Rotation-6

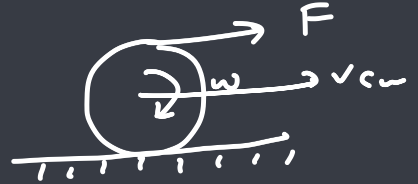
1

Ring, disc, Solid sphere

friction is present

$f = 0$

2

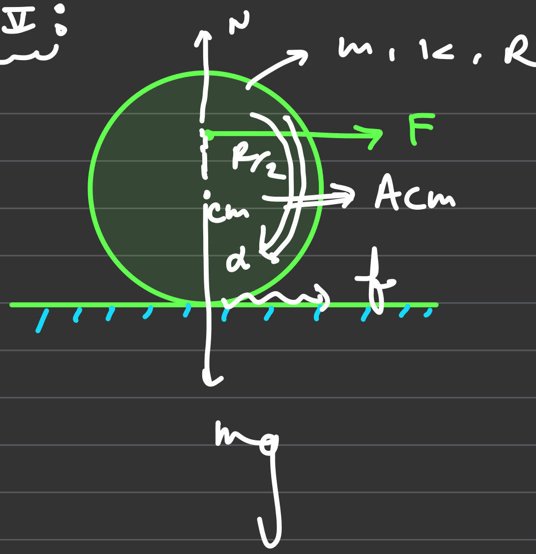


Ring = $f = 0$

disc = Right

solid = Right

Case V :



Pure rolling

$N = mg$

$$\begin{cases} F + f = m A_{cm} & \text{— (I)} \\ F \times \frac{R}{2} - f \times R = m k^2 \times \alpha & \text{— (II)} \\ A_{cm} - R\alpha = 0 & \text{— (III)} \end{cases}$$

$$\frac{3F}{2} = m A_{cm} + \frac{m k^2}{R^2} A_{cm}$$

$$A_{cm} = \frac{3F}{2m\left(1 + \frac{k^2}{R^2}\right)}$$

$$F + f = m A_{cm}$$

$$f = \cancel{m}\left[\frac{3F}{2\cancel{m}\left(1+\frac{k^2}{R^2}\right)} - F\right]$$

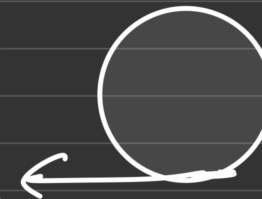
$$f = \frac{F\left(3 - 2\left(1+\frac{k^2}{R^2}\right)\right)}{2\left(1+k^2/R^2\right)}$$

$$f = \frac{F\left[3 - 2 - \frac{2k^2}{R^2}\right]}{2\left(1+k^2/R^2\right)}$$

$$f = \frac{F\left[1 - \frac{2k^2}{R^2}\right]}{2\left(1+k^2/R^2\right)}$$

Ring $k = R$

$$f = \frac{F(1-2)}{2(1+1)} = -\left(\frac{F}{4}\right)$$

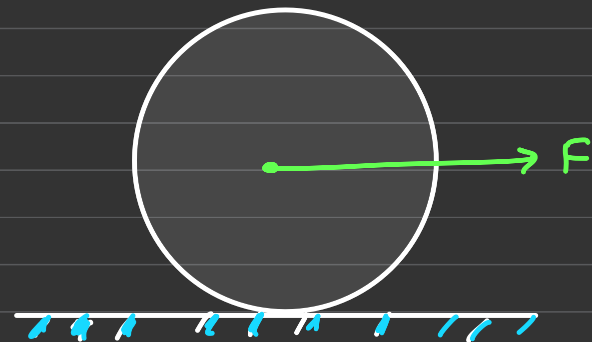


$f =$

disc $K = \frac{R}{\sqrt{2}}$ $f = 0$ 0

Solid sphere $K = \sqrt{\frac{2}{5}}R$ $f = +ve$ $\longrightarrow$

Case (VI):



