Kinematics -4

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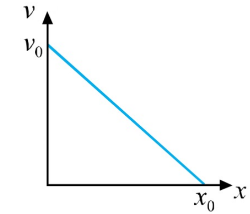
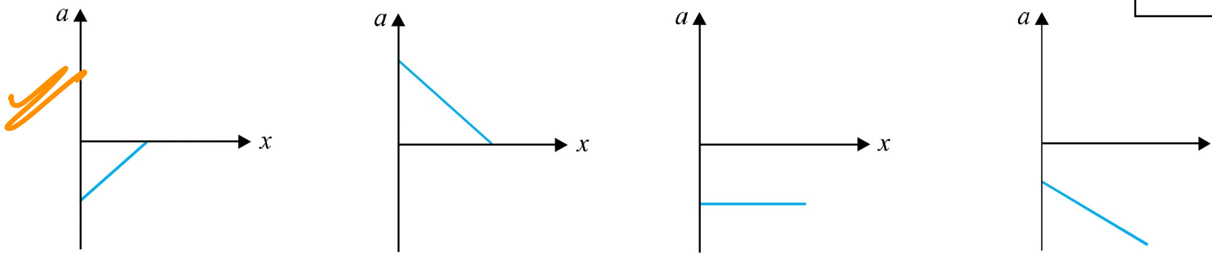
Illustration - 11 *The given graph shows the variation of velocity with displacement. Which one of the graph given below correctly represents the variation of acceleration with displacement ?*





(A) (B) (C) (D)

$$\Rightarrow \quad a = \frac{dv}{dt} = \frac{dv}{dt}\frac{dx}{dx}$$

$$a = \frac{dv}{dt} = v \cdot \frac{dv}{dx}$$

$$\Rightarrow \quad \frac{v}{v_0} + \frac{x}{x_0} = 1$$

$$v = v_0\left(1 - \frac{x}{x_0}\right)$$

$$v = v_0 - \frac{v_0 x}{x_0}$$

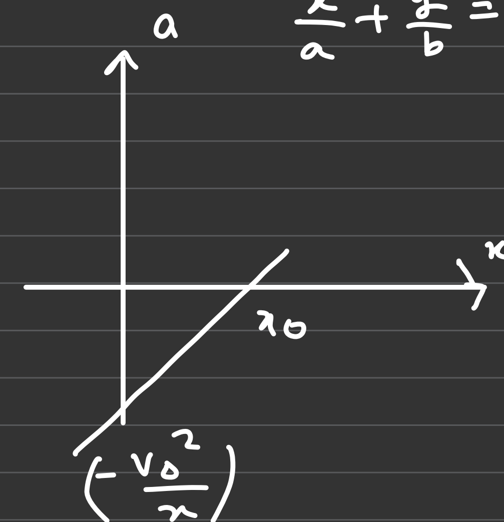
$$a = \left\{ v_0\left(1-\frac{x}{x_0}\right)\right\}\left\{-\frac{v_0}{x_0}\right\} \qquad \left\{\frac{dv}{dx} = \left(-\frac{v_0}{x_0}\right)\right\}$$

$$a = -\frac{v_0^2}{x_0}\left(1-\frac{x}{x_0}\right)$$

$$a = -\frac{v_0^2}{x_0} + \frac{v_0^2 x}{x_0^2}$$

$$\frac{x}{a}+\frac{y}{b}=1$$

$$a - \frac{v_0^2}{x_0^2}x = -\frac{v_0^2}{x_0}$$



$$\frac{a}{\left(-\frac{v_0^2}{x_0}\right)} + \frac{x}{x_0} = 1$$

Important result for integration :

1. (a) $\int t^n dt = \frac{t^{n+1}}{n+1} \quad (n \neq -1)$

   (b) $\int (at+b)^n dt = \frac{1}{a} \frac{(at+b)^{n+1}}{(n+1)}$

2. (a) $\int \frac{dt}{t} = \log t \Rightarrow \int_{t_1}^{t_2} \frac{dt}{t} = \log \frac{t_2}{t_1}$

