Gravitation-2

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IIT JEE | MEDICAL | FOUNDATION

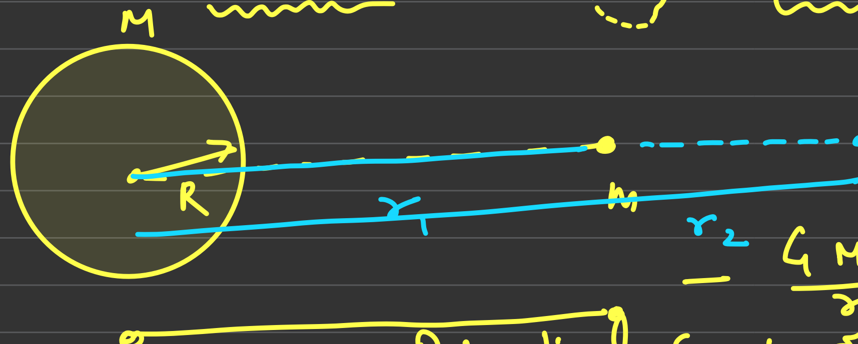
System of Particles (Potential)

$$U(r) = -\frac{G m_1 m_2}{r}$$

#

$U_{sp} = K$

$U_T = U_{sp} + U_p$



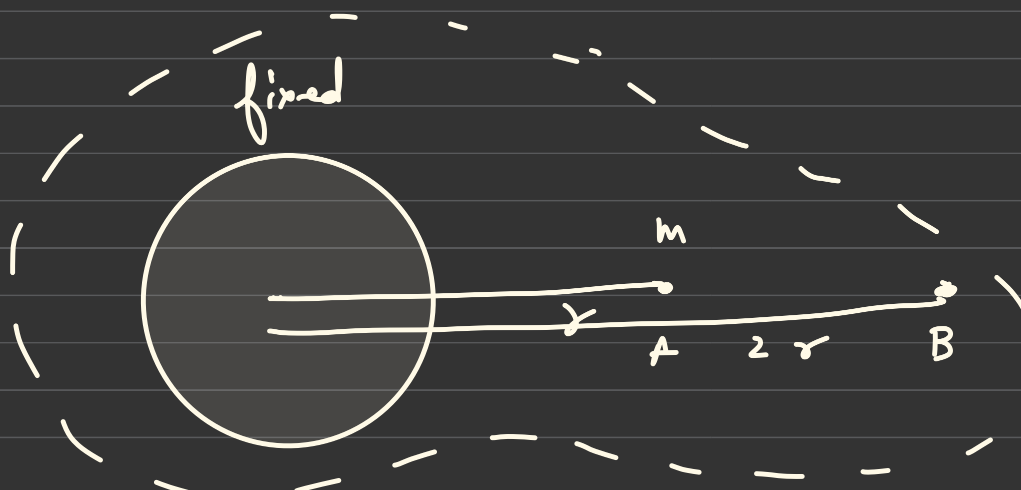
$$-\frac{GMm}{r}$$

Potential Energy of Point mass

if point object is outside the sphere then in that case we can assume sphere as point mass

Potential Energy of point object with respect sphere

$$U(r) = -\frac{GMm}{r}$$ only valid for if point object is outside

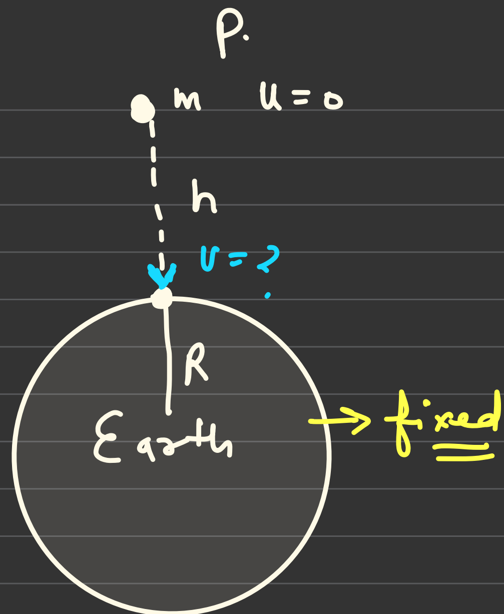


$$U_A = U_{sphere} + (U_p)_A$$

$$U_B = \cancel{U_{sphere}} + (U_p)_B$$

$$U_B - U_A = (U_p)_B - (U_p)_A$$

Q)



if Point object is released from rest the velocity of Point object when it hits the surface?

