Functions

Level - 0	CBSE Pattern

1. **(i)** We have, $f(x)=\dfrac{1}{\sqrt{1-\cos x}}$ $\because -1\le \cos x\le 1 \Rightarrow -1\le -\cos x\le 1 \Rightarrow 0\le 1-\cos x\le 2$

So, $f(x)$ is defined, if $1-\cos x\ne 0 \Rightarrow \cos x\ne 1 \Rightarrow x\ne 2n\pi\ \forall n\in Z$ $\therefore$ Domain of $f(x)=R-\{2n\pi : n\in Z\}$

(ii) We have, $f(x)=\dfrac{1}{\sqrt{x+|x|}}$ $\because$ $x+|x|=x-x=0,\ x<0\ ;\ x+x=2x,\ x\ge 0$

Hence, $f(x)$ is defined, $x>0$. $\therefore$ Domain of $f=R^+$

(iii) We have, $f(x)=x|x|$; Clearly, $f(x)$ is defined for any $x\in R$. $\therefore$ Domain of f = R

(iv) We have, $f(x)=\dfrac{x^3-x+3}{x^2-1}$; $f(x)$ is not defined, if $x^2-1=0 \Rightarrow (x-1)(x+1)=0 \Rightarrow x=-1,1$

$\therefore$ Domain of $f=R-\{-1,1\}$

(v) We have, $f(x)=\dfrac{3x}{28-x}$; Clearly, $f(x)$ is defined, if $28-x\ne 0\ ;\ \Rightarrow x\ne 28$ $\therefore$ Domain of $f=R-\{28\}$

2. **(i)** We have, $f(x)=\dfrac{3}{2-x^2}$; Let $y=f(x)$; Then, $y=\dfrac{3}{2-x^2} \Rightarrow 2-x^2=\dfrac{3}{y} \Rightarrow x^2=2-\dfrac{3}{y} \Rightarrow x=\sqrt{\dfrac{2y-3}{y}}$

x assumes real values, if $2y-3\ge 0$ and $y>0 \Rightarrow y\ge 3/2$ $\therefore$ Range of $f=[3/2,\infty)$

(ii) We know that, $|x-2|\ge 0 \Rightarrow -|x-2|\le 0 \Rightarrow 1-|x-2|\le 1 \Rightarrow f(x)\le 1$ $\therefore$ Range of $f=(-\infty,1]$

(iii) We know that, $|x-3|\ge 0 \Rightarrow f(x)\ge 0$ $\therefore$ Range of $f(x)=[0,\infty)$

(iv) We know that, $-1\le \cos 2x\le 1 \Rightarrow -3\le 3\cos 2x\le 3 \Rightarrow 1-3\le 1+3\cos 2x\le 1+3 \Rightarrow -2\le 1+3\cos 2x\le 1+3$

$\Rightarrow -2\le f(x)\le 4$ $\therefore$ Range of $f=[-2,4]$

3. Since, $|x-2|=\begin{cases} x-2 \text{ if } x\ge 2 \\ -(x-2) \text{ if } x<2 \end{cases}$ and $|2+x|=-(2+x),\ x<-2\ ;\ (2+x),\ x\ge -2$

$$\therefore\quad f(x)=|x-2|+|2+x|,\ -3\le x<3=\begin{cases} -(x-2)-(2+x) & , -3\le x<-2 \\ -(x-2)+2+x & , -2\le x<2 \\ x-2+2+x & , 2\le x\le 3 \end{cases}=\begin{cases} -2x & , -3\le x<-2 \\ 4 & , -2\le x<2 \\ 2x & , 2\le x\le 3 \end{cases}$$

4. We have, $f(x)=\dfrac{x-1}{x+1}$

(i) $f\left(\dfrac{1}{x}\right)=\dfrac{\frac{1}{x}-1}{\frac{1}{x}+1}=\dfrac{(1-x)/x}{(1+x)/x}=\dfrac{1-x}{1+x}=\dfrac{-(x-1)}{x+1}=-f(x)$

(ii) $f\left(-\dfrac{1}{x}\right)=\dfrac{-\frac{1}{x}-1}{-\frac{1}{x}+1}=\dfrac{(-1-x)/x}{(-1+x)/x}=f\left(-\dfrac{1}{x}\right)=\dfrac{-(x+1)}{x-1}$; Now, $\dfrac{-1}{f(x)}=\dfrac{-1}{\frac{x-1}{x+1}}=\dfrac{-(x+1)}{x-1}$ $\therefore$ $f\left(-\dfrac{1}{x}\right)=-\dfrac{1}{f(x)}$

5. We have, $f(x)=\sqrt{x}$ and $g(x)=x$ be two function defined in the domain $R^+\cup\{0\}$.

(i) $(f+g)(x)=f(x)+g(x)=\sqrt{x}+x$ **(ii)** $(f-g)(x)=f(x)-g(x)=\sqrt{x}-x$

(ii) $(fg)(x)=f(x).g(x)=\sqrt{x}\,.\,x=x^{3/2}$ **(iv)** $\left(\dfrac{f}{g}\right)(x)=\dfrac{f(x)}{g(x)}=\dfrac{\sqrt{x}}{x}=\dfrac{1}{\sqrt{x}}$

6. We have, $f(x) = \dfrac{1}{\sqrt{x-5}}$; $f(x)$ is defined, if $x-5>0 \Rightarrow x>5 \therefore$ Domain of $f = (5, \infty)$

Let $f(x) = y$, $\therefore y = \dfrac{1}{\sqrt{x-5}} \Rightarrow \sqrt{x-5} = \dfrac{1}{y} \Rightarrow x-5 = \dfrac{1}{y^2}$

$\therefore x = \dfrac{1}{y^2}+5 \quad \because x \in (5, \infty) \Rightarrow y \in R^+$, hence, range of $f = R^+$

7. We have, $f(x) = y = \dfrac{ax-b}{cx-a} \quad \therefore f(y) = \dfrac{ay-b}{cy-a} = \dfrac{a\left(\dfrac{ax-b}{cx-a}\right)-b}{c\left(\dfrac{ax-b}{cx-a}\right)-a}$

$= \dfrac{a(ax-b)-b(cx-a)}{c(ax-b)-a(cx-a)} = \dfrac{a^2x-ab-bcx+ab}{acx-bc-acx+a^2} = \dfrac{x(a^2-bc)}{a^2-bc} = x \quad \therefore f(y) = x$ Hence proved.

8.(D) We have, $n(A) = m$ and $n(B) = n$

$n(A\times B) = n(A)\,.\,n(B) = mn$; Total number of relations from A to B $= 2^{n(A\times B)}-1 = 2^{mn}-1$

9.(D) We have, $[x]^2-5[x]+6=0 \Rightarrow [x]^2-3[x]-2[x]+6=0 \Rightarrow [x]([x]-3)-2([x]-3)=0$

$\Rightarrow ([x]-3)([x]-2)=0 \Rightarrow [x]=2, 3 \therefore x \in [2, 4)$

10.(C) We know that, $-1 \le -\cos x \le 1 \Rightarrow -2 \le -2\cos x \le 2 \Rightarrow 1-2 \le 1-2\cos x \le 1+2 \Rightarrow -1 \le 1-2\cos x \le 3$

$\Rightarrow f(x) \le -1$ or $f(x) \ge 1/3 \Rightarrow$ Range of $f = (-\infty, -1] \cup [1/3, \infty)$

11.(C) We have, $f(x) = \sqrt{1+x^2}$; $f(xy) = \sqrt{1+x^2y^2}$; $f(x)\,.\,f(y) = \sqrt{1+x^2}\,.\,\sqrt{1+y^2}$

$= \sqrt{(1+x^2)(1+y^2)} = \sqrt{1+x^2+y^2+x^2y^2} \because \sqrt{1+x^2y^2} \le \sqrt{1+x^2+y^2+x^2y^2} \Rightarrow f(xy) \le f(x)\,.\,f(y)$

12.(B) Let $f(x) = \sqrt{a^2-x^2}$; $f(x)$ is defined, if $a^2-x^2 \ge 0 \Rightarrow x^2-a^2 \ge 0 \Rightarrow (x-a)(x+a) \le 0$

$\Rightarrow -a \le x \le a \quad [\because a>0] \quad \therefore$ Domain of $[-a, a]$

13.(B) We have, $f(x) = ax+b$; $f(-1) = a(-1)+b$; $-5 = -a+b$ .....**(i)** and, $f(3) = a(3)+b$; $3 = 3a+b$ ....**(ii)**

On solving equations (i) and (ii), we get : $a = 2$ and $b = -3$

14.(A) We have, $f(x) = \sqrt{4-x}+\dfrac{1}{\sqrt{x^2-1}}$; $f(x)$ is defined, if $4-x \ge 0$ or $x^2-1>0$; $x-4 \le 0$ or $(x+1)(x-1)>0$

$x \le 4$ or $x<-1$ and $x>1 \quad \therefore$ Domain of $f = (-\infty, -1) \cup (1, 4]$

15.(C) We have, $f(x) = \dfrac{4-x}{x-4}$; $f(x)$ is defined, if $x-4 \ne 0$ i.e. $x \ne 4 \quad \therefore$ Domain of $f = R-\{4\}$

Let $f(x) = y \therefore y = \dfrac{4-x}{x-4} \Rightarrow xy-4y = 4-x \Rightarrow xy+x = 4+4y \Rightarrow x(y+1) = 4(1+y) \therefore x = \dfrac{4(1+y)}{y+1}$

x assumes real values, if $y+1 \ne 0$ i.e., $y=-1 \quad \therefore$ Range of $f = R-\{-1\}$

16.(D) We have, $f(x) = \sqrt{x-1}$; $f(x)$ is defined, if $x-1 \ge 0 \Rightarrow x \ge 1 \quad \therefore$ Domain of $f = [1, \infty)$

Let $y = \sqrt{x-1} \quad \therefore y = \sqrt{x-1} \Rightarrow y^2 = x-1 \therefore x = y^2+1$

x assumes real values for $y \in R$; But $y \ge 0 \quad \therefore$ Range of $f = [0, \infty)$

17.(A) We have, $f(x) = \dfrac{x^2+2x+1}{x^2-x-6}$; $f(x)$ is not defined, if $x^2-x-6=0 \Rightarrow x^2-3x+2x-6=0$

$\Rightarrow x(x-3)+2(x-3)=0 \Rightarrow (x-3)(x+2)=0 \quad \therefore x = 3, -2 \quad \therefore$ Domain of $f = R-\{3, -2\}$

18.(B) We have, $f(x) = 2 - |x - 5|$; $f(x)$ is defined for all $x \in R$ $\therefore$ Domain of $f = R$

We know that, $|x-5| \geq 0 \Rightarrow -|x-5| \leq 0 \Rightarrow 2 - |x-5| \leq 2$ $\therefore$ $f(x) \leq 2$ $\therefore$ Range of $f = [-\infty, 2]$

19.(A) We have, $f(x) = 3x^2 - 1$ and $g(x) = 3 + x$; $f(x) = g(x) \Rightarrow 3x^2 - 1 = 3 + x \Rightarrow 3x^2 - x - 4 = 0$

$\Rightarrow 3x^2 - 4x + 3x - 4 = 0 \Rightarrow x(3x-4) + 1(3x-4) = 0 \Rightarrow (3x-4)(x+1) = 0$ $\therefore$ $x = -1, 4/3$

So, domain for which $f(x)$ and $g(x)$ are equal to $[-1, 4/3]$

20. We have $f = \{(0,1), (2,0), (3,4), (4,2), (5,1)\}$ and $g = \{(1,0), (2,2), (3,-1), (4,4), (5,3)\}$

$\therefore$ Domain of $f = \{0, 2, 3, 4, 5\}$ and Domain of $g = \{1, 2, 3, 4, 5\}$

$\therefore$ Domain of $(f \,.\, g) =$ Domain of, $f \cap$ Domain of $g = \{2, 3, 4, 5\}$

21. We have, $f = \{(2,4), (5,6), (8,-1), (10,-3)\}$ and $g = \{(2,5), (7,1), (8,4), (10,13), (11,5)\}$

So, $f - g$, $f + g$, $f \,.\, g$, $\dfrac{f}{g}$ are defined in the domain (domain of $f \cap$ domain of g)

i.e., $\{2, 5, 8, 10\} \cap \{2, 7, 8, 10, 11\} \Rightarrow \{2, 8, 10\}$

(i) $(f-g)(2) = f(2) - g(2) = 4 - 5 = -1$; $(f-g)(8) = f(8) - g(8) = -1 - 4 = -5$

$(f-g)(10) = f(10) - g(10) = -3 - 13 = -16$ $\therefore$ $f - g = \{(2,-1), (8,-5), (10,-16)\}$

(ii) $(f+g)(2) = f(2) + g(2) = 4 + 5 = 9$; $(f+g)(8) = f(8) + g(8) = -1 + 4 = 3$

$(f+g)(10) = f(10) + g(10) = -3 + 13 = 10$

(iii) $(f \,.\, g)(2) = f(2) = 4 \times 5 = 20$; $(f \,.\, g)(8) = f(8) \,.\, g(8) = -1 \times 4 = -4$

$(f \,.\, g)(10) = f(10) \,.\, g(10) = -3 \times 13 = -39$ $\therefore$ $fg = \{(2, 20), (8, -4), (10, -39)\}$

(iv) $\left(\dfrac{f}{g}\right)(2) = \dfrac{f(2)}{g(2)} = \dfrac{4}{5}$; $\left(\dfrac{f}{g}\right)(8) = \dfrac{f(8)}{g(8)} = \dfrac{-1}{4}$; $\left(\dfrac{f}{g}\right)(10) = \dfrac{f(10)}{g(10)} = \dfrac{-3}{13}$ $\therefore$ $\dfrac{f}{g} = \left\{\left(2, \dfrac{4}{5}\right), \left(8, -\dfrac{1}{4}\right), \left(10, \dfrac{-3}{13}\right)\right\}$

Hence, the correct matches are (i)-(c), (ii)-(d), (iii)-(b), (iv)-(a)

22.(C) $f(a - (x - a)) = f(a) f(x - a) - f(0) f(x)$ .....(i)

Put $x = 0, y = 0$; $f(0) = (f(0))^2 - [f(a)]^2 \Rightarrow f(a) = 0$ [$\because f(0) = 1$]. From (i), $f(2a - x) = -f(x)$.

23.(C) We know that, if A and B are two non-empty finite set containing m and n elements respectively, then the number of one-one and onto mapping from A to B is $\begin{cases} n!, & \text{if} \quad m = n \\ 0, & \text{if} \quad m \equiv n \end{cases}$

Given that, m = 5 and n = 6 $\therefore$ $m \neq n$ Number of mapping = 0

24.(D) Given that, A = {1, 2, 3, . . . , n} and B {a, b}.

We know that, if A and B are two non-empty finite sets containing m and n elements respectively, then the number of surjection from A into B is ${}^nC_m \times m!$, if $n \geq m$; 0, if $n < m$ Here, m = 2 $\therefore$ Number of surjection from A into B is ${}^nC_2 \times 2! = \dfrac{n!}{2!(n-2)!} \times 2! = \dfrac{n(n-1)(n-2)!}{2 \times 1(n-2)} \times 2! = n^2 - n$

25.(D) Given that, $f(x) = \dfrac{1}{x}, \forall x \in R$; For $x = 0$, $f(x)$ is not defined. Hence, $f(x)$ is a not definite function.

26.(B) Here, $f(x) = x + 2 \Rightarrow f(x_1) = f(x_2)$; $x_1 + 2 = x_2 + 2 \Rightarrow x_1 = x_2$

Let $y = x + 2$; $x = y - 2 \in Z, \forall y \in x$ Hence, $f(x)$ is one-one and onto.

