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Tuning a Proportional-Integral (PI) Controller

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Abstract
This study offers a fundamental understanding on system response using a Proportional Integral (PI) controller via the manual tuning method. The parameter gains (k_p and k_i) were randomly chosen in five (5) pairs, an open-loop transfer function of a second-order system was arbitrarily selected with a step input signal chosen for the system. The tuned results/responses were analysed in terms of rise time (T_r), percentage overshoot (%OS), settling time (T_s) and steady-state error (SSE). After analyses of tuned step responses, k_p = 1 and k_i = 4 tuning proved the best and had a rise time, (Tr) of 0.513 seconds, settling time, (T_s) of 3.41 seconds, a peak amplitude of 1.02 and an overshoot of 1.75% at 2.434 seconds. All codes were executed on command prompt in MATLAB environment.

Keywords: Proportional Integral Derivative (PID), Proportional gain (k_p), Integral gain (k_i), Open-loop transfer function (OLTF), Closed-loop transfer function (CLTF).

1.0 Introduction

A system normally operates to drive an error as minimal as possible. A proportional-integral-derivative (PID) controller calculates an error value as the difference between a measured process variable and a desired set-point. The controller attempts to minimize the error by adjusting the process through use of a manipulated variable. A PID controller is a control loop feedback mechanism (controller) widely used in industrial control systems.

Oku *et al.* (2018) demonstrated how a proportional integral (PI) controller could be implemented in a systematic way by pole placement technique as against the conventional method of tuning a PI controller using Ziegler-Nichols (Z-N) method. After analysing the PI controller application, it was observed that in both time and frequency domain, the PI controller showed a good performance for both set-point tracking and stability. Kambiz and Augustin (2012) presented the full classical approach to dealing with PID controllers. Different tuning methods, controller actions/modes were analysed. The study found out that many industrial processes are nonlinear and are thus complicated to describe mathematically. Ige (2018) made a study on automatic tuning of PID controllers.

The main objective of the work was the investigation of the various algorithms used for automatic tuning of PID controllers. The literature research on algorithms for automatic tuning of the PID controller and recursive system identification based methods for PID controller parameters calculation was performed, a simulated experiment of one or a few algorithms for automatic tuning of PID controllers and a laboratory experiment of the auto tuning method on quadruple tank process and/or the air heater was also presented. Haitao *et al.* (2018) presented a study on tuning method for PID controller for an integrating system with time delay. The proposed tuning method presented some formulae which considered all the factors. The proposed tuning method was tested by a practical circuit, which proved that the method could be used for several applications, especially for the inductor current control. The result revealed that the tuning method could be applied in the integrating process with time delay. The experiment results proved that the tuned PID controller obtained by the new method could also be applied in the practical control circuit for stability of the system.

Costa, (2011) introduced the basic fundamentals of PID control theory, and provided a brief overview of control theory and the characteristics of each of the PID control loops in the study. Rice, (2017) presented the basics for effective tuning of a PID controller. Mishra, (2011) worked on PID controller design for systems with time delay. Designing a PID controller to meet gain and phase margin specification is a well-known design technique. Interactive tuning tool of proportional-integral controllers for first order plus time delay (FOPTD) processes was presented by Ruz *et al.* (2018); the research presented an interactive tool focused on the study of PI controllers. The tool provides a set of tuning rules for both open-loop stable and unstable first order plus time delay processes. Hassaan, (2014) presented a tuning of a proportional derivative-proportional integral (PD-PI) controller used with a highly oscillating second-order process in which high oscillation in industrial processes is undesired and controller tuning has to solve these problems. Singh and Singh (2014) described a brief study of the Z-N method for tuning of PID controller. The conventional PID controller was replaced by Z-N tuning PID controller, the load-frequency control (LFC) was used to restore the balance between load and generation in each control area by means of speed control. The objective of Load-Frequency control (LFC) was to minimize the transient and steady state error to zero. Bansal *et al.* (2017) presented a review of the current as well as classical techniques used for PID tuning. The main goal of the study was to provide a comprehensive reference source for people working on PID controllers. Idoko *et al.* (2017) worked on design of tuning mechanism of PID controller for application in three phase induction motor speed control. The study presented a design of tuning mechanism of PID Controller for application in three phase induction motor speed controls. A review of PID controllers by Bhagwan *et al.* (2016) explained the different configurations of the PID controllers. The characteristics effect of the PID controller parameters were discussed and analysed.

It could be seen from the review that the effects of system response were not well articulated from previous works. This paper presents in clear and unambiguous terms demonstrated how rise time, settling time, overshoot and steady-state error affect a process/system.

