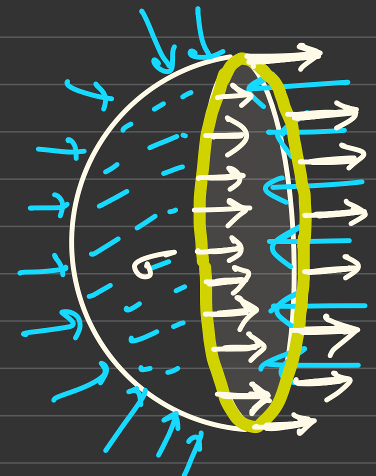
Liquid 5

VMC Vidyamandir Classes SINCE 1986

IIT JEE | MEDICAL | FOUNDATION

liquid drop:

Let us assume this liquid Drop is in equilibrium



$$P_0 R + \sigma 2 = P R$$

#Learn this

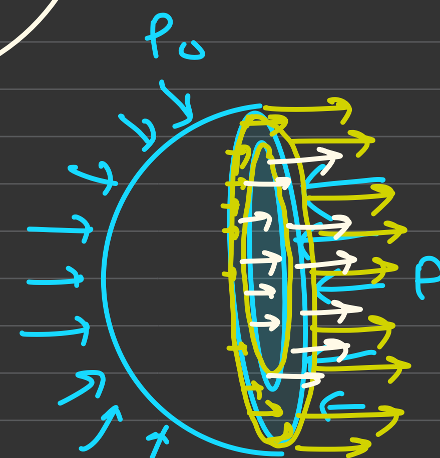
$$(P - P_0) = \frac{2\sigma}{R}$$

$$P_0 \times \pi R^2 + \sigma (2 \pi R) = P \times \pi R^3$$

force balance on Hemisphere

Soap bubble:

$$P_0 \pi R^2 + (\sigma\, 2\pi R) \times 2 = P \pi R^2$$



$$\boxed{(P - P_0) = \frac{4\sigma}{R}}$$

(a) liquid

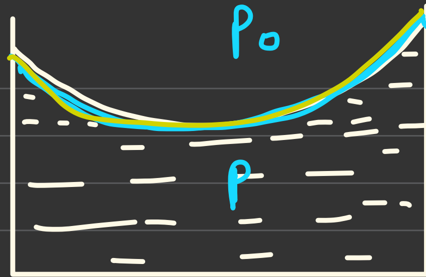
$$P - P_0 = \frac{2\sigma}{R}$$

(b) soap bubble

$$P - P_0 = \frac{4\sigma}{R}$$

" valid only for spherical boundary "

