VMC Vidyamandir Classes SINCE 1986

IIT JEE | MEDICAL | FOUNDATION

Straight Lines

Level - 1	Daily Tutorial Sheet - 1 to 6

1.(A) $\left(\dfrac{x_1+x_2}{2}, \dfrac{y_1+y_2}{2}\right) \equiv (3, 1) \Rightarrow (x_1 + x_2, y_1 + y_2) \equiv (6, 2)$ **...(i)**

and $\left(\dfrac{x_1+x_3}{2}, \dfrac{y_1+y_3}{2}\right) \equiv (5, 6)$

$\Rightarrow (x_1 + x_3, y_1 + y_3) \equiv (10, 12)$ **...(ii)**

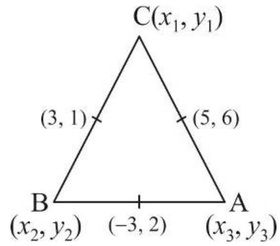
and $\left(\dfrac{x_2+x_3}{2}, \dfrac{y_2+y_3}{2}\right) \equiv (-3, 2)$

$\Rightarrow (x_2 + x_3, y_2 + y_3) \equiv (-6, 4)$ **...(iii)**

Using (i), (ii) and (iii) $\Rightarrow x_1 + x_2 + x_3 = 5$ ...**(iv)**

Use (i), (ii), (iii) and (iv) to get: $x_3 = -1, x_2 = -5, x_1 = 11$

and similarly: $y_3 = 7, y_2 = -3, y_1 = 5$



2.(B) $A \equiv (a + b, b - a)$; $B \equiv (a - b, a + b)$; $P(x, y)$ and $PA = PB \Rightarrow (PA)^2 = (PB)^2$

$\Rightarrow (x - (a + b))^2 + (y - (b - a))^2 = (x - (a - b))^2 + (y - (a + b))^2$

$\Rightarrow -2(a + b)x - 2y(b - a) = -2(a - b)x - 2(a + b)y \Rightarrow bx = ay$

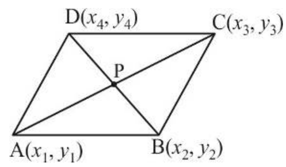
3.(C) $P \equiv (at^2, 2at)$; $Q \equiv \left(\dfrac{a}{t^2}, \dfrac{-2a}{t}\right)$; $S \equiv (a, 0)$

$$SP \equiv \sqrt{\left(at^2 - a\right)^2 + (2at)^2} = a\left(t^2 + 1\right);\ SQ \equiv \sqrt{\left(\frac{a}{t^2} - a\right)^2 + \left(\frac{-2a}{t}\right)^2} = a\left(\frac{1}{t^2} + 1\right)$$

$$\Rightarrow \frac{1}{SP} + \frac{1}{SQ} = \frac{1}{a}\left(\frac{1}{t^2+1} + \frac{t^2}{t^2+1}\right) = \frac{1}{a}$$

4.(A) $P \equiv \left[\dfrac{x_1+x_3}{2}, \dfrac{y_1+y_3}{2}\right] \equiv \left[\dfrac{x_2+x_4}{2}, \dfrac{y_2+y_4}{2}\right]$

$\Rightarrow x_1 - x_2 + x_3 - x_4 = 0$ and $y_1 - y_2 + y_3 - y_4 = 0$



5. **(i)(A) (ii)(A) (iii)(A) (iv)(B)**

Let $P \equiv (x, y)$ divides AB internally in $k : 1$ ratio.

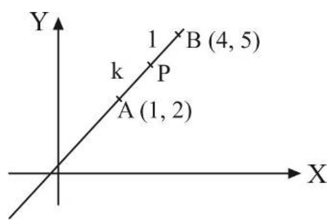
$$(x, y) \equiv \left(\frac{k(4) + 1(1)}{k+1}, \frac{k(5) + 1(2)}{k+1}\right)$$

For X-axis: Substitute $y = 0 \Rightarrow k = -\dfrac{2}{5}$ (Externally)

$\Rightarrow \dfrac{AP}{PB} = \dfrac{2}{5} \Rightarrow$ coordinates $\equiv (-1, 0)$

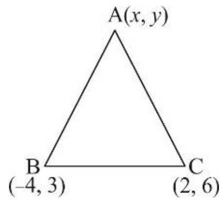
For Y-axis: Substitute $x = 0 \Rightarrow k = -\dfrac{1}{4}$ [i.e., 1: 4 externally]

$\Rightarrow \dfrac{AP}{PB} = \dfrac{1}{4} \Rightarrow$ coordinates $\equiv (0, 1)$



6.(C) Midpoint of BD = Midpoint of AC $\Rightarrow$ $\dfrac{a+b+a-b}{2}=\dfrac{2a+b+x}{2} \Rightarrow x=-b$

and $\dfrac{a-b+a+b}{2}=\dfrac{2a-b+y}{2} \Rightarrow y=b$



7.(B) $G=(x_a, y_a)\equiv(0,0)=\left(\dfrac{x-4+2}{3},\dfrac{y+3+6}{3}\right)$ $\Rightarrow$ $(x, y)\equiv(2,-9)$

8.(D) Statement - 2 is true (Basic fact)

For statement - 1, Given triangle is not equilateral.

9.(A) Statement - 2 is true (see theory)

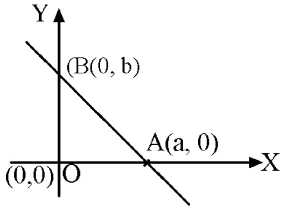
For statement - 1, Orthocentre of ΔOAB is at origin

[$\because$ $x-3y=0$ & $3x+y=0$ are $\perp r$]

or $3x-4y=0$ passes through origin

Hence through orthocentre.

Hence, both statements are true and statement - 2 explains statement – 1



10.(A) $A\equiv(6,3)$, $B\equiv(-3,5)$, $C\equiv(4,-2)$ and $P\equiv(x, y)$

Area of ΔPBC = $\dfrac{1}{2}\left|x(5+2)+(-3)(-2-y)+4(y-5)\right|=\dfrac{1}{2}\left|7x+7y-14\right|$

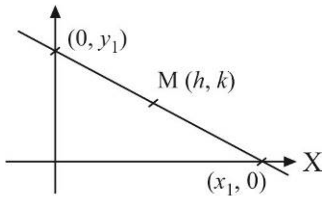
Area of ΔABC = $\dfrac{1}{2}\left|6(5+2)+(-3)(-2-3)+4(3-5)\right|=\dfrac{49}{2} \Rightarrow \dfrac{\text{Area of } \Delta PBC}{\text{Area of } \Delta ABC}=\dfrac{|x+y-2|}{7}$

11.(B) Let x_1 and y_1 be the x-intercept and y-intercept.

(h, k) is the midpoint of $(x_1, 0)$ and $(0, y_1)$

$\therefore$ $h=\dfrac{x_1}{2}$ and $k=\dfrac{y_1}{2}$

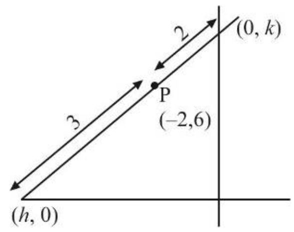
$\therefore$ Equation of line is: $\dfrac{x}{x_1}+\dfrac{y}{y_1}=1 \Rightarrow \dfrac{x}{h}+\dfrac{y}{k}=2$



12.(B) **Case I:** Coordinates of $P\equiv\left(\dfrac{2h+0}{5},\dfrac{0+3k}{5}\right)\equiv(-2,6)$

$\Rightarrow$ $(h, k)\equiv(-5, 10)$

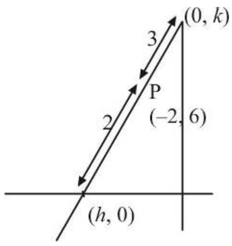
$\therefore$ Equation of line is: $\dfrac{x}{-5}+\dfrac{y}{10}=1$



Case II: Coordinates of $P\equiv\left(\dfrac{3h}{5},\dfrac{2k}{5}\right)\equiv(-2,6)$

$\Rightarrow$ $(h,k)\equiv\left(\dfrac{-10}{3},15\right)$

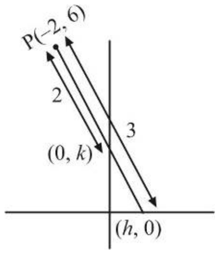
$\therefore$ Equation of line is: $\dfrac{x}{\frac{-10}{3}}+\dfrac{y}{15}=1$



Case III: Coordinates of $P\equiv\left(\dfrac{3\times0-2h}{3-2},\dfrac{3\times k-2\times0}{3-2}\right)\equiv(-2,6)$

$\Rightarrow$ $(-2h, 3k)\equiv(-2,6)$ $\Rightarrow$ $(h, k)\equiv(1,2)$

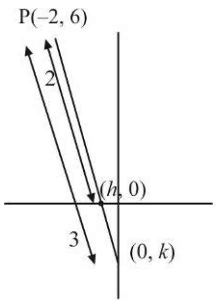
$\therefore$ Equation of line is: $\dfrac{x}{1}+\dfrac{y}{2}=1$



Case IV:

Coordinates of $P\equiv\left(\dfrac{3h-0}{1},\dfrac{-2k}{1}\right)\equiv(-2,6) \Rightarrow (h,k)\equiv\left(\dfrac{-2}{3},-3\right)$

$\therefore$ Equation of line is: $\dfrac{x}{-2/3}+\dfrac{y}{-3}=1$



13.(C) If $\frac{x}{a}+\frac{y}{b}=1$ is the required line then $a + b = 7$

(12, –1) lies on the line $\Rightarrow$ $\frac{12}{a}-\frac{1}{b}=1$

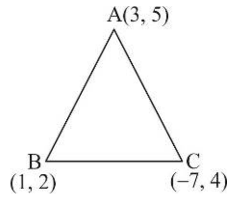
Solve the equation for a and b to get: [$a = 14$, $b = -7$] or [$a = 6$, $b = 1$]

Lines are: $\frac{x}{14}-\frac{y}{7}=1$ and $\frac{x}{6}+\frac{y}{1}=1$

14.(D) Equation of AB: $y-2=\frac{5-2}{3-1}(x-1)$ $\Rightarrow$ $3x-2y+1=0$ **...(i)**

Equation of BC: $y-4=\frac{4-2}{-7-1}(x+7)$ $\Rightarrow$ $x+4y-9=0$ **...(ii)**

Equation of CA: $y-5=\frac{5-4}{3+7}(x-3)$ $\Rightarrow$ $x-10y+47=0$ **...(iii)**



15.(B) Points A, B, C are collinear if slope AC = slope BC

$$\Rightarrow \quad \frac{2at_1}{at_1^2-a}=\frac{2at_2}{at_2^2-a} \Rightarrow t_1\left(t_2^2-1\right)=t_2\left(t_1^2-1\right) \Rightarrow \quad t_1t_2^2-t_1-t_2t_1^2+t_2=0 \Rightarrow \left(t_1t_2+1\right)\left(t_1-t_2\right)=0$$

Either $t_1 = t_2$ (but then A and B are same points) or $t_1 t_2 = -1$

16.(D) As EF is parallel to BC $\Rightarrow$ slope BC = slope EF = $\frac{+7+5}{-5+5}=\frac{12}{0}=\infty$ $\Rightarrow$ BC is | | to Y-axis.

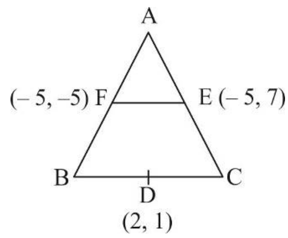
Similarly, slope AB = slope DE = $\frac{7-1}{-5-2}=\frac{-6}{7}$

slope CA = slope FD = $\frac{1+5}{2+5}=\frac{6}{7}$

Equation of AB: $y+5=-\frac{6}{7}(x+5)$ $\Rightarrow$ $6x+7y+65=0$

Equation of BC: $x-2=0$

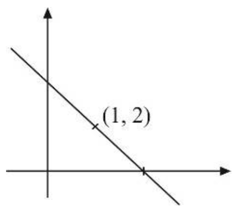
Equation of CA: $y-7=\frac{6}{7}(x+5)$ $\Rightarrow$ $6x-7y+79=0$



17.(A) $\frac{x}{a}+\frac{y}{b}=1$; $b=2a$ (Given)

It passes through (1, 2) $\Rightarrow$ $\frac{1}{a}+\frac{2}{2a}=1$ $\Rightarrow$ $a=2$

$\Rightarrow$ $\frac{x}{2}+\frac{y}{4}=1$ $\Rightarrow$ $2x+y=4$



18.(A) Let the equation be $\frac{x}{a}+\frac{y}{b}=1$

Also, $a + b = 14$; Line passes through (3, 4) $\Rightarrow$ $\frac{3}{a}+\frac{4}{b}=1$

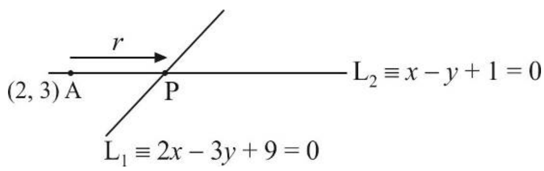
$\Rightarrow$ $\frac{3}{14-b}+\frac{4}{b}=1$ $\Rightarrow$ $b^2-15b+56=0$

$\Rightarrow$ $b = 7, 8$ and $a = 7, 6$ $\Rightarrow$ Equation are: $x + y = 7$ and $4x + 3y = 24$

19.(A) $P \equiv \left[2+\frac{r}{\sqrt{2}}, 3+\frac{r}{\sqrt{2}}\right]$

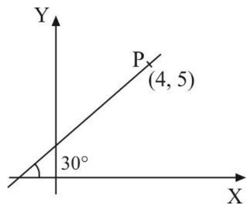
Since, P lies on $2x - 3y + 9 = 0$

$\Rightarrow$ $2\left[2+\frac{r}{\sqrt{2}}\right]-3\left[3+\frac{r}{\sqrt{2}}\right]+9=0$ $\Rightarrow$ $r=4\sqrt{2}$



20.(C) Points are: $(4 \pm 4 \cos 30°, 5 \pm 4 \sin 30°)$

$\equiv \left(4 \pm 2\sqrt{3}, 5 \pm 2\right) \equiv \left(4 + 2\sqrt{3}, 7\right)$ and $\left(4 - 2\sqrt{3}, 3\right)$

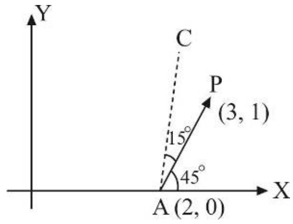


21.(B) $\dfrac{x - x_1}{\cos\theta} = \dfrac{y - y_1}{\sin\theta} = r$

22.(B) Equation of new line:

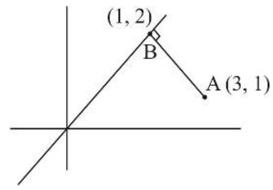
$\dfrac{y-0}{x-2} = \tan 60° = \sqrt{3}$

$\Rightarrow \quad \dfrac{x-2}{1/2} = \dfrac{y-0}{\sqrt{3}/2}$



23.(A) Line must be $\perp$ to AB.

$\Rightarrow$ slope of line $= \dfrac{-1}{\left(\dfrac{1-2}{3-1}\right)} = 2 \quad \Rightarrow \quad y - 2 = 2(x - 1) \Rightarrow y = 2x$



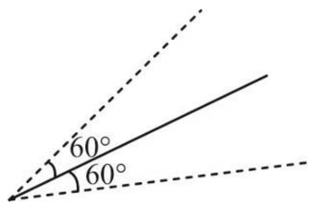
24.(C) Statement - 2 False. Determinant of coefficients should be zero.

$$\begin{vmatrix} 3 & 4 & 6 \\ \sqrt{2} & \sqrt{3} & 2\sqrt{2} \\ 4 & 7 & 8 \end{vmatrix} = 3\left(8\sqrt{3} - 14\sqrt{2}\right) - 4\left(8\sqrt{2} - 8\sqrt{2}\right) + 6\left(7\sqrt{2} - 4\sqrt{3}\right) = 0 \quad \Rightarrow \quad \text{lines are concurrent.}$$

25.(A) Given line: $y + 2 = \dfrac{x}{\sqrt{3}}$

Slope of lines inclined at 60° to the given lines:

$$m_1 = \frac{\dfrac{1}{\sqrt{3}} - \sqrt{3}}{1 + \dfrac{1}{\sqrt{3}}.\sqrt{3}} \text{ and } m_2 = \frac{\dfrac{1}{\sqrt{3}} + \sqrt{3}}{1 - \dfrac{1}{\sqrt{3}}.\sqrt{3}} \Rightarrow m_1 = \frac{-1}{\sqrt{3}} \text{ and } m_2 = \infty$$

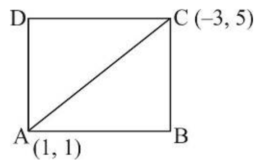


$\Rightarrow$ Equation of lines are: $\left(y - 9\right) = \dfrac{-1}{\sqrt{3}}\left(x - 7\right) \Rightarrow x + \sqrt{3}y = 7 + 9\sqrt{3}$ and $x = 7$

26.(C) Slope of diagonal $= \dfrac{5-1}{-3-1} = -1 \quad \Rightarrow \quad$ slope of $AB = 0$ and slope of $AD = \infty$

$\Rightarrow$ Equation of line AB: $y = 1$

and Equation of line AD: $x = 1$



27.(D) Diagonals are perpendicular. Hence, $PQRS$ must be a rhombus.

28.(B) $x(a + 2b) + y(a + 3b) = a + b \quad \Rightarrow \quad a(x + y - 1) + b(2x + 3y - 1) = 0$

$\Rightarrow$ Family of lines passing through the intersection of $x + y - 1 = 0$ and $2x + 3y - 1 = 0$

Solve to get $\equiv (2, -1)$

29.(A) Family of lines passing through intersection of given lines is:

$(2x + y - 1) + \lambda(3x + 2y - 5) = 0$

Since, line passes through origin $\Rightarrow \lambda = -\dfrac{1}{5} \quad \Rightarrow$ Equation of line: $7x + 3y = 0$

30.(D) Family of lines passing through intersection of given lines is: $(x + 2y - 5) + \lambda(3x + 7y - 17) = 0$

Since, it is perpendicular to $3x + 4y = 10 \Rightarrow -\dfrac{(1 + 3\lambda)}{2 + 7\lambda} \times \left(\dfrac{-3}{4}\right) = -1 \Rightarrow \lambda = -\dfrac{11}{37}$

$\Rightarrow$ Equation of line $\equiv 4x - 3y + 2 = 0$

31.(D) Equation of bisectors are:

$\dfrac{3x - 4y + 7}{\sqrt{25}} = \pm \dfrac{12x - 5y - 8}{\sqrt{169}} \Rightarrow 13\left(3x - 4y + 7\right) = \pm 5\left(12x - 5y - 8\right)$

Taking positive sign; we get: $21x + 27y - 131 = 0$

Taking negative sign; we get: $99x - 77y + 51 = 0$

32.(D) Area of $\Delta = \frac{1}{2}\begin{vmatrix} k+1 & k+2 & 1 \\ k & k+1 & 1 \\ k+1 & k & 1 \end{vmatrix} = (k+1)\left[k+1-k\right]-(k+2)\left[k-(k+1)\right]+\left[k^2-(k+1)^2\right]$

$\Rightarrow \quad (k+1)+(k+2)-(2k+1)=2$

33.(B) Roots of equation: $x^2+4x+3=0 \Rightarrow x=-1, -3$ and $x^2-x-6=0 \Rightarrow x=-2, 3$

$B \equiv (-3, 3)$ and $C \equiv (-1, -2)$

$A \equiv (3, -5)$

$AB = \sqrt{(3+3)^2+(-5-3)^2} = 10$; $AC \equiv \sqrt{(3+1)^2+(-5+2)^2} = 5$

$\therefore \quad \frac{BD}{DC} = \frac{AB}{AC} = \frac{10}{5} = 2:1$

$\therefore \quad D \equiv \left(\frac{2(-1)+1(-3)}{2+1}, \frac{2(-2)+1\times(3)}{2+1}\right) \equiv \left(\frac{-5}{3}, \frac{-1}{3}\right) \Rightarrow AD = \sqrt{\left(3+\frac{5}{3}\right)^2+\left(-5+\frac{1}{3}\right)^2} = \frac{14\sqrt{2}}{3}$

34.(A) Let $A \equiv (-a, 0)$, $B \equiv (a, 0)$

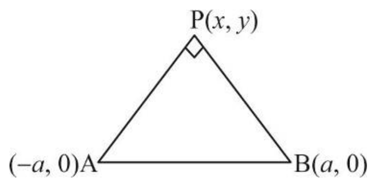
where a is a fixed value.

$P \equiv (x, y)$ is moving.

$\because \quad \angle APB$ is $90° \Rightarrow AP^2 + PB^2 = AB^2$

$\Rightarrow \quad (x+a)^2 + y^2 + (x-a)^2 + y^2 = (2a)^2$

$\Rightarrow \quad x^2 + y^2 = a^2 \Rightarrow$ Represents circle.



35.(C) Slope of the lines at an angle ϕ with $y = mx + b$ is:

$\frac{m+\tan\phi}{1-m\tan\phi}$ and $\frac{m-\tan\phi}{1+m\tan\phi}$

Since, the required line passes through origin, equation of the required line: $\frac{y}{x} = \frac{m \pm \tan\phi}{1 \mp m\tan\phi}$

36.(C) $SQ^2 + SR^2 = 2SP^2 \qquad S \equiv (x, y)$

$\Rightarrow \quad \left[(x+1)^2+(y-0)^2\right]+\left[(x-2)^2+y^2\right]=2\left[(x-1)^2+y^2\right]$

$\Rightarrow \quad 2x+1-4x+4 = 2[-2x+1] \quad \Rightarrow \quad x = \frac{-3}{2}$ [A straight line parallel to Y-axis]

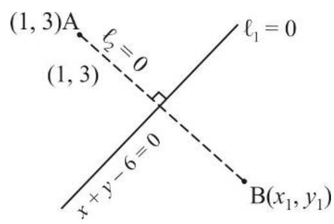
37.(A) To find the Image of a point in a line, we use following conditions:

(i) $\quad l_1$ is perpendicular to l_2

(ii) $\quad$ Mid-point of AB lies on l_1

$\Rightarrow \quad \frac{y_1-3}{x_1-1}(-1) = -1 \Rightarrow y_1 = x_1+2$ **...(i)**

and $\frac{x_1+1}{2}+\frac{y_1+3}{2}-6=0 \Rightarrow x_1+y_1=8$ **...(ii)**



Solving (i) and (ii) we get: $(x_1, y_1) \equiv (3, 5)$

38.(B) It is given that point A and B are fixed. Only point C is moving.

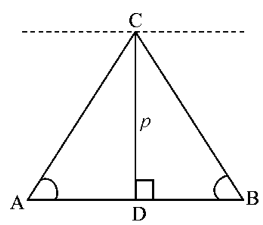
And, $\cot A + \cot B$ = constant = k

$\Rightarrow \quad \frac{AD}{P} + \frac{BD}{P} = K$ ..... [Using ΔACD and ΔBCD]

$\Rightarrow \quad P = \frac{AB}{K}$ = constant

Hence, C lies on a line which is always at a distance P from AB.

Locus of C is a straight line parallel to AB.



