Kinematics -3

Non-Uniformly Accelerated motion:

Acceleration should not Constant

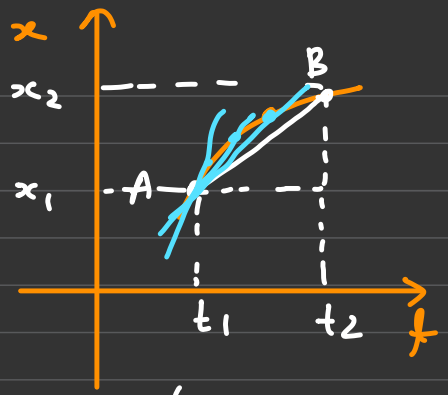
a) A particle moving along straight line as a function of time as $x = t^3 - 3t$ (m) then find time when particle is at origin?

$t = 2s$

$\frac{dx}{dt}$ = Velocity

$\frac{dv}{dt}$ = acceleration

$v = 3t^2 - 3$



$V_{avg} = \left(\frac{x_2 - x_1}{t_2 - t_1}\right) = \tan\theta$ Slope

Slope of chord in x-t is always V_{avg}

$\Delta t \to 0 \to$ Chord becomes tangent

$\Delta t \to 0 \quad \Delta x \to 0$

$a = 6t$

a) $0 = t^3 - 3t$

$t(t^2 - 3) = 0$

$t = 0 \quad t^2 = 3$

$t = \pm\sqrt{3}$

$\{ t = 0, \quad t = -\sqrt{3}, \quad t = +\sqrt{3} \}$

b) find position of particle when particle has zero velocity?

$v(t) = 3t^2 - 3 = 0$

$v_{avg} = \frac{\Delta x}{\Delta t}$

$v_{inst} = \frac{dx}{dt}$

differentiation ↑ w.r.t time

$x = t^n$

$\frac{dx}{dt} = n t^{n-1}$

$t^2 - 1 = 0$

$t = \pm 1$ sec

$x = t^3 - 3t$

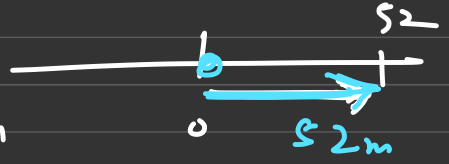
$x = 1 - 3 = -2$ m at $t = +1$ sec

$x = -1 + 3 = +2$ m $\quad t = -1$

c) find displacement from $0 \to 4$ sec

$x(0) = 0$

$x(4) = 64 - 12 = 52$ m



Displacement $= (52 - 0) = +52$ m

d) find distance covered by particle from 0 → 4sec

Steps to find distance in 1D:

① find position vector at extreme point

$$x(0) = 0$$
$$x(4) = 52$$

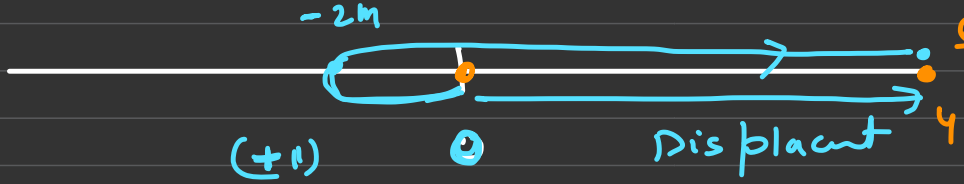
② find turning point ("where velocity is zero")

$t = (+1)$ -1 $0 \rightarrow 4$ sec

③ Draw time line

$x = t^3 - 3t$

$x(1) = 1 - 3 = -2$



$$\text{Distance} = +2 + 2 + 52$$
$$= \underline{56\,m}$$

Q) 1D $x(t) = 2t^3 - 15t^2 + 36t + 2$ find distance covered by the particle between 1 sec to 4 sec ?

