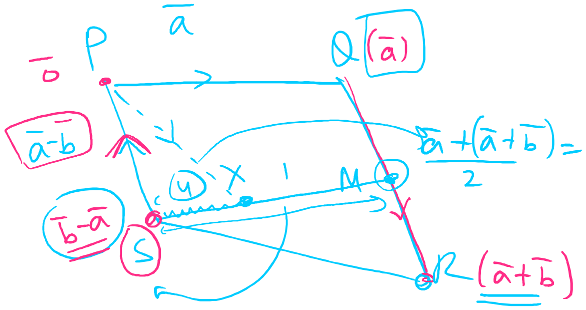


$\frac{\bar{a}+(\bar{a}+\bar{b})}{2} = \frac{2\bar{a}+\bar{b}}{2}$.

$PX = \frac{1}{5} PR$

$PX = \frac{3}{5} PR$

$$\overline{PX} = \frac{4\left(\frac{2\bar{a}+\bar{b}}{\cancel{2}}\right) + \bar{b} - \bar{a}}{5}$$

$$= \frac{4\bar{a} - \bar{a} + 3\bar{b}}{5}$$

$$= \frac{3}{5}(\bar{a}+\bar{b}) = \boxed{\frac{3}{5}\overline{PR}}$$

Binomial	$\boxed{\text{Level-2}}$	Assignment.
Circles	level-2	"
Complex	level-2	"
SL	level-2	"

MUST (at least) 3 topic level-2

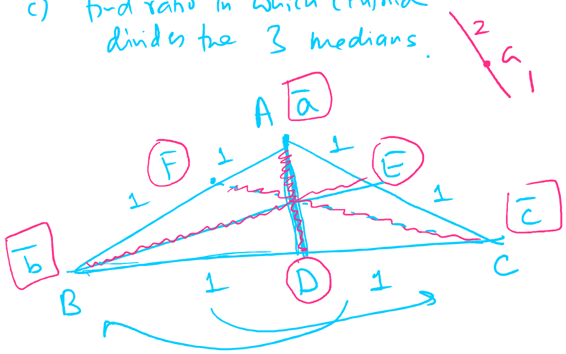
Ques

In a $\triangle ABC$, Prove

a) Medians are Concurrent.

b) Find point of Concurrency. $\boxed{\bar{g} = \frac{\bar{a}+\bar{b}+\bar{c}}{3}}$

c) Find ratio in which Centroid divides the 3 medians.



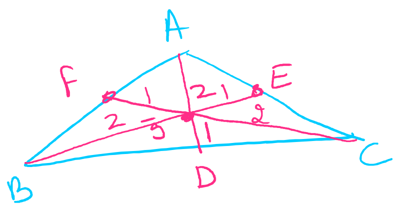
$\bar{d} = \frac{\bar{b}+\bar{c}}{2} \Rightarrow \bar{a} + 2\bar{d} = \bar{b}+\bar{c}+\bar{a}$

$\bar{e} = \frac{\bar{a}+\bar{c}}{2}$, $\bar{b} + 2\bar{e} = \bar{a}+\bar{c}+\bar{b}$

$\bar{f} = \frac{\bar{a}+\bar{b}}{2}$, $\bar{c} + 2\bar{f} = \bar{a}+\bar{b}+\bar{c}$

Make RHS same

$$\frac{\bar{a}+2\bar{d}}{3} = \frac{\bar{b}+2\bar{e}}{3} = \frac{\bar{c}+2\bar{f}}{3} = \frac{\bar{a}+\bar{b}+\bar{c}}{3}$$



$$\bar{g} = \frac{\bar{a}+2\bar{d}}{1+2} = \frac{\bar{b}+2\bar{e}}{1+2} = \frac{\bar{c}+2\bar{f}}{1+2} = \frac{\bar{a}+\bar{b}+\bar{c}}{3}$$

$$\boxed{\bar{g} = \frac{\bar{a}+\bar{b}+\bar{c}}{3}}$$

$\bar{a}$ — m — $\bar{c}$ — n — $\bar{b}$

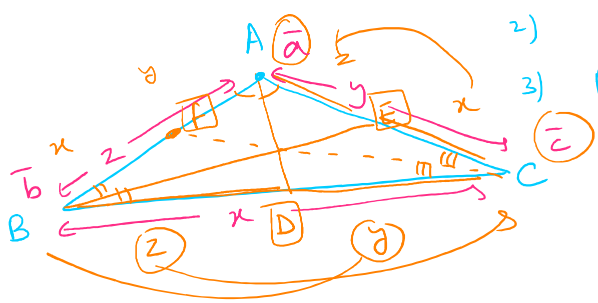
$\bar{c} = \frac{n\bar{a}+m\bar{b}}{n+m}$

Make RHS Same. In $\triangle ABC$

1) Prove angle bisector are concurrent

2) Ratio in which incentre divides 3 medians.

