Functions

Date Planned : __ / __ / __	CBSE Pattern
Actual Date of Attempt : __ / __ / __	Level – 0

1. Find the domain of each of the following functions given by :

(i) $f(x)=\dfrac{1}{\sqrt{1-\cos x}}$ **(ii)** $f(x)=\dfrac{1}{\sqrt{x+|x|}}$ **(iii)** $f(x)=x|x|$

(iv) $f(x)=\dfrac{x^3-x+3}{x^2-1}$ **(v)** $f(x)=\dfrac{3x}{28-x}$

2. Find the range of the following functions given by :

(i) $f(x)=\dfrac{3}{2-x^2}$ **(ii)** $f(x)=1-|x-2|$ **(iii)** $f(x)=|x-3|$ **(iv)** $f(x)=1+3\cos 2x$

3. Redefine the function $f(x)=|x-2|+|2+x|, -3\le x\le 3$.

4. If $f(x)=\dfrac{x-1}{x+1}$, $\forall\, x\in R-\{0,\pm 1\}$ then show that :

(i) $f\left(\dfrac{1}{x}\right)=-f(x)$ **(ii)** $f\left(-\dfrac{1}{x}\right)=\dfrac{-1}{f(x)}$

5. If $f(x)=\sqrt{x}$ and $g(x)=x$ be two functions defined in the domain $R^+\cup\{0\}$, then find the value of :

(i) $(f+g)(x)$ **(ii)** $(f-g)(x)$ **(iii)** $(fg)(x)$ **(iv)** $\left(\dfrac{f}{g}\right)(x)$

6. Find the domain and range of the function $f(x)=\dfrac{1}{\sqrt{x-5}}$.

7. If $f(x)=y=\dfrac{\alpha x-b}{cx-a}$, $\forall\, x\in R-\left\{\dfrac{a}{c}\right\}$ & $a^2\ne bc$ then prove that $f(y)=x$.

Choose the correct alternative. Only one choice is correct.

8. Let $n(A)=m$ and $n(B)=n$. Then, the total number of non-empty relations that can be defined from A to B is :

(A) m^n **(B)** n^m-1 **(C)** $mn-1$ **(D)** $2^{mn}-1$

9. If $[x]^2-5[x]+6=0$, where [.] denotes the greatest integer function, then :

(A) $x\in[3,4]$ **(B)** $x\in(2,3]$ **(C)** $x\in[2,3]$ **(D)** $x\in[2,4)$

10. Range of $f(x)=\dfrac{1}{1-2\cos x}$ is :

(A) $\left[\dfrac{1}{3},1\right]$ **(B)** $\left[-1,\dfrac{1}{3}\right]$ **(C)** $(-\infty,-1]\cup\left[\dfrac{1}{3},\infty\right)$ **(D)** $\left[-\dfrac{1}{3},1\right]$

11. Let $f(x)=\sqrt{1+x^2}$, then :

(A) $f(xy)=f(x)\,.\,f(y)$ **(B)** $f(xy)\ge f(x)\,.\,f(y)$

(C) $f(xy)\le f(x)\,.\,f(y)$ **(D)** None of these

12. Domain of $\sqrt{a^2 - x^2}\ (a > 0)$ is :

(A) $(-a, a)$ **(B)** $[-a, a]$ **(C)** $[0, a]$ **(D)** $(-a, 0]$

13. If $f(x) = ax + b$, where a and b are integers, $f(-1) = -5$ and $f(3) = 3$, then a and b are equal to :

(A) $a = -3, b = -1$ **(B)** $a = 2, b = -3$ **(C)** $a = 0, b = 2$ **(D)** $a = 2, b = 3$

14. The domain of the function f defined by $f(x) = \sqrt{4-x} + \dfrac{1}{\sqrt{x^2 - 1}}$ is equal to :

(A) $(-\infty, 1) \cup (1, 4]$ **(B)** $(-\infty, -1] \cup (1, 4]$ **(C)** $(-\infty, -1) \cup [1, 4]$ **(D)** $(-\infty, -1) \cup [1, 4)$

15. The domain and range of the real function f defined by $f(x) = \dfrac{4-x}{x-4}$ is given by :

(A) Domain = R, Range $= \{-1, 1\}$ **(B)** Domain = R, Range = R

(C) Domain = $R - \{4\}$, Range $R - \{-1\}$ **(D)** Domain = $R - \{4\}$, Range $= \{-1, 1\}$

16. The domain and range of real function f defined by $f(x) = \sqrt{x-1}$ is given by :

(A) Domain = $(1, \infty)$, Range = $(0, \infty)$ **(B)** Domain = $[1, \infty)$, Range = $(0, \infty)$

(C) Domain = $(1, \infty)$, Range = $[0, \infty)$ **(D)** Domain = $[1, \infty)$, Range = $[0, \infty)$

17. The domain of the function f given by $f(x) = \dfrac{x^2 + 2x + 1}{x^2 - x - 6}$.

(A) $R - \{3, -2\}$ **(B)** $R - \{-3, 2\}$ **(C)** $R - [3, -2]$ **(D)** $R - (3, -2)$

18. The domain and range of the function f given by $f(x) = 2 - |x - 5|$ is :

(A) Domain = R^+, Range = $(-\infty, 1]$ **(B)** Domain = R, Range $= [-\infty, 2]$

(C) Domain = R, Range = $(-\infty, 2)$ **(D)** Domain = R^+, Range = $(-\infty, 2]$

19. The domain for which the functions defined by $f(x) = 3x^2 - 1$ and $g(x) = 3 + x$ are equal to :

(A) $\left[-1, \dfrac{4}{3}\right]$ **(B)** $\left[1, \dfrac{4}{3}\right]$ **(C)** $\left[-1, -\dfrac{4}{3}\right]$ **(D)** $\left[-2, -\dfrac{4}{3}\right]$

20. Let f and g be two real functions given by :
$f = \{(0, 1), (2, 0), (3, -4), (4, 2), (5, 1)\}$ and $g = \{(1, 0), (2, 2), (3, -1), (4, 4), (5, 3)\}$, then the domain of $f \cdot g$ is given by _______.

21. Let $f = \{(2, 4), (5, 6), (8, -1), (10, -3)\}$ and $g = \{(2, 5), (7, 1), (8, 4), (10, 13), (11, 5)\}$ be two-real functions. Then, match the following:

Column - 1		**Column - 2**	
(i)	$f - g$	**(a)**	$\left\{\left(2, \dfrac{4}{5}\right), \left(8, \dfrac{-1}{4}\right), \left(10, \dfrac{-3}{13}\right)\right\}$
(ii)	$f + g$	**(b)**	$\{(2, 20), (8, -4), (10, -39)\}$
(iii)	$f \cdot g$	**(c)**	$\{(2, -1), (8, -5), (10, -16)\}$
(iv)	f / g	**(d)**	$\{(2, 9), (8, 3), (10, -10)\}$

The domain of $f - g,\ f + g,\ f \cdot g,\ \dfrac{f}{g}$ is domain of $f \cap$ domain of g. Then, find their images.

22. A real valued function $f(x)$ satisfies the function equation $f(x-y)=f(x)f(y)-f(a-x)f(a+y)$ where a is a given constant and $f(0)=1$, $f(2a-x)$ is equal to :

(A) $f(a)+f(a-x)$ **(B)** $f(-x)$ **(C)** $-f(x)$ **(D)** $f(x)$

23. If the set A contains 5 elements and the set B contains 6 elements, then the number of one-one and onto mappings from A to B is :

(A) 720 **(B)** 120 **(C)** 0 **(D)** None of these

24. If A = {1, 2, 3, ... , n} and B {a, b}. Then, the number of surjections from A into B is :

(A) ${}^nP_2$ **(B)** 2^n-2 **(C)** 2^n-1 **(D)** n^2-n

25. If $f: R\to R$ be defined by $f(x)=\frac{1}{x}, \forall\, x\in R$. Then, f is :

(A) one-one **(B)** into **(C)** bijective **(D)** f is not defined

26. Which of the following functions from Z into Z are bijections ?

(A) $f(x)=x^3$ **(B)** $f(x)=x+2$ **(C)** $f(x)=2x+1$ **(D)** $f(x)=x^2+1$

27. If $f: R\to R$ be the functions defined by $f(x)=x^3+5$, then $f^{-1}(x)$ is :

(A) $(x+5)^{1/3}$ **(B)** $(x-5)^{1/3}$ **(C)** $(5-x)^{1/3}$ **(D)** $5-x$

28. If $f: A\to B$ and $g: B\to C$ be the bijective function, then $(gof)^{-1}$ is :

(A) $f^{-1}og^{-1}$ **(B)** fog **(C)** $g^{-1}of^{-1}$ **(D)** gof

29. If $f: N\to R$ be the function defined by $f(x)=\frac{2x-1}{2}$ and $g: Q\to R$ be another function defined by $g(x)=x+2$. Then, $(gof)\frac{3}{2}$ is :

(A) 1 **(B)** 1 **(C)** $\frac{7}{2}$ **(D)** None of these

30. If $f:[0,1]\to[0,1]$ be defined by $f(x)=\begin{cases} x, & \text{if } x \text{ is rational} \\ 1-x, & \text{if } x \text{ is irrational}\end{cases}$, then $(fof)x$ is :

(A) constant **(B)** $1+x$ **(C)** x **(D)** None of these

31. If $f: R-\left\{\frac{3}{5}\right\}\to R$ be defined by $f(x)=\frac{3x+2}{5x-3}$, then :

(A) $f^{-1}(x)=f(x)$ **(B)** $f^{-1}(x)=-f(x)$ **(C)** $(fof)x=-x$ **(D)** $f^{-1}(x)=\frac{1}{19}f(x)$

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Functions

Date Planned : __ / __ / __	Daily Tutorial Sheet-1	Expected Duration : 90 Min
Actual Date of Attempt : __ / __ / __	Level-1	Exact Duration :________

Note (A): Questions having asterisk marked against them may have more than one correct answer.

