Rotational motion-2

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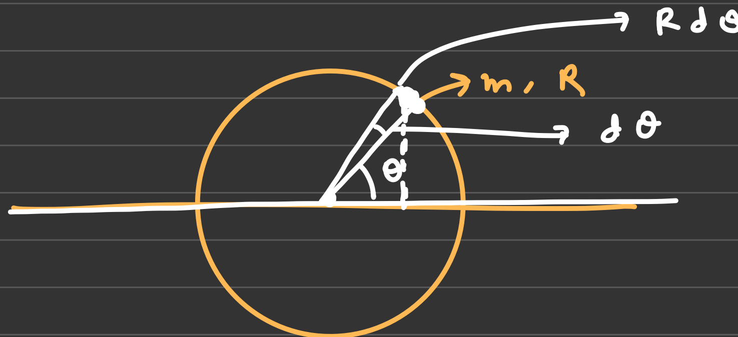
IIT JEE | MEDICAL | FOUNDATION

MOI of Ring along diameter:



$$\int dI = 2\int_0^{\pi} dm \times (R\sin\theta)^2$$

$$I = 2\int \left(\frac{m}{2\pi \cancel{R}} \cdot \cancel{R}\, d\theta\right)(R\sin\theta)^2$$

$2\pi R \longrightarrow m$

$1 \longrightarrow \frac{m}{2\pi R}$

$R\,d\theta \longrightarrow \frac{m}{2\pi R} \cdot R\,d\theta$

$$I = \cancel{2}\,\frac{mR^2}{\cancel{2}\pi}\int_0^{\pi} \sin^2\theta \cdot d\theta = \frac{mR^2}{\pi}\int_0^{\pi} \sin^2\theta \cdot d\theta$$

$\Rightarrow \cos^2\theta - \sin^2\theta = \cos 2\theta$

$$\frac{mR^2}{\pi}\int_0^{\pi}\left(\frac{1-\cos 2\theta}{2}\right)\cdot d\theta$$

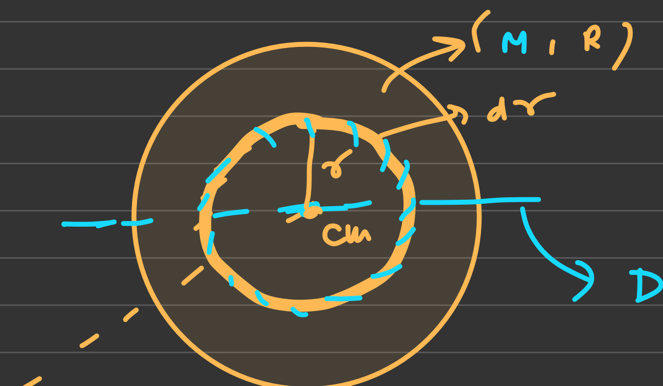
$$= \frac{mR^2}{2\pi}\left\{[\theta]_0^{\pi} - \left[\frac{\sin 2\theta}{2}\right]_0^{\pi}\right\}$$

$$= \left[\frac{mR^2}{2\pi}\pi\right] = \left(\frac{mR^2}{2}\right)$$

$\Rightarrow 1 - 2\sin^2\theta = \cos 2\theta$

$\sin^2\theta = \left(\frac{1-\cos 2\theta}{2}\right)$

Disc:



an axis passing through COM and $\perp$ to plane of disc

$I_{cm} = ?$

$I_D = ?$

$$\int dI = \int_0^R dm\,(r^2)$$

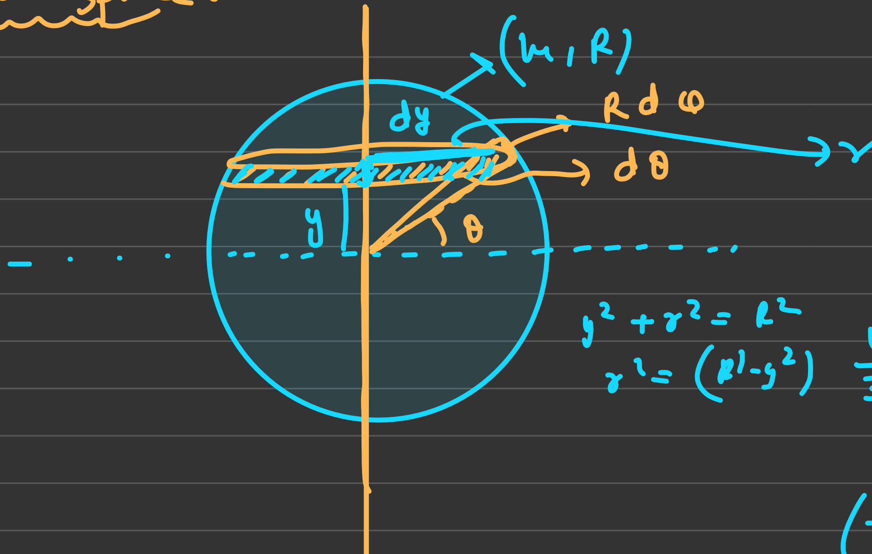
$$I_{cm} = \int_0^R \frac{M}{\pi R^2} \cdot (2\pi r \cdot dr) \cdot r^2$$

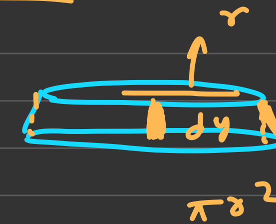
$$I_{cm} = \left(\frac{MR^2}{2}\right)$$

$$I_D = \int_0^R \frac{dm\, r^2}{2} = \frac{MR^2}{4} =$$

Solid sphere:

"Solid sphere"





$y^2 + r^2 = R^2$

$r^2 = (R^2 - y^2)$

$\frac{4}{3}\pi R^3 \longrightarrow m$

$1 \longrightarrow \frac{m}{\frac{4}{3}\pi R^3}$

$(\pi r^2 dy) \longrightarrow \frac{m}{\frac{4}{3}\pi R^3} \cdot \pi r^2 \cdot dy$

$$\int dI = \int \frac{dm\, r^2}{2}$$

$$I_{cm} = \int \left(\frac{m}{\frac{4}{3}\cancel{\pi} R^3} \cdot \cancel{\pi} r^2 \cdot dy \right) \cdot r^2$$

$$I_{cm} = \frac{3}{4} \frac{m}{R^3} \int r^4 \cdot dy$$

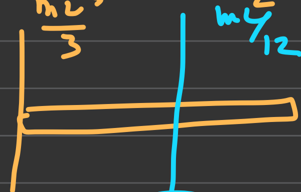
$$I_{cm} = 2\times\frac{3}{4}\frac{m}{R^3}\int_0^R (R^2-y^2)^2\,dy$$

$$I_{cm} = \frac{3}{2}\frac{m}{R^3}\int_0^R \left(R^4 + \frac{y^4}{4} + 2y^2R^2\right)\cdot dy$$

$$\boxed{I_{cm} = \frac{2}{5} m R^2}$$

Ans

(i) Rod →



(ii) Ring →

$\frac{mR^2}{2}$

$I_{cm} = mR^2$

(iii) disc: →

$I_{cm} = \frac{mR^2}{2}$

$I_D = \frac{mR^2}{4}$

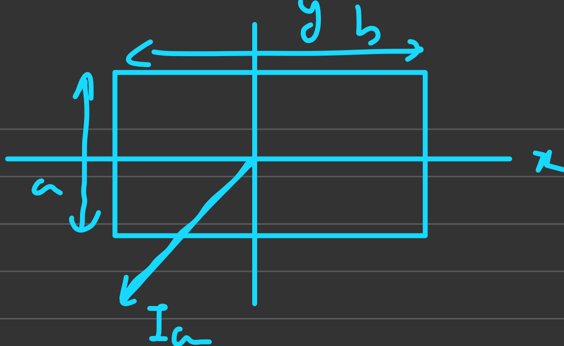
(iv) Solid sphere:

→ $\frac{2}{5}mR^2$

(v) Hollow sphere

$I_{cm} = \frac{2}{3}mR^2$

Ex 1



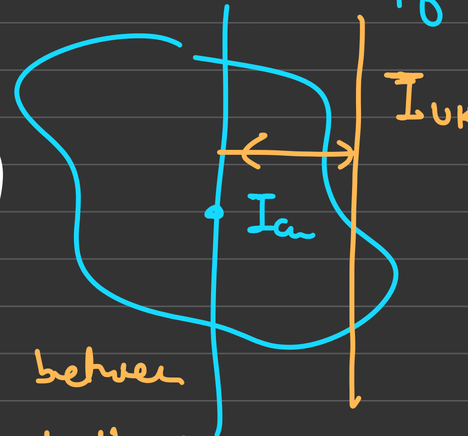
$$I_{cm} = \frac{m a^2}{12} + \frac{m b^2}{12}$$

Parallel axis and Perpendicular axis theorem:

① $\parallel$ axis theorem:

$$I = I_{cm} + m(d)^2$$

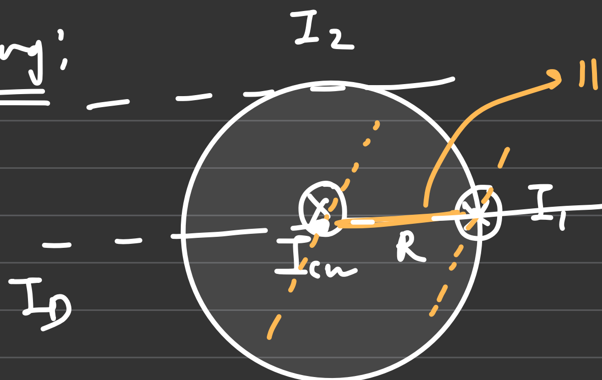
d : $\perp$ distance between C.M axis and $\parallel$ axis



