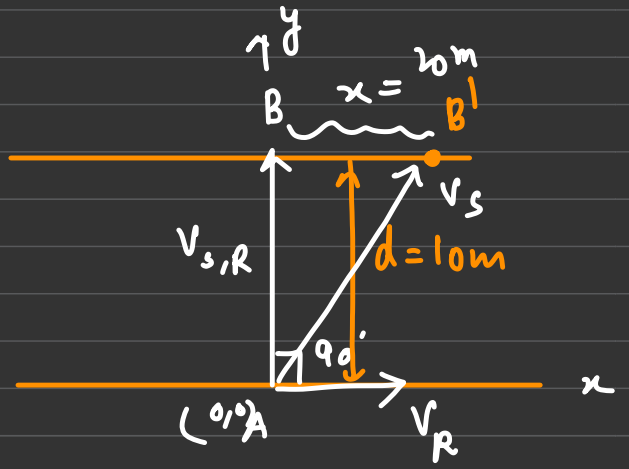
Introduction to Vector and Forces -5

Illustration - 31 *A person can swim in still water at the rate of 1.0 Km/hr. He tries to cross a river by swimming perpendicular to the river flowing at the rate of 2 Km/hr. If the width of the river is 10 m, find the location of the point where he lands on the other side of the river. Also find the time taken by him to cross the river.*

#



$V_{S,R}$ 1 Km/hr

V_R = 2 Km/hr

$\vec{V_{SR}} = \vec{V_S} - \vec{V_R}$

$\vec{V_S} = \vec{V_{SR}} + \vec{V_R}$

$V_{SR} = 1\,Km \times \frac{5}{18}$ m/s

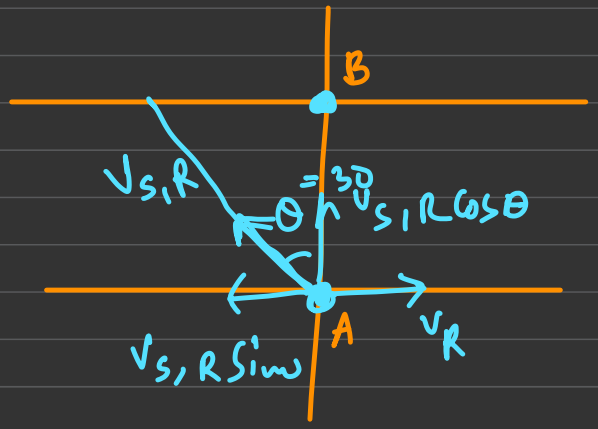
$x = \frac{2}{36} \times 2 \times 5$

$x = 20m$

$t_{min} = \frac{d}{V_{SR}} = \frac{16m \times 18}{5}$

$= 36$ sec

Illustration - 32 *A man who can swim with speed of 1 Km/hr in still water wants to cross a river. He starts from a point A from the river bank and wants to reach the point B which is directly opposite to A. In what direction should he try to swim ? Speed of the river flow is 0.5 Km/hr. Also find the time taken to cross the river if width is 10 m.*



$\overrightarrow{V_{S,R}} = 1\,km/h$

(min distance)

$\Rightarrow V_{S,R}\sin\theta = V_R$

$\sin\theta = \frac{0.5\,km/h}{1\,Km/h}$

$\sin\theta = \frac{1}{2}$

$\theta = 30$

$t = \frac{d}{V_{S,R}\cos\theta} = \frac{\frac{2}{10}\,Km}{\frac{1}{2}\times\frac{\sqrt{3}}{2}}$

$= \left(\frac{36\times 2}{\sqrt{3}}\right)$ sec

(2) Aeroplane – wind :

