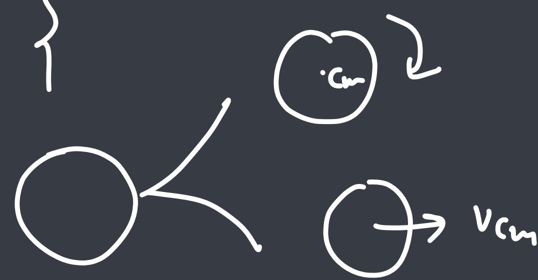
Rotation 5

GRBM

→ Pure rotation

→ Pure rotion



Dynamics of CRBM :

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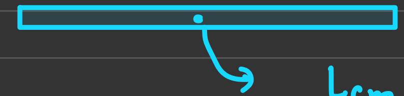
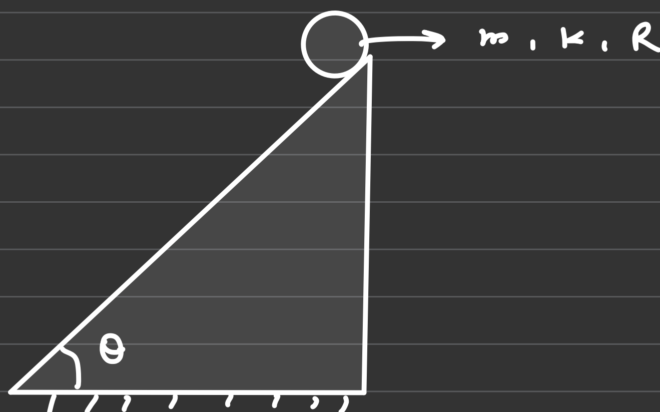


$$L_{cm} = I_{cm} \times \omega$$

$\frac{dL_{cm}}{dt} = I_{cm} \alpha$

$$\boxed{\tau_{cm} = I_{cm} \times \alpha}$$

Q) #



"No- slipping only rolling"

find A_{cm} and α of rolling body?

$\tau_{cm} = I_{cm} \times \alpha$

(Sirf COM ke Valid

hai)

(1) ✓

(2) ✓

$f = 0$ { No-acting friction

$f = +ve$ ✓ Sahi direction

$f = -ve$ ulti Direction

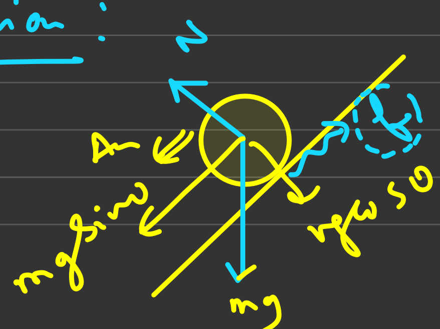
(1) FBD of body

(2) Acceleration Diagram (a_{cm}, α)

(3) Apply $F = ma$ and $\tau_c = I_c \alpha$

valid only in case of pure rolling

if Pure Translation:



$mg \sin\theta - f = mA_{cm}$ (1)

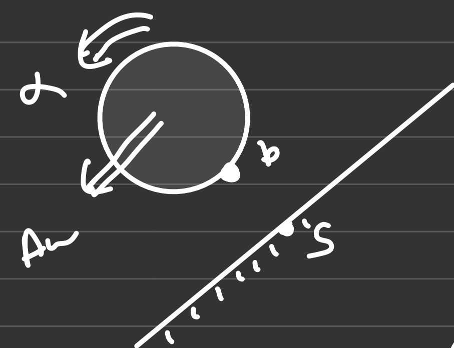
$N = mg\cos\theta$

if Pure rotation :

$$\Rightarrow \quad \tau_{cm} = I_{cm} \times \alpha$$

$$f \times R = mk^2 \times \alpha \quad — (11)$$

(3)



from pure rolling :