"Pure rolling"

find amount and direction of frictional force

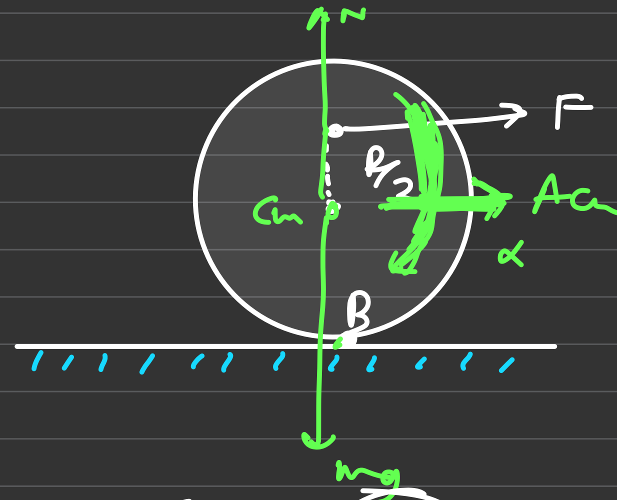
if a) Ring

b) disc

c) Solid sphere

Direct method to get direction of frictional force in pure rolling

Solid sphere



① assume no friction & then solve for Acm and α

$$A_{cm} = \frac{F}{m} \quad \text{— (I)}$$

$$F \times \frac{R}{2} = \frac{2}{5} m R^2 \times \alpha$$

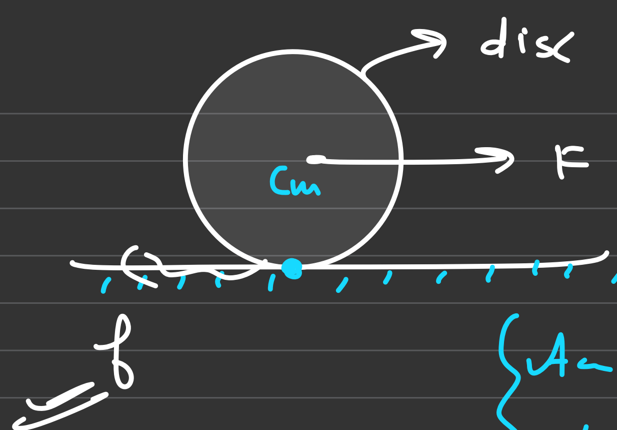
$$\alpha = \frac{5F}{4mR} \quad \text{— (II)}$$

$$a_B = A_{cm} - R\alpha$$

$$= \frac{F}{m} - R\left[\frac{5F}{4mR}\right]$$

friction# Rightward in case of "S.P"

Q)



Pure rolling

(Static nature)

① assume No friction

$$\begin{cases} a_{cm} = \dfrac{F}{m} \\ \alpha = 0 \end{cases}$$

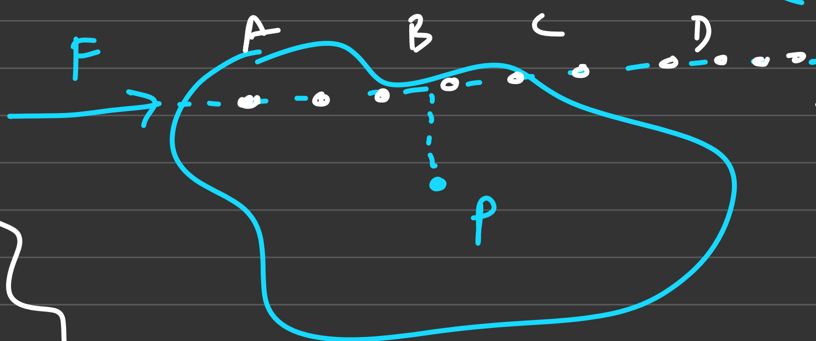
Law of Conservation of Angular momentum:

$$\frac{d}{dt}(L_P) = (\tau_{ext})_P$$

if $(\tau_{ext})_P = 0$ then

$L_P = \text{const}$

$(\tau_P)_F \neq 0$



$\tau_A = \tau_B = \tau_C$

$\tau_D = 0$

if $F_x = F_y = F_z = 0$

then $P_x = P_y = P_z = c$

Normal case

in case of collision

if $J_x = J_y = J_z = 0$

then $P_x = P_y = P_z$

$= \text{const}$

Q)

A string is doing circular motion over table top connected with string as shown.