2.0 Methodology

There are many available methods in determining acceptable values of the PI gains. The process of determining the gains is often called tuning. A common approach to tuning is the use of manual tuning methods, whereby the control gains are obtained by trial-and-error with minimal analytic analysis using step responses obtained via simulation, or in some cases, actual testing on the system and deciding on the gains based on observation and experience. The effects of PID tuning on systems from the literatures is surmised in Table 1.

Table 1: Effects of PID Tuning on Systems (Costa, 2011)

Control Term	T_r	% OS	T_s	SSE	System Stability
k_p	Decrease	Increase	Small change (minimal input)	Decrease	Decrease
k_i	Decrease	Increase	Increase	Eliminates (zero SSE)	Decrease
k_d	Small change	Decrease	Decrease	Minor effect (no impact)	Increase for small values of k_d

A faithful implementation of the PI controller equations will not result in a good controller. In practice, the controller equations need to be modified to deal with measurement of noise and step changes of the reference signal. Tuning PI controllers can seem a mystery, parameters that provide effective control over a process may one day fail to do so (Rice, 2017). The stability and responsiveness of a process seem to be at complete odds with each other. Controller equations include subtle differences that can baffle even the most experienced practitioners. Even so, the PI controller is the most widely used technology in industry for the control of production processes. The PID controller is seldom used because the derivative gain is sensitive to noise. The manual tuning method as reported in this work was achieved using MATLAB 2018b; codes were written in command prompt and results of step response of the process obtained.

The transfer function of the PI controller according to Singh and Singh (2014) is given as:

$$PI(s) = G_c(s) = k_p + \frac{k_i}{s} \qquad (1)$$

where,

$G_c$is open-loop transfer function of the controller. The process transfer function can be represented as thus;

$$G_p(s) = \frac{num(s)}{den(s)} = \frac{s+a_1}{as^2+bs+c} \qquad (2a)$$

The process transfer function $G_p(s)$ in this design is given as

$$G_p(s) = \frac{s+5}{s^2+2s+10} \qquad (2b)$$

The close loop transfer function of the system is

$$\text{CLTF} = \frac{G_p(s)}{1+G_p(s)G_c(s)} \qquad (3a)$$

Substituting equation (1) and (2a) in (3a) gives

$$\text{CLTF} = \frac{s^2k_p + s\left(k_i + k_pa_1\right) + k_ia_1}{as^3+s^2(b+k_p)+s(c+k_i+k_pa_1)+k_ia_1} \qquad (3b)$$

Equation 3b is the CLTF of the entire system.

