Gravitation-1

1 # Electrostatics → Analogous → gravitation

2 # 1- Newton's law of gravitation

2. gravitational field

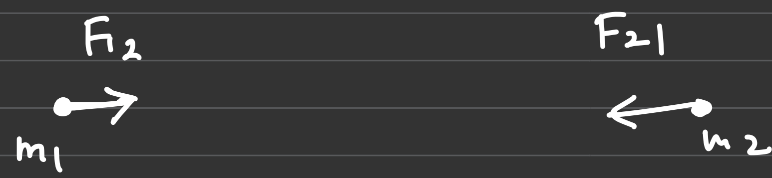
3. acc. due to gravity

4. Kepler law

gravitation escap

Newton's law of gravitation

① #



(attractive in nature

According to Newton

$$|F_{21}| = |F_{12}|$$

$$\begin{cases} F_{12} = F_{21} \propto m_1 m_2 \\ F_{12} = F_{21} \propto \frac{1}{r^2} \end{cases}$$

$$F_{12} = F_{21} = G\frac{m_1 m_2}{r^2}$$

Universal gravitation constant

{ modulus }

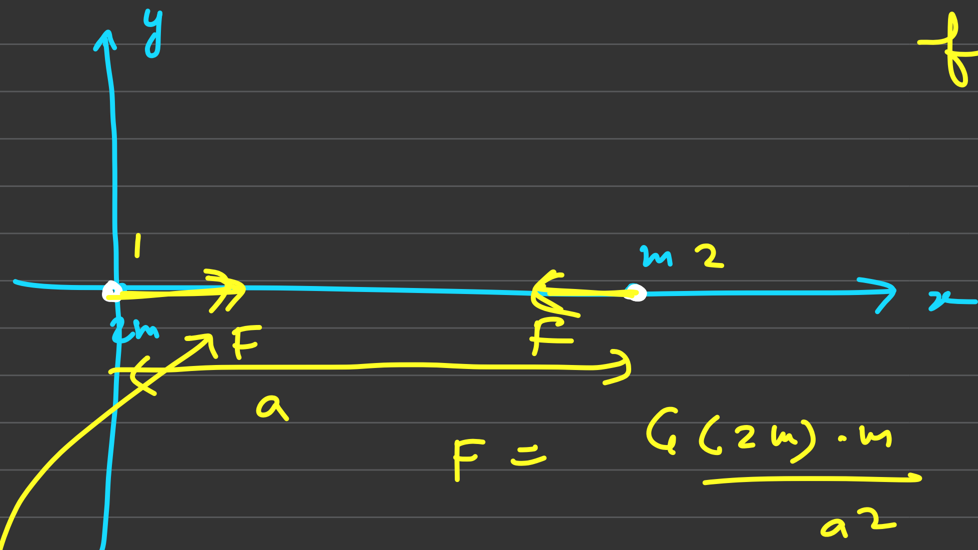
Direction:

$$\vec{F}_{21} = \frac{G m_1 m_2}{r^2} (\hat{r}_{12})$$

A unit vector from 1 to 2

Ex

1



$$F = \frac{G (2m) \cdot m}{a^2}$$

#

$$\vec{F}_{12} = \frac{2 G m^2}{a^2} (\hat{i})$$

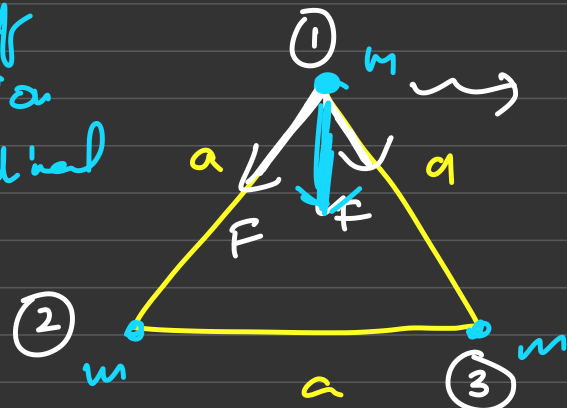
Force 1 due to 2

Force 2 due to 1 {

$$\vec{F}_{21} = \frac{2Gm^2}{r^2}(-\hat{i})$$

Gravitational force due to System of particle

Principle of Superposition of gravitational force



#

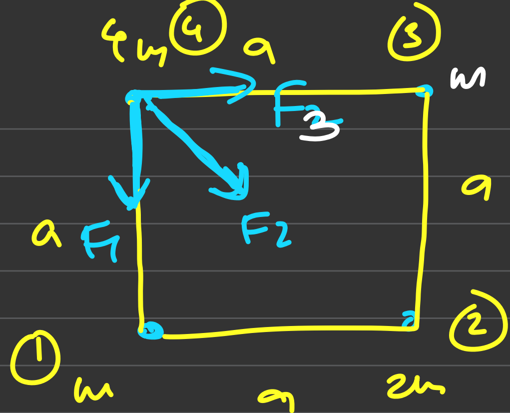
$$(F_{net}) = \sqrt{F^2+F^2+2F^2\cos 60}$$

$$F_{net} = \sqrt{2F^2+2F^2\cos 60}$$

$$= \sqrt{2F^2+F^2}$$

$$= \sqrt{3}F$$

Ex:



Find net force exp by (4m) ?

$$\vec{F}_{net} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3$$

$$F_1 = \frac{G\, m\, 4m}{a^2}$$

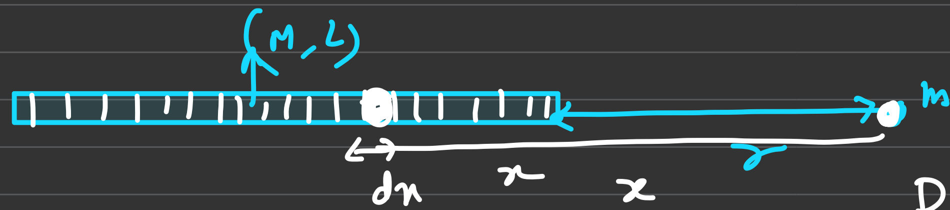
$$F_2 = \frac{G\, 2m \times 4m}{(\sqrt{2}\, a)^2}$$

$$F_3 = \frac{G\, 4m \cdot m}{a^2}$$

Ans

Force on point mass due to extended body:

(i)



$$\int dF = \int \frac{G\, m\, dm}{x^2}$$

as Direction of force on m due every small segment is same hence we can directly

$$F_{net} = G \int \frac{m\left(\frac{M}{L}dx\right)}{x^2}$$

$$F_{net} = \frac{G\, m\, M}{L} \int_{r}^{r+L} \frac{dx}{x^2}$$

$$= \frac{G\, m\, M}{L} \left[-\frac{1}{x}\right]_{r}^{r+L}$$

$L \rightarrow M$

$1 \rightarrow \dfrac{M}{L}$

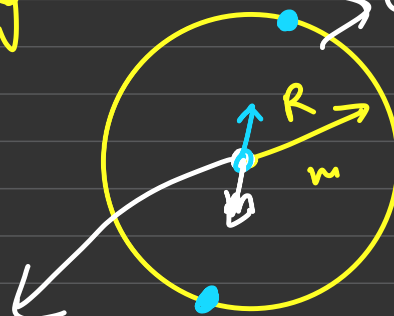
$dx \rightarrow \left(\dfrac{M}{L} \cdot dx\right)$

$$= -\frac{GmM}{L}\left[\frac{1}{x+L} - \frac{1}{x}\right]$$

$$F = \frac{GmM}{L}\left[\frac{1}{x} - \frac{1}{x+L}\right] \quad \text{Ans}$$

(ii) Ring

a)