39.(A) $l_1 \equiv 3x + 4y = 9$ **...(i)** $l_2 \equiv y - mx = 1$ **...(ii)**

Solving (i) and (ii) we get, $x = \dfrac{5}{4m+3}$

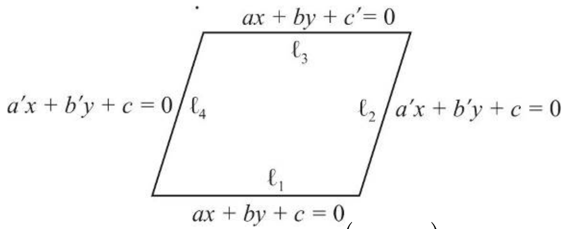
x is an integer when $m = -1, -2$.
Hence, two values of m are possible.

40.(C) It is given that diagonal of a parallelogram is perpendicular, it means it is a rhombus.

$\Rightarrow$ In a rhombus, distance between two parallel lines are same.

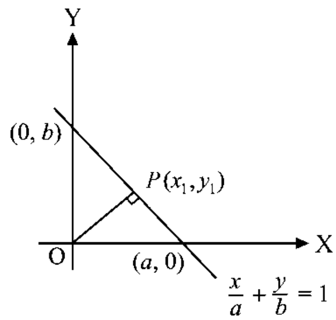
$\Rightarrow \dfrac{|c-c'|}{\sqrt{a^2+b^2}} = \dfrac{|c-c'|}{\sqrt{a'^2+b'^2}}$

$\Rightarrow a^2 + b^2 = a'^2 + b'^2$



41.(B) From the figure it is clear that line OP is perpendicular to given line and (x_1, y_1) lies on given line;

$\Rightarrow \dfrac{x_1}{a} + \dfrac{y_1}{b} = 1$ **...(i)**



Since the line $\dfrac{x}{a} + \dfrac{y}{b} = 1$ is perpendicular to the line joining (0, 0) and (x_1, y_1)

$\Rightarrow \dfrac{y_1}{x_1} \times \dfrac{-b}{a} = -1 \Rightarrow b = \dfrac{ax_1}{y_1}$ **...(ii)**

From (i) and (ii), we get:

$a = \dfrac{x_1^2 + y_1^2}{x_1}, \quad b = \dfrac{x_1^2 + y_1^2}{y_1}$ **...(iii)**

$\because \dfrac{1}{a^2} + \dfrac{1}{b^2} = \dfrac{1}{c^2} \Rightarrow \dfrac{x_1^2 + y_1^2}{\left(x_1^2 + y_1^2\right)^2} = \dfrac{1}{c^2} \Rightarrow x_1^2 + y_1^2 = c^2$

Hence the locus of foot of perpendicular is: $x^2 + y^2 = c^2$

42.(B) The family of the lines $(x + y - 1) + \lambda\ (2x + 3y - 5) = 0$
passes through intersection of

$x + y - 1 = 0$ **...(i)**

$2x + 3y - 5 = 0$ **...(ii)**

Solving (i) and (ii), we get $(x_1, y_1) \equiv (-2, 3)$

Family of the line $(3x + 2y - 4) + (x + 2y - 6) = 0$
Passes through Intersection of

$3x + 2y - 4 = 0$ **...(iii)**

$x + 2y - 6 = 0$ **...(iv)**

Solving (iii) and (iv), we get: $(x_2, y_2) \equiv \left(-1, \dfrac{7}{2}\right)$

Equation of line belonging to both the families will pass through (x_1, y_1) and (x_2, y_2)

$\Rightarrow y - 3 = \dfrac{\frac{7}{2} - 3}{-1+2}(x+2) \Rightarrow x - 2y + 8 = 0$ belongs to both the families.

43.(B) Suppose $f(x, y) = a^2x + aby + 1 \quad \forall\ a \in R, b > 0$

Origin and (1, 1) will lie on the same side if $f(0, 0)$ and $f(1, 1)$ have same sign.

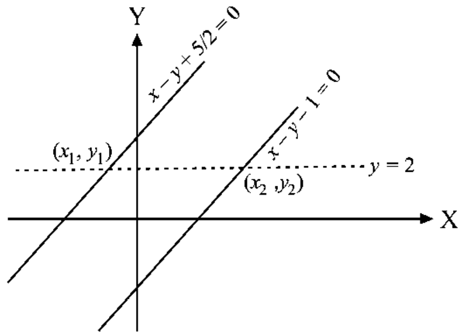
$\Rightarrow f(0, 0).f(1, 1) > 0 \Rightarrow 1.(a^2 + ab + 1) > 0 \quad \forall\ a \in R$

$\Rightarrow D < 0 \Rightarrow b^2 - 4 < 0 \Rightarrow b \in (-2, 2)$ **...(i)**

But $\quad b > 0$ **...(ii)**

$\Rightarrow$ Combining (i) and (ii), we have: $b \in (0, 2)$

44.(D) Let $\ell_1(x, y) = x - y - 1$ and $\ell_2(x, y) = x - y + 5/2$.



Then from figure (a, 2) will lie on $y = 2$ line.
Now solve $y = 2$ with $\ell_1 = 0$ we get:

$(x_1, y_1) \equiv (3,2)$ **...(i)**

Similarly solve $y = 2$ with $\ell_2 = 0$ we get:

$(x_2, y_2) \equiv (-1/2, 2)$ **...(ii)**

From (i) and (ii) range of $a \in (-1/2, 3)$

45.(A) Given Equations of lines are

$x - 2y + 3 = 0$ **...(i)**

$kx + 3y + 1 = 0$ **...(ii)**

$4x - ky + 2 = 0$ **...(iii)**

All three lines will be concurrent only if

$$\begin{vmatrix} 1 & -2 & 3 \\ k & 3 & 1 \\ 4 & -k & 2 \end{vmatrix} = 0 \Rightarrow 1(6 + k) + 2(2k - 4) + 3(-k^2 - 12) = 0 \Rightarrow -3k^2 + 5k - 38 = 0$$

$\Rightarrow \quad 3k^2 - 5k + 38 = 0$ **...(iv)**

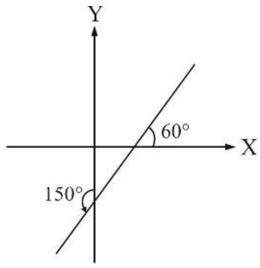
D = 25 – 3.4.48 < 0 $\Rightarrow$ No roots of equation (iv) is possible

Hence, number of possible values of k is zero.

46.(C) Given equation of line is $x = -2 + \dfrac{r}{\sqrt{10}}$ **...(i)** and $y = 1 + \dfrac{3r}{\sqrt{10}}$ **...(ii)**

Eliminate r from equation (i) and (ii) we get: $3x - y + 7 = 0$

Hence, slope of the line is 3



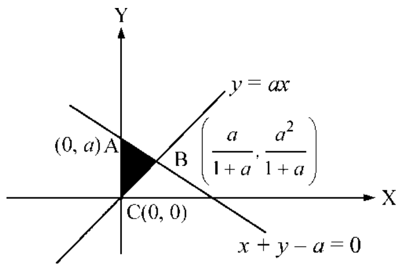
47.(B) Slope = $\tan 60° = +\sqrt{3}$; $y = \sqrt{3}x + C$

$\Rightarrow$ Distance from origin:

$$\frac{|C|}{\sqrt{1 + (\sqrt{3})^2}} = 7 \Rightarrow |C| = 14 \Rightarrow C \pm 14$$

$\Rightarrow \quad \sqrt{3}x - y \pm 14 = 0$

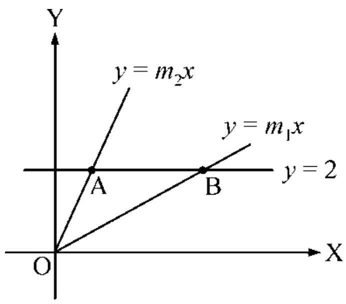
48.(D) The vertices of the triangle are:



$A \equiv (0, a)$, $B \equiv \left(\dfrac{a}{1+a}, \dfrac{a^2}{1+a}\right)$ and $C \equiv (0, 0)$

$$\Delta = \frac{1}{2} \times a \times \frac{a}{(1+a)} = \frac{a^2}{2(1+a)} = \frac{a^2}{2|1+a|}$$

49.(B) $A \equiv \left(\dfrac{2}{m_2}, 2\right)$; $B \equiv \left(\dfrac{2}{m_1}, 2\right)$

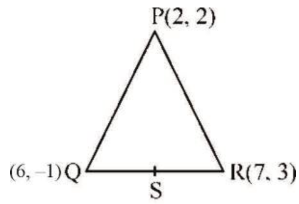


$$\text{Area} = \frac{1}{2}\begin{vmatrix} 0 & 0 & 1 \\ 2/m_1 & 2 & 1 \\ 2/m_2 & 2 & 1 \end{vmatrix} = 2\left|\frac{1}{m_1} - \frac{1}{m_2}\right|$$

$$= \frac{2\sqrt{m_1^2 + m_2^2 - 2m_1m_2}}{\sqrt{3} - 1} = \frac{2\sqrt{(m_1 + m_2)^2 - 4m_1m_2}}{m_1m_2}$$

$\Rightarrow \quad Area = \dfrac{2\sqrt{(\sqrt{3}+2)^2 - 4(\sqrt{3}-1)}}{\sqrt{3}-1} = \sqrt{33} + \sqrt{11}$

50.(D) Midpoint of QR is: $S \equiv \left(\dfrac{6+7}{2}, \dfrac{3-1}{2}\right) \equiv \left(\dfrac{13}{2}, 1\right)$

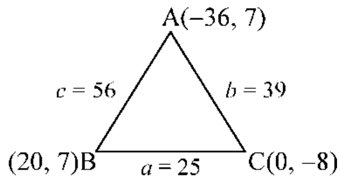


Slope of $PS = \dfrac{2-1}{2-\dfrac{13}{2}} = \dfrac{-2}{9}$

Equation of line parallel to PS and passing through $(1, -1)$ is:

$(y+1) = -\dfrac{2}{9}(x-1) \Rightarrow 2x + 9y + 7 = 0$

51.(B) Length of Sides BC, CA and AB are 25, 39 and 56 unit respectively

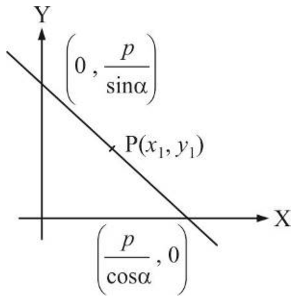


Incentre $(x', y') \equiv \left(\dfrac{ax_1 + bx_2 + cx_3}{a+b+c}, \dfrac{cy_1 + by_2 + cy_3}{a+b+c}\right)$

$\equiv \left(\dfrac{25 \times (-36) + 39 \times (20) + 56 \times (0)}{25 + 39 + 56}, \dfrac{25 \times 7 + 39 \times (7) + 56 \times (-8)}{25 + 39 + 56}\right)$

$\therefore \quad (x', y') \equiv (-1, 0)$

52.(B) The mid-point to intercepts between the axes are: $(2x_1, 2y_1) \equiv \left[\dfrac{p}{\cos\alpha}, \dfrac{p}{\sin\alpha}\right]$



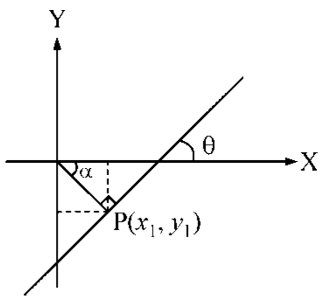
$\Rightarrow \quad x_1 = \dfrac{p}{2\cos\alpha}, \; y_1 = \dfrac{p}{2\sin\alpha}$

$\Rightarrow \quad \cos\alpha = \dfrac{p}{2x_1}, \; \sin\alpha = \dfrac{p}{2y_1}$

$\Rightarrow \quad \cos^2\alpha + \sin^2\alpha = 1 = \dfrac{p^2}{4x_1^2} + \dfrac{p^2}{4y_1^2}$

$\Rightarrow \quad \dfrac{1}{x_1^2} + \dfrac{1}{y_1^2} = \dfrac{4}{p^2} \xrightarrow[x_1 \to x, y_1 \to y]{\text{Replace}} \dfrac{1}{x^2} + \dfrac{1}{y^2} = \dfrac{4}{p^2}$

53.(B) $\tan\theta = \dfrac{3}{4} \Rightarrow \cot\alpha = \dfrac{3}{4}$



Apply $d = \dfrac{|C|}{\sqrt{A^2 + B^2}} = \dfrac{|25|}{5} = 5$

$(x_1, y_1) \equiv (5\cos\alpha, -5\sin\alpha) = (3, -4)$

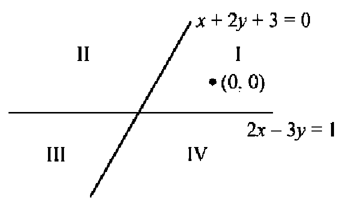
Another Approach:

It can be easily seen that P lies in 4th Quadrant and one option is of such type.

54.(A) Point $(1, 2)$ and $(2, 1)$ lie on same sides of $4x + 2y = 1$

$[\because 4(1) + 2(2) - 1 > 0, \; 4(2) + 2(1) - 1 > 0]$

55.(A) **For (2, –1):** $(2, -1) \Rightarrow \quad 2 + 2(-1) + 3 > 0, \; (0, 0)$



$\Rightarrow \quad 0 + 2(0) + 3 > 0$

$\therefore \quad (2, -1)$ lies on right of $x + 2y + 3 = 0$

$(2m - 1) \Rightarrow \quad 2(2) - 3(-1) - 1 > 0, (0, 0) \Rightarrow \quad 2(0) - 3(0) - 1 < 0$

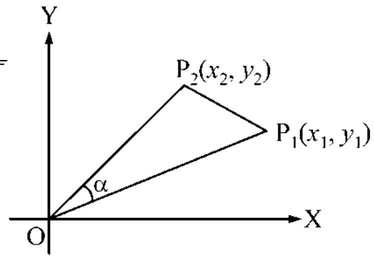
Hence $(2, -1)$ lies in 4th quadrant.

For (3, 2): $(1)(3) + 2(2) + 3 > 0$, point lies on right of $x + 2y + 3 = 0$.

Hence option (B) is incorrect.

For (–1, –2): $2(-1) - 3(-2) - 1 > 0$, point lies below $2x - 3y - 1 = 0$. Hence option (C) is incorrect.

56.(D) $\tan\alpha = \left|\dfrac{\dfrac{y_2}{x_2} - \dfrac{y_1}{x_1}}{1 + \dfrac{y_1 y_2}{x_1 x_2}}\right| \Rightarrow \cos\alpha = \dfrac{1}{\sqrt{1 + \left(\dfrac{y_2 x_1 - y_1 x_2}{x_1 x_2 + y_1 y_2}\right)^2}} = \dfrac{x_1 x_2 + y_1 y_2}{\sqrt{x_1^2 + y_1^2}\sqrt{x_2^2 + y_2^2}}$



$\therefore \quad OP_1 . OP_2 \cos\alpha = x_1 x_2 + y_1 y_2$

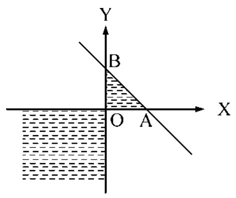
57.(C) Square and add the equations to get:

$(x\cos\alpha + y\sin\alpha)^2 + (x\sin\alpha - y\cos\alpha)^2 = a^2 + b^2 \quad \Rightarrow \quad x^2 + y^2 = a^2 + b^2$

58.(D) $(x - a_1)^2 + (y - b_1)^2 = (x - a_2)^2 + (y - b_2)^2 \Rightarrow (a_1 - a_2)x + (b_1 - b_2)y + \left[\dfrac{a_2^2 + b_2^2 - a_1^2 - b_1^2}{2}\right] = 0$

Hence, $c = \dfrac{a_2^2 + b_2^2 - a_1^2 - b_1^2}{2}$

59.(A)



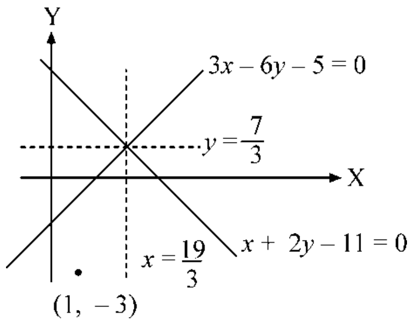
Hence, $P(x, y)$ can lie either inside ΔOAB or in third quadrant.

60.(D) Equation of line $\equiv \dfrac{y - a\sin^3\theta}{x - a\cos^3\theta} = \dfrac{\cos\theta}{\sin\theta} \left[\because \text{ slope } = \dfrac{-1}{\left(\dfrac{-\sec\theta}{\operatorname{cosec}\theta}\right)}\right] \Rightarrow x\cos\theta - y\sin\theta = a\cos 2\theta$

61.(A) The angle bisectors of two lines $x + 2y - 11 = 0$ and $3x - 6y - 5 = 0$ are :

$\dfrac{x + 2y - 11}{\sqrt{5}} = \pm \dfrac{3x - 6y - 5}{3\sqrt{5}} \quad \Rightarrow \quad 3x - 19 = 0; \; 3y - 7 = 0$

Hence, $3x = 19$ is the required angle bisector.



62.(B) Equation of side AB is: $4x - 3y - 17 = 0$

Equation of side BC is: $3x + 4y - 19 = 0$

(Slope of AB) (Slope of BC) $= (4/3)(-3/4) = -1 \Rightarrow \angle B = 90°$

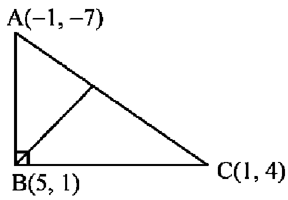
$\therefore$ Angle bisector of AB and BC: $\dfrac{(4x - 3y - 17)}{5} = \pm \dfrac{(3x + 4y - 19)}{5}$

$\Rightarrow \quad x - 7y + 2 = 0, \; 7x + y - 36 = 0$

For line $x - 7y + 2 = 0$

Points A and C lies on opposite sides.

$\therefore$ Internal angle bisector is: $x - 7y + 2 = 0$



63.(B) $ar(\Delta DBC) = \dfrac{1}{2} ar\ (\Delta ABC) \Rightarrow \dfrac{1}{2}\begin{vmatrix} x & 3x & 1 \\ -3 & 5 & 1 \\ 4 & -2 & 1 \end{vmatrix} = \dfrac{1}{2} \times \dfrac{1}{2}\begin{vmatrix} 6 & 3 & 1 \\ -3 & 5 & 1 \\ 4 & -2 & 1 \end{vmatrix}$

Or $\quad x(7) - 3x(-7) - 14 = \dfrac{1}{2}[6(7) - 3(-7) - 14]$

64.(C) $\sqrt{(x_1 + x_2)^2 + (y_1 - y_2)^2} = a, \sqrt{(x_2 - x_3)^2 + (y_2 - y_3)^2} = b$ and $\sqrt{(x_3 - x_1)^2 + (y_3 - y_1)^2} = c$

$$\left[\frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}\right]^2 = [ar\ (\Delta ABC)]^2 = s(s-a)\ (s-b)(s-c)$$

$[\because \Delta = \sqrt{s(s-a)\ (s-b)\ (s-c)}]$

65.(B) $3a + 2b - 13 = 0; 4b - a - 5 = 0 \quad \Rightarrow \quad a = 3, b = 2$

Hence, $P(3, 2), Q(2, 3) \Rightarrow$ Equation of line is $\frac{y-2}{x-3} = \frac{3-2}{2-3} = -1, \therefore x + y = 5$

66.(B) $\left[\frac{(\sin\alpha + \sin\beta)}{\sin(\alpha-\beta)}\right]\left[\frac{-(\cos\alpha+\cos\beta)}{\cos(\alpha-\beta)}\right] = -1 \Rightarrow \sin(\alpha+\beta) + \frac{\sin 2\alpha + \sin 2\beta}{2} = \frac{\sin 2(\alpha-\beta)}{2}$

$\Rightarrow \quad \sin 2\alpha + \sin 2\beta = \sin 2(\alpha-\beta) - 2\sin(\alpha+\beta)$

67.(D) Let centroid of ΔABC is (x_1, y_1) and vertex C be (x', y')

$(3x_1, 3y_1) \equiv [2 + 5 + x', 7 + 8 + y'] \Rightarrow y' = 3y_1 = 15, x' = 3x_1 - 7$

As (x', y') lies on the line 3x + 4y + 5 = 0, hence $3(3x_1 - 7) + 4(3y_1 - 15) + 5 = 0 \Rightarrow 9x_1 + 12y_1 - 76 = 0$

Replace $(x_1 y_1)$ by (x, y) to get: 9x + 12y – 76 = 0 $\Rightarrow$ Hence locus is parallel to $3x + 4y + 5 = 0$

68.(B) $\frac{x}{a} + \frac{y}{b} = 1\ ;\ d_1 = \frac{|0+0-1|}{\frac{1}{a^2}+\frac{1}{b^2}}$ (Distance from origin)

$\frac{x}{p} + \frac{y}{q} = 1\ ;\ d_2 \frac{1}{\sqrt{\frac{1}{p^2}+\frac{1}{q^2}}}$ (Distance from origin)

The perpendicular distance from the origin remains the same.

Hence, $d_1 = d_2 \Rightarrow \frac{1}{a^2} + \frac{1}{b^2} = \frac{1}{p^2} + \frac{1}{q^2}$

69.(B) $\left(\lambda^2 + 2\lambda + 5\right) + \left(\lambda^2 + 1\right) = 10 \Rightarrow 2\lambda^2 + 2\lambda - 4 = 0 \Rightarrow \lambda = 1, -2$

70.(A) Family of lines $\equiv (x - 3y + 1) + k(2x + 5y - 9) = 0 \quad \Rightarrow$ Slope of line $= -\frac{(1+2k)}{5k-3} = \infty \Rightarrow k = \frac{3}{5}$

$\therefore$ Equation of lines $(x - 3y + 1) + \frac{3}{5}(2x + 5y - 9) = 0 \Rightarrow x = 2$

71.(C) Family of lines passing through point

$A : (ax + by - 1) + k(bx + ay - 1) = 0$

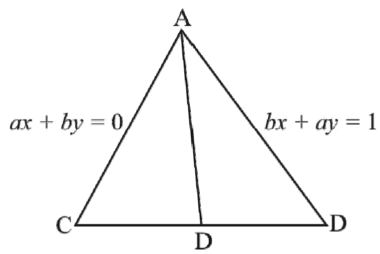
For median AD, $D(a, b)$ lies on-line

$(ax + by - 1) + k(bx + ay - 1) = 0$

$\therefore \quad (a^2 + b^2 - 1) + k(2ab - 1) = 0$

$\therefore$ Equation of median is:

$(1 - 2ab)(ax + by - 1) + (a^2 + b^2 - 1)(bx + ay - 1) = 0$



72.(C) $A(1, 2)$ when reflected in $y = x$ gives $B(2, 1)$

$B(2, 1)$ when reflected in X-axis gives $(2, 1) \equiv (\alpha, \beta)$

73.(C) Let the point equidistant from lines be $P(h, k)$ then:

$\frac{|4h + 3k + 10|}{5} = \frac{|5h - 12k + 26|}{13} = \frac{|7h + 24k - 50|}{25}$

Using hit and trial method, $(h, k) \equiv (0, 0)$ satisfies the above equations.