1. The domain of definition of the function $f(x)=\frac{1}{x^{\log_{10} x}}$ is:

(A) $(0,1)\cup(1,\infty)$ **(B)** $(0,\infty)$ **(C)** $[0,\infty)$ **(D)** $[0,1)\cup(1,\infty)$

2. If $f(x)$ is defined on (0, 1), then the domain of $g(x)=f\left(e^x\right)+f\left(\log_e |x|\right)$ is:

(A) $(-1,e)$ **(B)** $(1,e)$ **(C)** $(-e,-1)$ **(D)** $(-e,1)$

3. The domain of definitions of $f(x)=\underset{\rightarrow n \text{ times} \leftarrow}{\log_{10}\log_{10}\ldots\ldots\log_{10}} x$ is:

(A) $\left(10^n,\infty\right)$ **(B)** $\left(10^{n-1},\infty\right)$ **(C)** $\left(10^{n-2},\infty\right)$ **(D)** None of these

4. Let $f(x)=4\cos\sqrt{x^2-\frac{\pi^2}{9}}$. Then, the range of $f(x)$ is:

(A) $[-1,1]$ **(B)** $[-4,4]$ **(C)** $[0,1]$ **(D)** None of these

5. The function $f(x)=\frac{\sqrt{|\tan x|+\tan x}}{\sqrt{3x}}$ is defined for:

(A) R **(B)** $R-\left\{\frac{1}{3}\right\}$

(C) $R^+-\left\{n\pi+\frac{\pi}{2}\middle| n\in I^+\right\}$ **(D)** None of these

6. Let $f(x)=|x-2|+|x-3|+|x-4|$ and $g(x)=f(x+1)$. Then:

(A) $g(x)$ is an even function **(B)** $g(x)$ is an odd function

(C) $g(x)$ is neither even nor odd **(D)** $g(x)$ is periodic

7. The minimum value of $f(x)=|x-1|+|x-2|+|x-3|$ is equal to:

(A) 1 **(B)** 2 **(C)** 3 **(D)** 0

8. The domain of the function: $f(x)=\log_3\left[-\left(\log_3 x\right)^2+5\log_3 x-6\right]$ is:

(A) $(0,9)\cup(27,\infty)$ **(B)** $[9,27]$ **(C)** $(9,27)$ **(D)** None of these

9. The range of the function $f(x)=\sin\left[\log\left(\frac{\sqrt{4-x^2}}{1-x}\right)\right]$ is:

(A) [0, 1] **(B)** $(-1,0)$ **(C)** $[-1.1]$ **(D)** $(-1,1)$

10. The range of the function $f\left(x\right)=\dfrac{5}{3-x^2}$ is:

(A) $\left(-\infty, 0\right)\cup\left[\dfrac{5}{3}, \infty\right)$ **(B)** $\left(-\infty, 0\right)\cup\left(\dfrac{5}{3}, \infty\right)$

(C) $\left(-\infty, 0\right]\cup\left[\dfrac{5}{3}, \infty\right)$ **(D)** None of these

11. Which of the following when simplified reduces to unity?

I. $\log_{1.5}\log_4\log_{\sqrt{3}}81$ **II.** $\log_2\sqrt{6}+\log_2\sqrt{\dfrac{2}{3}}$

III. $-\dfrac{1}{6}\log_{\frac{\sqrt{3}}{2}}\left(\dfrac{64}{27}\right)$ **IV.** $\log_{3.5}\left(1+2+3\div 6\right)$

The correct choice is:

(A) I only **(B)** II and IV only **(C)** I and III only **(D)** All the above

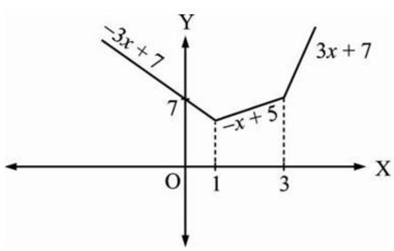
12. If $\log_6\log_2\left[\sqrt{4x+2}+2\sqrt{x}\right]=0,$ then x is:

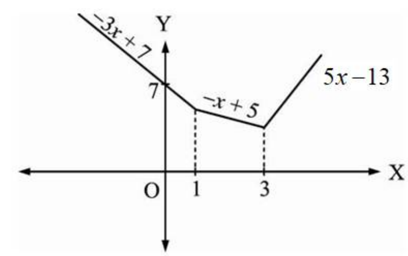
(A) 1/2 **(B)** 1/4 **(C)** 1/16 **(D)** None of these

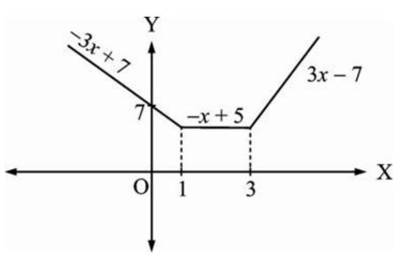
13. The number of values of x which satisfy: $\log 4+\left(1+\dfrac{1}{2x}\right)\log 3=\log\left(3^{\frac{1}{x}}+27\right)$.

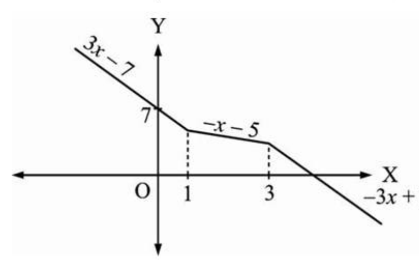
(A) 0 **(B)** 2 **(C)** 3 **(D)** None of these

14. The correct plot of $y=\left|x-1\right|+2\left|x-3\right|$ is:

(A)


(B)


(C)


(D)


15. The domain of the function $f\left(x\right)=\sqrt{\log_{10}\left(\dfrac{5x-x^2}{4}\right)}$ is $x\in$:

(A) [1, 4] **(B)** (1, 4) **(C)** (0, 5) **(D)** [0, 5]

Date Planned : __ / __ / __	Daily Tutorial Sheet-2	Expected Duration : 90 Min
Actual Date of Attempt : __ / __ / __	Level-1	Exact Duration :________

16. The domain of the function $f(x)=\dfrac{\sqrt{-\log_{0.3}(x-1)}}{\sqrt{-x^2+2x+8}}$ is:

(A) (1, 4) **(B)** (–2, 4) **(C)** [2, 4) **(D)** None of these

17. The range of $f(x)=\sqrt{|x|-x}$ is:

(A) $(0, \infty)$ **(B)** $[0, \infty)$ **(C)** $(-\infty, 0)$ **(D)** $(-\infty, 0]$

18. The range of $f(x)=\dfrac{\sin\pi[x^2-1]}{x^4+1}$ is {[.] represents greatest integer function} $f(x)\in$:

(A) R **(B)** [–1, 1] **(C)** {0, 1} **(D)** {0}

19. The domain of the function $f(x)=\dfrac{\tan 2x}{6\cos x+2\sin 2x}$ is:

(A) $R-\left\{(2n+1)\dfrac{\pi}{2} : n\in Z\right\}$

(B) $R-\left\{(2n+1)\dfrac{\pi}{4} : n\in Z\right\}$

(C) $R-\left\{\left\{(2n+1)\dfrac{\pi}{2} : n\in Z\right\}\cup\left\{(2n+1)\dfrac{\pi}{4} : n\in Z\right\}\right\}$

(D) None of these

20. If $f(x)=\log\dfrac{1+x}{1-x}$ and $g(x)=\dfrac{3x+x^3}{1+3x^2}$ then $(fog)(x)$ is equal to:

(A) $f(x)$ **(B)** $2f(x)$ **(C)** $3f(x)$ **(D)** $4f(x)$

21. $f(x)=\begin{cases}[x] & \text{if } -3<x\le -1\\ |x| & \text{if } -1<x<1\\ |[-x]| & \text{if } 1\le x<3\end{cases}$ then $\{x : f(x)\ge 0\}$ is equal to:

(A) (–1, 3) **(B)** [–1, 3) **(C)** (–1, 3] **(D)** [–1, 3]

22. If $f(x)=\dfrac{x-1}{x+1}$, then $f(2x)$ is:

(A) $\dfrac{f(x)+1}{f(x)+3}$ **(B)** $\dfrac{3f(x)+1}{f(x)+3}$ **(C)** $\dfrac{f(x)+3}{f(x)+1}$ **(D)** $\dfrac{f(x)+3}{3f(x)+1}$

