Kinematics -1

VMC Vidyamandir Classes SINCE 1986

IIT JEE | MEDICAL | FOUNDATION

Application of vector multiplication:

1. work done $= \underbrace{\vec{F} \cdot \vec{S}}_{\text{scalar}}$

2. torque $= \underbrace{\vec{r} \times \vec{F}}_{\text{vector}}$

3. Power $= \vec{F} \cdot \vec{v}$

4. Area of triangle

work done:

$W = \vec{F} \cdot \vec{s}$ → Displacement of body | Point mass • (Point Size)

$\vec{F}$ → Force Vector

$W = |F| |s| \cos\theta$ → angle between $\vec{F}$ and $\vec{s}$

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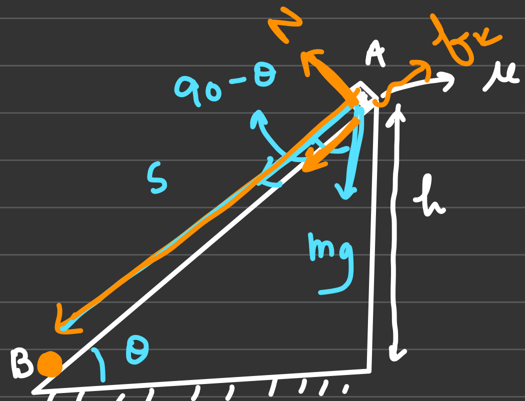


$mg \sin\theta > \mu_k mg \cos\theta$

find work done by mg, N, and frictional force

$$W_{mg} = \vec{F}\cdot\vec{s} = |F||s|\cos\theta$$

$$\sin\theta = \frac{h}{s}$$

$$= mg\,s \times \cos(90-\theta)$$

$$h = s\sin\theta$$

$$= (mg\,s\sin\theta)$$

$$s = \frac{h}{\sin\theta}$$

$$= \underbrace{\text{"}mgh\text{"}}_{+ve}$$

$$W_N = \vec{F}\cdot\vec{s} = N\,s\cos(90) = 0$$

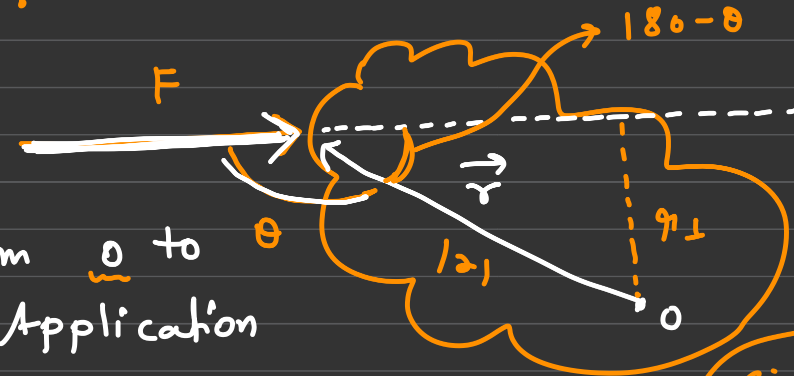
$$W_{f_k} = f_k\,s\cos(180)$$

$$= \mu_k N\,s\,(-1)$$

$$= \mu_k(mg\cos\theta)\frac{h}{\sin\theta}(-1) = \underbrace{-\mu_k mgh\cot\theta}_{-ve}$$

Torque: it is rotational capacity of body due to force.

$\vec{\tau}_0 = \vec{r} \times \vec{F}$



$|r| \sin\theta = r_\perp$

$\vec{r}$ = A Vector from O to the point of Application of force

$\sin(180-\theta) = \dfrac{r_\perp}{|r|}$

$\sin\theta = \dfrac{r_\perp}{|r|}$

$|\tau_0| = |r|\,|F| \sin\theta$ { angle between $\vec{r}$ and vector $\vec{F}$

$|r|\sin\theta\,|F| = (r_\perp F)$

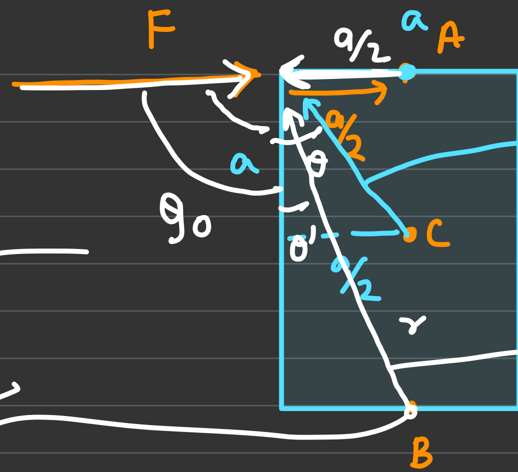
$\vec{r} \times \vec{F}$ = Direction $\vec{\tau}$

RHTR

$\vec{r}$ curl $\vec{F}$

Q)

find torque due to F about A, B and C?