27.(B) Given that, $f(x) = x^3 + 5$; Let $y = x^3 + 5 \Rightarrow x^3 = y - 5 \Rightarrow x = (y-5)^{1/3} \Rightarrow f(x)^{-1} = (x-5)^{1/3}$

28.(A) Given that, $f: A \to B$ and $g: B \to C$ be the bijective functions. $(gof)^{-1} = f^{-1} o g^{-1}$

29.(D) Given that, $f(x)=\dfrac{2x-1}{2}$ and $g(x)=x+2$; $(gof)=\dfrac{3}{2}=g\left[f\left(\dfrac{3}{2}\right)\right]=g\left(\dfrac{2\times\frac{2}{3}-1}{2}\right)=g(1)=1+2=3$

30.(C) Given that, $f:[0,1]\to[0,1]$ be defined by $f(x)=\begin{cases}x, & \text{if } x \text{ is rational}\\ 1-x, & \text{if } x \text{ is irrational}\end{cases}$ $\therefore$ $(fof)x=f(f(x))=x$

31.(A) Given that, $f(x)=\dfrac{3x+2}{5x-3}$; Let, $y=\dfrac{3x+2}{5x-3}$; $3x+2=5xy-3y$ $\Rightarrow$ $x(3-5y)=-3y-2$

$x=\dfrac{3y+2}{5y-3}$ $\Rightarrow$ $f^{-1}(x)=\dfrac{3x+2}{5x-3}$ $\therefore$ $f^{-1}(x)=f(x)$

Level - 1	**JEE Main Pattern**

1.(B) For $f(x)$ to be defined, $\Rightarrow$ $x^{\log_{10}x}\neq 0$ and $x>0$ $\Rightarrow x>0$ $\Rightarrow$ $x\in(0,\infty)$

2.(C) For domain of $g(x)$; $0<e^x<1$ $\Rightarrow$ $x\in(-\infty,0)$ **....(i)**

$0<\log_e|x|<1$ $\Rightarrow$ $|x|\in(1,e)$ $\Rightarrow$ $x\in(-e,-1)\cup(1,e)$ **....(ii)** From (i) and (ii), $x\in(-e,-1)$

3.(B) For $f(x)$ to be defined. $x>0$, $\log_{10}x>0$, $\log_{10}\log_{10}x>0$ .....

$\Rightarrow x>0, x>1, x>10, \ldots, x>10^{n-1}$ $\Rightarrow$ $x>10^{n-1}$ $\Rightarrow$ $x\in(10^{n-1},\infty)$

4.(B) $0\le\sqrt{x^2-\dfrac{\pi^2}{9}}<\infty$ $\Rightarrow$ $\cos\sqrt{x^2-\dfrac{\pi^2}{9}}\in[-1,1]$ $\Rightarrow$ $f(x)\in[-4,4]$

5.(C) *tan* x is defined, if $x\neq n\pi+\dfrac{\pi}{2}$ **...(i)** If` $\tan x>0$, then $|\tan x|+\tan x>0$ **...(ii)**

If $\tan x\le 0$, then $|\tan x|+\tan x=0$ **...(iii)**

$\therefore$ Numerator is defined for both equations (ii) and (iii) and non-zero $\sqrt{3x}$ is defined, $\forall\ x>0$

On combining equations (i), (ii), (iii) and (iv), we get : $D_f=R^+-\left\{n\pi+\dfrac{\pi}{2}\middle| n\in I^+\right\}$

6.(C) $g(x)=f(x+1)=|x-1|+|x-2|+|x-3|$ Now, $g(-x)=|x+1|+|x+2|+|x+3|$

Clearly, $g(x)\neq\pm g(-x)$ $\Rightarrow$ $g(x)$ is neither even nor odd.

7.(B) We get, $f(x)=\begin{cases}6-3x, & x<1\\ 4-x, & 1\le x<2\\ x, & 2\le x<3\\ 3x-6, & x\ge 3\end{cases}$. Draw the graph of $f(x)$ and get the minimum value of $f(x)=2$

8.(C) $f(x)$ defined, if $-(\log_3 x)^2+5\log_3 x-6>0$ and $x>0$ $\Rightarrow$ $(\log_3 x-3)(2-\log_3 x)>0$ and $x>0$

$\Rightarrow(\log_3 x-2)(\log_3 x-3)<0$ and $x>0$ $\Rightarrow 2<\log_3 x<3$ and $x>0$ $\Rightarrow$ $3^2<x<3^3$

$\Rightarrow$ $9<x<27$ Domain of $f(x)$ is $x\in(9,27)$

9.(C) For $f(x)$ to be defined, $\dfrac{\sqrt{4-x^2}}{1-x}>0$, $4-x^2>0$, $1-x\neq 0$; Since, $\sqrt{4-x^2}\not< 0$, we have $1-x>0$ and

$4-x^2>0$ $\Rightarrow$ $x<1$ and $(x-2)(x+2)<0$ $\Rightarrow$ $x<1$ and $-2<x<2$ $\Rightarrow$ $-2<x<1$

Since, $-\infty<\log\left(\dfrac{\sqrt{4-x^2}}{1-x}\right)<\infty$ $\Rightarrow$ $-1\le\sin\left[\log\left(\dfrac{\sqrt{4-x^2}}{1-x}\right)\right]\le 1$ $\therefore$ Range of $f=[-1,1]$

10.(A) For $f(x)$ to be defined $3-x^2 \neq 0$ *i.e,* $x \neq \pm\sqrt{3}$ $\therefore$ Domain of $f(x) = R\{\pm\sqrt{3}\}$

Now, let $y = \dfrac{5}{3-x^2} \Rightarrow x^2 = \dfrac{3y-5}{y} \Rightarrow x = \sqrt{\dfrac{3y-5}{y}} \Rightarrow$ For x to be defined

$\therefore$ $y < 0$ or $y \geq 5/3$ Hence, range of $f(x) = (-\infty, 0) \cup [5/3, \infty)$

11.(D) **(A)** $\log_{1.5}\log_4\log_{\sqrt{3}}81 = \log_{1.5}\log_4 8 = \log_{1.5}1.5 = 1$ **(B)** $\log_2\sqrt{6} + \log_2\sqrt{2/3} = \log_2 2 = 1$

(C) $-\dfrac{1}{6}\log_{\frac{\sqrt{3}}{2}}\left(\dfrac{64}{27}\right) = \dfrac{1}{6}\log_{\frac{\sqrt{3}}{2}}\left(\dfrac{27}{64}\right) = \dfrac{1}{6}.6 = 1$ **(D)** $\log_{3.5}(1+2+3\div 6) = \log_{3.5}3.5 = 1$

12.(C) $\log_6\log_2\left[\sqrt{4x+2}+2\sqrt{x}\right] = 0$; $x \geq 0$ **;** $\Rightarrow \log_2\left(\sqrt{4x+2}+2\sqrt{x}\right) = 1 \Rightarrow \sqrt{4x+2}+2\sqrt{x} = 2 \Rightarrow \sqrt{4x+2} = 2\left(1-\sqrt{x}\right)$

Squaring both sides $4x+2 = 4\left(1+x-2\sqrt{x}\right)$; $8\sqrt{x} = 2 \Rightarrow \sqrt{x} = 1/4 \Rightarrow x = 1/16$

13.(A) $\log 4 + \left(1+\dfrac{1}{2x}\right)\log 3 = \left(3^{\frac{1}{x}}+27\right) \Rightarrow \log\left(4.3^{1+\frac{1}{2x}}\right) = \log\left(3^{\frac{1}{x}}+27\right) \Rightarrow 12.3^{1/2x} = 3^{1/x}+27$

Let $3^{1/2x} = t$ $\therefore 12t = t^2+27 \Rightarrow t^2 - 12t + 27 = 0 \Rightarrow t = 3, 9$; $3^{1/2x} = 3 \Rightarrow 1/2x = 1 \Rightarrow x = 1/2$

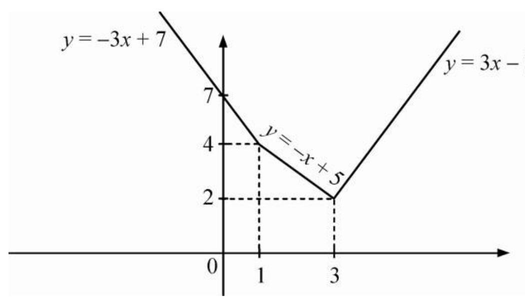
$3^{1/2x} = 9 \Rightarrow 1/2x = 2 \Rightarrow x = 1/4$

But x has to natural number (Since, $\sqrt[x]{3}$ is only defined, when x is natural number ≥ 2) $\therefore x \in \phi$

14.(B) $y = |x-1| + 2|x-3|$

$$y = \begin{cases} (-x+1)+2(-x+3) & x \leq 1 \\ (x-1)+2(-x+3) & 1 < x < 3 \\ (x-1)+2(x-3) & x \geq 3 \end{cases}$$

$$\Rightarrow \quad y = \begin{cases} -3x+7 & x \leq 1 \\ -x+5 & 1 < x < 3 \\ 3x-7 & x \geq 3 \end{cases}$$



15.(A) $\log_{10}\left(\dfrac{5x-x^2}{4}\right) \geq 0 \Rightarrow \dfrac{5x-x^2}{4} \geq 10^0 \Rightarrow 5x - x^2 \geq 4 \Rightarrow x^2 - 5x + 4 \leq 0 \Rightarrow (x-1)(x-4) \leq 0 \Rightarrow x \in [1, 4]$

Also, we need $\dfrac{5x-x^2}{4} > 0 \Rightarrow x^2 - 5x < 0 \Rightarrow x \in (0, 5)$ .....**(i)** Combining (i) and (ii), we get: $x \in [1, 4]$

16.(C) $f(x)$ is defined $\Rightarrow \log_{0.3}(x-1) \leq 0$ $-x^2+2x+8 > 0 \Rightarrow x-1 \geq 1, x^2-2x-8 < 0$

$\Rightarrow x \geq 2, (x+2)(x-4) < 0 \Rightarrow x \geq 2, -2 < x < 4 \Rightarrow 2 \leq x < 4$

17.(B) If $x \geq 0$ then $\sqrt{|x|-x} = \sqrt{x-x} = 0$; If $x < 0$ then $\sqrt{|x|-x} = \sqrt{-x-x} = \sqrt{-2x} > 0$ $\therefore$ Range = $[0, \infty)$

18.(D) $[x^2-1]$ is an integer $\Rightarrow \sin n\pi = 0 \ \forall\ x \in R \Rightarrow f(x) = 0 \ \forall\ x \in R$

19.(C) $f(x)$ is defined $\Rightarrow$ *tan* $2x$ is defined, $6\cos x + 2\sin 2x \neq 0$ $\tan 2x$ is defined $\Rightarrow 2x \neq (2n+1)\dfrac{\pi}{2} \Rightarrow x \neq (2n+1)\dfrac{\pi}{4}$

$6\cos x + 2\sin 2x \neq 0 \Rightarrow 6\cos x + 4\sin x\cos x \neq 0 \Rightarrow 2\cos x(3+2\sin x) \neq 0 \Rightarrow \cos x \neq 0 \Rightarrow x \neq (2n+1)\dfrac{\pi}{2}$

20.(C) $(fog)(x) = f[g(x)] = f\left[\dfrac{3x+x^3}{1+3x^2}\right] = \log\left[\dfrac{1+\dfrac{3x+x^3}{1+3x^2}}{1-\dfrac{3x+x^3}{1+3x^2}}\right] = \log\left[\dfrac{1+3x^2+3x+x^3}{1+3x^2-3x-x^3}\right] = \log\left(\dfrac{1+x}{1-x}\right)^3 = 3\log\dfrac{1+x}{1-x} = 3f(x).$

21.(A) $-3 < x \le -1 \Rightarrow f(x) = [x] < 0 \quad -1 < x < 1 \Rightarrow f(x) = |x| \ge 0\ ;\ 1 \le x < 3$

$\Rightarrow -3 < -x \le -1 \Rightarrow f(x) = |[-x]| > 0 \qquad \therefore \{x : f(x) \ge 0\} = (-1, 1) \cup [1, 3) = (-1, 3)$

22.(B) $f(x) = \dfrac{x-1}{x+1} \Rightarrow x = \dfrac{f(x)+1}{1-f(x)}\ ;\ f(x) = \dfrac{2x-1}{2x+1} = \dfrac{2\left[\dfrac{f(x)+1}{1-f(x)}\right]-1}{2\left[\dfrac{f(x)+1}{1-f(x)}\right]+1} = \dfrac{3f(x)+1}{f(x)+3}$

23.(A) $f(x) = x\ ;\ g(x) = |x|\ \forall x \in R\ ;\ [\phi(x) - f(x)]^2 + [\phi(x) - g(x)]^2 = 0$

If sum of two non-negative numbers is zero than each of the numbers should be zero.

$\Rightarrow \quad \phi(x) - f(x) = 0$ and $\phi(x) - g(x) = 0 \quad \Rightarrow \quad \phi(x) = f(x) = g(x)$

But $f(x) = g(x)$ is possible $\forall x \in [0, \infty)$; Hence $f(x) = x$ where $x \in [0, \infty)$

24.(B) $f(x)g(y) + f(y)g(x) = \dfrac{1}{2}(3^x + 3^{-x})\dfrac{1}{2}(3^y - 3^{-y}) + \dfrac{1}{2}(3^y + 3^{-y})\dfrac{1}{2}(3^x - 3^{-x})$

$= \dfrac{1}{4}\left[3^x 3^y - 3^x 3^{-y} + 3^{-x}3^y - 3^{-x}3^{-y} + 3^y 3^x - 3^y 3^{-x} + 3^{-y}3^x - 3^{-y}3^{-x}\right]$

$= \dfrac{1}{4}\left[2 \cdot 3^x 3^y - 2 \cdot 3^{-x}\ 3^{-y}\right] = \dfrac{3^{x+y} - 3^{-(x+y)}}{2} = g(x+y)$

25.(C) We have, for $n \in Z$, $|\sin x| + \sin x = \begin{cases} 2\sin x & \text{if } 2n\pi < x < (2n+1)\pi \\ 0 & \text{otherwise} \end{cases}$

Also, $2\sin x \ne 0$ if $2n\pi < x < (2n+1)\pi$. $\quad \therefore$ Domain of, f is $\bigcup\limits_{n \in Z} (2n\pi, (2n+1)\pi)$

26.(B) $\sin \log \dfrac{\sqrt{4-x^2}}{1-x}$ exists $\Rightarrow \dfrac{\sqrt{4-x^2}}{1-x} > 0 \Rightarrow 1-x > 0,\ 4-x^2 > 0 \Rightarrow 1 > x,\ x^2 - 4 < 0$

$\Rightarrow \quad 1 > x,\ -2 < x < 2 \Rightarrow -2 < x < 1 \quad \therefore$ Domain = (–2, 1)

27.(D) For $f(x)$ to be defined, $x - 2 > 0$ and $x - 3 > 0 \Rightarrow x > 2$ and $x > 3 \Rightarrow x \in (3, \infty)$

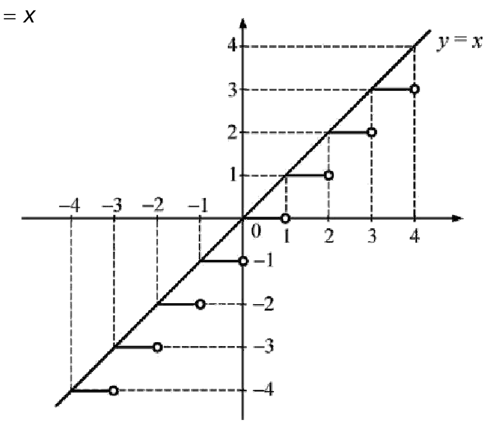
For $g(x)$ to be defined, $(x-2)(x-3) > 0 \Rightarrow x \in (-\infty, 2) \cup (3, \infty)$

Since $f(x)$ and $g(x)$ do not have the same domain, $f(x) \ne g(x)$

28.(A) For, $f(x) = |x-3| + |x-4| + |x-7| = (x-3) + (x-4) + (7-x) = x$

29.(D) For $f(x)$ to be defined, $x - [x] \ge 0$ or $x \ge [x]$

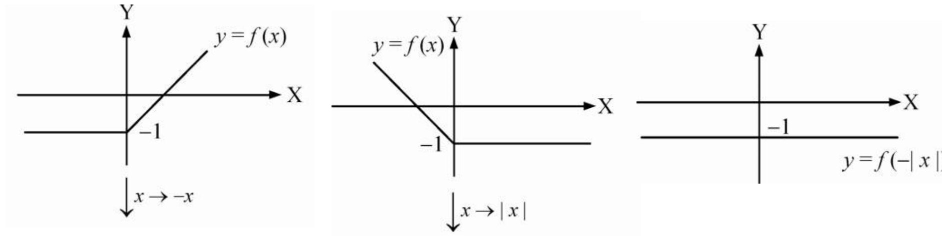
From the graphs we can clearly see that $x \ge [x]\ \forall x \in R$



30.(A) We have $f(x) = \sin[\pi^2]x + \sin[-\pi^2]x = \sin 9x + \sin(-10)x = \sin 9x - \sin 10x$

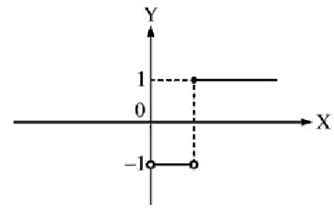
$\therefore \ f\left(\dfrac{\pi}{2}\right) = \sin\dfrac{9\pi}{2} - \sin 5\pi = 1 - 0 = 1 \Rightarrow \ f(\pi) = \sin 9\pi - \sin 10\pi = 0\ ;\ f\left(\dfrac{\pi}{4}\right) = \sin\dfrac{9\pi}{4} - \sin\dfrac{10\pi}{4} = \dfrac{1}{\sqrt{2}} - 1$

31.(D)



As it is a constant function, it is periodic. Other functions are not periodic.