$\vec{V_{A,w}}$ = Velocity of aeroplan w.r.t wind or in still wind

$\vec{V_{A,w}} = \vec{V_A} - \vec{V_w}$

w.r.t w.r.t

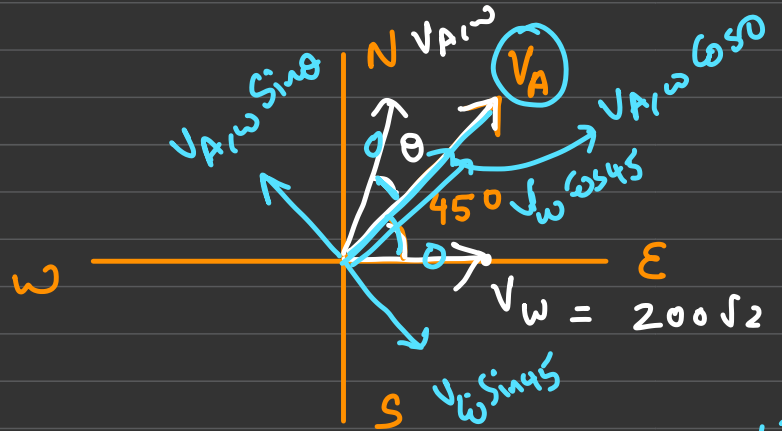
$\vec{V_A} = \vec{V_{A,w}} + \vec{V_w}$

Q) A wind starts blowing towards east with velocity of $200\sqrt{2}$ km/hr. In what direction the pilot should try to fly the aircraft such that he moves north east direct?

{given $\vec{V}_{A,w}$ = 400 km/hr (ground)

$\vec{V}_{A,w}$

#

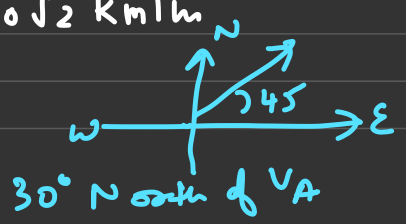


$V_{A,w}\sin\theta = V_w\sin45$

$400 \times \sin\theta = 200\sqrt{2} \times \frac{1}{\sqrt{2}}$

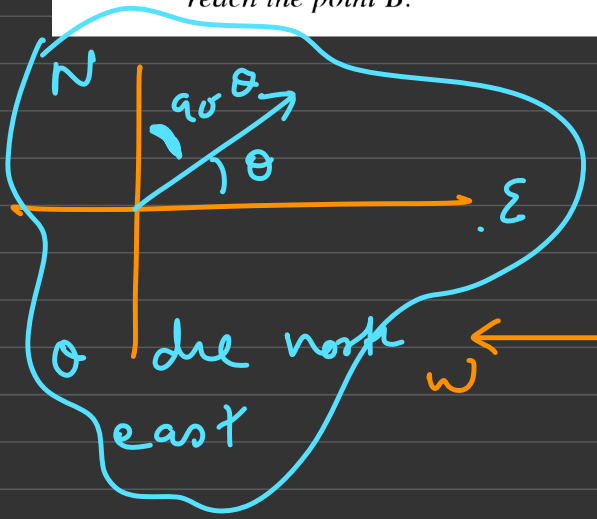
$\sin\theta = \frac{1}{2}$

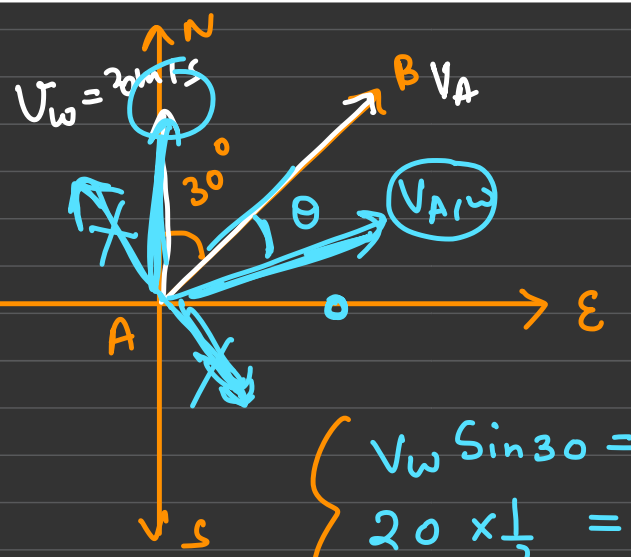
$\theta = 30°$



or 75° North of east

22. *An aeroplane has to go from a point A to another point B, 500 km away due 30° east of north. Wind is blowing due north at a speed of 20 m/s. The air-speed of the plane is 150 m/s. Find the direction in which the pilot should head the plane to reach the point B.*