$\sqrt{\frac{a^2}{4}+a^2} = \frac{\sqrt{5}\,a}{2}$

$\sqrt{(2a)^2+\frac{a^2}{4}} = \frac{\sqrt{17}\,a}{2}$

$\tau_C = Fa$ c-w

$\tau_B = 2Fa$ c-w

$\tau_A = \vec{r}\times\vec{F} = |r|\,|F|\sin(180)$

$= a/2\ F \times 0 = 0$

$\tau_C = \vec{r}\times\vec{F} = |r|\,|F|\sin\theta$

$= \frac{\sqrt{5}\,a}{2} F \times \sin(90+\theta)$

$= \frac{\sqrt{5}\,a}{2} F \cos(\theta) = \frac{\cancel{\sqrt{5}}a}{\cancel{2}} F \times \frac{a\,\cancel{2}}{\cancel{\sqrt{5}a}}$

$= (F\,a)$

'modulus of torque'

Clockwise or into plane

$F\,a$ clockwise

$$\tau_B = |r|\,|F|\sin(90+\theta')$$

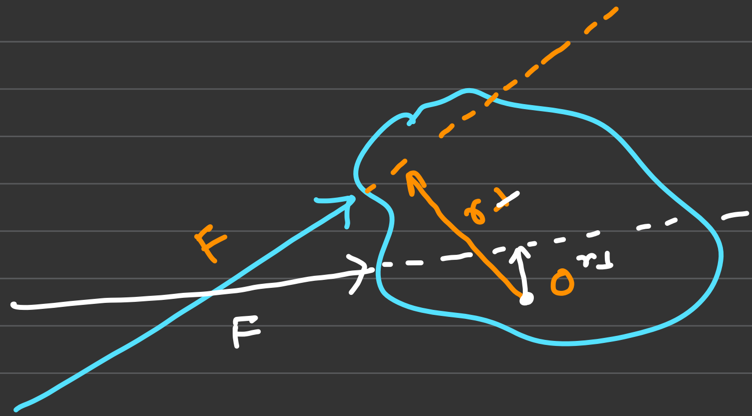
$$= \frac{\sqrt{17}\,a}{2}\,F\,\cos\theta' = \frac{\sqrt{\cancel{17}}\,a}{\cancel{2}}\,F \times \frac{2a\,\cancel{2}}{\sqrt{\cancel{17}}\,a}$$

$$= F\,2a$$

c.w or into plane

Direct method :