"if we MoI the com then

$$I_{cm} = I + md^2$$ X wrong

(i) **Ring:**



$$I_{cm} = mR^2$$

$$I_1 = mR^2 + mR^2$$
$$= 2mR^2$$

$$I_2 = (I_D)_{cm} + md^2$$

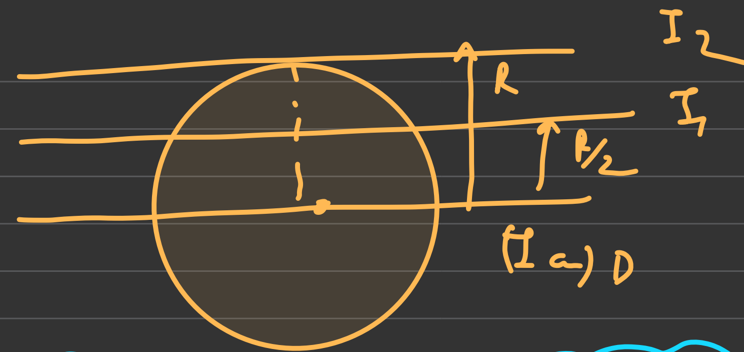
$$I_2 = \frac{mR^2}{2} + mR^2$$

$$I_2 = \left(\frac{3mR^2}{2}\right) \text{ Ans}$$

(ii) **disc:**

$$I' = \frac{mR^2}{4} + mR^2$$
$$= \frac{5}{4} mR^2 \text{ Ans}$$

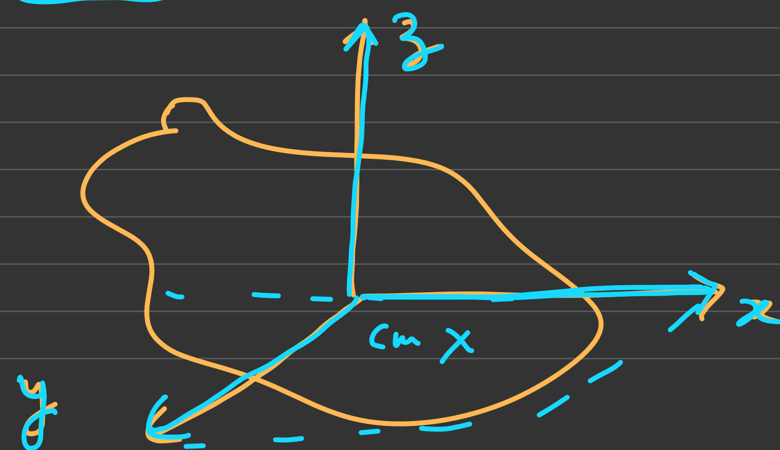
(iii) Solid Sphere:



$$\boxed{\begin{aligned} I_1 &= \frac{2}{5} mR^2 + m\left(R/2\right)^2 \\ I_2 &= 2/5\, mR^2 + mR^2 \end{aligned}}$$

$\perp$ axis theorem:

#

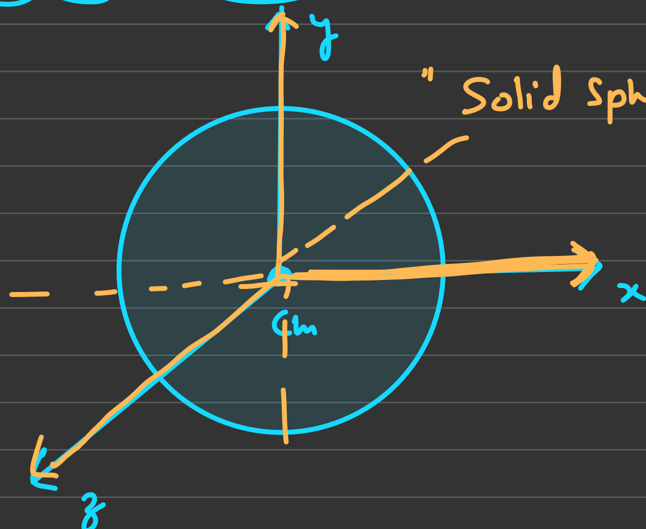


Valid only for
2D - object or lamina

$I_z = I_x + I_y$

{ x, y, z
mutually
⊥ }

Ex:



"Solid sphere"

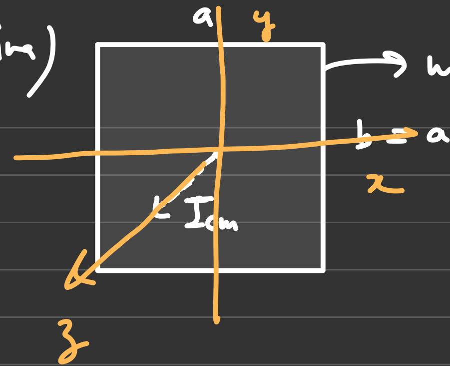
$I_y = I_x + I_z$

$= \frac{2}{5} mR^2 + \frac{2}{5} mR$

$I_y = \frac{4}{5} mR^2$ X

Ex #: Ring, disc, Lamina:

Square! (lamina)



