POM-3

VMC Vidyamandir Classes SINCE 1986

IIT JEE | MEDICAL | FOUNDATION

: Heat transfer :

med

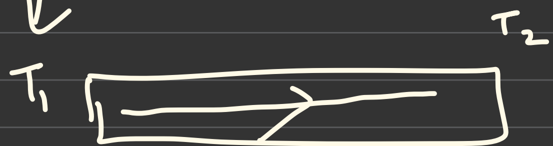
1) "Conduction" (mostly solid)

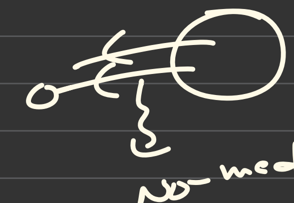
med

2) convection

3) Radiation

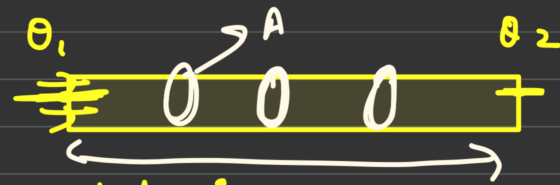
No-med





(i) Conduction :

$\theta_1 > \theta_2$



at steady state :

$$H = \frac{dQ}{dt} \propto \theta_1 - \theta_2$$

$H \propto A$ → Area of cross section ⊥ to Heat flow

$H \propto \frac{1}{l}$ → length of Heat flow

$$\frac{d\theta}{dt} = H = \frac{K(\theta_1 - \theta_2)A}{l}$$

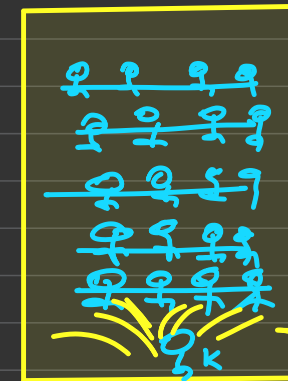
"Steady state"

Coefficient of thermal conductivity (This depends on material)

Temp of every cross-section is going to constant and rate of transfer of Heat is going to be constant

$\Downarrow$

"Steady State"



Constant rate

$\theta_A > \theta_B$

Unknown

Case I: Series

$H = ?$

$$H = \frac{K_1 A (\theta_A - \theta)}{l_1}$$

$$H = K_2 A (\theta - \theta_B)$$

$$\frac{H l_1}{K_1 A} = \theta_A - \cancel{\theta} \quad \text{—— (I)}$$

$$\frac{H l_2}{K_2 A} = \cancel{\theta} - \theta_B \quad \text{—— (II)}$$

$$H\left(\frac{l_1}{K_1 A} + \frac{l_2}{K_2 A}\right) = \theta_A - \theta_B$$

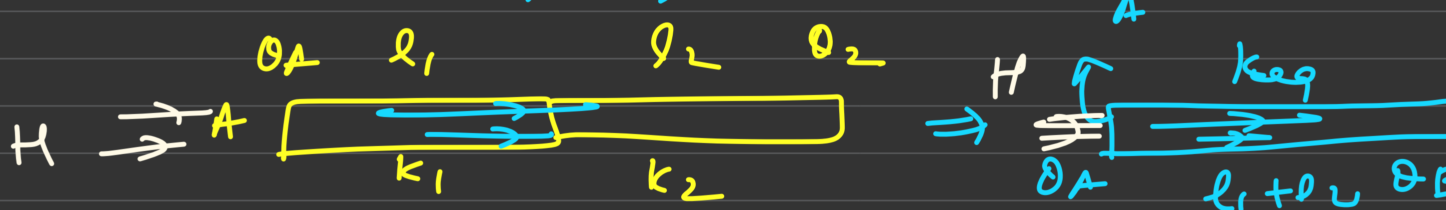
$$H = \frac{\theta_A - \theta_B}{\dfrac{l_1}{K_1 A} + \dfrac{l_2}{K_2 A}}$$