"then find speed of bob when radius of circle becomes $r/2$"

as $(\tau_0)_{ext} = 0$ hence $L_0 = \text{const}$ over time

$$\underbrace{m v r}_{\text{before}} = \underbrace{m(v') \times r/2}_{\text{after}}$$

$$\boxed{v' = 2v}$$

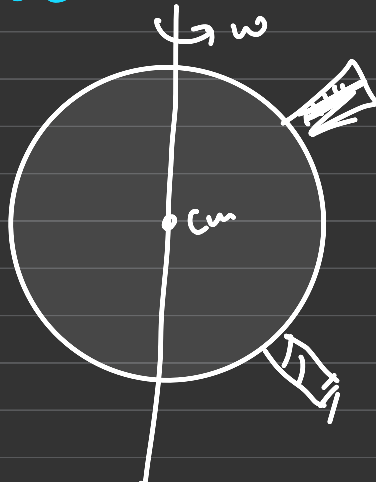
Ans

Real life example:

as $(\tau_{cm}) = 0$ of earth.

$L_{cm} = \text{Const} = I_{en} \times \omega$

(i)

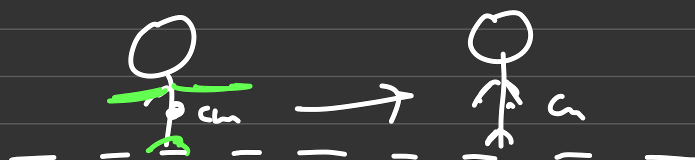


$I_{en}\,\omega = I_{en}' \times \omega'$

$I_{en}' > I_{en}$

then $\omega > \omega'$

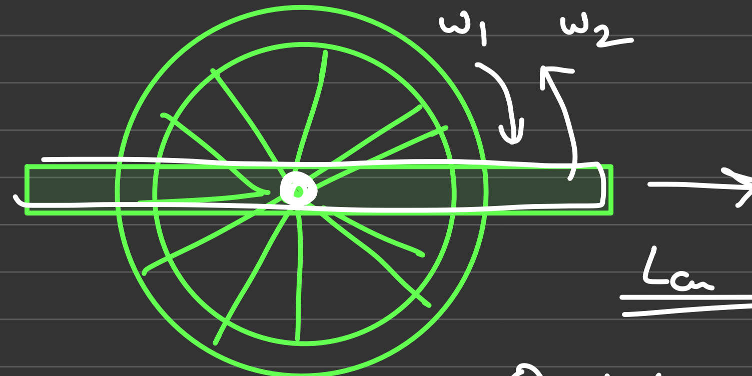
(ii)



$I_1 \omega_1 = I_2 \omega_2$

$I_2 < I_1 \Rightarrow \omega_2 > \omega_1$

(iii) Space-ship :

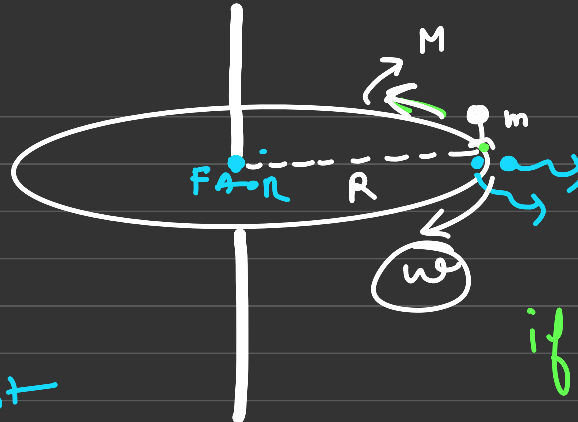


$$\tau_{cm} = 0$$

$$L_{cm} = \text{const}$$

$$0 + 0 = I_1 \omega_1 - I_2 \omega_2$$

8)