23. Let $f(x) = x$ and $g(x) = |x|$ for all $x \in R$. Then the function $\phi(x)$ satisfying $\left[\phi(x) - f(x)\right]^2 + \left[\phi(x) - g(x)\right]^2 = 0$ is:

(A) $\phi(x) = x,\ x \in [0, \infty)$ **(B)** $\phi(x) = x,\ x \in R$

(C) $\phi(x) = -x,\ x \in (-\infty, 0]$ **(D)** $\phi(x) = x + |x|,\ x \in R$

24. If $f(x) = \frac{1}{2}\left[3^x + 3^{-x}\right],\ g(x) = \frac{1}{2}\left[3^x - 3^{-x}\right]$, then $f(x)\,g(y) + f(y)\,g(x)$ is equal to:

(A) $f(x + y)$ **(B)** $g(x + y)$ **(C)** $2f(x)$ **(D)** $2g(x)$

25. The domain of $f(x) = \dfrac{1}{|\sin x| + \sin x}$ is:

(A) R **(B)** $\bigcup_{n \in Z} \left((2n+1)\pi, (2n+2)\pi\right)$

(C) $\bigcup_{n \in Z} \left(2n\pi, (2n+1)\pi\right)$ **(D)** ϕ

26. The domain of $\sin \log \left[\dfrac{\sqrt{4 - x^2}}{1 - x}\right]$ is:

(A) (–1, 1) **(B)** (–2, 1) **(C)** (–2, –1) **(D)** (1, 2)

<u>For Questions 27 - 29</u>

(A) Statement-1 is True, Statement-2 is True and Statement-2 is a correct explanation for Statement-1

(B) Statement-1 is True, Statement-2 is True and Statement-2 is NOT a correct explanation for Statement-1

(C) Statement-1 is True, Statement-2 is False

(D) Statement-1 is False, Statement-2 is True

27. **Statement 1 :** If $f(x) = \log(x - 2) + \log(x - 3)$ and $g(x) = \log(x - 2)(x - 3)$ then $f(x) = g(x)$.

Statement 2 : Two functions $f(x)$ and $g(x)$ are said to be equal if they are defined on same domain A and codomain B and $f(x) = g(x)\ \forall\ x \in A$.

28. **Statement 1 :** $f(x) = |x - 3| + |x - 4| + |x - 7|$ where $4 < x < 7$ is an identity function.

Statement 2 : $f : A \to A$ defined by $f(x) = x$ is an identity function.

29. **Statement 1 :** The domain of the function $f(x) = \sqrt{x - [x]}$ is R^+.

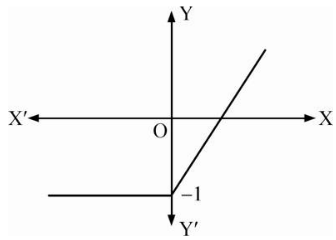
Statement 2 : The domain of the function $\sqrt{f(x)}$ is $\{x : f(x) \geq 0\}$.

30. If $f(x) = \sin\left[\pi^2\right]x + \sin\left[-\pi^2\right]x$, where [.] denotes the greatest integer function, then:

(A) $f\left(\frac{\pi}{2}\right) = 1$ **(B)** $f(\pi) = 2$ **(C)** $f\left(\frac{\pi}{4}\right) = 1$ **(D)** None of these

Date Planned : __ / __ / __	Daily Tutorial Sheet-3	Expected Duration : 90 Min
Actual Date of Attempt : __ / __ / __	Level-1	Exact Duration :________

31. Graph of $y = f(x)$ is as given below. Which function among the following is period?



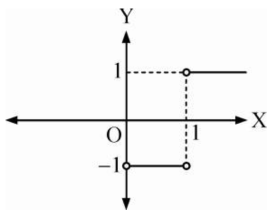
(A) $\frac{1}{2}\left(\left|f(x)\right| + f(x)\right)$

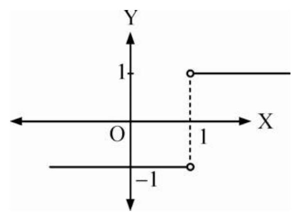
(B) $\frac{1}{2}\left(\left|f(x)\right| - f(x)\right)$

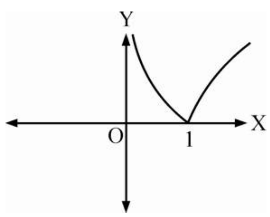
(C) $|f(x)|$

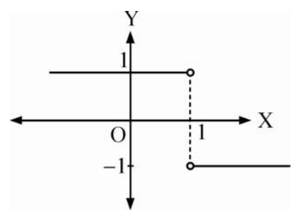
(D) $f(-|x|)$

32. The correct graph of $y = \frac{|\log_2 x|}{\log_2 x}$ is:

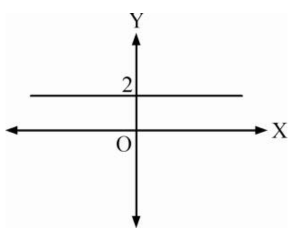
(A)


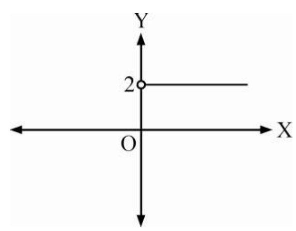
(B)


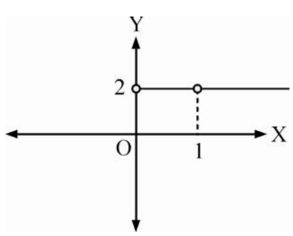
(C)


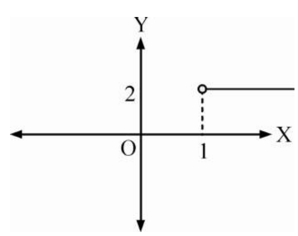
(D)


33. The graph of $y = x^{\log_x 2}$ is:

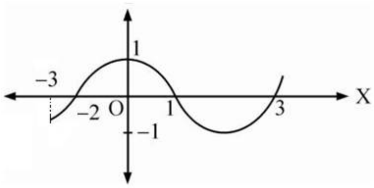
(A)


(B)


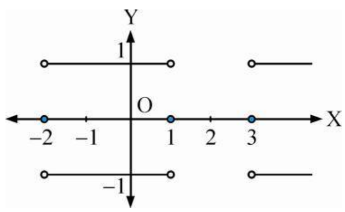
(C)


(D)


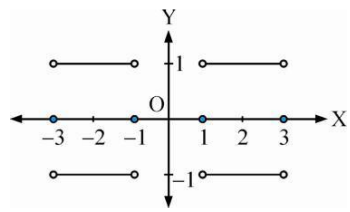
***34.** The graph of the function $y = f(x)$ is as shown in figure. Then which one of the following is correct?



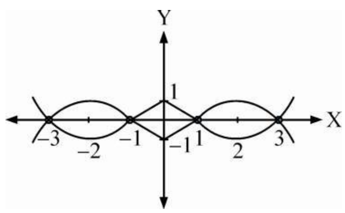
(A) $|y| = sgn\,(f(x))$



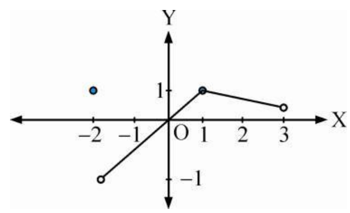
(B) $|y| = sgn\,(-f(x))$



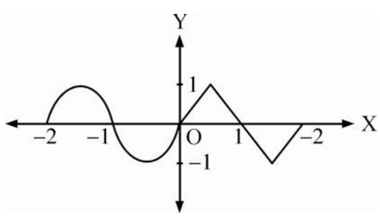
(C) $|y| = |f(|x|)|$



(D) $|y| = x^{\operatorname{sgn}(f(x))}$

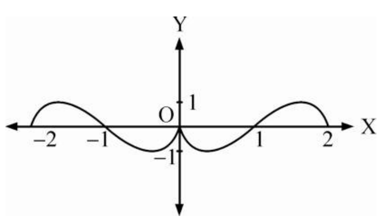


35. The graph $y = f(x)$ is as shown:

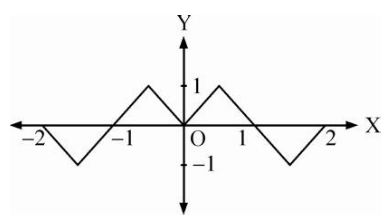


The graph of $y = f(-|x|)$ is:

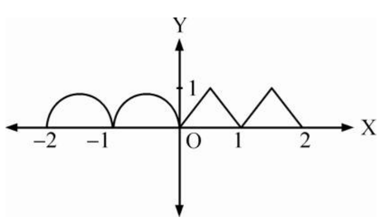
(A)



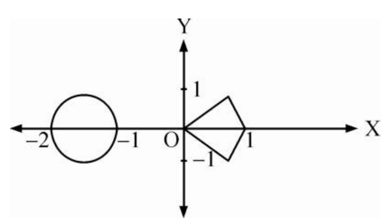
(B)