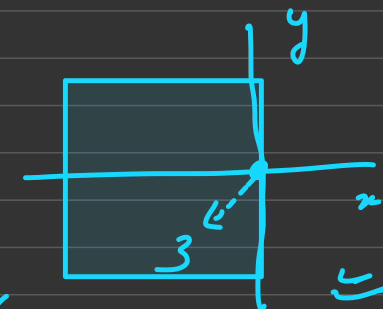
$$I_{cm} = \frac{ma^2}{12} + \frac{ma^2}{12}$$

$$I_{cm} = \frac{ma^2}{6}$$

$$(I_z) = I_x + I_y$$

$$I_z = 2I_x = 2I_y = \frac{ma^2}{6}$$

$$I_x = I_y = \frac{ma^2}{12}$$



$$I_z = I_x + I_y$$

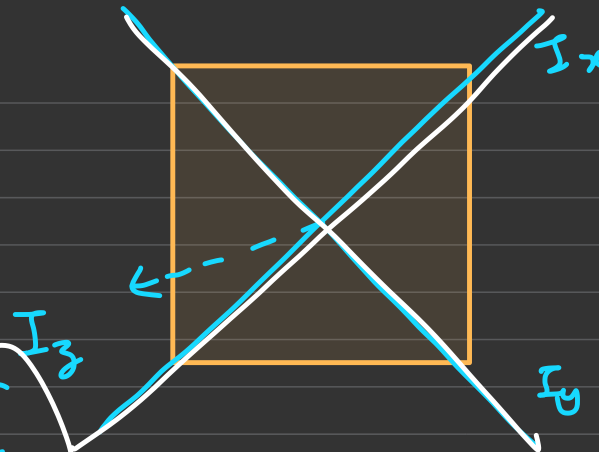
$$I_z = 2I_x = 2I_y \times$$

Square lamina!

