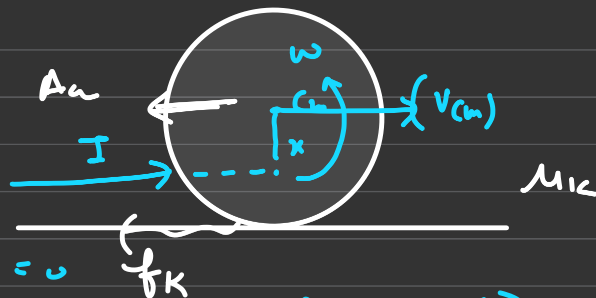
Rotation7

VMC Vidyamandir Classes SINCE 1986

IIT JEE | MEDICAL | FOUNDATION

Solution #

Solid sphere



find x for which solid sphere comes back?

$v_{cm} = 0$ and $\omega \neq 0$

$t = 0$

$$V_{cm} = \frac{I}{m} \quad \text{--- (I)}$$

$$Ix = \frac{2}{5} m R^2 \times \omega \Rightarrow \omega = \frac{5Ix}{2mR^2} \quad \text{--- (II)}$$

After time 't'

$$f_K \times R = \frac{2}{5} m R^2 \times \alpha$$

$$V_{cm}(t) = \frac{I}{m} - \frac{f_K}{m} \times t$$

$$\omega(t) = \frac{5Ix}{2mR^2} - \frac{5 f_K}{2mR} \times t$$

$$f_K = \mu_K m g$$

if this ball is going to come back then

$$V_{cm}(t) = 0 \Rightarrow \frac{I}{m} = \frac{f_k}{m} \times t$$

$$t = \frac{I}{f_k}$$

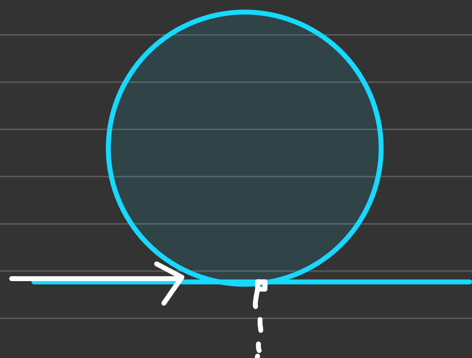
$$\omega(t) = \frac{\cancel{5}\cancel{I}x}{\cancel{2}m\cancel{R^2}} - \frac{\cancel{5}\cancel{f_k}}{\cancel{2}m\cancel{R}} \times \left[\frac{\cancel{I}}{\cancel{f_k}}\right] > 0$$

$$\frac{x}{R} - 1 > 0$$

$$\frac{x}{R} > 1$$

$$\boxed{x > R}$$

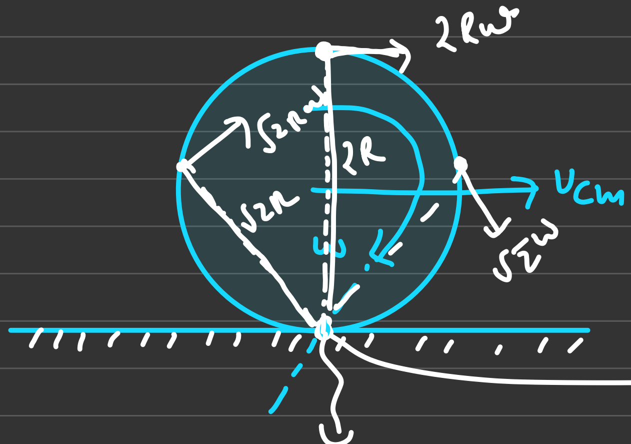
A



Instantaneous axis of rotation:

#

$V_{cm} - R\omega = 0$ Pure Rolling



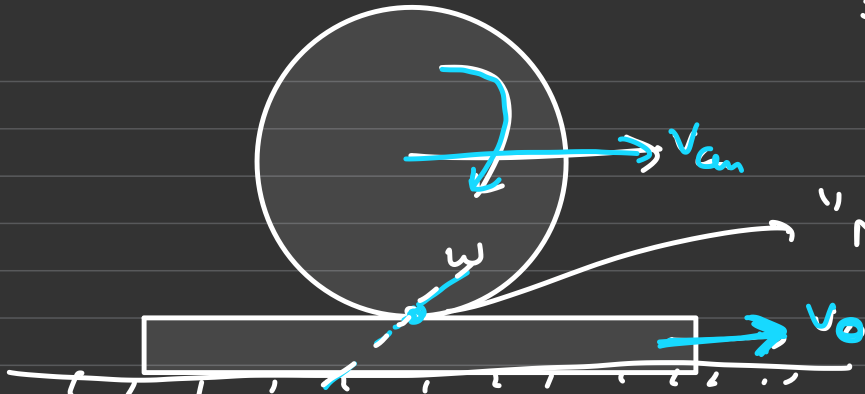
"Instantaneously at rest"