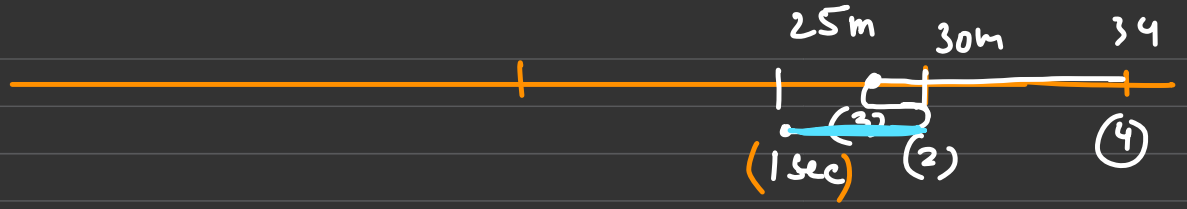
① $\begin{cases} x(1) = 25\,m \\ x(4) = 34\,m \end{cases}$

② turning point : $v(t) = 6t^2 - 30t + 36 = 0$

$$t^2 - 5t + 6 = 0$$
$$(t-2)(t-3) = 0$$
$$t = (2, 3)$$

$\begin{cases} x(2) = 30 \\ x(3) = 29 \end{cases}$

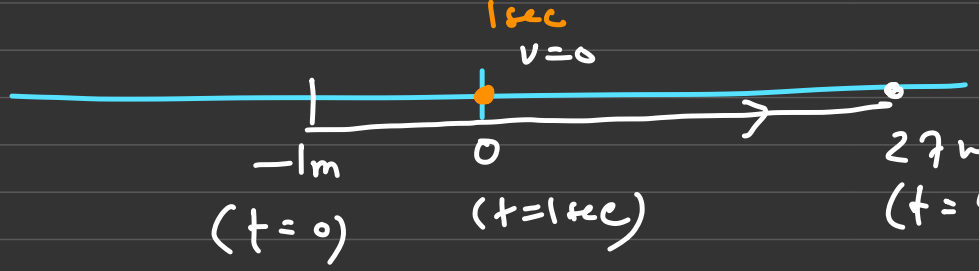
(3)



Distance = 5 + 1 + 5
= 11m Ans

Q) $x(t) = (t-1)^3$ then find distance covered between 0 → 4 sec ?

$\begin{cases} x(0) = -1m \\ x(4) = 27m \end{cases}$



|Displacement| = Distance

Ans

$$v(t) = 3(t-1)^2 = 0$$

$$t = 1 \text{ sec}$$

8) displacement in n^{th} sec =

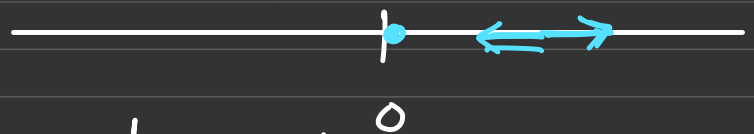
$$S_{nth} = u + \frac{1}{2} a (2n-1)$$

valid for const. Acc.

n^{th}

$t = 0 \quad u = 5 \text{ m/s}$

$a = -2 \text{ m/s}^2$



① find displacement in 3rd second

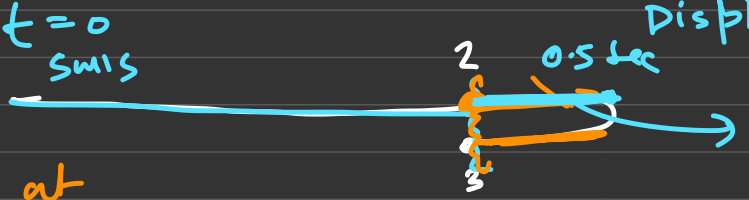
(2 → 3 second)

$$S_{nth} = 5 + \frac{1}{2}(-2)(3 \times 2 - 1)$$

$$= 5 - 1 \times 5$$

$= 0 \text{ m}$

(11) find distance covered in 3rd second?