$$a_b = A_{cm} - R\alpha$$

$$a_s = 0$$

Condition for pure rolling

$a_b = a_s$

$A_c - R\alpha = 0$

$$A_c = R\alpha \longrightarrow (11)$$

$$mg\sin\theta - f = mA_c$$

$$f \times R = mk^2 \times \frac{A_c}{R}$$

$$f = \frac{mk^2 A_c}{R^2}$$

$$mg\sin\theta - \frac{mk^2 A_c}{R^2} = mA_c$$

$$mg\sin\theta = mA_c\left(1 + \frac{k^2}{R^2}\right)$$

$$A_c = \frac{g\sin\theta}{1 + \frac{k^2}{R^2}}$$

Ring $\Rightarrow$ $k = R$ $\longrightarrow$ $A_c = \left(g\sin\theta/2\right)$

disc $=$ $k = \frac{R}{\sqrt{2}}$ $\longrightarrow$ $\frac{2}{3} g\sin\theta$

Solid sphe $=$ $k = \sqrt{\frac{2}{5}}\,R$ $\longrightarrow$ $5/7\, g\sin\theta$

if released from rest which one is going to first at bottom

$$S = \frac{1}{2} a t^2$$

$a_{max} \rightarrow t_{min}$

$(Ring)_{min} \rightarrow t_{max}$

$$v^2 = \cancel{u^2}_{0} + 2as$$

Solid sphere is going

b) find frictional force for pure rolling

$$f = mA_{cm}\frac{K^2}{R^2}$$

$$f = \frac{m(g\sin\theta)(K^2/R^2)}{1 + K^2/R^2}$$

$$f = \frac{mg\sin\theta}{\frac{R^2}{K^2} + 1}$$

$\Rightarrow$ Ring $\rightarrow$ $K = R$ $\longrightarrow$ $f = \left(\frac{\cancel{m}g\sin\theta}{2}\right) = \mu \cancel{m}g\cos\theta$

$$\underline{\mu = \left(\frac{\tan\theta}{2}\right)}$$

$$\mu_s \cancel{m}g\cos\theta = \frac{\cancel{m}g\sin\theta}{1 + \frac{R^2}{K^2}}$$

$$\Rightarrow \mu_s = \frac{\tan\theta}{1 + R^2/K^2}$$

$\downarrow$ for Ring = $\mu_s = \left(\frac{\tan\theta}{2}\right)$ {

disc = $\mu_s = \frac{\tan\theta}{3}$ {

Solid sphere = $\mu_s = \frac{2}{7}\tan\theta$ }

Q) Ring $\rightarrow$ $\mu_s = \left(\frac{\tan\theta}{4}\right)$ then

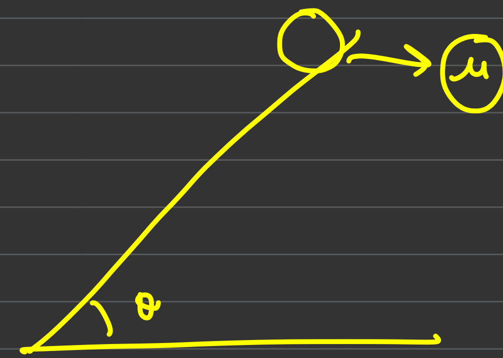
"friction is not sufficient for pure rolling"

