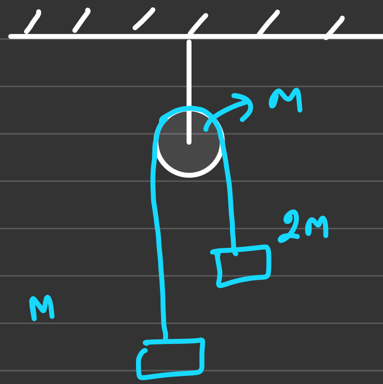
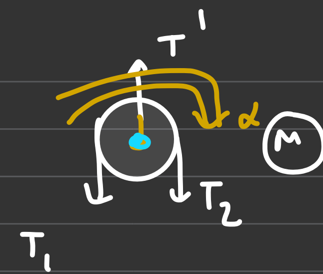
Rotational motion 4

VMC Vidyamandir Classes SINCE 1986

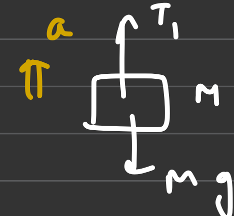
IIT JEE | MEDICAL | FOUNDATION

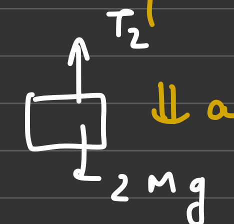
Q)





$$a = \frac{2g}{7}$$





$$2mg - T_2 = 2Ma \quad \text{— (I)}$$

$$T_1 - Mg = Ma \quad \text{— (II)}$$

$$\Rightarrow \tau_{FAOR} = I_{FAOR} \times \alpha$$

$$\Rightarrow T_2 \times R - T_1 \times R = I_{FAOR} \times \alpha \quad \text{— (III)}$$

$$T_2 R - T_1 \times R = \left(\frac{mR^2}{2}\right) \times \alpha$$

$$a = r\alpha \quad \text{— (IV)}$$

Radius of Gyration: "effective radius of bodies w.r.t to that axis"

$I = mR^2 = mK^2$

$K = R$

$I' = 2mR^2 = mK^2$

$K = \sqrt{2}\,R$

COM : axis

(I) Ring $= mR^2 = mK^2 \Rightarrow K = R$

(II) disc $= \frac{mR^2}{2} = mK^2 \Rightarrow K = \frac{R}{\sqrt{2}}$

(III) Solid Sphe $= \frac{2}{5} mR^2 = mK^2 \Rightarrow K = \sqrt{\frac{2}{5}}\, R$

(iv) Hollow sphere $= \frac{2}{3} mR^2 = mK^2 = K = \sqrt{\frac{2}{3}}\, R$

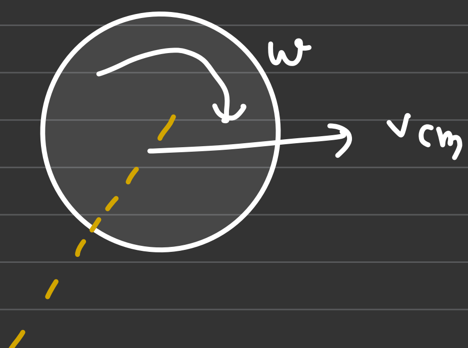
General Rigid body motion: (GRBM)

(I) Pure Translation

v_{cm} = velocity of other point

(II) Pure Rotation

mixed



GRBM

"earth rotation around sun"