"Instantaneous axis of rotation"

we can analyse this situation like fixed axis of rotation

Valid only for Pure rolling

#



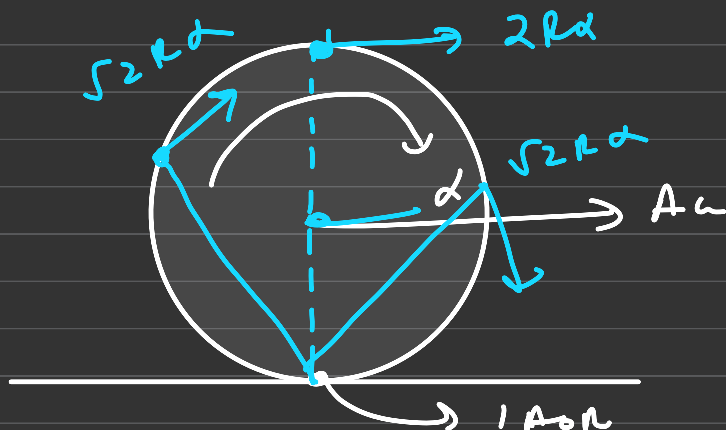
Pure rolling

IAOR should be at rest

"NOT IAOR

we can not assume FAOR at this instant"

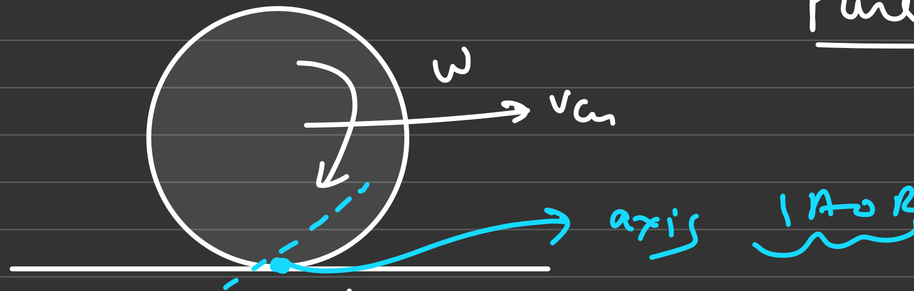
#



Pure rolling

Energy in IAOR:

Pure rolling



#

$$(KE)_{GRBM} = \frac{1}{2} M v_{cm}^2 + \frac{1}{2} I_{cm} \times \omega^2$$

$$= \frac{1}{2} M (R\omega)^2 + \frac{1}{2} MK^2 \times \omega^2$$

$$(KE)_{GRBM} = \frac{1}{2} M \omega^2 \left[R^2 + K^2 \right]$$

①

$$KE = \frac{1}{2} I_{FAOR} \times \omega^2$$

Using IAOR Concept!

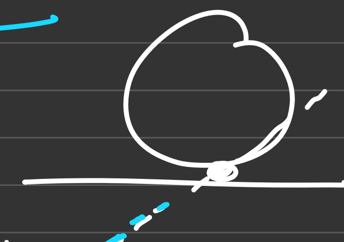
(2)

CRB

$\frac{1}{2} m v_{cm}^2 + \frac{1}{2} I_{cm} \omega^2$

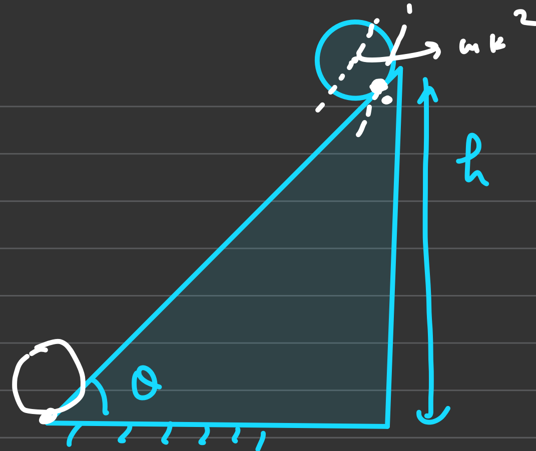
$$\Rightarrow (KE)_{body} = \frac{1}{2} I_{IAOR} \times \omega^2$$

$$= \frac{1}{2}(mk^2 + mR^2) \times \omega^2 \quad (3)$$



$$KE = \frac{1}{2} I_{IAOR} \times \omega^2$$

Q)