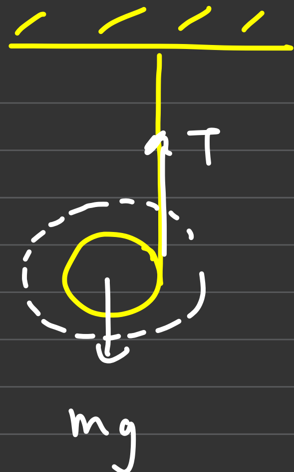
a) Pure rotation

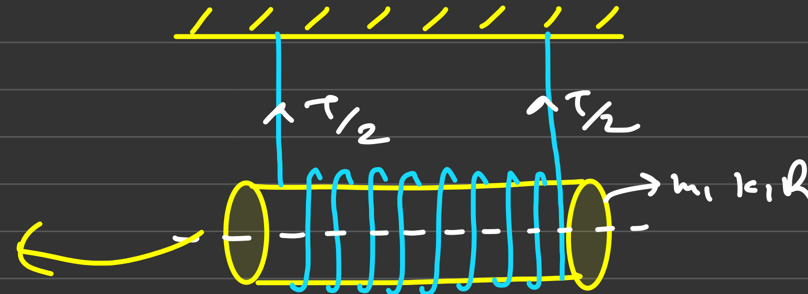
b) Pure Translation

c) rotation + Translation

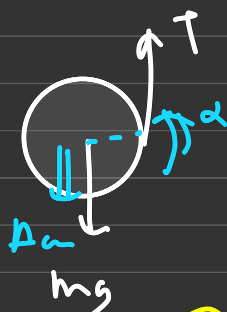
d) Pure rolling







No- slipping
Any where
then find an
A body



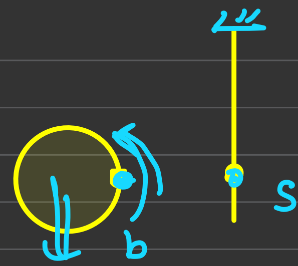
1 2 3

$mg - T = m A_{cm}$ — (1)

$T \times R = m k^2 \times \alpha$ — (II)

P. T { $\alpha = 0$ } assumption

P. R (assumption)



$a_s = 0$

$A_c = R\alpha$ — (iii)

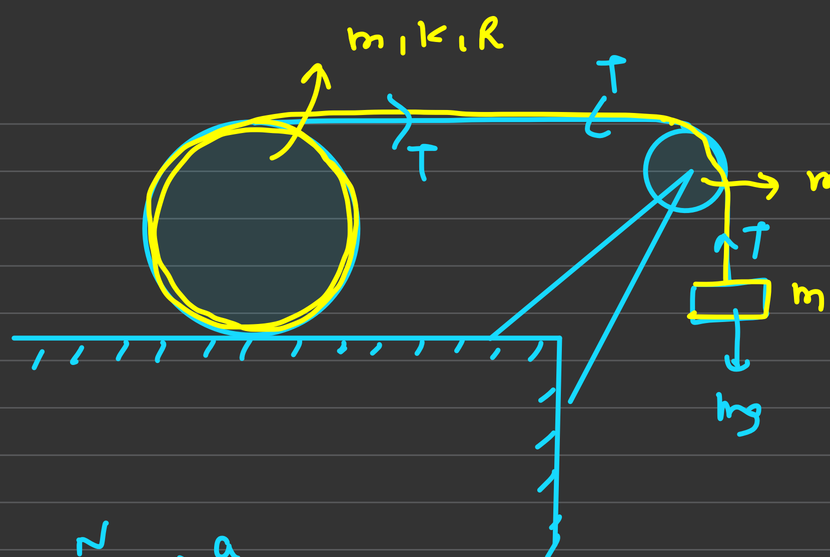
A_c

$a_b = A_c - R\alpha$

$a_s = a_b \Rightarrow$

$A_c - R\alpha = 0$

Q)

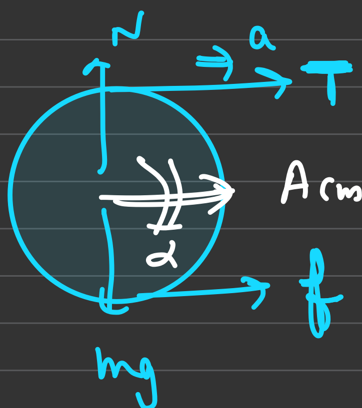


System is released from rest

No slipping

Pure rolling

#



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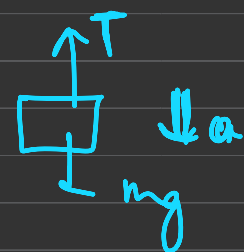