$90-\theta$ due East of north

$$V_w \sin 30 = V_{A,w} \sin\theta$$
$$20 \times \frac{1}{2} = 150 \sin\theta$$
$$\sin\theta = \frac{1}{15}$$
$$\theta = \sin^{-1}\left(\frac{1}{15}\right) \text{ due}$$

east of dB)

(3) Rain-man Problems

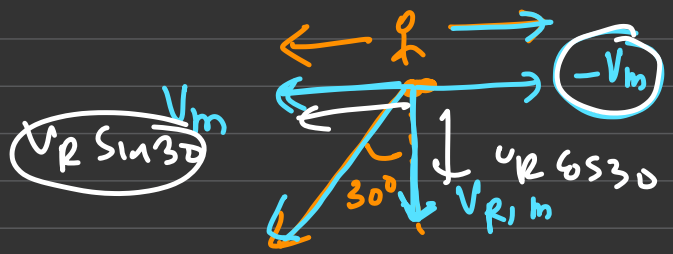
$\vec{V}_{R,m}$ = # Velocity of rain w.r.t man

Velocity of rain as seen man

$$\Rightarrow \vec{V}_{R/m} = \vec{V}_R - \vec{V}_m \quad (1)$$

$$= \vec{V}_R = \vec{V}_{R,m} + \vec{V}_m \quad (11)$$

Q)



$V_R = 20 m/s$

find speed and direction of man for which man can see rain falling over him

vertically downwards ?

(1) $\vec{v}_{R,m} = \vec{v}_R - \vec{v}_m$ 0

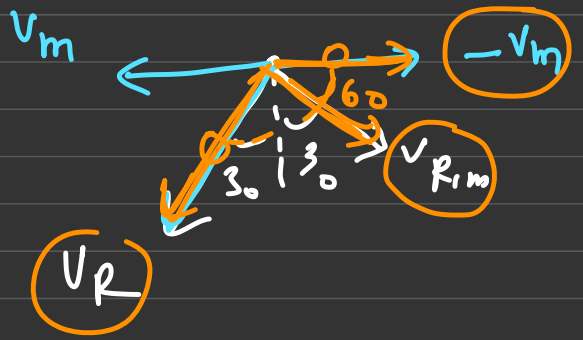
$\vec{v}_{R,m} = \vec{v}_R$ ⊥

(2) $\vec{v}_{R,m} = \vec{v}_R + (-\vec{v}_m)$ $\neq 0$

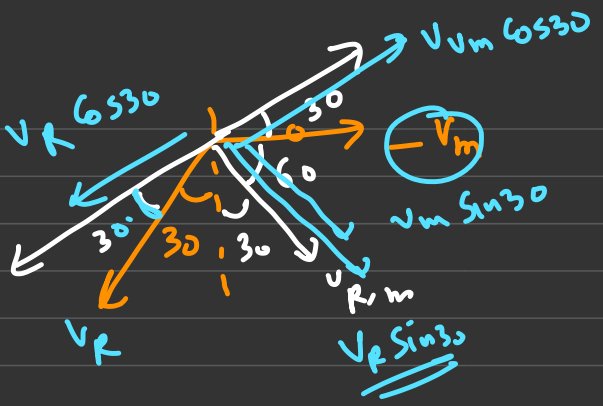
$v_R \sin 30 = v_m$

$20 \times \frac{1}{2} = v_m = 10 \text{ m/s}$ ← left

(ii)