$m_1 \overset{r}{\text{———}} m_2 \Rightarrow -\frac{Gm_1m_2}{r}$

loss in GPE = gain in KE

$$\Rightarrow -\frac{G\cancel{m}M}{(R+h)} - \left(-\frac{G\cancel{m}M}{R}\right) = \frac{1}{2}\cancel{m}v^2$$

$$\Rightarrow -\frac{GM}{(R+h)} + \frac{GM}{R} = \frac{1}{2}v^2$$

$$\Rightarrow mg = +\frac{GmM}{R^2}$$

$$\Rightarrow g = +\frac{GM}{R^2}$$

$$\underline{\underline{+GM = gR^2}}$$

$$\Rightarrow -\frac{gR^2}{R+h} + \frac{gR^2}{R} = \frac{1}{2}v^2$$

$$\Rightarrow +gR^2\left[\frac{1}{R} - \frac{1}{R+h}\right] = \frac{1}{2}v^2$$

$$= g\cancel{R^2}\left[\frac{\cancel{R}+h-\cancel{R}}{\cancel{R^2}+\cancel{R}h}\right] = \frac{1}{2}v^2$$

$$= \frac{ghR}{R+h} = \frac{1}{2}v^2$$

$$\underline{\underline{\{h << R\}}}$$

if we are near

$$gh = \frac{1}{2} v^2$$

$$\boxed{v = \sqrt{2gh}}$$

$v =$

: motion of Satellite :

① **Escape velocity** : "it is the $\overset{\text{min}}{\text{velocity}}$ with which it leaves gravitational field of planet"

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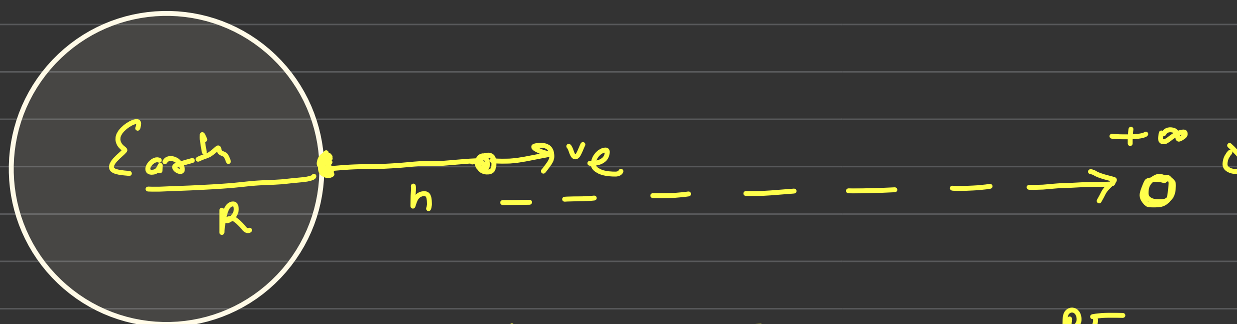


$\Rightarrow$ loss in KE = gain PE

$$= \left(\frac{1}{2}mv_e^2 - 0\right) = 0 - \left(-\frac{GMm}{R+h}\right)$$

$v_e = \sqrt{\dfrac{2GM}{(R+h)}}$ Height

if $h = 0$ (Surface)

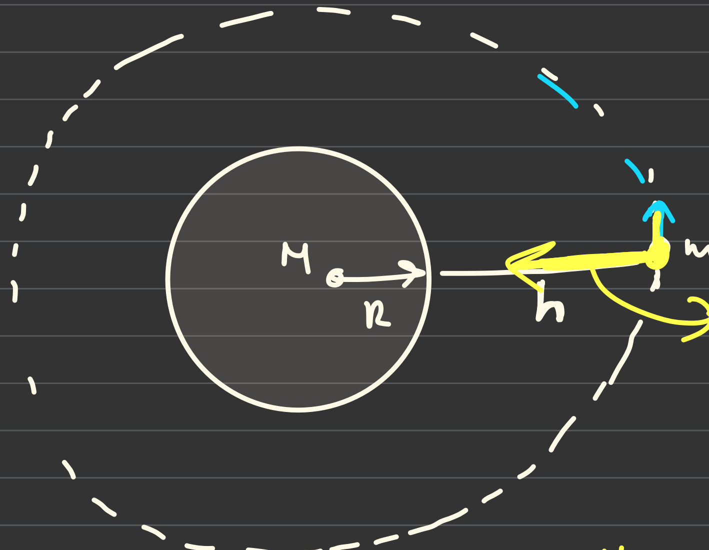
$v_e = \sqrt{\dfrac{2GM}{R}}$ surface

$v_e = \sqrt{\dfrac{2gR^2}{R}} = \sqrt{2gR}$

at the Surface of earth

$= 11.2$ km/sec

(ii) <u>Orbital velocity</u>: (Satellite)