Another Approach:

Find the internal angle bisectors of the triangle and solve them simultaneously.

74.(B) Given lines are: $4x - 3y + 7 = 0, 3x - 4y + 14 = 0$ where $a_1 a_2 + b_1 b_2 = 4(3) + (-3)(-4) > 0$

Hence negative sign gives acute angle bisector.

$$\frac{4x-3y+7}{5}=-\left(\frac{3x-4y+14}{5}\right)\Rightarrow x-y+3=0$$

75.(B) Let $PQ=r$ $\quad\therefore\quad$ $Q\equiv(x_1+r\cos\theta,\ y_1+r\sin\theta)$

Since point Q lies on the given line

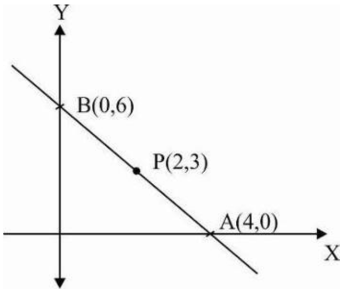
$\therefore\quad a(x_1+r\cos\theta)+b(y_1+r\sin\theta)+c=0\Rightarrow r=\dfrac{-(ax_1+by_1+c)}{a\cos\theta+b\sin\theta}\quad\Rightarrow\quad r=\left|\dfrac{ax_1+by_1+c}{a\cos\theta+b\sin\theta}\right|$

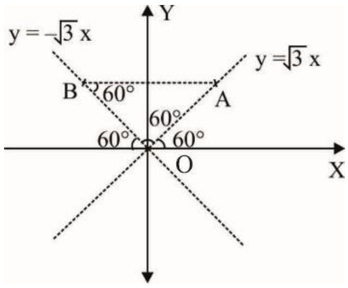
76.(B) As P is the mid-point of AB,

$\therefore$ A(4,0), B(0, 6)

Therefore, equation of line AB;

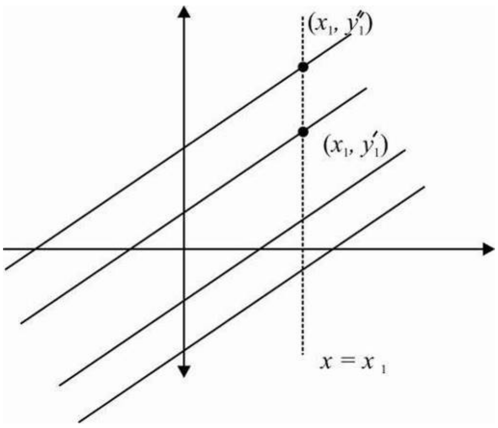
$\dfrac{x}{4}+\dfrac{y}{6}=1\Rightarrow 3x+2y=12$





77.(B) From figure, lines OA and OB are symmetrical about Y-axis and holds 60° angles with X-axis as shown in figure. Hence, line AB is parallel to X-axis. Equation of line is y = 2 as it passes through (2, 2).

78.(C) Lines are parallel to each other and cuts line $x=x_1$ at different points.

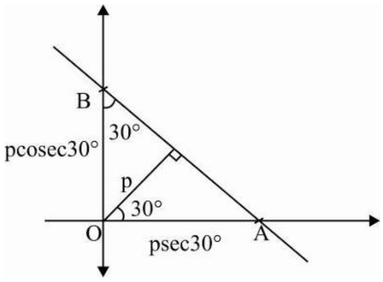


79.(B) $\dfrac{1}{2}p\sec 30°\times p\,\mathrm{cosec}\,30°=\dfrac{50}{\sqrt{3}}$

$\dfrac{1}{2}p^2\times\dfrac{2}{\sqrt{3}}\times 2=\dfrac{50}{\sqrt{3}}\Rightarrow p=5$

$x\cos 30°+y\sin 30°=5$

$\Rightarrow\quad \sqrt{3}x+y-10=0$



80.(B) Image of B(4, −1) is A(−1, 4) in $y=x$ line. Length of AB = $5\sqrt{2}$ units.

81.(A) Solve $x+2y-9=0$ and $3x+5y-5=0$ to get $x=-35$ and y = 22

For $ax+by-1=0$, we have $a(-35)+b(22)-1=0$

or $\quad 35a-22b+1=0$

Hence $35x-22y+1=0$ passes through (a, b).

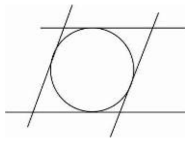
82.(B) Let variable line be $ax+by+c=0$

According to questions,

$\dfrac{2a+c}{\sqrt{a^2+b^2}}+\dfrac{2b+c}{\sqrt{a^2+b^2}}+\dfrac{a+b+c}{\sqrt{a^2+b^2}}=0\ \Rightarrow\ 3a+3b+3c=0\Rightarrow ax+by+c=0$ passes through point (1, 1).

83.(B) $\frac{b_1}{b_2} = \frac{c_1}{c_2} \Rightarrow \frac{c_1}{b_1} = \frac{c_2}{b_2}$. Both lines have same Y-intercept. Hence given family is family of concurrent lines having the same Y-intercept.

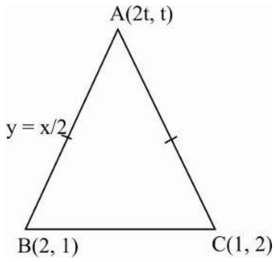
84.(B) $y = mx \pm a\sqrt{1+m^2}, y = nx \pm a\sqrt{1+n^2}$



These lines are parallel to each other and form a rhombus enclosing a circle

$x^2 + y^2 = a^2$.

85.(B) $\sqrt{(2t-2)^2 + (t-1)^2} = \sqrt{(2t-1)^2 + (t-2)^2}$



$\Rightarrow \quad 5(t-1)^2 = (2t-1)^2 + (t-2)^2 \Rightarrow t = 0$

$\Rightarrow \quad$ A(0, 0)

Equation of AC: $y = 2x$

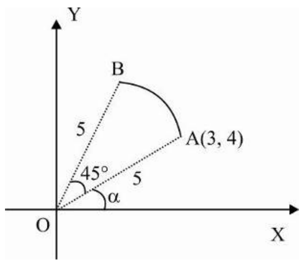
86.(B) If all three co-ordinates are rational numbers then side[2] will be a rational number as well.

Area = $= \frac{\sqrt{3}}{4}(side)^2$ = irrational number

Area = $\frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$ = rational number which is in contradiction to above.

Hence, third vertex cannot have rational coordinates.

87.(A) $(4,\ 1) \xrightarrow{1} (1,\ 4) \xrightarrow{2} (3,\ 4) \xrightarrow{3} \left(-\frac{7}{\sqrt{2}}, \frac{7}{\sqrt{2}}\right)$



$\therefore \quad B \equiv \left(5\cos(45° + \alpha), 5\sin(45° + \alpha)\right)$

$\equiv \left(\frac{1}{\sqrt{2}}, \frac{7}{\sqrt{2}}\right), \quad \cos\alpha = 4/5$

88.(A) $\sin\alpha - \frac{1}{\sqrt{2}}$ and $\frac{1}{\sqrt{2}} - \cos\alpha$ have same sign i.e.

$\left(\sin\alpha - \frac{1}{\sqrt{2}}\right)\left(\frac{1}{\sqrt{2}} - \cos\alpha\right) > 0$ or $\left(\sin\alpha - \frac{1}{\sqrt{2}}\right)\left(\cos\alpha - \frac{1}{\sqrt{2}}\right) < 0$

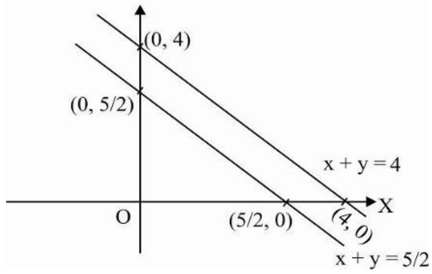
$\sin\alpha - \frac{1}{\sqrt{2}} > 0, \cos\alpha - \frac{1}{\sqrt{2}} < 0$ or $\sin\alpha - \frac{1}{\sqrt{2}} < 0, \cos\alpha - \frac{1}{\sqrt{2}} > 0$

$\alpha \in \left(\frac{\pi}{4}, \frac{3\pi}{4}\right) \qquad \alpha \in \left(-\frac{\pi}{4}, \frac{\pi}{4}\right)$

Combining, we get: $\alpha \in \left(-\frac{\pi}{4}, \frac{\pi}{4}\right) \cup \left(\frac{\pi}{4}, \frac{3\pi}{4}\right)$

89.(C) $x + x\tan^2\theta + y\tan^2\theta - 2 = 0 \quad \Rightarrow \quad (x-2) + \tan^2\theta(x+y) = 0$

$x = 2, y = -2$

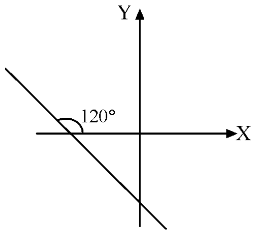


90.(A) Distance between | | lines $= \frac{\left|4 - \frac{5}{2}\right|}{\sqrt{2}} = \frac{3}{2\sqrt{2}}$

Hence, no point lies on $x + y = 4$ which is at a distance of 1 unit from $x + y = \frac{5}{2}$

Level – 2 & Numerical Value Type	Daily Tutorial Sheet - 7 to 12

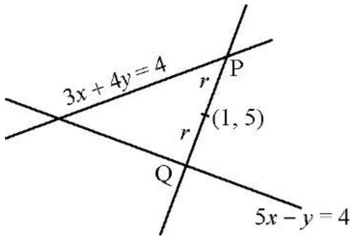
91.(C) Incentre $(x', y') \equiv \left[\dfrac{ab+0+0}{a+b+\sqrt{a^2+b^2}}, \dfrac{ab+0+0}{a+b+\sqrt{a^2+b^2}}\right]$.



92.(B) Slope = $tan\ 120° = -\sqrt{3}$

$\Rightarrow \quad \dfrac{y+5}{x-0} = -\sqrt{3} \Rightarrow y+\sqrt{3}x+5=0$

93.(C) $P \equiv (1 + r\cos\theta, 5 + r\sin\theta)$, $Q \equiv (1 - r\cos\theta, 5 - r\sin\theta)$



Since, P lies on $3x + 4y = 4$

$\Rightarrow \quad 3(1 + r\cos\theta) + 4(5 + r\sin\theta) = 4$

$\Rightarrow \quad 3r\cos\theta + 4r\sin\theta + 19 = 0$ **...(i)**

and Q lies on $5x - y - 4 = 0$

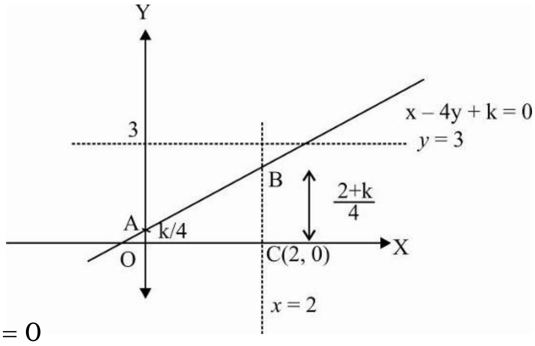
$\Rightarrow \quad 5(1 - r\cos\theta) - (5 - r\sin\theta) - 4 = 0$

$\Rightarrow \quad 5r\cos\theta - r\sin\theta + 4 = 0$ **...(ii)**

Solve (i) and (ii) to get: $r\cos\theta = -\dfrac{35}{23}$ and $r\sin\theta = -\dfrac{83}{23}$

$\Rightarrow \quad \tan\theta = \dfrac{83}{35} \quad \Rightarrow \quad$ Equation of line $\equiv 83x - 35y + 92 = 0$

94.(B) Area of trapezium $= \dfrac{1}{2}\left[\dfrac{k}{4} + \dfrac{2+k}{4}\right] \times 2 = \dfrac{1}{2} \times 2 \times 3$



$\Rightarrow \quad k = 5$

Equation of line is:

$x - 4y + 5 = 0$

95.(B) Let $Ax + By + C = 0$ be the equation of line.

Then, $\dfrac{Ax_1 + By_1 + C}{\sqrt{A^2+B^2}} + \dfrac{Ax_2 + By_2 + C}{\sqrt{A^2+B^2}} + \dfrac{Ax_3 + By_3 + C}{\sqrt{A^2+B^2}} = 0$

$\Rightarrow \quad A\left(\dfrac{x_1+x_2+x_3}{3}\right) + B\left(\dfrac{y_1+y_2+y_3}{3}\right) + C = 0$

$\Rightarrow \quad A(x_g) + B(y_g) + C = 0 \quad$ [where $(x_g, y_g) \equiv$ centroid of ΔABC]

Hence, line passes through the centroid of triangle.

96. (B) Suppose co-ordinate of centroid is (x_1, y_1) then:

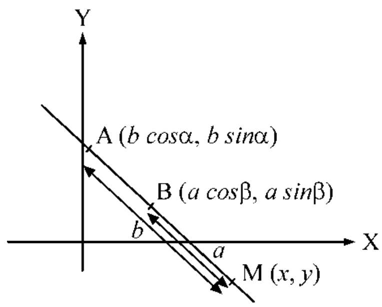
$\Rightarrow \quad x_1 = \dfrac{\cos\alpha + \sin\alpha + 1}{3} \quad \Rightarrow \quad \cos\alpha + \sin\alpha = 3x_1 - 1$ **...(i)**

and $\quad y_1 = \dfrac{\sin\alpha - \cos\alpha + 2}{3} \quad \Rightarrow \quad \sin\alpha - \cos\alpha = 3y_1 - 2$ **...(ii)**

Square and add (i) and (ii) $\quad \Rightarrow \quad (3x_1 - 1)^2 + (3y_1 - 2)^2 = 2$

Replace x_1 by x and y_1 by y $\quad \Rightarrow \quad 3(x^2 + y^2) - 2x - 4y + 1 = 0$

97.(B) Using section formula, we have:



$x = \dfrac{ab\cos\alpha - ab\cos\beta}{a-b}$ and $y = \dfrac{ab\sin\alpha - ab\sin\beta}{a-b}$

Dividing both to get:

$$\frac{x}{y}=\frac{-2\sin\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)}{2\cos\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha+\beta}{2}\right)} \quad \Rightarrow \quad x\cos\left(\frac{\alpha+\beta}{2}\right)+y\sin\left(\frac{\alpha+\beta}{2}\right)=0$$

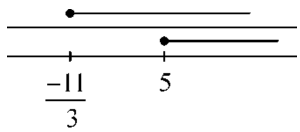
98.(A) For a triangle, sum of two sides is always greater than the third side.

(i) $(a^2+2a)+(2a+3)>a^2+3a+8 \;\Rightarrow\; a>5$

(ii) $(a^2+2a)+(a^2+3a+8)>2a+3$

$\Rightarrow \quad 2a^2+3a+5>0 \;\Rightarrow\; a\in R$

($\because$ Coefficient of x^2 > and $D<0$)



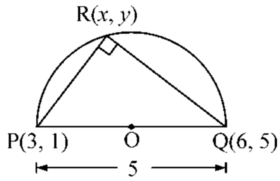
(iii) $(a^2+3a+8)+(2a+3)>a^2+2a$

$\Rightarrow \quad 3a+11>0 \;\Rightarrow\; a>\frac{-11}{3}$

99.(A) $ar(\Delta RQP)=7=\frac{1}{2}(5)(h) \;\Rightarrow h=2.8\ units$

From semicircle drawn, maximum height of Δ can be 2.5 units.

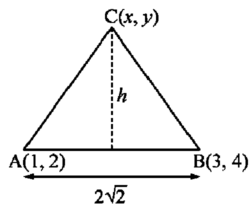
Hence no such $R(x, y)$ can exist



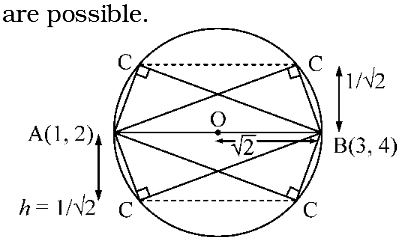
100.(B) $(x-1)(x-3)+(y-2)(y-4)=0 \quad \Rightarrow \quad \frac{(y-2)(y-4)}{(x-1)(x-3)}=-1$

$\Rightarrow \quad$ Slope $(AC)\times$ slope $(BC)=-1 \quad \Rightarrow \quad \angle ACB=90°$

$ar(\Delta ABC)=1 \quad \Rightarrow \quad \frac{1}{2}\times 2\sqrt{2}\times h=1 \quad \Rightarrow \quad h=\frac{1}{\sqrt{2}}$



Hence 4 positions of C are possible.

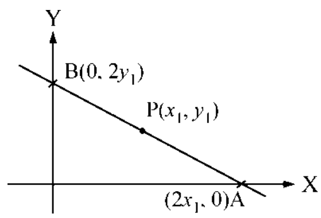


101.(C) Family of lines passing through the point of intersection of two given lines is:

$(x+2y-1)+k(2x-y-1)=0$

$\Rightarrow \quad$ X – intercept $=2x_1=\frac{k+1}{2k+1}$, Y-intercept $=2y_1=\frac{k+1}{2-k}$

Eliminate k to get: $x_1+3y_1=10x_1y_1 \xrightarrow[x_1\to x,\ y_1\to y]{\text{Replace}} x+3y=10xy$



102.(A) Area of equilateral $\Delta=\frac{\sqrt{3}}{4}(\text{side})^2\equiv$ Irrational Number

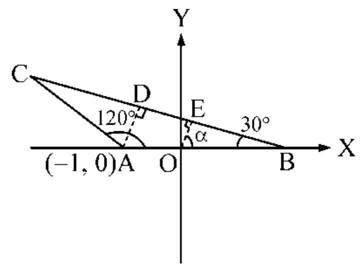
Area of any triangle $=\frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1\\ x_2 & y_2 & 1\\ x_3 & y_3 & 1\end{vmatrix}=$ Rational Number as $(x_1, y_1), (x_2, y_2), (x_3, y_3)$ are integers.

Hence Δ cannot be equilateral.

103.(A) $(5p-3q)^2-r^2=0 \quad \Rightarrow \quad (5p-3q+r)(5p-3q-r)=0$

Hence, $(5)p+(-3)q+r=0, (-5)p+(3)q+r=0$

So, $px+qy+r=0$ passes through $(5,-3), (-5,3)$



104.(C) $AD = BD\tan 30°$ $\Rightarrow$ $AD = 2\sqrt{3}\tan 30° = 2$ units .

$AC = AB = \sqrt{(2)^2 + (2\sqrt{3})^2} = 4$ units

In ΔABD, $OA = 1$, $AB = 4$ $\Rightarrow$ $OB = 3$

In ΔOEB, $OE = OB\sin 30° = \frac{3}{2}$ units

Also, $\alpha = 180° - (90° + 30°) = 60°$

$\therefore$ Equation of line in normal form is: $x\cos\alpha + y\sin\alpha = p$ $\Rightarrow$ $x + \sqrt{3}y = 3$

105.(C) Family of lines passing through point P is:

$(x\cos\alpha + y\sin\alpha - c) + k(x\sin\alpha - y\cos\alpha) = 0$

$(\cos\alpha + k\sin\alpha)x + (\sin\alpha - k\cos\alpha)y - c = 0$ **... (i)**

$ax + by + c = 0$ **... (ii)**

As (i) and (ii) are same, hence we have consistent lines.

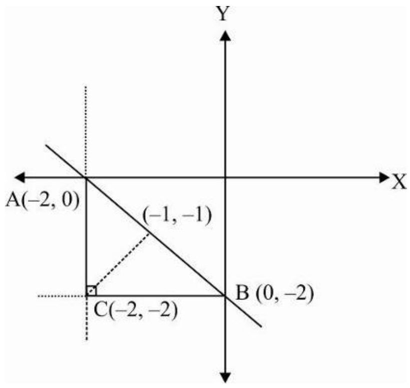
$$\frac{\cos\alpha + k\sin\alpha}{a} = \frac{\sin\alpha - k\cos\alpha}{b} = \frac{-c}{c} \Rightarrow \cos\alpha + k\sin\alpha = -a \text{ and } \sin\alpha - k\cos\alpha = -b$$

Square and add to get: $1 + k^2 = a^2 + b^2$

$$\text{Also } \left|\frac{\left(\frac{-\cos\alpha}{\sin\alpha}\right) - \left(\frac{-\cos\alpha - k\sin\alpha}{\sin\alpha - k\cos\alpha}\right)}{1 + \left(\frac{-\cos\alpha}{\sin\alpha}\right)\left(\frac{-\cos\alpha k\sin\alpha}{\sin\alpha - k\cos\alpha}\right)}\right| = \tan\frac{\pi}{4}$$

$$\left|\frac{-\cos\alpha\sin\alpha + k\cos^2\alpha + \sin\alpha\cos\alpha + k\sin^2\alpha}{\sin^2\alpha - k\sin\alpha\cos\alpha + \cos^2\alpha + k\sin\alpha\cos\alpha}\right| = 1 \Rightarrow \left|\frac{k}{1}\right| = 1$$

Hence, (iii) becomes $a^2 + b^2 = 1 + 1 = 2$

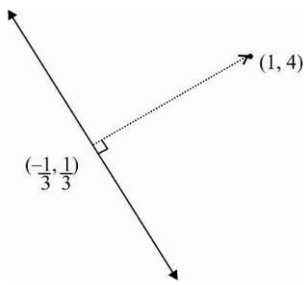


106.(B) $xy + 2x + 2y + 4 = 0 \Rightarrow (y+2)(x+2) = 0$

$(-1,-1)$ is equidistant from $(-2,0),(0,-2),(-2,-2)$

107.(B) $(x+y) + k(2x - y + 1) = 0$

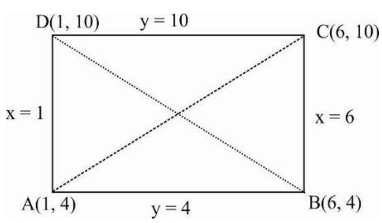
Solve $x + y = 0$ and $2x - y + 1 = 0 \Rightarrow x = -\frac{1}{3}, y = \frac{1}{3}$



Line through $\left(-\frac{1}{3},\frac{1}{3}\right)$ at a maximum distance (1, 4) is perpendicular to line joining $\left(-\frac{1}{3},\frac{1}{3}\right)$ and (1, 4) with slope $\frac{11}{4}$.

$$\therefore \quad \frac{y - \frac{1}{3}}{x + \frac{1}{3}} = -\frac{4}{11} \Rightarrow 11(3y - 1) + 4(3x + 1) = 0$$

$\Rightarrow$ $12x + 33y - 7 = 0$



108.(C) Equation of AC is: $\frac{y-4}{x-1} = \frac{10-4}{6-1}$

$\Rightarrow$ $6x - 5y + 14 = 0$

Equation of BD is: $\frac{y-4}{x-6} = \frac{10-4}{1-6}$

$\Rightarrow$ $6x + 5y = 60$

109.(D) Replace x by $-x$ in $ax^2+2hxy+by^2=0$

To get: $ax^2-2hxy+by^2=0$

110.(C) Given:

(i) A line through (1, 2) meets the coordinate axes at P and Q.

(ii) The area of ΔOPQ is minimum.

The slope of line PQ.

Let m be the slope of the line PQ, then the equation of PQ is $y-2=m(x-1)$

Now, PQ meets X-axis at $P\left(1-\frac{2}{m}, 0\right)$ and Y-axis at $Q(0, 2-m)$.

