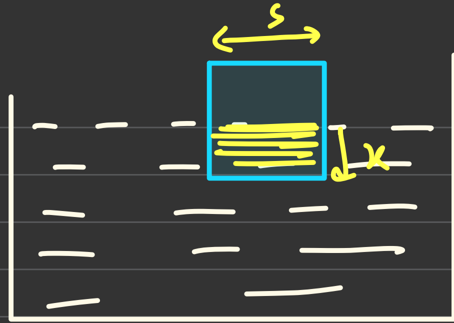
Liquid 3

VMC Vidyamandir Classes SINCE 1986

IIT JEE | MEDICAL | FOUNDATION

Q)

Case I: No-accn



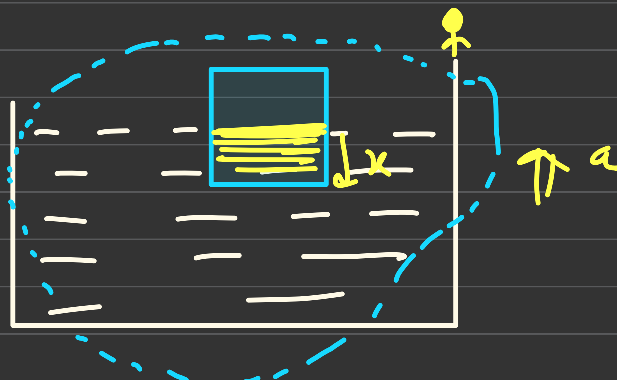
cubical block of sides

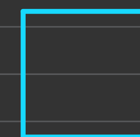
$$\Rightarrow mg = V_{im}\,\rho\,g$$

$$m\cancel{g} = (s^2 x)\,\rho\,\cancel{g} \quad \text{— (I)}$$

$$\Rightarrow \quad V_{im} = \frac{m}{\rho}$$

Case II: Accelerating upward



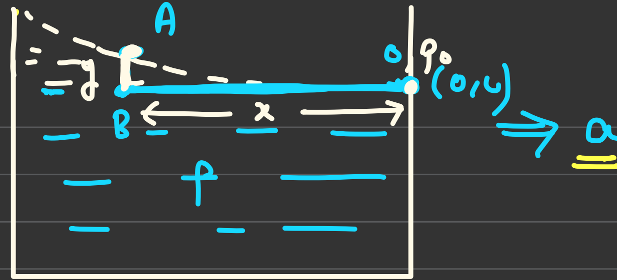


$$g_{eff} = -g - a$$
$$= -(g+a)$$
$$g_{eff} = (g+a) \Downarrow$$

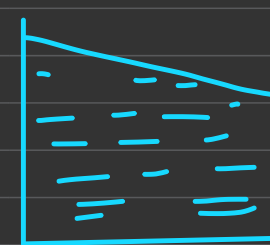
$$m\,\cancel{g_{eff}} = V_{im}\,\rho\,\cancel{g_{eff}}$$

$$\Rightarrow V_{im} = \frac{m}{\rho} = \quad \text{— (II)}$$

Case I:



find Locus of curve ?



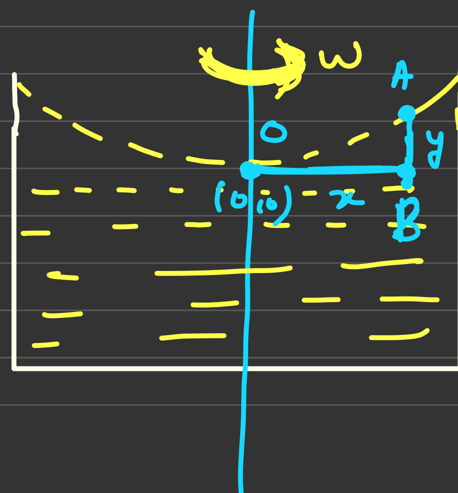
$$P_B - P_0 = \rho a x$$

$$\cancel{P_0} + \rho g y - \cancel{P_0} = \rho a x$$

$$\boxed{y = \frac{a x}{g}}$$

Straight line equation

Case II:



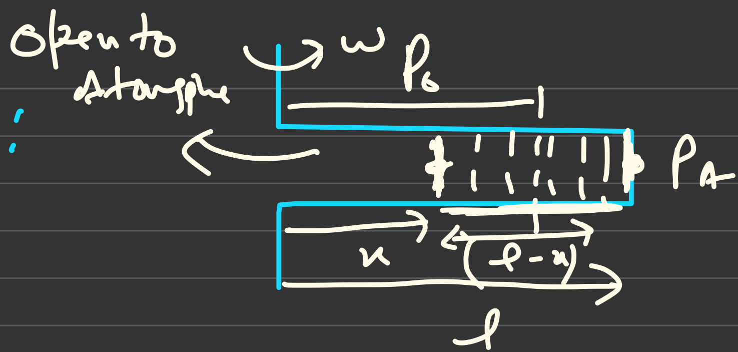
$$P_B - P_0 = \rho (a_c) x$$

$$P_B - P_0 = \rho \left(\omega^2 \frac{x}{2}\right) x$$

$$P_B - P_0 = \rho \frac{\omega^2 x^2}{2}$$

$$\cancel{P_0} + \rho g y - \cancel{P_0} = \rho \frac{\omega^2 x^2}{2}$$

Case III :



$$\frac{P}{\rho g}y = \rho \frac{\omega^2 x^2}{2}$$

$$y = \frac{\omega^2 x^2}{2g}$$

Parabola

$$P_A - P_0 = \rho\, a_c\, (l-x)$$

$$P_A - P_0 = \rho \omega^2 \left(\frac{l-x}{2} + x\right)(l-x)$$

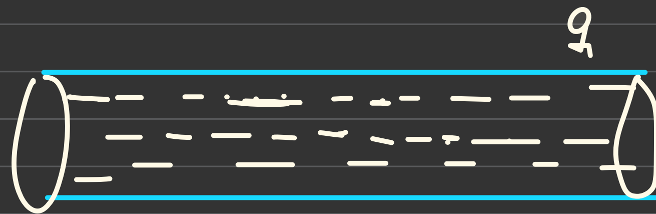
$$P_A - P_0 = \rho \omega^2 \left(\frac{l+x}{2}\right)(l-x)$$

$$P_A - P_0 = \rho \omega^2 \left(\frac{l^2 - x^2}{2}\right) \quad \text{Ans}$$

Ideal liquid:

1. Non - Viscous
2. incompressible
3. Streamline flow (Smooth)

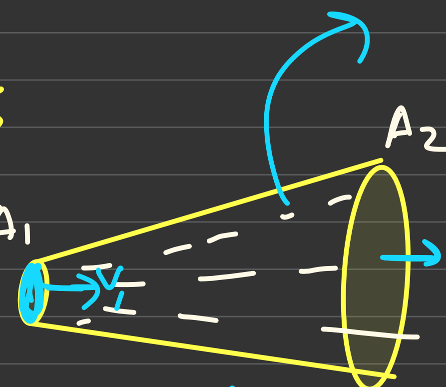
Hydrodynamics



"liquid is moving w.r.t container"

(a) Equation of Continuity :

"Volumetric flow of Ideal liquid must be same"



Q = const
↓
vol. flow

$$A_1 V_1 = A_2 V_2$$

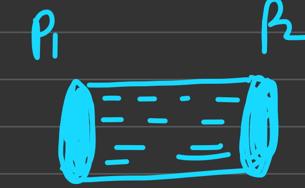
6

Bernoullies theorem:

$v_2 > v_1$

A_2, v_2

$P_1 > P_2$



$w_{p.D} = (P_1 A - P_2 A)\, dx$

$w_{p.d} = (P_1 - P_2) A\, dx$

$w_{p.d} = \underline{(P_1 - P_2)}\, \underline{dv}$

$w_{p.d} = (P_1 - P_2)\, \underline{\Delta V}$

work done due to Press. diff = Gain GPE of liquid + gain in KE

$$(P_1 - P_2)\Delta V = mg(h_2 - h_1) + \frac{1}{2}m(v_2^2 - v_1^2)$$

$$\left\{ (P_1 - P_2) \cancel{\Delta V} = (\cancel{\Delta V}\rho) g (h_2 - h_1) + \frac{1}{2}(\cancel{\Delta V}\rho)(v_2^2 - v_1^2) \right.$$

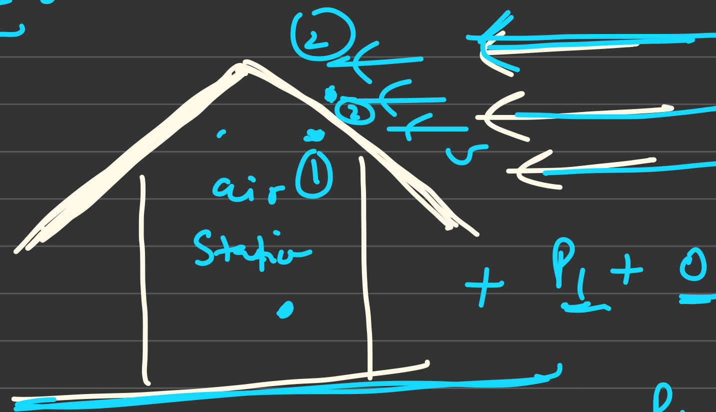
$$P_1 - P_2 = \rho g (h_2 - h_1) + \frac{1}{2}\rho (v_2^2 - v_1^2)$$

$$\underbrace{P_1}_{\text{Pre. H.}} + \underbrace{\rho g h_1}_{\text{Pot. Head}} + \underbrace{\frac{1}{2}\rho v_1^2}_{\text{Kine. Head}} = P_2 + \rho g h_2 + \frac{1}{2}\rho v_2^2$$

Pres + Pot + Kine = const

Ex# Real life:

1#

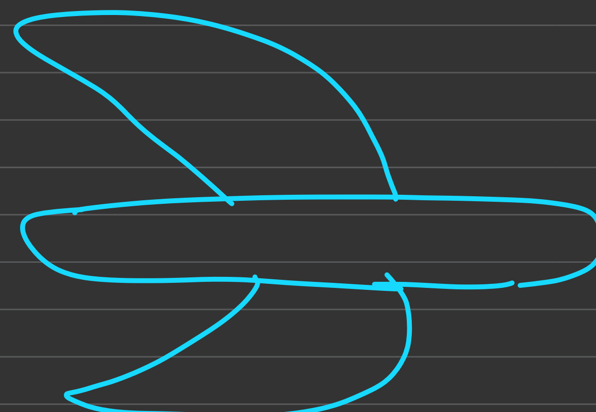


$$+ P_1 + 0 = \frac{1}{2}\rho v_1^2 + P_2$$

$$P_1 = \frac{1}{2}\rho v_1^2 + P_2$$

$$P_1 > P_2$$

2#



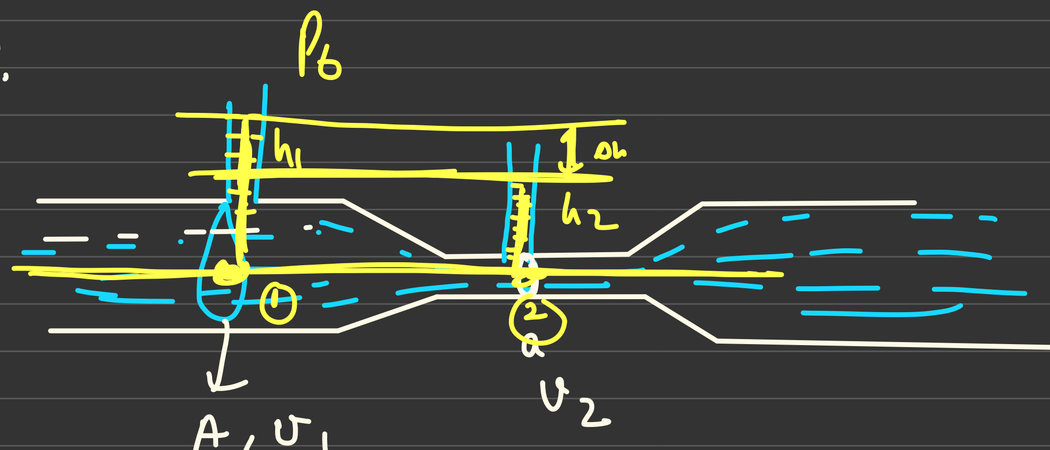
aeroplane

Instruments:

① Venturimeter:

is a device which measure volumetric flow of liquid / air

a) liquid:



$$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$$

$$\cancel{P_0} + \rho g h_1 + \frac{1}{2} \rho v_1^2 = \cancel{P_0} + \rho g h_2 + \frac{1}{2} \rho v_2^2$$

$$\cancel{\rho} g(h_1 - h_2) = \frac{1}{2} \cancel{\rho} (v_2^2 - v_1^2)$$

$$2g(h_1 - h_2) = v_2^2 - v_1^2 \quad \text{---} \ (1)$$

$$A \times v_1 = a v_2$$

$$2g(h_1 - h_2) = \left(\frac{A v_1}{a}\right)^2 - v_1^2$$

$$2g(h_1 - h_2) = \frac{A^2 v_1^2}{a^2} - v_1^2$$

$$A v_1 = A \sqrt{\frac{2g(h_1 - h_2)}{A^2/a^2 - 1}}$$

$$Q = \frac{A\,a}{}\sqrt{\frac{2g\,(\Delta h)}{A^2 - a^2}} \rightarrow$$

Δh = Reading

b) for gases: (To measure flow of gas)/air

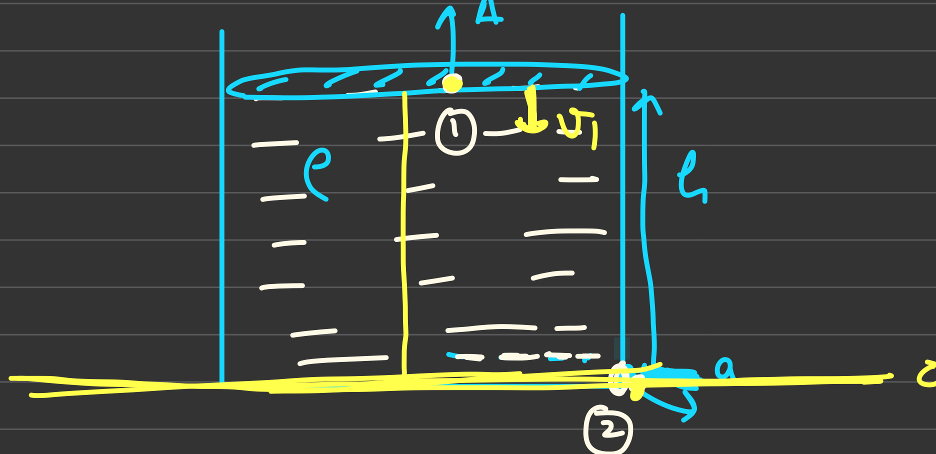
\# module \# H.W

Exam

c) Pitot Tube H.W

Velocity of efflux :

$a << A$



1 & 2 Apply Bernoulli's theorem

$$\cancel{P_0} + \rho g h + \frac{1}{2}\rho v_1^2 = \cancel{P_0} + 0 + \frac{1}{2}\rho v_2^2$$