$|\tau| = (r_\perp F)$



$r_\perp$ = $\perp$ length from O to line of App. of force

$\tau_O = r_\perp F$

modulus of Torque

C.W into plane

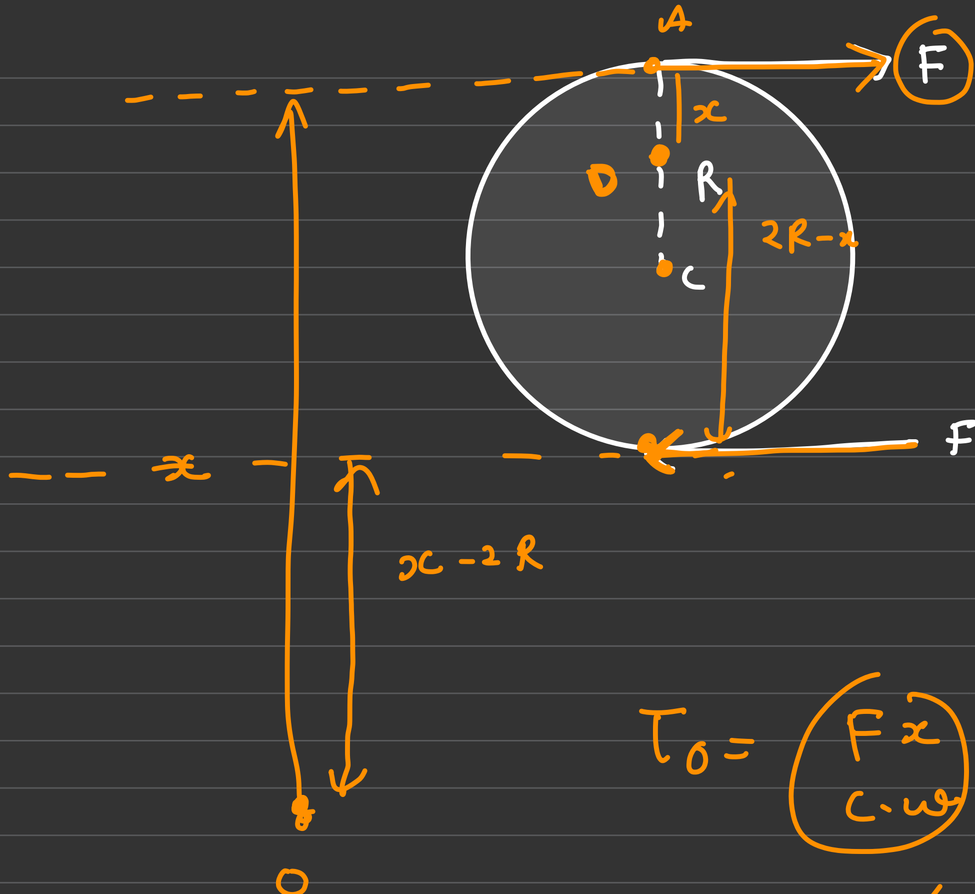
Ac.W out of plane

Q)

$\Sigma \tau_A = \{ \cancel{0} + (F\,2R)$ C.w B $\} = 2FR$ $\underline{\underline{Cw}}$

$\Sigma \tau_C = \underset{C.w}{FR} + \underset{C.w}{FR} = \underset{\underline{\underline{C.w}}}{2FR}$

$\Sigma \tau_B = \underline{\underline{2FR}}$ $\underline{\underline{C.w}}$



Imp.

Couple force

Torque about any point same

$F(2R-x)$ c.w Fx c.w

$F2R - \cancel{Fx} + \cancel{Fx}$

$= 2FR$ cw

$\tau_0 = Fx$ (c.w) $+ F(x-2R)$ (A.w)

$= \cancel{Fx} - \cancel{Fx} + F2R$

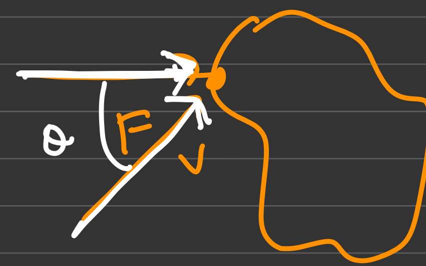
$= F2R$. cw

Power : (rate of doing of work)

$$P_{inst} = \frac{dW}{dt} = \frac{\vec{F}\cdot d\vec{s}}{dt} = \vec{F}\cdot\vec{v}$$

$$= |F||v|\cos\theta$$

$$=$$



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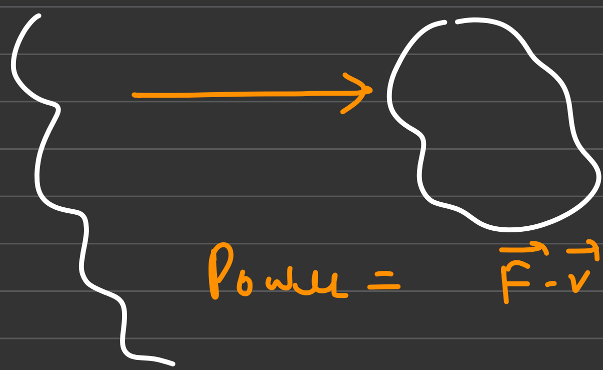