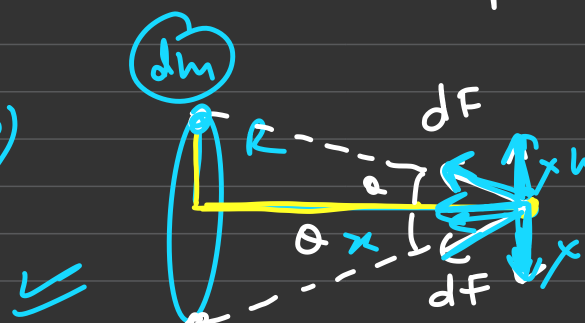
uniform

a) if we keep a point mass at its centre then net force exp by point mass

$F_{net} = 0$

Symmetrical body

b)



find net force exp by point mass due to ring?

$$F_{net} = \int dF \cos\theta \qquad dF = \frac{dm\, m\, G}{(\sqrt{R^2+x^2})^2} \times \cos\theta$$

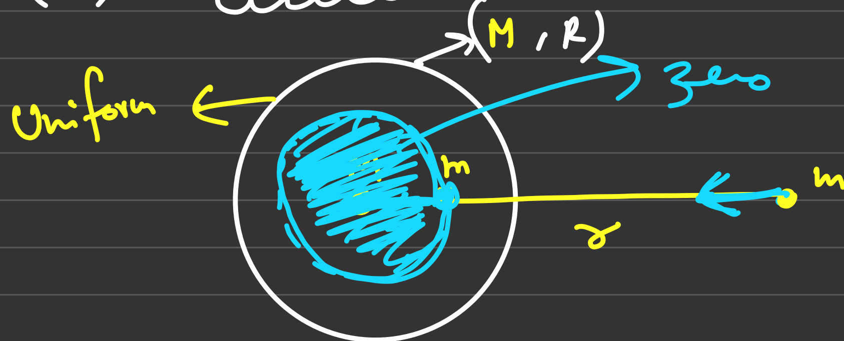
$$= \int \frac{dm\, m\, G}{(R^2+x^2)} \cdot \frac{x}{\sqrt{R^2+x^2}}$$

$$= \frac{G m x}{(R^2+x^2)^{3/2}} \int dm$$

$$= \frac{G m^2 x}{(R^2+x^2)^{3/2}}$$

# #

(iii) Hollow sphere:



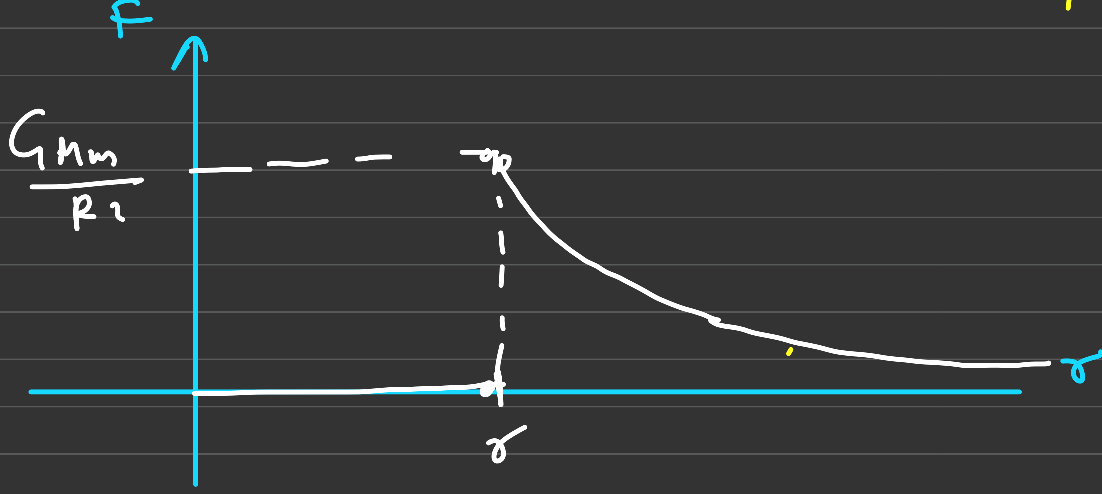
$$F(r) = \begin{cases} 0 & r < R \\ \dfrac{GmM}{r^2} & r > R \end{cases}$$