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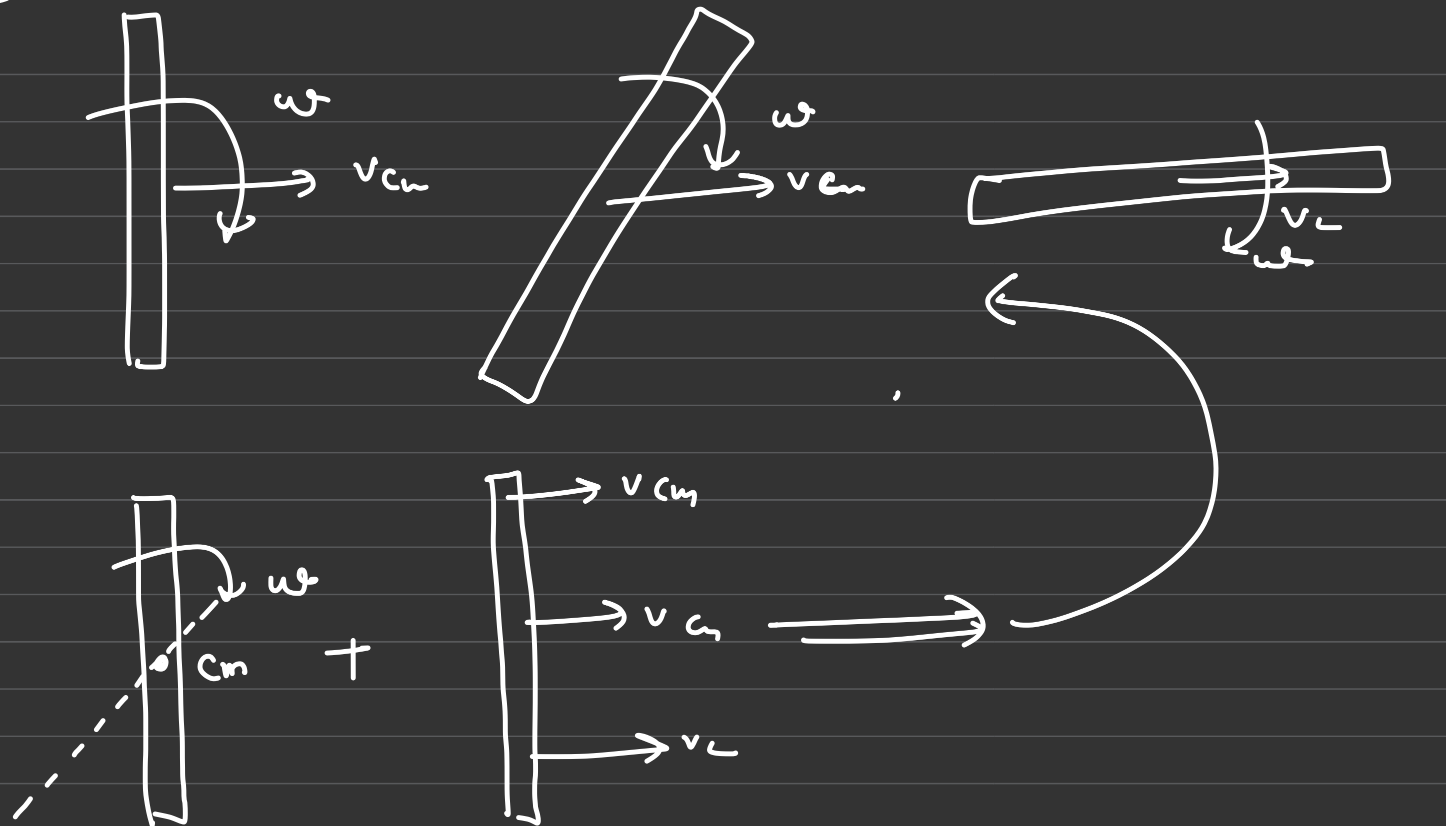
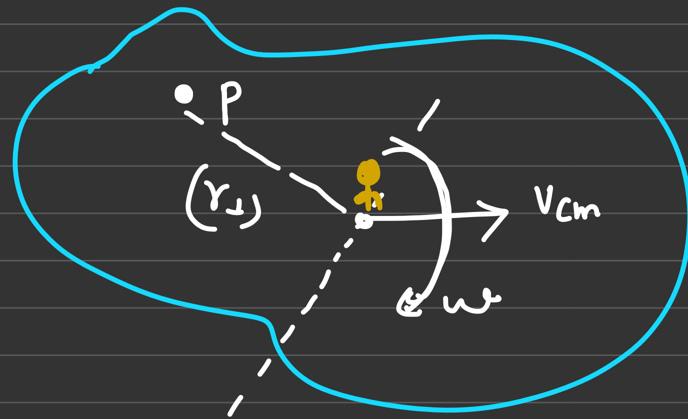