disc is free to rotate
initially system is at rest

$\tau_{FAOR} = 0$

$L_{FAOR} = $ const

if man start walking with const speed v_0 w.r.t ground then find time after which man (m anan) reaches the same point where he started from

along circumference

(a) w.r.t. disc

(b) w.r.t ground

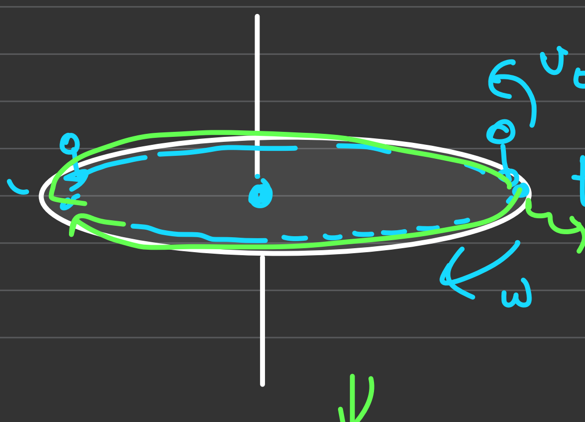
Solution: as $\tau_{FAoR} = 0 \Rightarrow L_{FAoR} = \text{const}$

w.r.t ground

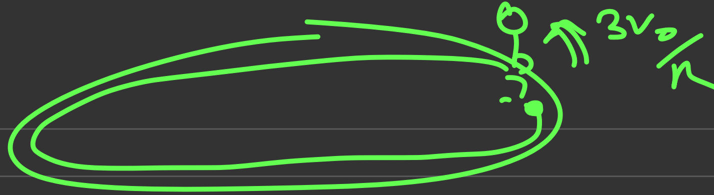
$$\underset{\text{man}}{0} + \underset{\text{(disc)}}{0} = \underset{\text{A.C.W}}{(m v_0 R)} - \underset{\text{C.W}}{I_{cm} \times \omega}$$

$$0 = m v_0 R - \frac{mR^2}{2} \times \omega$$

$$\omega = \left(\frac{2V_0}{R}\right)$$



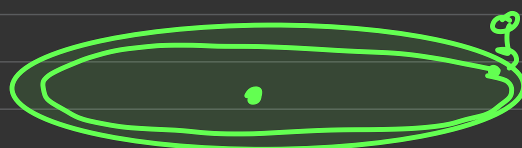
(i) w.r.t disc



$$\omega_{man,\,disc} = \left(\frac{v_0}{R} + \frac{2v_0}{R}\right)$$

$$= \left(\frac{3v_0}{R}\right)$$

$$t = \frac{2\pi}{(3v_0/R)} = \left(\frac{2\pi R}{3v_0}\right) \text{ Ans}$$



Time to reach top point w.r.t disc

b) w.r.t ground!

$$t = \frac{2\pi}{(v_0/R)} = \left(\frac{2\pi R}{v_0}\right) \text{ Ans}$$

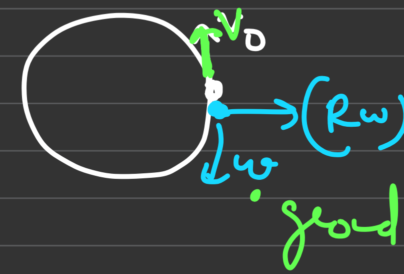
Q 2) in above Question:

if velocity of man is v_0 w.r.t disc then find time to reach same point

a) w.r.t disc

b) w.r.t ground

$$t = \left(\frac{2\pi R}{v_0}\right)$$



$$\# \quad 0 + 0 = \cancel{m}\left(v_0 - R\omega\right)\cancel{R} - \frac{\cancel{m}\cancel{R^2}}{2} \times \omega$$

$$\# \quad 0 = m v_0 R - m R^2 \omega - \frac{m R^2}{2} \times \omega$$

$$v_{m,d} = v_m - v_d$$

$$+v_0 = v_m - (-R\omega)$$

$$v_m = \underline{v_0 - R\omega}$$

$$v_0 - R\omega = \left(\frac{R\omega}{2}\right)$$

$$v_0 = R\omega + \frac{R\omega}{2} \Rightarrow \left(\frac{3R\omega}{2}\right)$$

$$\omega = \left(\frac{2v_0}{3r}\right)$$

$$v_{m,g} = v_0 - \cancel{R}\left(\frac{2v_0}{3\cancel{r}}\right) = \left(\frac{v_0}{3}\right)$$

$$t = \left(\frac{2\pi(l)\,3}{v_0}\right) \quad \underline{\underline{\text{Ans}}}$$