Multiple Slabs:

$$H = \frac{(\theta_A - \theta_B)A}{\frac{l_1}{K_1} + \frac{l_2}{K_2} + \cdots \frac{l_n}{K_n}}$$

#

$\theta_A > \theta_B$



$$H = \cancel{\frac{(\theta_A - \theta_B)A}{\frac{l_1}{K_1} + \frac{l_2}{K_2}}} \Rightarrow H = \cancel{\frac{K_{eq}(\theta_A - \theta_B)A}{l_1 + l_2}}$$

$$K_{eq} = \frac{l_1 + l_2}{\frac{l_1}{K_1} + \frac{l_2}{K_2}}$$

for same size slab $l_1 = l_1 = l$

$$K_{eq} = \frac{2l_1}{\frac{l}{K_1} + \frac{l}{K_2}} = \frac{2K_1K_2}{K_1+K_2}$$

for three slabs:

$$K_{eq} = \frac{l_1 + l_2 + l_2}{\frac{l_1}{K_1} + \frac{l_2}{K_2} + \frac{l_3}{K_3}}$$

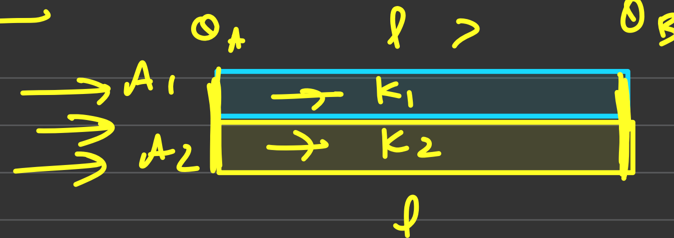
$$K_{eq} = \frac{l + l + l}{\frac{l}{K_1} + \frac{l}{K_2} + \frac{l}{K_3}}$$

$$K_{eq} = \frac{3K_1K_2K_3}{K_1K_2 + K_2K_3 + K_1K_3}$$

→ for same slab

Parallel Combination

Case II :



$$H = H_1 + H_2$$

$$H = \frac{K_1 A_1 (\theta_A - \theta_B)}{l} + \frac{K_2 A_2 (\theta_A - \theta_B)}{l}$$

Steady state

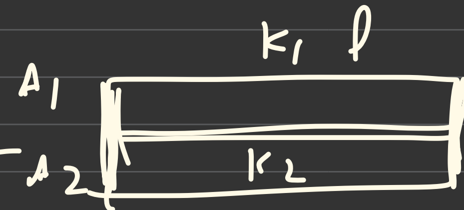
$$H = \left(\frac{\theta_A - \theta_B}{l}\right)\left(K_1 A_1 + K_2 A_2\right)$$

for n-Slabs!

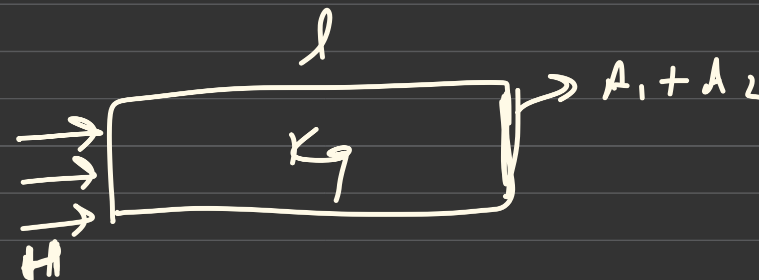
$$H = \left(\frac{\theta_A - \theta_B}{l}\right)\left(K_1 A_1 + K_2 A_2 + K_3 A_3 + \dots K_n A_n\right)$$

find K_{eq}?