3) P.V. of I.



$BD = z$

$x\bar{a} + (z+y)\bar{d} = y\bar{b} + z\bar{c} + x\bar{a}$

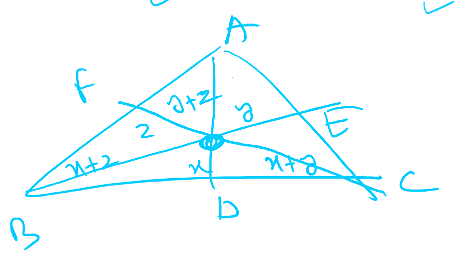
$y\bar{b} + (x+z)\bar{e} = x\bar{a} + z\bar{c} + y\bar{b}$

$$\frac{BD}{DC} = \frac{z}{y}$$

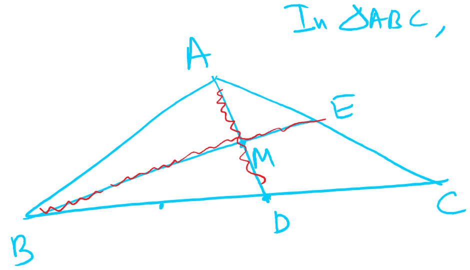
$$y\bar{b} + (x+z)\,\bar{e} = x\bar{a} + z\bar{c} + y\bar{b}$$

$$z\bar{c} + (x+y)\,\bar{f} = x\bar{a} + y\bar{b} + z\bar{c}$$

$$\overset{\bar{I}}{\frac{x\bar{a} + (y+z)\,\bar{d}}{x+y+z}} = \overset{\bar{I}}{\frac{y\boxed{\bar{b}} + (x+z)\,\boxed{\bar{e}}}{x+y+z}} = \overset{\bar{I}}{\frac{z\boxed{\bar{c}} + (x+y)\,\boxed{\bar{f}}}{z+x+y}} = \overset{\bar{I}}{\frac{x\bar{a}+y\bar{b}+z\bar{c}}{x+y+z}}.$$



In $\triangle ABC$, D divides BC in 2:1 ratio,
E divides CA in 2:1 ratio.
If AD & BE intersects at M,
a) find $\frac{AM}{MD}$ and $\frac{BM}{ME}$.

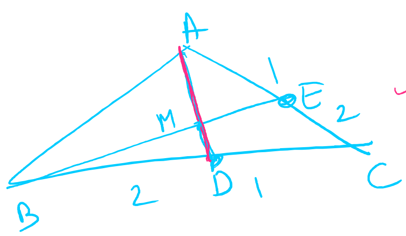


b) If $\bar{a}, \bar{b}, \bar{c}$ are pvs of A, B, C,
find $\bar{m}$ in terms of $\bar{a}, \bar{b}, \bar{c}$.

→ Can't use Equate pvs Method bec $\bar{o}$ is not there.

So Use Make RHS Same

(a) let us assume $\bar{a}, \bar{b}, \bar{c}$ be pvs of A, B, C.



$$3\bar{d} = \boxed{2\bar{c}} + \bar{b} \quad -①$$

$$3\bar{e} = 2\bar{a} + \boxed{\bar{c}} \quad -②$$

Multiply ② by 2.

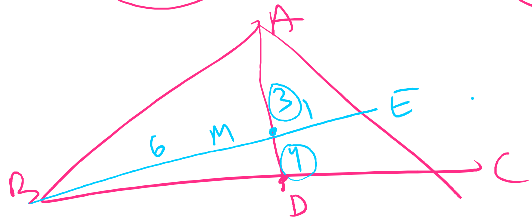
$$4\bar{a} + 3\bar{d} = 2\bar{c} + \bar{b} + 4\bar{a}$$

$$\bar{b} + 6\bar{e} = 4\bar{a} + 2\bar{c} + \bar{b}$$

Now $\bar{c}$ is same in 2 eqns.

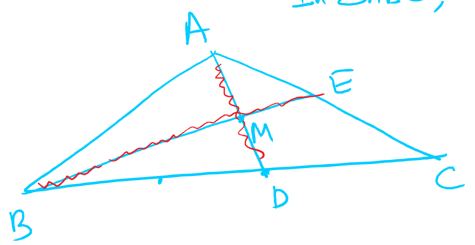
$$\overset{\bar{M}}{(4\bar{a} + 3\bar{d})} = \overset{\bar{m}}{(\bar{b} + 6\bar{e})} = \overset{\bar{m}}{(4\bar{a} + \bar{b} + 2\bar{c})}$$

$$\frac{4\bar{a}+3\bar{d}}{7} = \frac{\bar{b}+6\bar{e}}{7} = \frac{4\bar{a}+\bar{b}+2\bar{c}}{7}$$



Method-2 to solve above question. (Name of the method ≡ Equate PVs)