#



→ Spherical surface

→ cylindrical container

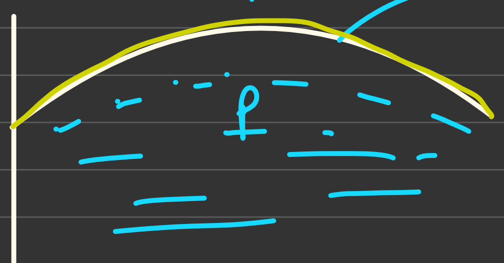
$$\underbrace{P_o > P_o}$$

\# $P_o - P = \frac{2\sigma}{R}$ Radius of meniscus

Po

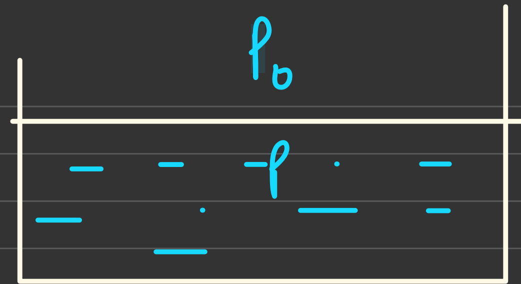
→ Spherical Surface

#



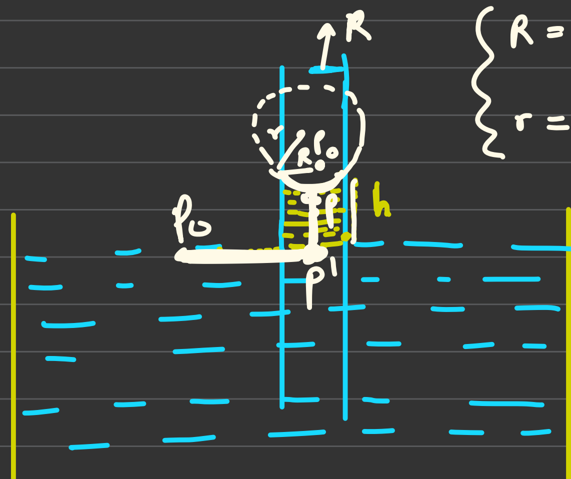
$$P - P_o = \frac{2\sigma}{R}$$

Radius of menisci



$P_0 = P$ flat surface

Capillary :



{ R = radius of cap
r = radius of meniscus.

Method 1:

"using Pr. diff Concept"

$$P_0 - P = \frac{2\sigma}{r}$$

$$P = P_0 - \frac{2\sigma}{r}$$

$$p' = \cancel{p_0} - \frac{2\sigma}{r} + \rho g h = \cancel{p_0}$$

$$\rho g h = \frac{2\sigma}{r}$$

$$\rho g h = \frac{2\sigma}{r}$$

Rise in Height

$$h = \frac{2\sigma}{r\rho g} = h = \frac{2\sigma\cos\theta}{R\rho g}$$

Method 2:

$$\sigma \cdot 2\pi R\cos\theta = mg$$

$$\sigma \cdot 2\pi R\cos\theta = \pi R^2 \times h \cdot g \times \rho$$

$$2\sigma \cdot \cos\theta \cdot \quad R h \rho g$$

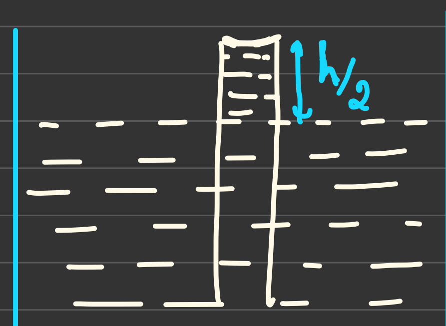
$$\cos\theta = \frac{R}{r}$$

$$2\sigma \frac{\cancel{R}}{r} = \cancel{R} \cdot h\rho g$$

$$\boxed{h = \frac{2\sigma}{\rho g r}}$$ Result

\# 'for given capillary and liquid Rise in Height of liquid is going to fixed"

"Direct voilation of law of Conservation"



"insufficient length of capillary"

"liquid should never overflow"

Radius of meniscus

$$h = \frac{2\sigma}{\rho g r}$$

$$hr = \left(\frac{2\sigma}{\rho g}\right) \Rightarrow h \cdot r = \frac{h}{2} \cdot r'$$

$$\boxed{r' = 2r}$$ Ans

$hr = \text{const}$ {in case insufficient length of capillary}

Angle of contact

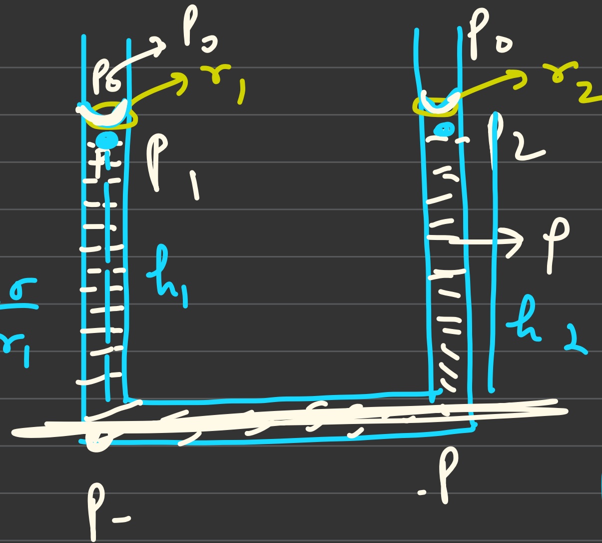
$$h = \frac{2\sigma \cos\theta}{\rho g R}$$