$$\frac{G\cancel{m}M}{(R+h)^{\cancel{2}}} = \frac{\cancel{m}v^2}{\cancel{(R+h)}}$$

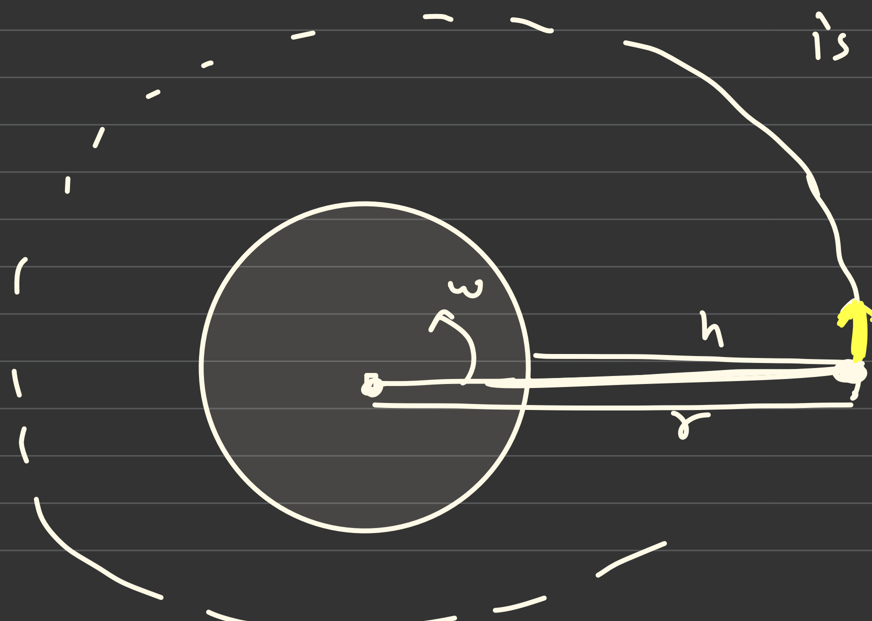
"Orbital Velocity"

$$v_0 = \sqrt{\frac{GM}{R+h}}$$

at surface = $(v_0) = \sqrt{\frac{GM}{R}}$

$\frac{v_e}{v_o} = \sqrt{2}$

(iii) Geostationary Satellite:



if time period of satellite is equal to time period of planet (in our case earth) same then geostationary

$$r = R + h$$

$$V_0 = \sqrt{\frac{GM}{(R+h)}}$$

$$T = \frac{2\pi(R+h)}{V_0}$$

$$R + h = r$$

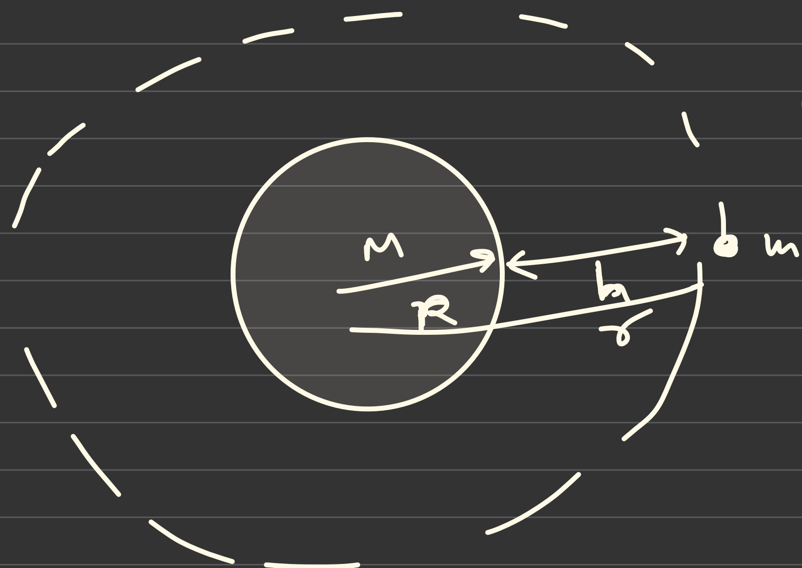
$$T = \frac{2\pi r}{\sqrt{\frac{GM}{r}}}$$

$$T^2 = \frac{4\pi^2 r^3}{GM} \Rightarrow r = 42000\text{ km}$$

center of earth

$$T^2 \propto r^3$$

(iv) Energy of Satellite!