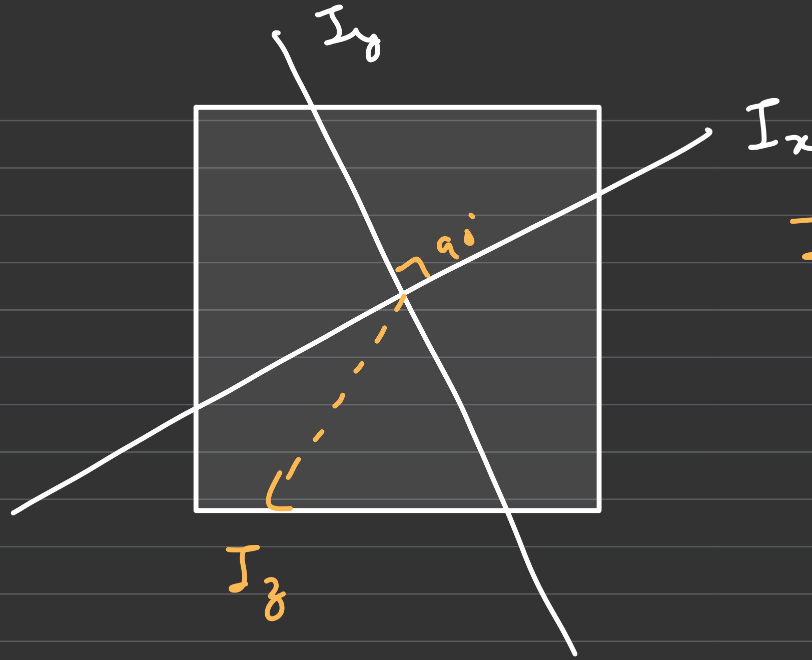
$$I_z = I_x + I_y$$

$$\frac{ma^2}{6} = 2I_x = 2I_y$$

$$\boxed{I_x = I_y = \frac{ma^2}{12}}$$



#



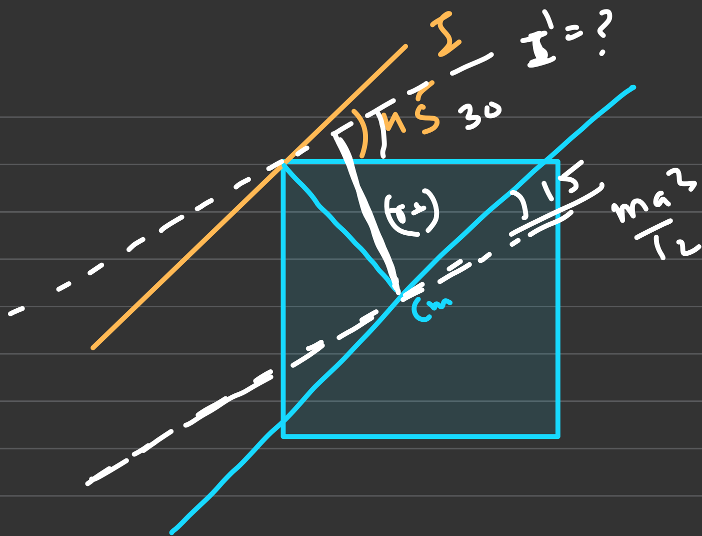
$I_z = I_x + I_y$

$I_x = I_y$

$I_z = 2I_x = 2I_y$

$I_x = I_y = \frac{I_z}{2}$

θ)



$$I_{cm} = \frac{ma^2}{12} + m\left(\frac{a}{\sqrt{2}}\right)^2$$

$$I_{cm} = \frac{ma^2}{12} + \frac{ma^2}{2}$$

$$I_{cm} = \frac{7ma^2}{12}$$

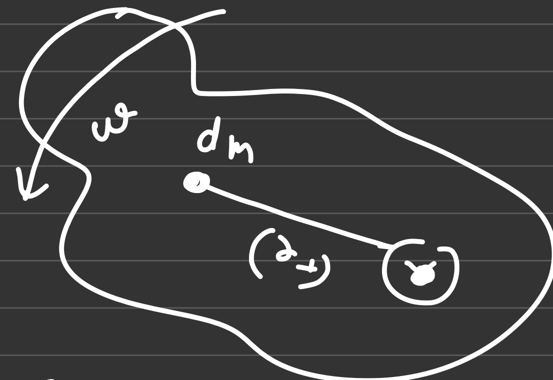
$$I_{cm} =$$

find I' which at 30° with edge?

$$\boxed{I' = \frac{ma^2}{12} + m(r_\perp)^2}$$

A

Kinetic Energy of body rotating about fixed:



$$dKE = \frac{1}{2}\, dm \times v^2$$

$$\int dKE = \int \frac{1}{2}\, dm\, (r_\perp \omega)^2$$

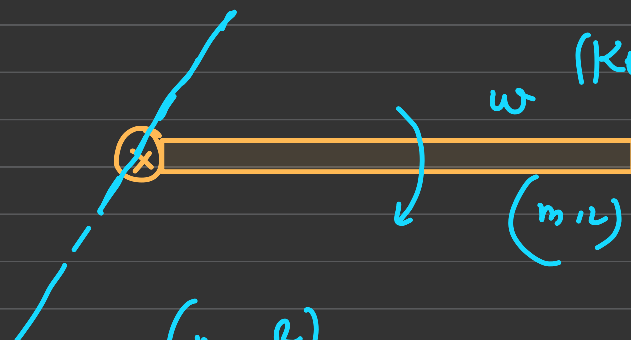
$$(KE_{body})_{FAOR} = \frac{1}{2}\,\omega^2 \left(\int dm\, (r_\perp)^2 \right)$$

$$= \frac{1}{2}\omega^2 I_{FAOR}$$

Direct formula of a body rotating about fixed axis $(KE) = \frac{1}{2} I_{FAOR} \times \omega^2$

body, FAOR

Ex# (1)