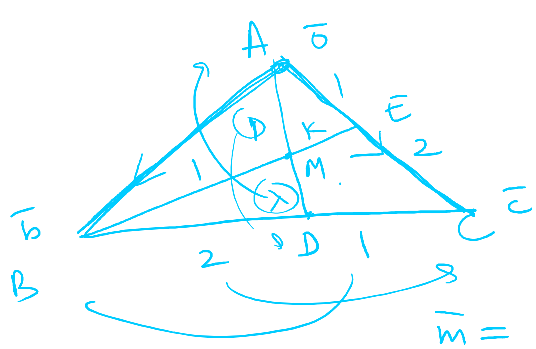
In $\triangle ABC$, D divides BC in 2:1 ratio, E divides CA in 2:1 ratio. If AD & BE intersects at M, find a) $\frac{AM}{MD}$ and $\frac{BM}{ME}$.



let $\bar{o}, \bar{b}, \bar{c}$ be pvs of A, B, C respectively.

(V. Imp. for Method-2, one of the vectors should be $\bar{o}$) It is must.

Assume $\frac{BM}{ME} = \frac{1}{k}$; $\frac{AM}{MD} = \frac{1}{\lambda}$.



Write pv of M on BE.

$$\bar{m} = \frac{k\bar{b}+\bar{e}}{k+1} = \frac{k\bar{b} + \frac{2\bar{o}+1(\bar{c})}{2+1}}{k+1} \quad \text{—(1)}$$

Write pv of M on AD

$$\bar{m} = \frac{\lambda\bar{o}+1(\bar{d})}{\lambda+1} = \frac{\lambda\bar{o} + \frac{2\bar{c}+\bar{b}}{3}}{\lambda+1} \quad \text{—(2)}$$

Using ① & ②, [equate $\bar{m}$ in ① & ②],

$$\frac{k\bar{b} + \frac{\bar{c}}{3}}{k+1} = \frac{2\bar{c}+\bar{b}}{3(\lambda+1)}$$

$$\frac{3k\bar{b} + \bar{c}}{\cancel{3}(k+1)} = \frac{2\bar{c}+\bar{b}}{\cancel{3}(\lambda+1)}$$

$\bar{a}, \bar{b}$ are non-zero, non-coll..

$x_1\bar{a} + y_1\bar{b} = x_2\bar{a} + y_2\bar{b}$

$\Rightarrow$ $x_1 = x_2$; $y_1 = y_2$

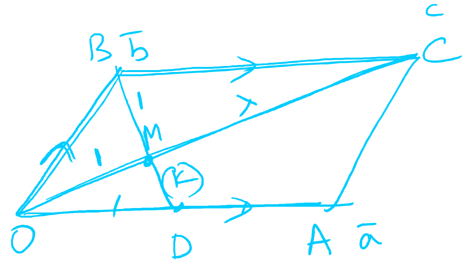
As $\bar{b}$ & $\bar{c}$ are non-collinear, we can do:

$$\boxed{\frac{3k}{k+1} = \frac{1}{\lambda+1}} \quad - ① \quad ; \quad \boxed{\frac{1}{k+1} = \frac{2}{\lambda+1}} \quad - ②$$

$\frac{①}{②}$ $\quad 3k = \frac{1/\cancel{\lambda+1}}{2/\cancel{\lambda+1}}$

$3k = \frac{1}{2}$. $\quad \underline{\underline{k = \frac{1}{6}}}$

In ||gm OACB, D is mid-point of OA. M is intersection of of OC & BD.. Find $\frac{BM}{MD}$ and $\frac{OM}{MC}$.



Equate PVs.

M on OC	M on BD $\quad \boxed{\bar{d} = \bar{a}/2}$
$\bar{m} = \frac{1\bar{c} + \lambda\bar{o}}{1+\lambda}$	$\bar{m} = \frac{k\bar{b} + 1\left(\frac{\bar{a}}{2}\right)}{k+1}$

$\underline{\bar{c} = \bar{a} + \bar{b}}$

or $\quad \overline{BC} = \overline{OA}$

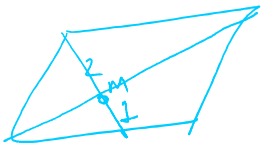
$\bar{c} - \bar{b} = \bar{a} - \bar{o}$

$\boxed{\bar{c} = \bar{a} + \bar{b}}$

Equate $\bar{m}$

$$\frac{\bar{c}}{1+\lambda} = \frac{k\bar{b} + \bar{a}/2}{k+1}.$$

$$\boxed{\frac{\bar{a}+\bar{b}}{1+\lambda}} = \frac{k\bar{b} + \bar{a}/2}{k+1}$$



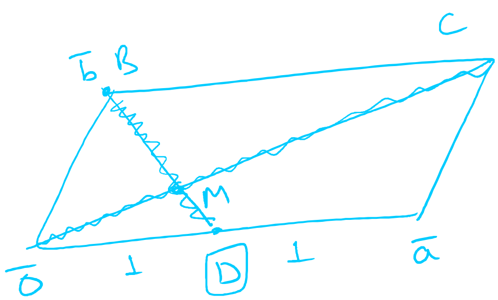
$$\frac{1}{1+\lambda} = \frac{1}{2(k+1)} \; ; \quad \frac{1}{1+\lambda} = \frac{k}{k+1}$$