for two slabs:



$$H = \frac{(\theta_A - \theta_B)}{l}(K_1 A_1 + K_2 A_2) \quad \text{--- (I)}$$



$$H = \frac{K_{eq}(A_1 + A_2)(\theta_A - \theta_B)}{l} \quad \text{--- (II)}$$

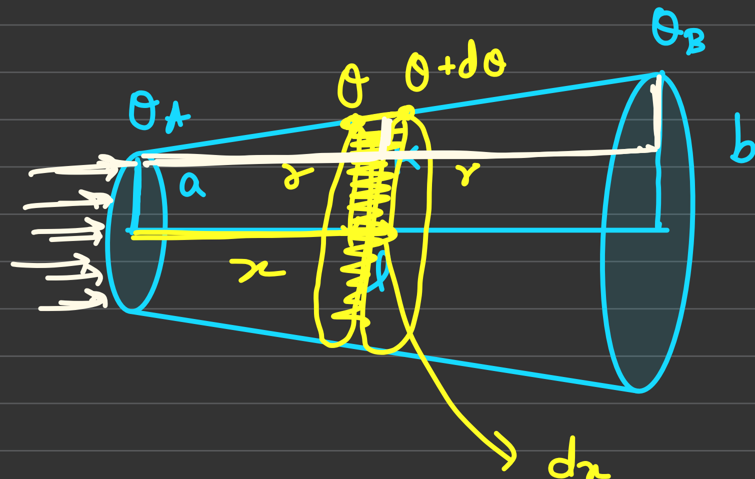
$$K_{eq}(A_1 + A_2) = K_1 A_1 + K_2 A_2$$

$$K_{eq} = \frac{K_1 A_1 + K_2 A_2}{A_1 + A_2}$$

if $A_1 = A_2 = A$ → for same area of cross-section

$$K_{eq} = \frac{K_1 + K_2}{2}$$

Frustum: (Heat transfer)



$$\frac{r-a}{x} = \frac{b-a}{l}$$

$$r = x\left(\frac{b-a}{l}\right) + a$$

$$H = \frac{K\pi r^2(\theta - \theta - d\theta)}{dx}$$

$$H = \frac{K\pi r^2(-d\theta)}{dx}$$

$$H\int \frac{dx}{r^2} = k\pi \int_{\theta_A}^{\theta_B} (-d\theta)$$

$$H \int \frac{dx}{\left\{\left(\frac{b-a}{l}\right)x + a\right\}^2} = k\pi \left(-\theta\right)_{\theta_A}^{\theta_B}$$

$$H \int \left\{Kx + a\right\}^{-2} \cdot dx = -k\pi\left(\theta_B - \theta_A\right)$$

$$-(H)\left[\frac{1}{\left(\frac{b-a}{l}\right)x + a}\right]_0^l \times \frac{l}{(b-a)} = k\pi\left(\theta_A - \theta_B\right)$$

$$-\left(\frac{Hl}{b-a}\right)\left[\frac{1}{b} - \frac{1}{a}\right] = k\pi\left(\theta_A - \theta_B\right)$$

$$\frac{H\ell}{(b-a)}\left(\frac{1}{a}-\frac{1}{b}\right) = k\pi(\theta_A - \theta_B)$$

$$\frac{H\ell}{(\cancel{b-a})}\left(\frac{\cancel{b-a}}{ab}\right) \quad k\pi(\theta_A - \theta_B)$$

$$\frac{H\ell}{ab} = k\pi(\theta_A - \theta_B)$$

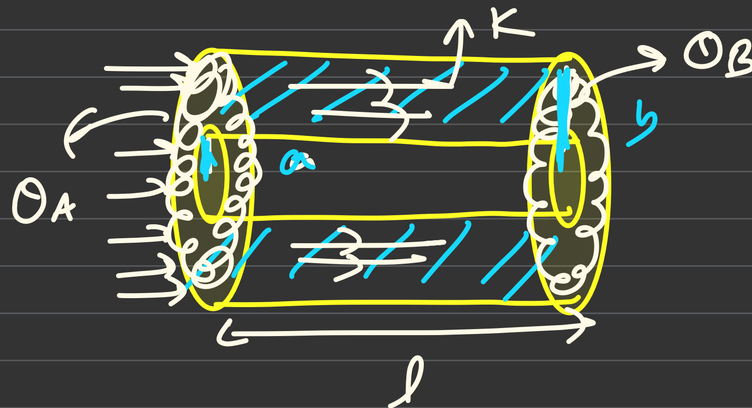
Remember

$$H = \frac{k\pi\, ab(\theta_A - \theta_B)}{\ell}$$

Cylindrical shell:

$\theta_A > \theta_B$

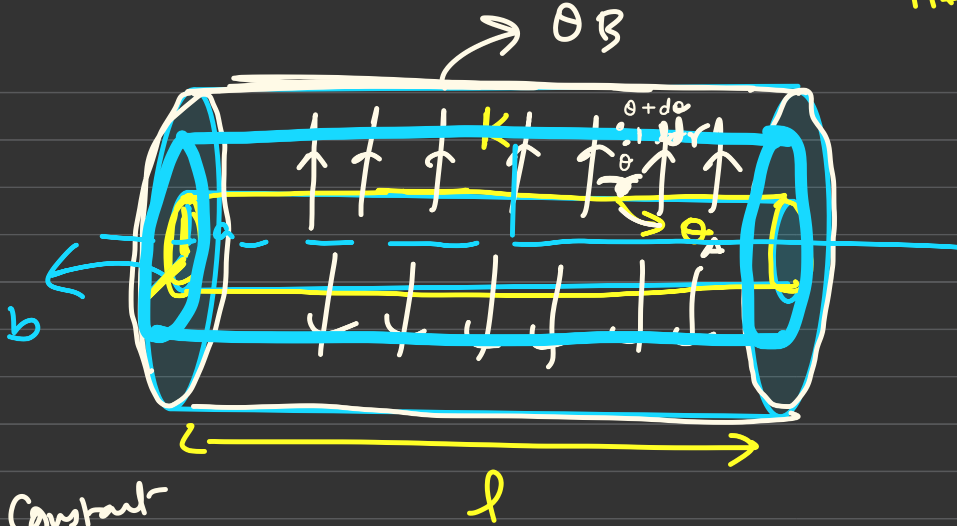
a)



$$H = \frac{K\pi(b^2 - a^2)(\theta_A - \theta_B)}{l}$$

b)

' Heat is flowing along radius "



Constant

$$H = \frac{K\, 2\pi r l\, (\theta - \theta - d\theta)}{dr}$$

$$H \cdot \int_a^b \frac{dr}{r} = K \cdot 2\pi l \int_{\theta_A}^{\theta_B} -d\theta$$

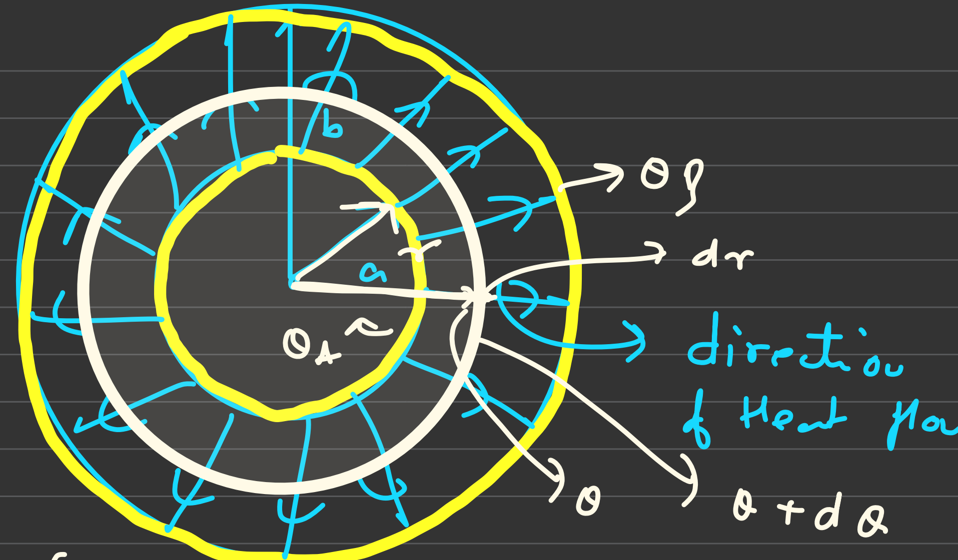
$$H\left[\ln\left(\frac{b}{a}\right)\right] = -k \cdot 2\pi l \,(\theta)_{\theta_A}^{\theta_B}$$

