Introduction to Vector and forces -1

VMC Vidyamandir Classes SINCE 1986
IIT JEE | MEDICAL | FOUNDATION

{ module #1 { → Revise Lecture same day

{ Homework { → module →
work book — DTS → Level 1
Level 2

Basics of vectors

Physical Quantity
(mass, velocity)

$\Downarrow$

Vectors — (Those Quantities having magnitude as well as Direction)

Scalars — { These Quantities only having magnitude }

$\vec{V} \to |\vec{V}|$ (modulus) {magnitude}

$\vec{V} \to \hat{V}$ (Direction) unit vector

$m = +5\,kg$

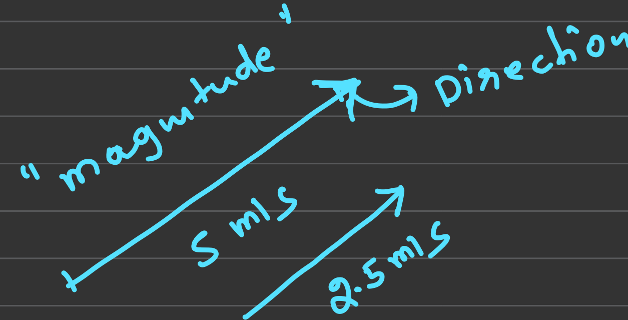
Temp = {+ve, −ve, 0}

Vectors: **representation:** 1) $\vec{a}$, $\tilde{a}$, $\hat{a}$ $\bar{a}$

$\vec{a} \rightarrow |\vec{a}|$ –

$\vec{a} \rightarrow \hat{a}$ –

Geometrical Representation:



#

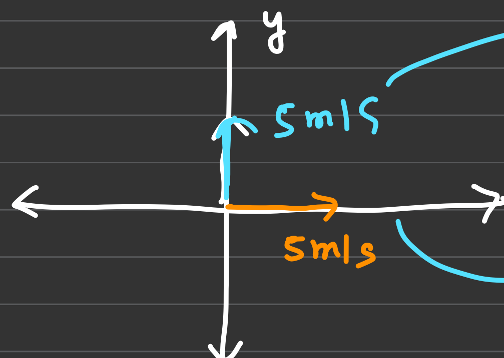
$$\begin{array}{ccc} & \vec{a} & \\ \swarrow & & \searrow \\ |\vec{a}| & & \hat{a} \end{array}$$

$\hat{a}$ = magnitude = 1

$$\vec{a} = |\vec{a}|\,\hat{a}$$

$$\vec{v} = 5\,m/s\;\hat{c}$$

#



$$\vec{v} = 5\,m/s\;\hat{j}$$

$$\vec{v} = 5\,\hat{i}$$

Vector operation:

Type of Vectors:

① Equal Vector : "magnitude as well Direction same"

$\vec{a}$ $\vec{b}$

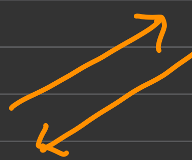
$\vec{a} = \vec{b}$ ✓

$|\vec{a}| = |\vec{b}|$, $\hat{a} = \hat{b}$

② Opposite Vectors:

$\vec{a}$, $\vec{b}$

$|\vec{a}| = |\vec{b}|$ $\hat{a} = -\hat{b}$



③ Unit Vector:

$\hat{a} \Rightarrow |\vec{a}| = 1$

$\hat{i} \qquad |\hat{i}| = 1 \qquad \longrightarrow x$

$$\vec{a} = |\vec{a}|\,\hat{a}$$

$$\hat{a} = \frac{\vec{a}}{|\vec{a}|}$$

① multiplication of a vector with scalar:

$\vec{a}$ (vector) $\qquad$ K (scalar)

Direction same magnitude changes by k

$\vec{a}k$ = Vector

$\vec{a}k = \begin{cases} \vec{a}k & k > 0 \\ \vec{a}k & k < 0 \end{cases}$

Direction changes magnitude k fing

$\vec{a}\,0 = \vec{0}$ (null vector)