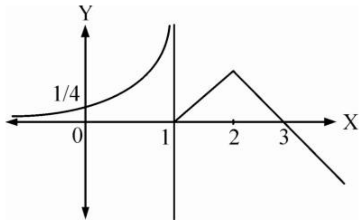
(C)



(D)

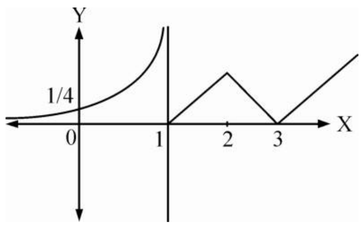


36. The graph of $y = f(x)$ is given below:

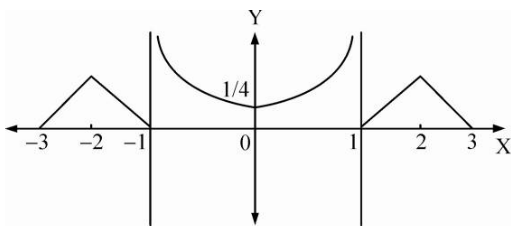


then the graph of $y = |f(|x|)|$ is:

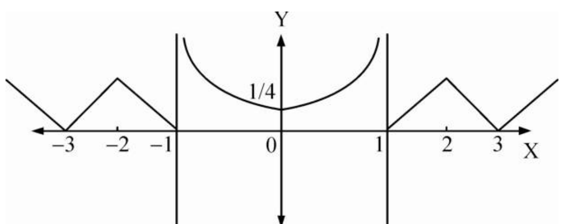
(A)



(B)



(C)



(D) None of these

37. Draw the following curves:

(i) $y = \left| |x^2| - 2x - 3 \right|$ **(ii)** $|x| + |y| = 1$

(iii) $|y| = \left| \log |x| \right|$ **(iv)** $y = \sqrt{2 - x^2}$

38. The range of the function $f(x) = \dfrac{1}{2 - \cos 3x}$ is:

(A) $y \in \left(\dfrac{1}{3}, 1\right)$ **(B)** $y \in \left[\dfrac{1}{3}, 1\right]$ **(C)** $y \in \left(-\dfrac{1}{3}, 1\right)$ **(D)** None of these

39. The range of the function $f\left(x\right)=\dfrac{x^2+2x+3}{x}$ is:

(A) $y \in R$ **(B)** $y \in \left[-2\sqrt{3}+2,\, 2\sqrt{3}+2\right]$

(C) $y \in \left(-\infty,\, -2\sqrt{3}+2\right] \cup \left[2\sqrt{3}+2,\, \infty\right)$ **(D)** None of these

40. The range of the function $f\left(x\right)=\dfrac{x^2-2}{x^2-3}$ is:

(A) $\left(-\infty,\, \dfrac{2}{3}\right] \cup \left(1,\, \infty\right)$ **(B)** $y \in R$

(C) $y \in \left(\dfrac{2}{3},\, 1\right)$ **(D)** None of these

41. The function $f\left(x\right)=\log\left(\dfrac{1+x}{1-x}\right)$ satisfies the equation:

(A) $f\left(x+2\right)-2f\left(x+1\right)+f\left(x\right)=0$ **(B)** $f\left(x+1\right)+f\left(x\right)=f\left(x\left(x+1\right)\right)$

(C) $f\left(x_1\right)f\left(x_2\right)=f\left(x_1+x_2\right)$ **(D)** $f\left(x_1\right)+f\left(x_2\right)=f\left(\dfrac{x_1+x_2}{1+x_1x_2}\right)$

42. The graph of the function $y = f(x)$ is symmetrical about the line $x = 2$, then:

(A) $f\left(x+2\right)=f\left(x-2\right)$ **(B)** $f\left(2+x\right)=f\left(2-x\right)$

(C) $f\left(x\right)=f\left(-x\right)$ **(D)** None of these

43. The range of the function $f\left(x\right)=\left[\sin x+\cos x\right]$ (where $[x]$ denotes the greatest integer function) is $f\left(x\right)\in$:

(A) $[-2, 1]$ **(B)** $\{-2, -1, 0, 1\}$ **(C)** $\{-1, 1\}$ **(D)** $\{-2, 0, -1\}$

44. The value of the function $f\left(x\right)=3\sin\left(\sqrt{\dfrac{\pi^2}{16}-x^2}\right)$ lies in the interval:

(A) $\left[-\dfrac{\pi}{4},\, \dfrac{\pi}{4}\right]$ **(B)** $\left[0,\, \dfrac{3}{\sqrt{2}}\right]$ **(C)** $\left(-3,\, 3\right)$ **(D)** None of these

45. If $f\left(1\right)=1,\ f\left(n+1\right)=2f\left(n\right)+1$ and $n \geq 1$, then $f(n)$ is equal to:

(A) 2^n+1 **(B)** 2^n **(C)** 2^n-1 **(D)** $2^{n-1}-1$

Date Planned : __ / __ / __	Daily Tutorial Sheet-4	Expected Duration : 90 Min
Actual Date of Attempt : __ / __ / __	Level-1	Exact Duration :________

46. The function $f(x) = \cos\left(\log\left(x + \sqrt{x^2 + 1}\right)\right)$ is:

(A) even **(B)** odd **(C)** constant **(D)** None of these

47. $f(x) = \left(\sin x^7\right) e^{x^5 \; Sgn x^9}$ is:

(A) an even function **(B)** an odd function

(C) neither even nor odd **(D)** None of these

48. Which of the following functions is an odd function:

(A) $f(x)$ = constant **(B)** $f(x) = \sin x + \cos x$

(C) $f(x) = \sin\left(\log\left(x + \sqrt{x^2 + 1}\right)\right)$ **(D)** $f(x) = 1 + x + 2x^3$

49. If $f(x) = \sin^2 x + \sin^2\left(x + \frac{\pi}{3}\right) + \cos x \cos\left(x + \frac{\pi}{3}\right)$ and $g\left(\frac{5}{4}\right) = 1$ then $(gof)(x)$ is equal to:

(A) 1 **(B)** 0 **(C)** $\sin x$ **(D)** None of these

50. The function $f(x) = \sin\left(\frac{\pi x}{n!}\right) - \cos\left(\frac{\pi x}{(n+1)!}\right)$ is:

(A) non-periodic **(B)** periodic, with period $2(n!)$

(C) periodic, with period $(n+1)$ **(D)** None of these

51. The period of the function $f(x) = \dfrac{\sin x + \sin 2x + \sin 4x + \sin 5x}{\cos x + \cos 2x + \cos 4x + \cos 5x}$ is:

(A) $\frac{\pi}{3}$ **(B)** $\frac{\pi}{4}$ **(C)** π **(D)** None of these

52. **Statement 1:** The function $f(x) = \sin x$ is symmetric about the line $x = 0$.

Statement 2: Every even function is symmetric about y-axis.

(A) Statement-1 is True, Statement-2 is True and Statement-2 is a correct explanation for Statement-1

(B) Statement-1 is True, Statement-2 is True and Statement-2 is NOT a correct explanation for Statement-1

(C) Statement-1 is True, Statement-2 is False

(D) Statement-1 is False, Statement-2 is True

53. Which of the following function has period π :

(A) $|\sin x| + |\cos x|$ **(B)** $\sin^4 x + \cos^4 x$

(C) $\sin(\sin x) + \sin(\cos x)$ **(D)** $\dfrac{1 + 2\cos x}{\sin x(2 + \sec x)}$

54. Which of the following functions is non-periodic?

(A) $f(x) = \tan(3x - 2)$

(B) $f(x) = \{x\}$, (where {.} denotes the fractional part of x)

(C) $f(x) = x + \cos x$

(D) $f(x) = 1 - \dfrac{\cos^2 x}{1 + \tan x} - \dfrac{\sin^2 x}{1 + \cot x}$

55. If $f(x) = \dfrac{1}{1-x}$, $g(x) = f[f(x)]$ and $h(x) = f[f\{f(x)\}]$, $\forall\ x \in R - \{0, 1\}$ then the value of $f(x) \cdot g(x) \cdot h(x)$ is:

(A) 1 **(B)** -1 **(C)** 0 **(D)** None of these

56. The period of function $\dfrac{|\sin x| + |\cos x|}{|\sin x - \cos x| + |\sin x + \cos x|}$ is:

(A) π **(B)** $\dfrac{\pi}{2}$ **(C)** 2π **(D)** $\dfrac{2\pi}{3}$

57. **Statement-1:** Function $f(x) = \sin(x + 3\sin x)$ is periodic

Statement-2: If $g(x)$ is periodic then $f(g(x))$ periodic

(A) Statement-1 is True, Statement-2 is True and Statement-2 is a correct explanation for Statement-1

(B) Statement-1 is True, Statement-2 is True and Statement-2 is NOT a correct explanation for Statement-1

(C) Statement-1 is True, Statement-2 is False

(D) Statement-1 is False, Statement-2 is True

58. The period of the function $f(x) = \sin^4 x + \cos^4 x$ is:

(A) π **(B)** $\dfrac{\pi}{2}$ **(C)** 2π **(D)** None of these

59. If $f(x) = \sin\left(\sqrt{[\lambda]}x\right)$ is a periodic function with period π, where $[\lambda]$ denotes the greatest integer less than or equal to λ, is π, then:

(A) $\lambda \in [4, 5)$ **(B)** $\lambda \in [4, 5]$ **(C)** $\lambda = 4, 5$ **(D)** None of these

***60.** If $f(x) = \dfrac{\sin \pi[x]}{\{x\}}$, then $f(x)$ is: {[.] denotes greatest integer function}.

(A) Periodic with fundamental period 1 **(B)** Even

(C) Range is singleton **(D)** None of these

Date Planned : __ / __ / __	Daily Tutorial Sheet-5	Expected Duration : 90 Min
Actual Date of Attempt : __ / __ / __	Level-1	Exact Duration :________

61. If $2f(x-1) - f\left(\dfrac{1-x}{x}\right) = x$, then $f(x)$ is:

(A) $\dfrac{1}{3}\left[2(1+x) + \dfrac{1}{1+x}\right]$ **(B)** $2(x-1) + \dfrac{1-x}{x}$

(C) $x^2 + \dfrac{1}{x^2} + 3$ **(D)** None of these

62. Suppose f is a real function $f(x + f(x)) = 4f(x)$ and $f(1) = 4$. Then the value of $f(21)$ is:

(A) 16 **(B)** 21 **(C)** 64 **(D)** 105

63. If $\sum_{r=0}^{21} f\left(\dfrac{r}{11} + 2x\right) =$ constant $\forall\, x \in R$ and $f(x)$ is periodic, then period of $f(x)$ is:

(A) 1 **(B)** 1/11 **(C)** 2 **(D)** 4

64. If $h(x) = \log_{10} x$ then the value of $\sum_{n=1}^{89} h(\tan n^\circ) =$

(A) 1 **(B)** 0 **(C)** -1 **(D)** None of these

65. If $f(x) + 2f\left(\dfrac{1}{x}\right) = 3x,\ x \neq 0$ and $S = \{x \in R : f(x) = f(-x)\}$; then S:

(A) contains exactly one element **(B)** contains exactly two elements

(C) contains more than two elements **(D)** is an empty set

66. Let $f(x) = min\,\{x, x^2\}$, for every $x \in R$. Then:

(A) $f(x) = \begin{cases} x, & x \geq 1 \\ x^2, & 0 \leq x < 1 \\ x, & x < 0 \end{cases}$ **(B)** $f(x) = \begin{cases} x^2, & x \geq 1 \\ x, & x < 1 \end{cases}$

(C) $f(x) = \begin{cases} x, & x \geq 1 \\ x^2, & x < 1 \end{cases}$ **(D)** $f(x) = \begin{cases} x^2, & x \geq 1 \\ x, & 0 \leq x < 1 \\ x^2, & x < 0 \end{cases}$

67. The function $f(x) = \max\{(1-x), (1+x), 2\},\ x \in (-\infty, \infty)$ is equivalent to:

(A) $f(x) = \begin{cases} 1-x, & x \leq -1 \\ 2, & -1 < x < 1 \\ 1+x, & x \leq -1 \end{cases}$ **(B)** $f(x) = \begin{cases} 1+x, & x \leq -1 \\ 2, & -1 < x < 1 \\ 1-x, & x \geq 1 \end{cases}$

(C) $f(x) = \begin{cases} 1-x, & x \leq -1 \\ 1, & -1 < x < 1 \\ 1+x, & x \geq 1 \end{cases}$ **(D)** None of these

Paragraph for Questions 68 – 70

Let a function $f(x)$ be such that $f(x) = \left| \left| x^2 - 3 \right| - 2 \right|$.

68. Equation $f(x) = \lambda$ has 2 solutions if :

(A) $\lambda > 2$ **(B)** $\lambda < 2$ **(C)** $1 < \lambda < 2$ **(D)** $\lambda \geq 2$

69. Equation $f(x) = \lambda$ has 4 solutions if :

(A) $\lambda = 2, 0$ **(B)** $\lambda \geq 2$ **(C)** $1 \leq \lambda \leq 2$ **(D)** $1 < \lambda < 2$

70. Equations $f(x) = \lambda$ has 8 solutions if :

(A) $\lambda < 1$ **(B)** $\lambda = 2$ **(C)** $1 < \lambda < 2$ **(D)** $0 < \lambda < 1$

71. For the equation $\log_{3\sqrt{x}} x + \log_{3x} \sqrt{x} = 0$, which of the following do not hold good?

(A) No real solution **(B)** One prime solution

(C) One integral solution **(D)** None of these

72. If $x \in [0, 2\pi]$, then $y_1 = \dfrac{\sin x}{|\sin x|}$, $y_2 = \dfrac{|\cos x|}{\cos x}$ are identical functions for $x \in$:

I. $\left(0, \dfrac{\pi}{2}\right)$ **II.** $\left(\dfrac{\pi}{2}, \pi\right)$ **III.** $\left(\pi, \dfrac{3\pi}{2}\right)$ **IV.** $\left(\dfrac{3\pi}{2}, 2\pi\right)$

(A) I, II **(B)** I, III **(C)** II, III **(D)** I, IV

73. The range of the function $f(x) = \sin\left[\log\left(\dfrac{\sqrt{4 - x^2}}{1 - x}\right)\right]$ is:

(A) $[0, 1]$ **(B)** $(-1, 0)$ **(C)** $[-1. 1]$ **(D)** $(-1, 1)$

74. If $f(x) + 2\,f(1 - x) = x^2 + 2, \forall\, x \in R$, then $f(x)$ is given as:

(A) $\dfrac{(x-1)^2}{3}$ **(B)** $\dfrac{(x-2)^2}{3}$ **(C)** $x^2 - 1$ **(D)** $x^2 - 2$

75. Let $f : R \to R$ be a function defined by $f(x) = \dfrac{|x|^3 + |x|}{1 + x^2}$, then the graph of $f(x)$ lies in the :

(A) I and II quadrants **(B)** I and III quadrants

(C) II and III quadrants **(D)** III and IV quadrants

Functions

Date Planned : __ / __ / __	Daily Tutorial Sheet-6	Expected Duration : 90 Min
Actual Date of Attempt : __ / __ / __	Level-2	Exact Duration :________

76. The domain of definition of $f(x) = \log_2\left(-\log_{1/2}\left(1+\frac{1}{x^{1/4}}\right)-1\right)$ is :

(A) (0, 1) **(B)** (0, 1] **(C)** $[1, \infty)$ **(D)** $(1, \infty)$

77. If $f(x) = \frac{x}{\sqrt{(1+x^2)}}$ then the value of $(fofof)(x)$ is :

(A) $\frac{2x}{\sqrt{1+3x^2}}$ **(B)** $\frac{x}{\sqrt{1+3x^2}}$ **(C)** $\frac{x}{1+3x^2}$ **(D)** $\frac{2x}{1+3x^2}$

***78.** The given function $f(x) = \frac{a^x - 1}{x^n(a^x+1)}$ is :

(A) Odd, if n is even **(B)** Even if n is odd
(C) Neither even nor odd **(D)** None of these

79. Draw the following curves :

(i) $y = [\sin[x]]\ \forall\ x \in [-2\pi, 2\pi]$ **(ii)** $y = [x] + \sqrt{x - [x]}$ **(iii)** $|y| = |1 + e^{|x|} - e - x|$

80. Construct the graph of the function $y = f(x-1) + f(x+1)$ where $f(x) = \begin{cases} 1-|x| & \text{where } |x| \le 1 \\ 0 & \text{where } |x| > 1 \end{cases}$

81. Let $f(x) = \frac{ax+b}{cx+d}$. Then the $fof(x) = x$ provided that :

(A) $d = -a$ **(B)** $d = a$ **(C)** $a = b = c = d = 1$ **(D)** $a = b = 1$

82. If $b^2 - 4ac = 0, a > 0$, then the domain of the function $f(x) = \log\left(ax^3 + (a+b)x^2 + (b+c)x + c\right)$ is :

(A) $R - \left\{-\frac{b}{2a}\right\}$ **(B)** $R - \left\{\left\{-\frac{b}{2a}\right\} \cup \{x \mid x \ge -1\}\right\}$

(C) $R - \left\{-\frac{b}{2a}\right\} \cap (-1, \infty)$ **(D)** None of these

83. If $f(x)$ is defined on [0, 1] by the rule $f(x) = \begin{cases} x, & \text{if } x \text{ is rational} \\ 1-x, & \text{if } x \text{ is irrational} \end{cases}$

Then for all $x \in [0,1]$, $f(f(x))$ is :

(A) constant **(B)** $1 + x$ **(C)** x **(D)** None of these

84. If $f(x)$ is a function such that $f(x+y) = f(x) + f(y)$ and $f(1) = 7$ then $\sum_{r=1}^{n} f(r)$ is equal to:

(A) $7n/2$ **(B)** $7(n+1)/2$ **(C)** $7n(n+1)$ **(D)** $7n(n+1)/2$

85. Let f be a real valued function defined by $f(x) = \frac{e^x - e^{-|x|}}{e^x + e^{|x|}}$, then range of $f(x)$ is :

(A) R **(B)** [0, 1] **(C)** [0, 1) **(D)** [0, 1/2)