Pure rolling

IAOR method

Released from rest

$$mgh = \frac{1}{2} I_{IAOR} \times \omega^2 - 0$$

$$I_{IAOR} = mk^2 + mR^2$$

$$2\cancel{m}gh = (\cancel{m}k^2 + \cancel{m}R^2) \times \omega^2$$

$$\sqrt{\frac{2gh}{k^2+R^2}} = \omega$$

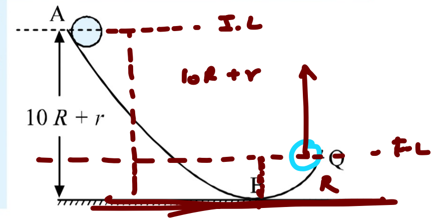
$v_{cm} - R\omega = 0$

$v_{cm} = ?$

Example - 4 *A solid sphere of radius r and mass m rolls without slipping down the track shown in the figure. At the end of its run at point Q its center-of-mass velocity is directed upward. The lower portion of track is circular of radius R.*

(a) *Determine the force with which the sphere presses against the track at B.*

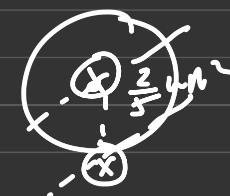
(b) *Upto what height does the CM rise after it leaves the track ?*



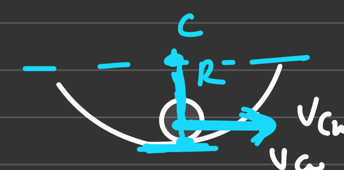
SOLUTION :

$$\Rightarrow \quad mg(10R) = \left\{ \frac{1}{2} I_{IAOR} \times \omega^2 \quad - 0 \right\}$$

$$mg\,10R = \frac{1}{2}\left\{ \left(\frac{7mr^2}{5} \right) \omega^2 \right\}$$

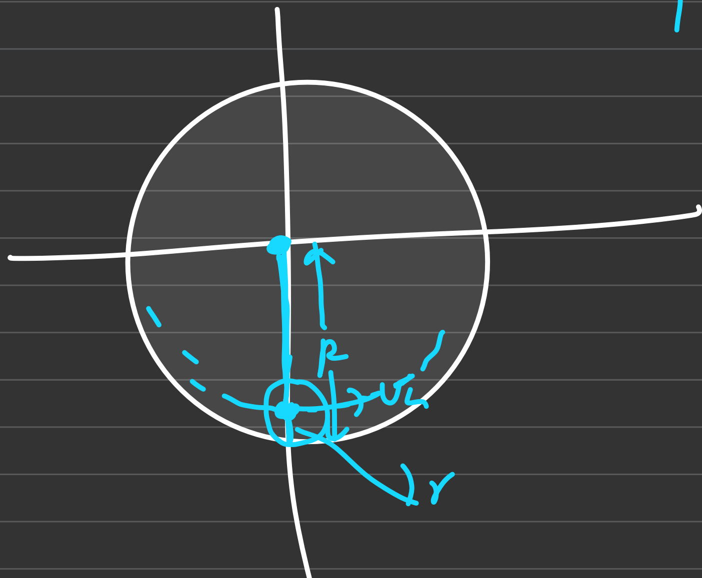


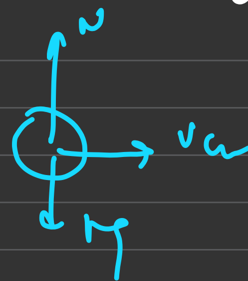
$$10\cancel{m}g\,\cancel{R} = \frac{7}{10}(mr^2)\,\omega^2$$



$$\omega = \sqrt{\frac{100\,gR}{7r^2}}$$

$$v_{cm} = r\omega = r\sqrt{\frac{100gR}{7r^2}}$$





$$v_a = \sqrt{\frac{100gR}{7}}$$

$$N - mg = \frac{m v_{cw}^2}{(R-r)}$$

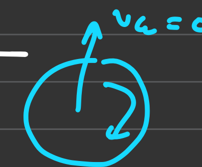
$$N - mg = \frac{m\left(\frac{100gR}{7}\right)}{R-r}$$

