Rotational Motion 1

Vidyamandir Classes SINCE 1986

IIT JEE | MEDICAL | FOUNDATION

motion

- Translation
- Rotational motion
  - Pure
  - general Rigid body motion

Translation: (Pure translation)

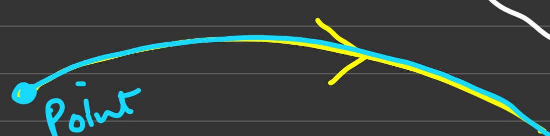
"mod + Direction"

Rigid bodies:

①

"if velocity of all point of body is same then body is called in Pure translation"

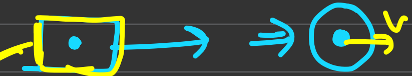
②



if body is in Pure translation we can assume this

3

body as point "mass"



m

$\Downarrow$

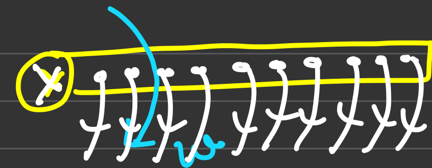
$= \frac{1}{2} v^2 \int dm \Rightarrow \frac{1}{2} m v^2$

$= \frac{1}{2}(v^2 m)$

Pure Rotation:

"if a point is fixed w.r.t ground and other part of body is moving w.r.t to that fixed then body is in pure rotation"

w.r.t ground fixed



Pure rotation

* # general Rigid body motion : "Translation + Rotation"

↑ ↑ ↑ ↑

<u>GRBM</u>

: bodies :

Rigid body

Non-Rigid body

#

t=0

t = t

" if seperation between any two point within the body is not changing w.r.t time the body is called Rigid body "

#

Sep. between any two point must be changing w.r.t time

Ex # : Rigid body

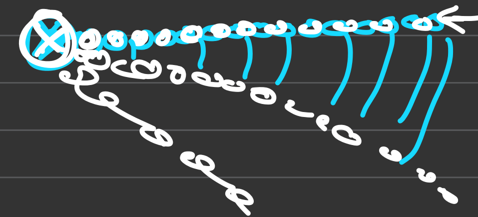
$\{\underline{\theta}, \underline{\omega}, \underline{\alpha}\}$ = Same

same "for Rigid body"

$\{x, v, a\}$ = diff

different

Ex #



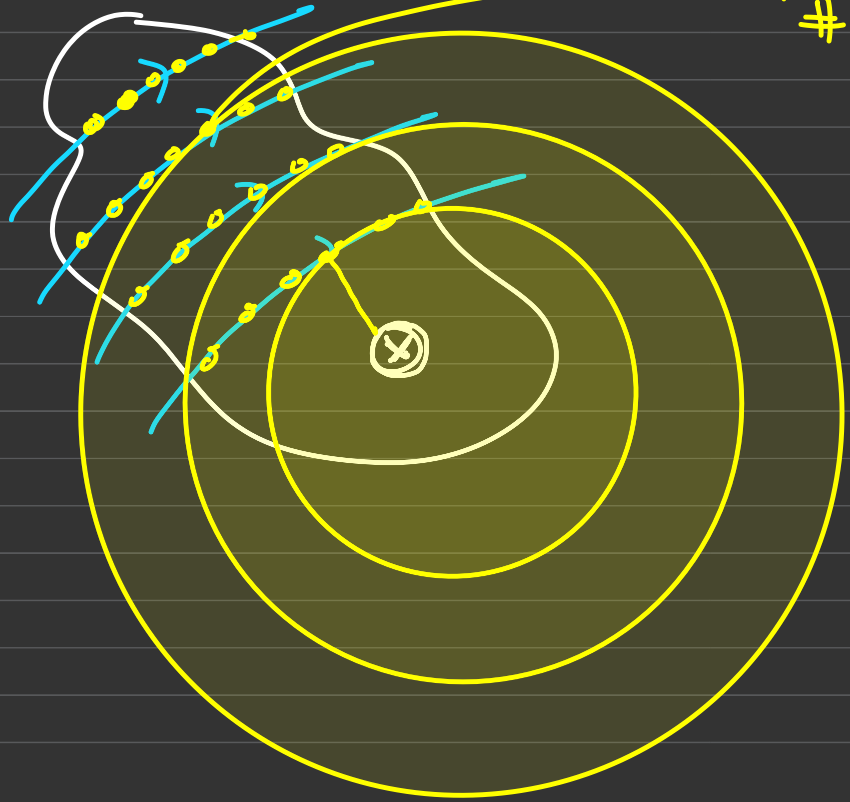
= (θ, ω, α) = diff

= (x, v, a) diff

" Rotation motion of "Rigid bodies" "