$k = 0$

Unknown

Vector Addition:

$\vec{a}$ $\quad$ $\vec{b}$

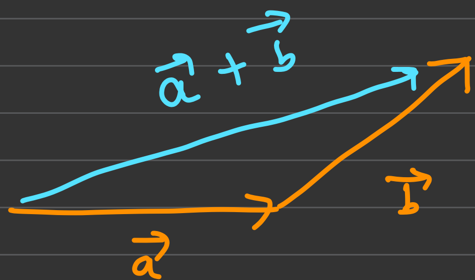
$\downarrow$ $\quad$ $\downarrow$

$|\vec{a}|$ $\quad$ $|\vec{b}|$

Head

① Graphical addition: (Valid only for 2 vectors)

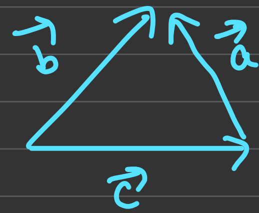
Vector Triangle law:



Tail

"if we join Head of 1st vector to tail of second vector then a vector from tail of 1st vector to Head of second vector is vector sum of $\vec{a}$ and $\vec{b}$"

Q1



find correct option

a) $\vec{b} + \vec{a} = \vec{c}$ ✗

b) $\vec{a} + \vec{c} = \vec{b}$ ✓ Ans

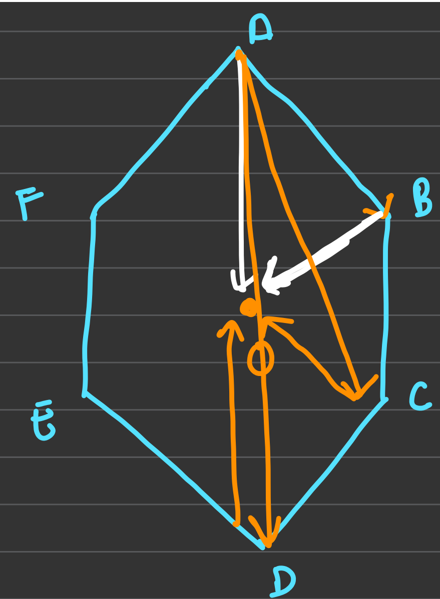
c) $\vec{b} + \vec{c} = \vec{a}$ ✗

d) None of above

$\{\vec{a} + \vec{c} = \vec{c} + \vec{a}$

85. $ABCDEF$ is regular hexagon with point O as centre. The value of $\overrightarrow{AB}+\overrightarrow{AC}+\overrightarrow{AD}+\overrightarrow{AE}+\overrightarrow{AF}$ is $n\times\overrightarrow{AO}$.

Find n. $\boxed{}$



$\Rightarrow$ $\Delta\, ABO$

$\overrightarrow{AB} + \overrightarrow{BO} = \overrightarrow{AO}$ — (I)

$\Rightarrow$ $\Delta\, ACO$

$\overrightarrow{AC} + \overrightarrow{CO} = \overrightarrow{AO}$ — (II)

$\overrightarrow{AD} + \overrightarrow{DO} = \overrightarrow{AO}$ — (III)

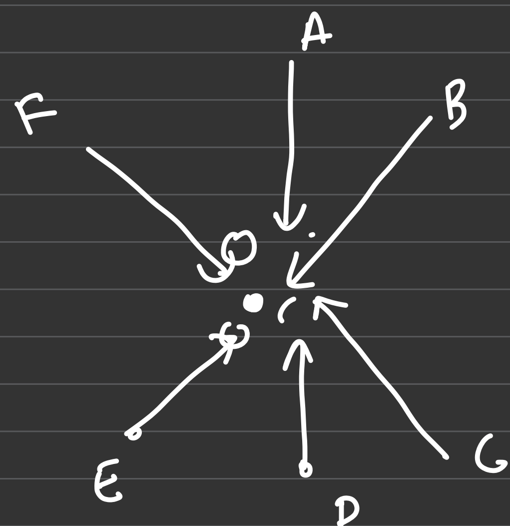
$\overrightarrow{AE} + \overrightarrow{EO} = \overrightarrow{AO}$ — (IV)