$\Rightarrow \quad OP=1-\frac{2}{m} \quad$ and $\quad OQ=2-m$

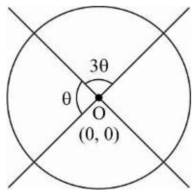
Also, area of $\Delta OPQ=\frac{1}{2}(OP)(OQ)=\frac{1}{2}\left|\left(1-\frac{2}{m}\right)(2-m)\right|=\frac{1}{2}\left|2-m-\frac{4}{m}+2\right|=\frac{1}{2}\left|4-\left(m+\frac{4}{m}\right)\right|$

Let $f(m)=4-\left(m+\frac{4}{m}\right) \Rightarrow \quad f'(m)=-1+\frac{4}{m^2}$

Now, $f'(m)=0 \quad \Rightarrow \quad m^2=4 \Rightarrow m=\pm 2 \quad \Rightarrow \quad f(2)=0$ and $f(-2)=8$

Since, the area cannot be zero, hence the required value of m is -2.

111.(A) Given equation of pair of lines is $\quad ax^2+2(a+x)xy+by^2=0$



Hence, $H=a+b,\ A=a,\ B=b$

Since, $4\theta=\pi \Rightarrow \theta=\frac{\pi}{4}$

Angle between lines is given by

$\tan\theta=\frac{2\sqrt{H^2-AB}}{A+B} \quad \Rightarrow \quad \tan\frac{\pi}{4}=1=\frac{2\sqrt{(a+b)^2-ab}}{a+b} \quad \Rightarrow \quad 3a^2+3b^2+2ab=0$

112.(B) Since, the triangle, whose vertices are $(a\cos t, a\sin t), (b\sin t, -b\cos t)$ and (1, 0).

Let the coordinates of centroid be (x, y)

Then $x=\frac{a\cos t+b\sin t+1}{3} \quad \Rightarrow \quad 3x-1=a\cos t+b\sin t \quad$ ...(i)

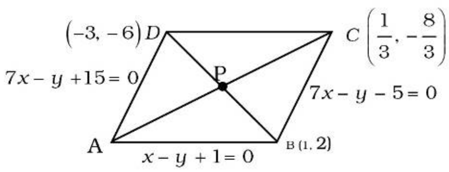
and $\quad y=\frac{a\sin t-b\cos t+0}{3} \quad \Rightarrow \quad 3y=a\sin t-b\cos t \quad$ ...(ii)

On squaring and adding equations (i) and (ii), we get:

$(3x-1)^2+(3y)^2=a^2(\cos^2 t+\sin^2 t)+b^2(\sin^2 t+\cos^2 t) \Rightarrow (3x-1)^2+(3y)^2=a^2+b^2$

$\left[\because \sin^2\theta+\cos^2\theta=1\right]$

113.(B)

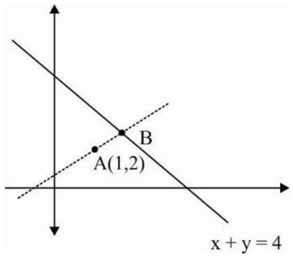


114.(B) Let line through A meets $x+y=4$ at point B such that AB makes an angle θ with +ve X-axis

Use parametric form,

$B\equiv\left(1+\frac{\sqrt{6}}{3}\cos\theta, 2+\frac{\sqrt{6}}{3}\sin\theta\right)$

$\Rightarrow \quad \left(1+\frac{\sqrt{6}}{3}\cos\theta\right)+\left(2+\frac{\sqrt{6}}{3}\sin\theta\right)=4$

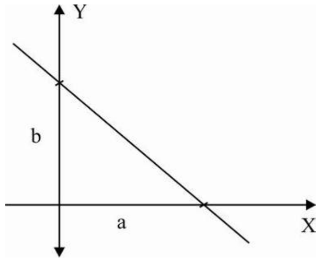


$\Rightarrow \quad \sin\theta+\cos\theta=\frac{3}{\sqrt{6}} \Rightarrow 1+\sin 2\theta=\frac{3}{2}$

$\Rightarrow \quad 2\theta = 30°$ or $2\theta = 150°$

$\theta = 15°$ $\quad \theta_{max} = 75°$

115.(B) Equation of line: $\frac{x}{a}+\frac{y}{b}=1$



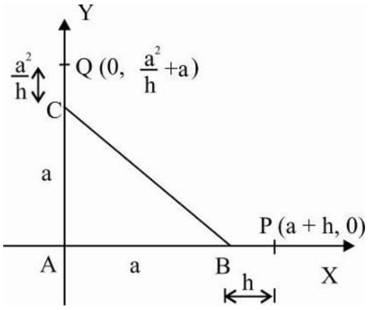
It passes through, (α, β) $\Rightarrow \frac{\alpha}{a}+\frac{\beta}{b}=1$

Now $\frac{\alpha}{a}+\frac{\beta}{b} \geq 2\sqrt{\frac{\alpha\beta}{ab}}$ $[A.M \geq G.M]$

$\Rightarrow \quad 1 \geq 2\sqrt{\frac{\alpha\beta}{ab}} \Rightarrow ab \geq 4\alpha\beta \Rightarrow \frac{1}{2}ab \geq 2\alpha\beta$

$\therefore$ Least area of triangle is $2\alpha\beta$.

116.(C) Let equal sides are along X-axis and Y-axis.



Equation of PQ is $\frac{x}{a+h}+\frac{y}{\frac{a^2}{h}+a}=1$

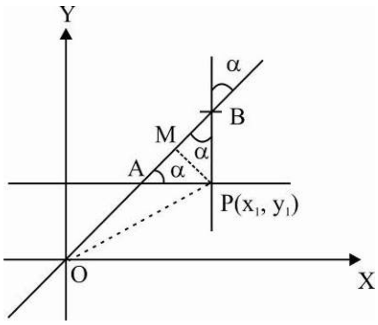
$\Rightarrow \quad \frac{x}{a+h}+\frac{yh}{a(a+h)}=1$

$\Rightarrow \quad ax+yh=a^2+ah$

$\Rightarrow \quad (ax-a^2)+h(y-a)=0$

$\Rightarrow$ Family of lines passing through the point of intersection of $x = a$ and $y = a$. $\Rightarrow (a, a)$

117.(A) $OP^2 = OM^2 + PM^2$



$x_1^2+y_1^2=\left(1+\frac{1}{2}\right)^2+\left(\frac{1}{2}\tan\alpha\right)^2$

$\Rightarrow \quad x_1^2+y_1^2=\frac{9+\tan^2\alpha}{4}$

Replace x_1, y_1 by x, y respectively

To get: $x^2+y^2=\frac{9+\tan^2\alpha}{4}$

118.(A) $y = mx$ cuts the curve $x^3+xy^2+2x^2+2y^2+3x+1=0$

$\therefore \quad x^3+x(mx)^2+2x^2+2(mx)^2+3x+1=0 \Rightarrow (1+m^2)x^3+(2+2m^2)x^2+3x+1=0$

Roots are in H.P.

$\therefore$ Replace x by $\frac{1}{t}$ to get, $(1+m^2)\frac{1}{t^3}+2(1+m^2)\frac{1}{t^2}+\frac{3}{t}+1=0$

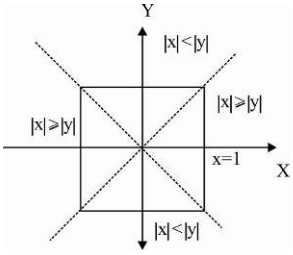
or $\quad t^3+3t^2+2(1+m^2)t+(1+m^2)=0$

Now, roots are in A.P.

Assume roots are $a-d, a, a+d$ so, $3a=\frac{-3}{1} \Rightarrow a=-1$

$\therefore \quad a^3+3a^2+2(1+m^2)a+(1+m^2)=0 \Rightarrow -1+3-2(1+m^2)+(1+m^2)=0 \Rightarrow m=\pm 1$

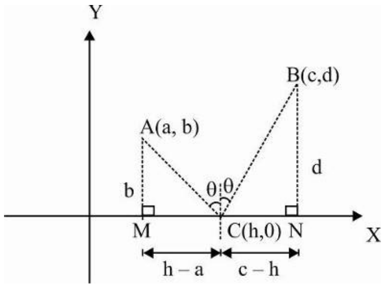
119.(B) $\max\{|x|,|y|\}=1$



$|x|=1; |x|\ge|y|, |y|=1; |x|<|y|$

Locus of point P is a square.

120.(D) $AC+CB$ is minimum if it is the path of light ray.

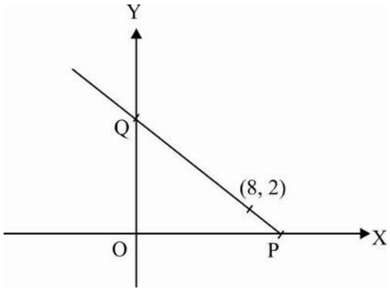


ΔAMC and ΔBNC are similar Δs ,

$\therefore \quad \dfrac{b}{d}=\dfrac{h-a}{c-h}$

$\Rightarrow \quad bc-bh=dh-ad \quad \Rightarrow \quad h=\dfrac{ad+bc}{b+d}$

121.(B) $y-2=m(x-8)$



X-int $= 8-\dfrac{2}{m}$

Y-int $= 2-8m$

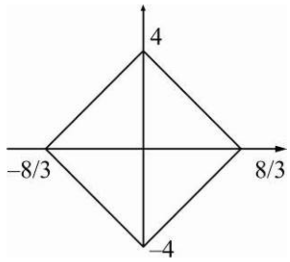
$OP + OQ = 8-\dfrac{2}{m}+2-8m=10-\dfrac{2}{m}-8m$

$\dfrac{\frac{-2}{m}-8m}{2}\ge\sqrt{\left(\dfrac{-2}{m}\right)(-8m)}=4 \quad (A.M\ge G.M)$

$10-\dfrac{2}{m}-8m\ge 18$

Hence, minimum value of $OP + OQ = 18$ units

122.(C) $f(x+y)=f(x)+f(y) \quad \Rightarrow \quad f(x)=a^x$ where $f(1) = 2$



$\Rightarrow \quad f(x) = 2^x$

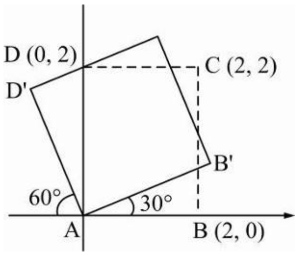
Area $= 4\times\left(\dfrac{1}{2}\times4\times\dfrac{8}{3}\right) = \dfrac{64}{3}$ square units $=\dfrac{f(6)}{3}$

123.(A) $y=|x-1|$

Replace x by $-x$

$y=|-x-1|=|x+1|$

124.(C) $B'\equiv(0+2\cos30°, 0+2\sin30°)\equiv(\sqrt{3},1)$



$D'\equiv(0+2\cos120°, 0+2\sin120°)\equiv(-1,\sqrt{3})$

$\dfrac{y-1}{x-\sqrt{3}}=\dfrac{\sqrt{3}-1}{-1-\sqrt{3}}=\dfrac{-(\sqrt{3}-1)^2}{(\sqrt{3}+1)(\sqrt{3}-1)}$

$\Rightarrow \quad \dfrac{y-1}{x-\sqrt{3}}=-\dfrac{(4-2\sqrt{3})}{2}=\sqrt{3}-2 \quad \Rightarrow \quad (2-\sqrt{3})x+y=2(\sqrt{3}-1)$

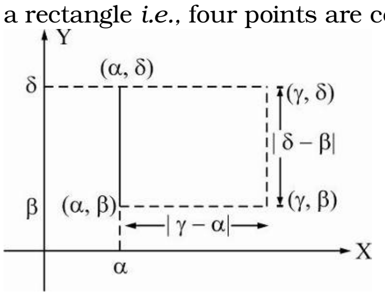
125.(A) $\dfrac{x}{t^3}=\dfrac{y}{t^2-3}=\dfrac{1}{t-1} \quad \Rightarrow \quad x=\dfrac{t^3}{t-1},\ y=\dfrac{t^2-3}{t-1}$

Let equation of line is: $\ell x+m\,y+n=0 \quad \Rightarrow \quad \ell\left(\dfrac{t^3}{t-1}\right)+m\left(\dfrac{t^2-3}{t-1}\right)+n=0$

$\Rightarrow \quad \ell t^3 + mt^2 + nt + (-3m - n) = 0 \quad \Rightarrow \quad a+b+c = \frac{-m}{\ell}, \; ab+bc+ca = \frac{n}{\ell}, \; abc = \frac{+(3m+n)}{\ell}$

$abc = -3(a+b+c) + (ab+bc+ca) \qquad \therefore \qquad abc + 3(a+b+c) = ab+bc+ca$

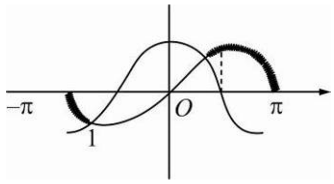
126.(D) Hence, four points form a rectangle *i.e.*, four points are concyclic.



127.(A) $3\left(\frac{\sin\alpha}{3} - 1\right) - 2\left(\frac{\cos\alpha}{2} - 1\right) + 1$ and $3(1) - 2(1) + 1$

have same sign *i.e.*, $\sin\alpha - \cos\alpha > 0 \quad \Rightarrow \quad \sin\alpha > \cos\alpha$

$\alpha \in \left[-\pi, \frac{-3\pi}{4}\right) \cup \left(\frac{\pi}{4}, \pi\right]$



128.(B) $x\cos\alpha + y\sin\alpha + \frac{\sin^2\alpha}{\cos\alpha} = 0$. So, $\quad p_1 = \left|m^2\cos\alpha + 2m\sin\alpha + \frac{\sin^2\alpha}{\cos\alpha}\right| = \left|\frac{(m\cos\alpha + \sin\alpha)^2}{\cos\alpha}\right|$,

$p_2 = \left|mm'\cos\alpha + (m+m')\sin\alpha + \frac{\sin^2\alpha}{\cos\alpha}\right| \Rightarrow p_2 = \left|\frac{(m\cos\alpha + \sin\alpha)(m'\cos\alpha + \sin\alpha)}{\cos\alpha}\right|$ and

$p_3 = \left|m'^2\cos\alpha + 2m'\sin\alpha + \frac{\sin^2\alpha}{\cos\alpha}\right| = \left|\frac{(m'\cos\alpha + \sin\alpha)^2}{\cos\alpha}\right| \Rightarrow p_1p_3 = p_2^2$

So, p_1, p_2 and p_3 are in G.P.

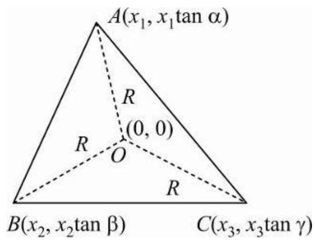
129.(D) $OA = OB = OC$

$x_1^2\left(1+\tan^2\alpha\right) = x_2^2\left(1+\tan^2\beta\right) = x_3^2\left(1+\tan^2\gamma\right) = R^2$

$x_1 = R\cos\alpha, x_2 = R\cos\beta, x_3 = R\cos\gamma$

Co-ordinates of Δ are:

$(R\cos\alpha, R\sin\alpha), (R\cos\beta, R\sin\beta), (R\cos\gamma, R\sin\gamma)$



Centroid divides orthocentre and circumcentre in the ratio of $2:1$ internally

$\left(\frac{\sum R\cos\alpha}{3}, \frac{\sum R\sin\alpha}{3}\right) \equiv \left[\frac{2(0)+1(a)}{3}, \frac{2(0)+1(b)}{3}\right] \qquad \Rightarrow \qquad \frac{a}{b} = \frac{\cos\alpha + \cos\beta + \cos\gamma}{\sin\alpha + \sin\beta + \sin\gamma}$

130.(C) Let a circle passes through (x_1, y_1), (x_2, y_2) and (x_3, y_3) where all of them are rational points:

Let equation of circle is: $x^2 + y^2 + 2x + 2fy + c = 0$

At is passes through (x_1, y_1), (x_2, y_2) and (x_3, y_3) then:

$x_1^2 + y_1^2 + 2gx_1 + 2fy_1 + c_1 = 0 \quad$ ...(i) $\quad$ and $\quad x_2^2 + y_2^2 + 2gx_2 + 2fy_2 + c_2 = 0 \quad$ ...(ii)

$x_3^2 + y_3^2 + 2gx_3 + 2fy_3 + c_3 = 0 \quad$ ...(iii)

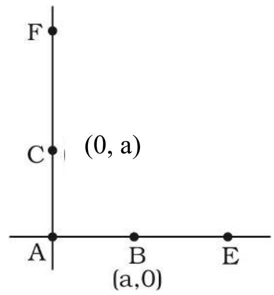
Above three equations are in g, f, c and (x_1, y_1), (x_2, y_2), (x_3, y_3) are rational then g, f, c (on solving) will also be rational.

Hence, a circle with three or more rational points has centre with rational co-ordinates.

Given circle has centre at $\left(0, \sqrt{3}\right)$. Hence, maximum of two rational points can lie on the circle.

131. Let A be the origin (0, 0), and B = $(a, 0)$ and $c \equiv (0, a)$

Given $BE \times CF = AB^2$



Say $BE = \lambda \quad \Rightarrow \quad CF = \frac{a^2}{\lambda}$.

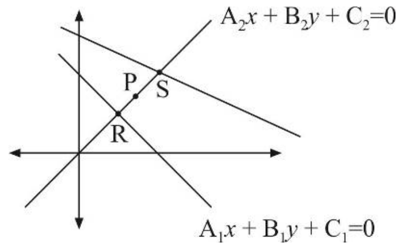
The equation to EF is $\frac{x}{a+\lambda} + \frac{y}{a + \frac{a^2}{\lambda}} = 1$

$\frac{x}{a+\lambda} + \frac{\lambda y}{a(a+\lambda)} = 1 \quad \Rightarrow \quad ax + \lambda y = a(a+\lambda)$

$\Rightarrow \quad a(x-1) + \lambda(y-a) = 0$ which is always passes through $(1, a)$

132. Let the fixed lines be $A_i x + B_i y + C_i = 0$ where $i = \{1, 2\}$

Transforming the equation into polar co-ordinates



$A_i r\cos\theta + B_i r\sin\theta + C_i = 0$

$r(A_i\cos\theta + B_i\sin\theta) = -C_i$

$\Rightarrow \quad \frac{1}{r} = -\frac{(A_i\cos\theta + B_i\sin\theta)}{C_i}$

$\Rightarrow \quad \frac{m+n}{r} = -m\frac{(A_1\cos\theta + B_1\sin\theta)}{C_1} - n\frac{(A_2\cos\theta + B_2\sin\theta)}{C_2}$

$\Rightarrow \quad m+n = -m\frac{(A_1x + B_1y)}{C_1} - n\frac{(A_2x + B_2y)}{C_2}$

which is a straight line passing through the intersection of the given lines.

133. Say the point of concurrency of the lines is origin O. Say that the fixed lines are $L_i, i = 1, 2\ldots, x$. Say that the line L_i makes an angle θ_i with the x-axis.

Say that the variable line $y = mx + c'$

Intersects L_i is A_i and let $OA_i = r_i$

Therefore, the coordinates of A_i are $(r_i\cos\theta_i, r_i\sin\theta_i)$

Where θ_i is a constant and r_i is a variable

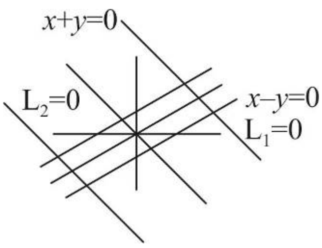
Now A_i lies on the variable line $\Rightarrow r_i\sin\theta_i = mr_i\cos\theta_i + c' \quad \Rightarrow \quad \frac{1}{r_i} = \frac{\sin\theta_i - m\cos\theta_i}{c'}$

$\sum\frac{1}{r_i} = c = \sum\frac{(\sin\theta_i) - m\sum\cos\theta_i}{c'}$

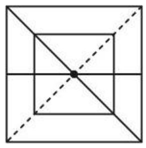
$\sum(\sin\theta_i) = m\sum(\cos\theta_i) + c\,c'$

Therefore, the line passes through the fixed point $\left(\frac{\sum(\cos\theta_i)}{c}, \sum\frac{(\sin\theta_i)}{c}\right)$

134.(6)



Since $2 \le d(p, L_1) + d(P, L_2) \le 4$



$$2 \le \frac{|x+y|+|x-y|}{\sqrt{2}} \le 4$$

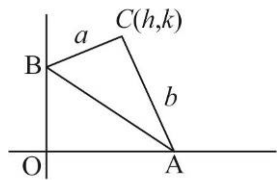
$$2\sqrt{2} \le |x+y|+|x-y| \le 4\sqrt{2}$$

$$2\sqrt{2} \le 2\max\{x, y\} \le 4\sqrt{2}$$

$$\sqrt{2} \le \max\{x, y\} \le 2\sqrt{2}$$

Required area $= (2\sqrt{2})^2 - \sqrt{2}^2 = 8 - 2 = 6$ square units

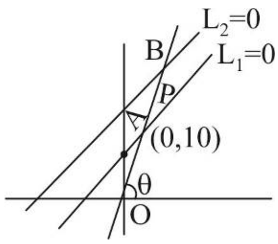
135. Let $C \equiv (h, k)$ and given $AC = b$ and $BC = a$.



Now $\frac{k}{h} = \frac{b}{a}$ or $y = \frac{b}{a}x$

Similarly, $y = -\frac{b}{a}x$

136.(D) $y - x = 10$ and $y - x = 20$



$$\frac{2}{OP} = \frac{1}{OA} + \frac{1}{OB}$$

Say $OA = r_1, OB = r_2$ and $OP = r$

$$r_1 \sin\theta - r_1 \cos\theta = 10$$

$\frac{1}{r_1} = \frac{\sin\theta - \cos\theta}{10}$ similarly, $\frac{1}{r_2} = \frac{\sin\theta - \cos\theta}{20}$ $\Rightarrow$ $\frac{2}{r} = \frac{\sin\theta - \cos\theta}{10} + \frac{\sin\theta - \cos\theta}{20}$

Therefore, the required locus is $2 = \frac{y-x}{10} + \frac{y-x}{20}$

$$40 = 2y - 2x + y - x$$

$$3y - 3x = 40$$

137.(C) $OP^2 = OA \times OB \Rightarrow r^2 = r_1 \cdot r_2 \Rightarrow r^2 = \frac{10}{(\sin\theta - \cos\theta)} \times \frac{20}{(\sin\theta - \cos\theta)} \Rightarrow (y-x)^2 = 200$

138.(A) $\frac{1}{r^2} = \frac{1}{r_1^2} + \frac{1}{r_2^2} \Rightarrow \frac{1}{r^2} = \frac{(\sin\theta - \cos\theta)^2}{100} + \frac{(\sin\theta - \cos\theta)^2}{400} \Rightarrow \frac{(y-x)^2}{100} + \frac{(y-x)^2}{200} = 1$

$\Rightarrow 5(y-x)^2 = 400 \Rightarrow (y-x)^2 = 80$

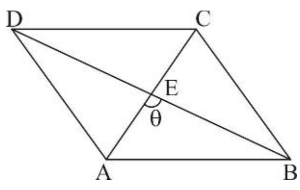
139.(C) The equation to AC is $x + 2y = 3$, its slope is $-\frac{1}{2}$

The equation to BD is $2x + y = 3,$ its slope is -2

Say the angle between the diagonals is θ, then $\tan\theta = \frac{3}{4} \Rightarrow \sin\theta = \frac{3}{5}$

Let the length of the other diagonal be d, then $\frac{1}{2} \cdot 4d \sin\theta = 8 \Rightarrow d = \frac{4}{\sin\theta} = \frac{20}{3}$ *units*.