   (b) $\int \frac{dt}{at+b} = \frac{1}{a} \ell og\,(at+b)$

3. (a) $\int e^t\, dt = e^t$

   (b) $\int e^{at}\, dt = \frac{e^{at}}{a}$

4. (a) $\int \sin t = -\cos t$

   (b) $\int \sin at = \frac{-\cos at}{a}$

5. (a) $\int \cos t = \sin t$

   (b) $\int \cos at\, dt = \frac{\sin at}{a}$

Non-Uniformly accelerated motion :

Conversion

$t=0$

$x(t)$
$\downarrow D$
$v(t)$
$\downarrow D$
$a(t)$

a)

$$a = -kv$$

$$\frac{dv}{dt} = -kv$$

$$\Rightarrow \int_{v=v_0}^{v} \frac{dv}{v} = \int_{t=0}^{t} -k\,dt$$

$$\Rightarrow \left[\ln(v)\right]_{v_0}^{v} = -k\left[t\right]_0^{t}$$

$$\Rightarrow \ln\left(\frac{v}{v_0}\right) = -kt \quad (\underline{\underline{\text{function}}})$$

$$\Rightarrow \frac{v}{v_0} = e^{-kt}$$

$$\Rightarrow \boxed{v = v_0 e^{-kt}}$$

{Q #1 $t = ?$ when $v = 0$

$$0 = v_0 e^{-kt}$$

$\{t = \infty\}$ (<u>after very long time</u>)

{Q #2 find time when velocity is half of initial velocity

$$v = v_0 e^{-kt}$$

$$\frac{\cancel{v_0}}{2} = \cancel{v_0}\, e^{-kt}$$

$$\frac{1}{2} = e^{-kt}$$

$$\ln\left(\frac{1}{2}\right) = \ln\left(e^{-kt}\right)$$

$$\ln(1) - \ln(2) = -kt \ln e$$

$$\checkmark \quad \boxed{t = \frac{\ln 2}{k}}$$

Case II: find all other possible function

$$v = v_0 e^{-kt}$$

$$\frac{dv}{dt} = a = v_0(-k)\, e^{-kt} = -k v_0 e^{-kt}$$

for $x(t) = ?$

$$v = v_0 e^{-kt}$$

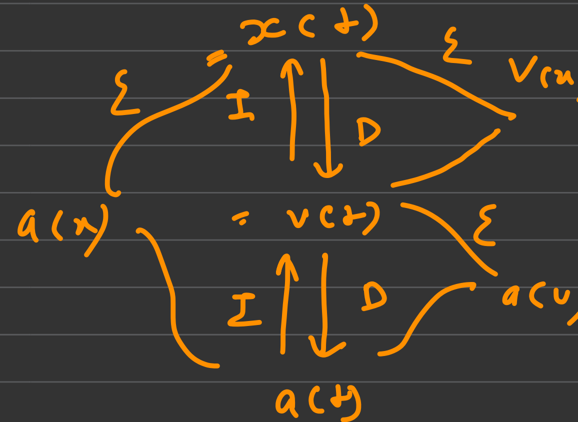
$$\frac{dx}{dt} = v_0 e^{-kt}$$

$$\int dx = \int v_0 e^{-kt} \, dt$$

$$[x]_0^x = v_0 \left[\frac{e^{-kt}}{-k} \right]_0^t$$

$$[x - 0] = \frac{v_0}{-k} \left[e^{-kt} - e^{-0} \right]$$

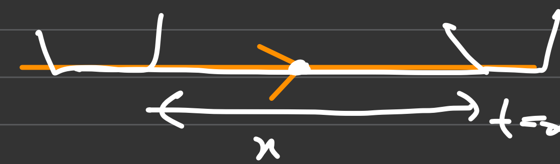
$$\boxed{x = \frac{v_0}{k} \left[1 - e^{-kt} \right]}$$