$$1 = \frac{1(k+1)}{2(k+1)\,k} = \boxed{k = \frac{1}{2}}$$

$$\lambda + 1 = \frac{k+1}{k}$$

$$= 1 + 2$$

$$\underline{\lambda = 2}$$



Find $\bar{m}$.

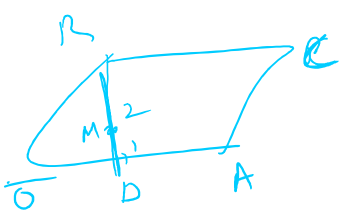
as $\frac{BM}{MD} = ?$.

$$\bar{d} = \frac{\bar{o}+\bar{a}}{2} \quad \Rightarrow \bar{b} + 2\bar{d} = \bar{a} + \bar{b}$$

Use Make RHS Same.

$$\bar{o} \qquad \bar{c} = \bar{a} + \bar{b}$$

$$\frac{\bar{b}+2\bar{d}}{3} = \frac{2\bar{o}+\bar{c}}{3} = \frac{\bar{a}+\bar{b}+\bar{o}}{3}$$



$$\boxed{\frac{\bar{b}+2\bar{d}}{3}} = \frac{\boxed{\bar{c}}}{3} = \frac{\bar{a}+\bar{b}}{3}$$

Scalar Triple product.

$\bar{a}\cdot\bar{b}$	$\bar{a}\times\bar{b}$

$$\underline{\bar{a}\cdot(\bar{b}\times\bar{c})} = \bar{a}\cdot\bar{b}\times\bar{c}$$

$$(2\times 3)\times 5$$

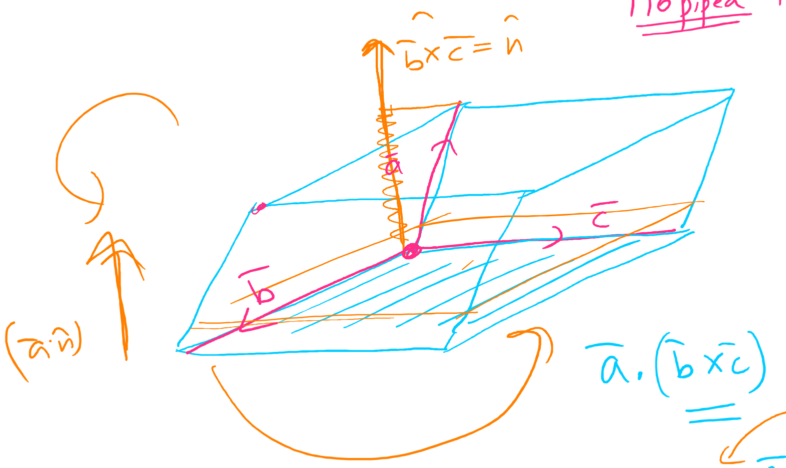
$$= \underline{2\times(3\times 5)}$$

$\bar{a}\cdot(\bar{b}\times\bar{c}) = \bar{a}\cdot\bar{b}\times\bar{c}$ $\qquad = 2\times(3\times5)$

= Scalar Triple product.

Geometrical Significance $\quad \bar{a}\cdot(\bar{b}\times\bar{c})$

llopiped is a Solid whose all faces are llgms.



$\bar{a}, \bar{b}, \bar{c}$ are adjacent edges of llopiped.

$$\bar{a}\cdot(\bar{b}\times\bar{c}) = \bar{a}\cdot\left[|\bar{b}\times\bar{c}|\ \widehat{b\times c}\right]$$

$$= \bar{a}\cdot\left(\text{Area of the llgm made by } \bar{b}\ \&\ \bar{c}\right)\hat{n}$$

$\bar{a}\cdot l\bar{b} = l(\bar{a}\cdot\bar{b})$

$l_1\bar{a}\cdot l_2\bar{b} = l_1 l_2(\bar{a}\cdot\bar{b})$

$= (\text{Area of llgm in base})\,(\bar{a}\cdot\hat{n})$

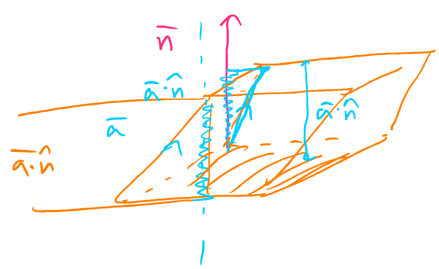
$= (\text{Area of llgm in base})\,(\text{projection of } \bar{a} \text{ along } \hat{n})$.