$v_2 = v_{efflux}$

$$\cancel{\rho} g h + \frac{1}{2}\cancel{\rho} v_1^2 = \frac{1}{2}\cancel{\rho} v_2^2$$

$$\Rightarrow \quad 2gh = v_2^2 - v_1^2$$

$$A \times v_1 = a \times v_2$$

$$2gh = v_2^2 - \left(\frac{a v_2}{A}\right)^2$$

$$V_2 = v_{eff} = \sqrt{\frac{2gh}{1 - a^2/A^2}}$$

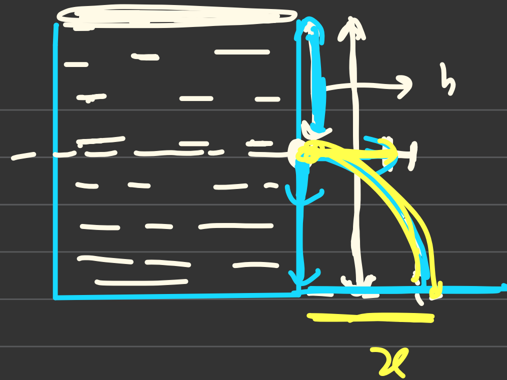
if $a \sim A$

$$1 >> \frac{a^2}{A^2}$$

if $a << A$

$$v_{eff} = \sqrt{2gh}$$

(b)



$a << A$

$$V_{eff} = \sqrt{2gh}$$

$$-(H-h) = -\frac{1}{2} g t^2$$

$$t = \sqrt{\frac{2(H-h)}{g}}$$

Range: $x = \sqrt{2\cancel{g}h} \times \sqrt{\frac{2(H-h)}{\cancel{g}}}$

$$x = \text{Range} = \sqrt{4h(H-h)}$$

find Height of Hole for which Range is maximum?

$$x = \sqrt{4h(H-h)}$$

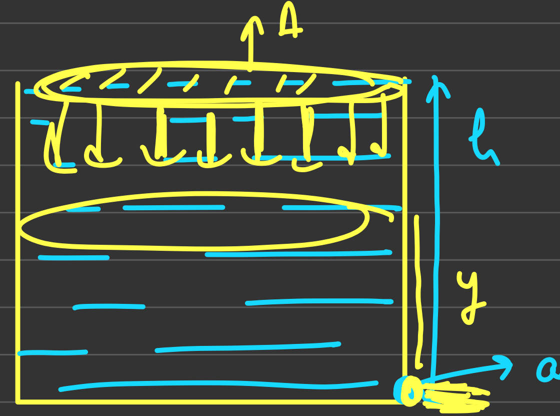
$f(h) = h(H-h)$

$H - 2h = 0$

$h = H/2$

maxim Range will be at $h = H/2$

c)



$a << A$

$t = h$

find time after which tank will be empty?

$$v = \sqrt{2gy} \quad — (1)$$

$$Q = \left(a\sqrt{2gy}\right) = A \times v$$

$$a\sqrt{2gy} = A \times v$$

$$v = -\frac{dy}{dt}$$

$$a\sqrt{2gy} = -A\frac{dy}{dt}$$

$$\int_{t=0}^{t=t} dt = \int_{h}^{0} -\frac{A}{a}\frac{dy}{\sqrt{2gy}}$$

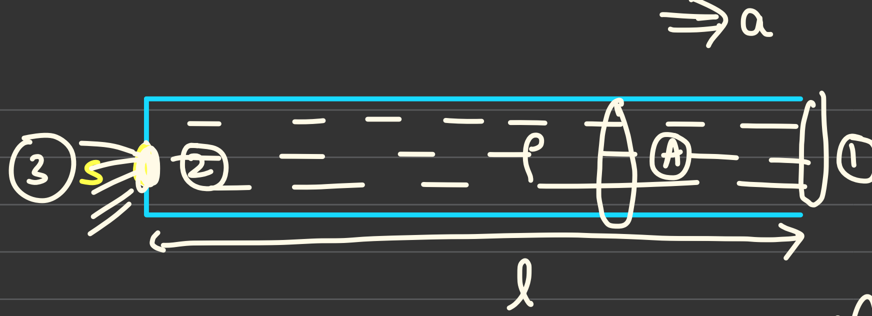
$$(t - 0) = -\frac{A}{a\sqrt{2g}} \int_h^0 \frac{dy}{\sqrt{y}}$$

$$t = -\frac{A}{a\sqrt{2g}} \left[\frac{\sqrt{y}}{(1/2)} \right]_h^0$$

$$t = +\frac{2A}{a\sqrt{2g}} \sqrt{h}$$

$$t = \frac{A}{a} \sqrt{\frac{2h}{g}}$$

Q)



find velocity with which liquid is coming out? at this inlet

we can not apply bernoulli's theo on points (1) and (3) as there will be some work done due to ext force "