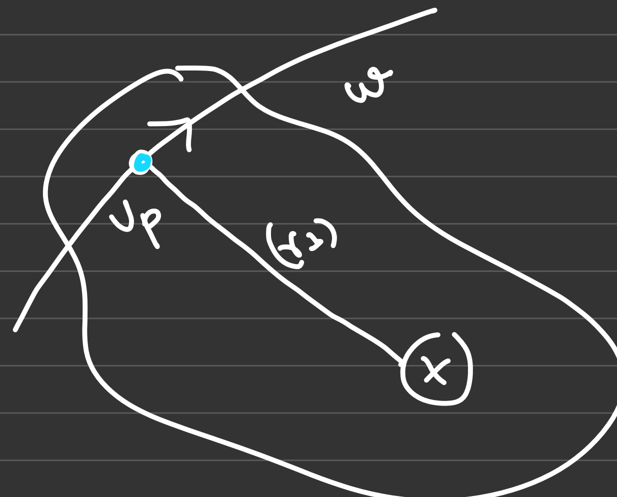
"A body rotating about a fixed axis:"

(FAOR)



ω = const
⇑
Uniform Circular
Non-Uniform Circular motion
ω = changing

(i) A body rotating about fixed axis (FAOR) : Kinematics:



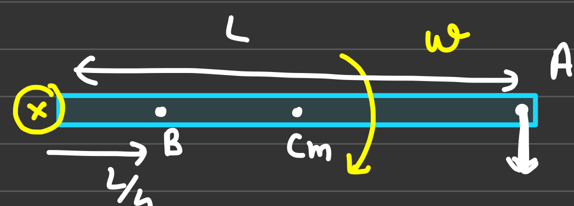
$v_p = (r_\perp)\,\omega$

min gap

it sep between point and axis of rotation

⊥ distance from Point to axis of rotation

Q)



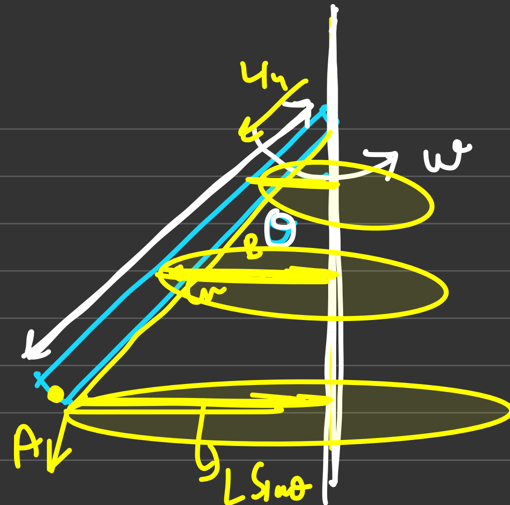
find |Velocity| of

$v_A = L\omega$

$v_{cm} = \frac{L}{2}\omega$

$v_B = \frac{L}{4}\omega$

θ)

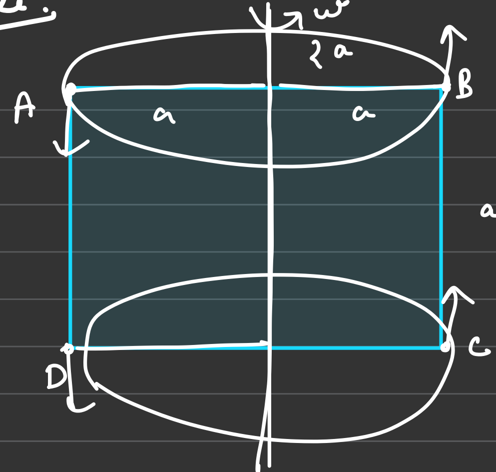


find |velocity| of

$$
\left\{
\begin{aligned}
v_A &= (L\sin\theta)\,\omega \\
v_{cm} &= \left(\frac{L}{2}\sin\theta\right)\omega \\
v_B &= \left(\frac{L}{4}\sin\theta\right)\omega
\end{aligned}
\right.
$$