↓
on point object

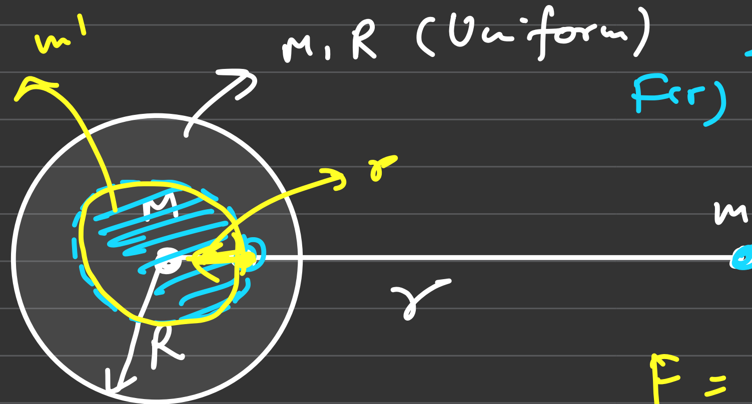
" if body is uniform and 3D spherical body (Hollow or Solid)

Case I: if point object is outside sphere (Hollow or Solid) then we can assume all mass of Hollow or Solid sphere at it com and we can directly use the result

Case II : if point object is inside sphere (Hollow or Solid) then we draw a spherical boundary enclosing that point and calculate enclosed mass and assume that mass at its centre of mass & then we can directly use the result



Solid sphere:



$$F(r) \begin{cases} \dfrac{GmM}{r^2} & r > R \\[2ex] \dfrac{Gm\left(\dfrac{Mr^3}{R^3}\right)}{r^2} & r < R \end{cases}$$

$$\Downarrow$$

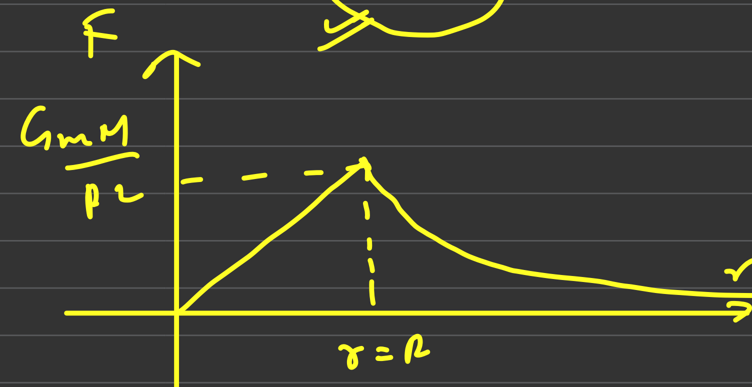
$$F = \frac{GmMr}{R^3} =$$

$$\frac{4}{3}\pi R^3 \longrightarrow M$$

$$1 \longrightarrow \frac{M}{\frac{4}{3}\pi R^3}$$

$$\frac{4}{3}\pi r^3 \longrightarrow \frac{M}{\cancel{\frac{4}{3}\pi} R^3} \times \cancel{\frac{4}{3}\pi} r^3$$

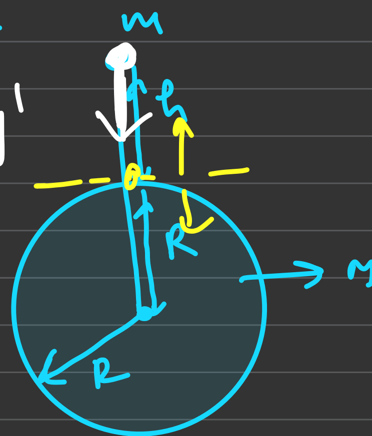
$$= \left(\frac{M r^3}{R^3}\right)$$



Acceleration Due to gravity: h from surface or ground

(i) above surface

$$F = \frac{GmM}{(R+h)^2} = mg'$$



$$g' = \frac{GM}{(R+h)^2}$$

$$g' = \frac{GM}{R^2 (1 + h/R)^2}$$

$$g' = \frac{g}{(1 + h/R)^2} \Rightarrow g' = g\left(1 + \frac{h}{R}\right)^{-2}$$