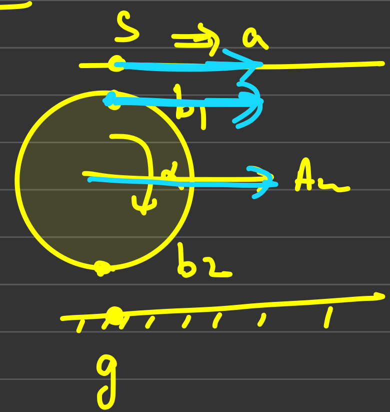
$mg - T = ma \quad \text{—(1)}$

$$T + f = m A_{cm} \quad \text{— (I)}$$

$$T \times R - f \times R = m k^2 \alpha \quad \text{— (II)}$$

$$a = A_{cm} + R\alpha \quad \text{— (IV)}$$

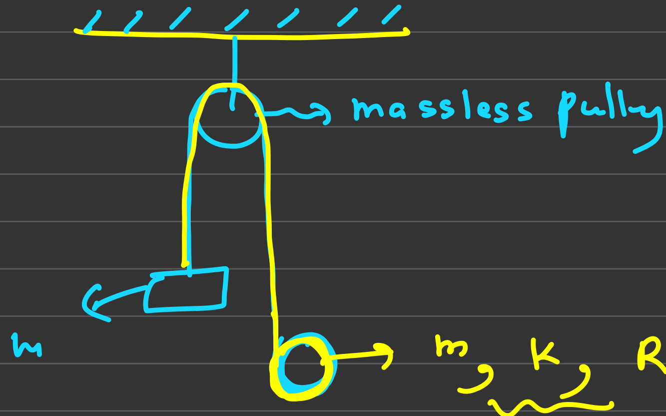
Pure rolling:



$$\left\{ \begin{aligned} a_s &= a \\ a_{b_1} &= A_{cm} + R\alpha \end{aligned} \right\}$$

$$\left. \begin{aligned} a_{b_2} &= A_{cm} - R\alpha \\ a_g &= 0 \end{aligned} \right\}$$

$$A_{cm} = R\alpha \quad \text{— (V)}$$



Pure rolling
released from rest
find Accelerations ?

#

P.T.

P.R

$mg - T = ma$ — (1)

(2) (1)

$mg - T = mA_m$ — (II) (3)

(4)

$T \times R = mK^2 \times \alpha$ — (III) from pure rolling

$\Rightarrow R\alpha - A_c = a$ — (IV)

$\Rightarrow$

if I assumed A_c upward then

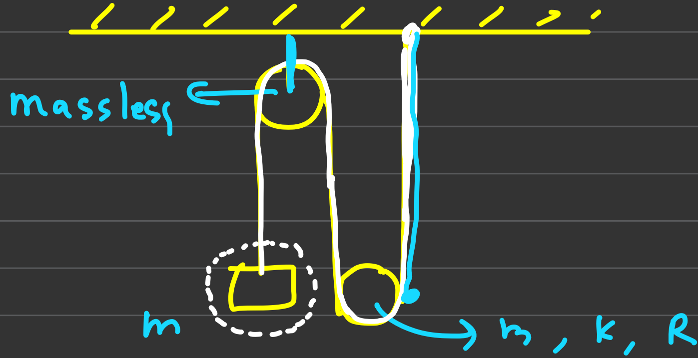
$T - mg = m A_c$ — (I)

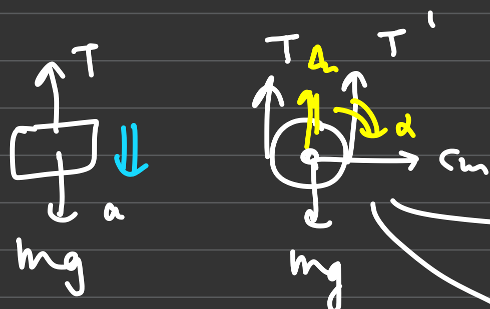
$T \times R = -m k^2 \times \alpha$ (II)

$A_c - R\alpha = a$ (III)

Q)

Releasing from rest
No-slipping anywhere
then find acc. of each block?