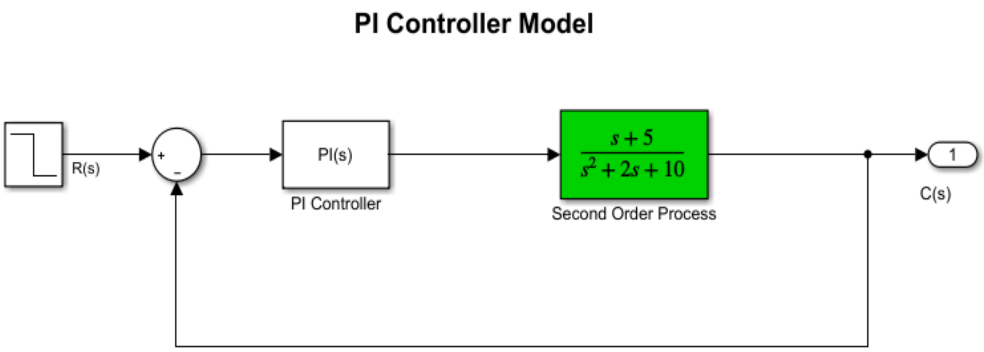

Figure 1: Block diagram of the system

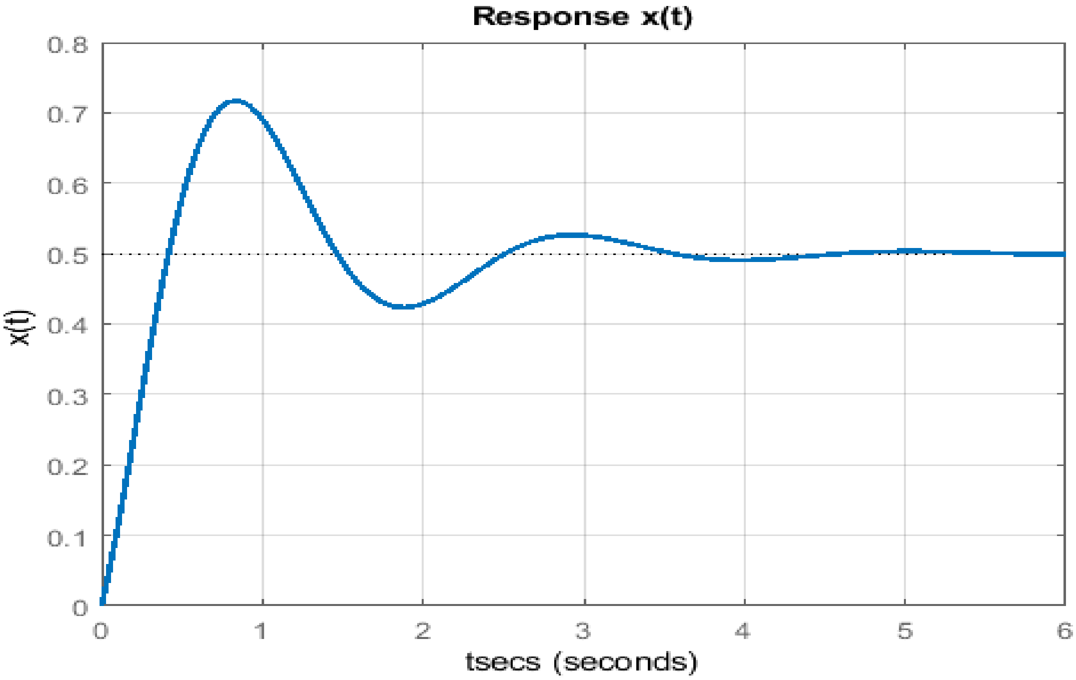

Figure 2: Process' Step Response

Note that Figure 1, and 2, equation 3b are generic.

The step response of the process transfer function has an under damped response. It has a rise time of 0.324 seconds, settling time of 3.36 seconds, a peak amplitude of 0.717, an overshoot of 43.5% at 0.829 seconds. However, adjusting the gains of the controller subsequently would show how a PI controller could be tuned manually to achieve some certain goals in a production process.

When the controller is on:

a- Proportional action (P-mode) – it adjusts the controller output according to the size of the error.

b- Integral action (I-mode) – eliminates steady-state error.

The effective tuning of a control system reduces to adjustment of the coefficients- k_p and k_i which is called tuning of the controller.

Table 2: Controller's Tuning Chart

Parameter	Tuning 1	Tuning 2	Tuning 3	Tuning 4	Tuning 5
k_p	1	2	4	1	1
k_i	1	1	1	2	4

3.0 Results

As seen, for a second-order plant, the PI controller will produce a higher response. There will be zero steady-state errors if the reference and disturbance inputs are either unchanging or have step changes. The process of introducing the integrator within the control loop is to reduce or eliminate steady-state errors.

From equation 3b, the CLTFs can be formed as follows:

For k_p =1 and k_i =1:

$$G_{c1}(s) = \frac{s+1}{s} \tag{4}$$

$$\mathrm{CLTF} = \frac{s^2 + 6s + 5}{s^3 + 3s^2 + 16s + 5} \tag{5}$$

For k_p = 2 and k_i = 1:

$$G_{c2}(s) = \frac{2s+1}{s} \quad (6)$$

$$\text{CLTF} = \frac{2s^2 + 11s + 5}{s^3 + 4s^2 + 21s + 5} \quad (7)$$

For $k_p = 4$ and $k_i = 1$:

$$G_{c3}(s) = \frac{4s+1}{s} \quad (8)$$

$$\text{CLTF} = \frac{4s^2 + 21s + 5}{s^3 + 6s^2 + 31s + 5} \quad (9)$$

For $k_p = 1$ and $k_i = 2$:

$$G_{c4}(s) = \frac{s+2}{s} \quad (10)$$

$$\text{CLTF} = \frac{s^2 + 7s + 10}{s^3 + 3s^2 + 17s + 10} \quad (11)$$

For $k_p = 1$ and $k_i = 4$:

$$G_{c5}(s) = \frac{s+4}{s} \quad (12)$$

$$\text{CLTF} = \frac{s^2 + 9s + 20}{s^3 + 3s^2 + 19s + 20} \quad (13)$$

The phase response of the above CLTFs of the system are shown in the Figure 3.