$$(KE)_s = \frac{1}{2} m (v_0)^2$$

$$= \frac{1}{2} m \left(\sqrt{\frac{GM}{r}}\right)^2$$

$$(KE)_s = \frac{GmM}{2r}$$

$$(PE)_s = -\frac{GmM}{r}$$

Total Energy = $(KE)_s + (PE)_s$

$$= \frac{GmM}{2r} - \frac{GmM}{r} = -\frac{GmM}{2r}$$

$$KE = -TE = -\frac{PE}{2}$$ Ans

Gravitational Potential !

it is defined at point

$$= -\frac{G\, m_1 m_2}{r}$$

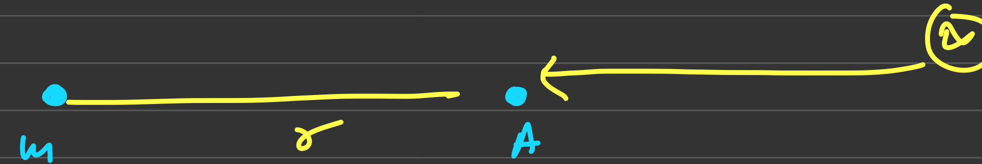
$V_{m_2} = -\frac{Gm_1}{r}$

$V_{m_1} = -\frac{Gm_2}{r}$

"amount of work done by external agent to bring A unit positive mass from infinite to that point slowly"

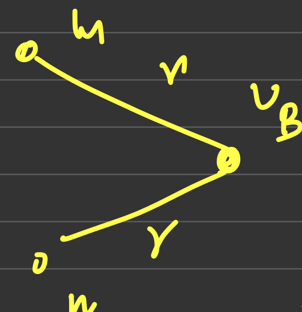
Q)

(i)



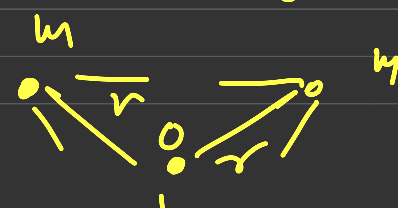
$$V_A = -\frac{GM}{r}$$

(ii)



$$V_B = -\frac{Gm}{r}\times 2$$

(iii)



$$V_0 = -3\frac{GM}{r}$$

$\backslash b^{r}/ \quad w$

1. *A satellite moves in a circular orbit round the earth at height $R_e/2$ from earth's surface when R_e is the radius of the earth. Calculate its period of revolution.*

Solution:

$$T^2 = \frac{4\pi^2}{GM} r^3$$

$$T = \frac{2\pi}{\sqrt{GM}} \left(\frac{3R_e}{2} \right)^3$$

$$T = \frac{2\pi}{\sqrt{gR_e^2}} \left(\frac{27 R_e^3}{8} \right)^{1/2}$$

Ans

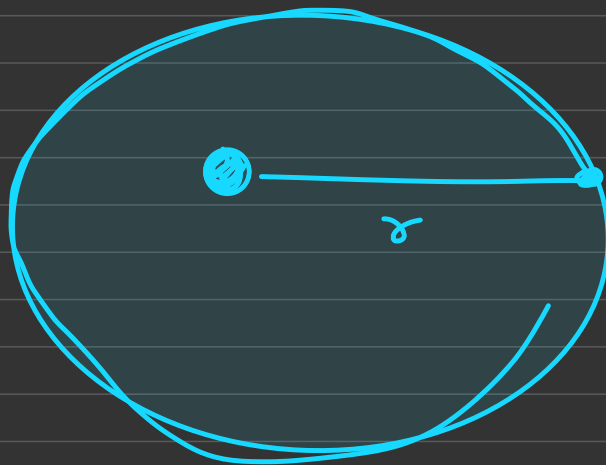
5. *If the earth is at one fourth of its present distance from the sun, the duration of the year will be :*