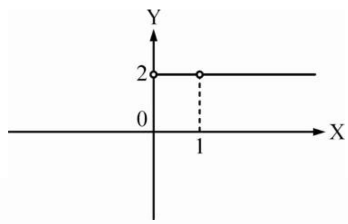
32.(A) $y = \dfrac{|\log_2 x|}{\log_2 x} = \begin{cases} 1 & \log_2 x \ge 0 \\ -1 & \log_2 x < 0 \end{cases} \quad \Rightarrow \quad y = \begin{cases} 1 & x \ge 1 \\ -1 & x < 1 \end{cases}$



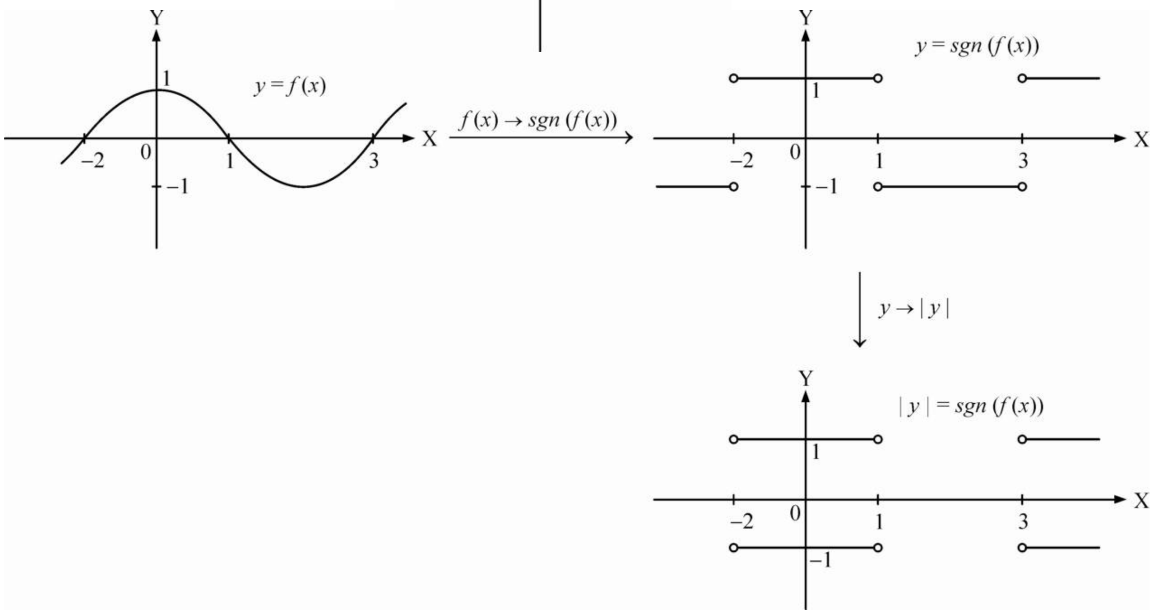
Note that $x > 0$ as $log_2 x$ is there

33.(C) $y = x^{\log_x 2} = 2$

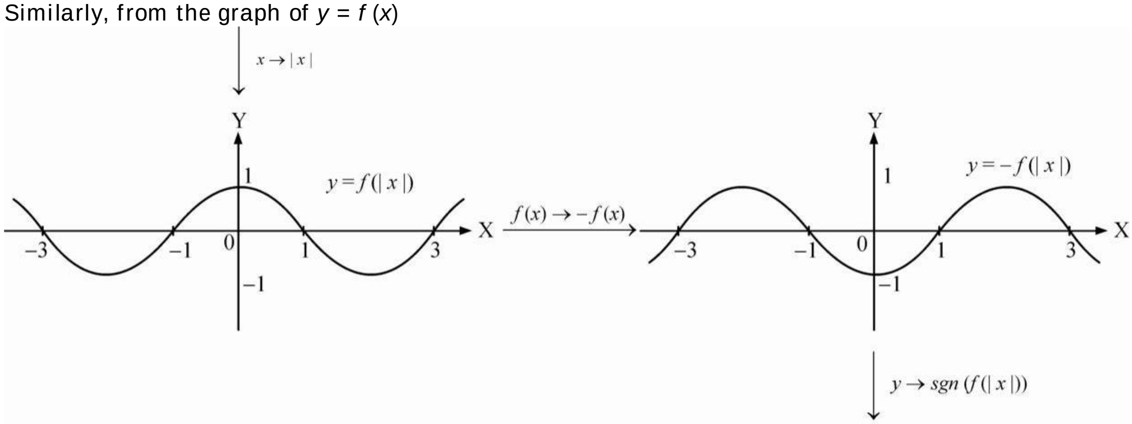
Also, $x > 0$ and $x \neq 1$

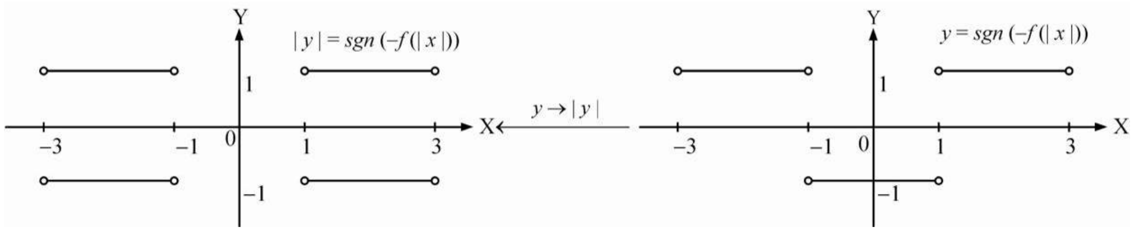


34.(AB)

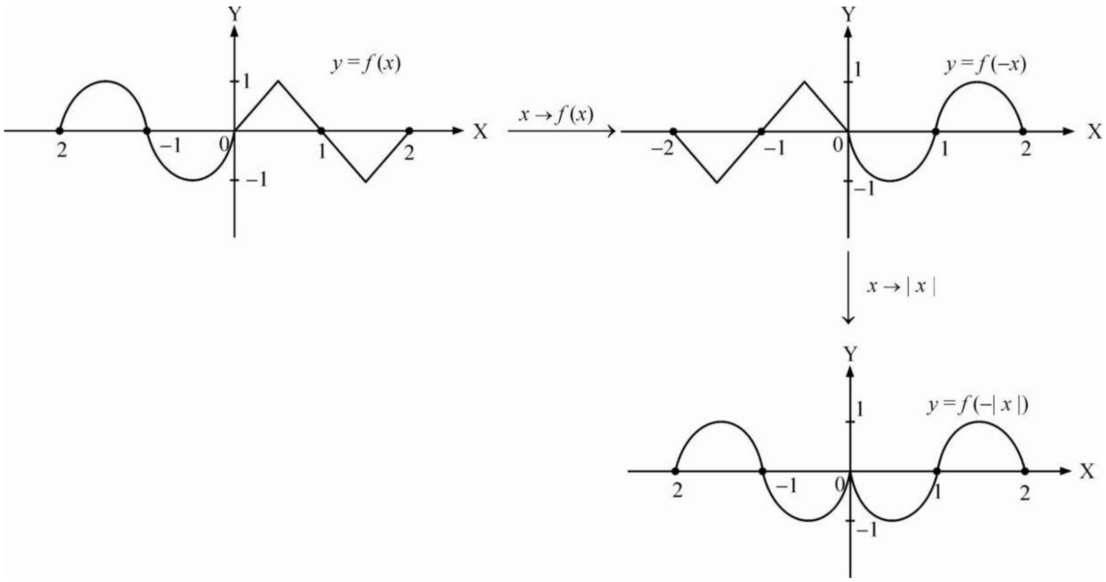


Similarly, from the graph of $y = f(x)$

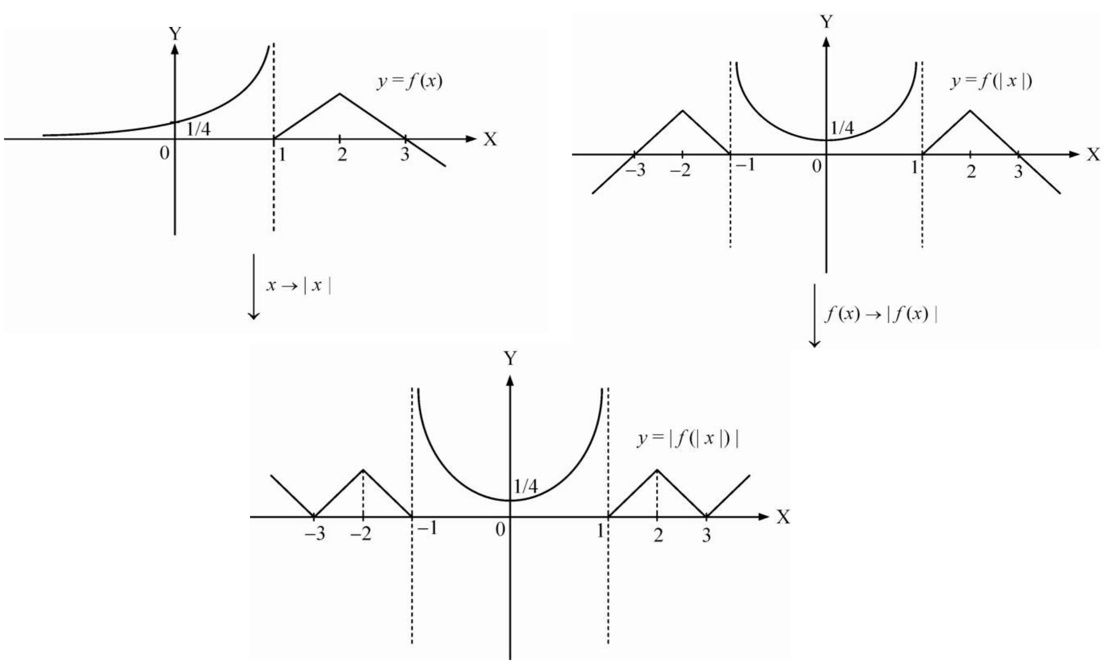




35.(A)



36.(C)



37. **(i)** $y = |\ |x^2| - 2x - 3\ | = |\ x^2 - 2x - 3\ |$ let $f(x) = x^2 - 2x - 3 = (x-3)(x+1)$

(ii) $|x| + |y| = 1$ **. . . .**

(iii) $|x| + y = 1$ **. . . . (ii)** $x + y = 1$ **. . . .(i)**

$\Uparrow \quad y \to |y|$ $\qquad \Uparrow \quad x \to |x|$

38.(B) $y=\dfrac{1}{2-\cos 3x}\quad -1\le \cos 3x\le 1 \Rightarrow -1\le -\cos 3x\le 1 \Rightarrow 1\le 2-\cos 3x\le 3 \Rightarrow 1\ge \dfrac{1}{2-\cos 3x}\ge \dfrac{1}{3} \Rightarrow y\in\left[\dfrac{1}{3},1\right]$

39.(C) $y=\dfrac{x^2+2x+3}{x} \Rightarrow x^2+(2-y)x+3=0\ ;\ D=(2-y)^2-4\times 3\ge 0 \Rightarrow y^2-4y-8\ge 0$

$\Rightarrow (y-2)^2\ge 12 \Rightarrow y-2\ge 2\sqrt{3}$ and $y-2\le -2\sqrt{3} \Rightarrow y\in(-\infty,2-2\sqrt{3}]\cup[2+2\sqrt{3},\infty)$

40.(A) $y=\dfrac{x^2-2}{x^2-3} \Rightarrow x^2=\dfrac{3y-2}{y-1}\ge 0 \Rightarrow y\in(-\infty,\dfrac{2}{3}]\cup(1,\infty)$

41.(D) $f(x)=\log\left(\dfrac{1+x}{1-x}\right) \Rightarrow f(x_1)=\log\left(\dfrac{1+x_1}{1+x_1}\right)$ and $f(x_2)=\log\left(\dfrac{1+x_2}{1-x_2}\right)$

$$\Rightarrow f(x_1)+f(x_2)=\log\left(\frac{1+x_1}{1-x_1}\right)+\log\left(\frac{1+x_2}{1-x_2}\right)=\log\left[\frac{1+x_1}{1-x_1}\cdot\frac{1+x_2}{1-x_2}\right]$$

$$=\log\left[\frac{1+x_1+x_2+x_1x_2}{1-x_1-x_2+x_1x_2}\right]=\log\left[\frac{1+\left(\frac{x_1+x_2}{1+x_1x_2}\right)}{1-\left(\frac{x_1+x_2}{1+x_1x_2}\right)}\right]=f\left(\frac{x_1+x_2}{1+x_1x_2}\right)$$

42.(B) Function is symmetric about $x=0$ line when $f(x)=f(-x) \Rightarrow f(0+x)=f(0-x)$

Here function is symmetrical about $x=2$ line $\Rightarrow f(2+x)=f(2-x)$

43.(B) $-\sqrt{2}\le \sin x+\cos x\le \sqrt{2} \Rightarrow \sin n\pi=0\ \forall\ x\in R \Rightarrow [\sin x+\cos x]=-2,-1,0,1$

44.(B) $f(x)=3\sin\left(\sqrt{\dfrac{\pi^2}{16}-x^2}\right) \Rightarrow \dfrac{-\pi}{4}\le x\le\dfrac{\pi}{4} \Rightarrow 0\le x^2\le\dfrac{\pi^2}{16} \Rightarrow \dfrac{-\pi^2}{16}\le -x^2\le 0 \Rightarrow 0\le\dfrac{\pi^2}{16}-x^2\le\dfrac{\pi^2}{16} \Rightarrow 0\le\sqrt{\dfrac{\pi^2}{16}-x^2}\le\dfrac{\pi}{4}$

As *sin y* is an increasing function $\forall\ y\in\left[0,\dfrac{\pi}{4}\right] \Rightarrow \sin 0\le \sin\sqrt{\dfrac{\pi^2}{16}-x^2}\le \sin\dfrac{\pi}{4} \Rightarrow 0\le 3\sin\sqrt{\dfrac{\pi^2}{16}-x^2}\le\dfrac{3}{\sqrt{2}}$

45.(C) Given, $f(1)=1$ and $f(n+1)=2f(n)+1, n\ge 1$; $f(2)=f(1+1)=2f(1)+1=2\times 1+1=3=2^2-1$

$f(3)=f(2+1)+2f(2)+1=2\times 3+1=7=2^3-1$; $f(4)=f(3+1)=2f(3)+1=2\times 7+1=15=2^4-1$