$$H \ln \frac{b}{a} = k \cdot 2\pi l \,(\theta_A - \theta_B)$$

$$H = \frac{2\pi k l\,(\theta_A - \theta_B)}{\ln(b/a)}$$

Rank

Spherical shell:



$$H = \frac{K\, 4\pi r^2 (\theta - \theta - d\theta)}{dr}$$

$$H \int_{r=a}^{r=b} \frac{dr}{r^2} = 4\pi K \int_{\theta_A}^{\theta_B} -d\theta$$

$$H\left[-\frac{1}{r}\right]_a^b = 4\pi k(\theta_A - \theta_B)$$

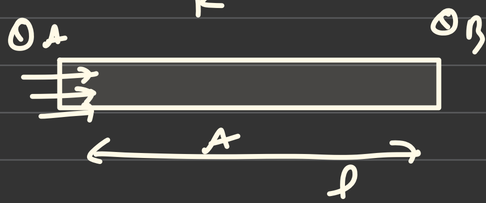
$$H\left[\frac{1}{a} - \frac{1}{b}\right] = 4\pi k(\theta_A - \theta_B)$$

$$H\left[\frac{b-a}{ab}\right] = 4\pi k(\theta_A - \theta_B)$$

Ans

$$H = \frac{4\pi ab k}{(b-a)}(\theta_A - \theta_B)$$

Slab



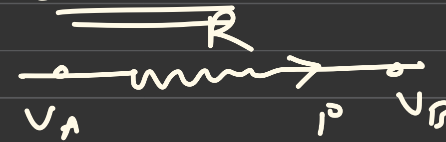
$$H = \frac{KA(\theta_A - \theta_B)}{l}$$