(ii) on the surface

$h = 0$

$$g = \frac{GM}{R^2} = 9.81\ m/s^2$$

at surface

$$gR^2 = GM$$

Radius of earth

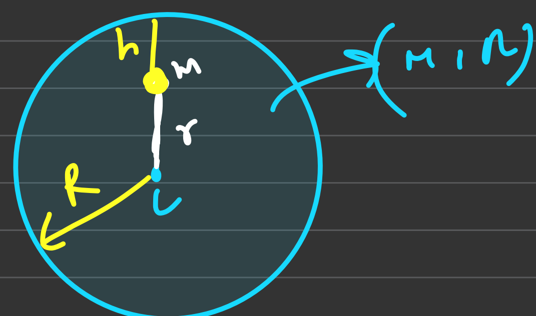
$$(1+x)^n = 1 + nx$$

$x << 1$

$$\frac{h}{R} << 1$$

$$g' = g\left(1 - \frac{2h}{R}\right)$$

(iii) below the surface:



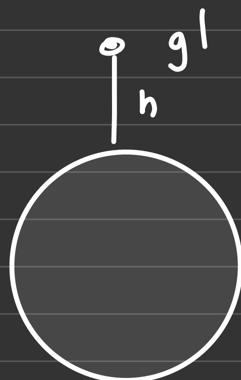
$$F = \frac{G m M (R-h)}{R^3}$$

$$\cancel{m} g'' = \frac{G \cancel{m} M (R-h)}{R^3}$$

$$g'' = \frac{GM}{R^2}\left[1 - \frac{h}{R}\right] \downarrow$$

$$g'' = g\left[1 - \frac{h}{R}\right]$$

Q)



at what depth value of g will be same "

{ use approx. relati…

$$g' = g\left(1 - \frac{2h}{R}\right)$$

$$g'' = g\left(1 - \frac{h'}{R}\right)$$

$$-2h = -h'$$

$$h' = 2h$$

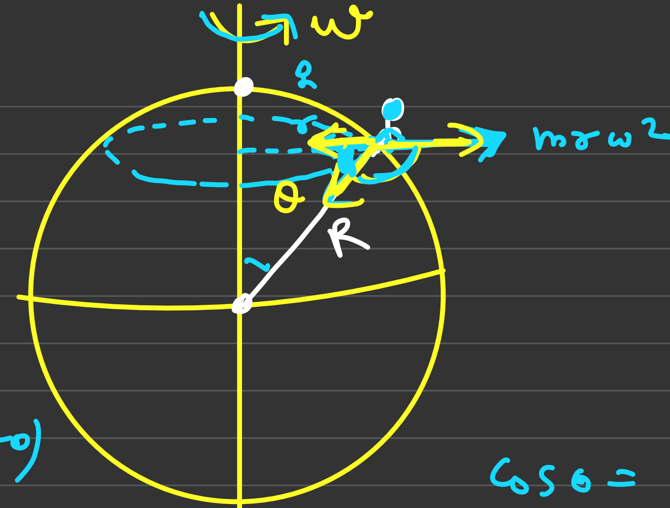
$$g' = g''$$

gravitational field:

"gravitation field at any point due to any object or sum of object or extended object is force is experience by a point unit mass at that point"

" g is to gravitational field has {

Acceleration due to gravity considering rotation of earth



$$F_{net} = \sqrt{(mg)^2 + (mr\omega)^2 + 2mg(mr\omega^2)\cos(180-\theta)}$$

$$\cos\theta = \frac{r}{R}$$

$$r = (R\cos\theta)$$

$$F_{net} = \sqrt{m^2g^2 + m^2r^2\omega^4 - 2m^2gr\omega^2\cos\theta}$$

$$\cancel{m}\, g_{eff} = \cancel{m}\sqrt{g^2 + r^2\omega^4 - 2gr\omega^2\cos\theta}$$

$$g_{eff} = \sqrt{g^2 + r^2\omega^4 - 2gr\omega^2\cos\theta}$$

$$g_{eff} = \sqrt{g^2 + R^2\cos^2\theta\,\omega^4 - 2gR\cos^2\theta\,\omega^2}$$

$$g_{eff} = \sqrt{g^2 + R^2\cos^2\theta\,\omega^4 - 2gR\cos^2\theta\,\omega^2}$$

According to our assumption of θ if we move from pole to equator then θ is going to decrease from $90 \to 0$