$\vec{F} = (2\hat{i} + 3\hat{j})\ N$

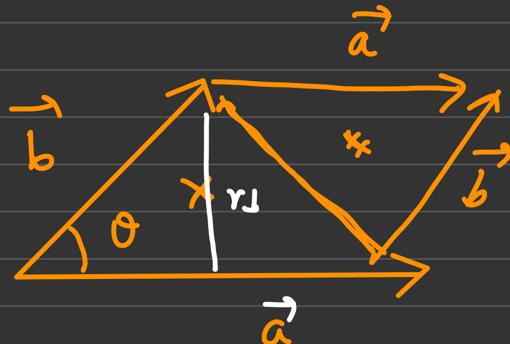
$\vec{v} = (2\hat{i} + 5\hat{k})\ m/s$

Power $= \vec{F}\cdot\vec{v} = (2\hat{i} + 3\hat{j})\cdot(2\hat{i} + 5\hat{k})$

$= 4$

$\vec{F} \cdot \vec{v} = 4$ watts

area of ||gram :



$\frac{1}{2} |r_\perp| \, |a|$ $= \frac{1}{2} b \sin\theta \, a$

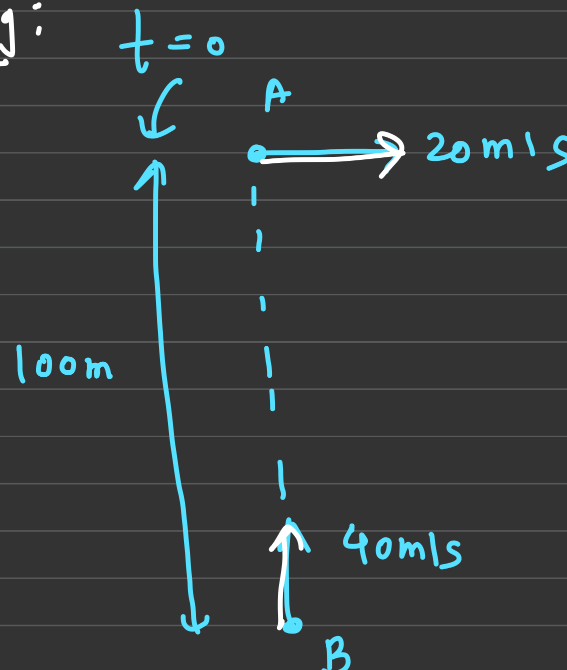
$= \frac{1}{2} (\vec{a} \times \vec{b})$ area of tri'angle

$\boxed{\text{Area of ||gram} = (\vec{a} \times \vec{b})}$

area is vector quantity

Q)

Problem Solving:



find min sep between A and B & find time after which this happens

Kinematics

mechanics

Kinematics

1D
2D

This is about

{Displacem, velocity
Acceleration}

"effect"

Dynamics

{force
cause}

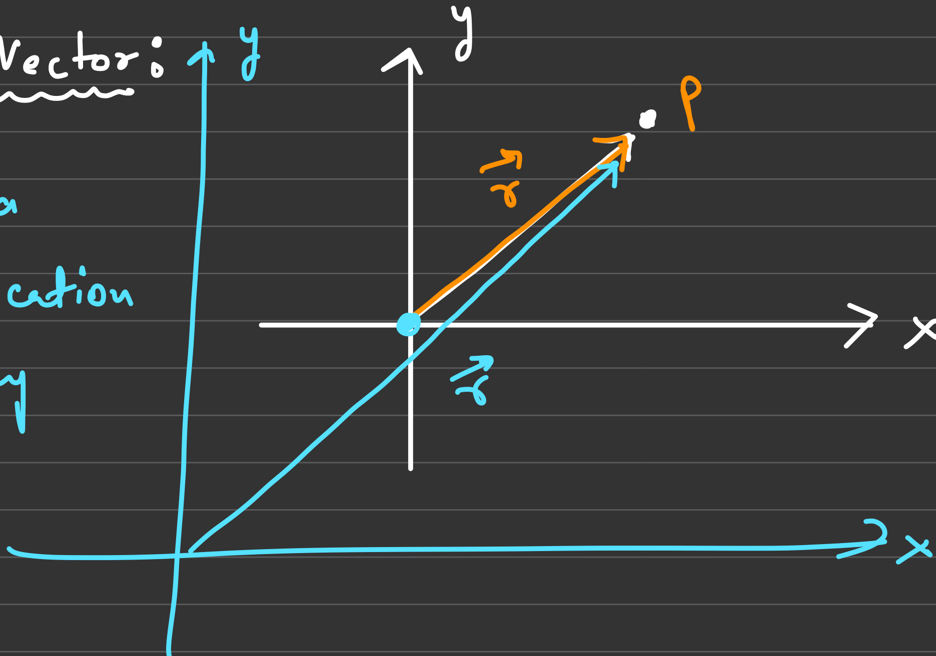
#

Kinematics: (1D-motion)

