Rotational motion 3

Vidyamandir Classes SINCE 1986

IIT JEE | MEDICAL | FOUNDATION

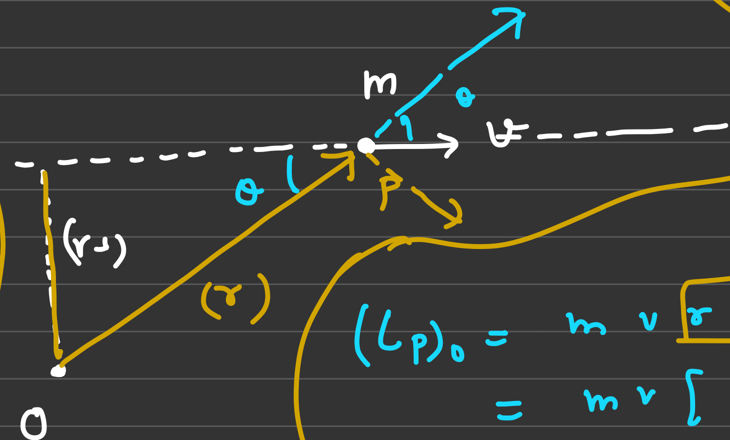
Angular momentum : (about some point)

of Point mass:

$(\vec{L_p})_o = m(\vec{r} \times \vec{v})$

"A vector from o to particle"

$\vec{v}$ is velocity of particle



$\sin\theta = \frac{r_\perp}{r}$

$(L_p)_o = (m r v_\perp)$

$(L_p)_o = m v \boxed{r \sin\theta}$

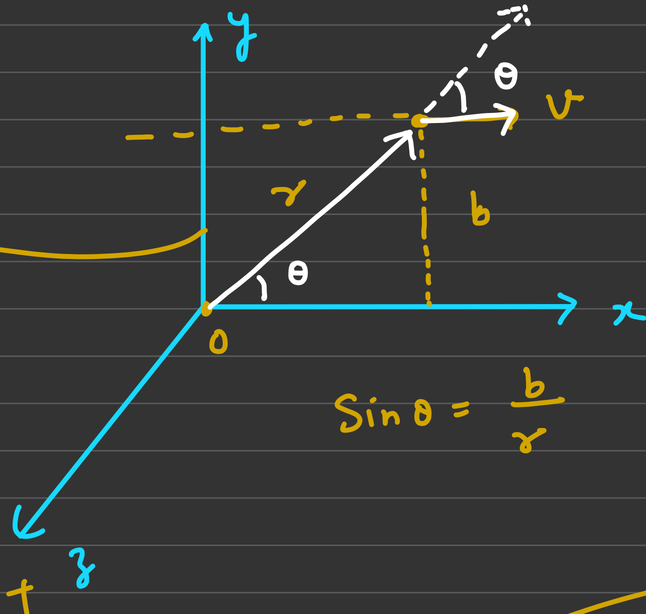
$= m v [r_\perp]$

$(L_p)_o = \underline{(m v r_\perp)}$ → is ⊥ distance from o to line of velocity

$|(L_p)_o| = (m r v \sin\theta)$

θ = angle between $\vec{r}$ and $\vec{v}$

e)