find $\vec{v}_m$ for given situation

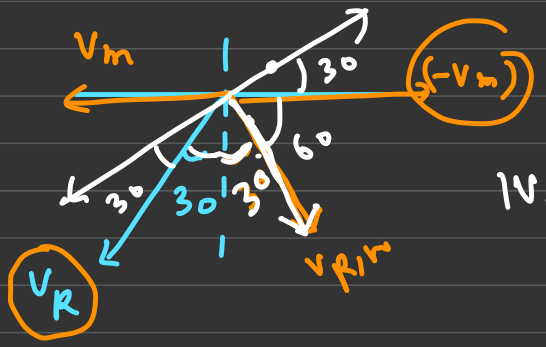


$$V_R \times \frac{\sqrt{3}}{2} = V_m \cos 30$$

$$V_R = V_m$$

$$|V_m| = 20\,m/s$$

Left

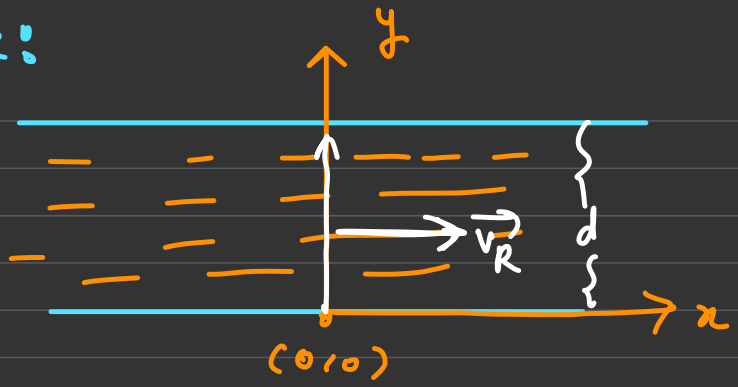


$$|V_R| \cos 30 = |V_m| \cos 30$$

$$|V_m| = 20\,m/s$$

Left

θ) Homework:



$$\vec{v}_R = v_0 \frac{y}{d} \hat{i}$$

$$|\vec{v}_{s,R}| = v_0$$

find total time taken by swimmer to cross the river in minimum distance ?

Vector Multiplication

$\vec{a}$ $\vec{b}$

Cross Product

$$\vec{a} \times \vec{b} = |\vec{a}|\,|\vec{b}| \sin\theta\, \hat{n}$$

Dot Product

$$\vec{a} \cdot \vec{b} = |\vec{a}|\,|\vec{b}| \cos\theta$$

Dot Product:
Scalar
(Multiplication)