$\overrightarrow{AF} + \overrightarrow{FO} = \overrightarrow{AO}$ (V)

$$\overrightarrow{AB}+\overrightarrow{AC}+\overrightarrow{AD}+\overrightarrow{AE}+\overrightarrow{AF}+\left\{\overrightarrow{BO}+\overrightarrow{CO}+\overrightarrow{DO}+\overrightarrow{EO}+\overrightarrow{FO}\right\}$$

$$\overrightarrow{AO} = 0$$

$$= 5\times\overrightarrow{AO}+\overrightarrow{AO}$$

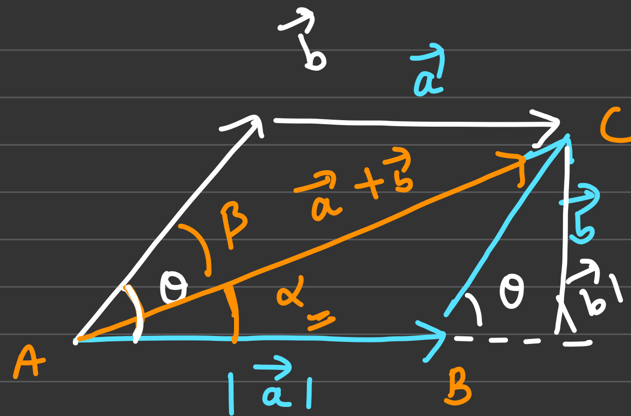


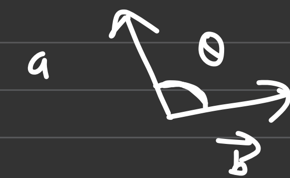
$$\overrightarrow{AB}+\overrightarrow{AC}+\overrightarrow{AD}+\overrightarrow{AE}+\overrightarrow{AF} = 6\,\overrightarrow{AO}$$

$n=6$ Ans

mathematically:

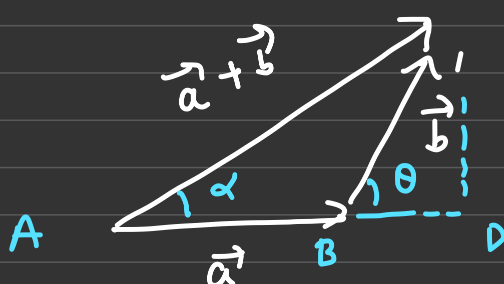
given: $\begin{cases} \vec{a} \\ \vec{b} \\ \theta \end{cases}$



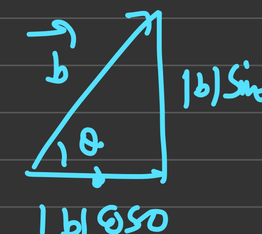


$\theta \leq 180$

ΔABC



ΔADC



$$AC^2 = AD^2 + DC^2$$

$$|\vec{a}+\vec{b}| = (|a| + |b| \cos\theta)^2 + (|b| \sin\theta)^2$$

$$|\vec{R}| = |\vec{a} + \vec{b}|$$

$$|\vec{R}| = \sqrt{(|a| + |b|\cos\theta)^2 + (|b|\sin\theta)^2}$$

$$|\vec{R}| = \sqrt{|a|^2 + \underline{|b|^2\cos^2\theta} + 2|\vec{a}||\vec{b}|\cos\theta + \underline{|b|^2\sin^2\theta}}$$

$$|\vec{R}| = \sqrt{|\vec{a}|^2 + |\vec{b}|^2(\sin^2\theta + \cos^2\theta) + 2|\vec{a}||\vec{b}|\cos\theta}$$

$$|\vec{a}| = a$$
$$|\vec{b}| = b$$

$$|\vec{R}| = \sqrt{|\vec{a}|^2 + |\vec{b}|^2 + 2|\vec{a}||\vec{b}|\cos\theta}$$

$$\tan\alpha = \frac{|b|\sin\theta}{|a| + |b|\cos\theta}$$

$$|\vec{R}| = \sqrt{a^2 + b^2 + 2ab\cos\theta}$$ Ar

Q)

