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IIT JEE | MEDICAL | FOUNDATION

IIT JEE - 2021

Solutions to Home Assignment - 2 | Functions | Mathematics

1.(B) $0 \le \sqrt{x^2 - \frac{\pi^2}{9}} < \infty \Rightarrow \cos\sqrt{x^2 - \frac{\pi^2}{9}} \in [-1, 1] \Rightarrow f(x) \in [-4, 4]$

2.(C) $g(x) = f(x+1) = |x-1| + |x-2| + |x-3|$ Now, $g(-x) = |x+1| + |x+2| + |x+3|$

Clearly, $g(x) \ne \pm g(-x) \Rightarrow g(x)$ is neither even nor odd.

3.(A) For $f(x)$ to be defined $3 - x^2 \ne 0$ *i.e*, $x \ne \pm\sqrt{3}$

$\therefore$ Domain of $f(x) = R\{\pm\sqrt{3}\}$

Now, let $y = \frac{5}{3 - x^2} \Rightarrow x^2 = \frac{3y-5}{y} \Rightarrow x = \sqrt{\frac{3y-5}{y}}$

$\Rightarrow$ For x to be defined $\therefore$ $y < 0$ or $y \ge \frac{5}{3}$

Hence, range of $f(x) = (-\infty, 0) \cup \left[\frac{5}{3}, \infty\right)$

4.(B) If $x \ge 0$ then $\sqrt{|x| - x} = \sqrt{x - x} = 0$

If $x < 0$ then $\sqrt{|x| - x} = \sqrt{-x - x} = \sqrt{-2x} > 0$ $\therefore$ Range = $[0, \infty)$

5.(C) $$(fog)(x) = f[g(x)] = f\left[\frac{3x + x^3}{1 + 3x^2}\right] = \log\left[\frac{1 + \frac{3x + x^3}{1 + 3x^2}}{1 - \frac{3x + x^3}{1 + 3x^2}}\right] = \log\left[\frac{1 + 3x^2 + 3x + x^3}{1 + 3x^2 - 3x - x^3}\right] = \log\left(\frac{1+x}{1-x}\right)^3 = 3\log\frac{1+x}{1-x}$$

$= 3f(x)$.

6.(B) $f(x) = \frac{x-1}{x+1} \Rightarrow x = \frac{f(x)+1}{1 - f(x)}$; $$f(x) = \frac{2x-1}{2x+1} = \frac{2\left[\frac{f(x)+1}{1-f(x)}\right] - 1}{2\left[\frac{f(x)+1}{1-f(x)}\right] + 1} = \frac{3f(x)+1}{f(x)+3}$$

7.(A) $f(x) = x$; $g(x) = |x|\ \forall\ x \in R$; $[\phi(x) - f(x)]^2 + [\phi(x) - g(x)]^2 = 0$

If sum of two non-negative numbers is zero than each of the numbers should be zero.

$\Rightarrow \quad \phi(x) - f(x) = 0$ and $\phi(x) - g(x) = 0 \quad \Rightarrow \quad \phi(x) = f(x) = g(x)$

But $f(x) = g(x)$ is possible $\forall\ x \in [0, \infty)$; Hence $f(x) = x$ where $x \in [0, \infty)$

8.(B) $f(x)g(y) + f(y)g(x) = \frac{1}{2}\left(3^x + 3^{-x}\right)\frac{1}{2}\left(3^y - 3^{-y}\right) + \frac{1}{2}\left(3^y + 3^{-y}\right)\frac{1}{2}\left(3^x - 3^{-x}\right)$

$= \frac{1}{4}\left[3^x 3^y - 3^x 3^{-y} + 3^{-x} 3^y - 3^{-x} 3^{-y} + 3^y 3^x - 3^y 3^{-x} + 3^{-y} 3^x - 3^{-y} 3^{-x}\right]$

$= \frac{1}{4}\left[2 \cdot 3^x 3^y - 2 \cdot 3^{-x}\ 3^{-y}\right] = \dfrac{3^{x+y} - 3^{-(x+y)}}{2} = g(x+y)$

9.(C) We have, for $n \in Z$, $|\sin x| + \sin x = \begin{cases} 2\sin x & \text{if} \quad 2n\pi < x < (2n+1)\pi \\ 0 & \text{otherwise} \end{cases}$

Also, $2\sin x \neq 0$ if $2n\pi < x < (2n+1)\pi$. $\quad \therefore \quad$ Domain of, f is $\bigcup\limits_{n \in Z} \left(2n\pi, (2n+1)\pi\right)$

10.(B) $\sin\log\dfrac{\sqrt{4-x^2}}{1-x}$ exists $\Rightarrow \dfrac{\sqrt{4-x^2}}{1-x} > 0 \Rightarrow 1-x > 0,\ 4-x^2 > 0 \Rightarrow 1 > x,\ x^2 - 4 < 0$

$\Rightarrow \quad 1 > x,\ -2 < x < 2 \Rightarrow -2 < x < 1 \quad \therefore \quad$ Domain = (–2, 1)