Collision:

linear

$$\Delta t \to 0$$

$$I = \int F\,dt = \Delta p$$

Angular

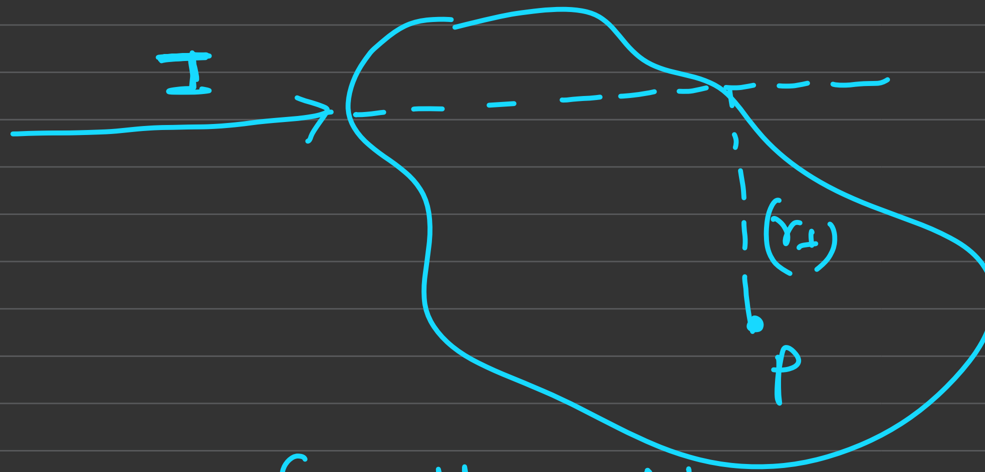
$$\Delta t \to 0$$

Remember

$$J_p = \int \tau_p \cdot dt = \Delta L_p$$

Angular

$$J_p = \int \frac{dL_p}{\cancel{dt}} \cdot \cancel{dt} = \int dL_p = \Delta L_p$$



$J_P = \int \tau_P \cdot dt = \Delta L_P$

$J_P = \int (r_\perp F\, dt) = \Delta L_P$

$J_P = r_\perp \left(\int F\, dt\right) = \Delta L_P = $

$J_P = r_\perp I = \Delta L_P$

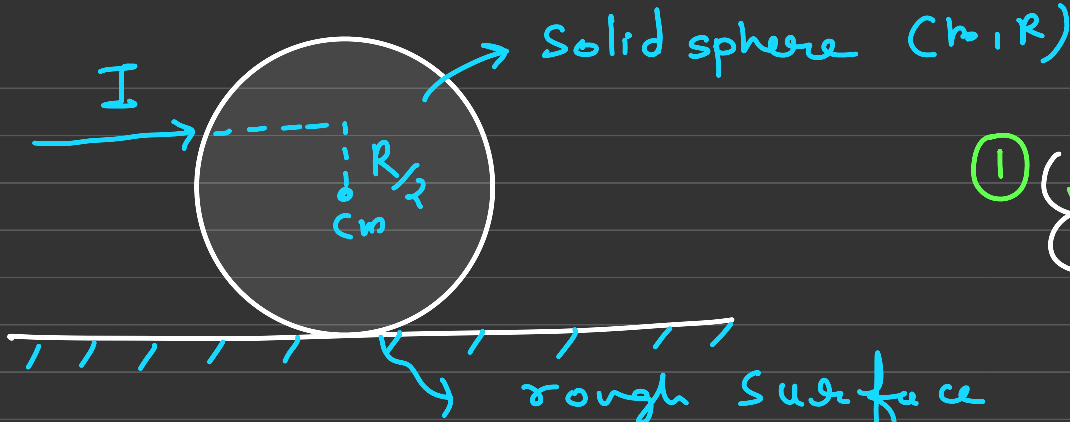
Angular Impulse theorem
Angular momentum

$$J_p = \underline{\underline{r_\perp I}} = \int \tau_p \cdot dt = \underline{\underline{\Delta L_p}}$$