(i) $g_{pole} \Rightarrow \quad \theta = 90 \Rightarrow g_{eff} = g$

(iii) $g_{equator} \Rightarrow \theta = 0$

$$g_{eff} = \sqrt{g^2 + R^2\omega^4 - 2gR\omega^2}$$

$$g_{eff} = \sqrt{(g - R\omega^2)^2}$$

$$\underline{g_{eff}} = (g - R\omega^2)$$

find ω of earth at which g_{eff} of earth become 0

$$0 = g - R\omega^2 \Rightarrow \omega = \left(\sqrt{g/R}\right)$$

" almost 14 times more
that
current value'
of a "

$=$

Gravitational Potential Energy :

(i)

A ——— B

$$\int F_{ext} \cdot dr = (\Delta U)_{AB}$$

slowly

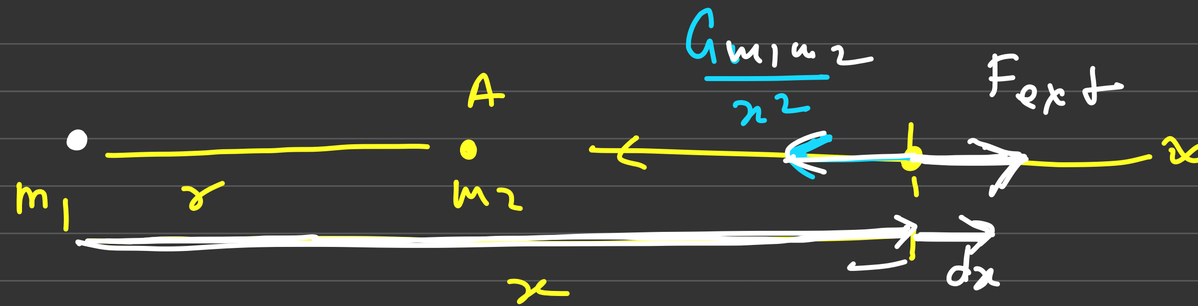
$\Delta K = 0$

(ii)

$$W_c = -(\Delta U)_{AB}$$

Potential of two point mass kept at some distance

$$U(r) = -\frac{Gm_1m_2}{r}$$



method 1: $(W_{F_{ext}})_{\infty \to A \text{ slowly}} = (\Delta U)_{\infty \to A}$

$$\int_{\infty}^{A} G\frac{m_1m_2}{x^2}\,dx = (\Delta U)_{\infty - A}$$

$$Gm_1m_2 \int \frac{dx}{x^2} = U_A - (U_\infty)$$

$$Gm_1m_2\left[-\frac{1}{r}\right]_{\infty}^{r}$$

$$-Gm_1m_2\left[\frac{1}{r}-\frac{1}{\infty}\right]=U_A-U_\infty$$

$$-\frac{Gm_1m_2}{r}=\left(U_A-\cancel{U_\infty}\right)$$

0

if $U_\infty=0$

$$U_A(r)=-\frac{Gm_1m_2}{r}$$

method 2:

$$W_c=-\Delta U$$

$$\int_{\infty}^{A} \frac{Gm_1m_2}{x^2}\,dx = f(\Delta u)$$

$$\Rightarrow \int_{\infty}^{A} \frac{Gm_1m_2}{x^2}\,dx = U_A - \cancel{U_\infty}$$