$f(5)=f(4+1)=2f(4)+1=2\times 15+1=31=2^5-1$; $f(n)=f((n+1)+1)=2f(n-1)+1=2^n-1$

46.(A) $f(x)=\cos\left(\log\left(x+\sqrt{x^2+1}\right)\right)$;

$$f(-x)=\cos\left(\log\left(-x+\sqrt{x^2+1}\right)\right)=\cos\left(\log\left(\frac{\left(\sqrt{x^2+1}\right)^2-x^2}{\sqrt{x^2+1}+x}\right)\right)=\cos\left(\log\left(\frac{1}{x+\sqrt{x^2+1}}\right)\right)$$

$=\cos\left(-\log\left(x+\sqrt{x^2+1}\right)\right)=f(x)$ $\quad[\because\ cos(-x)=cos(x)] \quad\Rightarrow\quad f(x)$ is an even function

47.(B) $f(x)=(\sin x^7)\times e^{x^5\,\mathrm{sgn}\,x^9}$

$\sin x^7\to$ odd function ; $x^5\to$ odd function

$\mathrm{sgn}\,x^n\to$ odd function $\Rightarrow x^5\,\mathrm{sgn}\,x^9=$ odd $*$ odd = even function

Hence $f(x)$ = odd $*$ even = odd function

48.(C) $f(x) = \sin x + \cos x$; $f(-x) = \sin(-x) + \cos(-x) = -\sin x + \cos x$; $f(x) =$ is neither even nor odd

$$g(x)=\sin\left[\log\left(x+\sqrt{x^2+1}\right)\right]$$

$$g(-x)=\sin\left[\log\left(-x+\sqrt{x^2+1}\right)\right]=\sin\left[-\log\left(x+\sqrt{x^2+1}\right)\right]=-\sin\left[\log\left(x+\sqrt{x^2+1}\right)\right]=-g(x);$$

[$g(x)$ is an odd function]

49.(A) $$f(x)=\sin^2 x+\left(\sin x\cos\frac{\pi}{3}+\cos x\sin\frac{\pi}{3}\right)^2+\cos x\left(\cos x\cos\frac{\pi}{3}-\sin x\sin\frac{\pi}{3}\right)$$

$$=\sin^2 x+\left(\frac{\sin x}{2}+\frac{\sqrt{3}\cos x}{2}\right)^2+\frac{\cos^2 x}{2}-\frac{\sqrt{3}}{2}\cos x\sin x$$

$$=\sin^2 x+\frac{\sin^2 x}{4}+\frac{3}{4}\cos^2 x+\frac{\sqrt{3}}{2}\sin x\cos x+\frac{\cos^2 x}{2}-\frac{\sqrt{3}}{2}\cos x\sin x=\frac{5}{4}\left(\sin^2 x+\cos^2 x\right)=\frac{5}{4}$$

$$\therefore\qquad [gof](x)=g[f(x)]=g\left(\frac{5}{4}\right)=1$$

50.(D) Period of $\sin\left(\frac{\pi x}{n!}\right)=\frac{2\pi}{\pi/n!}=2n!$; Period of $\cos\left[\frac{\pi x}{(n+1)!}\right]=\frac{2\pi}{\pi/(n+1)!}=2(n+1)!$

Period of $f(x)$ = LCM of $\{2n!, 2(n+1)!\}$ = 2 (n + 1) !

51.(A) We have,

$$f(x)=\frac{(\sin 5x+\sin x)+(\sin 4x+\sin 2x)}{(\cos 5x+\cos x)+(\cos 4x+\cos 2x)}=\frac{2\sin 3x\cos 2x+2\sin 3x\cdot\cos x}{2\cos 3x\cdot\cos 2x+2\cos 3x\cos x}=\frac{2\sin 3x(\cos 2x+\cos x)}{2\cos 3x(\cos 2x+\cos x)}=\tan 3x$$

Which is periodic with period $\frac{\pi}{3}$.

52.(D)

53.(D) The period of $|\sin x|+|\cos x|$ and $\sin^4 x+\cos^4 x$ is $\frac{\pi}{2}\cdot\sin(\sin x)+\sin(\cos x)$ has period 2π. The function $\frac{1+2\cos x}{\sin x(2+\sec x)}$ can be written in a simplified form as $\frac{\cos x}{\sin x}=\cot x$, so it has period π.

54.(C) $\tan(3x-2)$ is a periodic function with period $\frac{\pi}{3}$. The function $f(x)=\{x\}$ is periodic with period 1.

The function in (d) can be written as $f(x)=1-\frac{\cos^3 x}{\sin x+\cos x}-\frac{\sin^3 x}{\sin x+\cos x}=1-\frac{\sin^3 x+\cos^3 x}{\sin x+\cos x}$

$$=1-\frac{(\sin x+\cos x)\left(\sin^2 x+\cos^2 x-\sin x\cos x\right)}{(\sin x+\cos x)}=1-\left(1-\frac{1}{2}\sin 2x\right)=\frac{1}{2}\sin 2x$$

Which is periodic with period π. The function $x+\cos x$ is non-periodic as x non-periodic.

55.(B) We have, $g(x)=f[f(x)]=f\left(\frac{1}{1-x}\right)=\frac{1}{1-\frac{1}{1-x}}=\frac{x-1}{x}$

Also, $h(x)=f[f\{f(x)\}]=f\left(\frac{x-1}{x}\right)=\frac{1}{1-\frac{x-1}{x}}=x$ $\quad\therefore\ f(x)\cdot g(x)\cdot h(x)=\frac{1}{1-x}\cdot\frac{x-1}{x}\cdot x=-1$

56.(B) Fundamental period is $|\sin x+\cos x|\ \frac{\pi}{2}$, Now, $f\left(\frac{\pi}{2}+x\right)=\frac{|\cos x|+|\sin x|}{|\cos x-\sin x|+|\cos x+\sin x|}=f(x)$

57.(C) **Statement-1 :** $f(x+2\pi)=f(x) \Rightarrow T=2\pi$ is period ; **Statement-2 :** Obvious

58.(B) Given that, $f(x)=\sin^4 x+\cos^4 x$; $\therefore$ $f(x)=\left(\sin^2 x+\cos^2 x\right)^2-2\sin^2 x\cos^2 x$

$$=1-\frac{1}{2}(2\sin x\cos x)^2=1-\frac{1}{2}(\sin 2x)^2=1-\frac{1}{2}\left(\frac{1-\cos 4x}{2}\right)=\frac{3}{4}+\frac{1}{4}\cos 4x$$

$\therefore$ The period of $f(x)=\frac{2\pi}{4}=\frac{\pi}{2}$ [$\because$ $\cos x$ is periodic with period 2π]

59.(A) $\frac{2\pi}{\sqrt{[\lambda]}}=\pi \Rightarrow \sqrt{[\lambda]}=2 \Rightarrow [\lambda]=4 \Rightarrow \lambda\in[4,5)$

60.(ABC) We have, $f(x)=\frac{\sin\pi[x]}{\{x\}}$; Let T_1 be the period of $\sin\pi[x]$, Then, $\sin\pi[T_1+x]=\sin\pi[x]$

$\Rightarrow \pi[T_1+x]=2n\pi+\pi[x] \Rightarrow T_1+[x]=2n+[x] \therefore T_1=2n$ and minimum value is 2.

$T_1=2$; T_2 = Period of $\{x\}$ is 1. Hence, period of $f(x)$ = LCM of (T_1 and T_2) = 2

To find range of $f(x)=\sin\pi[x]$ is always 0. Hence, range of $f(x)=0$, which is a singleton set.

Since, $f(x)$ is always $0, \forall\, x\in R$ $\therefore$ $f(x)$ is an even function.

61.(A) Given expression is $2f(x-1)-f\left(\frac{1-x}{x}\right)=x$ **. . . .(i)**

Replace x by $\frac{1}{x}$, we get: $2f\left(\frac{1}{x}-1\right)-f(x-1)=\frac{1}{x} \Rightarrow 2f\left(\frac{1-x}{x}\right)-f(x-1)=\frac{1}{x}$ **. . . .(ii)**

Eliminate $f\left(\frac{1-x}{x}\right)$ from (i) and (ii), we get : $f(x-1)=\frac{1}{3}\left(2x+\frac{1}{x}\right)$ **. . . .(iii)**

Replace x by $x+1$ to get: $f(x)=\frac{1}{3}\left[2(1+x)+\frac{1}{1+x}\right]$

62.(C) $f(x+f(x))=4f(x)$**;** Put $x=1$, $f(1+f(1))=4f(1) \Rightarrow f(1+4)=4(4) \Rightarrow f(5)=16$

Again put $x=5$; $f(5+16)=4f(5) \Rightarrow f(21)=4(16)=64$

63.(C) $f(2x)+f\left(\frac{1}{11}+2x\right)+f\left(\frac{2}{11}+2x\right)+f\left(\frac{3}{11}+2x\right)+......+f\left(\frac{21}{11}+2x\right)=k$

Now, $2x\to 2x+\frac{1}{11}$; $f\left(2x+\frac{1}{11}\right)+f\left(2x+\frac{2}{11}\right)+f\left(2x+\frac{3}{11}\right)+f\left(2x+\frac{4}{11}\right)+......+f\left(2x+\frac{22}{11}\right)=k$

On subtracting $f(2x)=f(2x+2)$

64.(B) $h(x)=\log_{10}x=\sum_{n=1}^{89}\log_{10}(\tan n^\circ)$

$=\log_{10}\tan 1^\circ+\log_{10}\tan 2^\circ+\log_{10}\tan 3^\circ+\log_{10}\tan 4^\circ+.....+\log_{10}\tan 89^\circ$

$=\log_{10}(\tan 1^\circ\tan 2^\circ\tan 3^\circ\tan 4^\circ\tan 5^\circ......\tan 89^\circ)=\log_{10}(1)=0$

65.(B) $f(x)+2f\left(\frac{1}{x}\right)=3x,\quad x\neq 0$

$f\left(\frac{1}{x}\right)+2f(x)=\frac{3}{x}$; $3f(x)=\frac{6}{x}-3x$; $f(x)=\frac{2}{x}-x$; $f(x)=f(-x) \Rightarrow \frac{2}{x}-x=\frac{-2}{x}+x$

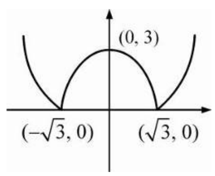
$P(E_1)=\frac{1}{6} \Rightarrow x^2=2 \Rightarrow x=\pm\sqrt{2}$

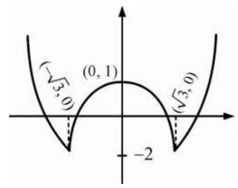
66.(A) $f(x) = \text{Min}\{x, x^2\}$; $f(x) = \begin{Bmatrix} x & x < 0 \\ x^2 & 0 \le x \le 1 \\ x & x > 1 \end{Bmatrix}$

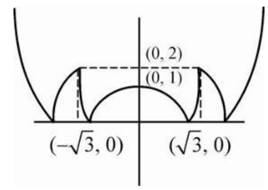
67.(A) $f(x) = \text{Max}\{(1-x), (1+x), 2\}$; See selected area : $f(x) = \begin{Bmatrix} 1-x & x \le -1 \\ 2 & -1 < x < 1 \\ 1+x & x \ge 1 \end{Bmatrix}$

68.(A) Let us draw the graph of $f(x)$. **69.(A)**

70.(D)

$\left|x^2-3\right|$ 

; $\left|x^2-3\right|-2$ 

; $\left|\left|x^2-3\right|-2\right|$ 

71.(C) $\dfrac{\log x}{\log 3\sqrt{x}} + \dfrac{\log \sqrt{x}}{\log 3x} = 0 \Rightarrow \dfrac{\log x}{\log 3 + \frac{1}{2}\log x} + \dfrac{1}{2}\dfrac{\log x}{(\log 3 + \log x)} = 0 \Rightarrow \log x\left[\dfrac{2}{2\log 3 + \log x} + \dfrac{1}{2\log 3 + 2\log x}\right] = 0$

$x > 0 \quad \log x = 0 \;\; or \;\; 4\log 3 + 4\log x = -2\log 3 - \log x$; $x = 1$ or $5\log x = -6\log 3$

One integral solution is $x = 1$ and One irrational solution $x = \dfrac{1}{3^{6/5}}$

72.(B) $x \in [0, 2\pi]$, $y_1 = \dfrac{\sin x}{|\sin x|}$, $y_2 = \dfrac{|\cos x|}{\cos x}$

When $x \in \left(0, \dfrac{\pi}{2}\right)$; $y_1 = \dfrac{\sin x}{\sin x}$, $y_2 = \dfrac{\cos x}{\cos x} = 1$ $\Rightarrow$ Identical

When $x \in \left(\dfrac{\pi}{2}, \pi\right)$; $y_1 = 1$, $y_2 = -1$ $\Rightarrow$ Not identical

When $x \in \left(\pi, \dfrac{3\pi}{2}\right)$; $y_1 = -1$, $y_2 = -1$ $\Rightarrow$ Identical

When $x \in \left(\dfrac{3\pi}{2}, 2\pi\right)$; $y_1 = -1$, $y_2 = 1$ $\Rightarrow$ Not identical

73.(C) For $f(x)$ to be defined, $\dfrac{\sqrt{4-x^2}}{1-x} > 0$, $4 - x^2 > 0$, $1 - x \ne 0$; Since, $\sqrt{4-x^2} \not< 0$ we have $1 - x > 0$ and

$4 - x^2 > 0 \Rightarrow x < 1$ and $(x-2)(x+2) < 0 \Rightarrow x < 1$ and $-2 < x < 2 \Rightarrow -2 < x < 1$

Since, $-\infty < \log\left(\dfrac{\sqrt{4-x^2}}{1-x}\right) < \infty \Rightarrow -1 \le \sin\left[\log\left(\dfrac{\sqrt{4-x^2}}{1-x}\right)\right] \le 1 \;\; \therefore$ Range of $f = [-1, 1]$

74.(B) Given, $f(x) + 2f(1-x) = x^2 + 2$ .... **(i)** Replace x by $1 - x$ in equation (i), we get :

$f(1-x) + 2f(x) = (1-x)^2 + 2$ **.......... (ii)** Now, multiplying equation (i) by 1 and equation (ii) by 2, then

subtracting each other, we get: $-3f(x) = x^2 + 2 - 2(1-x)^2 - 4 \Rightarrow 3f(x) = x^2 - 4x + 4 \Rightarrow f(x) = \dfrac{(x-2)^2}{3}$

75.(D) $f(x) = -\left(\dfrac{|x|^3 + |x|}{1 + x^2}\right) \Rightarrow f(x) < 0 \;\forall\; x \in R$; Hence $f(x)$ lies is III and IV quadrants only

Level - 2	JEE Advanced Pattern

76.(A) $f(x) = \log_2\left(-\log_{1/2}\left(1+\frac{1}{x^{1/4}}\right)-1\right)$ As $x^{1/4}$ is there, $x \ge 0$ **. . . .(i)**

$\frac{1}{x^{1/4}} \Rightarrow x^{1/4} \ne 0$ or $x \ne 0$ **. . . .(ii)** $-\log_{1/2}\left(1+\frac{1}{x^{1/4}}\right)-1 > 0 \Rightarrow \log_{1/2}\left(1+\frac{1}{x^{1/4}}\right) < -1$ **. . . .(iii)**

$\Rightarrow 1+\frac{1}{x^{1/4}} > 2$ or $\frac{1}{x^{1/4}} > 1 \Rightarrow x^{1/4} < 1$ or $x < 1$ **.....(iv)** Combining (i), (ii), (iii) and (iv), we get: $x \in (0, 1)$