Date Planned : __ / __ / __	Daily Tutorial Sheet-7	Expected Duration : 90 Min
Actual Date of Attempt : __ / __ / __	Level-2	Exact Duration :________

86. The domain of the function $f(x)=\sqrt{\left(\frac{1}{\sin x}-1\right)}$ is :

(A) $\left(2n\pi, 2n\pi+\frac{\pi}{2}\right), \forall\, n \in I$ **(B)** $\left(2n\pi, (2n+1)\pi\right), \forall\, n \in I$

(C) $\left((2n-1)\pi, 2n\pi\right), \forall\, n \in I$ **(D)** None of the above

87. Total number of solutions of $2^x+3^x+4^x-5=0$ is :

(A) 0 **(B)** 1 **(C)** 2 **(D)** Infinitely many

88. The number of roots of the equation $3^{|x|}\{|2-|x||\}=1$ is :

(A) 1 **(B)** 2 **(C)** 3 **(D)** 4

89. If $f: R \to R$ is a function satisfying the property $f(2x+7)+f(2x+3)=2, \forall\, x \in R$, then the period of $f(x)$ is :

(A) 2 **(B)** 4 **(C)** 8 **(D)** 12

90. Number of integral values of x which satisfying the equation, $9^{\log_3(\log_2 x)}=\log_2 x-(\log_2 x)^2+1$ is :

(A) 0 **(B)** 1 **(C)** 2 **(D)** None of these

91. The value of $E=81^{\log_{0.\bar{3}}\left(\frac{1}{\sqrt{4+2\sqrt{3}}-\sqrt{4-2\sqrt{3}}}\right)}$ is simplified to.

(A) 16 **(B)** 4 **(C)** 2 **(D)** 1/2

92. Range of $f(x)=\log_{\sqrt[3]{10}}\left(\sqrt{5}(2\sin x+\cos x)+5\right)$ is :

(A) $[0,1]$ **(B)** $[0,3]$ **(C)** $\left(-\infty, \frac{1}{3}\right]$ **(D)** None of these

93. Which of the following function is not periodic, where [.] denotes greatest integer function :

(A) $f(x)=1^{[x]}+(-1)^{[x]}$ **(B)** $g(x)=1^{[5x]}+(-1)^{[5x]}$

(C) $h(x)=2^{[x]}-(-2)^{[x]}$ **(D)** $\phi(x)=1^{[x]}-(-1)^{[x]}$

94. If A is domain of $f(x)=\ell n\left(\left(x^3-6x^2+11x-6\right)(x)\left(e^x-5\right)\right)$ and B is the range of $g(x)=\sin^2\frac{x}{4}+\cos\frac{x}{4}$. Then find $A \cap B$.

(A) (0, 2) **(B)** (0, 1) **(C)** (1, 2) **(D)** None of these

95. Let $f(k)=\frac{k}{2009}$ and $g(k)=\frac{f^4(k)}{[1-f(k)]^4[f(k)]^4}$, then the sum $\sum_{k=0}^{2009} g(k)$ is :

(A) 2009 **(B)** 2008 **(C)** 1005 **(D)** 1004

Date Planned : __ / __ / __	Daily Tutorial Sheet-8	Expected Duration : 90 Min
Actual Date of Attempt : __ / __ / __	Level-2	Exact Duration :________

96. A function f from the set of natural numbers to integers defined by $f(n)=\begin{cases}\dfrac{n-1}{2}, \text{ when } n \text{ is odd}\\ -\dfrac{n}{2}, \text{ when } n \text{ is even}\end{cases}$ is :

(A) one-one but not onto **(B)** onto but not one-one
(C) one-one and onto both **(D)** neither one-one nor onto

97. Let $g: R \to R$ be given by $g(x) = 3 + 4x$.

If $g^n(x) = gogo .....og(x)$, then $g^{-n}(x) =$ (where $g^{-n}(x)$ denotes inverse of $g^n(x)$)

(A) $(4^n - 1) + 4^n x$ **(B)** $(x+1)4^{-n} - 1$ **(C)** $(x+1)4^n - 1$ **(D)** $(4^{-n} - 1)x + 4^n$

98. Total number of solutions of the equation $\sin \pi x = \left| \ell n_e |x| \right|$ is :

(A) 8 **(B)** 10 **(C)** 9 **(D)** 6

99. Total number of solutions of the equation $x^2 - 4 - [x] = 0$ are : (where [.] denotes the greatest integer function)

(A) 1 **(B)** 2 **(C)** 3 **(D)** 4

100. The period of the function $f(x) = 4\sin^4\left(\dfrac{4x - 3\pi}{6\pi^2}\right) + 2\cos\left(\dfrac{4x - 3\pi}{3\pi^2}\right)$ is :

(A) $\dfrac{3\pi^2}{4}$ **(B)** $\dfrac{3\pi^3}{4}$ **(C)** $\dfrac{4\pi^2}{3}$ **(D)** $\dfrac{4\pi^3}{3}$

101. The range of $f(x) = \dfrac{2 + x - [x]}{1 - x + [x]}$ is :

(A) [0, 1) **(B)** $[2, \infty)$ **(C)** $[0, 1) \cup (1, 2]$ **(D)** R^+

102. The number of roots of the question $1 + \log_2(1 - x) = 2^{-x}$ is :

(A) 0 **(B)** 1 **(C)** 2 **(D)** many

103. The range of the function $f(x) = 3|\sin x| - 2|\cos x|$ is :

(A) $[-2, \sqrt{13}]$ **(B)** $[-2, 3]$ **(C)** $[3, \sqrt{13}]$ **(D)** $[-3, 2]$

104. Let $x = \left(\dfrac{81^{\frac{1}{\log_5 9}} + 3^{\frac{3}{\log_{\sqrt{6}} 3}}}{409}\right) \cdot \left((\sqrt{7})^{\frac{2}{\log_{25} 7}} - 125^{\log_{25} 6}\right)$, then value of $\log_2 x$ is equal to :

(A) 0 **(B)** 1 **(C)** -1 **(D)** None of these

***105.** If $f(x) = 0$ be a polynomial whose coefficients are all ± 1 and whose roots are all real, then degree of $f(x)$ can be :

(A) 1 **(B)** 2 **(C)** 3 **(D)** 4

Date Planned : __ / __ / __	Daily Tutorial Sheet-9	Expected Duration : 90 Min
Actual Date of Attempt : __ / __ / __	Level-2	Exact Duration :________

106. Let $f(x) = \sec^{-1}\left[1+\cos^2 x\right]$, where [.] denotes the greatest integer function. Then, the range of $f(x)$ is :

(A) $[1, 2]$ **(B)** $[0, 2]$ **(C)** $\left\{\sec^{-1}1, \sec^{-1}2\right\}$ **(D)** None of these

107. If T_1 is the period of the function $y = e^{3(x-[x])}$ and T_2 is the period of the function $y = e^{3x-[3x]}$ ([.] denotes the greatest integer function), then :

(A) $T_1 = T_2$ **(B)** $T_1 = \frac{T_2}{3}$ **(C)** $T_1 = 3T_2$ **(D)** None of these

108. If $f(x) = \frac{4^x}{4^x+2}$, then $f\left(\frac{1}{97}\right) + f\left(\frac{2}{97}\right) + ... + f\left(\frac{96}{97}\right)$ is equal to :

(A) 1 **(B)** 48 **(C)** -48 **(D)** -1

109. The domain of the function $f(x) = \log_{10}\left(\sqrt{x-4}+\sqrt{6-x}\right)$ is :

(A) $[4, 6]$ **(B)** $(-\infty, 6)$ **(C)** $(2, 3)$ **(D)** None of these

110. Given, $\log_a x = \alpha$; $\log_b x = \beta$; $\log_c x = \gamma$ & $\log_d x = \delta (x \neq 1)$, $a, b, c, d \in R^+ - \{1\}$ then $\log_{abcd} x$ has the value equal to :

(A) $\frac{1}{\alpha\beta\gamma\delta}$ **(B)** $\frac{1}{\alpha}+\frac{1}{\beta}+\frac{1}{\gamma}+\frac{1}{\delta}$ **(C)** $\frac{1}{\frac{1}{\alpha}+\frac{1}{\beta}+\frac{1}{\gamma}+\frac{1}{\delta}}$ **(D)** None of these

111. If $f(x) = \log\left(\frac{1+x}{1-x}\right)$, $-1 < x < 1$, then $f\left(\frac{3x+x^3}{1+3x^2}\right) - f\left(\frac{2x}{1+x^2}\right)$ is :

(A) $\left[f(x)\right]^3$ **(B)** $\left[f(x)\right]^2$ **(C)** $-f(x)$ **(D)** $f(x)$