$(L_p)_O = (m v b)$

$\sin\theta = \frac{b}{r}$

find $(\vec{L_p})_O =$

$$(\vec{L_p})_O = m(\vec{r} \times \vec{v})$$

$$= m r v \sin\theta$$

$$= m \frac{b}{\cancel{\sin\theta}} v \cancel{\sin\theta}$$

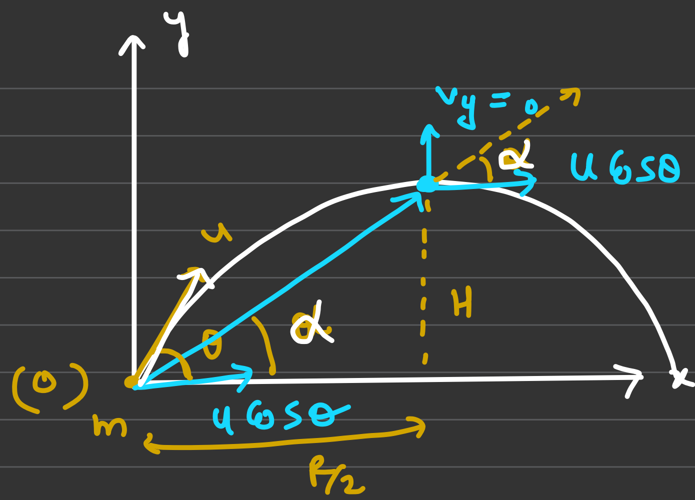
$$|(L_p)_O| = (m b v) \quad \text{Ans}$$

Direction: we get Direction of angular momentum using R.H.T. & "Cross or curl"

Vector

$$(\vec{L_p})_O = m b v (-\hat{k})$$

θ)



find angular moment of Particle about o when particle is at

1) ~~Height~~ Heighest point

H = max Height

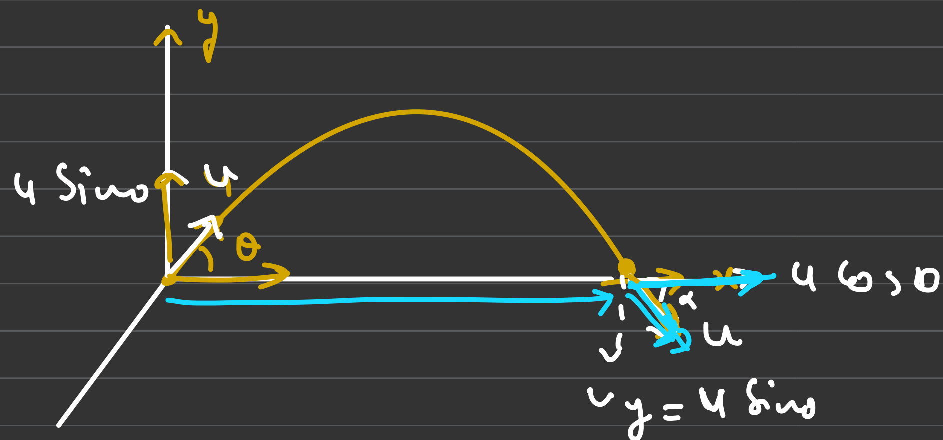
$$(\vec{L_P})_o = m(\vec{r} \times \vec{v})$$

$$= m\, r\, v \sin\alpha$$

$$= m\cancel{\sqrt{H^2 + \frac{R^2}{4}}} \times (u\cos\theta) \times \frac{H}{\cancel{\sqrt{H^2 + \frac{R^2}{4}}}}$$

$$= \underbrace{m\, u\cos\theta\, H}$$ into plane

(ii) just befor hitting the ground ?



$$R = \frac{u^2 \sin 2\theta}{2g}$$

$$(L_p)_O = m(\vec{r} \times \vec{v})$$

$$= m r v \sin\alpha$$

$$= m \times R \times u \times \sin\alpha$$

$$= (m u R \sin\theta)$$

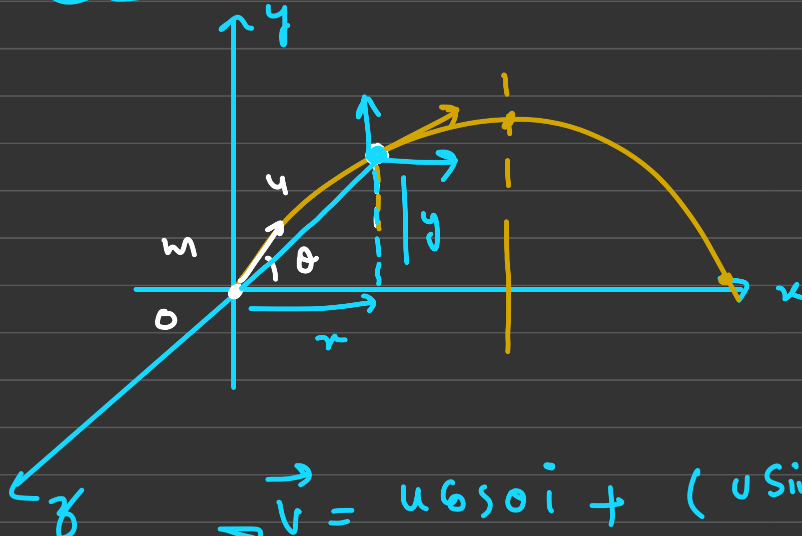
$$\tan\alpha = \frac{u \sin\theta}{u \cos\theta}$$