$R \rightarrow$ Radius of capillary

$$\frac{h}{\cos\theta} = \frac{h'}{\cos\theta'}$$

$\theta' \rightarrow$ New angle of contact

Q)



$$P_0 - P_1 = \frac{2\sigma}{r_1}$$

$$P_1 = P_0 - \frac{2\sigma}{r_1}$$

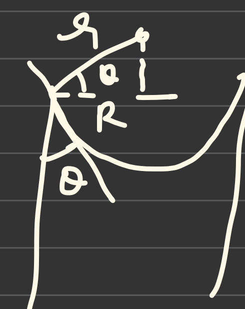
Angle of Contact θ°

"find diff in Height"

$r_1 > r_2$

$r_1 > r_2$

$P_1 > P_2$



$$\cos\theta = \frac{R}{r}$$

$$R = r$$

Radius of Capillaries :

$$P_1 + h_1 \rho g = P_2 + h_2 \rho g$$

$$\cancel{P_0} - \frac{2\sigma}{r_1} + h_1 \rho g = \cancel{P_0} - \frac{2\sigma}{r_2} + h_2 \rho g$$

$$(h_2 - h_1)\rho g = 2\sigma\left(\frac{1}{r_2} - \frac{1}{r_1}\right)$$

$$\underline{\Delta h} = \underline{(h_2 - h_1)} = \frac{2\sigma}{\rho g}\left(\frac{1}{r_2} - \frac{1}{r_1}\right)$$

$$\underline{h_2 > h_1}$$

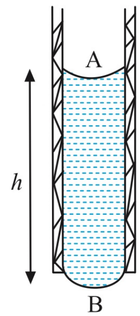
if $r_1 > r_2$

then $\left(\frac{1}{r_2} - \frac{1}{r_1}\right) = +ve$

Q)

Illustration - 34 *What is the maximum height of water column which a capillary of diameter 2r can hold without leaking ? The capillary is open at both ends.*

SOLUTION :



When water is filled in the open capillary, two meniscuses are formed one at top and the other at bottom end. Again going from A to B and adding pressure changes, we get :

$$P_A - \frac{2\sigma}{r} + h\rho g - \frac{2\sigma}{r} = P_B$$

and $$P_A = P_B = P_{atm.}$$

$$\Rightarrow \quad h\rho g = \frac{2\sigma}{r} + \frac{2\sigma}{r} \quad \Rightarrow \quad h = \frac{4\sigma}{r\rho g}$$

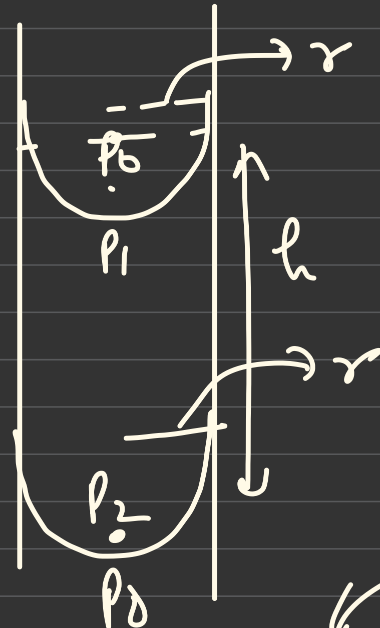
for maxim Height

force balan

$$\left\{ 2\times\sigma\times 2\cancel{\pi}\cancel{r} = \cancel{\pi}\, r^{\cancel{2}} \times h \times \rho \times g \right.$$