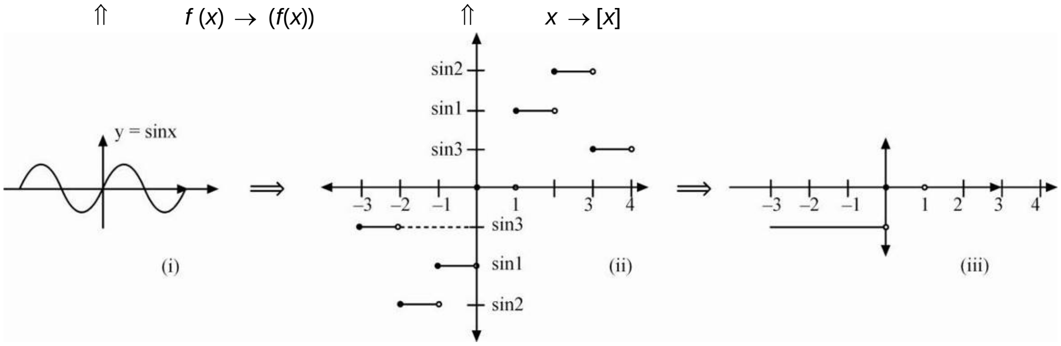
77.(B) $f(x) = \frac{x}{\sqrt{1+x^2}} \Rightarrow (fof)(x) \equiv f(f(x)) = \frac{f(x)}{\sqrt{1+(f(x))^2}} = \frac{\frac{x}{\sqrt{1+x^2}}}{\sqrt{1+\frac{x^2}{1+x^2}}} = \frac{x}{\sqrt{1+2x^2}}$

and $(f\ of\ of)(x) = f(f(f(x))) = f\left(\frac{x}{\sqrt{1+2x^2}}\right) = \frac{\frac{x}{\sqrt{1+2x^2}}}{\sqrt{1+\frac{x^2}{1+2x^2}}} = \frac{x}{\sqrt{1+3x^2}}$

78.(AB) $f(x) = \frac{a^x-1}{x^n(a^x+1)}$; $f(-x) = \frac{a^{-x}-1}{(-x)^n(a^{-x}+1)} = \frac{(-1)}{(-1)^n}\left(\frac{a^x-1}{x^n(a^x+1)}\right) \Rightarrow f(-x) = f(x)$ if n is odd and $f(-x) = -f(x)$ if n is even

79. **(i)** $y = [\sin[x]]$ **. . . . (iii)** $y = \sin[x]$ **. . . . (ii)** $y = \sin x$ **. . . . (i)**

$\Uparrow$ $f(x) \to (f(x))$ $\Uparrow$ $x \to [x]$



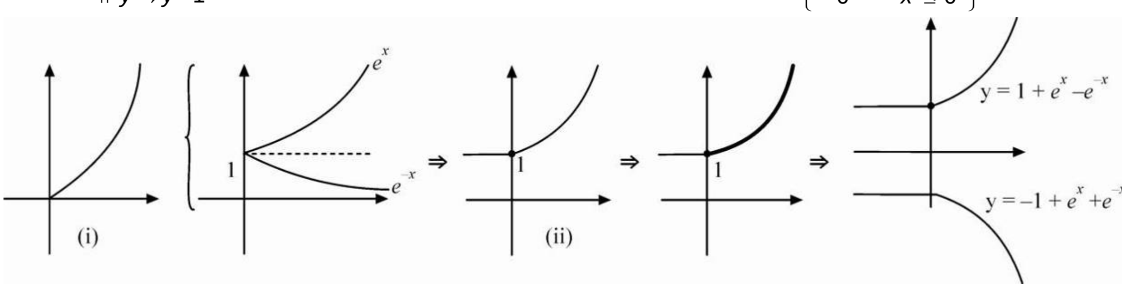
(ii) $y = [x] + \sqrt{x-[x]}$; $y = 0+\sqrt{x}$ $0 \le x < 1$; $= 1+\sqrt{x-1}$ $1 \le x < 2$; $= 2+\sqrt{x-1}$ $2 \le x < 3$

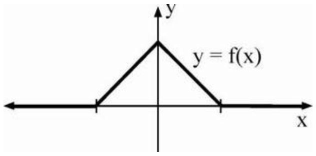
(iii) $|y| = |1 + e^{|x|} - e^{-x}|$ **. . . . (iv)** $y = 1 + e^{|x|} - e^{-x}$ **. . . . (iii)**

$\Uparrow y \to |y|$ $\Uparrow f(x) \to |f(x)|$

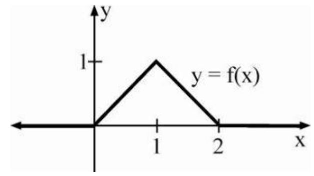
$y = 1 + e^{|x|} - e^{-x}$ **. . . . (ii)** $y = e^{|x|} - e^{-x} \Rightarrow y = \begin{cases} e^x - e^{-x} & x > 0 \\ 0 & x \le 0 \end{cases}$ **.... (i)**

$\Uparrow y \to y-1$

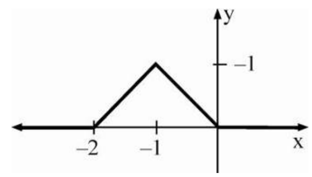


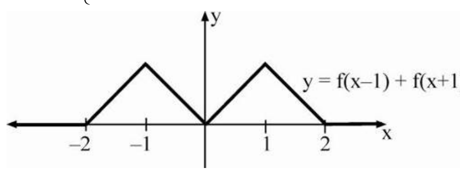
80. 

$$f(x-1)=\begin{cases}1-|x-1| & , \ |x-1|\le 1\\ 0 & , \ |x-1|>1\end{cases}=\begin{cases}0 & x<0\\ x & 0\le x\le 1\\ 2-x & 1\le x\le 2\\ 0 & x>2\end{cases} \quad \text{.....(i)}$$



$$f(x+1)=\begin{cases}1-|x+1| & , \ |x+1|\le 1\\ 0 & , \ |x+1|>1\end{cases}=\begin{cases}0 & x<-2\\ 2+x & -2\le x\le -1\\ -x & -1<x\le 0\\ 0 & x>0\end{cases} \quad \text{........(ii)}$$



Combining graph (1) and (2) 

81.(A) $f(x)=\dfrac{ax+b}{cx+d}$; $f(f(x))=\dfrac{af(x)+b}{cf(x)+d}=\dfrac{a\left(\dfrac{ax+b}{cx+d}\right)+b}{c\left(\dfrac{ax+b}{cx+d}\right)+d}=\dfrac{a^2x+ab+bcx+bd}{acx+cdx+bc+d^2}=x$ (Given)

$\Rightarrow \dfrac{(a^2+bc)x+(a+d)b}{c(a+d)x+(bc+d^2)}=x \Rightarrow c(a+d)=0$; $a^2+bc=bc+d^2$ and $(a+d)b=0$

$\Rightarrow a=-d$ and $a^2=d^2$ combining the two $\Rightarrow a=-d$

82.(C) $f(x) = \log(ax^3 + (a+b)x^2 + (b+c)x + c) \Rightarrow$ $ax^3 + (a+b)x^2 + (b+c) + xc > 0$

Let $g(x) = ax^3 + (a+b)x^2 + (b+c)x + c$; Clearly, $x = -1$ is the solution of $g(x)$

$\Rightarrow$ $g(x) = (x+1)(ax^2 + bx + c)$; But $g(x) > 0 \Rightarrow (ax^2 + bx + c)(x+1) > 0$

Consider $b^2 = 4ac$ (given) $\Rightarrow$ $ax^2 + bx + c$ is a perfect square. $\Rightarrow$ Hence, $x \in R \cap (-1, \infty)$

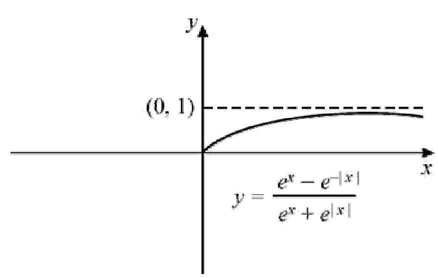
But $x \ne -b/2a$ as at $x=-\dfrac{b}{2a}$, we have $g(x) = 0$ $\Rightarrow$ $x \in R - \{-b/2a\} \cap (-1, \infty)$

83.(C) $f(x)=\begin{cases}x & ; \text{ if } x \text{ is rational}\\ 1-x & ; \text{ if } x \text{ is irrational}\end{cases}$. If $x \in$ Rational, $f(x) = x$ is rational ; $f(f(x)) = f(x) = x$

If $x \in$ Irrational, $1 - x$ is irrational $\Rightarrow$ $f(f(f(x) = f(1-x) = 1 - f(1-x) = x$ $\quad f(f(x)) = x\ \forall\ x\ [0, 1]$

84.(D) $f(x+y) \Rightarrow f(x)+f(y) \Rightarrow f(r)=rf(1) \Rightarrow \displaystyle\sum_{r=1}^{n} f(r) = f(1)\sum_{r=1}^{n} r = 7\frac{n(n+1)}{2}$

85.(D) $f(x)=\begin{cases}\dfrac{e^x-e^{-x}}{e^x+e^x}=\dfrac{e^{2x}-1}{2e^{2x}} & x>0\\ \dfrac{e^x-e^x}{e^x+e^{-x}}=0 & x\le 0\end{cases}$. Now for $x > 0$; $y=\dfrac{e^{2x}-1}{2e^{2x}}$



$\Rightarrow e^{2x}=\dfrac{1}{1-2y}\ge 1$; $\forall\ x>0 \Rightarrow \dfrac{1}{1-2y}-1\ge 0$

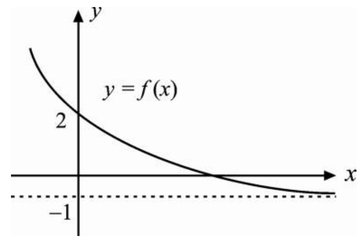
$\Rightarrow \dfrac{y}{2y-1}\le 0 \Rightarrow y \in \left[0, \dfrac{1}{2}\right)$; Range of the function is $\left[0, \dfrac{1}{2}\right)$

86.(B) $f(x)=\sqrt{\left(\frac{1}{\sin x}-1\right)}$

Domain of the function is $\frac{1}{\sin x}-1\geq 0 \Rightarrow \frac{\sin x-1}{\sin x}\leq 0 \Rightarrow \sin x(\sin x-1)\leq 0$

$\Rightarrow 0<\sin x\leq 1 \Rightarrow x\in(2n\pi\,.\,(2n+1)\pi),\ \forall\ n\in I$

87.(B) Let $f(x)=\left(\frac{2}{5}\right)^x+\left(\frac{3}{5}\right)^x+\left(\frac{4}{5}\right)^x-1$

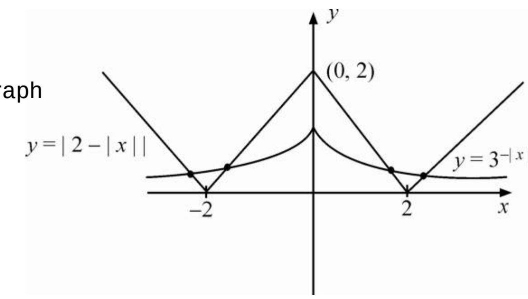


From the given figure it is clear that $f(x)$ is a decreasing function.

Also, as x approach ∞, $f(x)$ approaches -1 and as x approaches $-\infty$, $f(x)$ approaches to ∞

$\therefore$ The graph of $y = f(x)$ cuts the X-axis exactly once.

88.(D) Here $3^{|x|}\{|2-|x||\}=1$



We can re-write the equation $|2-|x||=3^{-|x|}$

Number of solution = number of point of intersection of graph $f(x)=|2-|x||$ and $g(x)=3^{-|x|}$

Graph of $f(x)$ and $g(x)$ are shown in figure.

Number of points of intersection of graph = 4

Hence number of solution = 4.

89.(A) $f(2x+3)+f(2x+7)=2$ . . . . . .**(i)**

Replace x by $x+2$

$f(2x+7)+f(2x+11)=2$ . . . . . .**(ii)**

Subtract (ii) from (i), we get : $f(2x+3)=f(2x+11)=f[2(x+4)+3]$

Period of $y=f(2x)$ is 4 ; Period of $y=f(x)=2\times$ period of $f(2x)=2\times 4=8$

90.(B) $2(\log_2 x)^2=\log_2 x+1$ $\quad(\log_2 x=t)$

$\Rightarrow 2t^2-t-1=0 \Rightarrow 2t^2-2t+t-1=0 \Rightarrow \log_2 x=1,\ \log_2 x=\frac{-1}{2} \Rightarrow x=2$

91.(A) $E=81^{\log_{0.\bar{3}}\left(\frac{1}{\sqrt{4+2\sqrt{3}}-\sqrt{4-2\sqrt{3}}}\right)}=81^{\log_{\frac{1}{3}}\left(\frac{1}{\sqrt{3}+1-(\sqrt{3}-1)}\right)}=81^{\log_{\frac{1}{3}}\frac{1}{2}}=81^{\log_3 2}=16$

92.(D) We know that $-\sqrt{5}\leq 2\sin x+\cos x\leq\sqrt{5},\ \forall\ x\in R$

$\Rightarrow -5\leq\sqrt{5}(2\sin x+\cos x)\leq 5 \Rightarrow 0\leq\sqrt{5}(2\sin x+\cos x)+5\leq 10$

$\Rightarrow -\infty<\log_{\sqrt[3]{10}}\left(\sqrt{5}(2\sin x+\cos x)+5\right)\leq 3$ Hence range is $(-\infty, 3]$

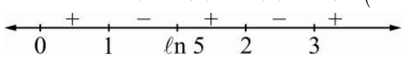
93.(C) **(A)** $f(x+T)=1^{[x+T]}+(-1)^{[x+T]}=1^{[x]+T}+(-1)^{[x]+T}$

Periodic for T is even ; Similarity for B and D

(C) $h(x+T)=2^{[x]+T}-(-2)^{[x]+T}=2^T\left(2^{[x]}-(-2)^{[x]}(-1)^T\right)\neq h(x)$

94.(B) $f(x) = \ell n\left((x-1)(x-2)(x-3)x\left(e^x - 5\right)\right)$

for defined $(x-1)(x-2)(x-3)x\left(e^x - 5\right) > 0$



A = Domain $(0, 1) \cup (\ell n 5, 2) \cup (3, \infty)$ and $g(x) = 1 - \cos^2\frac{x}{4} + \cos\frac{x}{4} = \frac{5}{4} - \left(\cos\frac{x}{4} - \frac{1}{2}\right)^2$

B = Range $\left[-1, \frac{5}{4}\right]$ $\qquad A \cap B = (0, 1)$

95.(C) We have, $f(k) = \frac{k}{2009}$ $\qquad \therefore\ f(2009-k) = \frac{2009-k}{2009} = 1 - \frac{k}{2009}$

We have, $g(k) = \frac{f^4(k)}{[1 - f(k)]^4 + [f(k)]^4}$

$$g(k) = \frac{\left(\frac{k}{2009}\right)^4}{\left(1 - \frac{k}{2009}\right)^4 + \left(\frac{k}{2009}\right)^4};\ g(k) = \frac{k^4}{(2009-4)^4 + k^4} \qquad \text{. . . .(i)}$$

$$\Rightarrow \qquad g(2009-k) = \frac{[f(2009-k)]^4}{[1 - f(2009-k)]^4 + [f(2009-k)]^4} = \frac{\left(1 - \frac{k}{2009}\right)^4}{\left(\frac{k}{2009}\right)^4 + \left(1 - \frac{k}{2009}\right)^4}$$

$$g(2009-k) = \frac{(2009-k)^4}{k^4 + (2009-k)^4} \qquad \text{. . . .(ii)}$$

Now, adding equations (i) and (ii), we get : $g(k) + g(2009-k) = 1$

We have to find $\sum_{K=0}^{2009} g(k) = [g(0) + g(2009)] + [g(1) + g(2006)] + \ldots$

$[g(1004) + g(1005)]$ i.e., $\underbrace{1 + 1 + 1 \ . \ . \ . + 1}_{1005 \text{ times}} = 1005$

96.(C) Given that, $f(n) = \begin{cases} \frac{n-1}{2}, \text{ where n is odd} \\ -\frac{n}{2}, \text{ where n is even} \end{cases}$ and $f : N \to I$, where N is the set of natural numbers and I is the set of integers.

Let $x, y \in N$ and both are even.

Then, $f(x) = f(y) \ \Rightarrow\ -\frac{x}{2} = -\frac{y}{2} \ \Rightarrow\ x = y$

Again, $x, y \in N$ and both are odd

Then, $f(x) = f(y) \quad \Rightarrow\ \frac{x-1}{2} = \frac{y-1}{2} \quad \Rightarrow\ x = y$

So, mapping is one-one.