$$H = \frac{\theta_A - \theta_B}{l/KA} = i = \frac{V_A - V_B}{R}$$

$$R = \frac{l}{KA}$$

for slab thermal resistance

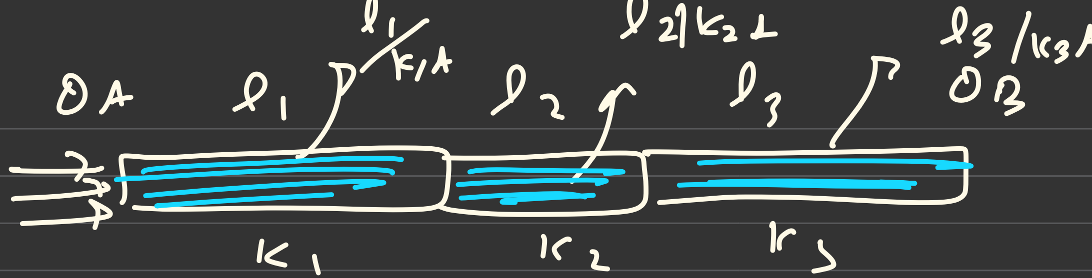
Ohm's law $V_A > V_B$



$$V_A - V_B = iR$$

$$H = \frac{\theta_A - \theta_B}{R_{eq}} \quad \text{also}$$

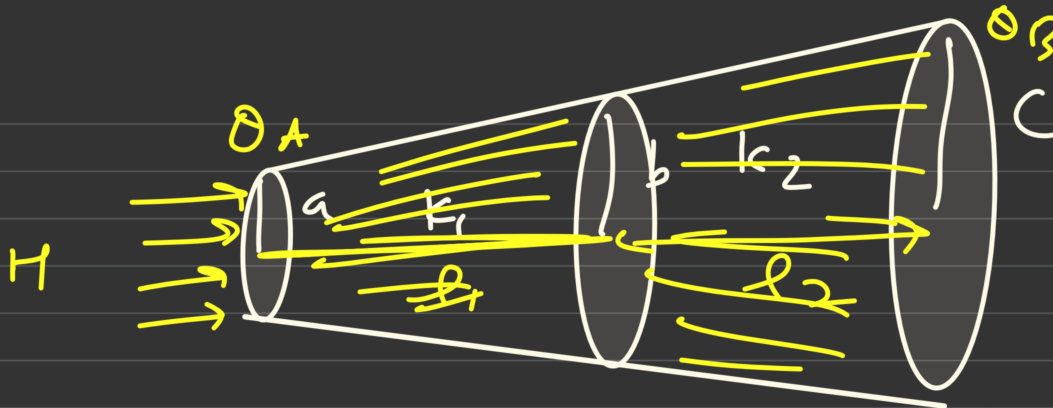
#

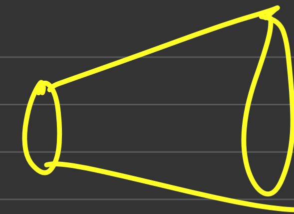


$$H = \frac{\theta_A - \theta_B}{\dfrac{l_1}{k_1 A} + \dfrac{l_2}{k_2 A} + \dfrac{l_3}{k_3 A}}$$