$$x = \frac{v_0}{k}\left(1 - e^{-kt}\right) \quad \text{— (I)}$$

$$v = v_0\, e^{-kt} \quad \text{— (II)}$$

$$a = -k v_0\, e^{-kt} \quad \text{— (III)}$$



$$x = \frac{v_0}{k}\left(1 - \frac{v}{v_0}\right)$$

$$x = \frac{v_0}{k} - \frac{v}{k}$$

$$v_0 - v = kx$$

$$\boxed{v = v_0 - kx}$$

Q) find distance covered by boat till it stops

$$0 = v_0 - kx$$

$$\left\{ \frac{t = a}{v =} \right\}$$

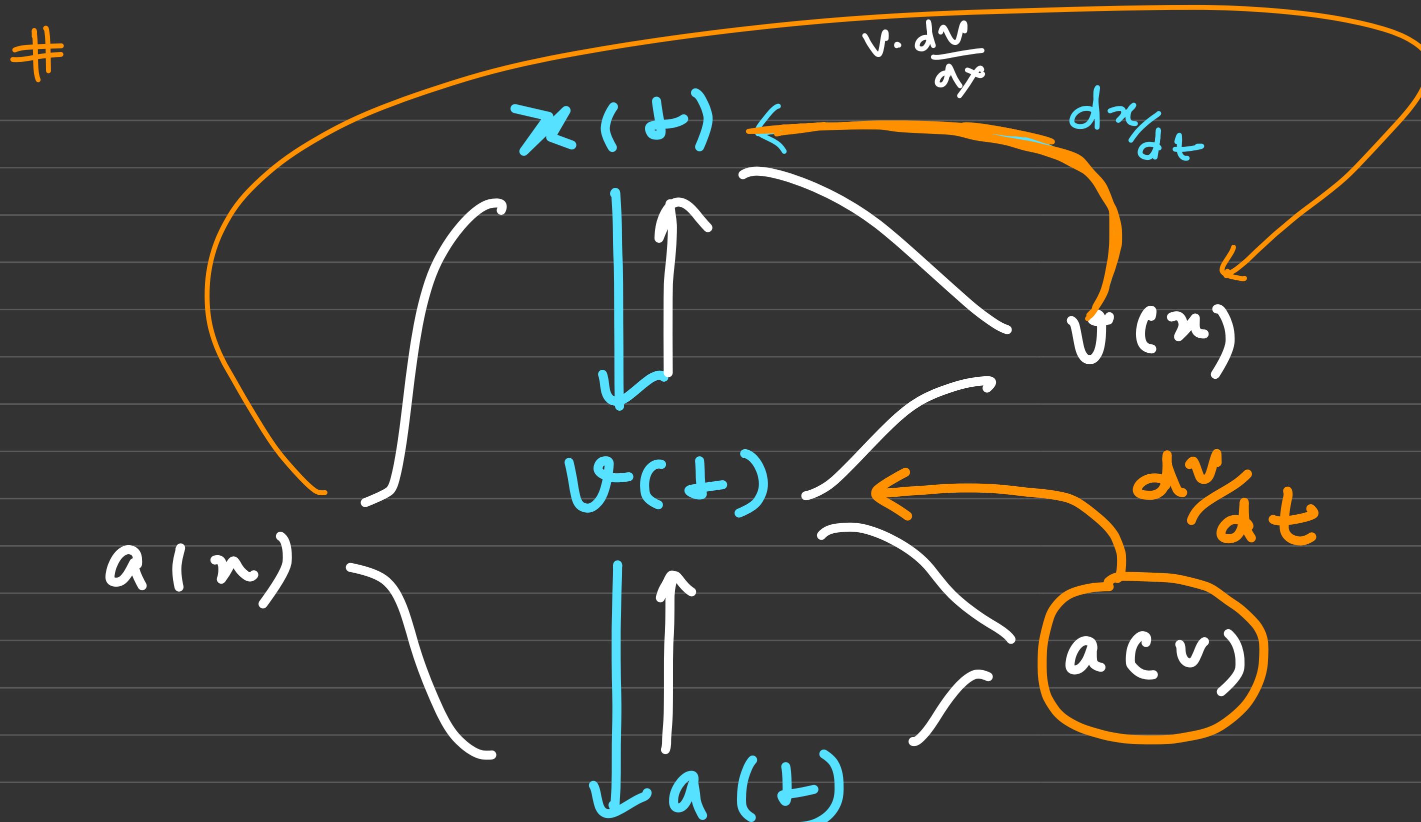
$$\boxed{x = \frac{v_0}{k}}$$

#

$$\frac{v}{a} = \frac{1}{-k}$$

$$a = -kv$$

$$\boxed{x = \frac{v_0}{k}\left(1 + \frac{a}{kv_0}\right)}$$


#
x(t)
v(t)
a(t)
v(x)
a(x)
a(v)
dx/dt
dv/dt
v·dv/dx

Q) if a particle is moving along x-axis as $v = 4 - 4x$ then find all other function

{given = $t = 0$, $x = 0$

$$\frac{dx}{dt} = 4 - 4x$$

$$\frac{dx}{4-4x} = dt \Rightarrow \int_{x=0}^{x} \frac{dx}{1-x} = \int_{t=0}^{t} 4\,dt$$

$$\frac{\ln(1-x)}{-1} = 4t$$

$$\left[\ln(1-x)\right]_{x=0}^{x=x} = \left(-4t\right)_{t=0}^{t}$$