P.R

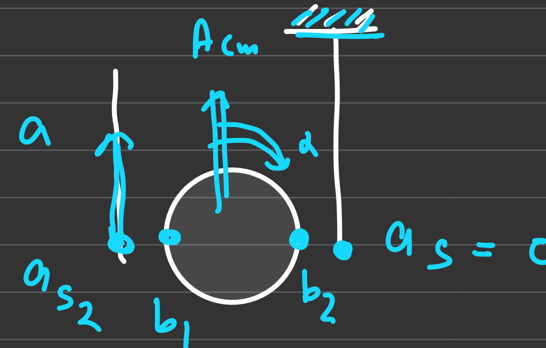
$$\underbrace{T\times R}_{C.W} - \underbrace{T'\times R}_{A.C.W} = mK^2\alpha \quad \text{— (III)}$$

$$mg - T = ma \quad \text{— (I)}$$

P.T

$$T + T' - mg = mA \quad \text{— (II)}$$

Pure rolling:



$$a_{b_2} = A_m - R\alpha$$

$$a_s = 0$$

$$A = R\alpha \quad \text{— (IV)}$$

$$a_{s_2} = a_{b_1}$$

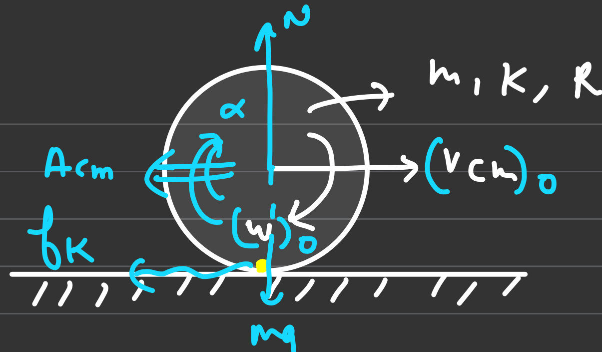
$$a = R\alpha + A_m \quad \text{— (V)}$$

No-slipping

H.W

"find Acceleration"

#



at $t = 0$ $\omega = \omega_0$ and $v_{cm} = (v_{cm})_0$

& $(v_{cm})_0 > R\omega_0$

find time after which it is going to do pure rolling?

Case I: if $(v_{cm})_0 > R(\omega)_0$

$$A_{cm} = \frac{\mu_K(\cancel{m}g)}{\cancel{m}}$$

$$A_{cm} = (\mu_K g) \quad \text{——— (1)}$$

$$f_K \times R = mK^2 \times \alpha$$

$$\mu_K \cancel{m}g \times R = \cancel{m}K^2 \times \alpha$$

$$\alpha = \frac{\mu_k g R}{k^2} \quad \text{— (ii)}$$

After time 't'

$$V_{cm} = (V_{cm})_0 + A_{cm} \times t$$

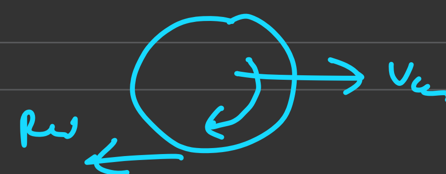
$$(V_{cm}) = (V_{cm})_0 - \mu_k g \times t \quad \text{— (i)}$$

$$\omega = \omega_0 + \alpha t$$

$$\omega = \omega_0 + \frac{\mu_k g R}{k^2} \times t \quad \text{— (ii)}$$

't'

$$V_{cm} - R\omega = 0$$



$V_c - R\omega = 0$

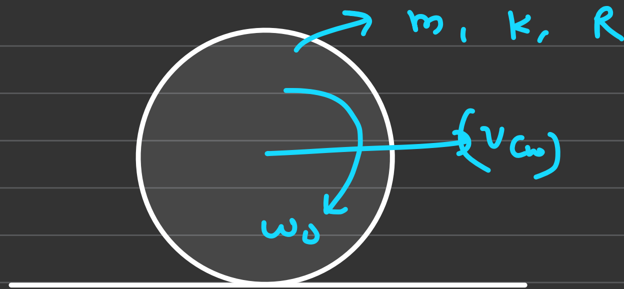
$$(V_{cm})_0 - \mu_k g t = R\left(\omega_0 + \frac{\mu_k g R}{K^2} t\right)$$

for pure rolling

$$\boxed{t = \frac{(V_{cm})_0 - R\omega_0}{\mu_k g + \mu_k g \frac{R^2}{K^2}}}$$

Case II:

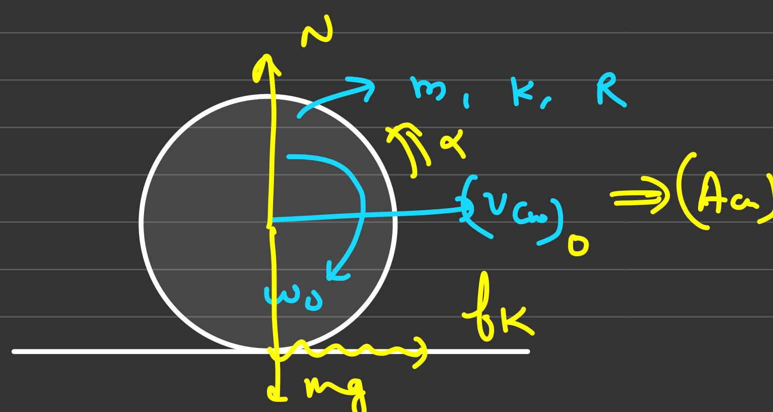
given



& given: $R\omega_0 > (v_{cm})_0$

then find time after which body is going to pure roll?

Solution:



$$\left\{ \begin{aligned} A_c &= \mu_k g \\ \alpha &= \frac{\mu_k g R}{k^2} \end{aligned} \right.$$

$A_c = \mu_k g \Rightarrow$ → This is going inc

$\alpha = \frac{\mu_k g R}{k^2}$ → This is going to reduce α

after time 't'

$$(V_c) = (V_c)_0 + \mu_k g \times t$$

$$\omega = \omega_0 - \frac{\mu_k g R}{k^2} \times t$$

$$(V_c)_0 + \mu_k g \times t = R\left(\omega_0 - \frac{\mu_k g t R}{k^2}\right)$$

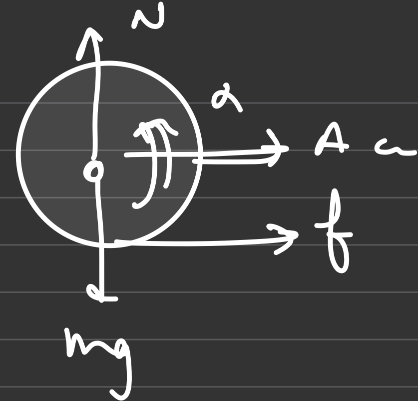
$$t = \frac{R\omega_0 - (V_c)_0}{\mu_k g + \frac{\mu_k R^2}{k^2}}$$

Case III :

if $(v_{cm})_0 = Rw_0$

then find value of frictional force acting on body?

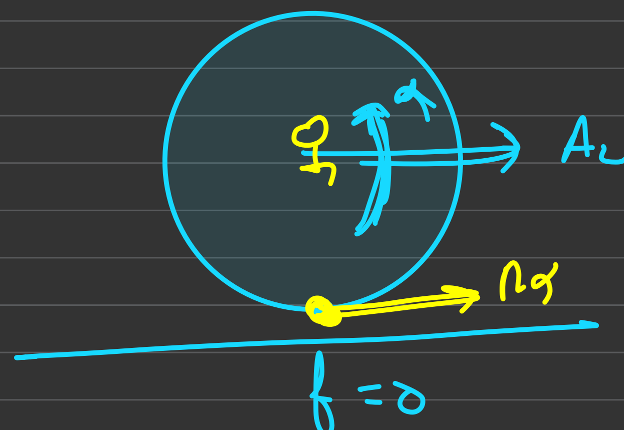
"Ye Sirf horizontal ke liye verify kiya hai"



$$A_c = \frac{f}{m} \quad \text{—(I)}$$

$$f \times R = m k^2 \times \alpha$$

$$\alpha = \frac{fR}{mk^2} \quad \text{—(II)}$$



"Condition for Pure rolling"