$$\tan\alpha = \tan\theta$$

$$\alpha = \theta$$

(iii) Angular momentum of particle about 'O' at any instant

$(L_P)_O = ?$



$\vec{r} = x\hat{i} + y\hat{j}$

Advanced

$\vec{v} = u\cos\theta\,\hat{i} + (u\sin\theta - gt)\hat{j}$

$\vec{r} = u\cos\theta\, t\,\hat{i} + (u\sin\theta\, t - \frac{1}{2}gt^2)\hat{j}$

$$\boxed{L(t) = m(\vec{r} \times \vec{v})}$$

HW

Homework: "Calculate range of time of L increasing or Decreasing"

Newton's Second Law of Rotation:

"Rate of change of angular momentum of point/body about any point is going to give us torque"

$$(\vec{L_p})_0 = m(\vec{r} \times \vec{v})$$

$$(\vec{\tau_p})_0 = \frac{d}{dt}(\vec{L_p})_0 = m\frac{d}{dt}(\vec{r} \times \vec{v})$$

$$\Rightarrow (\vec{\tau_p})_O = m \frac{d}{dt}(\vec{r}\times\vec{v})$$

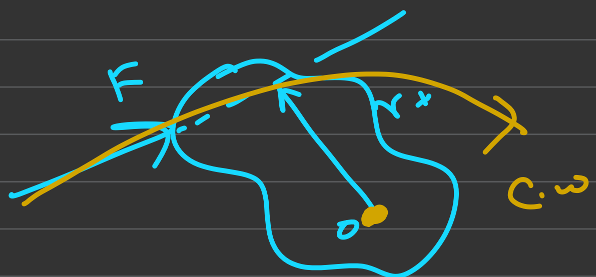
$$\Rightarrow (\vec{\tau_p})_O = m\left\{\underline{\frac{d}{dt}(\vec{r})\times\vec{v} + \vec{r}\times\frac{d}{dt}(\vec{v})}\right\}$$

$$(\vec{\tau_p})_O = m\left\{\underbrace{\cancel{\vec{v}\times\vec{v}}}_{0} + \underline{\vec{r}\times\vec{a}}\right\}$$