Displacement = 0 in 3rd second

$v = u + at$

$v = 5 \text{ m/s} - 2 \times 2$

$= 1 \text{ m/s}$

$s = ut + \frac{1}{2}at^2$

$s = 1 \times 0.5 + \frac{1}{2}(-2) \times (0.5)^2$

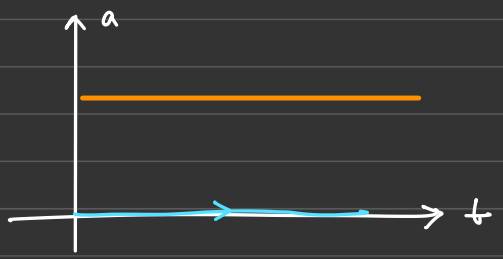
Distance $= 0.5 - 1 \times 0.25$

$= 0.25 \text{ m}$

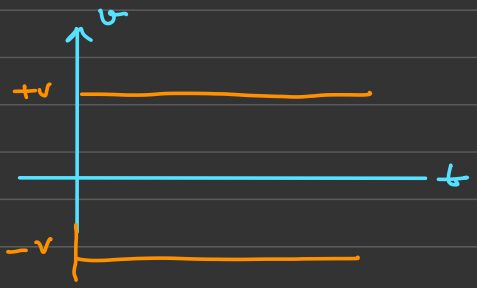
Total distance = 0.5 from 2sec to 3sec

: Graph :

① Uniform motion: Const velocity ⇒ $a = 0$



$a = 0$



$v = \text{Const}$

$x = v \times t$

$y = mx$

"
① $a-t$ is on time axis then $v-t$ must be $\parallel$ to time-axis "

② $v-t$ $\parallel$ to time axis the $x-t$ must be inclined line

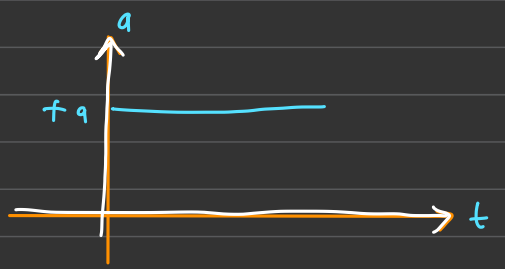
(3) area under a-t curve is always change in velocity

(4) area under v-t = displacement (final position − initial position)

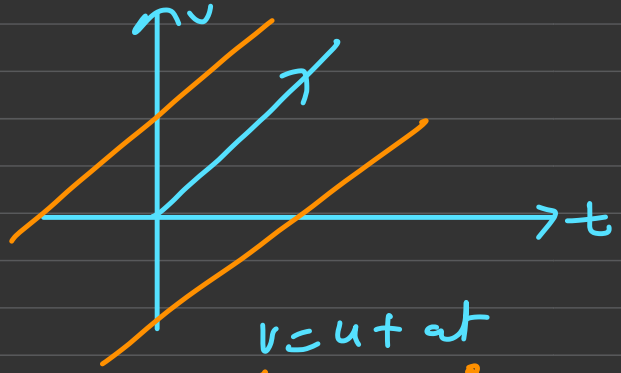
(5) Slope in v-t curve acceleration

(6) Slope in x-t → velocity

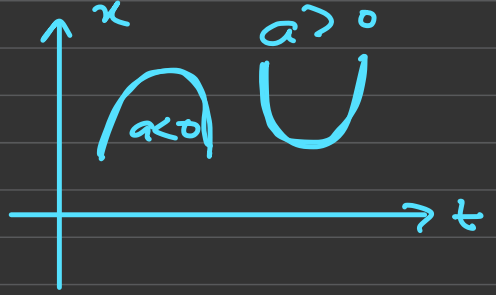
(2) Uniformly Accelerated motion: (a = const)