$$\left. \boxed{\frac{4\sigma}{\rho g r} = h} \right\}$$

#



$h >> r$

$$P_0 - P_1 = \frac{2\sigma}{r}$$

$$P_1 = P_0 - \frac{2\sigma}{r}$$

$$P_2 = P_0 - \frac{2\sigma}{r} + \rho g h$$

$$P_2 - P_0 = \frac{2\sigma}{r}$$

$$\cancel{P_0} - \frac{2\sigma}{r} + \rho g h - \cancel{P_0} = \frac{2\sigma}{r}$$

$$\rho g h = \frac{4\sigma}{r}$$

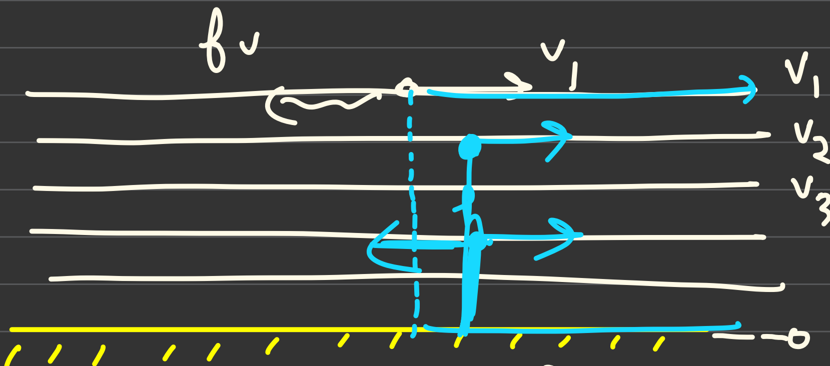
$$h_{max} = \frac{4\sigma}{r\rho g}$$

An

: Viscosity :

"non-Ideal fluid"

"friction in liquid"



$$\frac{\Delta v}{\Delta x} = \frac{v_1 - 0}{x_1 - 0}$$

$$\frac{\Delta v}{\Delta x} = \frac{v_1}{h}$$

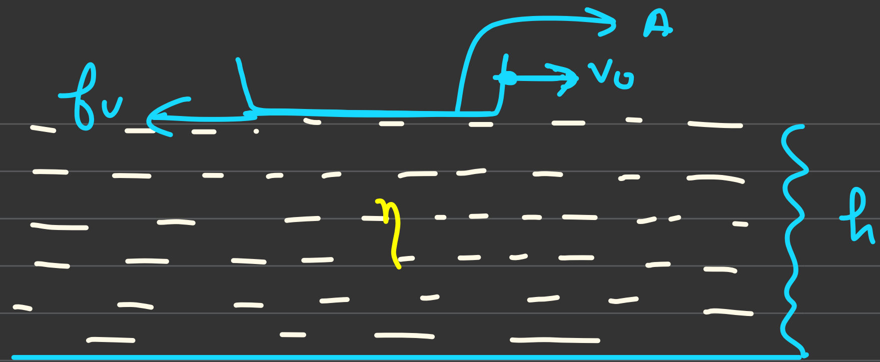
$$f_v = -\eta\, A \cdot \frac{dv}{dx}$$

A → Surface Area of liquid surface in contact

$\frac{dv}{dx}$ → velocity gradient

η → Coefficient of viscosity

f_v → Viscous force



"find ext force on boat for which boat moves with constant velocity?"

Velocity gradient.

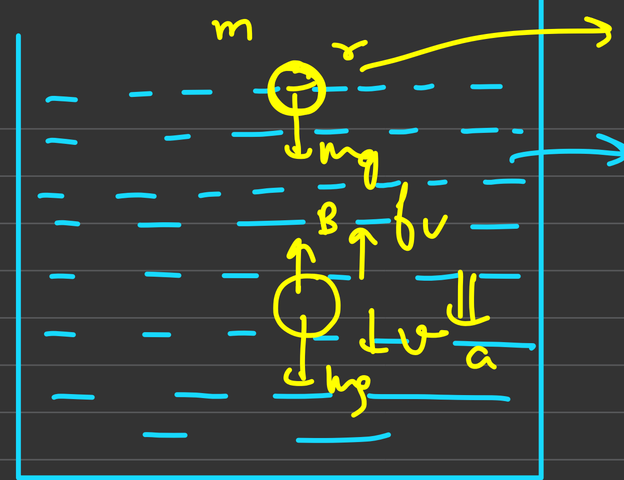
$$\frac{v_0}{h}$$

$$f_v = -\eta A \cdot \frac{v_0}{h}$$

$$|f_{ext}| = \eta \frac{A v_0}{h} \longrightarrow \underline{\underline{\text{Ans}}}$$