$$(\vec{\tau_p})_O = m(\vec{r}\times\vec{a})$$

$$(\vec{\tau_p})_O = \vec{r}\times(m\vec{a})$$

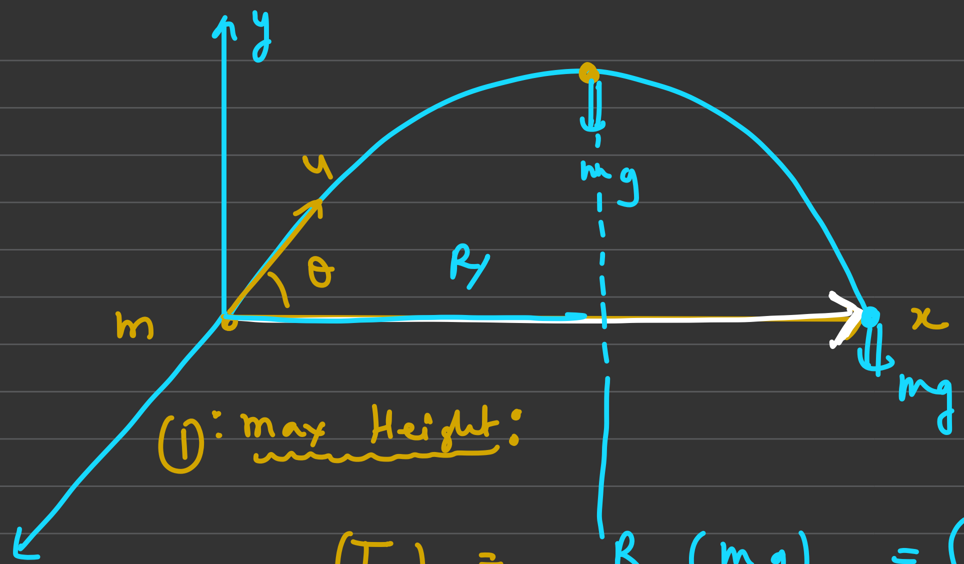
$$(\vec{\tau_p})_O = \vec{r}\times\vec{F}$$



$$\tau_O = r_\perp F$$

$$= |r||F|\sin\theta = (r_\perp)F$$

9)



① : max Height:

$$(\tau_0)_P = \frac{R}{2}(mg) = \left(\frac{mgR}{2}\right)$$

② Just before hitting the ground

$$(\tau_0)_P = \underline{mgR}$$

Angular momentum of body Rotating about fixed axis:

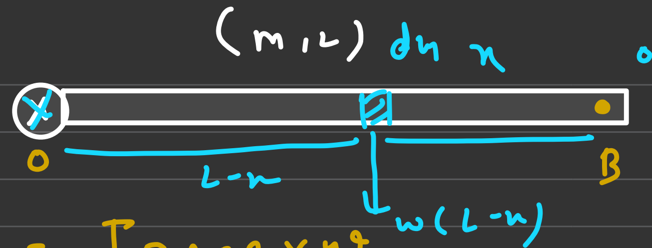
$$(dL_p)_O = (m\, v\, r_\perp)$$

$$\int (dL_p)_O = m\,(r_\perp \omega)\, r_\perp) = m\,\omega (r_\perp)^2$$

$$= \int \omega\,(m\, r_\perp)^2$$

$$(L_{body})_O = \omega$$

8)



(i) $(L_{body})_O = I_{FAOR} \times \omega$

$= \frac{mL^2}{3} \times \omega$

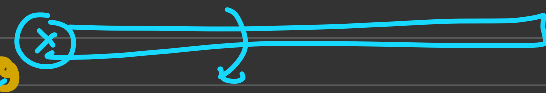
(ii) $I_B \times \omega = L_B$ ✗ (wrong)

* what is approach (Basic) *

$$\int (dL)_B = \int dm \times \{(\omega)(L-x)\} \times x$$

$$(L_{body})_B = \left(\frac{mL^2}{6}\omega\right)$$ Ans

Newton's second law of rotation for bodies:



$$L_{FAOR} = I_{FAOR} \times \omega$$

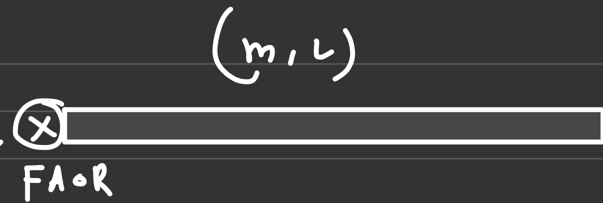
$$\frac{d}{dt}\left(L_{FAOR}\right) = I_{FAOR} \times \frac{d\omega}{dt}$$

$$\boxed{\tau_{FAOR} = I_{FAOR} \times \alpha}$$

Q)