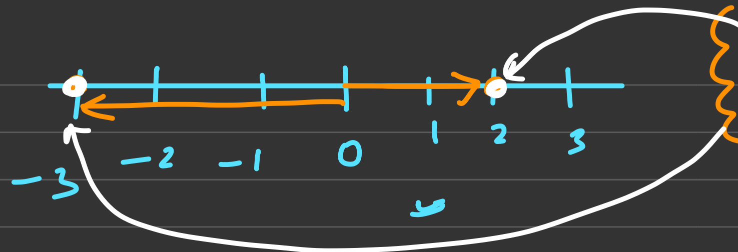
Basics of Kinematics:

① Position Vector:

"A vector from a reference to location of Particle at any instant"

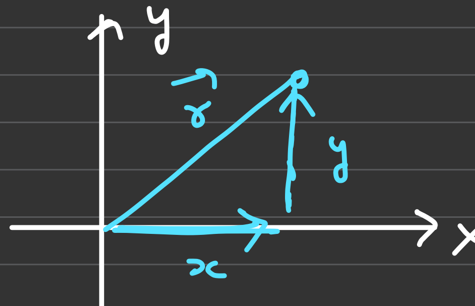


1D:



$$\left\{ \begin{aligned} x &= +2 \\ x &= -3 \end{aligned} \right\}$$

2D:



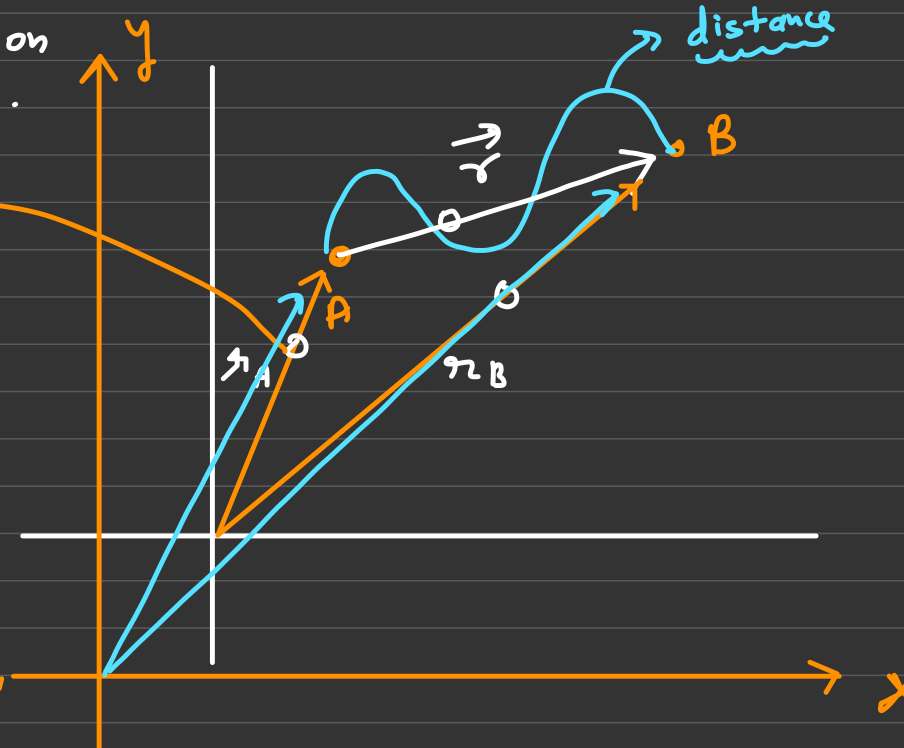
$\vec{r} = \underbrace{x\hat{i} + y\hat{j}}$ 2D

Displacement: is a vector from initial position vector to final position vector.