Stokes law:

"if a spherical body is moving in liquid then it must exp viscous force over it"



initially $mg > B$

and the value of viscous force is going to be for spherical body

$$f_v = 6\pi\eta r v$$

Stokes law

v → velocity of sphere

valid spherical body

r → radius of sphere

Direction of f_v is going to be in the opposite direction of velocity

at any time 't'

$$mg - B - f_v = ma$$

$$mg - V_{im} \rho g - 6\pi\eta r V = ma \quad \text{--- (1)}$$

" terminal velocity of Sphere "

when velocity of body
become const
or net force become
zero

$$mg - \frac{4}{3}\pi r^3 \times \rho \times g - 6\pi\eta r V_T = 0$$

$$V_T = \frac{\frac{4}{3}\pi r^3 \times d g - \frac{4}{3}\pi r^3 \rho g}{6\pi\eta r}$$

density of sphere

$$V_T = \frac{\frac{4}{3}\pi r^3 g}{\times 6\pi\eta r}\left[d - \rho\right]$$

$$V_T = \frac{2}{9}\frac{r^2 g}{\eta}\left[d - \rho\right]$$

Learn

find $V(t)$ before reaching terminal velocity.

$$\underbrace{\frac{mg - \frac{4}{3}\pi r^3 \times d \times g}{m}}_{C} - \underbrace{\frac{6\pi\eta r}{m}}_{D} V = a$$