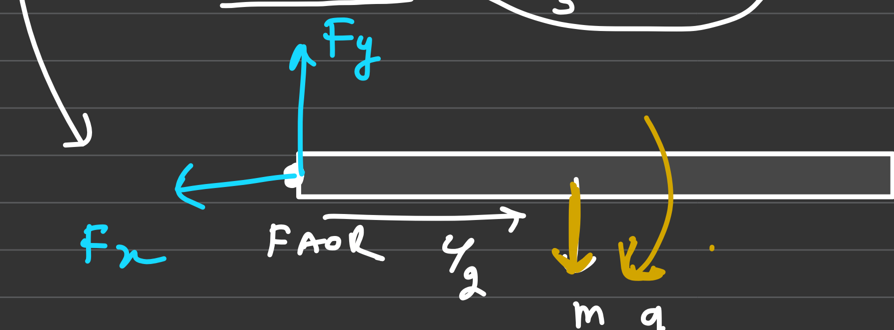
Released from rest

I then find α of the body?



$$\tau_{FAOR} = I_{FAOR} \times \alpha$$

$$\tau_{FAOR} = \frac{mL^2}{3} \times \alpha \quad \text{——} \; (1)$$

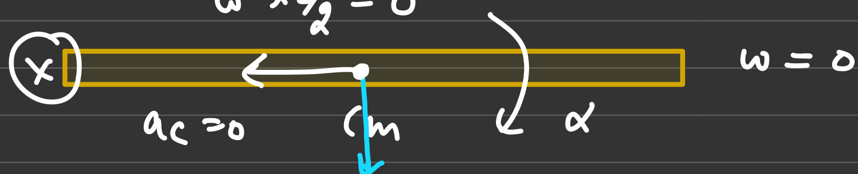


$$\tau_{FAOR} = \left(mg \times L/2 \right)$$

$$mg\frac{L}{2} = \frac{mL^2}{3} \times \alpha$$

$$\alpha = \frac{3g}{2L} \qquad \underline{\underline{c.w}}$$

(ii) find net Acceleration of com?



$$a_t = \frac{L}{2}\left(\frac{3g}{2L}\right)$$

$$a_{net} = \sqrt{a_t^2 + a_c^2}$$

$$a_{net} = \sqrt{(3g/4)^2 + (0)^2} = (3g/4)$$

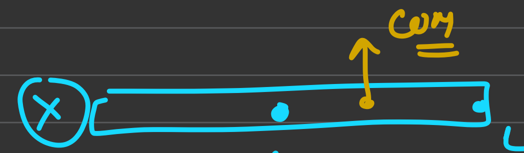
(ii) FAOR (m, L)

find α of rod? at this instant

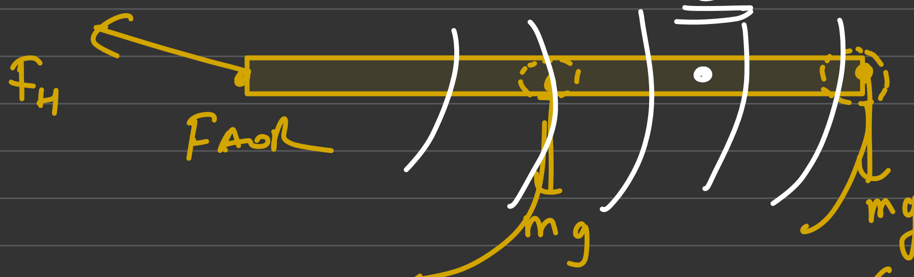
released from rest

$$\begin{cases} \tau_{FAOR} = I_{FAOR} \times \alpha \\ \tau_{FAOR} = \left(\frac{mL^2}{3} + mL^2\right)\alpha \quad \text{—(I)} \end{cases}$$

at the same time



$$\frac{m \times L/2 + m \times L}{2m} = \frac{3L}{4}$$



$$\tau_{FAOR} = \left(mg \times \frac{L}{2} + mg\, L\right) = \left(\frac{3mg\, L}{2}\right) \quad \text{—(II)} \quad \text{c.w}$$

$$\alpha \times \frac{4mL^2}{3} = \frac{3mgL}{2} \Rightarrow \alpha = \left(\frac{9g}{8L}\right) \underline{\underline{Ans}}$$

C.W

(ii) find net acceleration of COM?