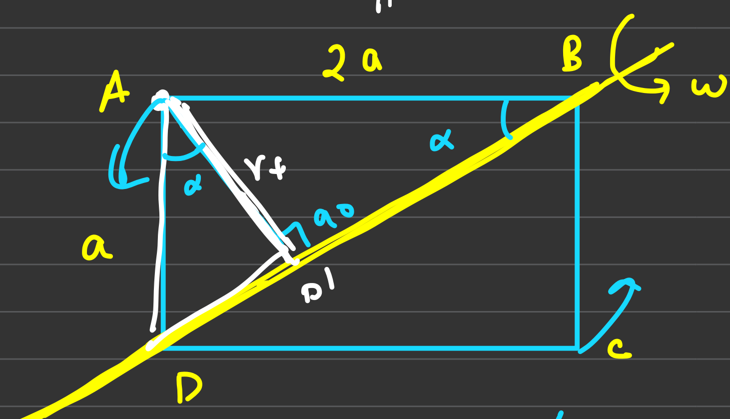
θ)

a) # Rectangle:



$$\left\{ v_A = v_B = v_C = v_D = a\omega \right.$$

b)



Ans

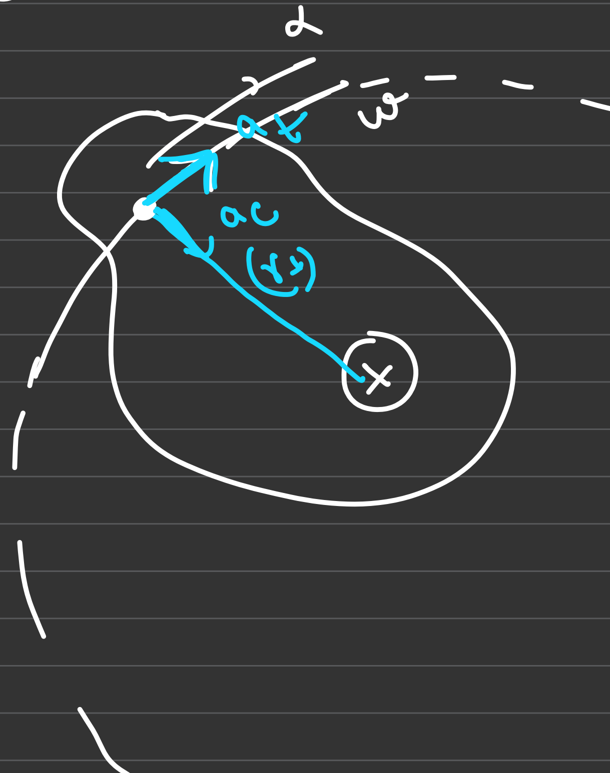
$$\left\{ \begin{aligned} v_A &= (r_\perp)\,\omega = \left(\frac{a}{\sqrt5}\,\omega\right) \\ v_B &= 0 \\ v_C &= \frac{a\omega}{\sqrt5} \quad \underline{\text{Ans}} \\ v_D &= 0 \end{aligned} \right.$$

AD'D ~ BDA

$$\left\{ \frac{a}{\sqrt5\,\cancel{a}} = \frac{(r_\perp)}{\cancel{a}} \right\} \Rightarrow (r_\perp) = \left(\frac{1}{\sqrt5}\,a\right) \quad \underline{\underline{\text{Ans}}}$$