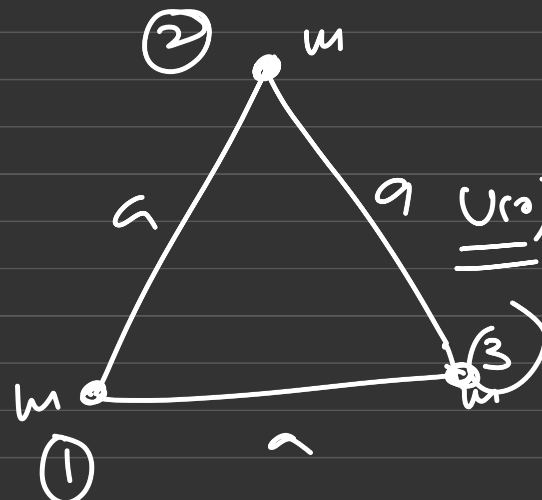
$$0$$

$$U_A = -\frac{Gm_1m_2}{r}$$

Ex# 1

$$U(r) = -\frac{Gm2m}{r}$$

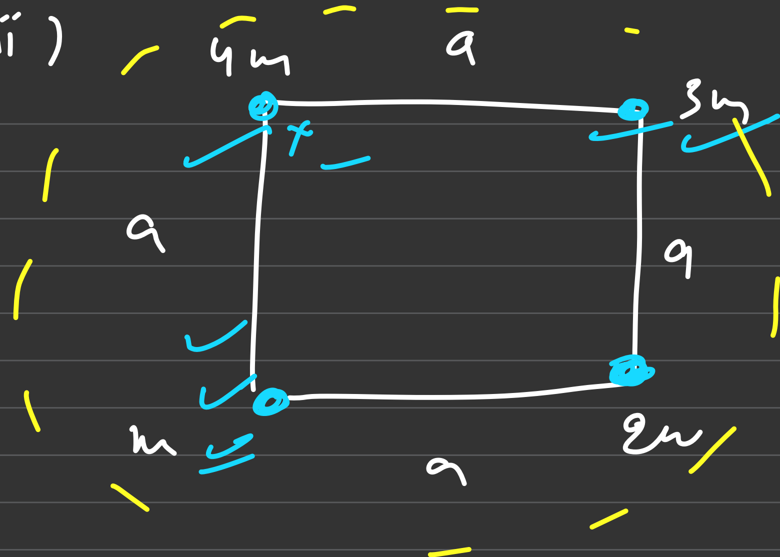
II



$$= -\frac{Gmm}{a} - \frac{Gmm}{a} - \frac{Gmm}{a}$$

$$U(r) = -\frac{3Gm^2}{a}$$

(iii)



find Potential Energy of System

$$U(r) = -\frac{G\,m\,(4m)}{a} - \frac{G\,3m\,(4m)}{a} - \frac{G\,m\,(3m)}{\sqrt{2}\,a}$$

$$-\frac{G\,2m\,m}{a} - \frac{G\,2m\,4m}{\sqrt{2}\,a} - \frac{G\,2m\,3m}{a}$$

Q) a) released from rest

find velocities of point object when sep between them become $r/2$?

$v_1 = 0$ $\quad$ $v_1 = 0$

m $\quad$ r $\quad$ $2m$

m $\quad$ $v_1 \rightarrow$ $\quad$ $\leftarrow v_2$ $\quad$ $2m$

$r/2$

$-\dfrac{Gm\,2m}{r}$

$-\dfrac{4Gm\,m}{r}$

$F_{ext} = 0$

loss GPE = gain KE

$$-\frac{2Gm^2}{r} + \frac{4Gm^2}{r} = \frac{1}{2} m v_1^2 + \frac{1}{2} 2m (v_2)^2$$

1 $\quad$ 2

as $(F_{ext})_{syst} = 0$ hence we can apply law of cons. of moment

$$0 + 0 = m(v_1) - 2m(v_2)$$

$$v_1 = 2v_2 \quad \text{— (11)}$$

Solve it for v_1 and v_2

Homework:

Solid sphere:

$U(r) = ?$