Note

If body is free to rotate then it will rotate about "COM" axis

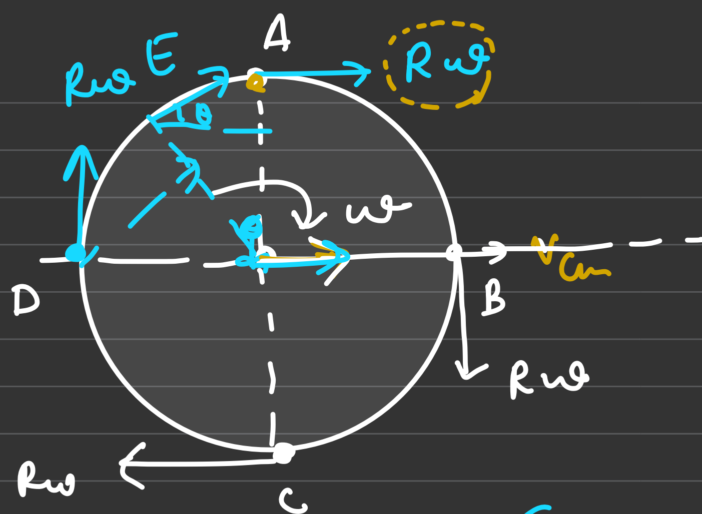
① Kinematics: (GRBM):



$$\vec{v}_{p,cm} = (r_\perp \omega)$$

$$\vec{v}_{p,g} = \vec{v}_{p,cm} + \vec{v}_{cm}$$

#



$$v_A = R\omega + v_{cm}$$

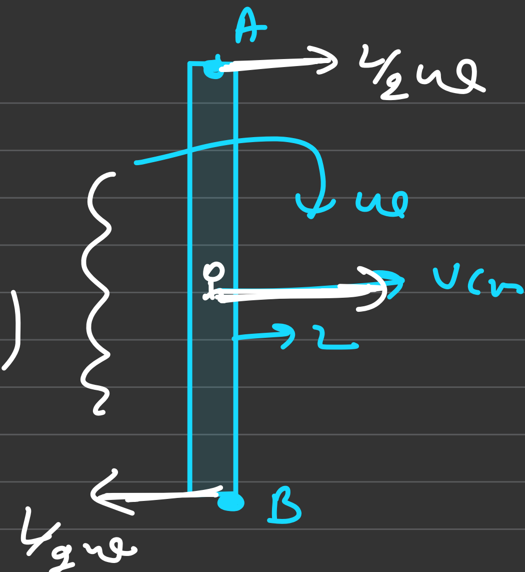
$$v_B = \sqrt{(v_{cm})^2 + (R\omega)^2}$$

$$v_C = (v_{cm} - R\omega)$$

$$v_D = \sqrt{(R\omega)^2 + (v_{cm})^2}$$

$$v_E = \sqrt{(R\omega)^2 + v_{cm}^2 + 2 v_{cm} R\omega \cos\theta}$$

Q) find velocity of A and B?



$V_A = \frac{L}{2}\omega + V_{cm}$

$V_B = \left(V_{cm} - \frac{L}{2}\omega\right)$

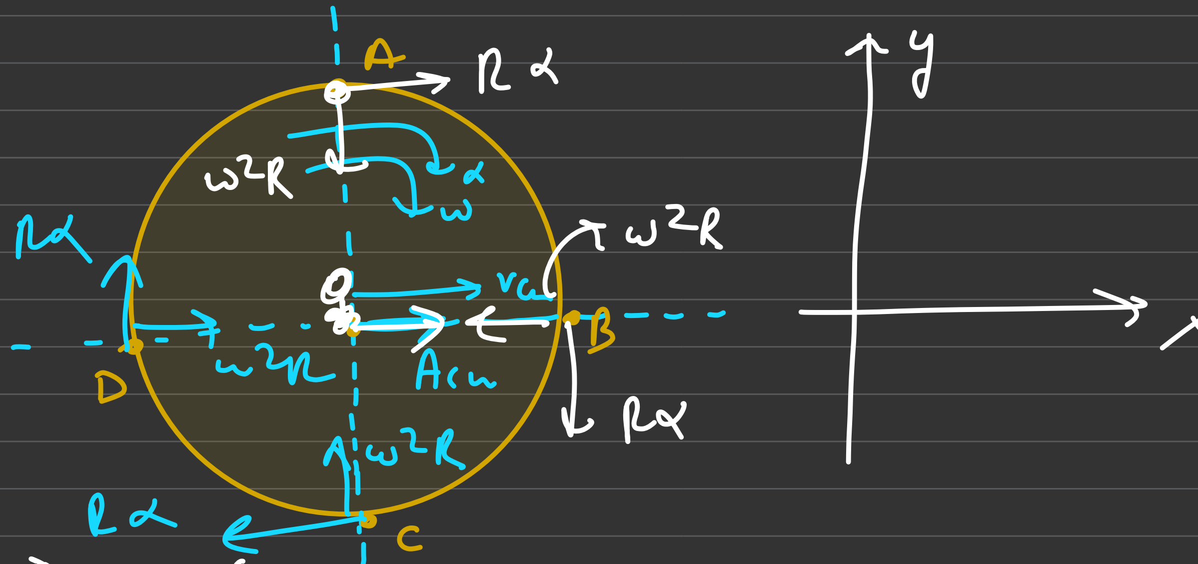
FAOR: Verification:

FAOR

GRBM: Verification

$$\begin{cases} v_{A,cm} = \frac{L}{2}\omega \\ v_A = \frac{L}{2}\omega + \frac{L}{2}\omega = L\omega \downarrow \quad \underline{\underline{Ans}} \end{cases}$$

Acceleration:



$$\vec{a}_A = (R\alpha + A_m)\,\hat{i} - \omega^2 R\,(\hat{j})$$

$$\left\{ \vec{a_B} = (A_t - \omega^2 R)\,\hat{i} - R\alpha\,\hat{j} \right.$$

$$\left\{ \vec{a_C} = (A_t - R\alpha)\,\hat{i} + \omega^2 R\,\hat{j} \right.$$

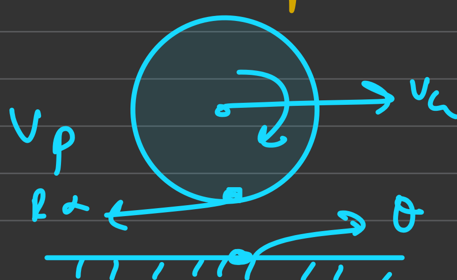
$$\left\{ \vec{a_D} = (\omega^2 R + A_t)\,\hat{i} + \underline{\underline{R\alpha\,\hat{j}}} \right.$$

Pure Rolling motion!

(1)

Find condition of Pure rolling?

$\vec{v}_P = \vec{v}_O$ ' Condition for pure or No-slipping "



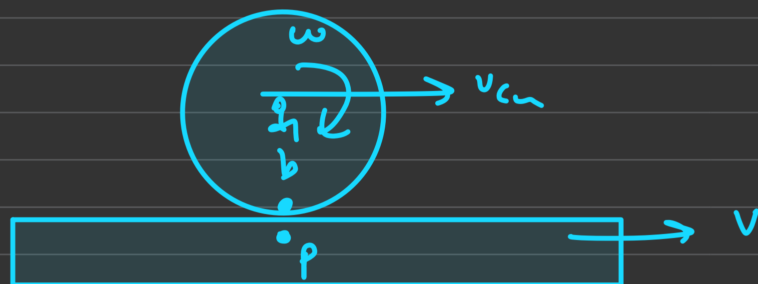
$$\begin{cases} v_P = v_{cm} - R\omega \\ v_O = 0 \end{cases}$$

$v_P = v_O$

$v_c - R\omega = 0$

$\boxed{V_{cm} = R\omega}$

(ii)



if body is pure rolling with plank then find relation among velocities

$\vec{V_b} \qquad \vec{V_p}$

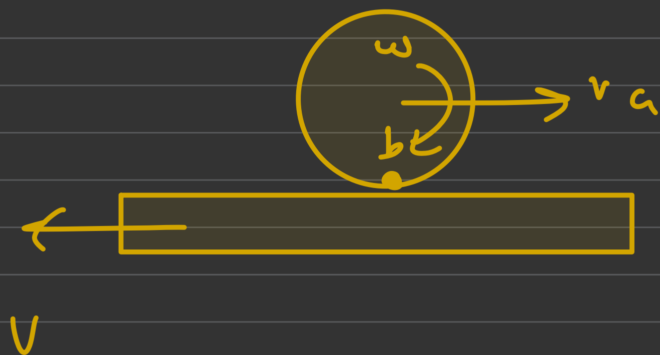
$\{ \vec{V_b} = \vec{V_p} \}$

$\vec{V_b} = (V_{cm} - R\omega) \longrightarrow$

$\vec{V_p} = V \longrightarrow$

for pure rolling $\boxed{V_a - R\omega = v}$

(iii)



find relation for pure rolling

$\vec{v}_p = V_a - R\omega \rightarrow$

$v_p = V \leftarrow$

$$\boxed{V_a - R\omega = -v}$$

Kinetic Energy of body (GRBM):