But we can apply bernoulli's theorem at (2) and (3) as there will almost negligible work done due to ext

∵ they are very to each other

$a \times v_{eff} = A \times v$

$v = \dfrac{a\, v_{eff}}{A}$

$a << A$

$P_2 - P_0 = \rho a x$

$\Rightarrow a$

$P_2 - P_0 = \rho a l$

$P_2 = P_0 + \rho a l$ — (1)

Apply Bernoulli theorem at this

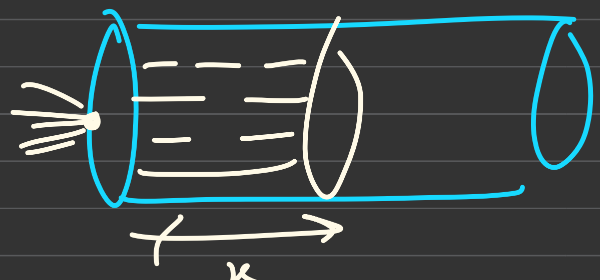
$P_0 + \frac{1}{2} \rho v_{eff}^2 = P_2 + \frac{1}{2} \rho v^2$

$$\cancel{P_0} + \frac{1}{2}\rho v_{eff}^2 = \cancel{P_0} + \rho a l + \cancel{\frac{1}{2}\rho v^2}$$

$$\frac{1}{2}\cancel{\rho} v_{eff}^2 = \cancel{\rho} a l$$

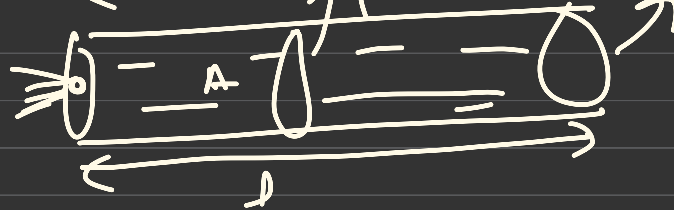
$$v_{eff} = \sqrt{2al}$$

#



$$v_{eff} = \sqrt{2ax}$$

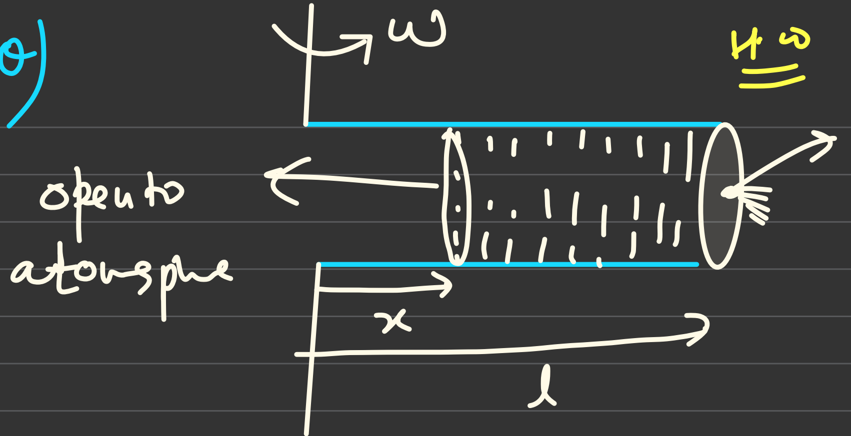
Q) small hole



find time after which tank will be empty?

(H.W)

Q)



H.W

small hole

then find velocity with which liquid is coming out

{ Homework DPS #3 → Level 1, → Level 2 }