$$\underline{\underline{R\alpha + A_c = 0}}$$

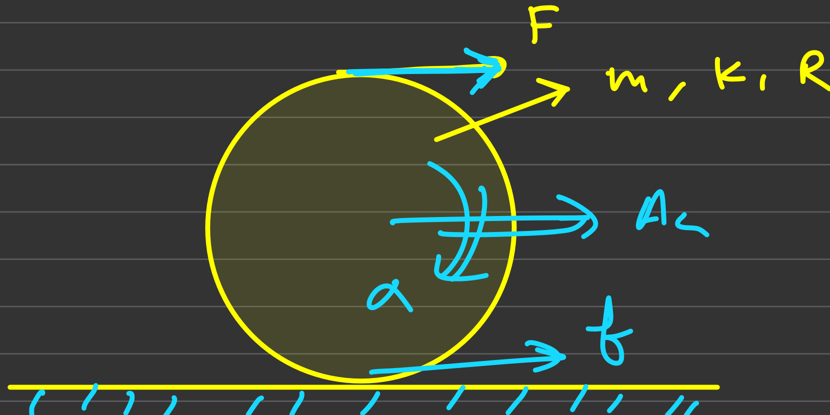
$$\Downarrow$$

$$\frac{f}{m} + \frac{R^2 f}{mk^2} = 0$$

$$f\left(\frac{1}{m} + \frac{R^2}{mk^2}\right) = 0$$

$f = 0$

Case IV : if it is doing pure rolling , find friction force ?



$$P.T \Rightarrow F + f = m A_c \quad \text{--- (I)}$$

$$P.R \Rightarrow F - f = \frac{m k^2}{R} \times \alpha \quad \text{--- (II)}$$

$$A a - R\alpha = 0 \quad \text{—— (III)}$$

$$F + \cancel{f} = m A a$$

$$F - \cancel{f} = m k^2 \left(\frac{A a}{R^2} \right)$$

$$2F = m A \left(1 + \frac{k^2}{R^2} \right)$$

$$A = \frac{2F}{m\left(1 + \frac{k^2}{R^2}\right)}$$

$$F + f = \frac{\cancel{u}}{\cancel{m}} \left[\frac{2F}{1 + k^2/n^2} \right]$$

$$f = \frac{2F}{1 + \frac{k^2}{n^2}} - F$$

$$f = F\left(\frac{2}{1 + \frac{k^2}{n^2}} - 1 \right)$$

$$f = F\left(\frac{2 - 1 - \frac{k^2}{n^2}}{1 + k^2/n^2} \right)$$

$$f = \frac{F\left(1 - \frac{K^2}{R^2}\right)}{1 + K^2/R^2}$$

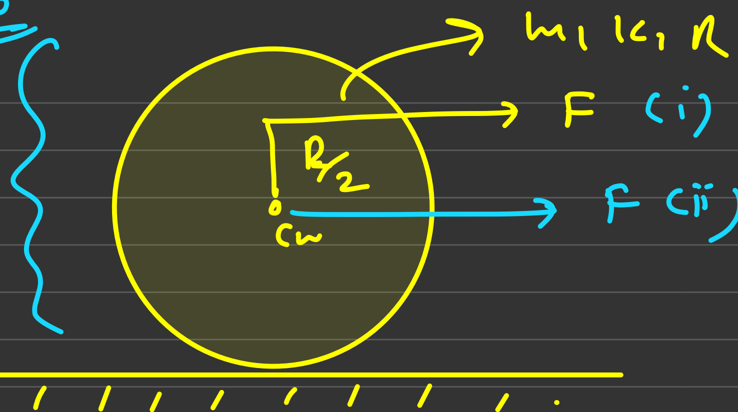
Ring: $K = R \Rightarrow f = 0$

disc: $K = \frac{R}{\sqrt{2}} \Rightarrow f = \frac{\frac{1}{2}F}{3/2} = +\left(F/3\right)$

Solid sph $K = \sqrt{\frac{2}{5}}R \Rightarrow f = +\left(\frac{3}{7}F\right)$

Case V : H.W

Pure rolling



find amount of frictional force (Direction)

(i) Ring

(ii) disc

(iii) Solid