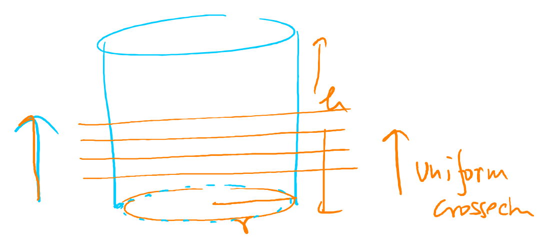
$= \underbrace{(\text{Area of llgm in base})}_{|\bar{b}\times\bar{c}|}\times\underbrace{\text{height of the llopiped}}_{(\bar{a}\cdot\hat{n})}$.

= Volume of the llopiped.

$$\boxed{\text{Volume of llopiped} = |\bar{a}\cdot(\bar{b}\times\bar{c})|}$$

= Scalar Triple product of 3 adjacent edges of the llopiped.



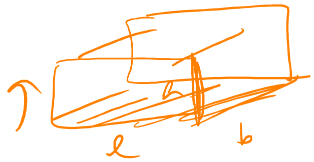


Volume $= (\pi r^2)\,h$

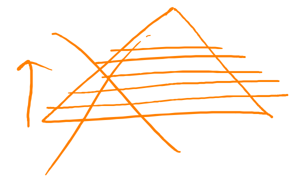
$V = \left[\text{Area of crossection that is uniform} \ldots\right] \times \left[\text{length in the direction}\right]$

$V = \left[\begin{array}{l}\text{Area of cross secn} \\ \text{that is uniform} \\ \text{in certain dir.}\end{array}\right] \times \left[\begin{array}{l}\text{length} \\ \text{in two directn} \\ \text{cross sectn is} \\ \text{uniform.}\end{array}\right]$

$a^2(a) = a^3$



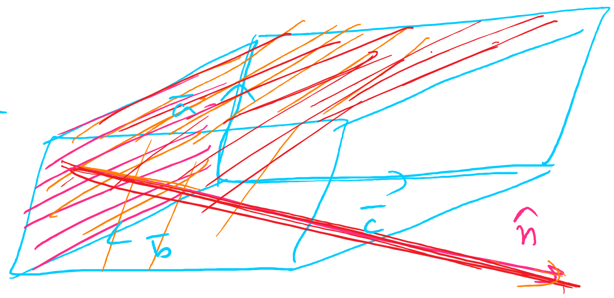
$(lb)h$

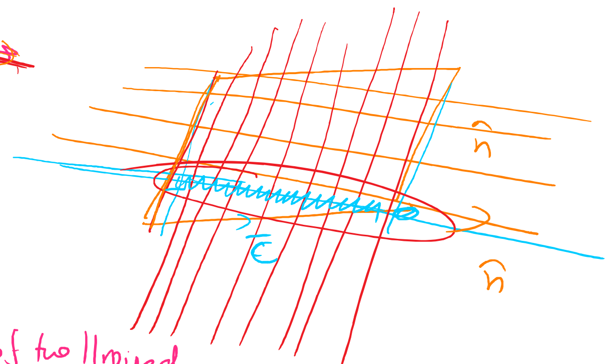


$\bar{a} \cdot (\bar{b} \times \bar{c})$

II I interchange position of $\cdot$ & $\times$ & see what happens.

$\bar{a} \times \bar{b} \cdot \bar{c}$





$(\bar{a} \times \bar{b}) \cdot \bar{c} = |\bar{a} \times \bar{b}| \, \widehat{a \times b} \cdot \bar{c}$

$= (\text{Area of } \| \text{gm on side surface}) \, \hat{n} \cdot \bar{c}$

$= (\text{Area of } \| \text{gm on side surface}) \times \text{width of the } \| \text{piped}.$

$= \text{Volume of } \| \text{piped}.$

$\bar{a} \cdot (\bar{b} \times \bar{c}) = (\bar{a} \times \bar{b}) \cdot \bar{c}$

$\Rightarrow$ In Scalar Triple product, $\cdot$ & $\times$ can be interchanged.

We have a new way of representation of Scalar Triple product.