$$\ln(1-x) - \ln(1) = -4t$$

$$\ln(1-x) = -4t$$

$$(1-x) = e^{-4t}$$

$$x = 1 - e^{-4t}$$

$$\begin{cases} v(t) = 4e^{-4t} \\ a(t) = -16e^{-4t} \end{cases}$$

find other (Hw)

Q) if $a = -4x$ then find all other function

$$v \cdot \frac{dv}{dx} = -4x$$

given:

$x = x_0, v = v_0$

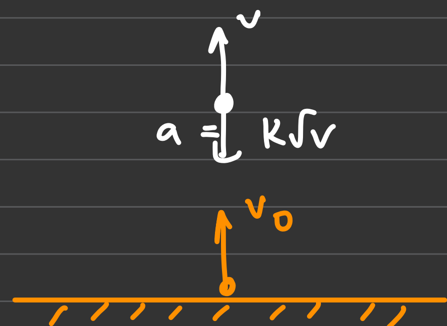
$$\int_{v_0}^{v=v} v \cdot dv = \int_{x=x_0}^{x=x} -4x \, dx$$

$$\left(\frac{v^2}{\cancel{2}}\right)_{v_0}^{v} = \left[-4\left(\frac{x^2}{\cancel{2}}\right)\right]_{x_0}^{x}$$

$$v^2 - {}_0^2 = -4\left[x^2 - x_0^2\right]$$

$$\Rightarrow v = \sqrt{v_0^2 - 4x^2 + 4x_0^2}$$

Illustration - 14 *A stone is thrown up with an initial speed v_0. There is a resisting acceleration $k\sqrt{v}$ due to air, where v is instantaneous velocity and k is some positive constant. Find the time taken to reach the highest point and the maximum height attained by the particle. Assume gravity to be absent.*