140.(A) $AE = 2$ and $BE = \frac{10}{3}$ and $\cos\theta = \frac{4}{5}$



Using cosine rule is ΔAEB, $\cos\theta = \frac{AE^2 + BE^2 - AB^2}{2AE \cdot BE}$

$$AB = \frac{2\sqrt{58}}{3}$$

141.(2) The given lines are concurrent. So,

$$\begin{vmatrix} 1 & -2 & -6 \\ 3 & 1 & -4 \\ \lambda & 4 & \lambda^2 \end{vmatrix} = 0$$

Or $\lambda^2 + 2\lambda - 8 = 0$ Or $\lambda = 2, -4$

Since $\lambda > 0$ $\therefore \lambda = 2$

142.(2) The mid-point of $(1, -2)$ and $(3, 4)$ will satisfy i.e. (2, 1)

$y - x - 1 + \lambda = 0$ Or $1 - 2 - 1 + \lambda = 0$ $\therefore$ $\lambda = 2$ *or* $|\lambda| = 2$

143.(4) The point of intersection of $x - y + 1 = 0$ and $3x + y - 5 = 0$ is (1, 2). It lies on the line

$x + y - 1 - \frac{\lambda}{2} = 0 \Rightarrow 1 + 2 - 1 - \frac{\lambda}{2} = 0$ Or $\lambda = 4$

144.(5) Let the two perpendiculars through the origin intersect $2x + y = 5$ at *A* and *B* so that the triangle *OAB* is isosceles.

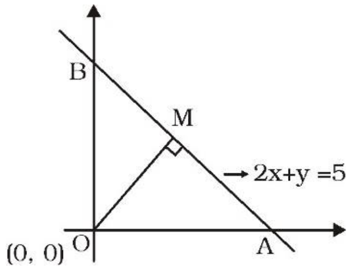
OM = Length of perpendicular from *O* to *AB*

$OM = \left|\frac{0 + 0 - 5}{\sqrt{5}}\right| = \sqrt{5}$

Also, $OM = AM = MB$

$\therefore AB = 2OM = 2 \times \sqrt{5} = 2\sqrt{5}$

$\therefore ar(\Delta OAB) = \frac{1}{2} \times AB \times OM = \frac{1}{2} \times 2\sqrt{5} \times \sqrt{5} = 5$ *sq. unit*



145.(5) Let the coordinates of *A* be (*a*, 0). Then the slope of the reflected ray is

$\frac{3 - 0}{5 - a} = \tan\theta$ ....(i)

Then the slope of the incident ray

$= \frac{2 - 0}{1 - a} = \tan(\pi - \theta)$ ....(ii)

From equations (i) and (ii), we get $\tan\theta + \tan(\pi - \theta) = 0$

$\Rightarrow \frac{3}{5 - a} + \frac{2}{1 - a} = 0 \Rightarrow 3 - 3a + 10 - 2a = 0$

$a = \frac{13}{5}$

Thus, the coordinate of *A* is $\left(\frac{13}{5}, 0\right)$ $\therefore k = 5$

146.(3) Let $\frac{AN}{BN} = \lambda$

Then, coordinate of *N* are $\left(\frac{a}{1 + \lambda}, \frac{a\lambda}{1 + \lambda}\right)$

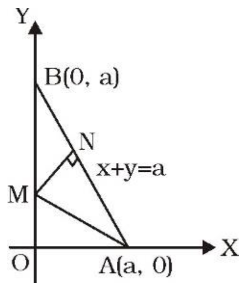
$\because$ Slope of $AB = -1$ $\therefore$ Slope of $MN = 1$

Equation of *MN* is

$y - \frac{a\lambda}{1 + \lambda} = x - \frac{a}{1 + \lambda} \Rightarrow x - y = a\left(\frac{1 - \lambda}{\lambda + 1}\right)$

So, the coordinates of *M* are $\left(0, a\left(\frac{\lambda - 1}{\lambda + 1}\right)\right)$

Therefore, area of $\Delta AMN = \frac{3}{8}$ area of ΔOAB



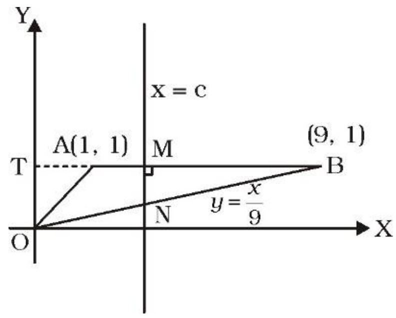
$\Rightarrow \quad \frac{1}{2} \cdot AN \cdot MN = \frac{3}{8} \cdot \frac{1}{2} a \cdot a$

$\Rightarrow \quad \frac{1}{2} \cdot \left| \frac{a\lambda\sqrt{2}}{1+\lambda} \cdot \frac{a\sqrt{2}}{1+\lambda} \right| = \frac{3}{8} \cdot \frac{1}{2} a \cdot a \quad \Rightarrow \quad \frac{a^2\lambda}{(1+\lambda)^2} = \frac{3}{8} \cdot \frac{1}{2} a^2 \quad \Rightarrow \quad 16\lambda = 3\,(\lambda^2 + 1 + 2\lambda)$

$3\lambda^2 - 10\lambda + 3 = 0 \qquad \therefore \qquad \lambda = 3 \text{ or } \lambda = \frac{1}{3}$

For $\lambda = \frac{1}{3}$, then M lies outside the segment OB and hence the required value of $\lambda = 3$.

147.(3) Let $O \equiv (0, 0)$, $A \equiv (1, 1)$ and $B \equiv (9, 1)$



Area of $\Delta OAB = \frac{1}{2} \times AB \times OT = \frac{1}{2} \times 8 \times 1 = 4$

It is clear that $1 < c < 9$

and $\quad M \equiv (c, 1)$ and $N \equiv \left(c, \frac{c}{9}\right)$

$\therefore$ Area of $\Delta BMN = 2 \quad \Rightarrow \quad \frac{1}{2} \times (9 - c) \times \left(1 - \frac{c}{9}\right) = 2$

or $\quad (9 - c)^2 = 36 \quad$ or $\quad 9 - c = \pm 6 \Rightarrow c = 3 \text{ or } 15$

but $\quad 1 < c < 9$

$\therefore \quad c = 3$

148.(3) Lines $5x + 3y - 2 + \lambda(3x - y - 4) = 0$ are concurrent at $(1, -1)$ and lines $x - y + 1 + \mu(2x - y - 2) = 0$ are concurrent at $(3, 4)$.

Thus equation of line common to both family is

$y + 1 = \frac{4+1}{3-1}(x - 1) \quad$ or $\quad 5x - 2y - 7 = 0 \quad \therefore \quad a = 5,\ b = -2 \Rightarrow a + b = 3$

149.(3) The equation of straight line through (2, 3) with slope m is

$y - 3 = m(x - 2)$ or $\quad mx - y = 2m - 3$

or $\quad \frac{x}{\left(\frac{2m-3}{m}\right)} + \frac{y}{(3-2m)} = 1 \qquad$ Here, $\quad OA = \frac{2m-3}{m} \text{ or } OB = 3 - 2m$

$\because$ The area of $\Delta OAB = 12 \quad \Rightarrow \quad \left| \frac{1}{2} \times OA \times OB \right| = 12$

or $\quad \frac{1}{2}\left(\frac{2m-3}{m}\right)(3 - 2m) = \pm 12 \quad$ or $\quad (2m - 3)^2 = \pm 24m$

Taking positive sign, we get $4m^2 - 36m + 9 = 0$

Here $D > 0$, This is a quadratic in m which given two value of m, and taking negative sign, we get $(2m + 3)^2 = 0$.

This gives one line of m as $\frac{-3}{2}$. $\qquad$ Hence, three straight lines are possible.

150.(6) $\because$ Point of intersection of $ax + 3y - 1 = 0$ and $ax + y + 1 = 0$ is $A\left(-\frac{2}{a}, 1\right)$ and point of intersection of $ax + 3y - 1 = 0$ and $x + 3y = 0$ is $B\left(\frac{1}{a-1}, -\frac{1}{3(a-1)}\right) \quad \Rightarrow \quad$ Slope of OA is $m_{OA} = -\frac{a}{2}$

and $\quad$ Slope of OB is $m_{OB} = -\frac{1}{3} \quad \because m_{OA} \times m_{OB} = -1 \quad \therefore -\frac{a}{2} \times -\frac{1}{3} = -1 \quad$ or $\quad a = -6 \quad \therefore \quad |a| = 6$

151.(5) Here, B is the image of A w.r.t line $y = x$

$\therefore \quad B \equiv (2, 1)$ and C is the image of A w.r.t line $x - 2y + 1 = 0$ if

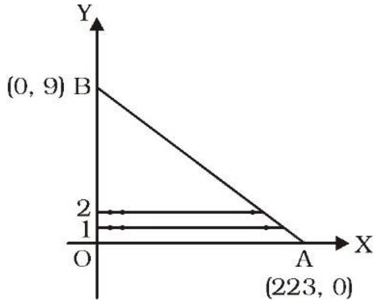
$C \equiv (\alpha, \beta)$, then

$\frac{\alpha-1}{1} = \frac{\beta-2}{-2} = \frac{-2(1-4+1)}{1+4}$ Or $\alpha = \frac{9}{5}$ and $\beta = \frac{2}{5}$ $\therefore$ $C \equiv \left(\frac{9}{2}, \frac{2}{5}\right)$ $\Rightarrow$ Equation of BC is

$y - 1 = \frac{\left(\frac{2}{5}-1\right)}{\left(\frac{9}{5}-2\right)}(x-2)$ or $3x - y - 5 = 0$ ($\because$ eq. of BC is $ax + by - 5 = 0$)

Here, $a = 3, b = -1$ $\therefore$ $a - 2b = 5$

152.(8) On the line $y = 1$, the number of lattice points is



$\left[\frac{2007-223}{9}\right] = 198$

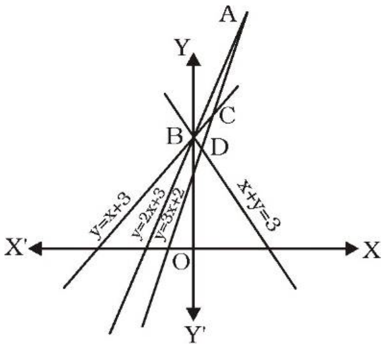
Hence, the total number of points

$= \sum_{y=1}^{8}\left[\frac{2007-223y}{9}\right] = 198 + 173 + 148 + 123 + 99 + 74 + 49 + 24 = 888$

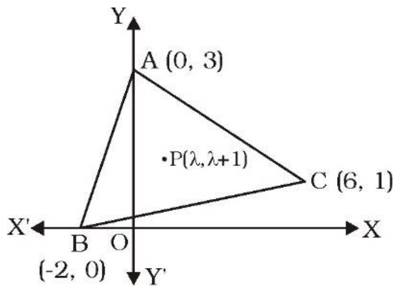
Hence, tens place digit is 8.

153.(3) A rough sketch of the lines is given.

There are three triangle namely ABC, BCD and ABD



154.(2) Equation of AB is $3x - 2y + 6 = 0$



Equation of BC is $x - 8y + 2 = 0$,

Equation of CA is $x + 3y - 9 = 0$

Let $P \equiv (\lambda, \lambda + 1)$

$\because$ B and P lie on one side of AC, then

$\frac{\lambda + 3(\lambda+1) - 9}{-2+0-9} > 0$ or $4\lambda - 6 < 0$ or $\lambda < \frac{3}{2}$

and C and P lie on one side of AB, then

$\frac{3\lambda - 2(\lambda+1) + 6}{18-2+6} > 0$ or $\lambda + 4 > 0$ or $\lambda > -4$ .....(ii)

Finally, A and P lie on one side of BC, then

$\frac{\lambda - 8(\lambda+1) + 2}{0-24+2} > 0$ or $-7\lambda - 6 < 0$ or $\lambda > -\frac{6}{7}$ .....(iii)

From equations (i), (ii) and (iii) we get $-\frac{6}{7} < \lambda < \frac{3}{2}$

Integral values of λ are 0 and 1.

Hence, number of integral values of λ is 2.

155.(4) Since, PQ is of fixed length.

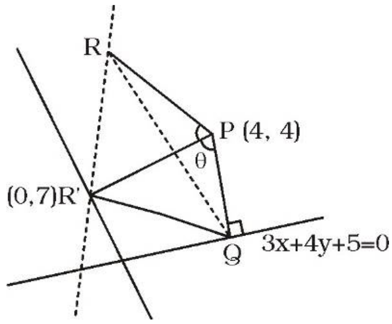
Area of $\Delta PQR = \frac{1}{2}|PQ||RP|\sin\theta$

This will be maximum, if $\sin\theta = 1$ and RP is maximum.

Since, line $y = mx + 7$ rotate about (0, 7), if PR' is perpendicular to the line than PR' is maximum value of PR.

$\therefore \qquad m = -\left(\frac{4-0}{4-7}\right) = \frac{4}{3}$

Hence, $3m = 4$



JEE Main Archive	**Daily Tutorial Sheet 1 to 3**

1.(C) Let coordinate of the intersection point in fourth quadrant be $(\alpha, -\alpha)$.
Since, $(\alpha, -\alpha)$ lies on both lines $4ax + 2ay + c = 0$ and $5bx + 2by + d = 0$.

$\therefore \qquad 4a\alpha - 2a\alpha + c = 0 \Rightarrow \alpha = \frac{-c}{2a} \qquad$ .... (i)

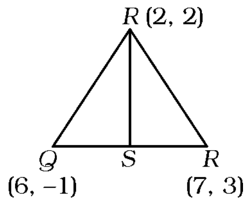
and $\qquad 5b\alpha - 2b\alpha + d = 0 \Rightarrow \alpha = \frac{-d}{3b} \qquad$ .... (ii)

From Equations (i) and (ii), we get $\frac{-c}{2a} = \frac{-b}{3b} \Rightarrow 3bc = 2ad \qquad \Rightarrow \qquad 2ad - 3bc = 0$

2.(B) Coordinate of $S = \left(\frac{7+6}{2}, \frac{3-1}{2}\right) = \left(\frac{13}{2}, 1\right)$

[$\because S$ is mid-point of line QR]

Slope of the line PS is $\frac{-2}{9}$.

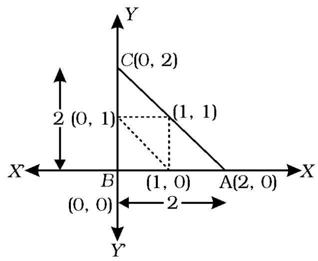


Required equation passes through (1, −1) and parallel to PS is $y + 1 = \frac{-2}{9}(x-1) \Rightarrow 2x + 9y + 7 = 0$

3.(B) Given mid-points of a triangle are (0, 1), (1, 1) and (1, 0).
Plotting these points on a graph paper and make a triangle.
So, the sides of a triangle will be 2, 2 and $\sqrt{2^2 + 2^2}$ i.e. $2\sqrt{2}$.

x-coordinate of incentre $= \frac{2 \times 0 + 2\sqrt{2} \cdot 0 + 2 \cdot 2}{2 + 2 + 2\sqrt{2}}$

$= \frac{2}{2+\sqrt{2}} \times \frac{2-\sqrt{2}}{2-\sqrt{2}} = 2 - \sqrt{2}$

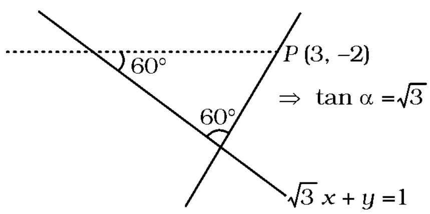


4.(B) A straight line passing through P and making an angle of $\alpha = 60°$, is given by

$\frac{y - y_1}{x - x_1} = \tan(\theta \pm \alpha)$

$\Rightarrow \qquad \sqrt{3}x + y = 1 \qquad \Rightarrow \qquad y = -\sqrt{3}x + 1$, then $\tan\theta = -\sqrt{3}$

$\Rightarrow \qquad \frac{y+2}{x-3} = \frac{\tan\theta \pm \tan\alpha}{1 \mp \tan\theta\tan\alpha}$

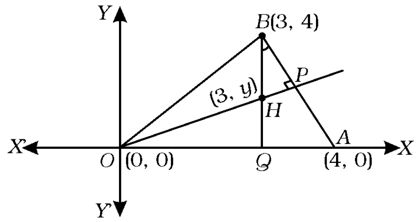


$\frac{y+2}{x-3} = \frac{-\sqrt{3}+\sqrt{3}}{1-(-\sqrt{3})(\sqrt{3})}$ and $\frac{y+2}{x-3} = \frac{-\sqrt{3}-\sqrt{3}}{1+(-\sqrt{3})(\sqrt{3})}$

$\Rightarrow \qquad y + 2 = 0$ and $\frac{y+2}{x-3} = \frac{-2\sqrt{3}}{1-3} = \sqrt{3} \qquad y + 2 = \sqrt{3}x - 3\sqrt{3}$

Neglecting, $y + 2 = 0$, as it does not intersect Y-axis.

5.(C) To find orthocentre of the triangle formed by (0, 0) (3, 4) and (4, 0).



Let H be the orthocentre of ΔOAB

$\therefore$ (slope of OP i.e. OH) (slope of BA) = –1

$\Rightarrow \quad \left(\dfrac{y-0}{3-0}\right)\cdot\left(\dfrac{4-0}{3-4}\right)=-1$

$\Rightarrow \quad -\dfrac{4}{3}y=-1 \qquad \Rightarrow \qquad y=\dfrac{3}{4} \qquad \therefore \qquad$ Required orthocentre $=(3,y)=\left(3,\ \dfrac{3}{4}\right)$

6.(A) On solving equations $3x+4y=9$ and $y=mx+1$, we get $x=\dfrac{5}{3+4m}$

Now, for x to be an integer, $3+4m=\pm 5$ or ± 1

The integral values of m satisfying these conditions are –2 and –1.

7.(B) Now, distance of origin from $4x+2y-9=0$ is $\dfrac{|-9|}{\sqrt{4^2+2^2}}=-\dfrac{9}{\sqrt{20}}$

and distance of origin from $2x+y+6=0$ is $\dfrac{|6|}{\sqrt{2^2+1^2}}=\dfrac{6}{\sqrt{5}}$

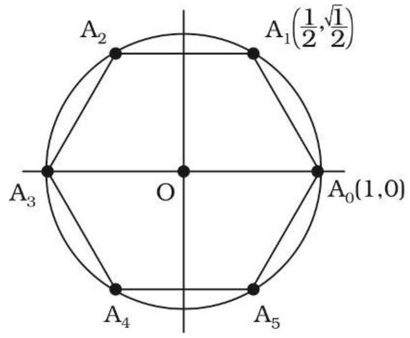
Hence, the required ratio $=\dfrac{9/\sqrt{20}}{6/\sqrt{5}}=\dfrac{3}{4}$

8.(D) Let the vertices of triangle be $A(1,\ \sqrt{3}), B(0,\ 0)$ and $C(2,\ 0)$. Here, $AB=BC=CA=2$.

Therefore, it is an equilateral triangle. So, the incentre coincides with centroid.

$\therefore \quad I=\left(\dfrac{0+1+2}{3},\ \dfrac{0+0+\sqrt{3}}{3}\right) \qquad \Rightarrow \qquad I=(1,\ 1/\sqrt{3})$

9.(C) Now, $(A_0A_1)^2=\left(1-\dfrac{1}{2}\right)^2+\left(0-\dfrac{\sqrt{3}}{2}\right)^2$



$=\left(\dfrac{1}{2}\right)^2+\left(\dfrac{\sqrt{3}}{2}\right)^2=\dfrac{1}{4}+\dfrac{3}{4}=1 \Rightarrow A_0A_1=1$

$(A_0A_2)^2=\left(1+\dfrac{1}{2}\right)^2+\left(0-\dfrac{\sqrt{3}}{2}\right)^2$

$=\left(\dfrac{3}{2}\right)^2+\left(-\dfrac{\sqrt{3}}{2}\right)^2=\dfrac{9}{4}+\dfrac{3}{4}=\dfrac{12}{4}=3$

$\Rightarrow \quad A_0A_2=\sqrt{3} \qquad$ and $\qquad (A_0A_4)^2=\left(1+\dfrac{1}{2}\right)^2+\left(0+\dfrac{\sqrt{3}}{2}\right)^2=\left(\dfrac{3}{2}\right)^2+\left(\dfrac{3}{4}\right)=\dfrac{9}{4}+\dfrac{3}{4}=\dfrac{12}{4}=3$

$\Rightarrow \quad A_0A_4=\sqrt{3} \qquad$ Thus, $\quad (A_0A_1)(A_0A_2)(A_0A_4)=3$

10.(C) $PQRS$ is a parallelogram if and only if the mid-point of the diagonal's PR is same as that of the mid-point of QS.

That is, if and only if

$\dfrac{1+5}{2}=\dfrac{4+a}{2}$ and $\dfrac{2+7}{2}=\dfrac{6+b}{2} \quad \Rightarrow \quad a=2$ and $b=3$.

11.(D) Slope of line $x+3y=4$ is $-1/3$

and slope of line $6x-2y=7$ is 3.

Here, $3\times\left(\frac{-1}{3}\right)=-1$

Therefore, these two lines are perpendicular which show that both diagonals are perpendicular.
Hence, *PQRS* must be a rhombus.

12.(C) Orthocentre of right-angled triangle is at the vertex of right angle. Therefore, orthocentre of the triangle is at (0, 0).

13.(D) Let the coordinate of *S* be (*x*, *y*).

$\because \quad SQ^2+SR^2=2SP^2 \quad \Rightarrow \quad (x+1)^2+y^2+(x-2)^2+y^2=2[(x-1)^2+y^2]$

$\Rightarrow \quad x^2+2x+1+y^2+x^2-4x+4+y^2=2(x^2-2x+1+y^2) \Rightarrow 2x+3=0 \Rightarrow x=-\frac{3}{2}$

Hence, it is a straight line parallel to *Y*-axis.

14.(A) The point $O(0, 0)$ is the mid-point of $A(-a, -b)$ and $B(a, b)$. Therefore, *A*, *O*, *B* are collinear and equation of line *AOB* is $y=\frac{b}{a}x$

Since, the fourth point $D(a^2, ab)$ satisfies the above equation.

Hence, the four points are collinear.

15. $y=10^x$ is reflection of $y=\log_{10}x$ about $y = x$.

16.(T) Since, $a_1x+b_1y+c_1=0$ and $a_2x+b_2y+c_2=0$ cuts the coordinate axes at concyclic points.

$\Rightarrow \quad a_1a_2=b_1b_2 \quad$ or $\quad a_1b_2+b_1a_2=0$

Given lines are, $2x+3y+19=0$ and $9x+6y-17=0$

Here, $a_1=2, b_1=3, c_1=19$ and $a_2=9, b_2=6, c_2=-17$

$\therefore \quad a_1a_2=18$ and $b_1b_2=18 \quad \Rightarrow \quad a_1a_2=b_1b_2$. Thus, points are concyclic.