$$\boxed{N = mg + \frac{100mgR}{7(R-r)}}$$

b) $$mg(9R+r) = \left\{\frac{1}{2} I_{,\text{con}} \times w^2 - 0\right\}$$

$$mg(9R+r) = \frac{1}{2}\left[\frac{7}{5} m r^2 \times w^2\right]$$

$$w = \sqrt{\frac{10g(9R+r)}{7r^2}}$$



$$v_{cm} = r \times w$$

$$\cancel{v^2} = u^2 + 2gs$$

$$h = \frac{u^2}{2g}$$

$$v_{cm} = \sqrt{\frac{\cancel{r^2}\,10g(9R+r)}{7\cancel{r^2}}}$$

$$\underline{\underline{v_{cm}}} = \sqrt{\frac{10g(9R+r)}{7}}$$

$$h_{max} = \frac{10\cancel{g}(9R+r)}{7 \times 2\cancel{g}}$$

$$= \frac{5(9R+r)}{7}$$

$$h_{max} = \frac{45R}{7} + \frac{5r}{7} + R$$ Ans

w.r.t ground:

$$h_{max} = \frac{52R}{7} + \frac{5r}{7}$$ Ans

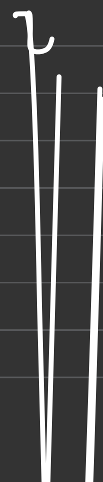
Dynamics (IAOR)

IAOR

$$\tau_{FAOR} = I_{FAOR} \times \alpha$$

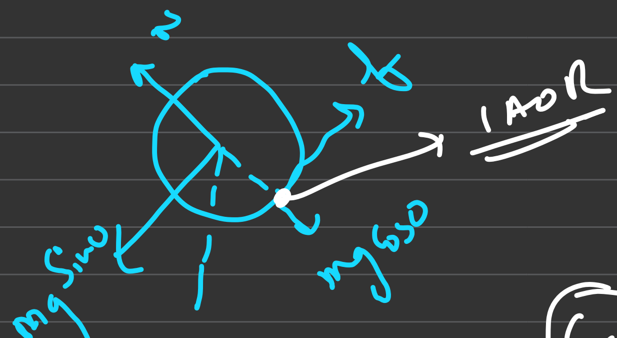
Sim. $\tau_{IAOR} = I_{IAOR} \times \alpha$

$I\alpha$ Bhi valid hai "



Q)

find acceleration at any instant using IAOR?



$$\Rightarrow \tau_{IAOR} = I_{IAOR} \times \alpha$$

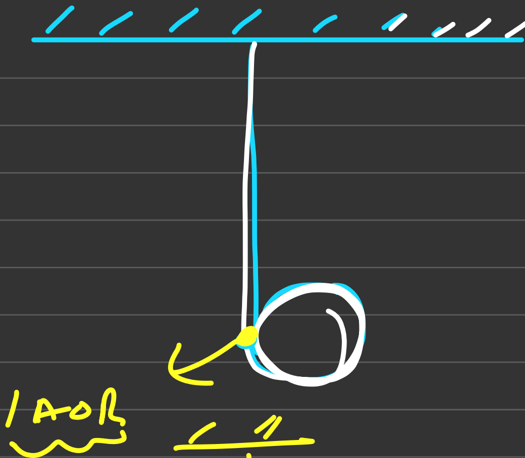
$$mg\sin\theta \times R = \frac{7}{5} mR^2 \alpha$$

$$\boxed{\alpha = \frac{5g\sin\theta}{7R}}$$

$$A_c - R\alpha = 0$$

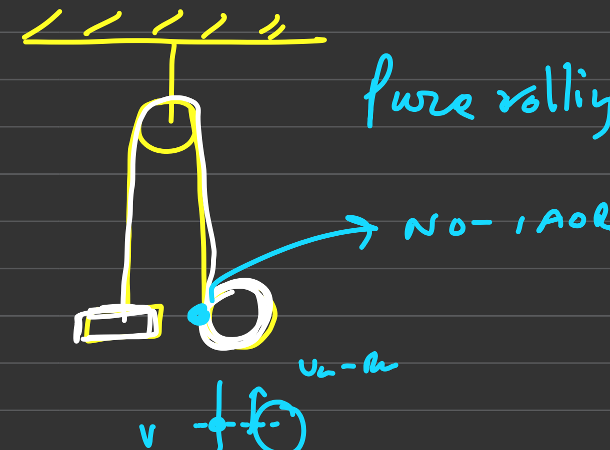
$$A_c = \frac{5g\sin\theta}{7}$$

Identify IAOR??