Since each negative integer is an image of even natural number and positive integer is an image of odd natural number. So, mapping is onto. Hence, mapping is one-one onto.

97.(B) Here, $g^2(x) = gog(x) = g\{g(x)\} = g(3+4x)$

$= 15 + 4^2x = \left(4^2 - 1\right) + 4^2x$ **. . . . . (i)**

$g^3(x) = g\left(15 + 4^2x\right) = 63 + 4^3x = \left(4^3 - 1\right) + 4^3x$ **. . . . . (ii)**

Generalizing, we get: $g^n(x) = \left(4^n - 1\right) + 4^n x = y \ (say)$

$\because \quad g^n(x) = y \Rightarrow x = g^{-n}(y)$ **. . . . . (iii)**

Then, $x = \left(4^{-n} - 1\right) + 4^{-n}y = (y+1)4^{-n} - 1$

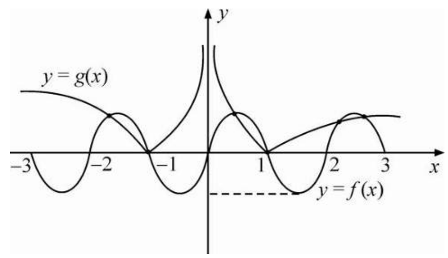
Using (iii), $g^{-n}(y) = (y+1)\,4^{-n} - 1$ Replace y by x $\Rightarrow$ $g^{-n}(x) = (x+1)\,4^{-n} - 1$

98.(D) Graph of the functions are shown in figure.
Suppose

$f(x) = \sin \pi x$

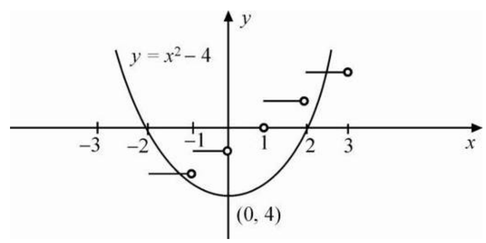
$g(x) = |\ell n\, e^{|x|}|$

Number of solution = 6



99.(B) Given equation is $x^2 - 4 - [x] = 0$

Number of solution is same as number of Points of intersection of $y = x^2 - 4$ and $y = [x]$.
Number of solution = 2.



100.(B) Given function is $f(x) = \left\{2\sin^2\left(\dfrac{4x - 3\pi}{6\pi^2}\right)\right\}^2 + 2\cos\left(\dfrac{4x - 3\pi}{3\pi^2}\right) = \left\{1 - \cos\left(\dfrac{4x - 3\pi}{3\pi^2}\right)\right\}^2 + 2\cos\left(\dfrac{4x - 3\pi}{3\pi^2}\right)$

$= 1 + \cos^2\left(\dfrac{4x - 3\pi}{3\pi^2}\right) = \dfrac{1}{2}\left[2 + 1 + \cos\left(\dfrac{8x - 6\pi}{3\pi^2}\right)\right] = \dfrac{3}{2} + \dfrac{1}{2}\cos\left(\dfrac{8x - 6\pi}{3\pi^2}\right)$

Hence period of $f(x) = \dfrac{2\pi}{8/3\pi^2} = \dfrac{3\pi^3}{4}$

101.(B) $f(x) = \dfrac{2 + x - [x]}{1 - x + [x]} = \dfrac{2 + \{x\}}{1 - \{x\}}$ where $\{x\}$ is the fractional part of x.

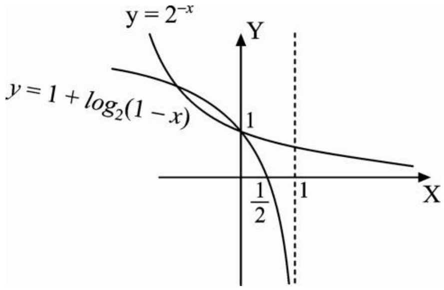
Since, $\{x\} \in [0, 1)$, $2 \le 2 + \{x\} < 3$ and $0 < 1 - \{x\} \le 1$

$\Rightarrow \quad 2 \le \dfrac{2 + \{x\}}{1 - \{x\}} < \infty \qquad \therefore$ Range of $f(x) = [2, \infty)$

102.(C) Consider the function $f(x) = 1 + \log_2(1 - x)$ and $g(x) = 2^{-x}$

In this question we are supposed to find number of roots of $f(x) = g(x)$. Number of roots of $f(x) = g(x)$ is same as number of points of intersection of $y = f(x)$ and $y = g(x)$.
Graph of $f(x)$ and $g(x)$ is shown in figure.

From the graph it is clear that number of points of intersection of $y = f(x)$ and $y = g(x)$ is 2.



Hence number of solutions = 2

Note : In the given equation one side is exponential and one side is logarithmic. Analytical solution is not possible in this case. Graphical method is better approach for these types of problems.

103.(B) Given function is $f(x) = 3\,|\sin x| - 2\,|\cos x|$

$f(x)$ is continuous function and $|\sin x|$ *and* $|\cos x|$ are always + ve.

Hence, $f(x)$ is minimum when $|\sin x| = 0$ and $|\cos x| = 1$

min value = 0 – 2 = – 2

and $f(x)$ is maximum when $|\sin x| = 1$ and $|\cos x| = 0$

Maximum value = 3 – 0 = 3 $\quad\therefore\quad$ required range = $[-2, 3]$

***Note* :** In this case max of $|\sin x|$ occurs at the point where $|\cos x|$ is min and vice versa. This might not be the case with other functions. So, think before applying above logic.
* You can check the range using graph as well.

104.(A) $$x = \left(\frac{81^{\frac{1}{\log_5 9}} + 3^{\frac{3}{\log_{\sqrt{6}} 3}}}{409}\right)\cdot\left(\left(\sqrt{7}\right)^{\frac{2}{\log_{25} 7}} - 125^{\log_{25} 6}\right)$$

$$x = \left(\frac{81^{\frac{1}{\log_9 5}} + 3^{\frac{3}{\log_3 \sqrt{6}}}}{409}\right)\cdot\left(7^{\frac{2}{\log_7 25}} - 6^{\log_{25} 125}\right) = \frac{25+6\sqrt{6}}{409}\left(25-6\sqrt{6}\right) = 1 \Rightarrow \quad \log_2 x = \log_2 1 = 0$$

105.(ABC) Let the equation be $a_0x^n + a_1x^{n-1} + a_2x^{n-2} + \ldots + a_n$

Now, $\quad \Sigma\alpha_i = \pm 1, \Sigma\alpha_i\alpha_j = \pm 1 \quad \Rightarrow \quad \Sigma\alpha_i^2 \le 1 + 2 = 3 \quad \therefore \quad n \ge 3$

106.(C) $1+\cos^2 x \in [1, 2] \quad \Rightarrow \quad \left[1+\cos^2 x\right] = 1, 2 \quad \therefore \quad \sec^{-1}\left[1+\cos^2 x\right] = \sec^{-1}1, \sec^{-1}2$

107.(C) Let $g(x) = e^{3\{x\}} \Rightarrow T_1 = 1$ and $f(x) = e^{\{3x\}} \Rightarrow T_2 = 1/3 \quad \therefore \quad T_1 = 3T_2$

108.(B) Since, $f(x) = \dfrac{4^x}{4^x+2} \quad \therefore \quad f(1-x) = \dfrac{4^{1-x}}{4^{1-x}+2} = \dfrac{4}{4+2\cdot 4^x} = \dfrac{2}{2+4^x}$

$\Rightarrow \quad f(x) + f(1-x) = 1$

Putting, $x = \dfrac{1}{97}, \dfrac{2}{97}, \ldots, \dfrac{48}{97}$, we get: $f\left(\dfrac{1}{97}\right), + f\left(\dfrac{2}{97}\right) + \ldots + f\left(\dfrac{96}{97}\right) = 48$

109.(A) For the given function, we must have

$x - 4 \ge 0$ and $6 - x \ge 0 \Rightarrow \quad x \ge 4$ and $x \le 6$; $\quad$ Therefore, the domain is $[4, 6]$.

110.(C) $\log_a x = \alpha, \log_b x = \beta, \log_c x = \gamma, \log_d x = \delta \Rightarrow a = x^{\frac{1}{\alpha}}, b = x^{\frac{1}{\beta}}, c = x^{\frac{1}{\gamma}}, d = x^{\frac{1}{\delta}}$

$$\log_{abcd} x = \frac{1}{\frac{1}{\alpha}+\frac{1}{\beta}+\frac{1}{\gamma}+\frac{1}{\delta}}$$

111.(D) Given, $f(x) = \log\left(\frac{1+x}{1-x}\right)$ then, $f\left(\frac{3x+x^3}{1+3x^2}\right) - f\left(\frac{2x}{1+x^2}\right)$

$$= \log\left(\frac{1+\left(\frac{3x+x^3}{1+3x^2}\right)}{1-\left(\frac{3x+x^3}{1+3x^2}\right)}\right) - \log\left(\frac{1+\frac{2x}{1+x^2}}{1-\frac{2x}{1+x^2}}\right) = \log\left(\frac{1+x}{1-x}\right)^3 - \log\left(\frac{1+x}{1-x}\right)^2 = \log\left(\frac{1+x}{1-x}\right) = f(x)$$

112.(C) The period of the function is found as follows

Given, $f(x) + f(x+4) = f(x+2) + f(x+6)$ **...(i)**

$\therefore$ Replacing x by $x+2$ we get: $f(x+2) + f(x+6) = f(x+4) + f(x+8)$ **...(ii)**

From equations **(i)** and **(ii)**, we get:

$f(x) + f(x+4) = f(x+4) + f(x+8) \Rightarrow f(x) = f(x+8) \Rightarrow$ period of $f(x) = 8$

113.(B) $y = f(x) = \begin{cases} \frac{x^2}{1+x^2}, & x < 0 \\ 0, & x = 0 \\ \frac{-x^2}{1+x^2}, & x > 0 \end{cases}$

Case I : $x < 0 \Rightarrow y = \frac{x^2}{1+x^2} \Rightarrow x^2(1-y) = y \Rightarrow x = -\sqrt{\frac{y}{1-y}} \quad \because (x < 0)$

Case II : $x > 0 \Rightarrow y = \frac{-x^2}{1+x^2} \Rightarrow x^2 = \frac{-y^2}{1+y} \Rightarrow x = +\sqrt{\frac{-y}{1+y}} \Rightarrow f^{-1}(y) = x = \text{sgn}(-y)\sqrt{\frac{|y|}{1-|y|}}$

114.(C) $4\log_{x/2}\sqrt{x} + 2\log_{4x} x^2 = 3\log_{2x} x^3 \qquad \left(x > 0, x \neq \frac{1}{2}, \frac{1}{4}, 2\right)$

$\Rightarrow 2\log_{x/2} x + 4\log_{4x} x = 9\log_{2x} x \Rightarrow \frac{2}{\log_x (x/2)} + \frac{4}{\log_x 4x} = \frac{9}{\log_x 2x}$

$\Rightarrow \frac{2}{1-\log_x 2} + \frac{4}{\log_x 4x} = \frac{9}{\log_x 2x} \Rightarrow \frac{2}{1-t} + \frac{4}{2t+1} = \frac{9}{t+1}$ (Let, $\log_x 2 = t$; $(x \neq 1)$)

$\Rightarrow 6(t+1) = 9(t - 2t^2 + 1) \Rightarrow 18t^2 - 3t - 3 = 0 \Rightarrow 6t^2 - t - 1 = 0 \Rightarrow t = \frac{1}{2}, -\frac{1}{3} \Rightarrow x = 4, \frac{1}{8}$

Now, checking for $x = 1$

$x = 1$ satisfies the original equation $\quad \therefore$ Integral solution are $\{4, 1\}$

115.(A) $f(x)$ is defied, if $x^2 - 5x + 6 \neq 0$, $\left[x + \frac{1}{2}\right] > 0, \left[x + \frac{1}{2}\right] \neq 1$

$x^2 - 5x + 6 \neq 0, \Rightarrow (x-2)(x-3) \neq 0 \Rightarrow x \neq 2, 3$ **....(i)**

$\left[x + \frac{1}{2}\right] > 0 \Rightarrow x \geq \frac{1}{2}$ **...(ii)**

$\left[x+\frac{1}{2}\right] \neq 1 \quad \Rightarrow \quad x \notin \left[\frac{1}{2}, \frac{3}{2}\right)$ **...(iii)**

From Eq. **(i)**, **(ii)** and **(iii)**, we get domain of $f = \left[\frac{3}{2}, 2\right) \cup (2, 3) \cup (3, \infty)$.

116.(C) Suppose $S = \sum_{r=1}^{100} \left[\frac{1}{2} + \frac{r}{100}\right]$

$$\Rightarrow \quad S = \underbrace{\left[\frac{1}{2}+\frac{1}{100}\right] + \left[\frac{1}{2}+\frac{2}{100}\right] + \ldots + \left[\frac{1}{2}+\frac{49}{100}\right]}_{=0} + \underbrace{\left[\frac{1}{2}+\frac{50}{100}\right] + \ldots + \left[\frac{1}{2}+\frac{100}{100}\right]}_{51 \text{ terms}} = 0 + 51 = 51.$$

Hence summation of series is 51.

117.(A) $\sqrt{\cos(\sin x)}$ is defined for $x \in R$ as $\sin x \in [-1, 1]$

$\cos(\sin x)$ is always +ve as $[-1, 1]$ lies between $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$

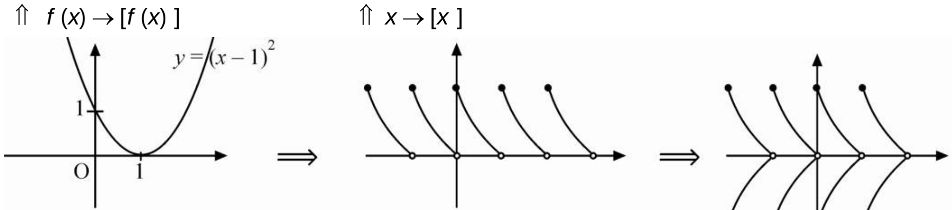
Consider $\sin^{-1}\left(\frac{1+x^2}{2x}\right) \Rightarrow -1 \le \frac{1+x^2}{2x} \le 1 \Rightarrow \frac{1+x^2}{2x} \ge -1 \Rightarrow \frac{1+x^2+2x}{2x} \ge 0 \Rightarrow \frac{(1+x)^2}{2x} \ge 0 \Rightarrow x > 0$

Equality holds at $x = +1$ and $\frac{1+x^2}{2x} \le 1 \Rightarrow \frac{1+x^2}{2x} - 1 \le 0 \Rightarrow \frac{(1-x)^2}{2x} \le 0$

$\Rightarrow x < 0 \Rightarrow$ Equality holds at $x = +1$. Combining, we can say $x = \pm 1$

118. $|y| = \{x\} - 1)^2$ **.... (iii)** $\quad y = (\{x\} - 1)^2$ **. . . . (ii)** $\quad y = (x-1)^2$ **. . . . (i)**

$\Uparrow f(x) \to [f(x)]$ $\quad \Uparrow x \to [x]$



(i) (ii) (iii)

119.(D)

120.(C) $f(x)$ defined, if $-(\log_3 x)^2 + 5\log_3 x - 6 > 0$ and $x > 0$

$\Rightarrow (\log_3 x - 3)(2 - \log_3 x) > 0$ and $x > 0 \Rightarrow (\log_3 x - 2)(\log_3 x - 3) < 0$ and $x > 0$

$\Rightarrow 2 < \log_3 x < 3$ and $x > 0 \Rightarrow 3^2 < x < 3^3 \Rightarrow 9 < x < 27$; Domain of $f(x)$ is $x \in (9, 27)$

121.(A) Since, the function $\sec x$ is an even function and $\log\left(x + \sqrt{1+x^2}\right)$ is odd function, therefore the function $\sec\left[\log\left(x + \sqrt{+x^2}\right)\right]$ is an even function.