$$(KE)_{Rod} = \frac{1}{2} I_{FAOR} \times \omega^2$$

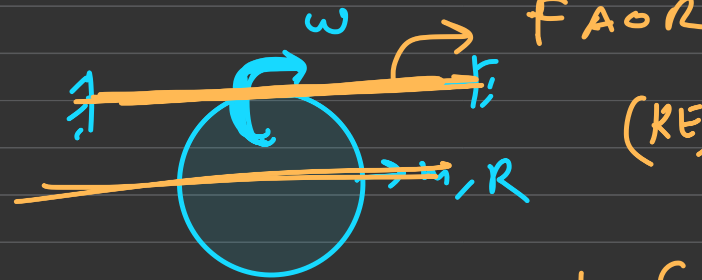
$$= \frac{1}{2}\left(\frac{mL^2}{3}\right) \times \omega^2$$

$$= \frac{mL^2\omega^2}{6}$$

(2) a)

$$(KE)_{Ring} = \frac{1}{2} mR^2 \times \omega^2 = \left(\frac{mR^2\omega^2}{2}\right) \quad \underline{Ans}$$

b)



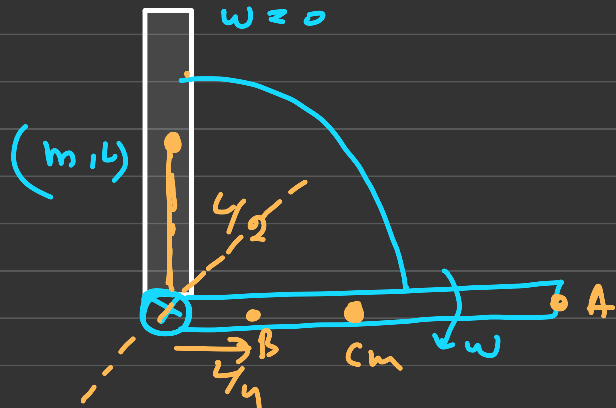
$$(KE)_{Ring} = \frac{1}{2} I_{FAOR} \times \omega^2$$

$$= \frac{1}{2}\left[\frac{3}{2} mR^2\right] \times \omega^2$$

$$= \frac{3}{4} mR^2\omega^2 \quad \underline{Ans}$$

Law of Conservation of Energy for Rigid bodies!

Q)



find ω of the rod when rod is horizontal?

loss in GPE = gain KE of Rod Rotating about fixed

$$mg\left(\frac{L}{2}\right) = \left\{\frac{1}{2} I_{FAOR} \times \omega^2\right\}_F - \{0\}$$

$$\cancel{m} g \frac{L}{\cancel{2}} = \frac{1}{\cancel{2}} \frac{\cancel{m} L^2}{3} \times \omega^2 - 0$$

$$\omega = \sqrt{\frac{3g}{L}}$$

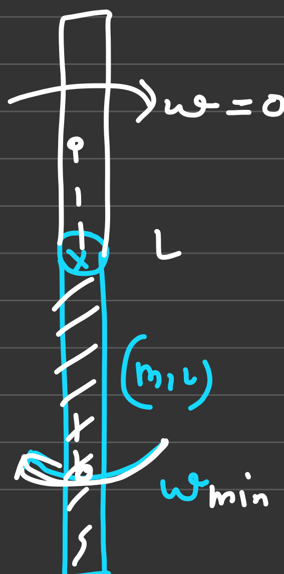
Ans

at this instant

\# $v_A = \omega L$

\# $v_{cm} = \frac{L}{2}\omega$

\# $v_B = \frac{L}{4}\omega$



θ #

find ω_{min} of rod for which rod just complete circular motion?