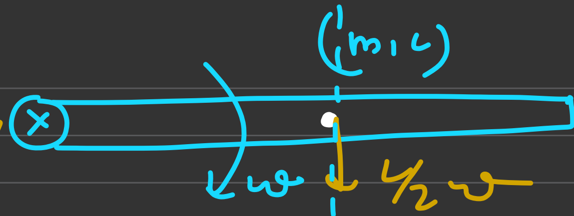
KE of this body

$$(KE) = \frac{1}{2} m v_{cm}^2 + \frac{1}{2} I_{cm} \times \omega^2$$

Ex#

$$= \frac{1}{2} m (v_{cm})^2 + \frac{1}{2} m R^2 \times \omega^2$$

$$= \frac{1}{2} m v_{cm}^2 + \frac{1}{2} m R^2 \omega^2$$



$$(KE)_{body} = \frac{1}{2}\,\frac{mL^2}{3}\times\omega^2 = \left(\frac{mL^2\omega^2}{6}\right) \text{ Ans}$$

GRBM: $$= \frac{1}{2} m v_{cm}^2 + \frac{1}{2} I_{cm}\times\omega^2$$

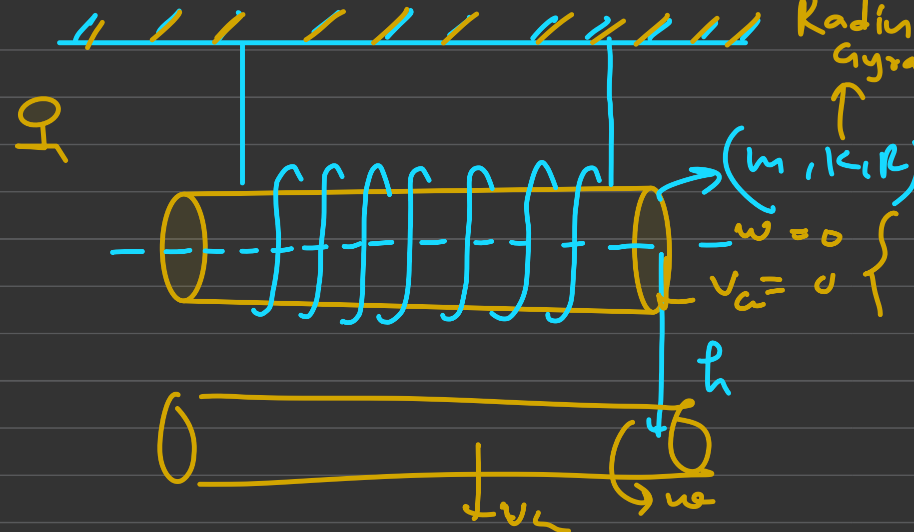
$$= \frac{1}{2} m \left(\frac{L}{2}\omega\right)^2 + \frac{1}{2}\left(\frac{mL^2}{12}\right)\omega^2$$

$$= \frac{mL^2\omega^2}{8} + \frac{mL^2}{24}\omega^2$$

$$= \frac{4mL^2\omega^2}{24} = \left(\frac{mL^2\omega^2}{6}\right) \text{ Ans}$$

Q)

Radius of Gyration



Released from rest

if body goes down by h then find ω of the body?

No-slipping

angular

$$mgh = \left(\frac{1}{2} m v_c^2 + \frac{1}{2} I_c \omega^2\right) - (0 - 0)$$

$$mgh = \frac{1}{2} m v_c^2 + \frac{1}{2} m K^2 \omega^2$$

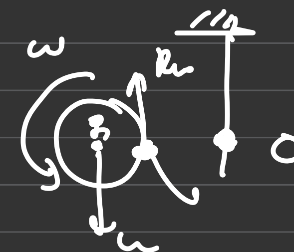
$$mgh = \frac{1}{2} m v_{cm}^2 + \frac{m k^2 \times \omega^2}{2}$$