a = const



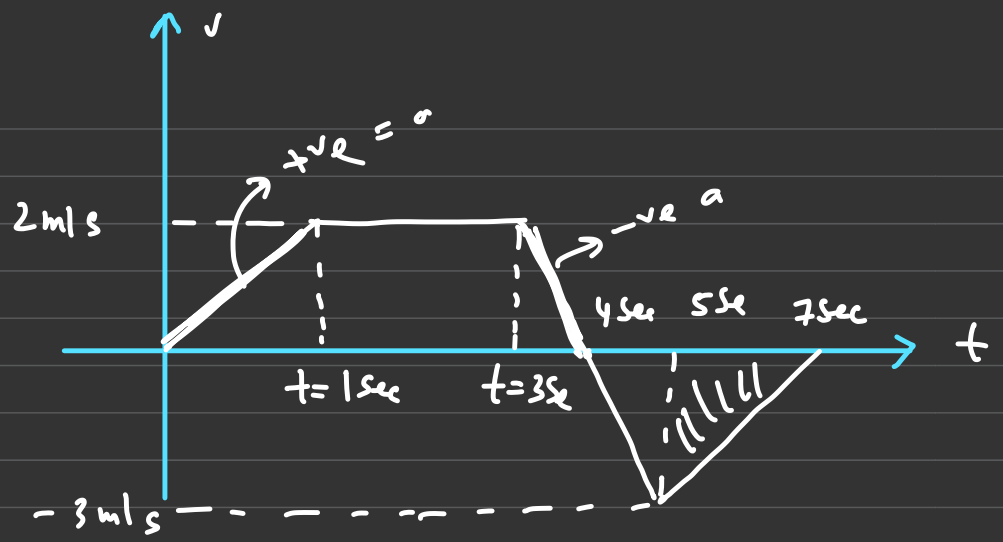
$v = u + at$



$x = ut + \frac{1}{2}at^2$

: Conversion of graph: $x(t) \rightarrow v(t) \rightarrow a(t)$

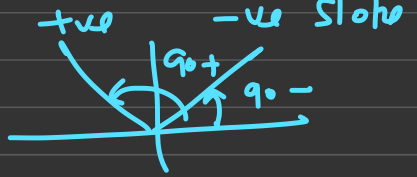
Θ)



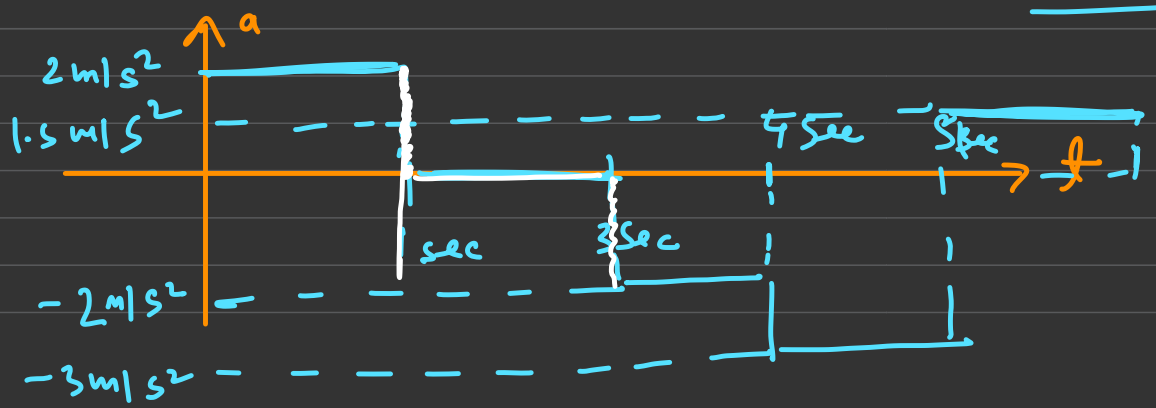
1D

v is changing as time according to this graph.