122.(C) Put, $x = y = 0$

$\Rightarrow f(0) + f(0) = 2(f(0))^2 \quad \Rightarrow \quad f(0) = 0, 1 \quad \Rightarrow k = 0, 1$

Put, $x = 0$, we get, $f(y) + f(-y) = 2f(0)f(y)$

If, $f(0) = 0 \quad \Rightarrow \quad f(y) + f(-y) = 0 \quad \Rightarrow \quad f$ is odd

If, $f(0) = 1 \quad \Rightarrow \quad f(y) + f(-y) = 2f(y) \quad \Rightarrow \quad f$ is even

123.(A) $f(15+x) = f(15-x)$ ......**(i)**

And $f(30+x) = -f(30-x)$ ......**(ii)**

Replace $x \to 15-x$ in **(i)** $\Rightarrow$ $f(30-x) = f(x)$

By **(ii)** $f(x) = -f(30+x)$ ......**(iii)**

Replace $x \to x+30$ **;** Then $f(x+30) = -f(x+60)$ ......**(iv)**

From **(iii)** & **(iv)** $f(x) = f(x+60)$ $\Rightarrow$ $f(x)$ is periodic with period 60

Again $f(30-x) = f(x)$

$x \to x+30$ $\Rightarrow f(30+x) = f(-x)$ From **(ii)** $f(-x) = -f(x)$; $f(x)$ is odd

124.(A) $y = f(x) = \sin\{[x+5] + \{x - \{x - \{x\}\}\}\} = \sin\{x - \{[x]\}\} = \sin\{x - 0\} = \sin x$

$\because\ 0 < x < \dfrac{\pi}{4}$ $\therefore\ x = \sin^{-1} y$ or $f^{-1}(x) = \sin^{-1} x$

125.(A) Here, $f(x+10) = \dfrac{f(x)-5}{f(x)-3}$ $\Rightarrow$ $f(x+20) = \dfrac{2f(x)-5}{f(x)-2}$; $f(x+30) = \dfrac{3f(x)-5}{f(x)-1}$;

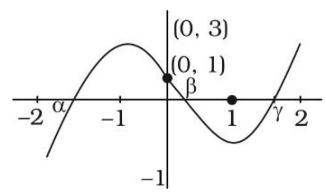
$f(x+40) = \dfrac{3f(x+10)-5}{f(x+10)-1} = f(x)$ $\therefore$ $f(x)$ is periodic with period 40 and $f(10) = f(50)$

Numerical Value Type	**JEE Main Pattern**

126.(26) $f(x) - 2x + 1 = (x-1)(x-2)(x-3)(x-4)(x-5)(2009x - \alpha)$

127.(7) $f(x) = x^3 - 3x + 1$; $f(f(x)) = 0$

Let $f(x) = t$ $\Rightarrow f(t) = 0$ $\Rightarrow$ $t = \alpha, \beta, \gamma$ $\Rightarrow$ $f(x) = \alpha,\ \alpha \in (-2, -1)$



No. of solution = 1

$f(x) = \beta,\ \beta \in (0, 1)$

No. of solution = 3 ; $f(x) = \gamma,\ \gamma \in (1, 2)$; No. of solution = 3

128.(1) $-1 \le \dfrac{2x}{3} \le 1$ $\Rightarrow$ $\dfrac{-3}{2} \le x \le \dfrac{3}{2}$; $12 - 3^x - \dfrac{27}{3^x} \ge 0 \Rightarrow (3^x - 3)(3^x - 9) \le 0 \Rightarrow 1 \le x \le 2$

129.(1) $\sin^{-1}(0) + \cos^{-1}(-1) = \pi$ $\quad 0 \le x^2 < \dfrac{4}{9}$

$\sin^{-1}(1) + \cos^{-1}(0) = \pi$ $\quad \dfrac{4}{9} \le x^2 < \dfrac{13}{9}$

130.(5) Clearly $[\sin x] = 0, 1$ or -1, $[\cos x] = 0, 1$ or -1 and $[\sin x + \cos x] = 0, 1, -1$ or -2

$\therefore$ Least value & Maximum value of $[\sin x] + [\cos x] + [\sin x + \cos x]$ may be –4 and 3 respectively.

Clearly $[\sin x]$ and $[\cos x]$ cannot be 1 together.

$\therefore$ total possible elements in required range are 5 i.e. $0, -1, -2, 1$ and 2.

131.(1) $f(f(x)) = \dfrac{1}{\sqrt[2011]{1 - \dfrac{1}{1-x^{2011}}}} = \dfrac{\sqrt[2011]{1-x^{2011}}}{-x}$; $f(f(f(x))) = \dfrac{\sqrt[2011]{\left(1 - \dfrac{-1}{1-x^{2011}}\right)}}{\dfrac{-1}{\sqrt[2011]{1-x^{2011}}}} = \dfrac{\dfrac{-x}{\sqrt[2011]{1-x^{2011}}}}{\dfrac{-1}{\sqrt[2011]{1-x^{2011}}}} = x$

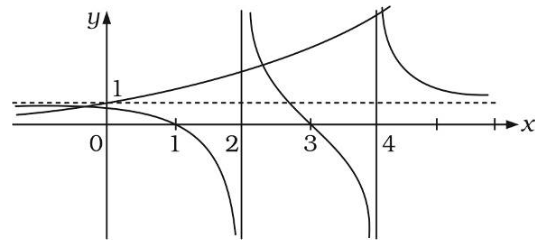
132.(4) $(f(x, y))^2 - (g(x, y))^2 = \dfrac{1}{2}$; $f(x, y) \cdot g(x, y) = \dfrac{\sqrt{3}}{4}$ $\Rightarrow$ $f(x, y) = x^2 - y^2 = \pm\dfrac{\sqrt{3}}{2}$; $g(x, y) = 2xy = \pm\dfrac{1}{2}$

133.(6) Clearly $f(x)=\dfrac{x+5}{\sqrt{x^2+1}}$ is maximum when $x=\dfrac{1}{5} \Rightarrow f(x)=\sqrt{26}<6$

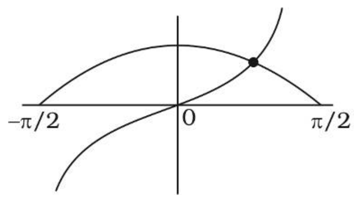
$\therefore$ $f(x)\le k$ must have a solution if $k=6$

134.(7) $f(x)=\dfrac{(x-1)(x-3)}{(x-2)(x-4)}-e^x$; $f(x)=0$ has three solutions

$f(-x)=\dfrac{(x+1)(x+3)}{(x+2)(x+4)}-e^{-x}=0$ has three solutions.



$x^3=\cos x$



one solution

There are total 7 solutions.

135.(6) $f(x)=x^2-bx+c=0 \begin{cases} p_1 \\ p_2 \end{cases}$

$p_1+p_2=b$ (odd no.)

$p_1p_2=c$ **;** $b+c=(p_2+2)+2p_2=35 \;\Rightarrow\; p_2=11 \;\Rightarrow\; f(x)=x^2-13x+22$; $\lambda=f(x)_{\min}=-\dfrac{81}{4}$

136.(4) $f(x)=f(-x)=f\left(\dfrac{x+1}{x+2}\right) \Rightarrow x=\dfrac{x+1}{x+2} \Rightarrow x^2+x-1=0 \Rightarrow x=\dfrac{-1\pm\sqrt{5}}{2}$

and $-x=\dfrac{x+1}{x+2} \Rightarrow x^2+3x+1=0 \Rightarrow x=\dfrac{-3\pm\sqrt{5}}{2}$ Therefore four real values of x.

137.(8) $\displaystyle\sum_{r=1}^{n}[\log_2 r]=0+1+1+(2+2+2+2)+\underbrace{(3+3+....+3)}_{8\text{ times}}+.....$

$=2\cdot1+4\cdot2+8\cdot3+....+ \quad\Rightarrow\quad T_r=2^r(r)$ **,** $S_n=\displaystyle\sum_{r=1}^{n}T_r=2010 \;\Rightarrow\; n=512$

138.(2) Let $p(x)=x^3+ax^2+bx+100$

Now: $x^3+ax^2+bx+100=(x^2-5x+6)Q_1+2Ax+2B$ and $x^3+ax^2+bx+100=(x^2-5x+4)Q_2+Ax+B$

Now substitute $x=2,3$ in first equation and $x=4,1$ in second equation and solve to get $a=45, b=-248$

$\Rightarrow \quad p(x)=x^3+45x^2-248x+100 \;\Rightarrow\; p(5)=110$

139.(2) $f(\theta)=0 \Rightarrow \theta=-5\pm\sqrt{5} \;\Rightarrow\; f(f(f(x)))=-5\pm\sqrt{5}$

Since $f(x)=(x+5)^2-5$

$f(f(f(x)))=-5\pm\sqrt{5}$; $((f(f(f(x))))+5)^2=-5\pm\sqrt{5}$; $(f(f)+5)^2=\sqrt{5}$; $f(f)+5=\pm5^{1/4}$

$f(f)=-5\pm5^{1/4}$; $(f+5)^2-5=-5\pm5^{1/4}$; $(f+5)^2=5^{1/4}$; $f+5=\pm5^{1/8}$

140.(4) $P(x)=(x-3)Q_1(x)+6 \Rightarrow P(3)=6$

$P(x)=(x^2-9)Q(x)+(ax+b)$; $P(3)=3a+b=6$

If equation of odd degree polynomial, then $b=0, a=2$

Archive	**JEE Main**

1.(A) $f(x) = x + \frac{1}{x}$ i.e. $y = x + \frac{1}{x}$ $\Rightarrow$ $xy = x^2 + 1$ $\Rightarrow x^2 - xy + 1 = 0$ $\therefore$ $x = \frac{y \pm \sqrt{y^2 - 4}}{2}$

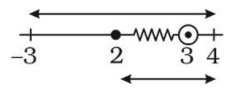
Since $y \in [2, \infty)$, so $f^{-1}(x) = \frac{x + \sqrt{x^2 - 4}}{2}$

2.(D) $f(x) = \cos(\log x)$

$f(x)f(y) - \frac{1}{2}\left[f\left(\frac{x}{y}\right) + f(xy)\right]$ $= \cos(\log x)\cos(\log y) - \frac{1}{2}[\cos(\log x - \log y) + \cos(\log x + \log y)]$

$= \cos(\log x)\cos(\log y) - \frac{1}{2}[2\cos(\log x)\cos(\log y) = 0$

3.(B) $f(x) = \frac{p(x)}{q(x)}(say)$



Then domain of $f(x)$ is $D_f p(x) \cap D_f q(x), q(x) \neq 0$

Now D_f of $p(x)$ is $-\frac{\pi}{2} \leq \sin^{-1}(x-3) \leq \frac{\pi}{2}$ $\Rightarrow$ $-\sin\frac{\pi}{2} \leq x - 3 \leq \sin\frac{\pi}{2}$ $\Rightarrow$ $2 \leq x \leq 4$ ....(i)

Again $9 - x^2 > 0 \Rightarrow x^2 < 9$; $|x| < 3$ i.e. $-3 < x < 3$ .....(ii) From (i) and (ii), we have $\therefore$ $2 \leq x < 3$

4.(A) If $y = \sin^{-1} a$, then $-1 \leq a \leq 1$

$\therefore$ $-1 \leq \log_3\left(\frac{x}{3}\right) \leq 1$ $\left[\text{as } y = \sin^{-1}\left[\log_3\left(\frac{x}{3}\right)\right]\right]$ $\Rightarrow$ $\frac{1}{3} \leq \frac{x}{3} \leq 3^1$ $\Rightarrow 1 \leq x \leq 9$

5.(A) Here, $f(x) = \frac{b-x}{1-bx}$ where $0 < b < 1, 0 < x < 1$

For function to be invertible it should be one-one onto. $\therefore$ Check range

Let $f(x) = y$ $\Rightarrow$ $y = \frac{b-x}{1-bx}$ $\Rightarrow y - bxy = b - x$ $\Rightarrow$ $x(1-by) = b - y$ $\Rightarrow$ $x = \frac{b-y}{1-by}$

where $0 < x < 1$ $\therefore$ $0 < \frac{b-y}{1-by} < 1$

$\frac{b-y}{1-by} > 0$ and $\frac{b-y}{1-by} < 1$ $\Rightarrow$ $y < b$ or $y > \frac{1}{b}$ **. . . .(i)**

$\frac{(b-1)(y+1)}{1-by} < 0 - 1 < y < \frac{1}{b}$ **. . . .(ii)** From, $f(x)$ is not invertible.

6.(B) Given, $f(x) = x^2, g(x) = \sin x$ $(gof)(x) = \sin x^2$

Now, $go(gof)(x) = \sin(\sin^2)$ $\Rightarrow$ $(fogogof)(x) = \left[\sin(\sin^2)\right]^2$ **......(i)**

Again, $(gof)(x) = \sin x^2$; $(gogof)(x) = \sin(\sin x^2)$ **.......(ii)**

Given, $(fogogof)(x) = (gogof)(x)$

$\Rightarrow$ $\left(\sin(\sin x^2)\right)^2 = \sin(\sin x^2)$ $\Rightarrow$ $\sin(\sin x^2)\left\{\sin(\sin x^2) - 1\right\} = 0$

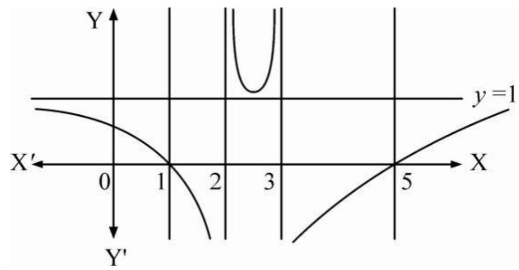
$\Rightarrow$ $\sin(\sin x^2) = 0$ *or* $\sin(\sin x^2) = 1$ $\Rightarrow \sin x^2 = \frac{\pi}{2}$ or $\sin x^2 = \frac{\pi}{2}$

$\therefore$ $x^2 = n\pi$ (i.e. not possible as $-1 \leq \sin\theta \leq 1$) $\Rightarrow$ $x = \pm\sqrt{n\pi}$

7.(D) Let $A \cap B = \phi, A, B \subset S \quad \Rightarrow \quad 3^4 = \frac{81+1}{2} = 41 \qquad \therefore \quad \frac{3^4 +}{2} = 41$

8.(B) Given, $f(x) = \frac{(x-1)(x-5)}{(x-2)(x-3)}$

The graph of $f(x)$ is shown



(i) If $-1 < x < 1 \Rightarrow 0 < f(x) < 1$

(ii) If $1 < x < 2 \Rightarrow f(x) < 0$

(iii) If $3 < x < 5 \Rightarrow f(x) < 0$

(iv) If $x > 5 \Rightarrow 0 < f(x) < 1$

Hence (i) $\to$ (p), (ii) $\to$ (q), (iii) $\to$ (q), (iv) $\to$ (p)

$X = 1$

9.(D) Let $\phi(x) = f(x) - g(x) = \begin{cases} x, & x \in Q \\ -x, & x \notin Q \end{cases}$

Now, to check one-one

Take any straight line parallel to X-axis which will intersect $\phi(x)$ only at one point.

$\Rightarrow \phi(x)$ is one-one.

To check onto

As, $f(x) = \begin{cases} x, x \in Q \\ -x, x \notin Q \end{cases}$, which shows

$y = x$ and $y = -x$ for rational and irrational values.

$\Rightarrow \quad y \in$ real numbers $\qquad \therefore \qquad$ Range = Codomain $\quad \Rightarrow \quad$ onto

Thus, $f - g$ is one-one and onto.

10.(C) Since, only option (C) satisfy given definition $\qquad$ i.e., $f\left\{f^{-1}(B)\right\} = B \qquad$ Only, if $B \subseteq f(x)$

11.(D) Given, $F(x) = \int \sin^2 \times dx = \int \frac{1-\cos 2x}{2} dx \quad \Rightarrow \quad F(x) = \frac{1}{2}(2x - \sin 2x) + C$

Since, $F(x+\pi) \neq F(x)$. Hence, Statement I is incorrect.

But Statement II is correct as $sin^2 x$ is periodic with period π.

