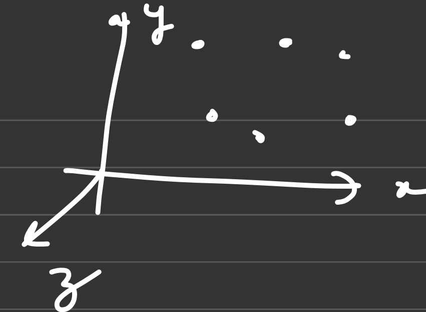
$$M_{sp} = \rho \times \frac{4}{3}\pi R^3 = M$$

$$-M_{sp} = -\rho \frac{4}{3}\pi (R/2)^3 = -\frac{M}{8}$$

$$x_{cm} = \frac{M\times 0 - \frac{M}{8}\times \frac{R}{2}}{M - \frac{M}{8}} = -\frac{R}{16}\times\frac{8}{7}$$

$$= \left(-\frac{R}{14}\right)$$

Application of COM:



(I) $$\vec{r}_{cm} = \frac{m_1 \vec{r}_1 + m_2 \vec{r}_2 + \cdots}{m_1 + m_2 + \cdots}$$

(II) $$\vec{v}_{cm} = \frac{m_1 \vec{v}_1 + m_2 \vec{v}_2 + \cdots}{m_1 + m_2 + \cdots}$$

$$\underline{m_1 \vec{v}_1 + m_2 \vec{v}_2 + \cdots m_n \vec{v}_n} = \left(\sum m_i\right) v_{cm}$$

$$\left(\vec{P}_{system}\right) = \underline{\underline{\left(\sum m_i\right)}}\ \underline{\underline{v_{cm}}}$$

$$\vec{v}_{cm} = \frac{\vec{P}_{system}}{\sum m_i}$$

$\Rightarrow$

method 1:

$$\vec{V}_{cm} = \frac{m_1\vec{v}_1 + m_2\vec{v}_2 + \ldots m_n\vec{v}_n}{m_1 + m_2 + \ldots m_n}$$

method:

a) $\vec{V}_{cm} = \dfrac{\vec{P}_{system}}{\Sigma m_i}$

b) $\vec{P}_{system} = \vec{V}_{cm}(\Sigma m_i)$

Q)

at t = 0

① find $\underline{v_{cm}}$ ($t=0$) $= \dfrac{\cancel{m}\times 3 + 2\cancel{m}\times 6}{3\cancel{m}} = \dfrac{15}{3} = 5$ m/s

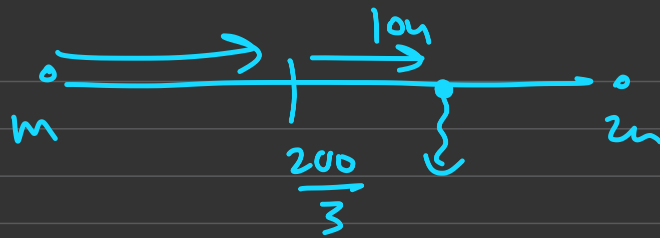
$v_{cm} = \frac{5}{3}\times 3 = 5$ m/s

② $x_{cm} = \dfrac{m\times 0 + 2m\times 100}{3m} = \dfrac{200}{3}$ m

③ find location of centre of after 2 sec?

displacement of com after 2 sec

$= 5\times 2 = 10$ m



"New Location (COM w.r.t origin given)" $\frac{200}{3} + 10 = \left(\frac{230}{3}\right)$ m =

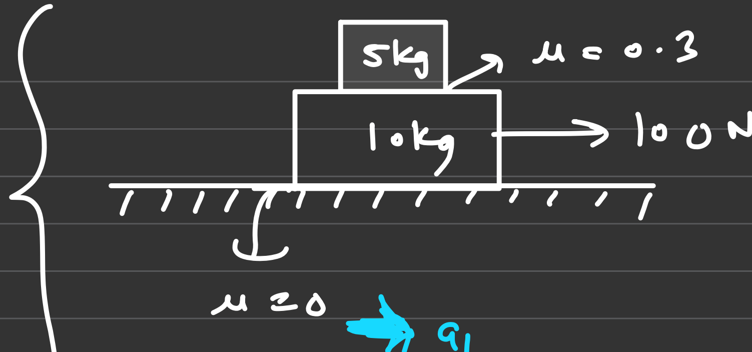
(ii) # $$\vec{V}_{cm} = \frac{m_1\vec{v_1} + m_2\vec{v_2} + \cdots m_n v_n}{m_1 + m_2 + \cdots m_n}$$

method I # $$\vec{a}_{cm} = \frac{m_1\vec{a_1} + m_2\vec{a_2} + \cdots m_n\vec{a_n}}{m_1 + m_2 + \cdots m_n}$$

$$\vec{a}_{cm}\left(\sum m_i\right) = \underbrace{m_1\vec{a_1} + m_2\vec{a_2} + \cdots}_{\vec{F}_{ext}}$$