112. The period of the function $f(x)$ which satisfies the relation $f(x) + f(x+4) = f(x+2) + f(x+6)$ is :

(A) 6 **(B)** 7 **(C)** 8 **(D)** None of these

113. If $f(x) = -\frac{x|x|}{1+x^2}$, then $f^{-1}(x)$ equals :

(A) $\sqrt{\frac{|x|}{1-|x|}}$ **(B)** $(\text{sgn}(-x))\sqrt{\frac{|x|}{1-|x|}}$ **(C)** $-\sqrt{\frac{x}{1-x}}$ **(D)** $(\text{sgn}(x))\sqrt{\frac{|x|}{1+|x|}}$

114. The number of integral solutions of the equation $4\log_{x/2}\left(\sqrt{x}\right) + 2\log_{4x}\left(x^2\right) = 3\log_{2x}\left(x^3\right)$ is :

(A) 0 **(B)** 1 **(C)** 2 **(D)** None of these

115. The domain of the function $f(x) = \log_{\left[x+\frac{1}{2}\right]}\left|x^2-5x+6\right|$ is: (where [.] denotes the greatest integer function)

(A) $x \in \left[\frac{3}{2}, 2\right) \cup (2, 3) \cup (3, \infty)$ **(B)** $x \in \left[\frac{3}{2}, \infty\right)$

(C) $x \in \left[\frac{1}{2}, \infty\right)$ **(D)** None of these

Date Planned : __ / __ / __	Daily Tutorial Sheet-10	Expected Duration : 90 Min
Actual Date of Attempt : __ / __ / __	Level-2	Exact Duration :________

116. If [.] denotes the greatest integer function, then the value of $\sum_{r=1}^{100}\left[\frac{1}{2}+\frac{r}{100}\right]$ is :

(A) 49 **(B)** 50 **(C)** 51 **(D)** 52

117. The function $f(x)=\sqrt{\cos(\sin x)}+\sin^{-1}\left(\frac{1+x^2}{2x}\right)$ is defined for :

(A) $x\in\{-1,1\}$ **(B)** $x\in[-1,1]$ **(C)** $x\in R$ **(D)** $x\in(-1,1)$

118. Draw the graph of $|y|=(\{x\}-1)^2$.

119. The function $f(x)=\frac{\sec^{-1}x}{\sqrt{x-[x]}}$, where [x] denotes the greatest integer less than or equal to x is defined for all x belonging to :

(A) R

(B) $R-\{(-1,1)\cup\{n \mid n\in Z\}\}$

(C) $R^{+}-(0,1)$

(D) $R^{+}-\{n \mid n\in N\}$

120. The domain of the function : $f(x)=\log_3\left[-(\log_3 x)^2+5\log_3 x-6\right]$ is :

(A) $(0,9)\cup(27,\infty)$ **(B)** $[9,27]$ **(C)** $(9,27)$ **(D)** None of these

121. The function $f(x)=\sec\left[\log\left(x+\sqrt{1+x^2}\right)\right]$ is:

(A) Even **(B)** Odd **(C)** Constant **(D)** None of these

122. Let $f(x+y)+f(x-y)=2f(x)f(y),\ \forall\ y\in R$ and $f(0)=k$, then :

I. $f(x)$ is even, if $k=1$

II. $f(x)$ is odd, if $k=0$

III. $f(x)$ is always odd

IV. $f(x)$ is neither even nor odd for any value of k

The correct choice is :

(A) I, III **(B)** II, III **(C)** I, II **(D)** III, IV

123. Let f be a real valued function such that for any real x

$$f(15+x) = f(15-x) \text{ and } f(30+x) = -f(30-x)$$

Then which of the following statements is true ?

(A) f is odd and periodic
(B) f is odd but not periodic
(C) f is even and periodic
(D) f is even nut not periodic

124. If $f(x) = \sin\{[x+5]+\{x-\{x-\{x\}\}\}\}$ for $x \in \left(0, \frac{\pi}{4}\right)$ is invertible, where {.} and [.] represent fractional part and greatest integer functions respectively, then $f^{-1}(x)$ is :

I. $\sin^{-1} x$ **II.** $\frac{\pi}{2} - \cos^{-1} x$ **III.** $\sin^{-1}\{x\}$ **IV.** $\cos^{-1}\{x\}$

The correct choice is :

(A) I, II, III
(B) II, III
(C) III, IV
(D) None of these

125. **Statement 1 :** Let $f : r - \{3\}$ be a function given by $f(x+10) = \frac{f(x)-5}{f(x)-3}$, then $f(10) = f(50)$.

Statement 2 : $f(x)$ is a periodic function.

(A) Statement-1 is True, Statement-2 is True and Statement-2 is a correct explanation for Statement-1
(B) Statement-1 is True, Statement-2 is True and Statement-2 is NOT a correct explanation for Statement-1
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True

Functions

Date Planned : __ / __ / __	**Daily Tutorial Sheet - 11**	**Expected Duration : 90 Min**
Actual Date of Attempt : __/__/__	**Numerical Value Type for JEE Main**	**Exact Duration :________**

126. Let $f(x)$ be a polynomial of degree 6 with leading coefficient 2009. Suppose further, that $f(1)=1,\ f(2)=3,\ f(3)=5,\ f(4)=7,\ f(5)=9,\ f'(2)=2,$ then the sum of all the digits of $f(6)$ is:

127. Let $f(x)=x^3-3x+1$. Find the number of different real solution of the equation $f(f(x))=0$

128. If the domain of $f(x)=\sqrt{12-3^x-3^{3-x}}+\sin^{-1}\left(\frac{2x}{3}\right)$ is $[a,b]$, then $a=$........

129. The number of elements in the range of the function :
$y=\sin^{-1}\left[x^2+\frac{5}{9}\right]+\cos^{-1}\left[x^2-\frac{4}{9}\right]$ where [.] denotes the greatest integer function is:

130. The number of integers in the range of function $f(x)=[\sin x]+[\cos x]+[\sin x+\cos x]$ is
(where [.] = denotes greatest integer function)

131. For all real number x, *let* $f(x)=\frac{1}{\sqrt[2011]{1-x^{2011}}}$. Find the number of real roots of the equation $f(f(......(f(x)).......)=\{-x\}$. Where f is applied 2013 times and {.} denotes fractional part function.

132. Let $f(x,y)=x^2-y^2$ and $g(x,y)=2xy$.
Such that $(f(x,y))^2-(g(x,y))^2=\frac{1}{2}$ and $f(x,y)\cdot g(x,y)=\frac{\sqrt{3}}{4}$. Find the number of ordered pairs (x,y)?

133. Let $f(x)=\frac{x+5}{\sqrt{x^2+1}}\ \forall x\in R$, then the smallest integral value of k for which $f(x)\le k\ \forall x\in R$ is :

134. The number of roots of equation : $\left(\frac{(x-1)(x-3)}{(x-2)(x-4)}-e^x\right)\left(\frac{(x+1)(x+3)e^x}{(x+2)(x+4)}-1\right)(x^3-\cos x)=0$

135. Let $f(x)=x^2-bx+c$, b is an odd positive integer. Given that $f(x)=0$ has two prime numbers as roots and $b+c=35$. If the least value of $f(x)\forall x\in R$ is λ, then $\left[\left|\frac{\lambda}{3}\right|\right]$ is equal to
(where [.] denotes greatest integer function)

136. If $f(x)$ is an even function, then the number of distinct real numbers x such that $f(x)=f\left(\frac{x+1}{x+2}\right)$ is :

137. If $\sum_{r=1}^{n}[\log_2 r]=2010$, where [.] denotes greatest integer function, then the sum of the digits of n is:

138. Let $P(x)$ be a cubic polynomial with leading co-efficient unity. Let the remainder when $P(x)$ is divided by x^2-5x+6 equals 2 times the remainder when $P(x)$ is divided by x^2-5x+4. If $P(0)=100$, find the sum of the digits of $P(5)$:

139. Let $f(x)=x^2+10x+20$. Find the number of real solution of the equation $f(f(f(f(x))))=0$

140. Polynomial $P(x)$ contains only terms of odd degree. When $P(x)$ is divided by $(x-3)$, then remainder is 6. If $P(x)$ is divided by (x^2-9) then remainder is $g(x)$. Find the value of $g(2)$.