Hence, given statement is true.

17.(T) Since, $(1, \sqrt{3}), (1, -\sqrt{3})$ and $(3, \sqrt{3})$ form a right angled triangle at $(1, \sqrt{3})$

$\therefore$ Equation of circumcircle taking $(3, \sqrt{3})$ and $(1, -\sqrt{3})$ as and points of diameter.

$\therefore \quad (x-3)(x-1)+(y-\sqrt{3})(y+\sqrt{3})=0 \Rightarrow x^2-4x+3+y^2-3=0 \Rightarrow x^2+y^2-4x=0$

At point $\left(\frac{5}{2}, 1\right)$, $S_1=\frac{25}{4}+1-10<0$

$\therefore$ Point (5/2, 1) lies inside the circle.

Hence, no tangent can be drawn. Hence, given statement is true.

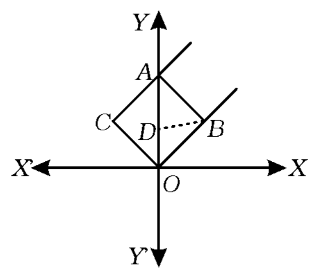
18.(T) The point of intersection of $x+2y=10$ and $2x+y+5=0$ is $\left(-\frac{20}{3}, \frac{25}{3}\right)$ which clearly satisfy $5x+4y=0$.

Hence, given statement is true.

19.(D) Let lines $OB: y=mx$

$CA: y=mx+1$

$BA: y=nx+1$ and $OC: y=nx$



The point of intersection *B* of *OB* and *AB* has *x* coordinate $\frac{1}{m-n}$.

Now, area of a parallelogram $OBAC=2\times$ area of ΔOBA

$=2\times\frac{1}{2}\times OA\times DB=2\times\frac{1}{2}\times\frac{1}{m-n} = \frac{1}{m-n}=\frac{1}{|m-n|}$

Depending upon whether $m>n$ or $m<n$.

20.(D) Since, vertices of a triangle are (0, 8/3), (1, 3) and (82, 30)

Now, $\dfrac{1}{2}\begin{vmatrix} 0 & 8/3 & 1 \\ 1 & 3 & 1 \\ 82 & 30 & 1 \end{vmatrix} = \dfrac{1}{2}\left[-\dfrac{8}{3}(1-82)+1(30-246)\right] = \dfrac{1}{2}[216-216]=0$

$\therefore$ Points are collinear.

21.(A) The points of intersection of three lines are $A(1,1)$, $B(2,-2)$, $C(-2,2)$.

Now, $|AB| = \sqrt{1+9} = \sqrt{10}$,

$|BC| = \sqrt{16+16} = 4\sqrt{2}$,

and $|CA| = \sqrt{9+1} = \sqrt{10}$ $\therefore$ Triangle is an isosceles.

22.(C) Given lines, $x+2y-3=0$ and $3x+4y-7=0$ intersect at (1, 1), which does not satisfy $2x+3y-4=0$ and $4x+5y-6=0$.

Also, $3x+4y-7=0$ and $2x+3y-4=0$ intersect at (5, –2) which does not satisfy $x+2y-3=0$ and $4x+5y-6=0$.

Lastly, intersection point of $x+2y-3=0$ and $2x+3y-4=0$ is $(-1,2)$ which satisfy $4x+5y-6=0$.

Hence only three lines are concurrent.

Fill in the blanks

23. $\left(\dfrac{\mathbf{3}}{\mathbf{4}}, \dfrac{\mathbf{1}}{\mathbf{2}}\right)$ The set of lines $ax+by+c=0$, where $3a+2b+4c=0$ or $\dfrac{3}{4}a+\dfrac{1}{2}b+c=0$ are concurrent at $\left(x=\dfrac{3}{4}, y=\dfrac{1}{2}\right)$ i.e. comparing the coefficients of x and y.

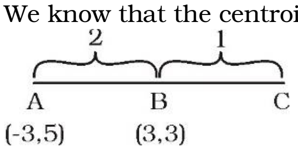
Thus, point of concurrency is $\left(\dfrac{3}{4}, \dfrac{1}{2}\right)$.

Alternate Solution

As, $ax+by+c=0$, satisfy $3a+2b+4c=0$ which represents system of concurrent lines whose point of concurrency could be obtained by comparison as,

$ax+by+c \equiv \dfrac{3a}{4}+\dfrac{2}{4}b+c \;\Rightarrow\; x=\dfrac{3}{4}, y=\dfrac{1}{2}$ is point of concurrency. $\therefore$ $\left(\dfrac{3}{4}, \dfrac{1}{2}\right)$ is the required point.

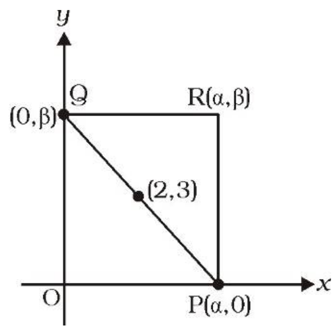
24.(D) We know that the centroid divides orthocenter and circumcentre in the ratio 2 : 1



$AC = \dfrac{3}{2}AB = \dfrac{3}{2}\sqrt{6^2+2^2} = \dfrac{3}{2}\cdot 2\sqrt{10} = 3\sqrt{10}$

Radius of the circle with AC as diameter $= \dfrac{3}{2}\sqrt{10} = 3\sqrt{\dfrac{5}{2}}$

25.(D) The equation of the given line is $\dfrac{x}{\alpha}+\dfrac{y}{\beta}=1$ ...(i)



As (2,3) lies on (i),

$\therefore \dfrac{2}{\alpha}+\dfrac{3}{\beta}=1 \Rightarrow 2\beta+3\alpha-\alpha\beta=0$

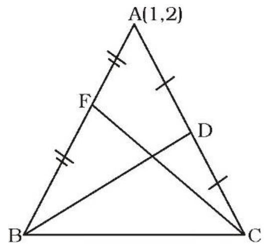
Changing (α,β) *to* (x,y) we have the locus of R as $3x+2y-xy=0$

26.(B) The equation of median BD is $x+y=5$

$\because$ B lies on it, therefore co-ordinates of B be $(x_1, 5-x_1)$

$\therefore$ Co-ordinates of $F = \left(\dfrac{x_1+1}{2}, \dfrac{5-x_1+2}{2}\right)$

Also, F lies on x = 4, $\therefore \dfrac{x_1+1}{2}=4 \Rightarrow x_1=7$



$\Rightarrow$ Co-ordinates of B = (7,-2)

Similarly, let $C = (4, y_1)$

$\because$ D is the mid-point of AC, $\therefore$ $D = \left(\frac{4+1}{2}, \frac{y_1+2}{2}\right)$

Now D lies on $x + y = 5 \Rightarrow \frac{5}{2} + \frac{y_1+2}{2} = 5 \Rightarrow y_1 = 3$

$\therefore$ Co-ordinates of C = (4,3)

Now, are of $\Delta ABC = \frac{1}{2}\begin{vmatrix} 1 & 2 & 1 \\ 7 & -2 & 1 \\ 4 & 3 & 1 \end{vmatrix} = \frac{1}{2}[1(-2-3) - 2(7-4) + 1(21+8) = \frac{1}{2}[18] = 9$

27.(B) Let coordinates of A be (0,a)

The diagonals intersect of P(1,2)

We know that the diagonals will be parallel to the angle bisectors of the two sides $y = x + 2$ & $y = 7x + 3$

i.e., $\frac{x-y+2}{\sqrt{2}} = \pm\frac{7x-y+3}{5\sqrt{2}} \Rightarrow 5x - 5y + 10 = \pm(7x - y + 3)$

$\Rightarrow 2x + 4y - 7 = 0$ & $12x - 6y + 13 = 0$ $\Rightarrow m_1 = -\frac{1}{2}$ & $m_2 = 2$

(where m_1 & m_2 are the slopes of the given two lines)

Let one diagonal be parallel to $2x + 4y - 7 = 0$ and other be parallel to $12x - 6y + 13 = 0$

The vertex A could be on any of the two diagonals, Hence, slope of AP is either $-\frac{1}{2}$ *or* 2.

$\Rightarrow \frac{2-a}{1-0} = 2$ *or* $\frac{2-a}{1-0} = -\frac{1}{2} \Rightarrow a = 0$ *or* $a = \frac{5}{2}$

But $a \neq 0$ $\therefore$ $a = \frac{5}{2}$.

Thus, ordinate of A is $\frac{5}{2}$.

28.(A) Given, $3x + y = \lambda\ (\lambda \neq 0) \Rightarrow 3x + y - \lambda = 0$

Foot of perpendicular from (x_1, y_1) to $ax + by + c = 0$ is given by $\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{-(ax_1 - by_1 + c)}{a^2 + b^2}$

$\Rightarrow \frac{x-0}{3} = \frac{y-0}{1} = \frac{-(3 \times 0 + 0 - \lambda)}{3^2 + 1^2}\ [\because (x_1, y_1) = (0,0)]$ $\Rightarrow \frac{x}{3} = \frac{y}{1} = \frac{\lambda}{10}$

Hence, foot of perpendicular is $P\left(\frac{3\lambda}{10}, \frac{\lambda}{10}\right)$

Now, line meets x-axis where y = 0, so $3x + 0 = \lambda \Rightarrow x = \frac{\lambda}{3}$

Hence, coordinates of A are $\left(\frac{\lambda}{3}, 0\right)$

Similarly, coordinates of B are $(0, \lambda)$

$$\therefore \frac{BP}{PA} = \frac{\sqrt{\left(\frac{3\lambda}{10} - 0\right)^2 + \left(\frac{\lambda}{10} - \lambda\right)^2}}{\sqrt{\left(\frac{3\lambda}{10} - \frac{\lambda}{3}\right)^2 + \left(\frac{\lambda}{10} - 0\right)^2}} = \frac{\sqrt{\frac{9\lambda^2}{100} + \frac{81\lambda^2}{100}}}{\sqrt{\frac{\lambda^2}{900} + \frac{\lambda^2}{100}}} = \frac{\sqrt{\frac{90\lambda^2}{100}}}{\sqrt{\frac{10\lambda^2}{900}}} = \frac{\sqrt{\frac{9}{10}}}{\sqrt{\frac{1}{90}}} = \frac{\sqrt{81}}{1} = \frac{9}{1} \Rightarrow BP : PA = 9 : 1$$

29.(C) As area is given to be 56, we have $\begin{vmatrix} k & -3k & 1 \\ 5 & k & 1 \\ -k & 2 & 1 \end{vmatrix} = \pm 56$

Expanding, we get $k(k-2) - 5(-3k-2) - k(-3k-k) = \pm 56$

$\Rightarrow k^2 - 2k + 15k + 10 + 3k^2 + k^2 = \pm 56$ $\Rightarrow 5k^2 + 13k + 10 = \pm 56$

Taking the positive sign $5k^2 + 13k - 46 = 0$ $\Rightarrow (5k + 23)(k - 2) = 0$ $\therefore k = 2$ is an integer

Taking the negative sign $5k^2+13k+66=0 \Rightarrow D=13^2-4.5.66<0$

Thus there is no solution in this case.

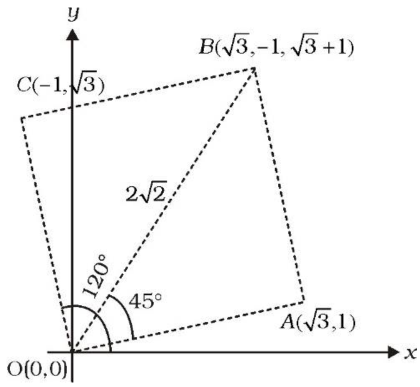
So the vertices are $A(2,-6), B(5,2)$ & $C(-2,2)$.

The equation of altitude from A is x = 2 and the equation of altitude from C is $y-2=-\frac{3}{8}(x+2)$

i.e., $3x+8y-10=0$

Solving the two we get the orthocenter as $\left(2,\frac{1}{2}\right)$

30.(D) Side of square = 2



$\frac{x}{\cos 30°}=\frac{y}{\sin 30°}=2$

$\Rightarrow \quad x=\frac{2\sqrt{3}}{2}=\sqrt{3}$

and y = 1

$\frac{x}{\cos 120°}=\frac{y}{\sin 120°}=2$

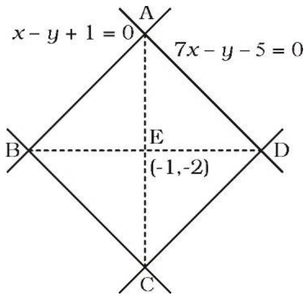
$\Rightarrow \quad x=-1,\ y=\sqrt{3}$

$\frac{x}{\cos 75°}=\frac{y}{\sin 75°}=2\sqrt{2}$

$\Rightarrow \quad x=\sqrt{3}-1$ & $y=\sqrt{3}+1$

$\therefore$ Required sum $=0+\sqrt{3}+\sqrt{3}-1+(-1)=2\sqrt{3}-2$

31.(C) Coordinates of $A(1,2) \therefore$ *Slope of* $AE=2$



$\Rightarrow$ Slope of $BD=-\frac{1}{2}$

$\Rightarrow$ Equation of BD is $\frac{y+2}{x+1}=-\frac{1}{2}$

$\Rightarrow \quad x+2y+5=0$

$\therefore$ Co-ordinates of $D=\left(\frac{1}{3},\frac{-8}{3}\right)$

32.(A) Given equations of lines can be written as $4x+3y-12=0$ & $3x+4y-12=0$

Equation of line passing through the intersection of these two lines is given by $(4x+3y-12)+\lambda(3x+4y-12)=0 \quad \Rightarrow \quad x(4+3\lambda)+y(3+4\lambda)-12(1+\lambda)=0$

Above line meets the coordinate axes at points A and B.

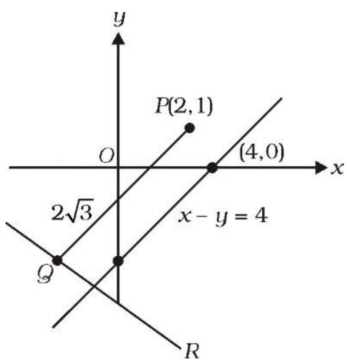
Now, coordinates of point A are $\left(\frac{12(1+\lambda)}{4+3\lambda}, 0\right)$ and coordinates of point B are $\left(0,\frac{12(1+\lambda)}{3+4\lambda}\right)$

$\therefore$ Coordinates of mid-point of AB are given by

$h=\frac{6(1+\lambda)}{4+3\lambda}$ ...(i) and $k=\frac{6(1+\lambda)}{3+4\lambda}$ ...(ii)

Eliminating λ from (i) and (ii), we get, $6(h+k)=7hk \quad \therefore$ Locus of the mid-point of AB is, $6(x+y)=7xy$

33.(D) We have, $L: x-y=4$



Now, slope of L = 1

Since, line L is perpendicular to QR $\therefore$ Slope of QR = -1

Let equation of QR be

$y=mx+c$

$\Rightarrow \quad y=-x+c \Rightarrow x+y-c=0$

Now, distance of QR from point (2,1) is $2\sqrt{3}$ units

$\therefore \ 2\sqrt{3}=\frac{|2+1-c|}{\sqrt{2}} \Rightarrow 2\sqrt{6}=|3-c|$

$\Rightarrow \quad c-3=\pm 2\sqrt{6}$ *or* $x+y=3-2\sqrt{6}$

34.(C) Length of $\perp$ from O(0,0) to $4x+3y=10$ is $p_1=\frac{|4(0)+3(0)-10|}{\sqrt{4^2+3^2}}=\frac{10}{5}=2$

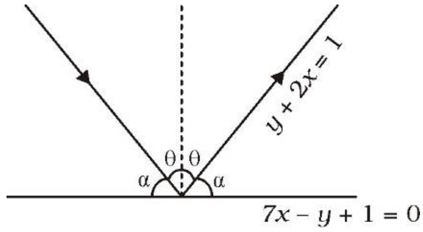
Length of $\perp$ from O(0,0) to $8x+6y+5=0$ is $p_2=\frac{|8(0)+6(0)+5|}{\sqrt{8^2+6^2}}=\frac{5}{10}=\frac{1}{2}$

Lines are parallel to each other $\Rightarrow$ ratio will be 4 : 1 or 1 : 4.

35.(C) Let slope of incident ray be m

Now angle of incidence = angle of reflection

$\therefore \left|\frac{m-7}{1+7m}\right|=\left|\frac{-2-7}{1-14}\right|=\frac{9}{13} \Rightarrow \frac{m-7}{1+7m}=\frac{9}{13}$ *or* $\frac{m-7}{1+7m}=-\frac{9}{13}$



$\Rightarrow 13m-91=9+63m$ *or* $13m-91=-9-63m$

$\Rightarrow 50m=-100$ *or* $76m=82 \Rightarrow m=-2, m=\frac{41}{38}$

$\therefore$ Equation of incident line at (0, 1) are

$y-1=-2(x-0)$ *or* $y-1=\frac{41}{38}(x-0)$

i.e., $2x+y-1=0$ *or* $38y-38-41x=0$

JEE Advanced Archive	**Daily Tutorial Sheet 1 to 4**

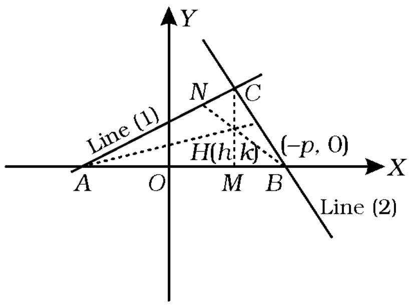
1.(D) Given, lines are $(1+p)x-py+p(1+p)=0$ .... (i)

and $(1+q)x-qy+q(1+q)=0$ .... (ii)

on solving Equations (i) and (ii), we get $C\{pq,(1+p)(1+q)\}$

$\therefore$ Equation of altitude CM passing through C and perpendicular to AB is

$x=pq$ .... (iii)



$\because$ Slope of line (ii) is $\left(\frac{1+q}{q}\right)$.

$\therefore$ Slope of altitude BN (as shown in figure) is $\frac{-q}{1+q}$.

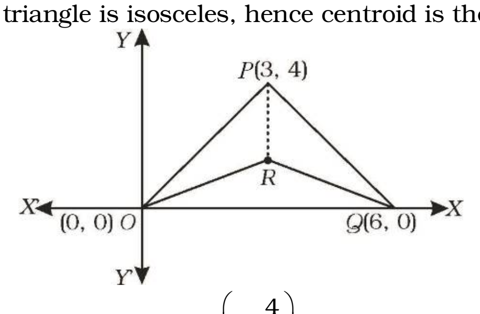
$\therefore$ Equation of BN is $y-0=\frac{-q}{1+q}(x+p) \Rightarrow y=\frac{-q}{(1+q)}(x+p)$ .... (iv)

Let orthocentre of triangle be $H(h,k)$, which is the point of intersection of equations (iii) and (iv).

On solving equations (iii) and (iv), we get $x=pq$ and $y=-pq \Rightarrow h=pq$ and $k=-pq$

$\therefore h+k=0 \quad \therefore$ Locus of $H(h,k)$ is $x+y=0$.

2.(C) Since, triangle is isosceles, hence centroid is the desired point.



$\therefore$ Coordinates of $R\left(3, \frac{4}{3}\right)$.

3.(ACD) Since, the coordinates of the centroid are $\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$, then the centroid is always a rational point. Also, the equations of perpendicular bisectors of sides and altitudes would have rational coefficients so circumcentre and orthocentre are also rational points.

4.(D) Let $y = \cos x \cos(x+2) - \cos^2(x+1) \quad = \cos(x+1-1)\cos(x+1+1) - \cos^2(x+1)$

$= \cos^2(x+1) - \sin^2 1 - \cos^2(x+1) \Rightarrow y = -\sin^2 1$

This is a straight line which is parallel to X-axis.

It passes through $(\pi/2, -\sin^2 1)$.

5.(A) By the given conditions, we can take two perpendicular lines as x and y axes. If (h, k) is any point on the locus, then $|h| + |k| = 1$. Therefore, the locus is $|x| + |y| = 1$. This consist of a square of side $\sqrt{2}$.

Hence, the required locus is a square.

6.(B) Since, the origin remains the same. So, length of the perpendicular from the origin on the line in its position $\frac{x}{a} + \frac{y}{b} = 1$ and $\frac{x}{p} + \frac{y}{q} = 1$ are equal.

Therefore, $\frac{1}{\sqrt{\frac{1}{a^2} + \frac{1}{b^2}}} = \frac{1}{\sqrt{\frac{1}{p^2} + \frac{1}{q^2}}} \Rightarrow \frac{1}{a^2} + \frac{1}{b^2} = \frac{1}{p^2} + \frac{1}{q^2}$

7.(C) Let B, C, D be the position of the point $A(4, 1)$ after the three operations I, II and III, respectively. Then, B is $(1, 4)$, $C(1+2, 4)$ i.e. $(3, 4)$. The point D is obtained from C by rotating the coordinate axes through an angle $\pi/4$ in anti-clockwise direction.

Therefore, the coordinates of D are given by $X = 3\cos\frac{\pi}{4} - 4\sin\frac{\pi}{4} = -\frac{1}{\sqrt{2}}$ and $Y = 3\sin\frac{\pi}{4} + 4\cos\frac{\pi}{4} = \frac{7}{\sqrt{2}}$

$\therefore$ Coordinates of D are $\left(-\frac{1}{\sqrt{2}}, \frac{7}{\sqrt{2}}\right)$.

8.(AC) As $a > b > c > 0$, $a - c > 0$ and $b > 0$

$\Rightarrow$ $a + b - c > 0$ .... (i)

$a - b > 0$ and $c > 0$ .... (ii)

$a + c - b > 0$ $\therefore$ (a) and (b) are correct.

Also, the point of intersection for $ax + by + c = 0$ and $bx + ay + c = 0$

i.e. $\left(\frac{-c}{a+b}, \frac{-c}{a+b}\right)$

The distance between (1, 1) and $\left(\frac{-c}{a+b}, \frac{-c}{a+b}\right)$

i.e. less than $2\sqrt{2}$. $\Rightarrow \sqrt{\left(1 + \frac{c}{a+b}\right)^2 + \left(1 + \frac{c}{a+b}\right)^2} < 2\sqrt{2}$

$\Rightarrow \left(\frac{a+b+c}{a+b}\right)\sqrt{2} < 2\sqrt{2} \Rightarrow a + b + c < 2a + 2b \Rightarrow a + b - c > 0$

From Equations (i) and (ii), option (C) is correct.

9.(AC) Since, $3x + 2y \geq 0$ .... (i)

Where (1, 3) (5, 0) and (–1, 2) satisfy equation (i).

$\therefore$ Option (a) is true.

Again, $2x + y - 13 \geq 0$

Is not satisfied by (1, 3), $\therefore$ Option (b) is false. $2x - 3y - 12 \leq 0$

is satisfied for all points, $\therefore$ Option (c) is true.

and $-2x + y \geq 0$

is not satisfied by (5, 0), $\therefore$ Option (d) is false.

Thus, (a) and (c) are correct answers.

10.(1,1) Let the variable straight line be $ax + by + c = 0$ .... (i)

where, algebraic sum of perpendiculars from (2, 0), (0, 2) and (1, 1) is zero.