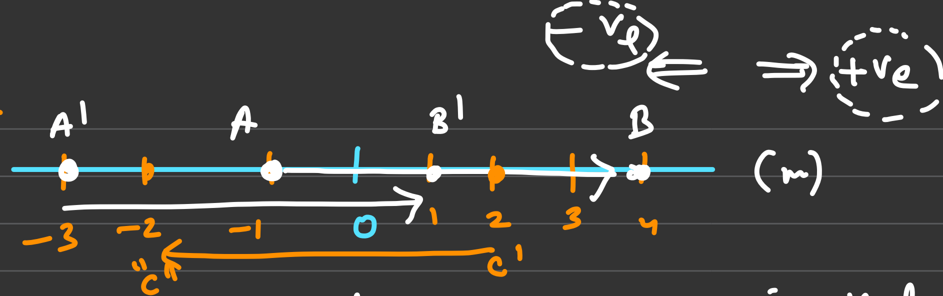
$\vec{r} + \vec{r}_A = \vec{r}_B$

$\vec{r} = \vec{r}_B - \vec{r}_A$

"Kya displacement origin pe depend karta hai"



1 D :



x_{AB} = final position − initial position vector

$x_{AB} = x_B - x_A = +4 - (-1) = +5$

Right

$x_{A'B'} = x_{B'} - x_{A'}$

$= +1m - (-3m)$

$= +4\ m$ → Right

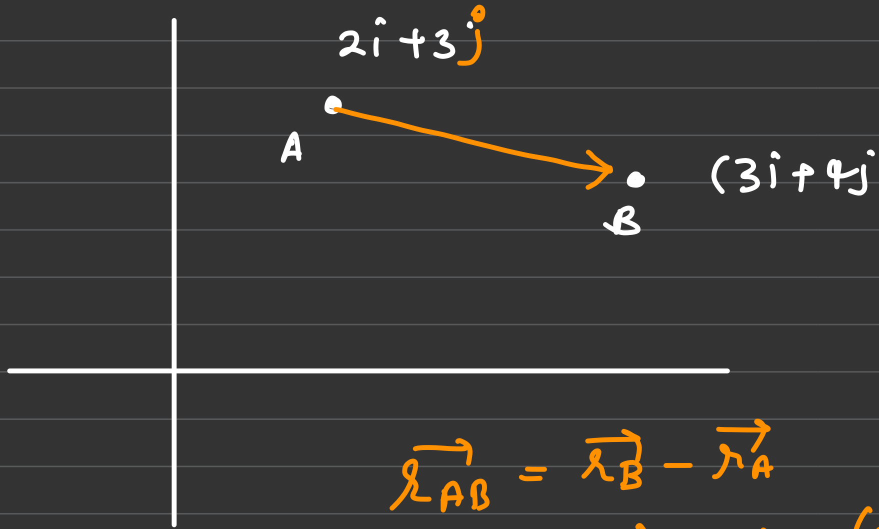
$\underline{\underline{\vec{x}_{c'c''}}} = \vec{x}_{c''} - \vec{x}_{c'}$

$= -2 - (+2)$

$= -4\,m$

↳ left

Q)

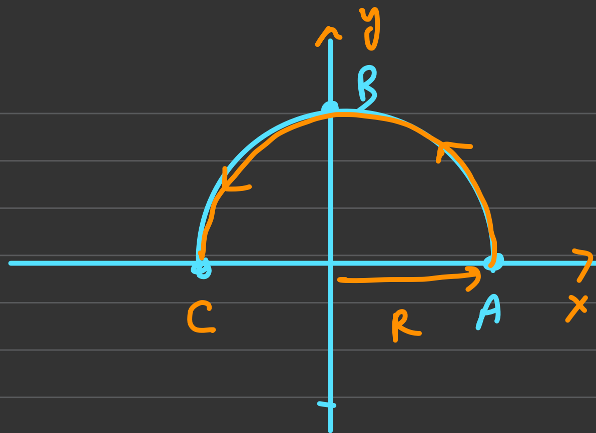


$\vec{r}_{AB} = \vec{r}_B - \vec{r}_A$

$= (3\hat{i} + 4\hat{j}) - (2\hat{i} + 3\hat{j})$

$= \underline{\underline{\hat{i} + \hat{j}}}$ Displacement

Q)