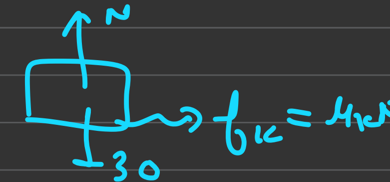
method 2: # $$\vec{F}_{ext} = \left(\sum m_i\right)\vec{a}_{cm}$$

Q)



find acceleration

5kg 10kg



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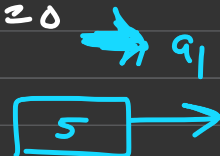
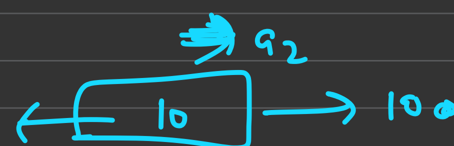


$f_k = 0.3 \times 50 = 15\ N$

$15 = 5 \times a_1$

$a_1 = 3\ m/s^2$ — (I)



$15\ N = f_k$

$100 - 15 = 10 \times a_2$ — (II)

$a_2 = 8.5\ m/s^2$

if in this case:

$a_2 > a_1$ then assum is correct

if $a_2 < a_1$ then then they are moving together "to iska matlab phir se solve karo"

$\boxed{5} \rightarrow 3\,m/s^2$

$\boxed{10\,K} \Rightarrow 8.5\,m/s^2$

method 1

$$a_{cm} = \frac{5\times 3 + 10\times 8.5}{5+10} = \frac{15+85}{15} = \frac{100}{15}\,m/s^2$$

method 2:

#

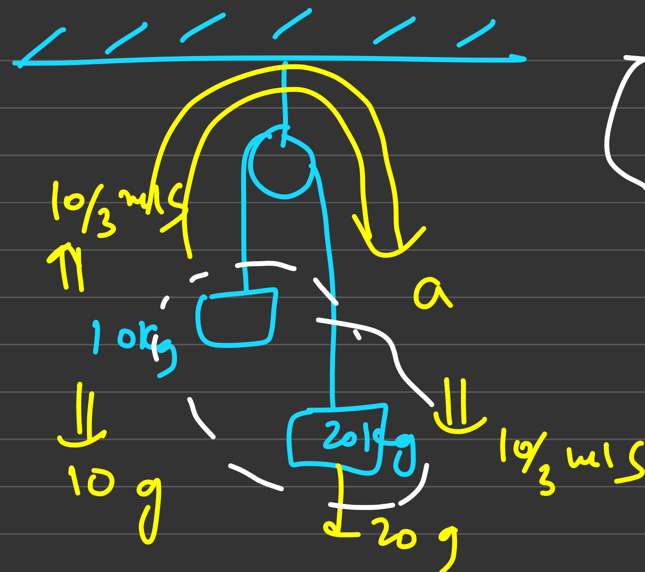
$F = 100\,N$

$$a_{cm} = 100/15\ m/s^2$$

#

released from rest

find a_{cm} of 10kg and 20kg block?



$10/3 \ m/s^2$

$$20g - 10g = 30 \times a$$

$$a = \frac{10g}{30} = g/3 = 10/3 \ m/s^2$$

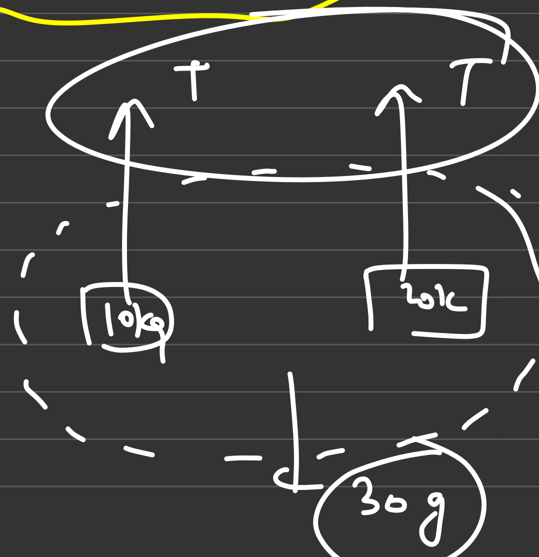
$$a_{cm} = \frac{+10 \times \frac{10}{3} - 20 \times \frac{10}{3}}{30}$$

$$a_n = \frac{\frac{100}{3} - \frac{200}{3}}{30} = -\frac{100}{3\times 30} = -\frac{10}{9}\ \text{m/s}^2$$

$$a = -\frac{10}{9}\ \text{m/s}^2$$

Ans

method?!



$$a_n = \frac{30g - 2T}{30}$$

"Calculate T and verify Answer"

$$a_c = \frac{300 - \frac{2 \times 400}{3}}{30}$$

$$= \frac{1\cancel{0}0}{3 \times 3\cancel{0}} = +\frac{10}{9} \downarrow$$

Complete module & workbook

"moment of inertia"