loss in KE of rod = gain PE

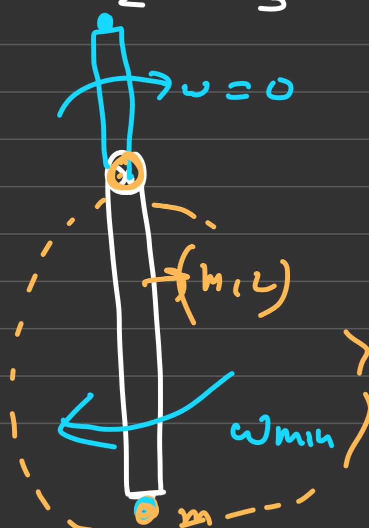
$$\Rightarrow \left\{ \frac{1}{2} I_{FAOR} \times \omega^2 - 0 \right\} = mgL$$

$$\Rightarrow \frac{1}{2} I_{FAOR} \times \omega^2 = mgL$$

$$\Rightarrow \frac{1}{2} \frac{\cancel{m}L^2}{3} \times \omega^2 = \cancel{m}gL$$

$$\omega_{min} = \sqrt{\frac{6g}{L}} \quad \text{Ans}$$

Q)



Wmin for which rod completes circular motion ?

loss KE of rod = gain GPE of rod

$$\left(\frac{1}{2} I_{FAOR} \times \omega^2 + \frac{1}{2} I_{FAOR} \times \omega^2\right) - (0 + 0) =$$

$$\left(\frac{1}{2} \frac{ml^2}{3} \times \omega^2 + \frac{1}{2} m l^2 \times \omega^2\right) = mgL + mg(2L)$$

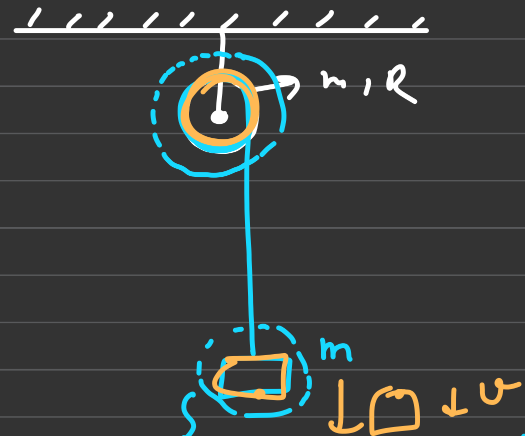
$$\frac{\cancel{m} l^2 \omega^2}{6} + \frac{\cancel{m} l^2 \omega^2}{2} = 3\cancel{m} g l$$

$$\frac{4}{6} L\omega^2 = 3g$$

$$\omega = \sqrt{\frac{9}{2}\frac{g}{L}}$$

$$\underline{\underline{Ans}}$$

θ)



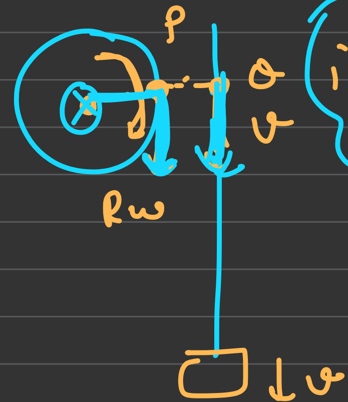
if System is released from rest and no Slipping between string and pulley then find $\underline{\underline{v}}$? of m if it goes down by h?

loss in GPE of m = gain KE of m and disc (Pulley)

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}\underline{I_{FAORX}\,\omega^2}$$

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}\frac{mR^2}{2}\times\omega^2 \quad \text{—(1)}$$

No-Slipping Condition :



if No-Slipping between two points then $\vec{v}_P = \vec{v}_Q$

$$v = R\omega$$

$$mgh = \frac{1}{2} m v^2 + \frac{1}{2} \frac{mR^2}{2} \left(\frac{v}{R}\right)^2$$

$$\cancel{m}gh = \frac{3}{4}\cancel{m}v^2$$

$$v = \sqrt{\frac{4gh}{3}}$$

\# Kinematic (FAOR)

\# Energy (FAOR)

\# Dynamics $\longrightarrow$ \# Torque \#

$\Downarrow$

Angular momentum

τ

Hero

2DTS

Level 1 $\rightarrow$

Level 2 $\leftarrow$

Angular momentum of bodies | point mass:

INE = A