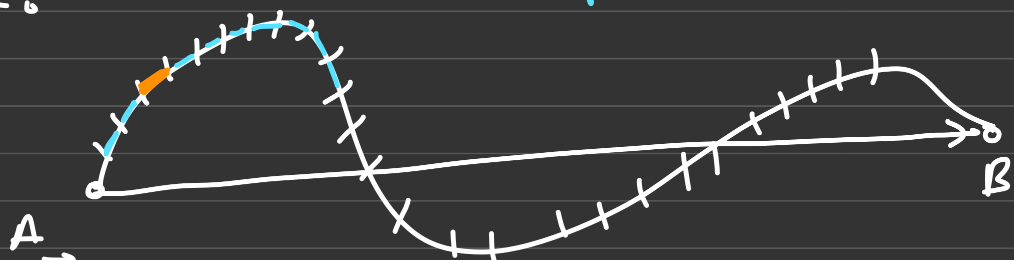
find

$$\vec{r}_{AB} = \vec{r}_B - \vec{r}_A = R\hat{j} - R\hat{i}$$

$$\vec{r}_{BC} = \vec{r}_C - \vec{r}_B = -R\hat{i} - R\hat{j}$$

$$\vec{r}_{AC} = \vec{r}_C - \vec{r}_A = -R\hat{i} - R\hat{i} = -2R\hat{i}$$

Distance : Actual path length:



$\vec{S}_{ins}$ S_{inst}

|Displacement| $\leq$ Distance

$$\left\{ \, |\vec{ds}| = ds \, \right\}$$

$|\vec{ds}|$ ↓ mod of inst Displacement

ds ↓ inst Dist

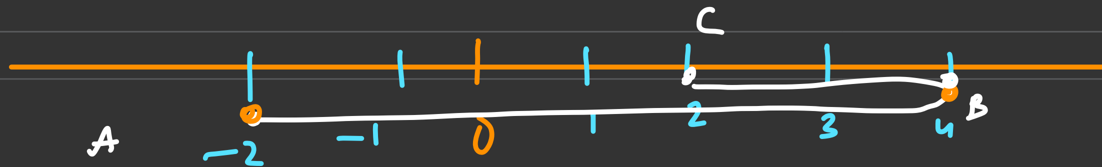
Velocity:

- Ins Velocity
- Avg Velocity

Average Velocity:

$$\vec{V}_{avg} = \left\{ \frac{\vec{r}_f - \vec{r}_i}{\Delta t} \right\}$$

1D:



$t_{AB} = 2\ \text{sec}$
$t_{BC} = 1\ \text{sec}$ } given

find

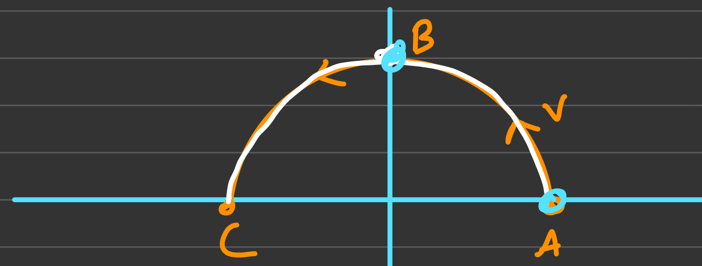
$$(V_{avg})_{AB} = \frac{+4-(-2)}{2} = \frac{+6}{2} = +3\ \text{m/s} \rightarrow$$

$$(V_{avg})_{BC} = \frac{+2-4}{1} = -2\ \text{m/s} \leftarrow$$

$$(V_{avg})_{AC} = \frac{+2-(-2)}{3} = +\frac{4}{3}\ \text{m/s} \rightarrow$$

2D :

Speed = $\frac{\text{distance}}{\text{time}}$



"find $(V_{avg})_{AB}$" $= \frac{R\hat{j} - R\hat{i}}{\left(\pi R/2v\right)}$

$= \frac{2v(\hat{j} - \hat{i})}{\pi}$ m/s

modulus $= \frac{2v\sqrt{2}}{\pi}$

$(V_{avg})_{BC} =$

$(V_{avg})_{AD} =$

$$V_{BC} = \frac{-R\hat{i} - R\hat{j}}{\left(\frac{\pi R}{2v}\right)} \text{ m/s}$$

$$V_{AC} = \frac{-2R\hat{i}}{\left(\pi R/v\right)} \text{ m/s}$$

Homework: { "Differentiation & Integration" (Revision) }

Vector:
{module + work book}

Level 1 → main
Level 2 → Ad