$\bar{a} \cdot \bar{b} \times \bar{c} = \bar{a} \times \bar{b} \cdot \bar{c} = [\bar{a} \; \bar{b} \; \bar{c}]$

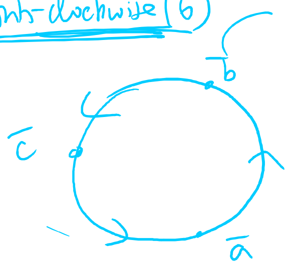
$\bar{a} \cdot \bar{b} \times \bar{c}$

$\bar{a} \times \bar{b} \cdot \bar{c}$

$\bar a, \bar b, \bar c = \boxed{}_{3} \circ_{2} \boxed{}_{2} \circ_{1} \boxed{}_{1}$

$3\times2\times2\times1\times1 = 12$

Anti-clockwise (6)



Anti-clockwise

① $\boxed{\bar a\cdot(\bar b\times\bar c)}$

② $(\bar a\times\bar b)\cdot\bar c$

③ $\bar b\cdot(\bar c\times\bar a)$

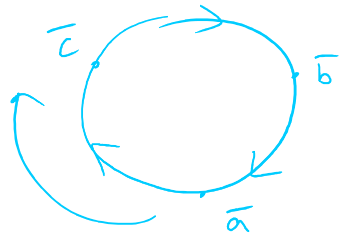
④ $\boxed{(\bar b\times\bar c)\cdot\bar a}$

⑤ $\bar c\cdot(\bar a\times\bar b)$

⑥ $(\bar c\times\bar a)\cdot\bar b$

Equal

Clockwise (6)



$\bar b\cdot(\bar a\times\bar c)$

$(\bar b\times\bar a)\cdot\bar c$

$\bar c\cdot(\bar b\times\bar a)$

$\bar c\times\bar b\cdot\bar a$

$\bar a\cdot(\bar c\times\bar b)$

$(\bar a\times\bar c)\cdot\bar b$

Equal

①=②, ③=④, ⑤=⑥.

①=④

②=⑤

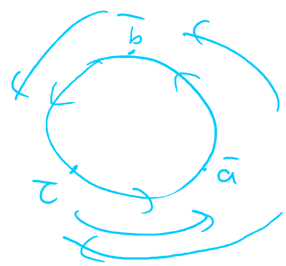
$\bar a\cdot(\bar b\times\bar c) = (\bar b\times\bar c)\cdot\bar a$

$\bar a\cdot\bar b = \bar b\cdot\bar a$

Anti-clockwise $= (\bar a\times\bar b)\cdot\bar c = -(\bar b\times\bar a)\cdot\bar c$

$\boxed{\text{Anti-clockwise} = -\text{Clockwise order}}$

$\bar c\cdot(\bar a\times\bar b) = [\bar a\cdot(\bar b\times\bar c)]$



$= -(\bar a\times\bar c)\cdot\bar b$

Clockwise order.

$$\bar{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$$
$$\bar{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$$
$$\bar{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}.$$

$$\boxed{[\bar{a}\ \bar{b}\ \bar{c}] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}}$$

$$\bar{a} \cdot \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = \bar{a} \cdot \left[(b_2c_3 - c_2b_3)\hat{i} - \hat{j}[b_1c_3 - b_3c_1] + \hat{k}(b_1c_2 - b_2c_1)\right]$$

$$= [a_1\hat{i} + a_2\hat{j} + a_3\hat{k}] \cdot \left[(b_2c_3 - c_2b_3)\hat{i} - [b_1c_3 - b_3c_1]\hat{j} + (b_1c_2 - b_2c_1)\hat{k}\right]$$

$$= \left[(b_2c_3 - c_2b_3)a_1 - (b_1c_3 - b_3c_1)a_2 + (b_1c_2 - b_2c_1)a_3\right]$$

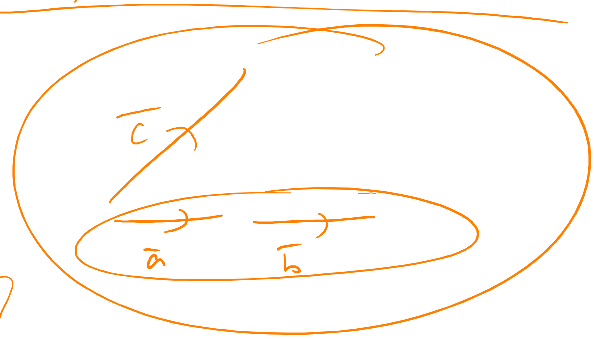
$$= \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

When is $[\bar{a}\ \bar{b}\ \bar{c}] = 0$? Volume of llpiped with $\bar{a}, \bar{b}, \bar{c}$ $= 0$

$$\bar{a} \times \bar{b} \cdot \bar{c} = 0$$

a) Either $\bar{a} = \bar{0}$ or $\bar{b} = \bar{0}$ or $\bar{c} = \bar{0}$.

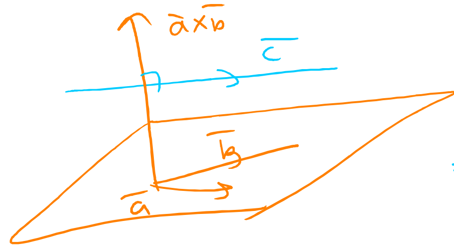
b) $\bar{a} \times \bar{b} = \bar{0}$ ⇒ $\bar{a}$ & $\bar{b}$ are Collinear.
⇒ $\bar{a}, \bar{b}, \bar{c}$ are Coplanar
⇒ No llpiped (No Solid)
Volume $= 0$.