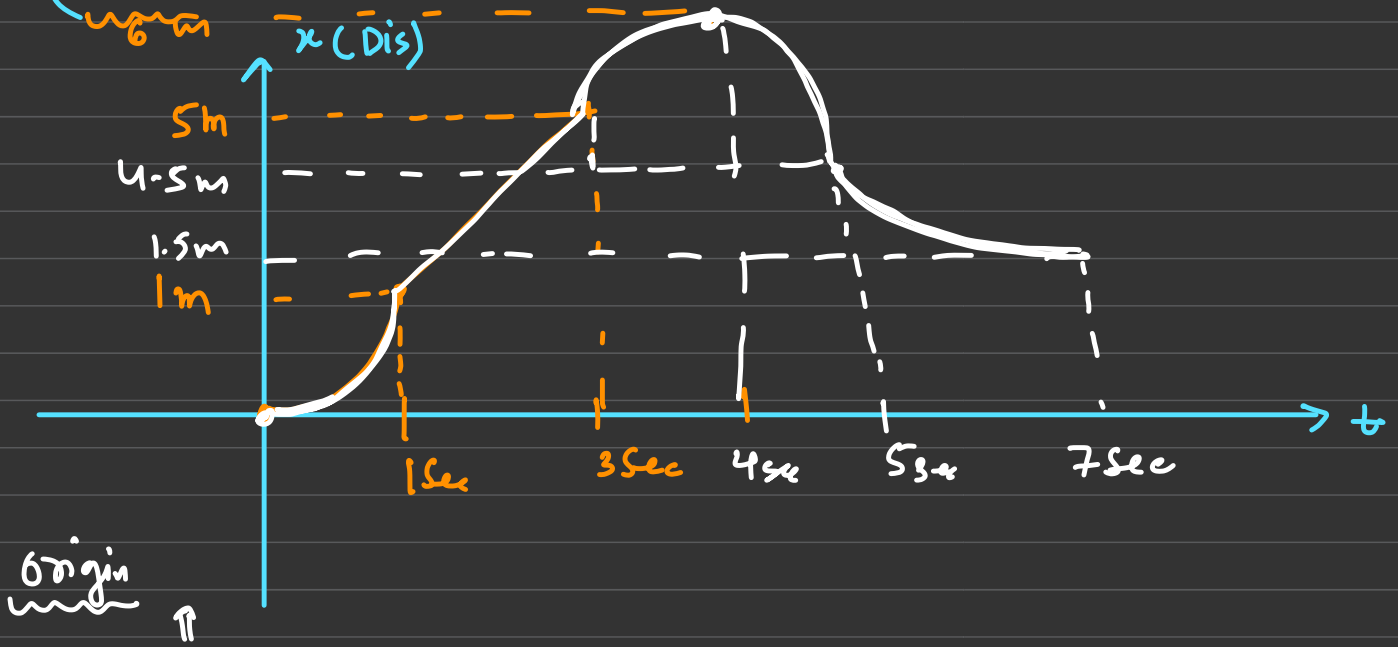
Slope = tanθ = 2m/s²



Convert this graph: 1) a-t