A

#



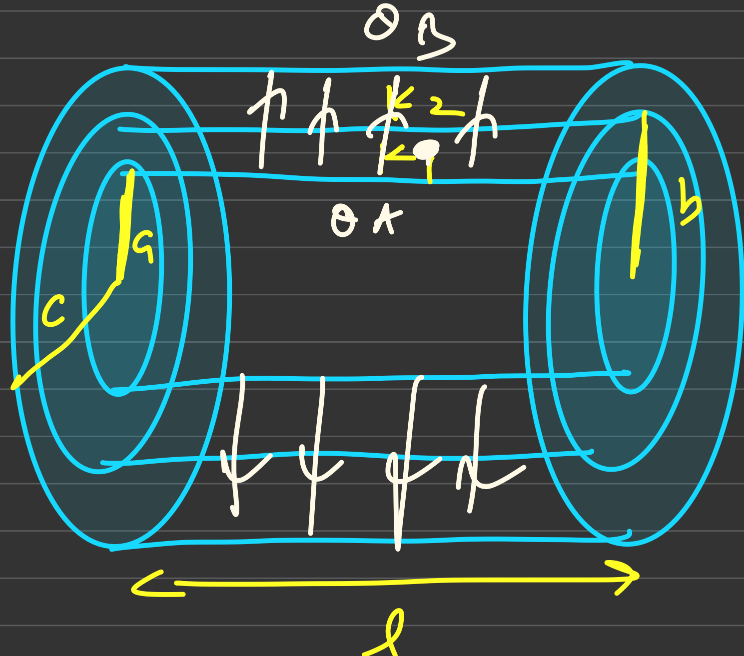


$$H = \frac{\theta_A - \theta_B}{\left(\frac{l}{K\pi ab}\right)}$$

A

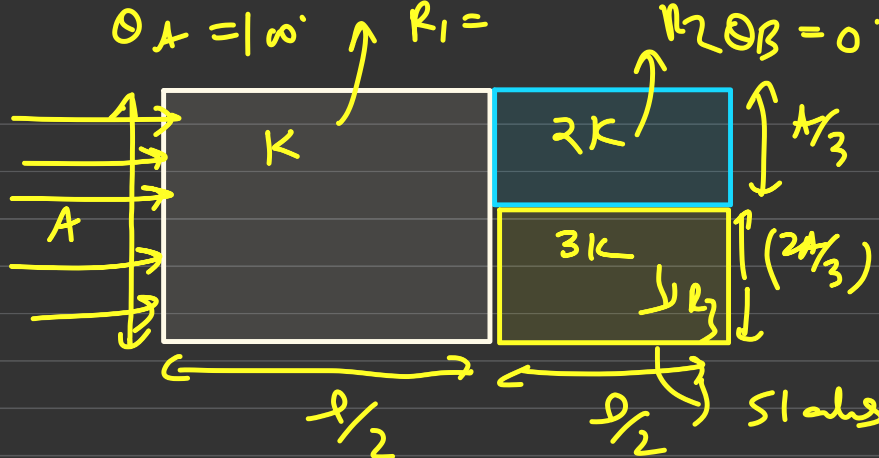
$$H = \frac{\theta_A - \theta_B}{\frac{l_1}{K_1 \pi ab} + \frac{l_2}{K_2 \pi bc}}$$

for two concentric cylindrical shells:

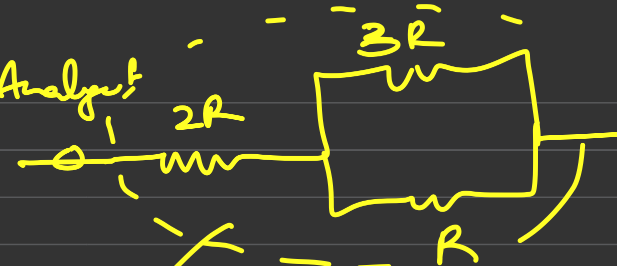


$$H = \frac{\theta_A - \theta_B}{\frac{\ln(b/a)}{2\pi k_1 l} + \frac{\ln(c/b)}{2\pi k_2 l}}$$

Q)



Analogue



$$R_1 = \frac{l}{2KA} = 2R$$

$$R_2 = \frac{l \cdot 3}{2(2K)A} = \frac{3l}{4KA} = 3R$$

$$R_3 = \frac{l \cdot \cancel{3}}{2\cdot\cancel{3}K\cdot 2A}$$

$$R_3 = \frac{l}{4KA} = R$$

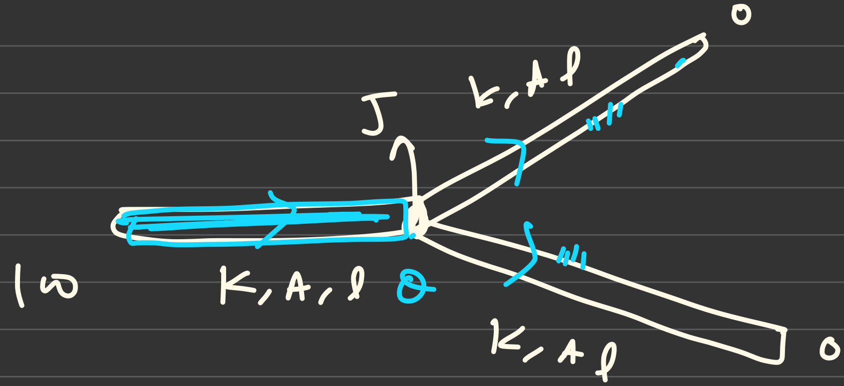
$$H = \frac{100 - 0}{(R_{eq})}$$

$$H = \frac{1600KA}{11l}$$

Ans

Q#

find temp of J ?