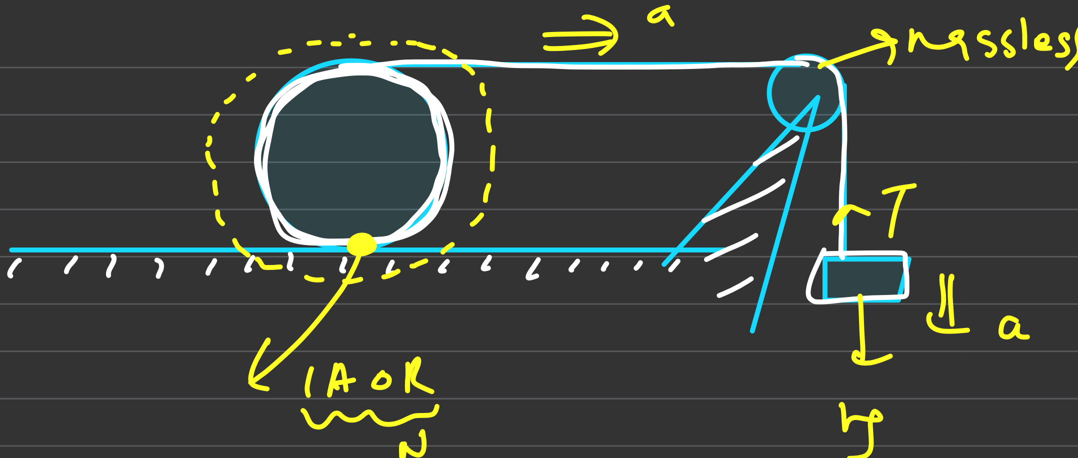


Released from rest

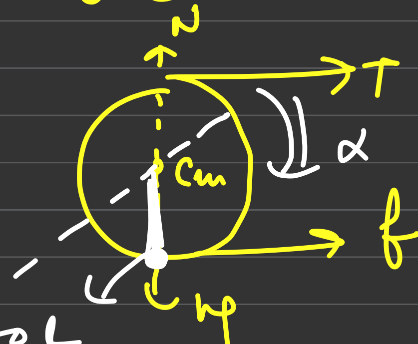
#



Q)

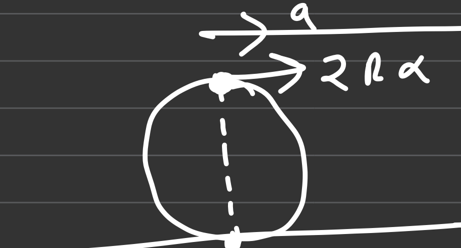


$mg - T = ma$ — (1)



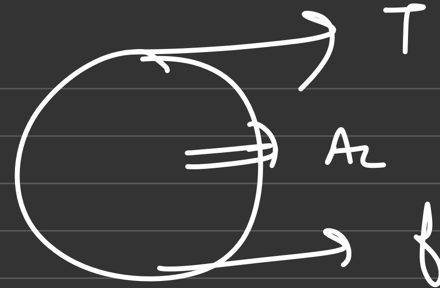
$\tau_{IAOR} = I_{IAOR} \times \alpha$

$T \times 2R = (mK^2 + mR^2) \times \alpha$ (3) — (II)



Pure rolling

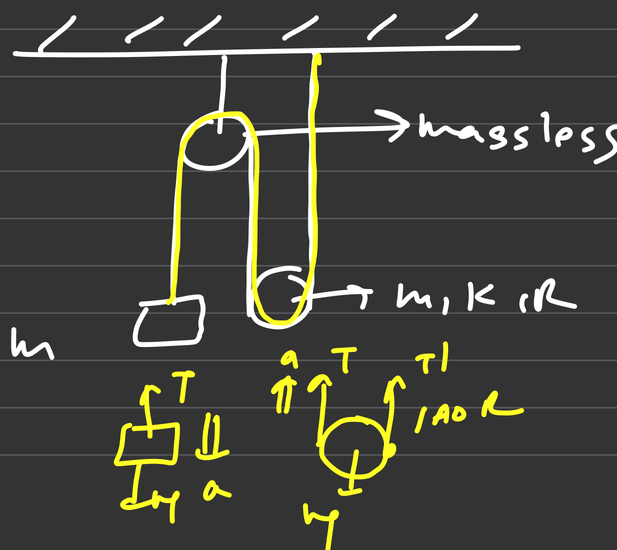
$2R\alpha = a$ — (III)