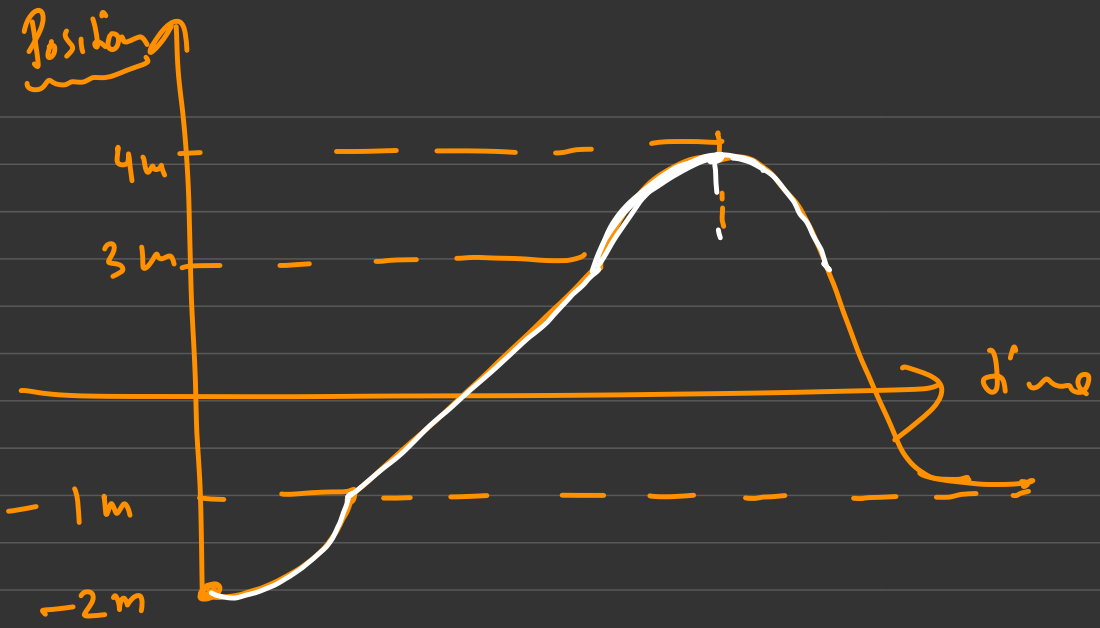


2) $(x-t)$

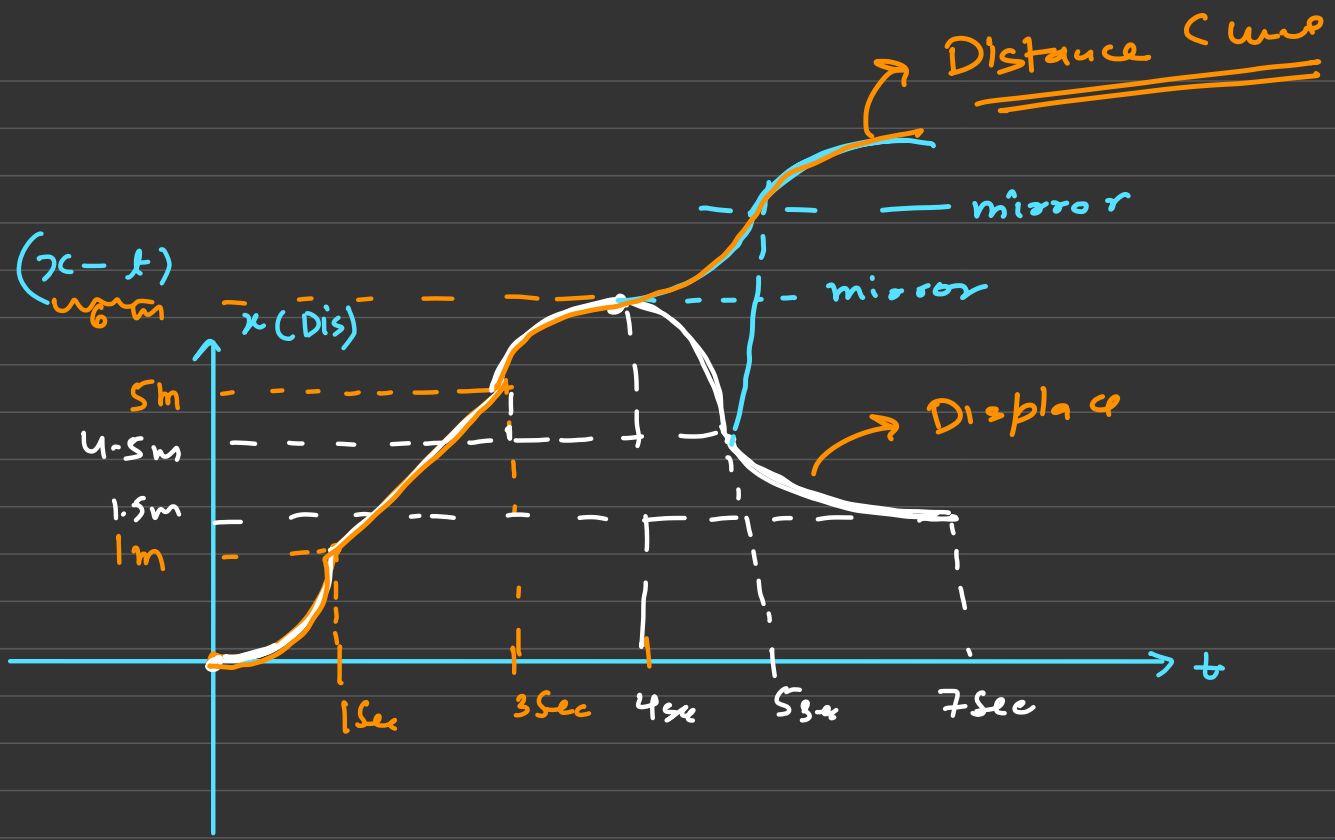


Position - time!