Functions

Date Planned : __ / __ / __	Daily Tutorial Sheet – 1	Expected Duration : 90 Min
Actual Date of Attempt : __ / __ / __	JEE Main (Archive)	Exact Duration: __________

1. If $f:[1,\infty]\to[2,\infty]$ is given by $f(x)=x+\dfrac{1}{x}$, then $f^{-1}(x)$ equals: [2001]

(A) $\dfrac{x+\sqrt{x^2-4}}{2}$ **(B)** $\dfrac{x}{1+x^2}$ **(C)** $\dfrac{x-\sqrt{x^2-4}}{4}$ **(D)** $1+\sqrt{x^2-4}$

2. If $f(x)=\cos(\log x)$, then $f(x)f(y)-\dfrac{1}{2}\left[f\left(\dfrac{x}{y}\right)+f(xy)\right]$ has the value: [1983]

(A) -1 **(B)** $\dfrac{1}{2}$ **(C)** -2 **(D)** None of these

3. The domain of the function $f(x)=\dfrac{\sin^{-1}(x-3)}{\sqrt{9-x^2}}$ is: [2004]

(A) [1, 2] **(B)** [2, 3) **(C)** [2, 3] **(D)** [1, 2)

4. The domain of $\sin^{-1}\left[\log_3\left(\dfrac{x}{3}\right)\right]$ is: [2002]

(A) [1, 9] **(B)** [–1, 9] **(C)** [–9, 1] **(D)** [–9, –1]

5. Let $f:(0,1)\to R$ be defined by $f(x)=\dfrac{b-x}{1-bx}$, where b is a constant such that $0<b<1$. Then: [2011]

(A) f is not invertible on (0, 1)

(B) $f\neq f^{-1}$ on (0, 1) and $f'(b)=\dfrac{1}{f'(0)}$

(C) $f\neq f^{-1}$ on (0, 1) and $f'(b)=\dfrac{1}{f'(0)}$

(D) f^{-1} is differentiable on (0, 1)

6. Let $f(x)=x^2$ and $g(x)=\sin x$ for all $x\in R$. Then the set of all x satisfying $(fogogof)(x)=(gogof)(x)$, where $(fog)(x)=f(g(x))$ is : [2011]

(A) $\pm\sqrt{n\pi},\ n\in\{0,1,2,\ldots\}$ **(B)** $\pm\sqrt{n\pi},\ n\in\{1,2,\ldots\}$

(C) $\dfrac{\pi}{2}+2n\pi,\ n\in\{-2,-1,0,1,2,\ldots\}$ **(D)** $2n\pi,\ n\in\{\ldots,-2,-1,0,1,2,\ldots\}$

7. Let S = {1, 2, 3, 4}. The total number of unordered pairs of disjoint subsets of S is equal to : [2010]

(A) 25 **(B)** 34 **(C)** 42 **(D)** 41

8. Let $f(x)=\dfrac{x^2-6x+5}{x^2-5x+6}$.

Match the Column I with Column II and make the correct option from the codes given below. [2007]

	Column I		Column II
I.	If $-1<x<1$, then $f(x)$ satisfies	**(p)**	$0<f(x)<1$
II.	If $1<x<2$, then $f(x)$ satisfies	**(q)**	$f(x)<0$
III.	If $3<x<5$, then $f(x)$ satisfies	**(r)**	$f(x)>0$
IV.	If $x>5$, then $f(x)$ satisfies	**(s)**	$f(x)<1$

Codes :

	I	II	III	IV		I	II	III	IV
(A)	q	p	r	r	**(B)**	p	q	q	p
(C)	s	p	q	r	**(D)**	p	r	q, r	s

9. Let $f(x)=\begin{cases}x, & \text{if } x \text{ is rational}\\ 0, & \text{if } x \text{ is irrational}\end{cases}$ and $g(x)=\begin{cases}0, & \text{if } x \text{ is rational}\\ x, & \text{if } x \text{ is irrational}\end{cases}$

Then, $f-g$ is : [2005]

(A) one-one and onto
(B) neither one-one nor onto
(C) many one and onto
(D) one-one and onto

10. If X and Y are two non-empty sets, where $f: X \to Y$, is function is defined such that [2005]

$f(C)=\{f(x): x \in C\}$ for $C \subseteq X$ and $f^{-1}(D)=\{x : f(x) \in D\}$ $D \subseteq Y$. For any $A \subseteq Y$ and $B \subseteq Y$, then

(A) $f^{-1}\{f(A)\}=A$
(B) $f^{-1}\{f(A)\}=A$ only if $f(X)=Y$
(C) $f\{f^{-1}(B)\}=B$ only if $B \subseteq f(x)$
(D) $f^{-1}\{f(B)\}=B$

For Questions 11- 13 [2007]

(A) Statement-1 is True, Statement-2 is True and Statement-2 is a correct explanation for Statement-1
(B) Statement-1 is True, Statement-2 is True and Statement-2 is NOT a correct explanation for Statement-1
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True

11. Let F (x) be an indefinite integral of $\sin^2 x$.

Statement 1 : The function F(x) satisfies $F(x+\pi)=F(x)$ for all real x.

Statement 2 : $\sin^2(x+\pi)=\sin^2 x$ for all real x.

12. Let $f(x)=2+\cos x$ for all real x.

Statement 1 : For each real t, there exists a point c in $[t, t+\pi]$ such that $f'(c)=0$.

Statement 2 : $f(t)=f(t+2\pi)$ for each real t.

13. **Statement 1 :** The curve $y = -\frac{x^2}{2} + x + 1$ is symmetric with respect to the line $x = 1$

Statement 2 : A parabola is symmetric about its axis.

14. Let f be a real-valued function defined on the interval $(-1,1)$ such that

$e^{-x} f(x) = 2 + \int_0^x \sqrt{t^4 + 1}.dt, \forall x \in (-1,1)$ and let f^{-1} be the inverse function of f. Then $[f^{-1}(2)]'$ is equal to:

(A) 1 **(B)** 1/3 **(C)** 1/2 **(D)** 1/e [2010]

15. Let $E_1 = \left\{x \in \mathbb{R} : x \neq 1 \text{ and } \frac{x}{x-1} > 0\right\}$ and $E_2 = \left\{x \in E_1 : \sin^{-1}\left(\log_e\left(\frac{x}{x-1}\right)\right) \textit{is a real number}\right\}$.

(Here, the inverse trigonometric function $\sin^{-1} x$ assumes values in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.) Let $f : E_1 \to \mathbb{R}$ be the function defined by $f(x) = \log_e\left(\frac{x}{x-1}\right)$ and $g : E_2 \to \mathbb{R}$ be the function defined by

$g(x) = \sin^{-1}\left(\log_e\left(\frac{x}{x-1}\right)\right)$. [2018]

List- I		List- II	
P.	The range of f is	**1.**	$\left(-\infty, \frac{1}{1-e}\right] \cup \left[\frac{e}{e-1}, \infty\right)$
Q.	The range of g contains	**2.**	(0, 1)
R.	The domain of f contains	**3.**	$\left[-\frac{1}{2}, \frac{1}{2}\right]$
S.	The domain of g is	**4.**	$(-\infty, 0) \cup (0, \infty)$
		5.	$\left(-\infty, \frac{e}{e-1}\right]$
		6.	$(-\infty, 0) \cup \left(\frac{1}{2}, \frac{e}{e-1}\right]$

The correct option is:

(A) P → 4; Q → 2; R → 1; S → 1 **(B)** P → 3; Q → 3; R → 6; S → 5

(C) P → 4; Q → 2; R → 1; S → 6 **(D)** P → 4; Q → 3; R → 6; S → 5

Functions

Date Planned : __ / __ / __	Daily Tutorial Sheet – 1	Expected Duration : 90 Min
Actual Date of Attempt : __ / __ / __	JEE Advanced (Archive)	Exact Duration: __________

1. Let $f(x) = \sin\left(\frac{\pi}{6}\sin\left(\frac{\pi}{2}\sin x\right)\right)$ for all $x \in R$ and $g(x) = \frac{\pi}{2}\sin x$ for all $x \in R$. Let (*fog*) (*x*) denotes $f\left[g(x)\right]$ and (*gof*) (*x*) denotes $g\left[f(x)\right]$. Then, which of the following is (are) true ? [2015]

(A) Range of *f* is $\left[-\frac{1}{2}, \frac{1}{2}\right]$

(B) Range of *fog* is $\left[-\frac{1}{2}, \frac{1}{2}\right]$

(C) $\lim_{x \to 0} \frac{f(x)}{g(x)} = \frac{\pi}{6}$

(D) There is $x \in R$ such that (*gof*) (*x*) = 1

2. Let $f : \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \to R$ be given by $f(x) = \left[\log(\sec x + \tan x)\right]^3$. Then: [2008]

(A) *f* (*x*) is an odd function

(B) *f* (*x*) is a one-one function

(C) *f* (*x*) is an onto function

(D) *f* (*x*) is an even function

3. Find the range of values of *t* for which $2\sin t = \frac{1-2x+5x^2}{3x^2-2x-1}, t \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$. [2005]

4. If function $f(x) = x^3 + e^{x/2}$ and $g(x) = f^{-1}(x)$, then the value of $g'(1)$ is [2009]

5. A function $f : IR \to IR$, where IR, is the set of real numbers, is defined by $f(x) = \frac{\alpha x^2 + 6x - 8}{\alpha + 6x - 8x^2}$.
Find the interval of values of α for which *f* is onto. Is the functions one-to-one for $\alpha = 3$? Justify your answer. [1996]

6. Let *f* be a one-one function with domain $\{x, y, z\}$ and range $\{1, 2, 3\}$. It is given that exactly one of the following statements is true and the remaining two are false $f(x) = 1, f(y) \neq 1, f(z) \neq 2$ determine $f^{-1}(1)$. [1982]

7. Find the natural number '*a*' for which $\sum_{k=1}^{n} f(a+k) = 16(2^n - 1)$, where the function '*f*' satisfies the relation $f(x+y) = f(x)f(y)$ for all natural numbers x, y and further $f(1) = 2$. [1992]

8. Show that the value of $\frac{\tan x}{\tan 3x}$, wherever defined never lies between $\frac{1}{3}$ and 3. [1992]