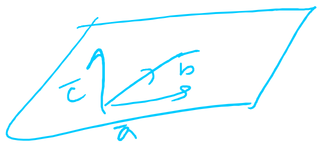


(c) $\bar{a} \times \bar{b} \cdot \bar{c} = 0$

non-zero   non-zero

$\bar{a} \times \bar{b} \perp \bar{c}$





$\bar{c} \perp$ to Normal $(\bar{a}\times\bar{b})$

$\Rightarrow \bar{c}$ is $\parallel$ to the plane of $\bar{a}$ & $\bar{b}$.

$\bar{c}$ is free vector.

$\Rightarrow \bar{c}$ lies in the plane $\bar{a}$ & $\bar{b}$.

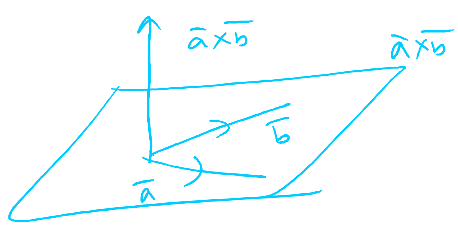
$\Rightarrow \bar{a}, \bar{b}, \bar{c}$ are Coplanar.

$\bar{a}\cdot(\bar{b}\times\bar{a}) = 0$

$$\bar{a}\cdot(\bar{b}\times\bar{a}) = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ a_1 & a_2 & a_3 \end{vmatrix} = 0.$$

$$(\bar{b}\times\bar{a})\cdot\bar{a} = \bar{b}\cdot(\bar{a}\times\bar{a}) = \bar{b}\cdot\bar{0} = 0.$$

$\bar{a}\cdot(\bar{b}\times\bar{a}) = 0$



If in a Scalar Triple product, a vector repeats, then Scalar Triple product is zero.

$\bar{x}\cdot(\bar{y}\times\bar{x}) = 0$

$(\bar{m}\times\bar{n})\cdot\bar{n} = 0$

$\bar{p}\times\bar{q}\cdot\bar{p} = 0$

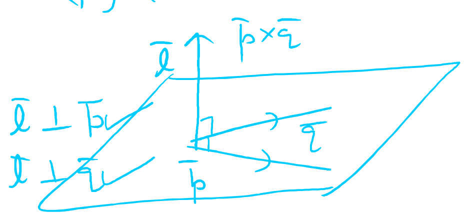
Vector Triple Product

$\bar{l} = \bar{a} \times (\bar{b} \times \bar{c})$

$\bar{a}$: isolated vector

$(\bar{b} \times \bar{c})$ ⇒ vectors in bracket.

$\bar{l} = (\bar{p}) \times (\bar{q})$



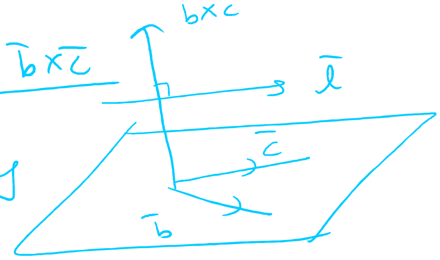
$\bar{l} \perp \bar{p}$

$\bar{l} \perp \bar{q}$

Geometrical Significance

$\bar{l} \perp \bar{a}$ ⇒ $\bar{l}$ is $\perp$ to $\bar{a}$

$\bar{l} \perp \bar{b} \times \bar{c}$



$\bar{l}$ is $\parallel$ to the plane of $\bar{b}$ & $\bar{c}$.

⇒ $\bar{l}$ lies in the plane of $\bar{b}$ & $\bar{c}$

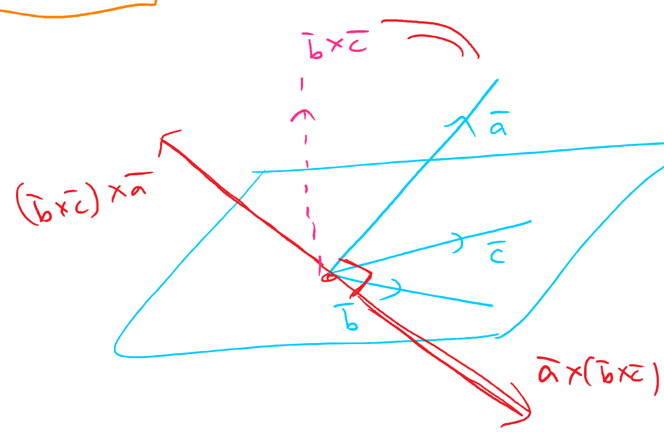
$\boxed{\bar{l} = \bar{a} \times (\bar{b} \times \bar{c})}$

$\bar{l}$ is $\perp$ to $\bar{a}$ & also $\bar{l}$ lies in the plane of $\bar{b}$ & $\bar{c}$.

or

Vector Triple product is $\perp$ to Isolated vector & lies in the plane of vectors in bracket.