$$T + f = m A_z \quad \text{— (IV)}$$

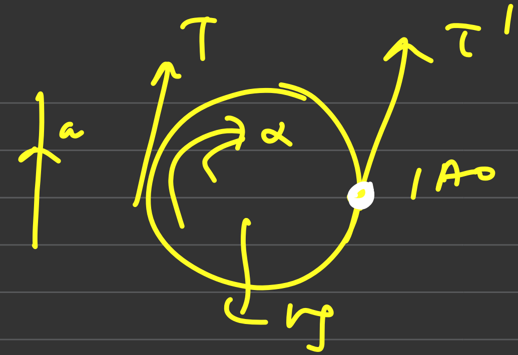
$$A_z = R\alpha \quad \text{(V)}$$

#



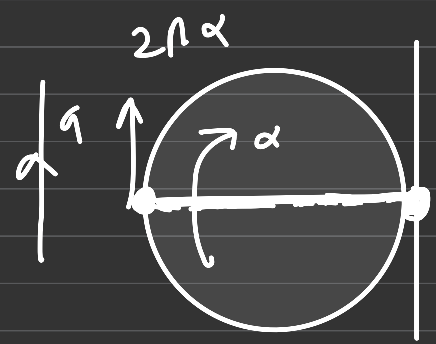
"Pure rolling"
Rolling without slipping

$mg - T = ma$ — ①



$\tau_{IAOR} = I_{IAOR} \times \alpha$

$T \times 2R - mgR = (mk^2 + mR^2)\alpha$

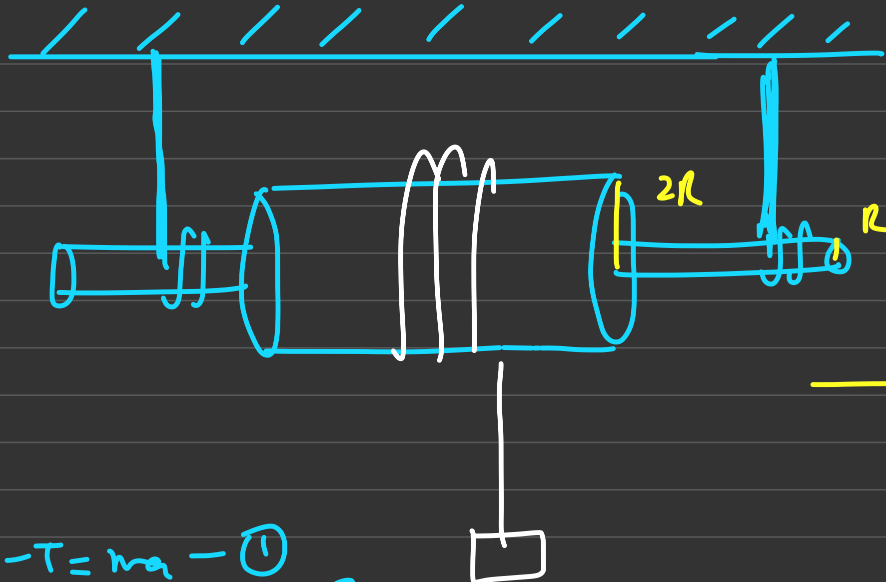


$2R\alpha = a$ — (III)

Q)