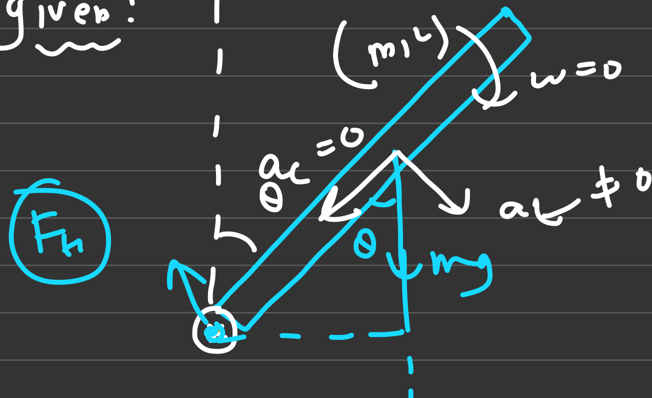
$a_c = 0$

$$a_{net} = a_{net} = \frac{3L}{4}\left[\frac{9g}{8L}\right) = \frac{27g}{32}$$

$\underline{\underline{Ans}}$

Q) given:

Released from rest



find α of rod at that instant

$$\tau_{F.A.O.R} = I_{F.A.O.R} \times \alpha$$

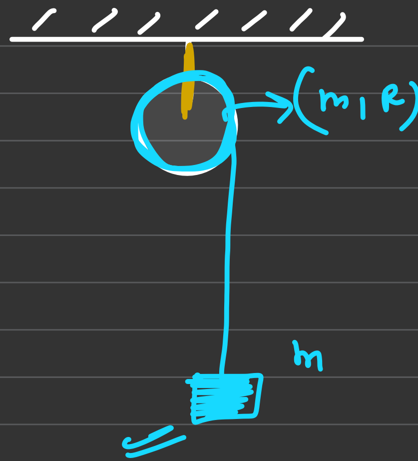
$$mg \times \frac{L}{2} \sin\theta = \frac{mL^2}{3} \times \alpha$$

$$\alpha = \frac{3g\sin\theta}{2L}$$

$$a_{net} = a_t = \frac{L}{2}\left(\frac{3g\sin\theta}{2L}\right)$$

$$a_{net} = a_t \left(\frac{3g\sin\theta}{4}\right)\frac{L}{L}$$

Q)

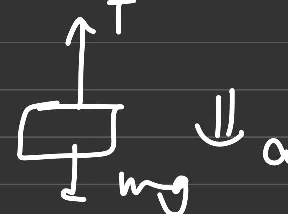


Radius of Gyration: (K)

there is no slipping between string and disc?

Released from rest then find acceleration of block?

① FBD:



$$\tau_F = I_F \times \alpha \quad \text{— (II)}$$

$$mg - T = ma \quad \text{— (I)}$$

$$T \times R = \frac{mR^2}{2} \times \alpha \quad \text{—— (11)}$$

(3)

$$T \cancel{R} = \frac{m \cancel{R^2}}{2} \times \frac{a}{\cancel{R}}$$

$$T = \frac{ma}{2}$$

$$a_p = R\alpha \downarrow$$

$$a_\theta = a \downarrow$$

$$a = R\alpha$$

$\downarrow a = g/2$

$$mg - \frac{ma}{2} = ma$$

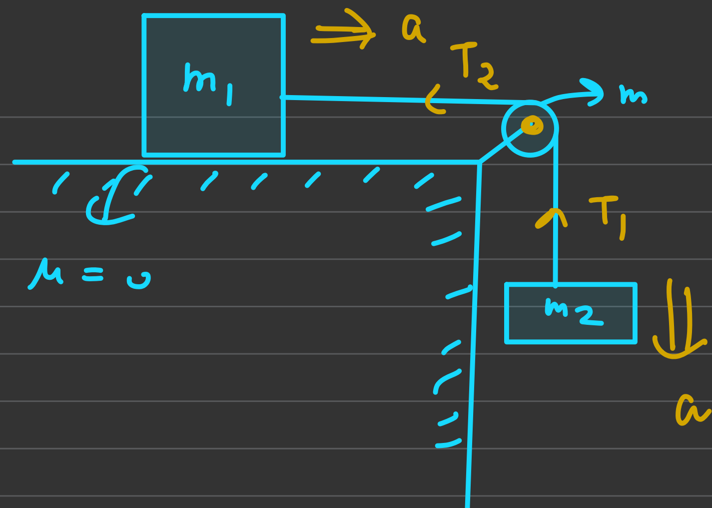
" tangential acceleration should remain"