\# $a = -k\sqrt{v}$

\# $v(t) = ?$

\# $v(x) = ?$

$$\Rightarrow \frac{dv}{dt} = -k\sqrt{v}$$

$$\Rightarrow \int_{v_0}^{v} \frac{dv}{\sqrt{v}} = \int_{t=0}^{t} -k\,dt$$

$$\left[2\sqrt{v}\right]_{v_0}^{v} = \left(-kt\right)_0^t$$

$$2\left[\cancel{\sqrt{v}}^{\,0} - \sqrt{v_0}\right] = -kt$$

at $v = 0$

max Height

$t = ?$

$$t = \frac{2\sqrt{v_0}}{k}$$

$$a = -k\sqrt{v}$$

$$\left\{v \cdot \frac{dv}{dx}\right\} = -k\sqrt{v}$$

$$\sqrt{v} \cdot dv = -k\,dx$$

$$\left(\frac{v^{3/2}}{3/2}\right)_{v_0}^{v} = \left(-kx\right)_0^x$$

$$\Rightarrow \frac{2}{3}\cancel{v^{3/2}} - \frac{2}{3}v_0^{3/2} = -kx$$

$v = 0$

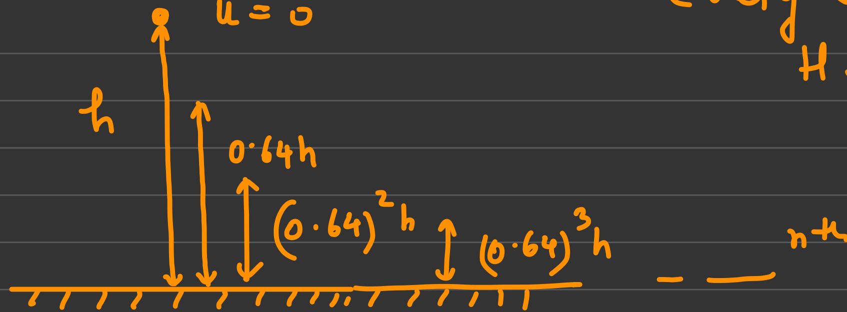
location

maximum

$$\Rightarrow \boxed{\frac{2}{3}\frac{v_0^{3/2}}{k} = x}$$

θ)

every time it rebounds to Height 0.64 times of Previous Height ?



Sign

↑ +ve

↓ −ve

1 # find the no. of rebounds before it stops ?

nth rebound

$$h_{nth} = (0.64)^n h$$

$$0 = (0.64)^n h$$

$$\boxed{n = \infty}$$

Displacement $= -h$

Distance :

$$\Rightarrow h + (0.64)\times 2\times h + (0.64)^2 h\times 2 + \ldots \infty$$

$$\Rightarrow h + 2(0.64)h\left[1 + (0.64) + (0.64)^2 + \ldots \infty\right]$$

$$\Rightarrow h + 2\times 0.64h\left[\frac{1}{1-0.64}\right]$$

$$\Rightarrow h + \frac{2\times 0.64 h}{0.36}$$

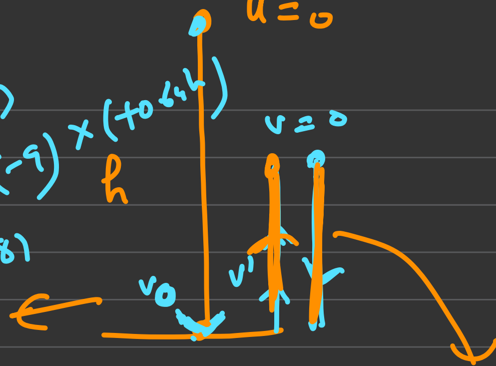
$$\Rightarrow \frac{41h}{9} \quad \underline{Ans}$$