$$mgh = \frac{1}{2} m (R\omega)^2 + \frac{m k^2 \omega^2}{2}$$

$$\omega = \sqrt{\frac{2gh}{R^2 + k^2}} \quad \text{Ans}$$

$$v_{cm} - R\omega = 0$$

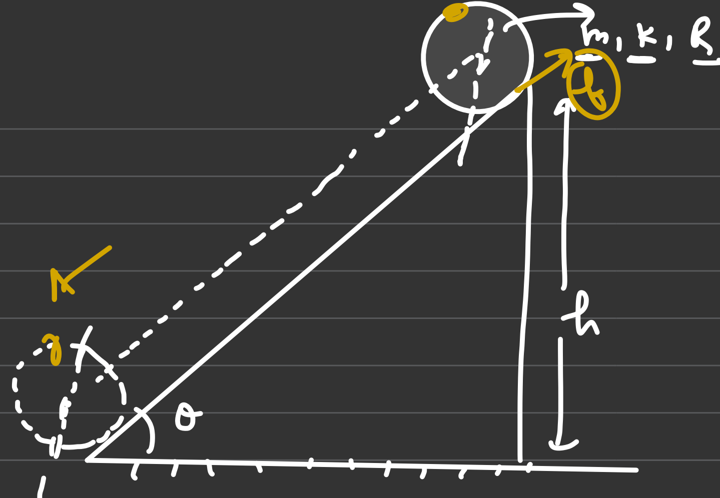
$$v_{cm} = R\omega$$



$$\vec{v}_{body} = \vec{v}_s$$

w.r.t ground

θ)



Released from rest

it comes down pure rolling

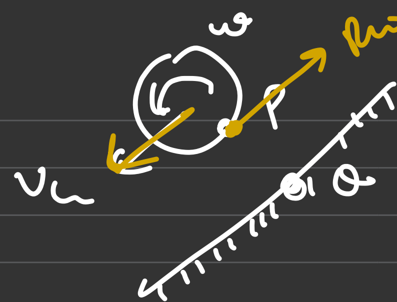
find velocity of body when it is at bottom of inclined body

loss in GPE = gain KE

$$mgh = \left\{\frac{1}{2} m v_{cm}^2 + \frac{1}{2} I_{axis}\omega^2\right\} - (0+0)$$

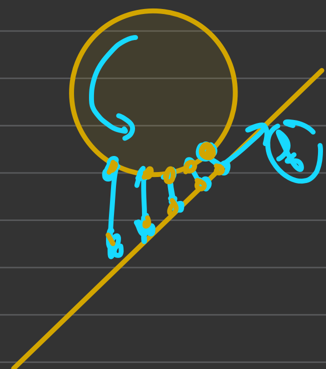
$mgh = \frac{1}{2} m v_{cm}^2 + \frac{1}{2} m k^2 \times \omega^2$ — (1)

$\boxed{mgh = \frac{1}{2} m v_{cm}^2 + \frac{1}{2} m k^2 \times \left(\frac{v_{cm}}{R}\right)^2}$ Ans



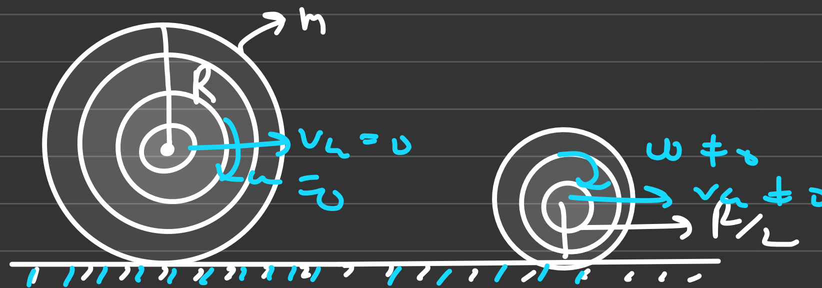
$$\begin{cases} v_P = v_c - R\omega \\ v_Q = 0 \end{cases}$$

$$v_c = R\omega \quad (1)$$



$$W = f \times \text{dist}$$
$$= f \times 0$$
$$= 0$$

H.W



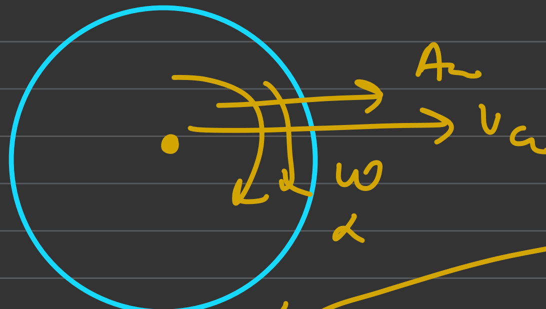
Pure rolling

find V_{cm} and ω of the body when Radius is $R/2$?