"from p on line of impulse"

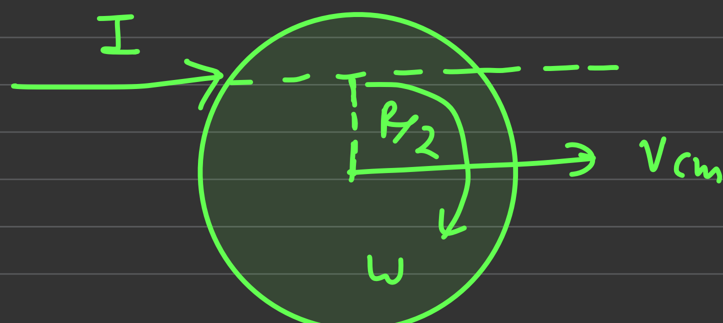
—

Q)



① Just After giving Impulse find v_{cm} of body and ω of body?

"Body is going to perform GRBM"



$$I = (m v_{cm} - 0)$$

$$v_{cm} = \frac{I}{m} \quad \text{--- (1)}$$

"$\perp$ from point to line of impulse"

$$J_{cm} = \underline{r_\perp I} = \int \tau_{cm} \cdot dt = \underline{\underline{\Delta L_P}}$$

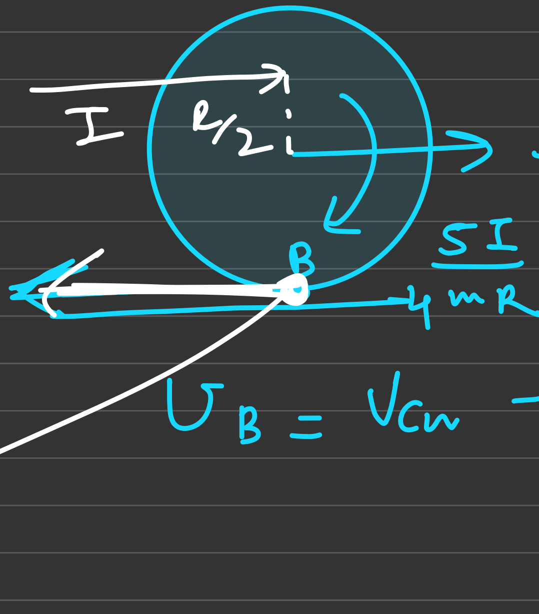
$$r_\perp I = \Delta L_P$$

$$\frac{R}{2} I = (I_{cm} \times \omega - 0)$$

$$\omega = \frac{\frac{RI}{2}5}{2mR^2}$$

$$\omega = \left(\frac{5I}{4mR}\right) \quad \text{--- (II)}$$

(2)



$$\frac{I}{m} = v_{cm}$$

$$\frac{5I}{4mR} = \omega$$

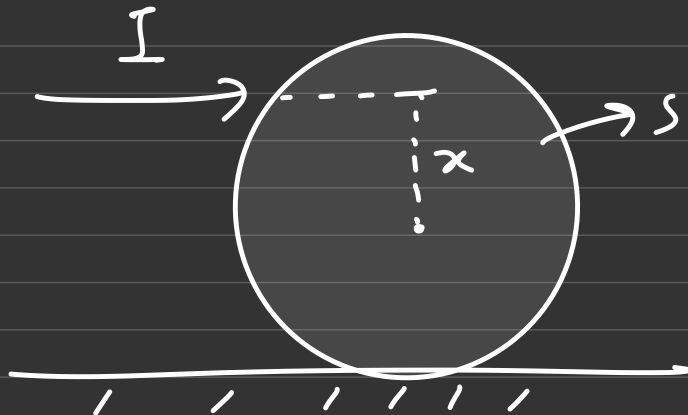
$$v_B = v_{cm} - R\omega = \frac{I}{m} - R\left(\frac{5I}{4mR}\right)$$