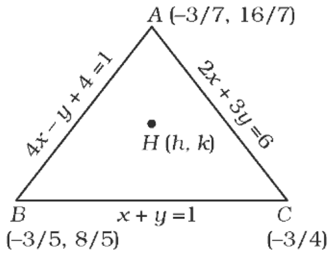
$\therefore \quad \dfrac{2a+0+c}{\sqrt{a^2+b^2}}+\dfrac{0+2b+c}{\sqrt{a^2+b^2}}+\dfrac{a+b+c}{\sqrt{a^2+b^2}}=0$

$\Rightarrow \quad 3a+3b+3c=0 \quad \Rightarrow \quad a+b+c=0$ .... (ii)

From equations (i) and (ii) $ax+by+c=0$ always passes through a fixed point (1, 1).

11.(1st) Let $H(h, k)$ be orthocentre.

$\Rightarrow$ (slope of AH). (Slope of BC) = – 1



$\Rightarrow \quad \left(\dfrac{k-\frac{16}{7}}{h+\frac{3}{7}}\right)\cdot(-1)=-1 \quad \Rightarrow \quad k-\dfrac{16}{7}=h+\dfrac{3}{7}$

$\Rightarrow \quad h-k=-\dfrac{19}{7}$ .... (i)

Also, (slope of CH) (slope of AB) = – 1

$\Rightarrow \quad \dfrac{k-4}{h+3}\cdot(3)=-1 \quad \Rightarrow \quad 4k-16=-h-3 \quad \Rightarrow \quad h+4k=13$ .... (ii)

On solving equations (i) and (ii), we get $h=\dfrac{3}{7}, k=\dfrac{22}{7}$ $\quad \therefore$ Orthocentre $\left(\dfrac{3}{7}, \dfrac{22}{7}\right)$

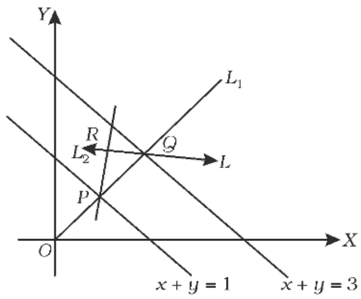
Hence, this coordinate lies in the first quadrant.

12.(1, –2) Since, a,b,c are in AP.

$\therefore \quad 2b=a+c$ or $a-2b+c=0$ which satisfy $ax+by+c=0$

$\therefore \quad ax+by+c=0$ always pass through a fixed point (1, –2).

13. Let the equation of straight-line L be $y=mx$



$P\equiv\left(\dfrac{1}{m+1},\dfrac{m}{m+1}\right)$

$Q=\left(\dfrac{3}{m+1},\dfrac{3m}{m+1}\right)$

Now, equation of $L_1: y-2x=\dfrac{m-2}{m+1}$ .... (i)

and equation of $L_2: y+3x=\dfrac{3m+9}{m+1}$ .... (ii)

By eliminating m from equations (i) and (ii), we get locus of R as $x-3y+5=0$, which represents a straight line.

14.(18) Let $L:(y-2)=m(x-8), m<0$

The points P and Q are $\left(8-\dfrac{2}{m}, 0\right)$ and $(0, 2-8m)$, respectively.

Then, $OP+OQ=\left(8-\dfrac{2}{m}\right)+(2-8m)=10+\left[-\dfrac{2}{m}+(-8m)\right]$ [using $AM \ge GM$]

$\Rightarrow \quad \left(\dfrac{2}{-m}\right)+(-8m)\ge 2\sqrt{16}$ $\quad [\because \dfrac{2}{m}$ and $-8\ m$ are positive]

$\Rightarrow \quad -\left(\dfrac{2}{m}+8m\right)\ge 8 \quad \Rightarrow \quad 10-\left(\dfrac{2}{m}+8m\right)\ge 10+8 \quad \Rightarrow \quad OP+OQ\ge 18$

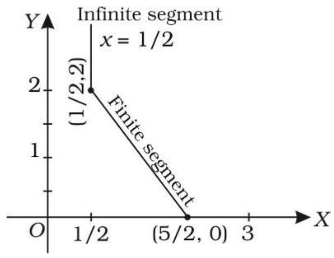
15. Now, let $P(x, y)$ be any point in the first quadrant. We have

$d(P, 0)=|x-0|+|y-0|=|x|+|y|=x+y$ $\quad [\because x,y>0]$

$d(P, A)=|x-3|+|y-2|$ [given]

$d(P, 0)=d(P, A)$ [given]

$\Rightarrow \quad x+y=|x-3|+|y-2|$ .... (i)



Case I When $0 < x < 3, 0 < y < 2$

In this case, Eq. (i) becomes $x + y = 3 - x + 2 - y \Rightarrow 2x + 2y = 5$

or $x + y = 5/2$

Case II When $0 < x < 3, y \geq 2$

Now, Eq. (i) becomes $x + y = 3 - x + y - 2$

$\Rightarrow \quad 2x = 1 \Rightarrow x = 1/2$

Case III When $x \geq 3, 0 < y < 2$

Now, Eq. (i) becomes $x + y = x - 3 + 2 - y \quad \Rightarrow \quad 2y = -1 \quad$ or $\quad y = -1/2$

Hence, no solution.

Case IV When $x \geq 3, y \geq 2$

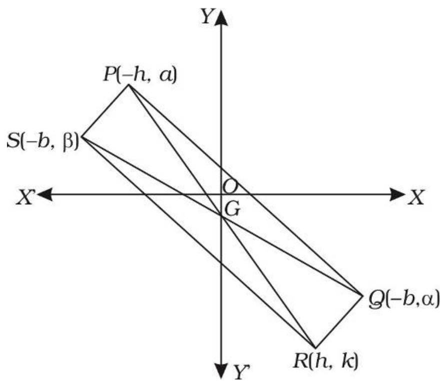
In this case, case I changes to $x + y = x - 3 + y - 2 \Rightarrow 0 = -5$

Which is not possible.

Hence, the solution set is $\{(x, y) \mid x = 1/2, y \geq 2\} \cup \{(x, y)\} \mid$

$x + y = 5/2, 0 < x < 3, 0 < y < 2\}$

The graph is given in adjoining figure.



16. Let the coordinates of Q be (b, α) and that of S be $(-b, \beta)$. Suppose, PR and SQ meet in G. Since, G is mid-point of SQ. its x-coordinate must be 0. Let the coordinates of R be (h, k).

Since, G is mid-point of PR, the x-coordinate of P must be $-h$ and as P lies on the line $y = a$, the coordinates of P are $(-h, a)$. Since, PQ is parallel to $y = mx$, slope of

$PQ = m \quad \Rightarrow \quad \dfrac{\alpha - a}{b + h} = m$ .... (i)

Again, $RQ \perp PQ$

Slope of $RQ = -\dfrac{1}{m} \quad \Rightarrow \quad \dfrac{k - \alpha}{h - b} = -\dfrac{1}{m}$ .... (ii)

From Eq. (i), we get $\alpha - a = m(b + h)$

$\Rightarrow \quad \alpha = a + m(b + h)$ .... (iii)

And from Eq. (ii), we get $k - \alpha = -\dfrac{1}{m}(h - b) \Rightarrow \quad \alpha = k + \dfrac{1}{m}(h - b)$ .... (iv)

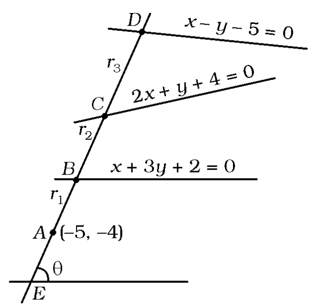
From equations (iii) and (iv), we get $a + m(b + h) = k + \dfrac{1}{m}(h - b)$

$\Rightarrow \quad am + m^2(b + h) = km + (h - b) \Rightarrow \quad (m^2 - 1)h - mk + b(m^2 + 1) + am = 0$

Hence, the locus of vertex is $(m^2 - 1)x - my + b(m^2 + 1) + am = 0$

17. Let equation of line AC is $\dfrac{y + 4}{\sin\theta} = \dfrac{x + 5}{\cos\theta} = r$

Let line AE make angle θ with X-axis and intersects $x + 3y + 2 = 0$ at B at a distance r_1 and line $2x + y + 4 = 0$ at C at a distance r_2 and line $x - y - 5 = 0$ at D at a distance r_3.



$\therefore \quad AB = r_1, AC = r_2, AD = r_3.$

$$r_1 = -\frac{-5 - 3 \times 4 + 2}{1 \cdot \cos\theta + 3 \cdot \sin\theta}\left[\because\ r = -\frac{I'}{(a\cos\theta + b\sin\theta)}\right]$$

$\Rightarrow \quad r_1 = \dfrac{15}{\cos\theta + 3\sin\theta}$ .... (i)

Similarly, $r_2 = -\dfrac{2 \times (-5) + 1(-4) + 4}{2\cos\theta + 1 \cdot \sin\theta}$

$\Rightarrow \quad r_2 = \dfrac{10}{2\cos\theta + \sin\theta}$ .... (ii)

And $\quad r_3 = -\dfrac{-5\times 1 - 4(-1) - 5}{\cos\theta - \sin\theta} \quad \Rightarrow \quad r_3 = \dfrac{6}{\cos\theta - \sin\theta}$ .... (iii)

But it is given that, $\left(\dfrac{15}{AB}\right)^2 + \left(\dfrac{10}{AC}\right)^2 = \left(\dfrac{6}{AD}\right)^2 \quad \Rightarrow \quad \left(\dfrac{15}{r_1}\right)^2 + \left(\dfrac{10}{r_2}\right)^2 = \left(\dfrac{6}{r_3}\right)^2$

$\Rightarrow \quad (\cos\theta + 3\sin\theta)^2 + (2\cos + \sin\theta)^2 = (\cos\theta - \sin\theta)^2$ [From Eqs. (i), (ii) and (iii)]

$\Rightarrow \quad \cos^2\theta + 9\sin^2\theta + 6\cos\theta\sin\theta + 4\cos^2\theta + \sin^2\theta + 4\cos\theta\sin\theta = \cos^2\theta + \sin^2\theta - 2\cos\theta\sin\theta$

$\Rightarrow \quad 4\cos^2\theta + 9\sin^2\theta + 12\sin\theta\cos\theta = 0 \quad \Rightarrow \quad (2\cos\theta + 3\sin\theta)^2 = 0 \quad \Rightarrow \quad 2\cos\theta + 3\sin\theta = 0$

$\Rightarrow \quad \cos\theta = -(3/2)\sin\theta$

On substituting this in equation of AC, we get $\dfrac{y+4}{\sin\theta} = \dfrac{x+5}{-\frac{3}{2}\sin\theta}$

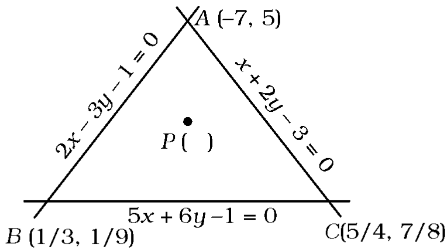
$\Rightarrow \quad -3(y+4) = 2(x+5) \quad \Rightarrow \quad -3y - 12 = 2x + 10$

$\Rightarrow \quad 2x + 3y + 22 = 0$ which is the equation of required straight line.

18. Given lines are $2x + 3y - 1 = 0$ .... (i)

$x + 2y - 3 = 0$ .... (ii)

$5x - 6y - 1 = 0$ .... (iii)



On solving equations (i), (ii) and (iii), we get the vertices of a triangle are $A(-7, 5)$, $B\left(\dfrac{1}{3}, \dfrac{1}{9}\right)$ and $C\left(\dfrac{5}{4}, \dfrac{7}{8}.\right)$

Let $P(\alpha, \alpha^2)$ be a point inside the ΔABC. Since, A and P are on the same side of $5x - 6y - 1 = 0$, both $5(-7) - 6(5) - 1$ and $5\alpha - 6\alpha^2 - 1$ must have the same sign, therefore, $5\alpha - 6\alpha^2 - 1 < 0$

$\Rightarrow \quad 6\alpha^2 - 5\alpha + 1 > 0 \quad \Rightarrow \quad (3\alpha - 1)(2\alpha - 1) > 0$

$\Rightarrow \quad \alpha < \dfrac{1}{3}$ or $\alpha > \dfrac{1}{2}$ .... (iv)

Also, since $P(\alpha, \alpha^2)$ and $C\left(\dfrac{5}{4}, \dfrac{7}{8}\right)$ lie on the same side of $2x + 3y - 1 = 0$, therefore both $2\left(\dfrac{5}{4}\right) + 3\left(\dfrac{7}{8}\right) - 1$ and $2\alpha + 3\alpha^2 - 1$ must have the same sign. Therefore, $2\alpha + 3\alpha^2 - 1 > 0$

$\Rightarrow \quad (\alpha + 1)\left(\alpha - \dfrac{1}{3}\right) > 0 \quad \Rightarrow \quad \alpha < -1 \cup \alpha > 1/3$ .... (v)

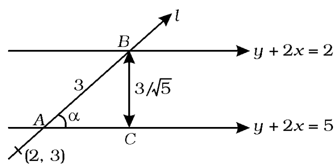
and lastly $\left(\dfrac{1}{3}, \dfrac{1}{9}\right)$ and $P(\alpha, \alpha^2)$ lie on the same side of the line therefore, $\dfrac{1}{3} + 2\left(\dfrac{1}{9}\right) - 3$ and $\alpha + 2\alpha^2 - 3$ must have the same sign.

Therefore, $2\alpha^2 + \alpha - 3 < 0 \Rightarrow \quad 2\alpha(\alpha - 1) + 3(\alpha - 1) < 0 \quad \Rightarrow \quad (2\alpha + 3)(\alpha - 1) < 0 \Rightarrow -\dfrac{3}{2} < \alpha < 1$

On solving equations (i), (ii) and (iii), we get the common answer is $-\dfrac{3}{2} < \alpha < -1 \cup \dfrac{1}{2} < \alpha < 1.$

19. Let l makes an angle α with the given parallel lines and intercept AB is of 3 units.

Now, distance between parallel lines $= \dfrac{|5-2|}{\sqrt{1^2 + 2^2}} = \dfrac{3}{\sqrt{5}}$



$\therefore \quad \sin\alpha = \dfrac{1}{\sqrt{5}}, \cos\alpha = \dfrac{2}{\sqrt{5}}$ and $\tan\alpha = \dfrac{1}{2}$

$\Rightarrow$ Equation of straight line passing through (2, 3) and making an angle α with $y + 2x = 5$ is

$$\frac{y-3}{x-2} = \tan(\theta+\alpha) \Rightarrow \frac{y-3}{x-2} = \frac{\tan\theta + \tan\alpha}{1-\tan\theta\tan\alpha} \quad \text{and} \quad \frac{y-3}{x-2} = \frac{\tan\theta - \tan\alpha}{1+\tan\theta\tan\alpha}$$

$$\Rightarrow \quad \frac{y-3}{x-2} = -\frac{3}{4} \quad \text{and} \quad \frac{y-3}{x-2} = \frac{1}{0} \quad \Rightarrow \quad 3x+4y=18 \quad \text{and} \quad x=2$$

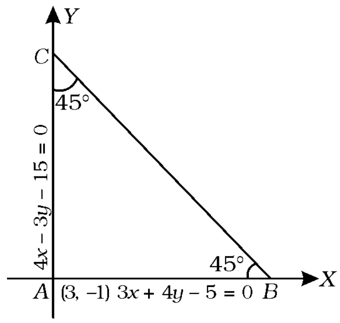
20. Let m_1 and m_2 be the slopes of the lines $3x + 4y = 5$ and $4x - 3y = 15$, respectively.

Then, $m_1 = -\dfrac{3}{4}$ and $m_2 = \dfrac{4}{3}$

Clearly, $m_1 m_2 = -1$. So, lines AB and AC are at right angle. Thus, the ΔABC is a right angled isosceles triangle.

Hence, the line BC through (1, 2) will make an angle of 45° with the given lines. So, the possible equations of BC are $(y-2) = \dfrac{m \pm \tan 45°}{1 \mp m\tan 45°}(x-1)$

where, m = slope of $AB = -\dfrac{3}{4}$



$$\Rightarrow \quad (y-2) = \frac{-\frac{3}{4} \pm 1}{1 \mp \left(-\frac{3}{4}\right)}(x-1) \quad \Rightarrow \quad (y-2) = \frac{-3 \pm 4}{4 \mp 3}(x-1)$$

$$\Rightarrow \quad (y-2) = \frac{1}{7}(x-1) \quad \text{and} \quad (y-2) = -7(x-1)$$

$$\Rightarrow \quad x - 7y + 13 = 0 \quad \text{and} \quad 7x + y - 9 = 0$$

21. The equation of the line AB is $\dfrac{x}{7} + \dfrac{y}{-5} = 1$ .... (i)

$\Rightarrow \quad 5x - 7y = 35$

Equation of line perpendicular to AB is $7x + 5y = \lambda$ .... (ii)

It meets X-axis at $P(\lambda/7, 0)$ and Y-axis at $Q(0, \lambda/5)$.

The equations of lines AQ and BP are $\dfrac{x}{7} + \dfrac{5y}{\lambda} = 1$ and $\dfrac{7x}{\lambda} - \dfrac{y}{5} = 1$, respectively.

Let $R(h, k)$ be their point of intersection of lines AQ and BP.

Then, $\dfrac{h}{7} + \dfrac{5k}{\lambda} = 1$ and $\dfrac{7h}{\lambda} - \dfrac{k}{6} = 1$

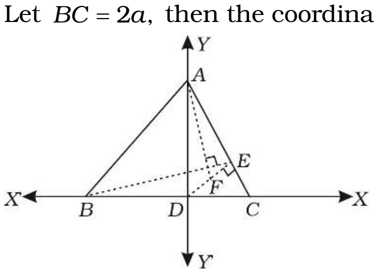
$\Rightarrow \quad \dfrac{1}{5k}\left(1 - \dfrac{h}{7}\right) = \dfrac{1}{7h}\left(1 + \dfrac{k}{5}\right)$ [on eliminating λ] $\Rightarrow h(7-h) = k(5+k) \Rightarrow h^2 + k^2 - 7h + 5k = 0$

Hence, the locus of a point is $x^2 + y^2 - 7x + 5y = 0$.

22. Let BC be taken as X-axis with origin at D, the mid-point of BC and DA will be Y-axis.

Given, $AB = AC$

Let $BC = 2a$, then the coordinates of B and C are $(-a, 0)$ and $(a, 0)$ let $A(0, h)$.



Then, equation of AC is $\frac{x}{a}+\frac{y}{h}=1$ .... (i)

and equation of $DE \perp AC$ and passing through origin is $\frac{x}{h}-\frac{y}{a}=0 \Rightarrow x=\frac{hy}{a}$ .... (ii)

On solving, equations (i) and (ii), we get the coordinates of point E as follows $\frac{hy}{a^2}+\frac{y}{h}=1$

$\Rightarrow \quad y=\frac{a^2h}{a^2+h^2} \quad \therefore \quad$ Coordinate of $E=\left(\frac{ah^2}{a^2+h^2}, \frac{a^2h}{a^2+h^2}\right)$

Since, F is mid-point of DE. $\quad \therefore \quad$ Coordinate of $F\left[\frac{ah^2}{2(a^2+h^2)}, \frac{a^2h}{2(a^2+h^2)}\right]$

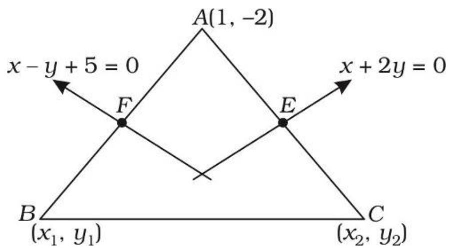
$$\therefore \quad \text{Slope of } AF,\ m_1=\frac{h-\frac{a^2h}{2(a^2+h^2)}}{0-\frac{ah^2}{a(a^2+h^2)}}=\frac{2h(a^2+h^2)-a^2h}{-ah^2} \Rightarrow m_1=\frac{-(a^2+2h^2)}{ah} \quad .... \text{(iii)}$$

$$\text{And slope of } BE,\ m_2=\frac{\frac{a^2h}{a^2+h^2}-0}{\frac{ah^2}{a^2+h^2}+a}=\frac{a^2h}{ah^2+a^3+ah^2} \Rightarrow m_2=\frac{ah}{a^2+2h^2} \quad .... \text{(iv)}$$

From equations (iii) and (iv), $m_1m_2=-1 \Rightarrow AF \perp BE$

23. Let the coordinates of B and C be (x_1, y_1) and (x_2, y_2) respectively. Let m_1 and m_2 be the slopes of AB and AC, respectively. Then, $m_1=$ slope of $AB=\frac{y_1+2}{x_1-2}$

and $\quad m_2=$ slope of $AC=\frac{y_2+2}{x_2-1}$



Let F and E be the mid-point of AB and AC, respectively. Then, the coordinates of E and F are $E\left(\frac{x_2+1}{2}, \frac{y_2-2}{2}\right)$ and $F\left(\frac{x_1+1}{2}, \frac{y_1-2}{2}\right)$, respectively.

Now, F lies on $x-y+5=0$. $\Rightarrow \frac{x_1+1}{2}-\frac{y_1-2}{2}=-5 \Rightarrow x_1-y_1+13=0$ .... (i)

Since, AB is perpendicular to $x-y+5=0$.

$\therefore$ (slop of AB). (slope of $x-y+5=0$) $=-1$.

$\Rightarrow \frac{y_1+2}{x_1-1}\cdot(1)=-1 \Rightarrow y_1+2=-x_1+1 \Rightarrow x_1+y_1+1=0$ .... (ii)

On solving equations (i) and (ii), we get $x_1=-7, y_1=6$.

So, the coordinates of B are (–7, 6).

Now, E lies on $x+2y=0$. $\therefore \frac{x_1+1}{2}+2\left(\frac{y_2-2}{2}\right)=0$

$\Rightarrow x_2+2y_2-3=0.$ .... (iii)

Since, AC is perpendicular to $x+2y=0$ $\therefore$ (slope of AC). (slope of $x+2y=0$) $=-1$

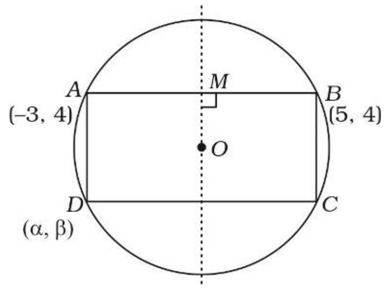
$\Rightarrow \frac{y_2+2}{x_2-1}\cdot\left(-\frac{1}{2}\right)=-1 \Rightarrow 2x_2-y_2=4$ .... (iv)

On solving equations (iii) and (iv), we get $x_2=\frac{11}{5}$ and $y_2=\frac{2}{5}$

So, the coordinates of C are $\left(\frac{11}{5}, \frac{2}{5}\right)$.

$y-6=\frac{2/5-6}{11/5+7}(x+7) \Rightarrow \quad -23(y-6)=14(x+7) \quad \Rightarrow \quad 14x+23y-40=0$

24. Let O be the centre of circle and M be mid-point of AB.



Then, $OM \perp AB \Rightarrow M(1, 4)$

Since, slope of $AB = 0$

Equation of straight-line MO is $x=1$ and equation of diameter is $4y=x+7$.