1

$v^2 = u^2 + 2as$

$0 = v'^2 + 2\times(-g)\times(+0.64h)$

$v' = \sqrt{2gh}\,(0.8)$



$s = \frac{1}{2}at^2$

$-h = \frac{1}{2}(-g)\times t^2$

$t = \sqrt{\frac{2h}{g}}$

$-0.64h = \frac{1}{2}(-g)\times t'^2$

$t = \sqrt{\frac{2\times 0.64h}{g}}\times 2$

$v^2 = u^2 + 2as$

$v^2 = 0 + 2\times(-g)(-h)$

$v_0 = \sqrt{2gh}$

$v_0 = \sqrt{2gh}$ ↓ ↑ $0.8\sqrt{2gh}$

$0 = 0.8\sqrt{2gh}\times t - \frac{1}{2}gt^2$

$t = 2\times 0.8\sqrt{\frac{2gh}{g}} = 2\times\sqrt{\frac{0.64h\times 2}{g}}$

$\downarrow \sqrt{2gh} \qquad \uparrow 0.8\sqrt{2gh} \qquad \uparrow (0.8)^2\sqrt{2gh} \qquad \uparrow (0.8)^3\sqrt{2gh} \ldots \infty$

$$= \sqrt{\frac{2h}{g}} + 2\sqrt{\frac{2\times 0.64}{g}} + 2\times\sqrt{\frac{2\times (0.64)^2 h}{g}} + \ldots \infty$$

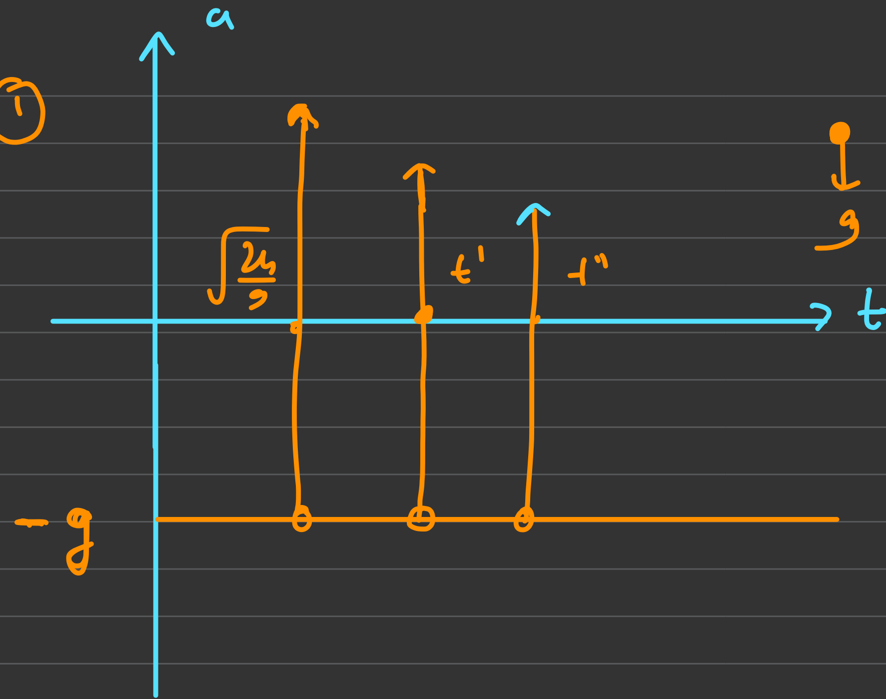
$$= \sqrt{\frac{2h}{g}} + 2\sqrt{\frac{2\times 0.64}{g}}\left[1 + 0.8 + (0.8)^2 + (0.8)^3 + \ldots \infty\right]$$

$$= \sqrt{\frac{2h}{g}} + 2\times\sqrt{\frac{2\times 0.64}{g}}\left(\frac{1}{1-0.8}\right) = \frac{49}{9}\sqrt{\frac{2h}{g}}$$

Ans

graph :