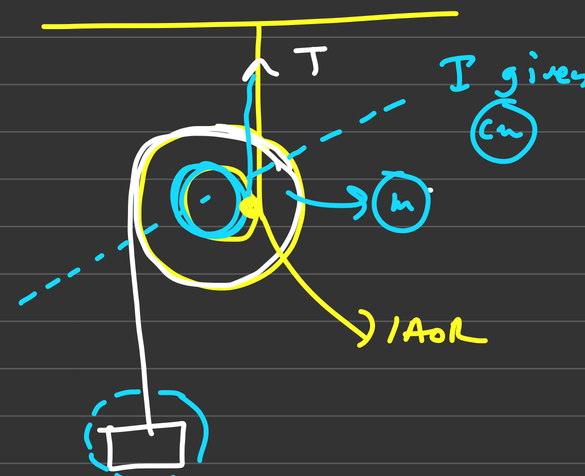
Release of from rest

Pure rolling



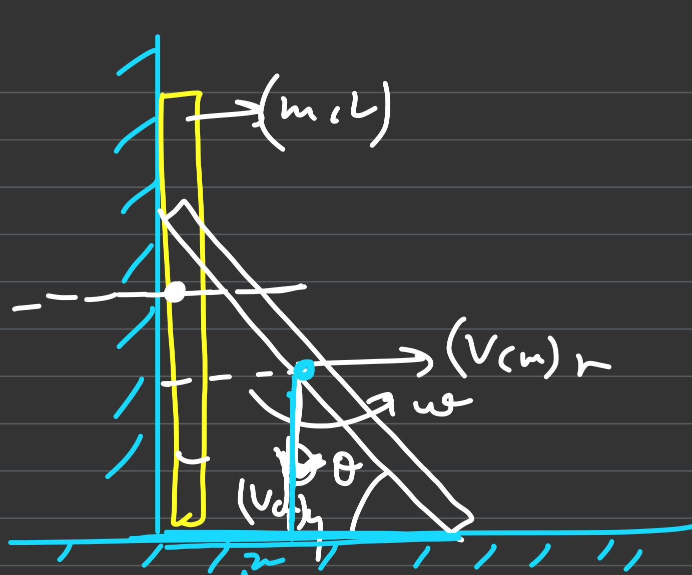
$$mg - T = ma \quad \text{— (1)}$$

$$a = 3R\alpha \quad \text{— (iii)}$$



$$T \times 3R + mg \times R = (I + mR^2)\alpha \quad \text{— (ii)}$$

θ)



" all the surfaces are smooth "

given [m, L, θ]

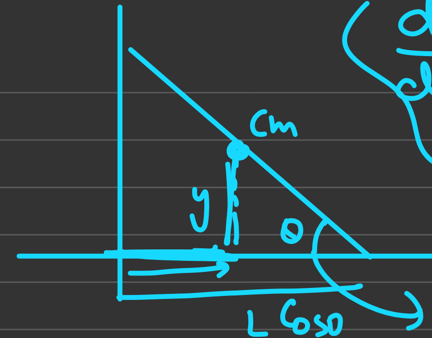
find ω, (V_{cm}) = ? at this θ

Energy Conservation:

$$mg\left(\frac{L}{2}\right) - mg\left(\frac{L}{2}\right)\sin\theta = \frac{1}{2} m V_{cm}^2 + \frac{1}{2} I_{cm} \times \omega^2 \quad (1)$$

$$x_{cm} = \frac{L}{2}\cos\theta$$

$$y_{cm} = \frac{L}{2}\sin\theta$$



$\frac{L}{2}\cos\theta$

$$\frac{d\theta}{dt} = \text{Sign}$$

$\omega \uparrow \quad \theta \downarrow$

$$\frac{d\theta}{dt} = -\omega$$

$\omega \uparrow \quad \theta \uparrow$

$$\frac{d\theta}{dt} = +\omega$$

$$(V_{cm})_x = \frac{L}{2}(-\sin\theta)\cdot\frac{d\theta}{dt}$$

$$(V_{cm})_y = \frac{L}{2}\cos\theta\cdot\frac{d\theta}{dt}$$

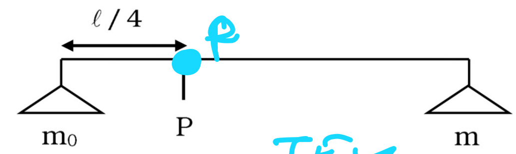
$$(V_{cm})_x = \frac{L}{2}(+\sin\theta)\cdot\omega$$

$$(V_{cm})_y = -\frac{L}{2}\cos\theta\,\omega$$

$$(V_{cm})_{net} = \sqrt{(V_{cm})_x^2 + (V_{cm})_y^2} = \sqrt{\left(\frac{L}{2}\sin\theta\cdot\omega\right)^2 + \left(\frac{L}{2}\cos\theta\right)^2}$$

$$V_{cm} = \frac{L}{2}\omega \qquad \text{(1)}$$

17. A light rod of length ℓ is pivoted at distance $\ell / 4$ from the left end and has two masses m_0 and m attached to its ends such that rod is in equilibrium. Find m in terms of m_0.



(A) $\frac{m_0}{2}$ (B) $\frac{m_0}{3}$ (C) $2m_0$ (D) m_0

TE →
RE →

Rotation
Translation

Equilibrium $\begin{cases} A_{cm} = 0 \\ \alpha = 0 \end{cases}$

if only $\alpha = 0$ then that body is in Rotation Eq/b

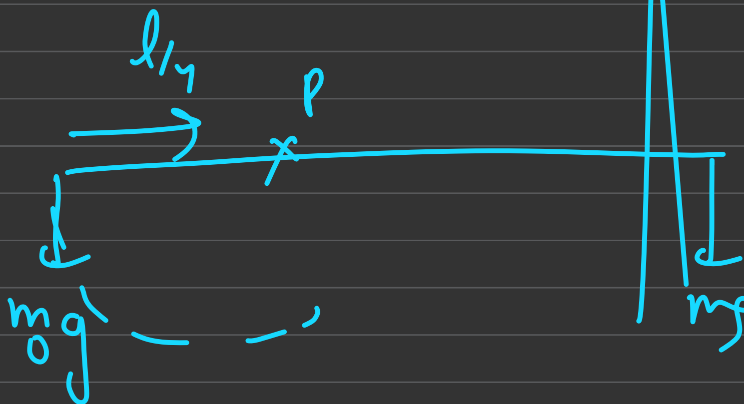
if only Acceleration of all points are zero then is Translational Eqb if any body