12.(B) Given, $f(x) = 2 + \cos x, \forall x \in R$

Statement-I There exists a point $c \in [t, t+\pi]$, where $f'(c) = 0$

Hence, Statement I is correct.

Statement-II $f(t) = f(t+2\pi)$ is true. But Statement-II is not a correct explanation for Statement-I.

13.(A) Given, $y = -\frac{x^2}{2} + x + 1 \quad \Rightarrow \quad y - \frac{3}{2} = -\frac{1}{2}(x-1)^2 \quad \Rightarrow$ It is symmetric about

14.(B) We have, $e^{-x} f(x) = 2 + \int_0^x \sqrt{t^4+1}\, dt, \forall x \in (-1, 1)$

On differentiating w. r. t. x, we get $e^{-x}[f'(x) - f(x)] = \sqrt{x^4+1}$

$\Rightarrow \qquad f'(x) = f(x) + \sqrt{x^4+1}.e^x \qquad \because \qquad f^{-1}$ is the inverse of f.

$\therefore \quad f^{-1}\{f(x)\} = x \quad \Rightarrow [f^{-1}\{f(x)\}]' f'(x) = 1 \Rightarrow [f^{-1}\{f(x)\}]' = \frac{1}{f'(x)} \Rightarrow [f^{-1}\{f(x)\}]' = \frac{1}{f(x) + \sqrt{x^4+1}.\, e^x}$

At $x = 0, f(x) = 2 \quad \Rightarrow \quad \{f^{-1}(2)\}' = \frac{1}{2+1} = \frac{1}{3}$

15.(A) $E_1 : \dfrac{x}{x-1} > 0$, $x \in (-\infty, 0) \cup (1, \infty)$ and $E_2 = \sin^{-1}\left[ln\,\dfrac{x}{x-1}\right] \Rightarrow -1 \le ln\left(\dfrac{x}{x-1}\right) \le 1$

$\Rightarrow \dfrac{1}{e} \le \dfrac{x}{x-1} \le e \Rightarrow \dfrac{1}{e} \le 1 + \dfrac{1}{x-1} \le e \Rightarrow \dfrac{1-e}{e} \le \dfrac{1}{x-1} \le e-1 \Rightarrow x-1 \in \left(-\infty, \dfrac{e}{1-e}\right] \cup \left[\dfrac{1}{e-1}, \infty\right)$

$\Rightarrow x \in \left(-\infty, \dfrac{1}{1-e}\right] \cup \left[\dfrac{e}{e-1}, \infty\right)$

$f(x) = ln\left(\dfrac{x}{x-1}\right) \Rightarrow$ Domain of *f*: $(-\infty, 0) \cup (1, \infty)$; Range of *f:* $(-\infty, \infty) - \{0\}$

$g(x) = \sin^{-1}\left(ln\,\dfrac{x}{x-1}\right) \Rightarrow$ Domain of *g*: $\left(-\infty, \dfrac{1}{1-e}\right] \cup \left[\dfrac{e}{e-1}, \infty\right)$; Range of *g*: $\left[-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right] - \{0\}$

P-4, Q-2, R-1, S-1

Archive	**JEE Advanced**

1.(ABC) (A) $f(x) = \sin\left(\dfrac{\pi}{6}\sin\left(\dfrac{\pi}{2}\sin x\right)\right), x \in R$

$= \sin\left(\dfrac{\pi}{6}\sin\theta\right), -\dfrac{\pi}{2} \le \theta \le \dfrac{\pi}{2} = \sin\alpha, -\dfrac{\pi}{6} \le \alpha \le \dfrac{\pi}{6} \quad \therefore$ Range of $f(x) \in \left[-\dfrac{1}{2}, \dfrac{1}{2}\right]$

(B) $(fog)(x) = f[g(x)] = f(t), -\dfrac{\pi}{2} \le t \le \dfrac{\pi}{2} = \sin\left[\dfrac{\pi}{6} . \sin\left(\dfrac{\pi}{2}\sin t\right)\right], -\dfrac{\pi}{2} \le t \le \dfrac{\pi}{2}$

$\therefore \quad f[g(x)] = f(t)$ has same range of *f* (*x*) $\quad \therefore \quad$ Range of $(fog)(x) \in \left[-\dfrac{1}{2}, \dfrac{1}{2}\right]$

(C) $\lim\limits_{x \to 0} \dfrac{\sin\left(\dfrac{\pi}{6}\sin\left(\dfrac{\pi}{2}\sin x\right)\right)}{\dfrac{\pi}{2}\sin x} \Rightarrow \lim\limits_{x \to 0} \dfrac{\sin\left(\dfrac{\pi}{6}\sin\left(\dfrac{\pi}{2} \cdot \sin x\right)\right)}{\dfrac{\pi}{6}\sin\left(\dfrac{\pi}{2} \cdot \sin x\right)} \cdot \dfrac{\dfrac{\pi}{6} \cdot \sin\left(\dfrac{\pi}{2}\sin x\right)}{\left(\dfrac{\pi}{2} \cdot \sin x\right)} \Rightarrow 1 \times \dfrac{\pi}{2} \times 1 = \dfrac{\pi}{6}$

(D) $(gof)(x) = g[f(x)] = \dfrac{\pi}{2}\sin[f(x)] \quad \therefore \quad (gof)(x) = 1$

2.(ABC) (i) If $f'(x) > 0, \forall x \in (a, b)$, then *f* (*x*) is an increasing function is (*a*, *b*) and thus, *f* (*x*) is one-one function in (*a*, *b*).

(ii) If range of *f* (*x*) = codomain of *f* (*x*), then *f* (*x*) is an onto function.

(iii) A function *f* (*x*) is said to be odd function, if $f(-x) = -f(x), \forall x \in R$

i.e., $f(-x) + f(x) = 0, \forall x \in R$

Given, $f(x) = [\ln(\sec x + \tan x)]^3$; $f'(x) = \dfrac{3[\ln(\sec x + \tan x)]^2(\sec x \tan x + \sec^2 x)}{(\sec x + \tan x)}$

$f'(x) = 3\sec x[\ln(\sec x + \tan x)]^2 > 0, \forall x \in \left(-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right)$

f (*x*) is an increasing function. $\quad \therefore \quad$ *f* (*x*) is an one-one function.

$(\sec x + \tan x) = \tan\left(\dfrac{\pi}{4} + \dfrac{\pi}{2}\right)$, as $x \in \left(-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right)$, then $0 < \tan\left(\dfrac{\pi}{4} + \dfrac{x}{2}\right) < \infty$

$0 < \sec x + \tan x < \infty$

$\Rightarrow \quad -\infty < \ln(\sec x + \tan x) < \infty;\ -\infty < [\ln(\sec x + \tan x)]^3 < \infty \quad \Rightarrow \quad -\infty < f(x) < \infty$

$\therefore$ Range of $f(x)$ is R and thus $f(x)$ is an onto function.

$$f(-x) = [\ln(\sec x - \tan x)]^3 = \left[\ln\left(\frac{1}{\sec x + \tan x}\right)\right]^3$$

$f(-x) = -[\ln(\sec x + \tan x)]^3$ Now, $f(x) + f(-x) = 0 \Rightarrow f(x)$ is an odd function.

3. Given, $2\sin t = \dfrac{1-2x+5x^2}{3x^2-2x-1}, t \in \left[-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right]$

Put, $2\sin t = y \quad \Rightarrow \quad -2 \le y \le 2 \quad \therefore y = \dfrac{1-2x+5x^2}{3x^2-2x-1} \quad \Rightarrow \quad (3y-5)x^2 - 2x(y-1) - (y+1) = 0$

Since, $x \in R - \{1, -1/3\}$ $\quad\left(\text{As } 3x^2 - 2x - 1 \ne 0 \quad \Rightarrow \quad (x-1)(x+1/3) \ne 0\right)$

$\therefore \quad D \ge 0$

$\Rightarrow 4(y-1)^2 + 4(3y-5)(y+1) \ge 0 \Rightarrow y^2 - y - 1 \ge 0 \Rightarrow \left(y - \dfrac{1}{2}\right)^2 - \dfrac{5}{4} \ge 0$

$\Rightarrow \left(y - \dfrac{1}{2} - \dfrac{\sqrt{5}}{2}\right)\left(y - \dfrac{1}{2} + \dfrac{\sqrt{5}}{2}\right) \ge 0 \Rightarrow y \le \dfrac{1-\sqrt{5}}{2}$ or $y \ge \dfrac{1+\sqrt{5}}{2} \Rightarrow 2\sin t \le \dfrac{1-\sqrt{5}}{2}$ or $2\sin t \ge \dfrac{1+\sqrt{5}}{2}$

$\Rightarrow \sin t \le \sin\left(-\dfrac{\pi}{10}\right)$ or $\sin t \ge \sin\left(\dfrac{3\pi}{10}\right) \Rightarrow t \le -\dfrac{\pi}{10}$ or $t \ge \dfrac{3\pi}{10}$ Hence, range of t is $\left[-\dfrac{\pi}{2}, \dfrac{\pi}{10}\right] \cup \left[\dfrac{3\pi}{10}, \dfrac{\pi}{2}\right]$

4.(2) Given, $g\{f(x)\} = x \quad \Rightarrow \quad g'\{f(x)\}f'(x) = 1$

If $f(x) = 1 \quad \Rightarrow \quad x = 0, f(0) = 1$

Substituting $x = 0$ in Eq. (i), we get $g'(1) = \dfrac{1}{f'(0)} \Rightarrow g'(1) = 2 \quad \left[\because\ f'(x) = 3x^2 + \dfrac{1}{2}e^{x/2} \Rightarrow f'(0) = \dfrac{1}{2}\right]$

Alternate Solution

Given, $f(x) = x^3 + e^{x/2} \Rightarrow f'(x) = 3x^2 + \dfrac{1}{2}e^{x/2}$

For $x = 0$, $f(0) = 1, f'(0) = \dfrac{1}{2}$ and $g(x) = f^{-1}(x)$

Replacing x by $f(x)$, we have $g(f(x)) = x \Rightarrow g'(f(x)).f'(x) = 1$; Put $x = 0$, we get, $g'(1) = \dfrac{1}{f(0)} = 2$

5.($2 \le \alpha \le 14$ **, No)**

Let $y = \dfrac{\alpha x^2 + 6x - 8}{\alpha + 6x - 8x^2} \quad \Rightarrow \quad \alpha y + 6xy - 8x^2y = \alpha x^2 + 6x - 8 \quad \Rightarrow \quad -\alpha x^2 - 8x^2y + 6xy - 6x + ay + 8 = 0$

$\Rightarrow \quad \alpha x^2 + 8x^2y - 6xy + 6x - \alpha y - 8 = 0 \quad \Rightarrow \quad x^2(\alpha + 8y) + 6x(1-y) - (8 + \alpha y) = 0$

Since, x is real.

$\Rightarrow B^2 - 4AC \le 0 \Rightarrow 63(1-y)^2 + 4(\alpha + 8y)(8 + \alpha y) \ge 0 \Rightarrow 9(1 - 2y + y^2) + [8\alpha + (64 + \alpha^2)y + 8\alpha y^2] \ge 0$

$\Rightarrow \quad y^2(9 + 8\alpha) + y(46 + \alpha^2) + 9 + 8\alpha \ge 0 \quad$ .....(i)

$\Rightarrow \quad A > 0, D \le 0, \quad \Rightarrow \quad 9 + 8\alpha > 0$ and $(46 + \alpha^2)^2 - 4(9 + 8\alpha)^2 \le 0 \quad \Rightarrow \quad \alpha > -9/8$

and $\quad [46 + \alpha^2 - 2(9 + 8\alpha)][46 + \alpha^2 + 2(9 + 8\alpha)] \le 0 \quad \Rightarrow \quad \alpha > -9/8$

and $\quad (\alpha^2 - 16\alpha + 28)(\alpha^2 + 16\alpha + 64) \le 0 \Rightarrow \alpha > -9/8$ and $[(\alpha-2)(\alpha-14)](\alpha+8)^2 \le 0 \quad \Rightarrow \quad \alpha > -9/8$

and $\quad (\alpha - 2)(\alpha - 14) \le 0 \qquad [\because\ (\alpha + 8)^2 \ge 0]$

$\Rightarrow \quad \alpha > -9/8$ and $2 \le \alpha \le 14 \quad \Rightarrow \quad 2 \le \alpha \le 14$

Thus, $f(x) = \dfrac{\alpha x^2 + 6x - 8}{\alpha + 6x - 8x^2}$ will be onto, if $2 \le \alpha \le 14$

Again, when $\alpha = 3$; $f(x) = \dfrac{3x^2 + 6x - 8}{3 + 6x - 8x^2}$, in this case $f(x) = 0 \Rightarrow 3x^2 + 6x - 8 = 0$

$\Rightarrow x = \dfrac{-6 \pm \sqrt{36+93}}{6} = \dfrac{-6 \pm \sqrt{132}}{6} = \dfrac{1}{3}(-3 \pm \sqrt{33})$. This shows that $f\left[\dfrac{1}{3}(-3+\sqrt{33})\right] = f\left[\dfrac{1}{3}(-3-\sqrt{33})\right] = 0$

Therefore, f is not one-to-one.

6. $f^{-1}(1) = y$

It gives three cases

Case I When $f(x) = 1$ is true.

In this case, remaining two are false. $\quad\therefore\quad f(y) = 1$ and $f(z) = 2$

This means x and y have the same image, so $f(x)$ is not an injective, which is a contradiction.

Case II When $f(y) \ne 1$ is true.

If $f(y) \ne 1$ is true, then the remaining statements are false. $\quad\therefore\quad f(x) \ne 1$ and $f(z) = 2$

i.e. both x and y are not mapped to 1. So, either both associate to 2 or 3. Thus, it is not injective.

Case III When $f(z) \ne 2$ is true.

If $f(z) \ne 2$ is true, then remaining statements are false. $\therefore$ If $f(x) \ne 1$ and $f(y) = 1$

But f is injective. Thus, we have $f(x) = 2, f(y) = 1$ and $f(z) = 3$ Hence, $f^{-1}(1) = y$

7.(3) Given that $f(x+y) = f(x)f(y)\ \forall\ x, y \in N$ and $f(1) = 2$.

To find 'a' such that $\sum_{k=1}^{n} f(a+k) = 16(2^n - 1)$ ....(1)

We start with $f(1) = 2$ ....(2)

Then $f(2) = f(1+1) = f(1)f(1) \Rightarrow f(2) = 2^2$ [Using (2)]

Similarly, we get $f(3) = 2^3, f(4) = 2^4, \ldots, f(n) = 2^n$

Now equation (1) can be written as $f(a+1) + f(a+2) + f(a+3) + \ldots + f(a+n) = 16(2^n - 1)$

$\Rightarrow f(a)f(1) + f(a)f(2) + f(a)f(3) + \ldots + f(a)f(n) = 16(2^n - 1)$

$\Rightarrow f(a)[f(1) + f(2) + f(3) + \ldots + f(n)] = 16[2^n - 1] \Rightarrow f(a)\left[\dfrac{2(2^n - 1)}{2-1}\right] = 16[2^n - 1]$

$\Rightarrow f(a) = 8 = 2^3 = f(3) \Rightarrow a = 3$

8. $\dfrac{\tan x}{\tan 3x} = \dfrac{\tan x}{\left(\dfrac{3\tan x - \tan^3 x}{1 - 3\tan^2 x}\right)}$

Now $\tan x \ne 0$, otherwise the given function is not defined.

Cancelling $\tan x$, we have $\dfrac{\tan x}{\tan 3x} = \dfrac{1 - 3\tan^2 x}{3 - \tan^2 x} = r(say)$

$\therefore\ 1 - 3\tan^2 x = 3r - r\tan^2 x$ or $(r-3)\tan^2 x = 3r - 1$ or $\tan^2 x = \dfrac{3r-1}{r-3}$

As L.H.S. ≥ 0, for the R.H.S. to be ≥ 0, r cannot lie between $\dfrac{1}{3}$ and 3. i.e. $\dfrac{\tan x}{\tan 3x}$ cannot lie between $\dfrac{1}{3}$ and 3.