$\bar{a} \times (\bar{b} \times \bar{c})$



V.V. Important ≡ Vector Triple product formula.

$\boxed{\bar{a}} \times (\bar{b} \times \bar{c}) = (\bar{a} \cdot \bar{c})\bar{b} - (\bar{a} \cdot \bar{b})\bar{c}$. Linear Combination of vectors in bracket

$= (\bar{a} \cdot \bar{c})\bar{b} - (\bar{a} \cdot \bar{b})\bar{c}$

$x + y = 1$

$2x - 3y = 2$

2x - 3y = 2

$$= (a \cdot c)\,b - (\bar{a} \cdot b)\,\bar{c}$$

$$= (\bar{a} \cdot \bar{c})\,\bar{b} - (\bar{a} \cdot \bar{b})\,\bar{c}.$$

$$\bar{p} \times (\bar{q} \times \bar{r}) = (\bar{p} \cdot \bar{r})\,\bar{q} - (\bar{p} \cdot \bar{q})\,\bar{r}.$$

$$\bar{a} \times (\bar{b} \times \bar{c}) = (\bar{a} \cdot \bar{c})\,\bar{b} - (\bar{a} \cdot \bar{b})\,\bar{c}$$

$$(\bar{a} \times \bar{b}) \times \bar{c} = (\bar{c} \cdot \bar{a})\,\bar{b} - (\bar{c} \cdot \bar{b})\,\bar{a}$$

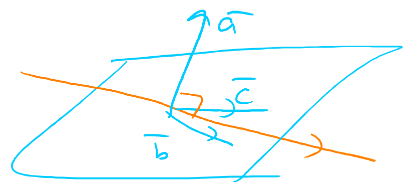
Are these equal?

$$\bar{a} \times (\bar{b} \times \bar{c}) \neq (\bar{a} \times \bar{b}) \times \bar{c}$$

Vector Triple product does not follow associated law.

When is $\bar{l} = \bar{a} \boxed{\times} (\bar{b} \times \bar{c}) = \bar{0}$?

(a) When $\bar{a} = \bar{0}$ or $\bar{b} = \bar{0}$ or $\bar{c} = \bar{0}$

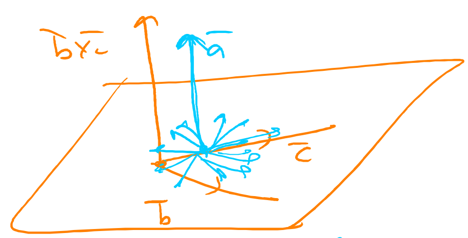


(b) $\bar{b} \times \bar{c} = \bar{0} \Rightarrow \bar{b}$ & $\bar{c}$ are Collinear.

No plane defined by $\bar{b}$ & $\bar{c}$.

(c) $\bar{a}$ is Collinear with $(\bar{b} \times \bar{c})$.

$\Rightarrow \bar{a}$ is normal of the plane of $\bar{b}$ & $\bar{c}$.

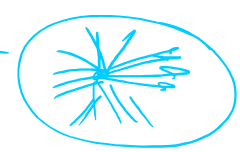


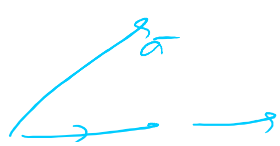
$$0\,\bar{a} = \bar{0}$$
$$0\,\bar{b} = \bar{0}$$
$$0\,\bar{c} = \bar{0}$$

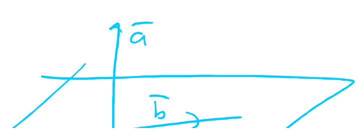
Vector Triple product lies in the plane of $\bar{b}$ & $\bar{c}$ & $\perp$ to $\bar{a}$.

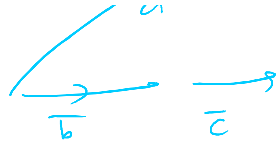
There are infinite such Vectors in the plane.

$\bar{0}$ has infinite direction.

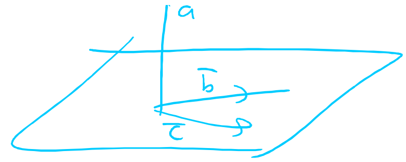








$\bar{a} \times (\bar{b} \times \bar{c}) = \bar{0}$



$\bar{a} \times (\bar{b} \times \bar{c}) = \bar{0}$

$\bar{l} = \boxed{\bar{a}} \times (\bar{a} \times \bar{b}) \neq \bar{0}$.

$\bar{l} \perp \bar{a}$

$\bar{l}$ in the plane of $\bar{a}$ & $\bar{b}$.