TE ✓ RE ✓

$\tau_{\text{about any point}} = 0$

TE ✗ Rot Eq ✓

$\tau_{cm} = 0$



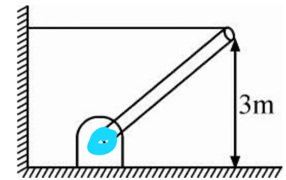
$\tau_P = 0$

$\cancel{m_0 g} \times \frac{\cancel{l}}{4} - m\cancel{g} \times \frac{3\cancel{l}}{4} = 0$

A.C.W C.W

$m_0 = 3m$

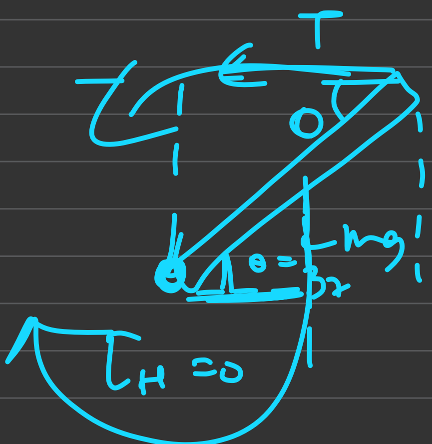
18. A uniform rod of mass 15 kg and length 5 m is pivoted at one end and is held stationary with the help of a light string as shown in the figure. The tension in the string is:



(A) 150 N **(B)** 255 N

(C) 100 N **(D)** None of the above

$\left\{ \begin{array}{l} TE \leftarrow \\ RE \circlearrowleft \end{array} \right.$



$\sin\theta = \frac{3}{5} = 37$

$mg \times \frac{5}{2}(\sin 37) - T(5 \sin 37) = 0$

$\cancel{150}^{75} \times \frac{\cancel{5}}{\cancel{2}} \times \frac{\cancel{4}^{2}}{\cancel{5}} - T \times 5 \times \frac{3}{5} =$

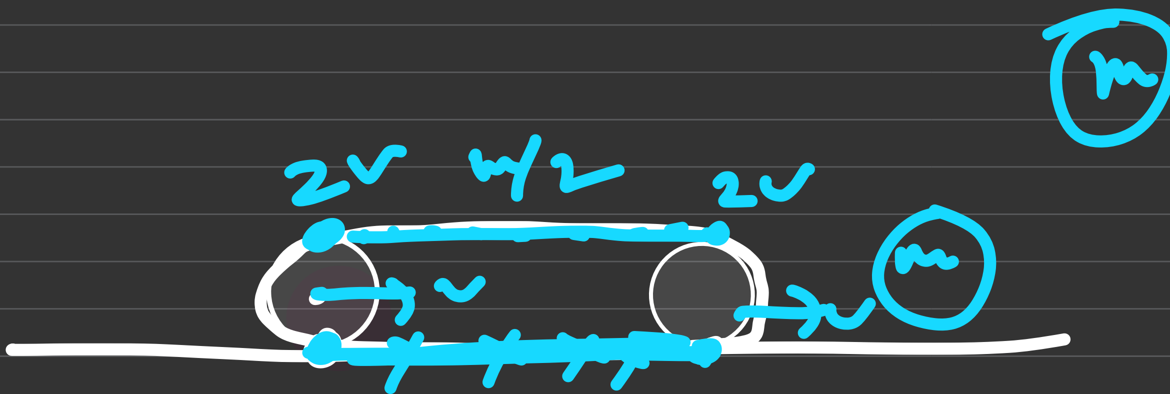
$$150 \times \frac{\cancel{5}}{\cancel{2}} \times \frac{\cancel{4}^{2}}{\cancel{5}} - T \times 5 \times \frac{3}{5} = 0$$

$$300 - 3 \times T = 0$$

$$\underline{\underline{T = 100\ N}}$$

Complete work book & module

Except { Toppling }



$$= \frac{1}{2}\left(\frac{m}{2}\right)\cdot(2v)^2$$

$$= mv^2$$