$|\vec{a}| = 5$ Unit

$|\vec{b}| = 10$ Unit

$\theta = 60^\circ$

find Sum of $(\vec{a} + \vec{b})$

$$|\vec{R}| = \sqrt{(5)^2 + (10)^2 + 2 \times 5 \times 10 \times \cos\theta}$$

$$|\vec{R}| = \sqrt{25 + 100 + 100 \times \frac{1}{2}}$$

$$|\vec{R}| = \sqrt{175}$$

$$\tan\alpha = \frac{|b|\sin\theta}{|a| + |b|\cos\theta}$$

{Angle of resultant w.r.t $\vec{a}$ }

$$\tan\alpha = \frac{|b|\sin\theta}{|a| + |b|\cos\theta}$$

$$= \frac{10 \times \sin 60}{5 + 10 \times \frac{1}{2}} =$$

$$\tan\alpha = \frac{5\sqrt{3}}{5+5} = \left(\frac{\sqrt{3}}{2}\right)$$

$$\alpha = \tan^{-1}\left(\sqrt{3}/2\right)$$

w.r.t $\vec{a}$

$$R = \sqrt{a^2 + b^2 + 2ab\cos\theta}$$ for fixed a and b

for what value of θ R is going to maximum and minimum ?

$R \rightarrow$ maximum when $\cos\theta \Rightarrow$ maximum

$R \rightarrow$ minimum when $\cos\theta \Rightarrow$ minimum

min $\cos\theta = -1$ $\qquad \theta = 180$

max $\cos\theta = +1$ $\qquad \theta = 0$

$$R_{max} = \sqrt{a^2 + b^2 + 2ab} = \sqrt{(a+b)^2}$$

$$= a + b$$

$|a| = 5$ unit

$|b| = 10$ unit

in above $R_{max} = 15$

$R_{min} = \sqrt{a^2 + b^2 - 2ab}$ $\qquad \theta = 180$

$R_{min} = \sqrt{(a-b)^2}$

$= |(a-b)|$

$R_{min} = 5$

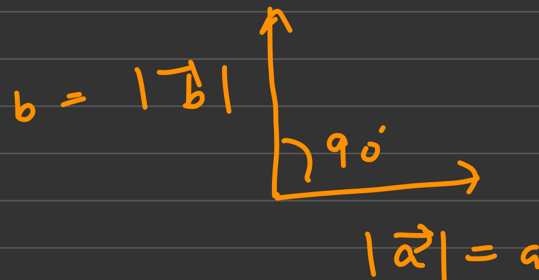
$5 \leq R \leq 15$

Or gen:

$$|a-b| \leq R \leq |a+b|$$

$\underline{\underline{A}}$

$\theta = 90$ (Perpendicular Vector)



$$R = \sqrt{a^2 + b^2 + 2ab\cos 90}$$

$$R = \sqrt{a^2 + b^2}$$

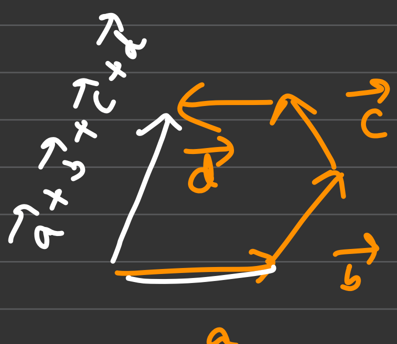
$\Rightarrow \theta = 0^\circ$

- * Collinear vector $\Rightarrow R = a + b$ {Same Direction}
- * Collinear vector $\Rightarrow R = a - b$ {opposite}