①



g

g

dt

$\downarrow \sqrt{2gh}$ $\uparrow 0.8\sqrt{2gh}$

dt

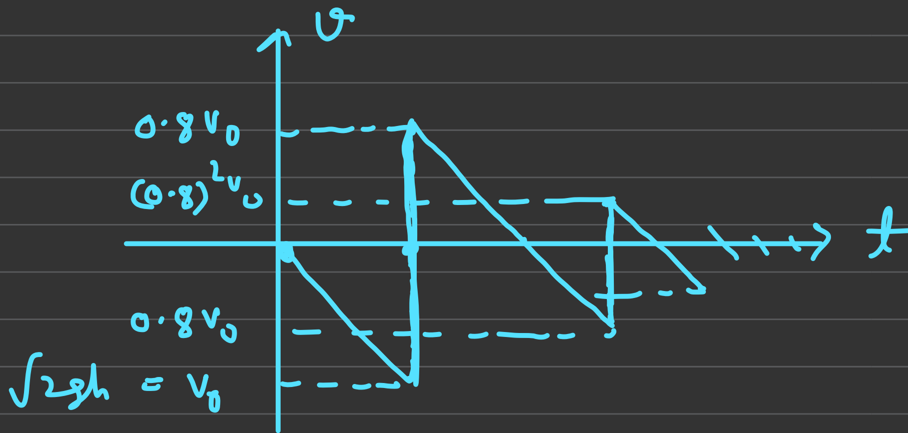
$$= \left(\frac{v_f - v_i}{dt}\right)$$

$$= \frac{(0.8\sqrt{2gh}) - (-\sqrt{2gh})}{dt}$$

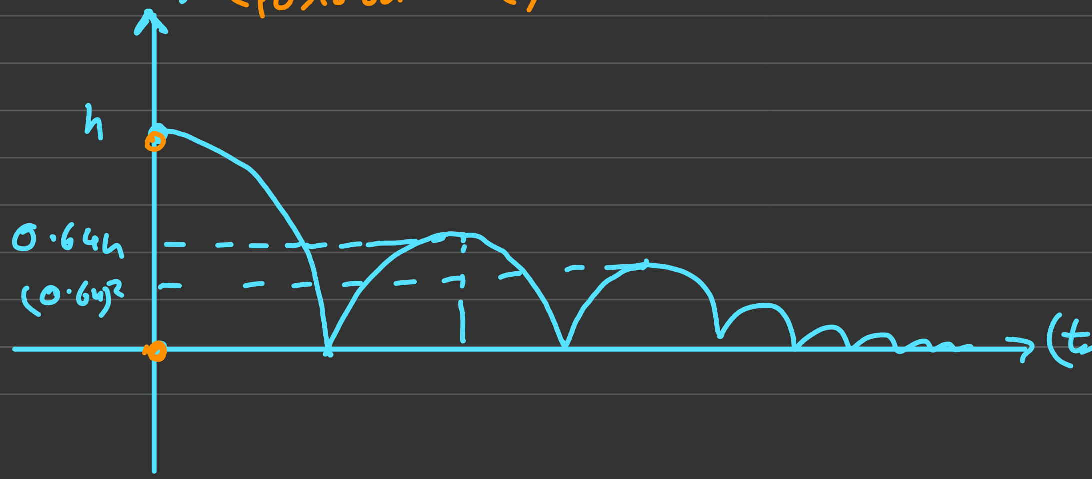
$$= \left(\frac{1.8\sqrt{2gh}}{dt}\right) \uparrow$$

large value

② ($v - t$)



③ x (Position time)

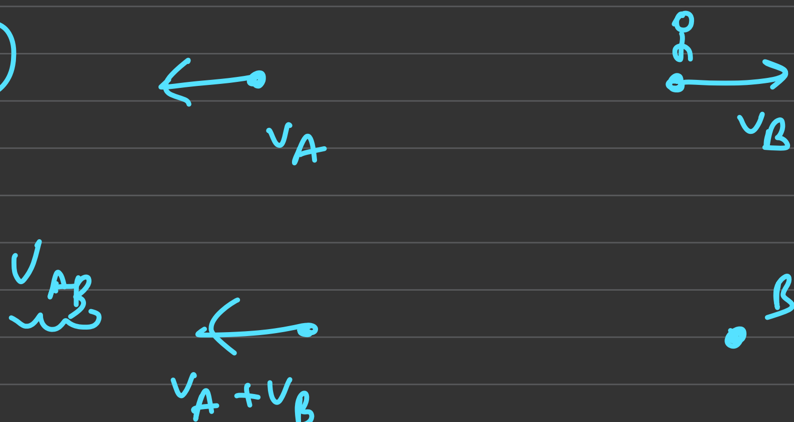


Relative motion :