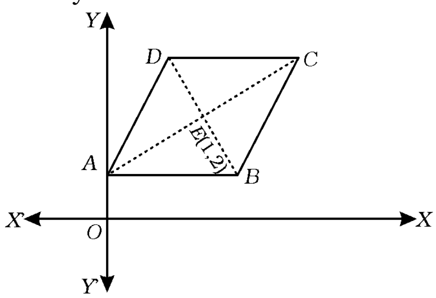
$\Rightarrow$ Centre is (1, 2).

Also, O is mid-point of BD

$\Rightarrow \left(\frac{\alpha+5}{2}, \frac{\beta+4}{2}\right)=(1, 2) \Rightarrow \alpha=-3, \beta=0 \therefore AD=\sqrt{(-3+3)^2+(4-0)^2}=4$ and $AB=\sqrt{64+0}=8$

Thus, area of rectangle $=8\times 4=32$ sq units

25. Let the coordinates of A be $(0, \alpha)$. Since, the sides AB and AD are parallel to the lines $y=x+2$ and $y=7x+3$, respectively.



$\therefore$ The diagonal AC is parallel to the bisector of the angle between these two lines. The equation of the bisectors are given by $\frac{x-y+2}{\sqrt{2}}=\pm\frac{7x-y+3}{\sqrt{50}}$

$\Rightarrow \quad 5(x-y+2)=\pm(7x-y+3) \quad \Rightarrow \quad 2x+4y-7=0$ and $12x-6y+13=0$.

Thus, the diagonals of the rhombus are parallel to the lines $2x+4y-7=0$ and $12x-6y+13=0$.

$\therefore$ Slope of $AE=-\frac{2}{4}$ or $\frac{12}{6} \Rightarrow \quad \frac{2-\alpha}{1-0}=-\frac{1}{2}$ or $\frac{2-\alpha}{1-0}=2 \Rightarrow \quad \alpha=\frac{5}{2}$ or $\alpha=0$.

Hence, the coordinates are $(0, 5/2)$ or $(0, 0)$.

26. The equation of any line passing through (1, – 10) is $y+10=m(x-1)$.

Since, it makes equal angles, say θ, with the given lines,

therefore $\tan\theta=\frac{m-7}{1+7m}=\frac{m-(-1)}{1+m(-1)} \Rightarrow m=\frac{1}{3}$ or -3

Hence, the equations of third side are $y+10=\frac{1}{3}(x-1)$ or $y+10=-3(x-1)$

i.e. $\quad x-3y-31=0$ or $3x+y+7=0$

27. Let ABC be a triangle whose vertices are $A[at_1t_2, a(t_1+t_2)]$, $B(at_2t_3, a(t_2+t_3))$ and $C[at_1t_3, a(t_1+t_3)]$.

Then, Slope of $BC = \dfrac{a(t_2+t_3)-a(t_1+t_3)}{at_2t_3-at_1t_3} = \dfrac{1}{t_3}$

Slope of $AC = \dfrac{a(t_1+t_3)-a(t_1+t_2)}{at_1t_3-at_1t_2} = \dfrac{1}{t_1}$

So, the equation of a line through A perpendicular to BC is $y-a(t_1+t_2) = -t_3(x-at_1t_2)$ .... (i)

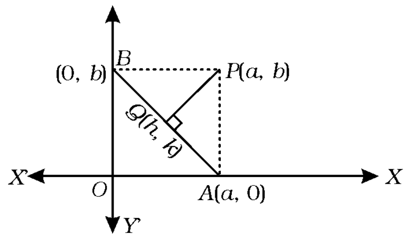
And the equation of a line through B perpendicular to AC is $y-a(t_2+t_3) = -t_3(x-at_1t_2)$ .... (ii)

The point of intersection of equations (i) and (ii), is the orthocentre.
On subtracting equations (ii) from equation (i), we get $x=-a$.
On subtracting equation (i), we get
$y = a(t_1+t_2+t_3+t_1t_2t_3)$
Hence, the coordinates of the orthocentre are $[-a, a(t_1+t_2+t_3+t_1t_2t_3)]$.

28. Let $OA = a$ and $OB = b$. Then, the coordinates of A and B are $(a, 0)$ and $(0, b)$ respectively and also, coordinates of P are (a, b). Let θ be the foot of perpendicular from P on AB and let the coordinates of $Q(h, k)$. Here, a and b are the variable and we have to find locus of Q.

Given, $AB = c \Rightarrow AB^2 = c^2 \Rightarrow OA^2 + OB^2 = c^2 \Rightarrow a^2+b^2=c^2$ .... (i)

Since, PQ is perpendicular to AB.



$\Rightarrow$ Slope of AB. Slope of $PQ = -1$

$\Rightarrow \dfrac{0-b}{a-0}\cdot\dfrac{k-b}{h-a} = -1$

$\Rightarrow bk - b^2 = ah - a^2$

$\Rightarrow ah - bk = a^2 - b^2$ .... (ii)

Equation of line AB is $\dfrac{x}{a}+\dfrac{y}{b} = 1$.

Since, Q lies on AB, therefore $\dfrac{h}{a}+\dfrac{k}{b}=1 \Rightarrow bh + ak = ab$ .... (iii)

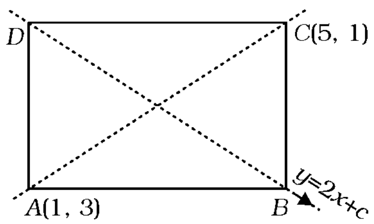
On solving equations (ii) and (iii), we get $\dfrac{h}{ab^2+a(a^2-b^2)} = \dfrac{k}{-b(a^2-b^2)+a^2b} = \dfrac{1}{a^2+b^2}$

$\Rightarrow \dfrac{h}{a^3} = \dfrac{k}{b^3} = \dfrac{1}{c^2}$ [From Eq. (i)]

$\Rightarrow a = (hc^2)^{1/3}$ and $b = (kc^2)^{1/3}$

On substituting the values of a and b in $a^2+b^2=c^2$,
We get $h^{2/3}+k^{2/3} = c^{2/3}$
Hence, locus of a point is $x^{2/3}+y^{2/3}=c^{2/3}$.

29. Since, diagonals of rectangle bisect each other, so mid-point of (1, 3) and (5, 1) must satisfy $y = 2x + c$, i.e. (3, 2) lies on it.



$\Rightarrow 2 = 6 + c \Rightarrow c = -4 \quad \therefore$ Other two vertices lie on $y = 2x-4$

Let the coordinate of B be $(x, 2x-4)$.

$\therefore$ Slope of AB. Slope of $BC = -1 \Rightarrow \left(\dfrac{2x-4-3}{x-1}\right)\cdot\left(\dfrac{2x-4-1}{x-5}\right) = -1$

$\Rightarrow (x^2-6x+8) = 0 \Rightarrow x = 4, 2 \Rightarrow y = 4, 0$

Hence, required points are (4, 4), (2, 0).

30. Let the coordinates of third vertex be $C(a, b)$.

Since, CH is $\perp AB$,

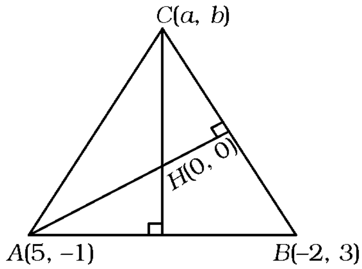
$\therefore \quad \left(\frac{b}{a}\right)\left(\frac{4}{-7}\right) = -1$

$\Rightarrow \quad 4b = 7a \quad$ .... (i)

Also, $AH \perp BC$

$\therefore \quad \left(-\frac{1}{5}\right)\left(\frac{3-b}{-2-a}\right) = -1$

$\Rightarrow \quad 3 - b = -10 - 5a \quad$ .... (ii)



On solving equations (i) and (ii), we get $a = -4, b = -7 \quad \therefore \quad$ C has co-ordinates $(-4, -7)$

31. Since, the side AB is perpendicular to AD.

$\therefore \quad$ Its equation is of the form $7x - 4y + \lambda = 0$

Since, it passes through $(-3, 1)$.

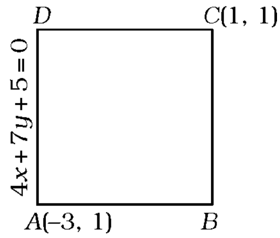
$\therefore \quad 7(-3) - 4(1) + \lambda = 0.$

$\Rightarrow \quad \lambda = 25$

$\therefore \quad$ Equation of AB is $7x - 4y + 25 = 0$

Now, BC is parallel to AD. Therefore, its equation is

$4x + 7y + \lambda = 0$



Since, it passes through $(1, 1)$.

$\therefore \quad 4(1) + 7(1) + \lambda = 0 \Rightarrow \lambda = -11 \quad \therefore \quad$ Equation of BC is $4x + 7y - 11 = 0$

Now, equation of DC is $7x - 4y + \lambda = 0$

$\Rightarrow \quad 7(1) - 4(1) + \lambda = 0 \quad \Rightarrow \quad \lambda = -3 \quad \therefore \quad 7x - 4y - 3 = 0$

Integer Answer Type:

32. Let $P(x, y)$ is the point in first quadrant.

Now, $\quad 2 \le \left|\frac{x-y}{\sqrt{2}}\right| + \left|\frac{x+y}{\sqrt{2}}\right| \le 4$

$2\sqrt{2} \le |x - y| + |x + y| \le 4\sqrt{2}$

Case I $\quad x \ge y$

$2\sqrt{2} \le (x - y) + (x + y) \le 4\sqrt{2} \Rightarrow x \in [\sqrt{2}, 2\sqrt{2}]$

Case II $\quad x < y$

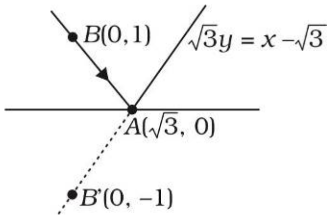
$2\sqrt{2} \le y - x + (x + y) \le 4\sqrt{2}$

$y \in [\sqrt{2}, 2\sqrt{2}] \Rightarrow A = (2\sqrt{2})^2 - (\sqrt{2})^2 = 6$ sq units

33.(B) Take any point $B(0, 1)$ on given line.

Equation of AB' is $y - 0 = \frac{-1-0}{0-\sqrt{3}}(x - \sqrt{3})$

$\Rightarrow \quad -\sqrt{3}y = -x + \sqrt{3} \Rightarrow \quad x - \sqrt{3}y = \sqrt{3} \Rightarrow \quad \sqrt{3}y = x - \sqrt{3}$



34.(D) $\Delta = \begin{vmatrix} -\sin(\beta-\alpha) & -\cos\beta & 1 \\ \cos(\beta-\alpha) & \sin\beta & 1 \\ \cos(\beta-\alpha+\theta) & \sin(\beta-\theta) & 1 \end{vmatrix}$

Clearly, $\Delta \ne 0$ for any value of α, β, θ. Hence, points are non-collinear.

35.(C) The line segment QR makes an angle of 60° with the positive direction of X-axis

So, the bisector of the angle PQR will make an angle of 60° with the negative direction of X-axis it will therefore have angle of inclination of 120° and so, its equation is

$y - 0 = \tan 120°(x - 0) \quad \Rightarrow \quad y = -\sqrt{3}x \Rightarrow y + \sqrt{3}x = 0$

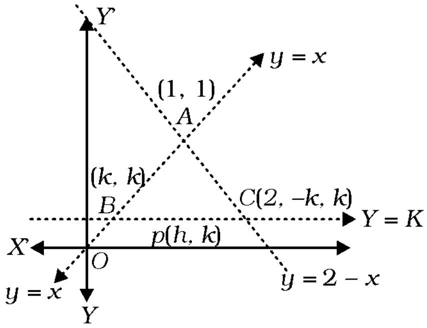
Fill in the blank.

36. Let BD bisects angle ABC and D lies on AC, now $\frac{BC}{BA} = \frac{CD}{DA} \Rightarrow \frac{CD}{DA} = \frac{1}{2}$

$\therefore \quad D \equiv \left(\frac{1}{3}, \frac{1}{3}\right)$, so equation of BD is: $y - 1 = \frac{2/3}{14/3}(x-5) \quad \Rightarrow \quad 7y = x + 2$

Analytical and Descriptive Questions:

37. Here, the triangle formed by a line parallel to X-axis passing through $P(h, k)$ and the straight line $y = x$ and $y = 2 - x$ could be as shown below :



Since, area of $\Delta ABC = 4h^2$

$\therefore \quad \frac{1}{2} AB \cdot AC = 4h^2$

where, $\quad AB = \sqrt{2}\,|k-1|$

and $\quad AC = \sqrt{2}(|k-1|)$

$\Rightarrow \quad \frac{1}{2} \cdot 2(k-1)^2 = 4h^2$

$\Rightarrow \quad 4h^2 = (k-1)^2 \Rightarrow \quad 2h = \pm(k-1)$

The locus of a point is $2x = \pm(y-1)$.

38. Given equation of lines are $x - 2y + 4 = 0$ and $4x - 3y + 2 = 0$

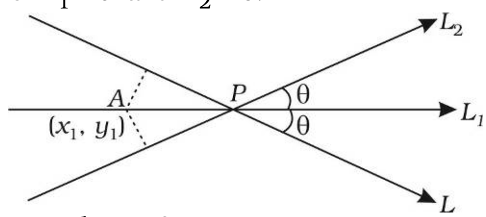
Here, $\quad a_1a_2 + b_1b_2 = 1(4) + (-2)(-3) = 10 > 0$

For obtuse angle bisector, we take negative sign.

$\therefore \quad \frac{x-2y+4}{\sqrt{5}} = -\frac{4x-3y+2}{5} \quad \Rightarrow \quad \sqrt{5}(x-2y+4) = -(4x-3y+2)$

$\Rightarrow \quad (4+\sqrt{5})x - (2\sqrt{5}+3)y + (4\sqrt{5}+2) = 0$

39. Since, the required line L passes through the intersection of $L_1 = 0$ and $L_2 = 0$.



So, the equation of the required line L is

$L_1 + \lambda L_2 = 0$.

i.e. $(ax + by + c) + \lambda(lx + my + n) = 0 \quad$ .... (i)

where, λ is a parameter.

Since, L_1 is the angle bisector of $L = 0$ and $L_2 = 0$.

$\therefore \quad$ Any point $A(x_1, y_1)$ on L_1 is equidistant from $L = 0$ and $L_2 = 0$.

$\Rightarrow \quad \frac{|lx_1 + my_1 + n|}{\sqrt{l^2+m^2}} = \frac{|(ax_1+by_1+c)+\lambda(lx_1+my_1+n)|}{\sqrt{(a+\lambda l)^2+(b+\lambda m)^2}} \quad$ .... (ii)

But, $A(x_1, y_1)$ lies on L_1. So, it must satisfy the equation of L_1, i.e., $ax_1 + by_1 + c = 0$ in Eq. (ii) we get

$\frac{|lx_1+my_1+n|}{\sqrt{l^2+m^2}} = \frac{|0+\lambda(lx_1+my_1+n)|}{\sqrt{(a+\lambda l)^2+(b+\lambda m)^2}} \Rightarrow \lambda^2(l^2+m^2) = (a+\lambda l)^2 + (b+\lambda m)^2 \therefore \quad \lambda = -\frac{(a^2+b^2)}{2(al+bm)}$

On substituting the value of λ in Eq. (i), we get $(ax+by+c) - \frac{(a^2+b^2)}{2(al+bm)}(lx+my+n) = 0$

$\Rightarrow \quad 2(al+bm)(ax+by+c) - (a^2+b^2)(lx+my+n) = 0$ which is required equation of line L.

40.(ABC) Given lines $px + qy + r = 0$, $qx + ry + p = 0$ and $rx + py + q = 0$ are concurrent.

$\therefore \quad \begin{vmatrix} p & q & r \\ q & r & p \\ r & p & q \end{vmatrix} = 0$

Applying $R_1 \to R_1 + R_2 + R_3$ and taking common from R_1

$$(p+q+r)\begin{vmatrix}1 & 1 & 1\\ q & r & p\\ r & p & q\end{vmatrix}=0 \Rightarrow (p+q+r)(p^2+q^2+r^2-pq-qr-pr)=0 \Rightarrow p^3+q^3+r^3-3pqr=0$$

Therefore, (a), (B) and (c) are the answers.

41. (A $\to$ s, B $\to$ p, q, C $\to$ r, D $\to$ p, q, s)

(A) Solving equation L_1 and L_3,

$$\frac{x}{-36+10}=\frac{y}{-25+12}=\frac{1}{2-15} \quad \therefore \quad x=2, y=1$$

L_1, L_2, L_3 are concurrent, if point (2, 1) lies on L_2

$\therefore \quad 6-k-1=0 \Rightarrow k=5$

(B) Either L_1 is parallel to L_2, or L_3 is parallel to L_2, then

$$\frac{1}{3}=\frac{3}{-k} \text{ or } \frac{3}{5}=\frac{-k}{2} \Rightarrow k=-9 \text{ or } k=\frac{-6}{5}$$

(C) L_1, L_2, L_3 form a triangle, if they are not concurrent, or not parallel.

$$\therefore \quad k \neq 5, -9, -\frac{6}{5} \Rightarrow k=\frac{5}{6}$$

(D) L_1, L_2, L_3 do not form a triangle, if $k=5, -9, -\frac{6}{5}$.

42.(F) Since, $\begin{vmatrix}x_1 & y_1 & 1\\ x_2 & y_2 & 1\\ x_3 & y_3 & 1\end{vmatrix}=\begin{vmatrix}a_1 & b_1 & 1\\ a_2 & b_2 & 1\\ a_3 & b_3 & 1\end{vmatrix}$ represents triangles are equal in area, which does not imply triangles are congruent. Hence, given statement is false.

43. $$\frac{\frac{1}{2}\begin{vmatrix}x & y & 1\\ -3 & 5 & 1\\ 4 & -2 & 1\end{vmatrix}}{\frac{1}{2}\begin{vmatrix}6 & 3 & 1\\ -3 & 5 & 1\\ 4 & -2 & 1\end{vmatrix}}=\left|\frac{7x+7y-14}{49}\right|=\left|\frac{x+y-2}{7}\right|$$

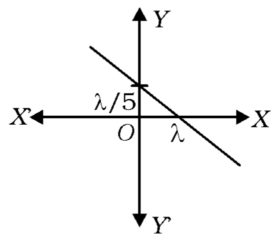
44.(AD) A straight line perpendicular to $5x-y=1$ is $x+5y=\lambda$.

Since, area of triangle = 5

$\Rightarrow \quad \frac{1}{2}\left|\lambda \cdot \frac{\lambda}{5}\right|=5 \Rightarrow \lambda^2=50$

$\Rightarrow \quad |\lambda|=5\sqrt{2}.$

$\therefore$ Equation of the line L is, $x+5y=\pm 5\sqrt{2}$



45.(B) Let a and b non-zero-real numbers.

Therefore, the given equation $(ax^2+by^2+c)(x^2-5xy+6y^2)=0$

$\Rightarrow \quad (x-2y)(x-3y)=0$

$\Rightarrow \quad x=2y$ and $x=3y$

Represent two straight lines passing through origin or $ax^2+by^2+c=0$ when $c=0$ and a and b are of same signs, then $ax^2+by^2+c=0$, $c=0$ and $y=0$.

Which is a point specified as the origin.

When, $a=b$ and c is of sign opposite to that of a, $ax^2+by^2+c=0$ represents a circle.

Hence, the given equation, $(ax^2+by^2+c)(x^2-5xy+6y^2)=0$ may represent two straight lines and a circle.

46.(B) Let S be the mid-point of QR and given ΔPQR is an isosceles.

Therefore, $PS \perp QR$ and S is mid-point of hypotenuse, therefore S is equidistant from P, Q, R.

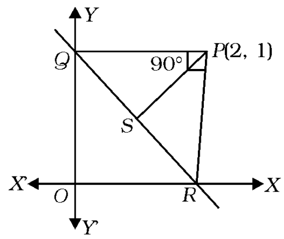
$\therefore \quad PS = QS = RS$

Since, $\angle P = 90°$ and $\angle Q = \angle R$

But $\angle P + \angle Q + \angle R = 180°$

$\therefore \quad 90° + \angle Q + \angle R = 180°$

$\Rightarrow \quad \angle Q = \angle R = 45°$



Now, slope of QR is –2. [Given]

But $QR \perp PS$.

$\therefore \quad$ Slope of PS is $1/2$.

Let m be the slope of PQ.

$\therefore \quad \tan(\pm 45°) = \dfrac{m - 1/2}{1 + m(1/2)} \quad \Rightarrow \quad \pm 1 = \dfrac{2m-1}{2+m} \quad \Rightarrow \quad m = 3, -1/3$

$\therefore \quad$ Equations of PQ and PR are $y - 1 = 3(x-2)$ and $y - 1 = -\dfrac{1}{3}(x-2)$ or $3(y-1) + (x-2) = 0$

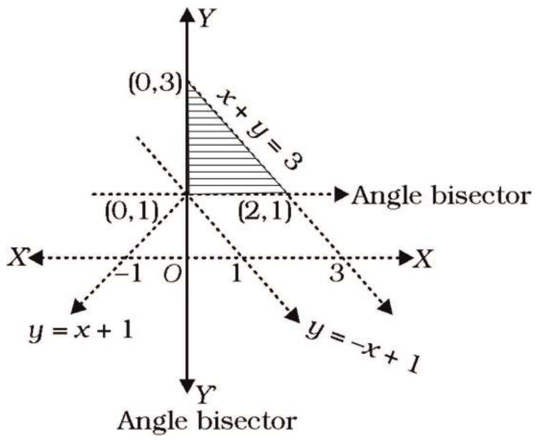
Therefore, joint equation of PQ and PR is $[3(x-2) - (y-1)][(x-2) + 3(y-1)] = 0$

$\Rightarrow \quad 3(x-2)^2 - 3(y-1)^2 + 8(x-2)(y-1) = 0 \quad \Rightarrow \quad 3x^2 - 3y^2 + 8xy - 20x - 10y + 25 = 0$

47.(A) Given, $x^2 - y^2 + 2y = 1 \quad \Rightarrow \quad x^2 = (y-1)^2 \quad \Rightarrow \quad x = y - 1$

and $\quad x = -y + 1$

From the graph, it is clear that equation of angle bisectors are $y = 1$



and $\quad x = 0$

$\therefore \quad$ Area of region bounded by $x + y = 3, x = 0$

and $y = 1$ is $\Delta = \dfrac{1}{2} \times 2 \times 2 = 2$ sq units

48. The given curve is $3x^2 - y^2 - 2x + 4y = 0$ .... (i)

Let $y = mx + c$ be the chord of curve (i) which subtend right angle at origin. Then, the combined equation of lines joining points of intersection of curve (i) and chord $y = mx + c$ to the origin, can be obtained by the equation of the curve homogenous, i.e.

$$3x^2 - y^2 - 2x\left(\frac{y - mx}{c}\right) + 4y\left(\frac{y - mx}{c} = 0\right) \Rightarrow \quad 3cx^2 - cy^2 - 2xy + 2mx^2 + 4y^2 - 4mxy = 0$$

$\Rightarrow \quad (3c + 2m)x^2 - 2(1 + 2m)y + (4 - c)y^2 = 0$

Since, the lines represented are perpendicular to each other.

$\therefore \quad$ Coefficient of $x^2 +$ Coefficient of $y^2 = 0 \quad \Rightarrow \quad 3c + 2m + 4 - c = 0 \Rightarrow \quad c + m + 2 = 0$

On comparing with $y = mx + c \quad \Rightarrow \quad y = mx + c$ passes through (1, –2).