(A) *half the present year*

(B) *one-eighth the present year*

(C) *one-fourth the present year*

(D) *one-sixth the present year*



$$T^2 \propto r^3$$

$$T_1 = K\,(r)^{3/2}$$

$$T_2 = K\left(\frac{r}{4}\right)^{3/2}$$

$$\frac{T_1}{T_2} = (4)^{3/2}$$

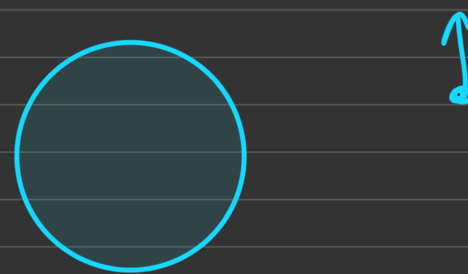
$$\frac{T_1}{T_2} = 8$$

$$T_2 = \left(\frac{T_1}{8}\right)$$

9. *An artificial satellite moving is circular orbit around the earth has a total (kinetic + potential) energy* E_0. *Its potential energy is :*

(A) $-E_0$ **(B)** $1.5\,E_0$ ~~**(C)**~~ $2E_0$ **(D)** E_0

$TE = E_0$



$TE = -KE = +\frac{PE}{2}$

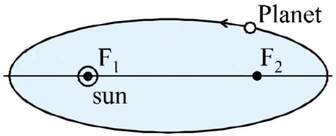
E_0

$\underline{PE = 2E_0}$

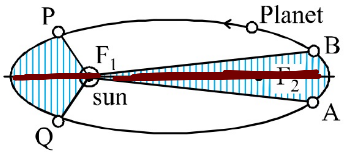
Keplers laws of Planetary motion!

1. **Law of Orbits :** Each planet revolves around the sun in an elliptical orbit with the sun at one focus of the ellipse.

2. **Law of Areas :** This law states that the radius vector from the sun to the planet sweeps out equal areas in equal time intervals.



Both shaded areas are equal if the time from A to B is equal to the time from P to Q.



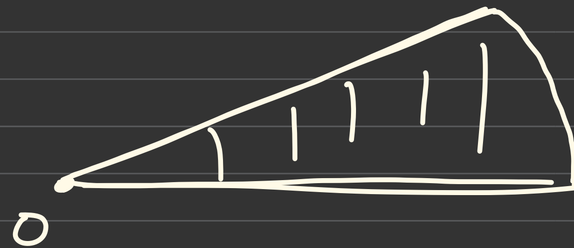
3. **Law of Periods :** It states that the square of the time taken by the planet about the sun is proportional to the cube of the planet's mean distance from the sun.

If T be the time period of the planet and r be the mean distance of planet from the sun (average of maximum and minimum distances from sun)

$$r = \frac{r_{min} + r_{max}}{2} \Rightarrow T^2/r^3 \text{ is same for all planets}$$

$$T^2 \propto r^3 \qquad \frac{r_{min} + r_{max}}{2}$$

(2) law of areas # " $\frac{dA}{dt}$ = Cont for any planet "



$$A = \left(\frac{1}{2} r^2 \theta\right)$$

$$2\pi \longrightarrow \pi R^2$$

$$1 \longrightarrow \frac{\pi R^2}{2\pi}$$

$$\theta \longrightarrow \left(\theta \cdot \frac{\cancel{\pi} R^2}{2\cancel{\pi}}\right) = \frac{1}{2} R^2 \theta$$

$$\frac{dA}{dt} = \frac{d}{dt}\left(\frac{1}{2} r^2 \theta\right) = \text{Cnt}$$

$$\Rightarrow \frac{1}{2} r^2 \frac{d\theta}{dt} = K$$

$$\Rightarrow \frac{m\, r^2 \omega}{m\ 2} = K$$

$\Rightarrow \frac{I\omega}{2m} = K$

$L_{planet} = K$

w.r.t Centre

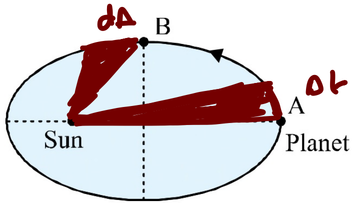
angular momentum

10. *A planet revolves in an elliptical orbit around the sun. Then out of following physical quantities the one which remains constant is :*