Acceleration:

if ω is changing then α is true

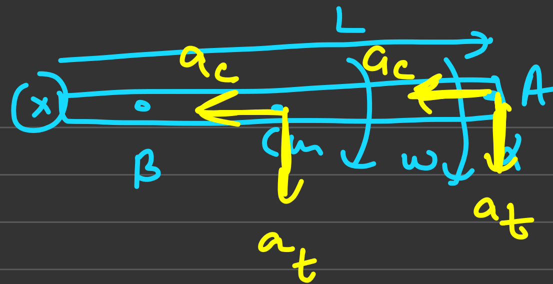


$$a_{net} = \sqrt{a_t^2 + a_c^2}$$

$$\begin{cases} a_t = \dfrac{dv}{dt} = (r_\perp)\alpha \\ a_c = \dfrac{v^2}{(r_\perp)} = \omega^2(r_\perp) \end{cases}$$

$$a_{net} = \sqrt{((r_\perp)\alpha)^2 + (\omega^2 r_\perp)^2}$$

#



find net acceleration of A

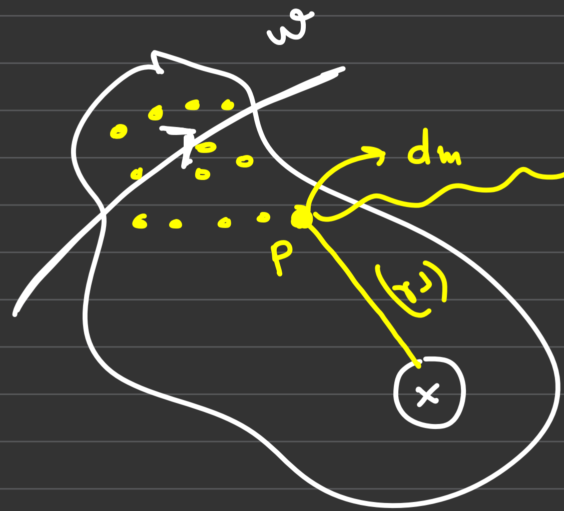
$$a_A = \sqrt{(\omega^2 L)^2 + (L\alpha)^2}$$

$$a_{cm} = \sqrt{\left(\omega^2 \frac{L}{2}\right)^2 + \left(\frac{L}{2}\alpha\right)^2}$$

$$a_B = \sqrt{\left(\omega^2 \frac{L}{4}\right)^2 + \left(\frac{L}{4}\alpha\right)^2}$$

Kinetic Energy of body rotating about fixed axis:

(FAOR)



$$dKE = \frac{1}{2}\, dm\, (v_p)^2$$

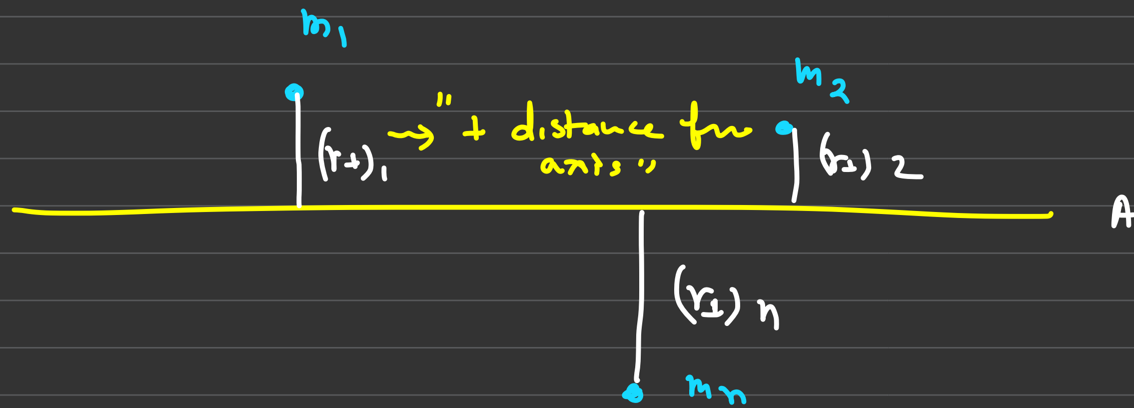
$$\int dKE = \int \frac{1}{2}\, dm\, (r_\perp \omega)^2$$

$$(KE)_{FAOR} = \frac{\omega^2}{2}\left[\underbrace{\int dm (r_\perp)^2}_{\text{moment of inertia}}\right]$$

$$\underline{(KE)_{FAOR} = \frac{\omega^2}{2}\left[I_{FAOR} \right]}$$

moment of inertia: "Rotational inertia of body"