$$\vec{a} \cdot \vec{b} = |\vec{a}|\,|\vec{b}| \cos\theta$$

$$\vec{r} = a\hat{i} + bj + ck$$

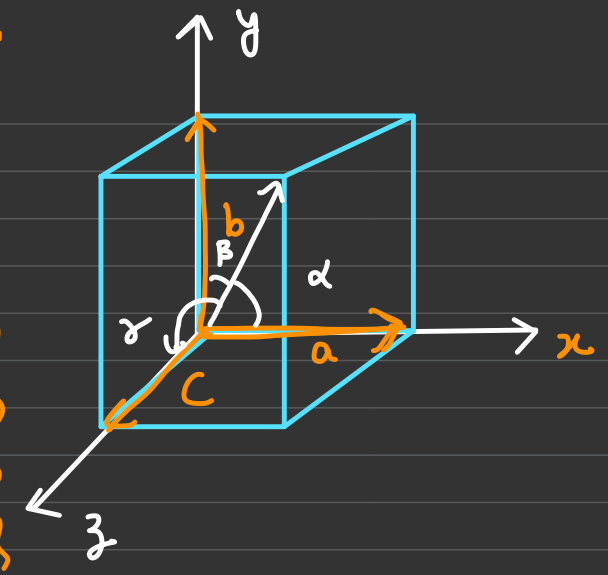
$$|\vec{r}| = \sqrt{a^2 + b^2 + c^2}$$

$\cos\alpha = \dfrac{a}{\sqrt{a^2+b^2+c^2}}$

$\vec{r}$ ω.r.t. x-axis

$\cos\beta = \dfrac{b}{\sqrt{a^2+b^2+c^2}}$

$\cos\gamma = \dfrac{c}{\sqrt{a^2+b^2+c^2}}$



① $\begin{cases} \vec{a} = 3\hat{i} + 4\hat{j} + 5\hat{k} \\ \vec{b} = 3\hat{i} + 3\hat{k} - 5\hat{k} \end{cases}$

$\vec{a}\cdot\vec{b} = 9 + 12 - 25$

$= 21 - 25 = -4$

$\begin{cases} \hat{i}\cdot\hat{i} = 1 \\ \hat{i}\cdot\hat{j} = 0 \end{cases}$

$\begin{cases} \vec{a}\cdot\vec{b} = |a||b|\cos 0 \\ = 1 \times 1 \times \cos 0 \\ = 1 \end{cases}$

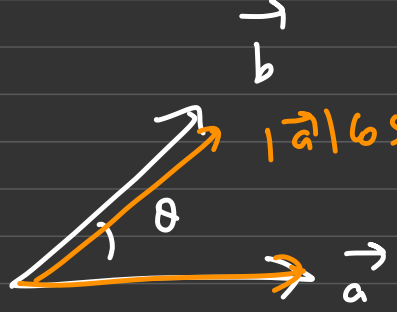
(11) $\vec{a}\cdot\vec{b} = |a||b|\cos\theta$

$\hat{i}\cdot\hat{j} = 1\times1\times\cos 90 = 0$

$$\cos\theta = \frac{\vec{a}\cdot\vec{b}}{|a||b|} = \frac{-4}{\sqrt{3^2+4^2+5^2}\sqrt{3^2+3^2+5^2}}$$

$$\theta = \cos^{-1}\left(\frac{-4}{\sqrt{50}\times\sqrt{43}}\right)$$

(111)



find Projection of $\vec{a}$ on $\vec{b}$

$\vec{a}\cdot\vec{b} = |a||b|\cos\theta$

$|\vec{a}|\cos\theta = \frac{\vec{a}\cdot\vec{b}}{|b|}$

$$|b|\cos\theta = \frac{\vec{a}\cdot\vec{b}}{|a|}$$

Projection of $\vec{b}$ over $\vec{a}$

Q) $\vec{a} = 2i + 3j$ find Projection $\vec{a}$ over $\vec{b}$

$\vec{b} = 2i - 3j$

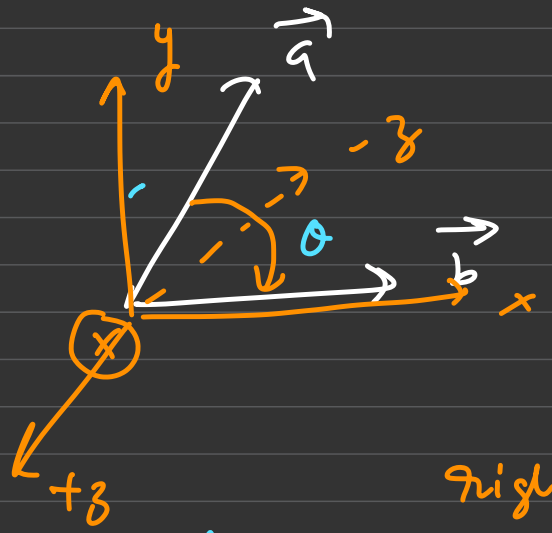
$\vec{a}\cdot\vec{b} = 4 - 9 = -5$

$$|\vec{a}|\cos\theta = \frac{\vec{a}\cdot\vec{b}}{|b|} = \frac{-5}{\sqrt{4+9}}$$