$$\Rightarrow \quad a = C - DV \qquad v(t) = ?$$

$$\frac{dv}{dt} = C - DV$$

$$\int \frac{dv}{C - DV} = \int dt$$

$$\left[\frac{\ln(c - Dv)}{-D}\right]_{v=0}^{v} = \left[t\right]_{t=0}^{t}$$

$$\ln(c - Dv) - \ln(c) = -Dt$$

$$\frac{c - Dv}{c} = e^{-Dt}$$

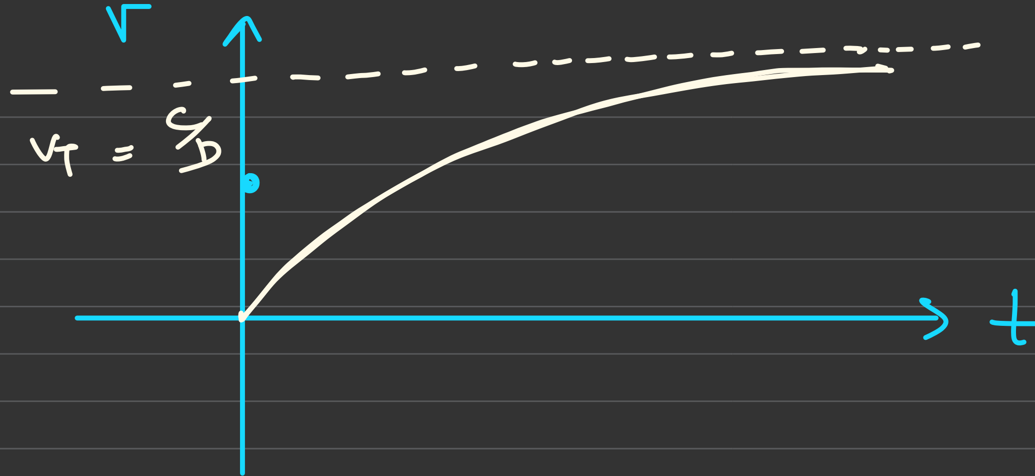
$$a(t) = c\, e^{-Dt}$$

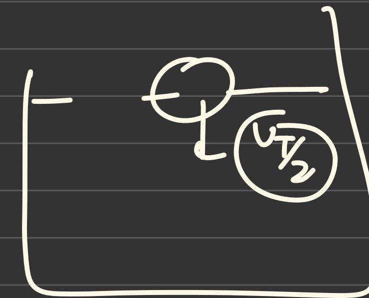
$$a = 0 \text{ at } t = \infty$$

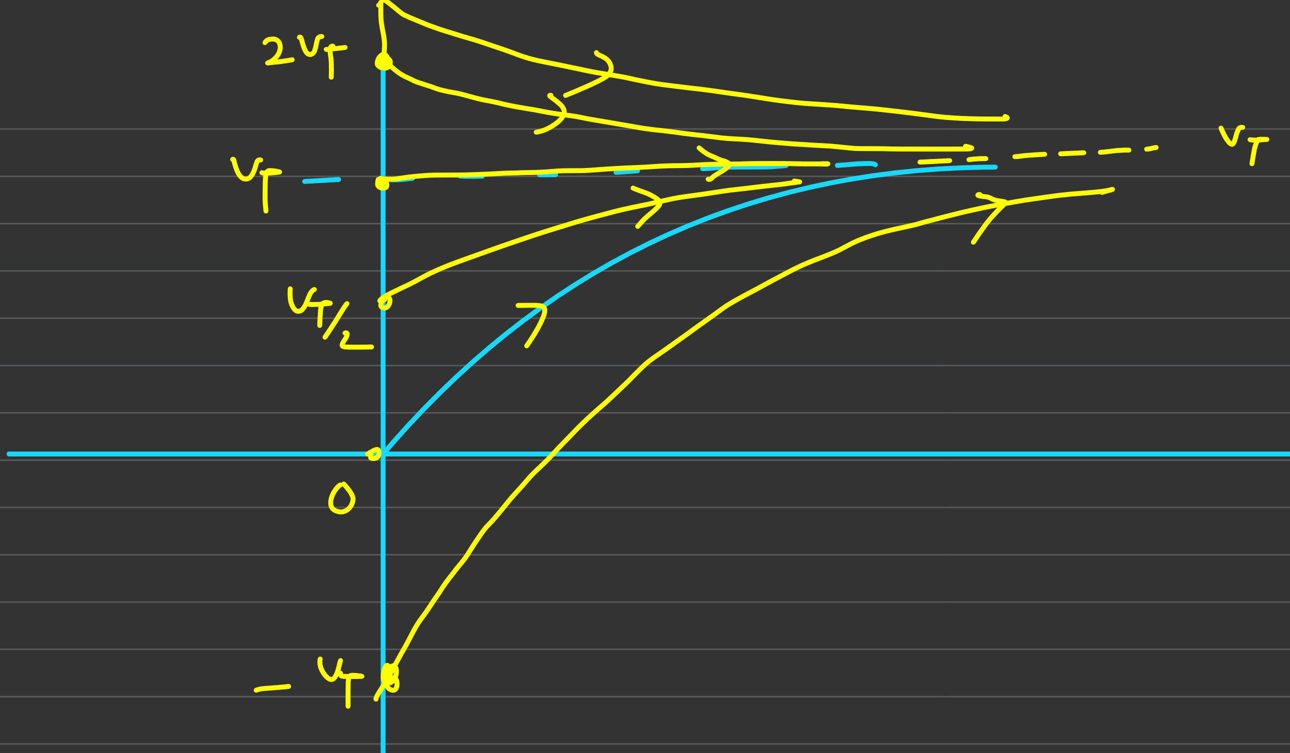
$$c - Dv = c\, e^{-Dt}$$

$$v(t) = \frac{c}{D}\left(1 - e^{-Dt}\right)$$

$$y = e^{x}$$


v
$v_T = g/b$
t


$v_T/2$



$2v_T$
v_T
v_T
$v_T/2$
0
$-v_T$