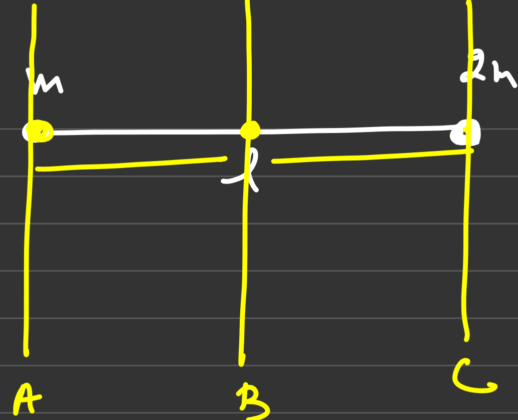
(i)



$$I_A = m_1 (r_\perp)_1^2 + m_2 (r_\perp)_2^2 + \dots m_n (r_\perp)_n^2$$

Scalar Quantity

Q)



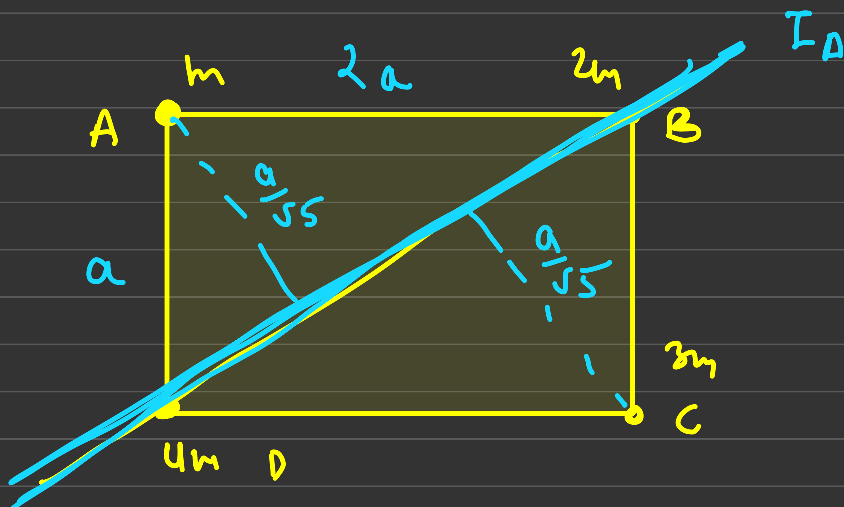
find moment of inertia?

a) $I_A = 2m(l)^2 + m(0)^2$

b) $I_B = m(l/2)^2 + 2m(\frac{l}{2})^2$

c) $I_C = 2m(0)^2 + m(l)^2$

Q)



$I_A = m\left(\frac{a}{\sqrt{5}}\right)^2 + \cancel{2m(0)^2}$

$+ \cancel{4m(0)^2} + 3m\left(\frac{a}{\sqrt{5}}\right)^2$

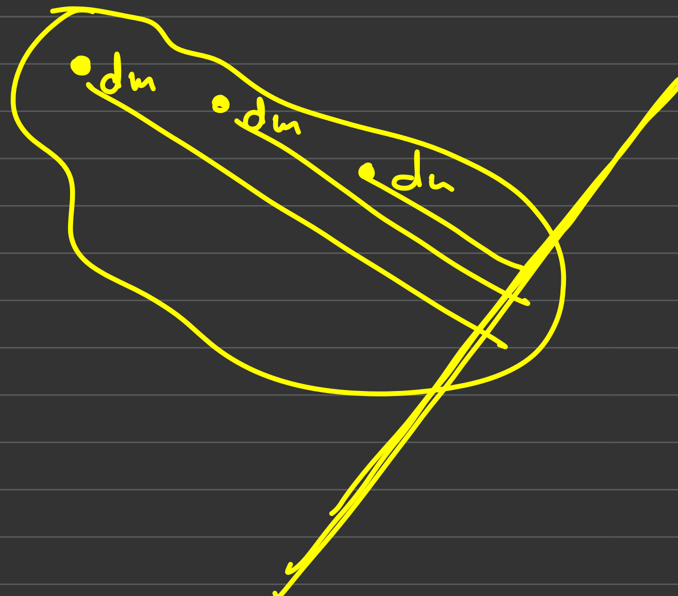
$I_A = \frac{ma^2}{5} + \frac{3ma^2}{5}$