  $\Downarrow 180^\circ$
- # Perpendicular vector $\Rightarrow R = \sqrt{a^2 + b^2}$ $\theta = 90$

(ii) Polygon law :

① graphical Addition

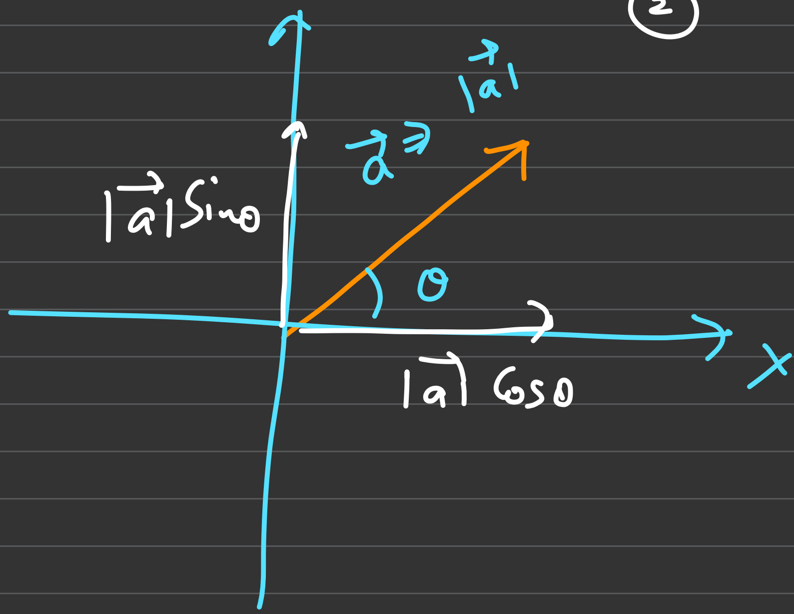


Gb if we join the Head of first vector to tail of second vector then Head of second vector to tail of 3rd vector and so--- then a vector from tail of first vector to head of last vector is vector

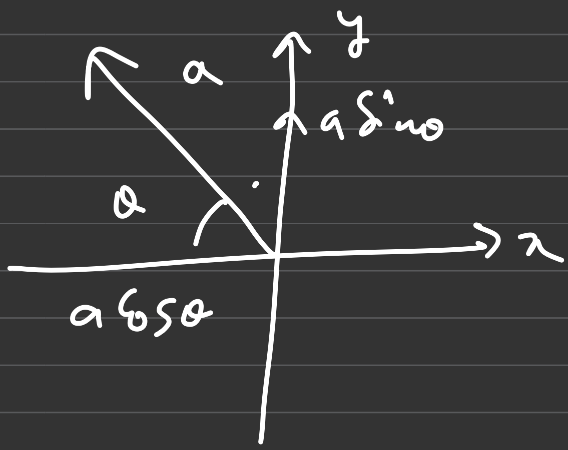
Sum of all the vector?

Addition of more than two vector using Component method:

①



②

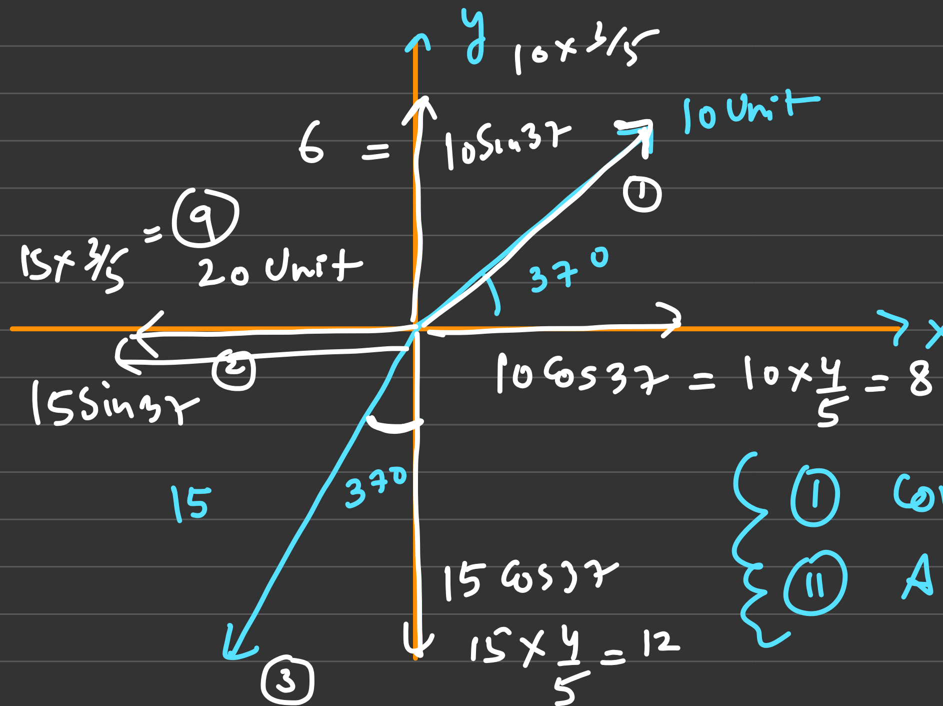


Q)

find sum of all three vectors

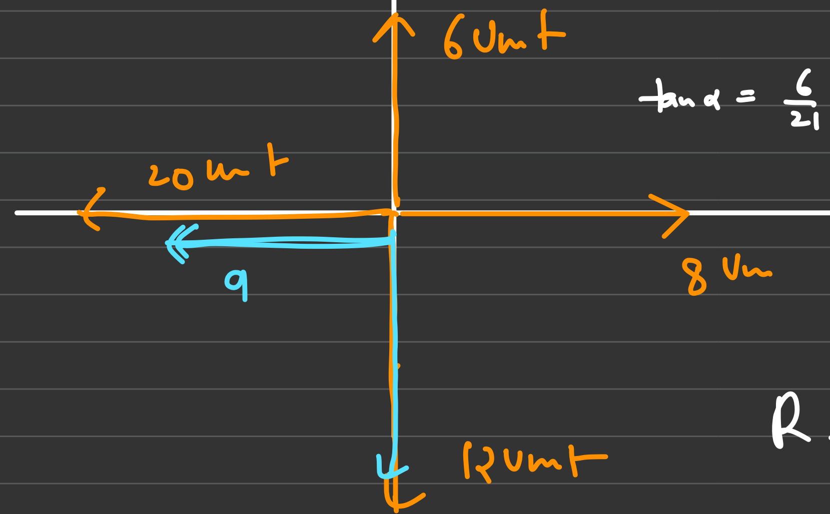
$\sin 37 = 3/5$

$\cos 37 = 4/5$

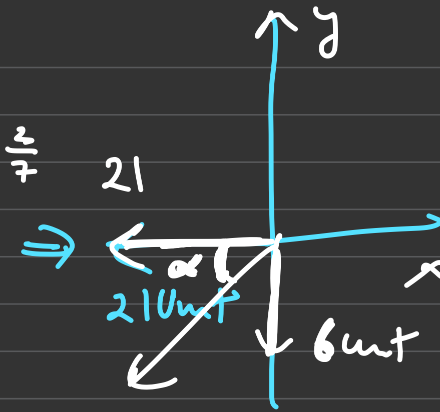


(I) Component

(II) Add



$\tan\alpha = \frac{6}{21} = \frac{2}{7}$



$R = \sqrt{(21)^2 + (6)^2}$

$R = \sqrt{441 + 36}$

$R = \sqrt{477}$

$\alpha = \tan^{-1}\left(\frac{2}{7}\right)$ w.r.t

− x-axis

Section 1: module # illustration . In chapter exercise

A

work book! Level 1. DPS # 1 15

Level 2

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