(A) *velocity* (B) *kinetic energy* (C) *momentum* (D) *angular momentum*

Ans

10. A planet is moving round the sun in an elliptical orbit as shown. As the planet moves from A to B :



(A) its kinetic energy will decrease

(B) its potential energy will remain unchanged

(C) its angular momentum about centre of sun will remain unchanged

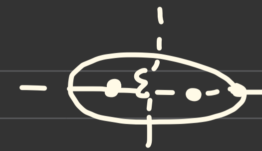
(D) its speed is minimum at A

Trajectory of a Satellite for different speeds :

Let v be the velocity given to a satellite. Let V_c represent the velocity for a circular orbit and V_e be the escape velocity.

$$V_c = \sqrt{\frac{GM}{r}} \quad \text{and} \quad V_e = \sqrt{\frac{2GM}{r}}$$

Remember



Where r is the distance of the satellite from centre of the earth.

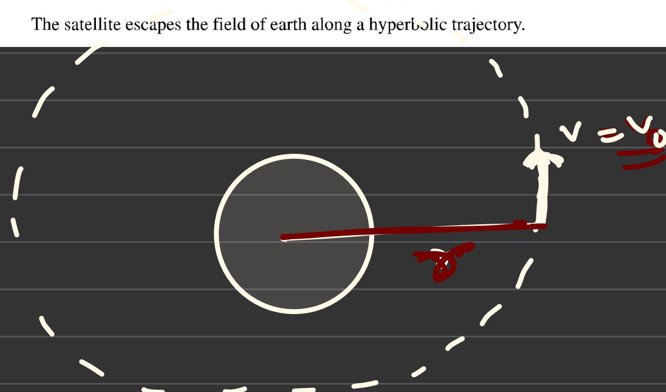
(i) $v < V_c$ — The satellite follows an elliptical path with centre of earth as the farther focus. In this case, if satellite is projected from near the surface of earth, it will hit the earth's surface without completing the orbit.

(ii) $v = V_c$ — The satellite follows a circular orbit with the centre of earth as the centre of orbit.

(iii) $V_c < v < V_e$ — The satellite follows an elliptical orbit with the centre of earth as the focus nearer to the point of projection.

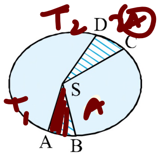
(iv) $v = V_e$ — The satellite escapes from the field of earth along a parabolic trajectory.

(v) $v > V_e$ — The satellite escapes the field of earth along a hyperbolic trajectory.



$$V_0 = \sqrt{\frac{GM}{r}}$$

5. The figure represents an elliptical orbit of a planet around sun. The planet takes time T_1 to travel from A to B and it takes time T_2 to travel from C to D. If the area CSD is double that of area ASB, then :



(A) $T_1 = T_2$ (B) $T_1 = 2T_2$
(C) $T_1 = 0.5\,T_2$ (D) Data insufficient

$$\frac{dA}{dt} =$$

$$\Rightarrow \frac{\cancel{A}}{T_1} = \frac{2\cancel{A}}{T_2}$$

$$T_2 = 2T_1$$

$$T_1 = 0.5\,T_2$$

2. A space vehicle approaching a planet has a speed v, very long way out and is on a trajectory which would miss the centre of the planet by a distance R if it continued in a straight line. If the planet has a mass M and radius r, what is the smallest value of R in order that the resulting orbit will just miss the surface :

(A) $R = \dfrac{2GMv}{r}$

(B) $R = vr\left[1 + \dfrac{2GM}{r}\right]$

(C) $R = \dfrac{r}{v}\left[v^2 + \dfrac{2GM}{r}\right]$

(D) $R = \dfrac{r}{v}\left[v^2 + \dfrac{2GM}{r}\right]^{1/2}$

loss in GPE = gain KE

$$\frac{GmM}{r} = \frac{1}{2}mv' - \frac{1}{2}mv^2 \quad \text{— (I)}$$

Angular momentum about centre of planet is constant

$$mvR = mv'r \quad \text{— (II)}$$

Solve it

4. Assuming that the earth is spherical and of radius R, gravitational acceleration on its surface is g and mass m, then its mean density is :

(A) $\dfrac{3g}{4\pi GR}$ (B) $\dfrac{4\pi GR}{3g}$

(C) $\dfrac{4\pi^2 G}{3Rg}$ (D) $\dfrac{3g\,GR}{4\pi^2}$

Density $= \dfrac{M}{\frac{4}{3}\pi R^3}$

$GM = gR^2$

$= \dfrac{3gR^2}{G\,4\pi R^3}$

$= \left(\dfrac{3g}{4G\pi R}\right)$

Doubt