$I_A = \frac{4ma^2}{5}$ Ans

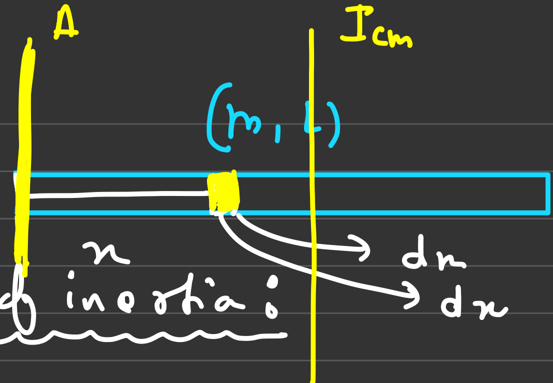
: moment of inertia of bodies | Extended bodies :

"about any axis"

$$I_0 = \int dm \, (r_\perp)^2$$



Ex# : **Rod :**



Uniform

find moment of inertia :

$$dm = \left(\frac{m}{L}\,dx\right)$$

#

$$I_A = \int dm\,(x)^2$$

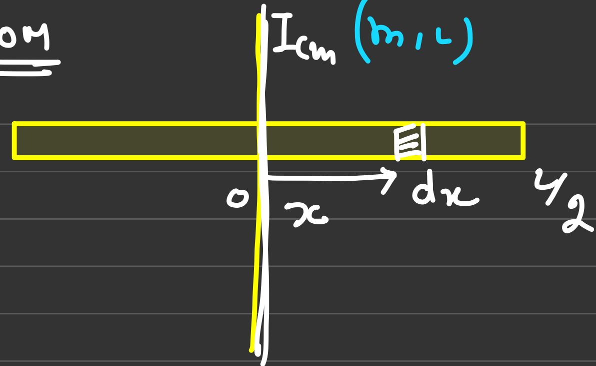
$$I_A = \int \left(\frac{m}{L} \cdot dx\right) x^2$$

$$I_A = \frac{m}{L} \int_0^L x^2 \cdot dx$$

$$I_A = \frac{m}{L} \left[\frac{x^3}{3}\right]_0^L = \left(\frac{mL^2}{3}\right)$$

{ Learn {

about COM



$$\int dI = 2\int_0^{L/2} dm\, x^2$$

$$I_{cm} = 2\int_0^{L/2} dm\, x^2$$

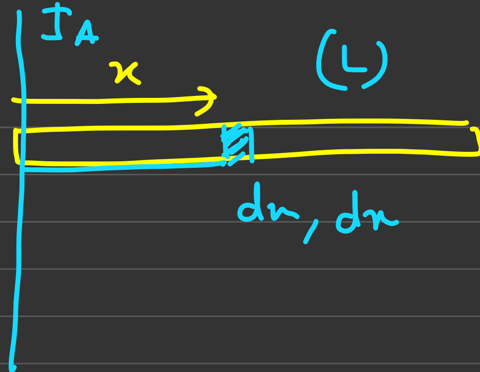
$$I_{cm} = 2\frac{m}{L}\left[\frac{x^3}{3}\right]_0^{L/2}$$

$$= 2\frac{m}{L}\left[\frac{L^3}{24}\right]$$

$$= \left(\frac{mL^2}{12}\right)$$

learn

e)