Dynamics: (GRBM):

(I) $\tau_{FAOR} = I_{FAOR} \times \alpha$

$L_{FAOR} = I_{FAOR} \times \omega$



(II) $L_{cm} = I_{cm} \times \omega$

$$\Downarrow$$

$\tau_{cm} = I_{cm} \times \alpha$

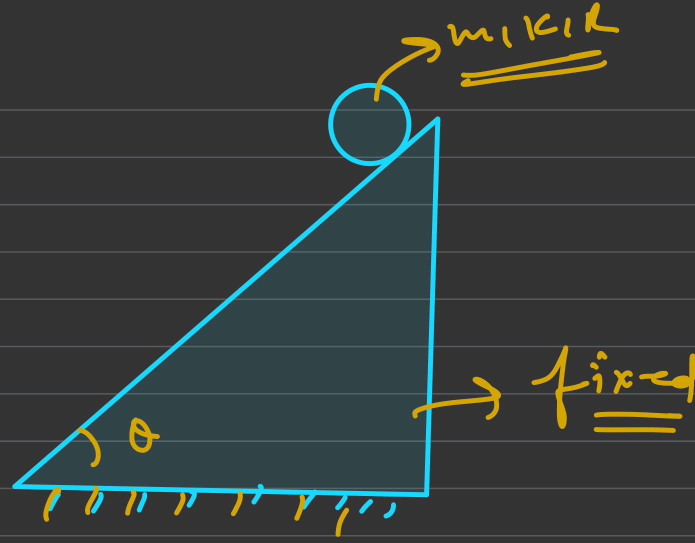
Sln:

$L_{cm} = I_{cm} \times \omega$ —— (I)

Remember

$\tau_{cm} = I_{cm} \times \alpha$ —— (II)

Q)

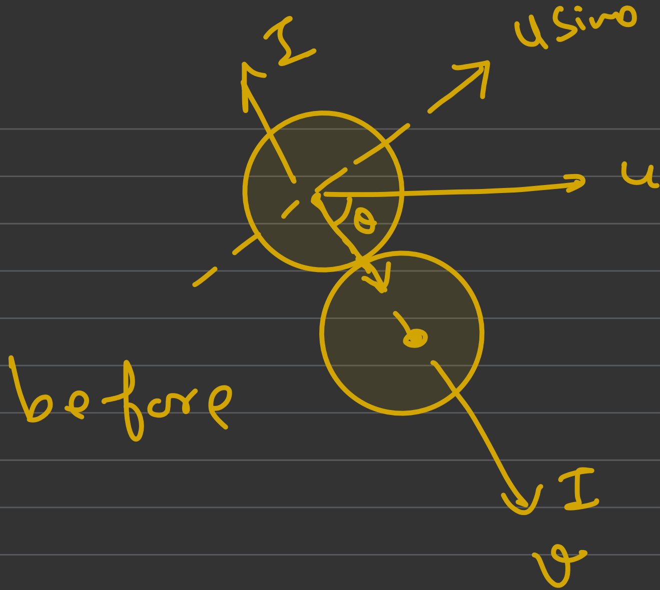


find α of body if it is doing pure rolling?

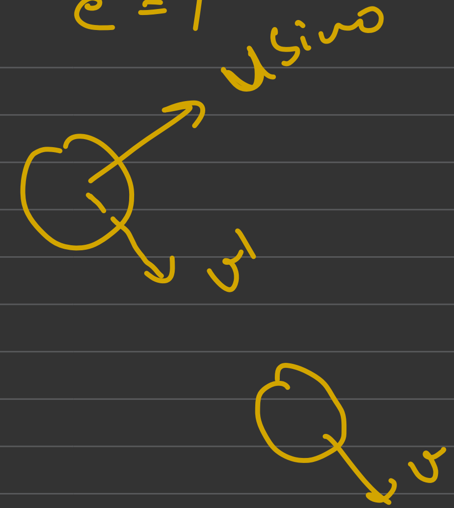
H.W = INE A #
INT B #

Level 1, 2 { DTS 1,2,3

a)



$e = 1$



$v - v' = 1 \times u\cos\theta \quad — (1)$