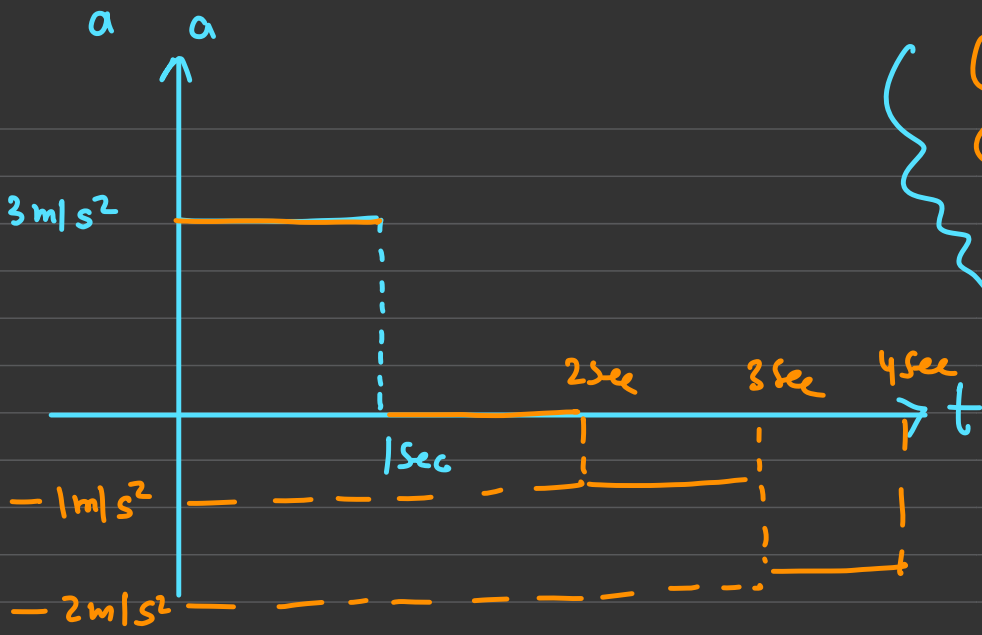
Draw position time graph: # $\{t=0 \quad x=-2\}$



Distance: { +ve and increasing {


Distance Curve
mirror
mirror
(x – t)
graph
x (Dis)
5m
4.5m
1.5m
1m
Displace
t
1Sec
3Sec
4Sec
5Sec
7Sec

θ)



① Draw v-t $t = -2m/s$

② a) Displacement time

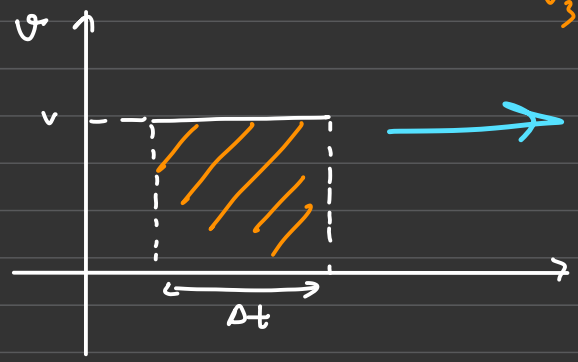
b) Draw position time

$t = 0 \quad x = -1m$

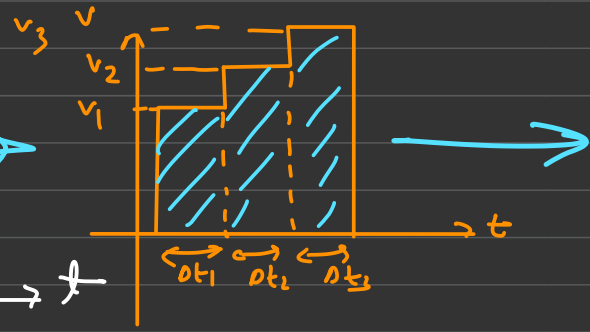
③ Distance time

Homework

Integration: (area under curve ⇒ integration)

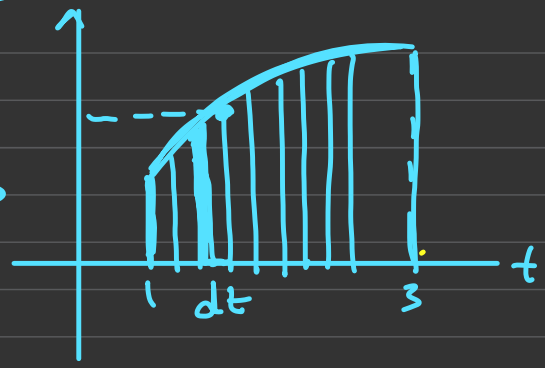


$s = v\,\Delta t$



$s = v_1 \Delta t_1 + v_2 \Delta t_2 + v_3 \Delta t_3$

$v = t^2$



$$\int_{s_i}^{s_f} ds = \int v\,dt$$

$$\Rightarrow \int_{s_1}^{s_f} ds = \int_{1\,sec}^{3} t^2\,dt$$

$$\Delta s = s_f - s_i = \left[\frac{t^3}{3}\right]_1^3$$

Integration : (Revise)

find time after which it (boat) stops

Q)

Home till DTS 3 → Level 1 / Level 2

module grater scale