(1)

Velocity of approach = $(v_A + v_B)$

(2)



Velocity of sep = $v_A + v_B$

③

v_A → v_B →

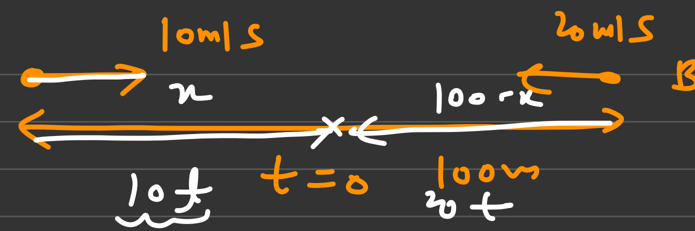
$v_A > v_B$

$\underline{\underline{V_{app}}} = v_A - v_B$ if $v_A > v_B$

$\underline{\underline{V_{sep}}} = v_B - v_A$ if sep $v_B > v_A$

Q)

← −ve   → +ve



10×3.33

A ⇒ $x = 33.3\,m$

B = 67.7

ground frame:

$$x = 10t \quad \text{—(I)}$$

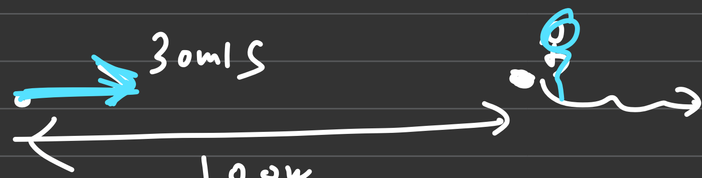
$$100 - x = 20 \times t \quad \text{—(II)}$$

$$30t = 100$$

$$t = \frac{100\,sec}{30} = 3.3\ sec$$

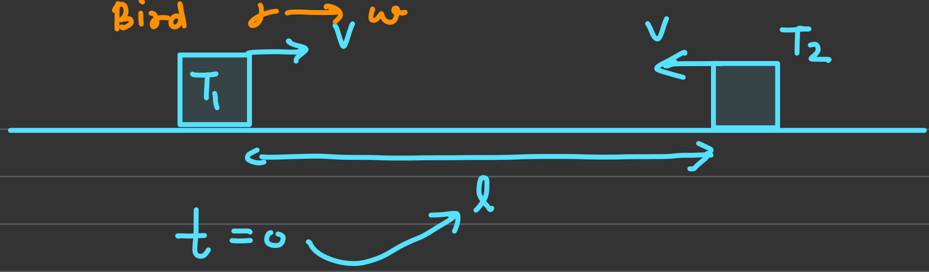
Relative:

w.r.t B



$$t = \frac{100}{30} = 3.3$$

Q)



ω > v

Bird is moving to and fro

Example - 4 *Two trains are approaching each other on a long straight track with constant speed of v km/hr each. When the trains are ℓ km apart, a bird just in front of one train flies at a speed ω km/hr (ω > v) towards the other train. When it arrives just in front of that train, it turns and flies back towards the first train. In this way, it flies back and forth between the two trains until the final moment when it is sandwiched between the trains.*

(a) *Find the total distance travelled by the bird.*

(b) *Taking ℓ = 20 km, v = 50 km/hr, ω = 70 km/hr, draw the v-t and x-t graphs for the problem.*

" find total distance travelled by bird "

Homework

Complete workbook
Complete module