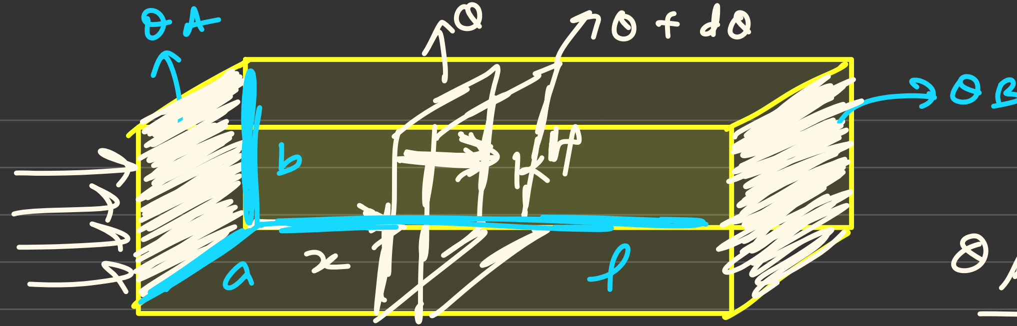
$$H = H_1 + H_2$$

$$\frac{\cancel{K}A(100-\theta)}{\cancel{l}} = \frac{2\cancel{K}A(\theta-0)}{\cancel{l}}$$

$$100 - \theta = 2\theta \Rightarrow 3\theta = 100$$

$$\theta = 33.3^\circ \ \text{Ans}$$

K - Variable :



$\theta_A > \theta_B$

$$K = K_0\left(1 + \frac{x}{l}\right)$$

$x = 0 \Rightarrow K_0$
$x = l \Rightarrow 2K_0$

$$H = \frac{K_0\left(1 + \frac{x}{l}\right)(-d\theta)\, ab}{dx}$$

$$H \int_{x=0}^{x=l} \frac{dx}{\left(1 + x/l\right)} = K_0\, ab \int_{\theta_A}^{\theta_B} -d\theta$$

$$\Rightarrow \quad H l \left[\ln\left(1 + \frac{x}{l}\right) \right]_0^l = k_0 a b \left[-\theta \right]_{\theta_A}^{\theta_B}$$

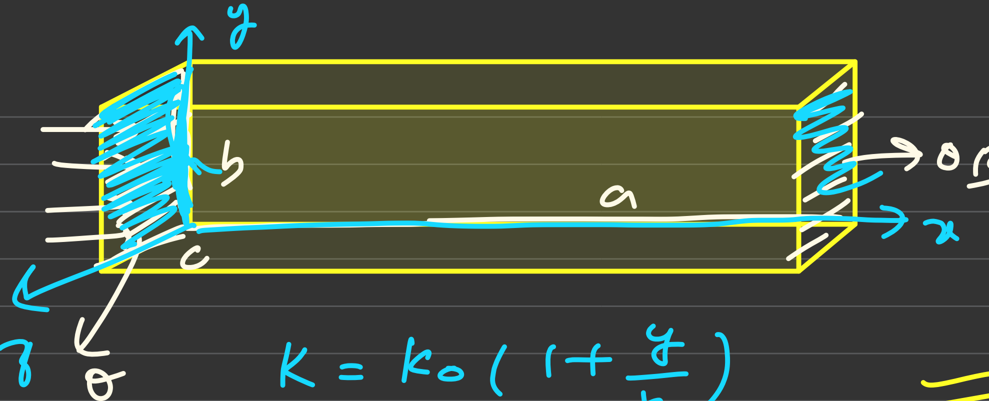
$$\Rightarrow \quad H l \left[\ln(2) - 0 \right] = k_0 a b \left(\theta_A - \theta_B \right)$$

$$H l \ln(2) = k_0 a b \left(\theta_A - \theta_B \right)$$

$$\boxed{H = \frac{k_0 a b}{(\ln 2)\, l} \left(\theta_A - \theta_B \right)} \quad \text{Ans}$$

Home work :

find K = ?



$$K = K_0\left(1 + \frac{y}{b}\right)$$

Home → Level 1 } DTS 1, 2, 3
Level 2

module → INE #1