$$= I/m - \frac{5I}{4m}$$

$$= I/m - 5I/4m = \frac{-I}{4m}$$

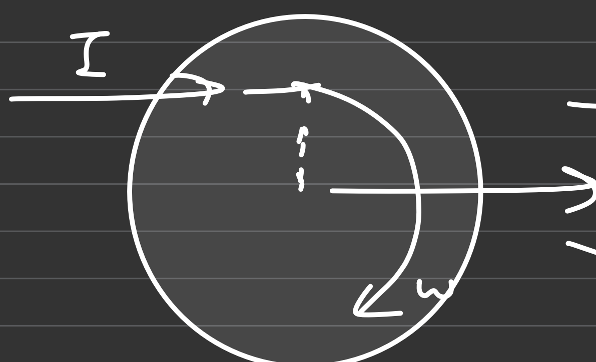
This point is slipping just after given the impulse.

(3)



Solid sphere of m, R

find x for which just after giving the impulse Solid sphere will pure roll.



$$V_{cm} = \frac{I}{m} \quad \text{(I)}$$

$$Ix = \frac{2}{5} m R^2 \omega \quad \text{(II)}$$

$$\omega = \frac{5Ix}{2mR^2}$$

$$V_a - R\omega = 0$$

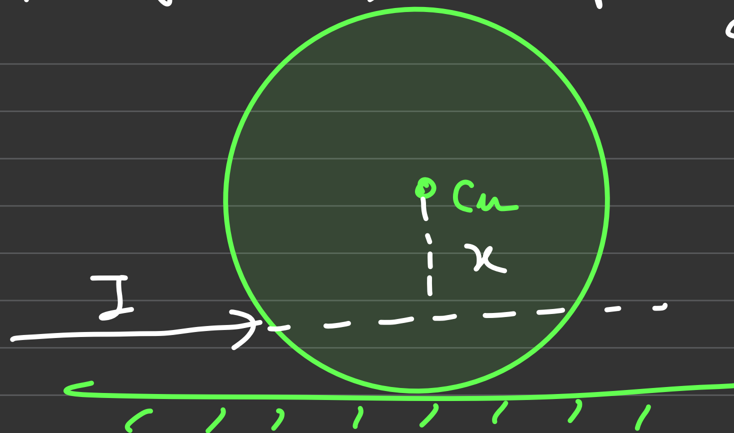
$$\frac{\cancel{I}}{m} - \cancel{R}\left(\frac{5\cancel{I}x}{2mR^{\cancel{2}}}\right) = 0$$

$$1 - \frac{5x}{2R} = 0$$

$$x = \frac{2R}{5}$$

if $x = \frac{2R}{5}$ then it is definitely going to pure roll.

Q) Just After giving the impulse



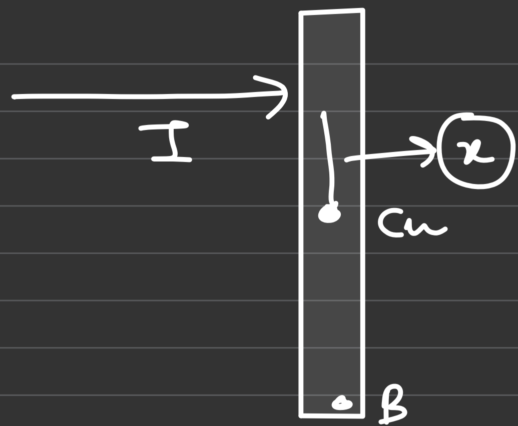
a) find x for which

$V_{cm} > R\omega$

b) find x for which

$V_{cm} < R\omega$

Q



"kept on horizontal smooth surface"

"find x for which velocity of B = 0"

{ DTS # 4 → Level 1, Level 2 }