$$\vec{a_p} = \vec{a_\theta}$$

$$\frac{3ma}{2} = g$$

$$a = \frac{2g}{3}$$

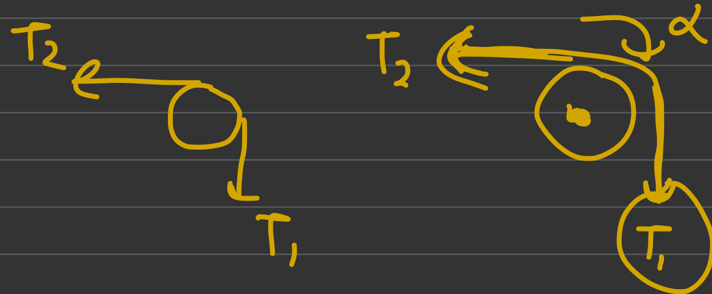
Q)



Released from rest

find acceleration of m_1 and m_2

No-slipping between string and pulley.

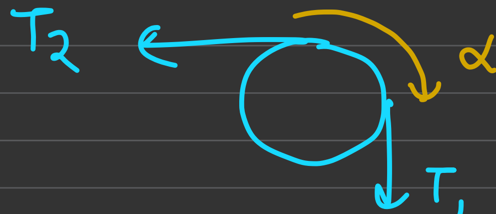


$$\tau_{FAOR} = T_1 \times R - T_2 \times R$$
$$= I\alpha$$

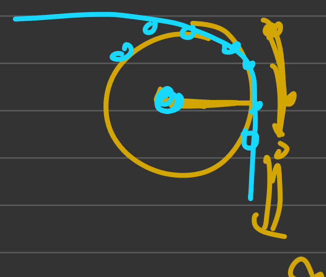
In case of mass-wali pulley, we will assume different tension at its end

$T_2 = m_1 a$ — (I)

$m_2 g - T_2 = m_2 a$ — (II)



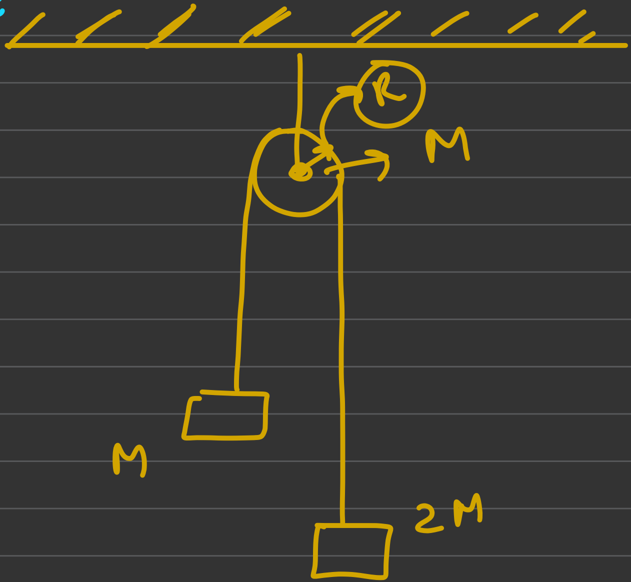
$T_1 \times R - T_2 \times R = \frac{m R^2}{2} \times \alpha$ — (III)



$a = R\alpha$ — (IV)

(No-Slipping)

H-W



Released from rest

find Acceleration of ends

No- Slipping between string and pulley