$\lambda = \lambda_0 \frac{x}{L}$

$$I_A = \int (dm)\, x^2$$

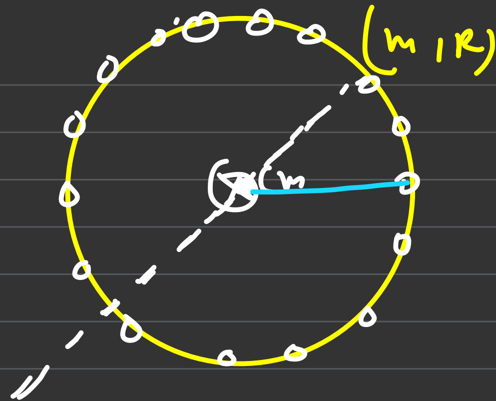
$$I_A = \int x^2 \left[\left(\lambda_0 \frac{x}{L}\right)(dx) \right]$$

$$I_A = \frac{\lambda_0}{L} \int_0^L x^3\, dx = \frac{\lambda_0}{L}\left[\frac{x^4}{4}\right]_0^L = \left(\frac{\lambda_0 L^3}{4}\right) =$$

✓

Q)

a)



An axis passing COM and $\perp$ to plane of ring

$$\int dI = \int dm\, R^2$$

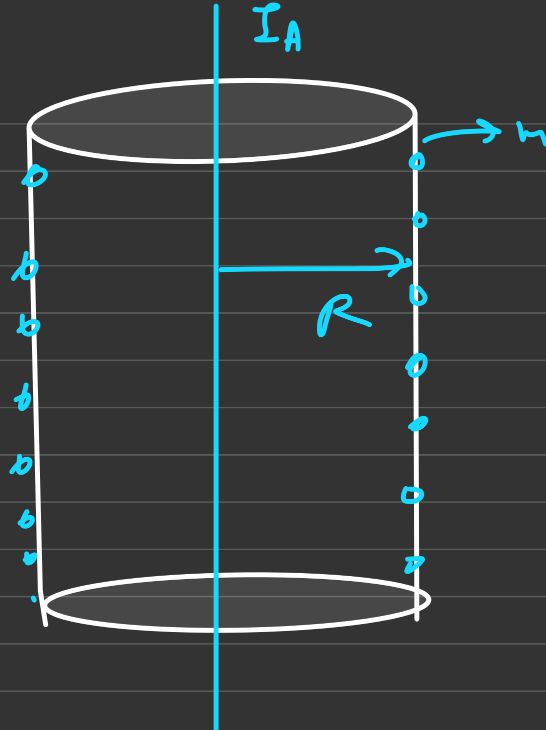
$$I_{cm} = R^2 \int dm = (mR^2)$$

if $\perp$ distance of all point mass is same then moment of inertia

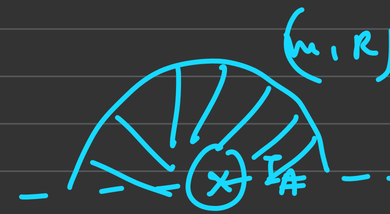
is going to be

$$I = \underbrace{d^2}_{\substack{\downarrow \\ \perp \text{ distance from axis}}} \underbrace{\sum m_i} = \underbrace{d^2}_{\substack{\downarrow \\ \perp \text{ distan.}}} \left(\int dm\right)$$

Q) "Hollow cylinder"

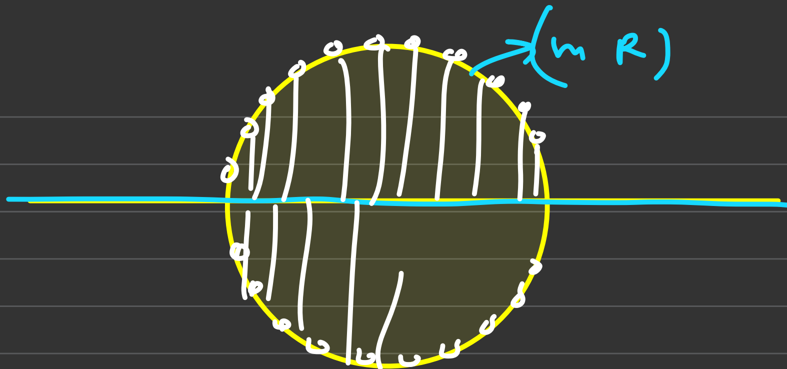


$$\underline{\underline{I = mR^2}}$$



$$I = mR^2$$

θ)



along diameter on plane of ring

I_B

find moment of inertia about B-axis

{momentum} Velocity DTS #1

→ Level 1

→ Level 2

INEA !

"revise torque"

rotation