Projection $\vec{a}$ over $\vec{b}$ $= -\frac{5}{\sqrt{13}}$

Cross Product:

$$\vec{a} \times \vec{b} = \underbrace{|a|\,|b| \sin\theta}_{\text{modulus}}\ \underbrace{\hat{n}}_{\text{Dir}}$$



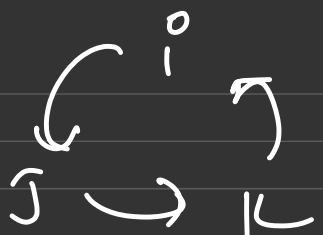
$$\vec{a} \times \vec{b} = -a\,b \sin\theta\ \hat{k}$$

Direction

"Right hand thumb rule"

$\vec{a} \times \vec{b}$

"if we curl our right hand fingers from $\vec{a}$ to $\vec{b}$ (θ = smaller) then Direction thumb ^ represents Direction of $\vec{a} \times \vec{b}$?



$\Rightarrow \hat{i} \times \hat{j} = \hat{k}$

$\Rightarrow \hat{j} \times \hat{k} = \hat{i}$

$\hat{k} \times \hat{i} = \hat{j}$

$\hat{j} \times \hat{i} = |\hat{i}||\hat{i}| \sin\theta \times$

$\hat{j} \times \hat{j} = 0$

$\hat{k} \times \hat{k} = 0$

$\vec{a} = 2\hat{i} + 3\hat{j}$

$\vec{b} = 2\hat{i} - 3\hat{j}$

$\Rightarrow \vec{a} \times \vec{b} = (2\hat{i} + 3\hat{j}) \times (2\hat{i} - 3\hat{j})$

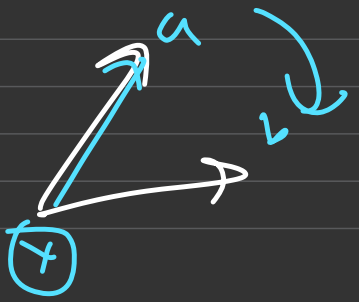
$= -6\hat{k} - 6\hat{k} + 0$

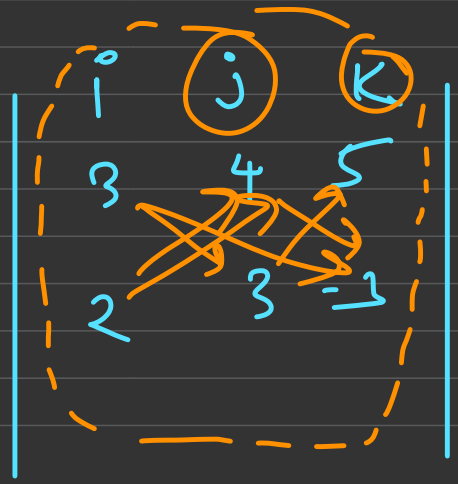
$\vec{a} \times \vec{b} = -12\hat{k}$

$$\begin{cases} \vec{a} = 3\hat{i} + 4\hat{j} + 5\hat{k} \\ \vec{b} = 2\hat{i} + 3\hat{j} - 3\hat{k} \end{cases}$$

$\vec{a} \times \vec{b}$

$\vec{a} \times \vec{b} =$





$$\vec{a} \times \vec{b} = -\vec{b} \times \vec{a}$$

$$= \hat{i}(4 \times (-3) - 5 \times 3) - \hat{j}(-9-1$$

$$+ \hat{k}(9 - 8)$$

$$\vec{a} \times \vec{b} = -27\hat{i} + 19\hat{j} + 1\hat{k}$$

:Revision:

Applications:

1. work done
2. Power
3. torque

for doubts:

msg on my Personal no. 8884945513

"Kinematics"

Doubt

Level 1:
Level 2:

main aim
Advanced

Homework calc:

1. Complete module
2. Complete work Level 1

50 Q Level 2

main