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IIT JEE | MEDICAL | FOUNDATION

IIT JEE - 2021

Solutions to Home Assignment - 1 | Function | Mathematics

1.(B) For $f(x)$ to be defined, $\Rightarrow$ $x^{\log_{10} x} \neq 0$ and $x > 0$ $\Rightarrow x > 0$ $\Rightarrow$ $x \in (0, \infty)$

2.(C) For domain of $g(x)$

$0 < e^x < 1 \quad \Rightarrow \quad x \in (-\infty, 0) \quad$ ....**(i)**

$0 < \log_e |x| < 1 \quad \Rightarrow \quad |x| \in (1, e) \quad \Rightarrow \quad x \in (-e, -1) \cup (1, e) \quad$ ....**(ii)**

From (i) and (ii), $x \in (-e, -1)$

3.(C) *tan x* is defined, if $x \neq n\pi + \frac{\pi}{2}$ **...(i)**

If` $\tan x > 0$, then $|\tan x| + \tan x > 0$ **...(ii)**

If $\tan x \leq 0$, then $|\tan x| + \tan x = 0$ **...(iii)**

$\therefore$ Numerator is defined for both equations (ii) and (iii) and non-zero $\sqrt{3x}$ is defined, $\forall\, x > 0$

On combining equations (i), (ii), (iii) and (iv), we get : $D_f = R^+ - \left\{ n\pi + \frac{\pi}{2} \middle| n \in I^+ \right\}$

4.(B) We get, $f(x) = \begin{cases} 6 - 3x, & x < 1 \\ 4 - x, & 1 \leq x < 2 \\ x, & 2 \leq x < 3 \\ 3x - 6, & x \geq 3 \end{cases}$. Draw the graph of $f(x)$ and get the minimum value of $f(x) = 2$

5.(C) $f(x)$ defined, if $-(\log_3 x)^2 + 5\log_3 x - 6 > 0$ and $x > 0$

$\Rightarrow \quad (\log_3 x - 3)(2 - \log_3 x) > 0$ and $x > 0 \quad \Rightarrow \quad (\log_3 x - 2)(\log_3 x - 3) < 0$ and $x > 0$

$\Rightarrow \quad 2 < \log_3 x < 3$ and $x > 0 \quad \Rightarrow \quad 3^2 < x < 3^3 \quad \Rightarrow \quad 9 < x < 27$

Domain of $f(x)$ is $x \in (9, 27)$

6.(D) **(A)** $\log_{1.5} \log_4 \log_{\sqrt{3}} 81 = \log_{1.5} \log_4 8 = \log_{1.5} 1.5 = 1$ **(B)** $log_2 \sqrt{6} + \log_2 \sqrt{\frac{2}{3}} = \log_2 2 = 1$ **(C)**

$-\frac{1}{6} \log_{\frac{\sqrt{3}}{2}} \left(\frac{64}{27}\right) = \frac{1}{6} \log_{\frac{\sqrt{3}}{2}} \left(\frac{27}{64}\right) = \frac{1}{6} . 6 = 1$ **(D)** $\log_{3.5}(1 + 2 + 3 \div 6) = \log_{3.5} 3.5 = 1$

7.(C) $\log_6 \log_2 \left[\sqrt{4x + 2} + 2\sqrt{x}\right] = 0$; $x \geq 0$

$\Rightarrow \quad \log_2\left(\sqrt{4x+2} + 2\sqrt{x}\right) = 1 \quad \Rightarrow \quad \sqrt{4x+2} + 2\sqrt{x} = 2 \quad \Rightarrow \quad \sqrt{4x+2} = 2\left(1 - \sqrt{x}\right)$

Squaring both sides $4x + 2 = 4\left(1 + x - 2\sqrt{x}\right)$

$8\sqrt{x} = 2 \quad \Rightarrow \quad \sqrt{x} = \frac{1}{4} \quad \Rightarrow \quad x = \frac{1}{16}$

8.(A) $\log_{10}\left(\frac{5x-x^2}{4}\right) \geq 0 \quad \Rightarrow \quad \frac{5x-x^2}{4} \geq 10^0$

$\Rightarrow \quad 5x - x^2 \geq 4 \quad \Rightarrow \quad x^2 - 5x + 4 \leq 0 \quad \Rightarrow \quad (x-1)(x-4) \leq 0 \quad \Rightarrow \quad x \in [1, 4]$

Also, we need $\frac{5x-x^2}{4} > 0 \quad \Rightarrow \quad x^2 - 5x < 0 \quad \Rightarrow \quad x \in (0, 5)$ **. . . .(i)**

Combining (i) and (ii), we get: $x \in [1, 4]$

9.(C) $f(x)$ is defined $\Rightarrow \log_{0.3}(x-1) \leq 0 \;\; -x^2 + 2x + 8 > 0 \;\Rightarrow\; x - 1 \geq 1, x^2 - 2x - 8 < 0$

$\Rightarrow \quad x \geq 2, (x+2)(x-4) < 0 \Rightarrow x \geq 2, -2 < x < 4 \quad \Rightarrow \quad 2 \leq x < 4$

10.(C) $f(x)$ is defined $\Rightarrow$ *tan* $2x$ is defined, $6\cos x + 2\sin 2x \neq 0$

tan $2x$ is defined $\Rightarrow \quad 2x \neq (2n+1)\frac{\pi}{2} \;\Rightarrow\; x \neq (2n+1)\frac{\pi}{4}$

$6\cos x + 2\sin 2x \neq 0 \;\Rightarrow\; 6\cos x + 4\sin x \cos x \neq 0 \;\Rightarrow\; 2\cos x(3 + 2\sin x) \neq 0$

$\Rightarrow \quad \cos x \neq 0 \;\Rightarrow\; x \neq (2n+1)\frac{\pi}{2}$