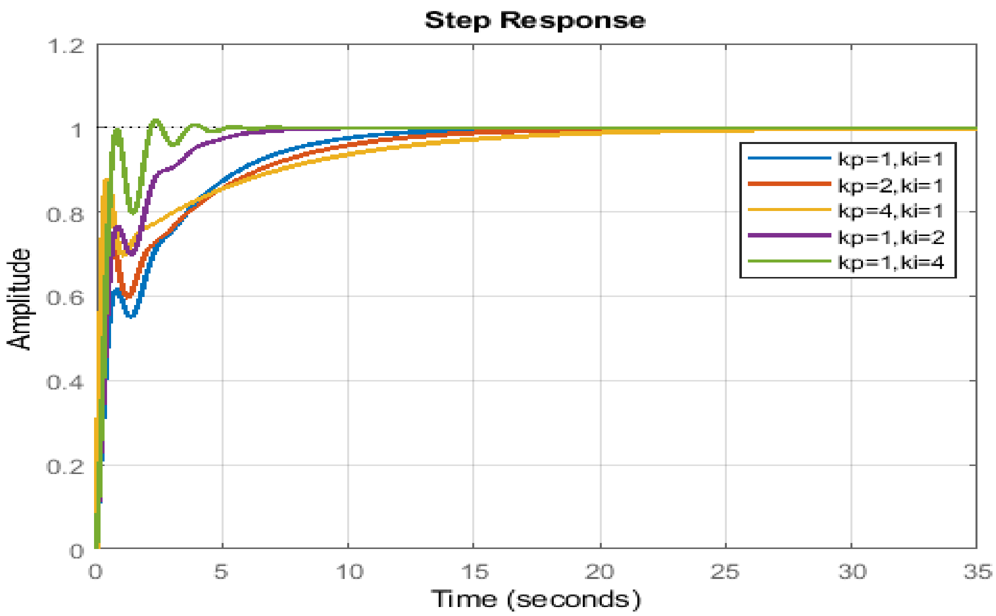

Figure 3: Step Response of the PI Controller Tuning

4.0 Discussion

The discussion of these results would be analysed in terms of T_r, % OS, T_S, SSE. The first tuning is that which both the proportional and integral gains are tuned to 1 as the reference tuning for this study. It has an over damped response and takes up to 5.61 seconds to rise and attain 90% of the final value. It takes it 10.60 seconds to track the reference input. There is no steady-state error as tuned because the controller has tracked the reference signal perfectly.

Tuning the controller to a proportional gain of 2 and an integral gain of 1 as shown in Figure 3, the system would take up to 6.40 seconds to reach 90% of its final value. As seen, the initial system response does not exceed the set-point, hence there is no overshoot. At 12.90 seconds, the system settles, hence this is the time it takes the system to track the reference input signal. It has a small steady-state error and peak amplitude of 0.99 at 25 seconds.

With the proportional gain tuned to 4 and the integral gain tuned to 1, the system has a rise time of 7.21 seconds; for such parameters, it takes the system 7.21 seconds to reach 90% of the system's final value. It has no overshoot and settles at 16.90 seconds tracking the reference input. This is in contrast with Oku *et al.* (2018) because the integral gain which helps in cancelling the steady-state error is rather been reduced.

With the proportional gain tuned to 1 and the integral gain tuned to 2, the system has an over damped response under this parameter adjustment; it takes the system up to 2.65 seconds to reach the set-point value. Its initial response does not exceed the set-point value, hence no presence of an overshoot. As shown in Table 1, Costa, (2011) stated the effects of controller gains on step response. However, the step response for this tuning is a combined effect of both proportional and integral actions of the controller, yet similar arguments with those of Costa, (2011) could be seen here. The reduction/decrease in proportional gain has resulted to a quicker rise time as compared with other tunings. This tuning has a long settling time as it takes the system up to 5.36 seconds to reach and stay within the range of 3% of the reference input value. After settling, the system continuously tracks the reference signal with a little difference, hence there is a reduced steady-state error.

With the proportional gain tuned to 1 and the integral gain tuned to 4, the system has an underdamped response under this tuning. It has a rise time of 0.513 seconds. The system's output exceeds its final (steady-state) value, hence it has an overshoot of 1.75%. Increasing the integral gain has resulted to the controller producing large, slowly damping oscillations. It has a settling time of 3.41 seconds; hence it takes the system 3.41 seconds to converge to the set-point value. Its steady-state error is near zero and can offer little or no effect (negative) to the operation of the system.

The analyses from tuning in terms of rise time, overshoot, settling time and steady-state error are shown in Table 3.

Table 3: Combined Closed-loop Tuning (Chart)

Tuning	T_r (seconds)	% OS	T_s (seconds)	SSE
k_p =1, k_i =1	5.61	0	10.60	Tolerable
k_p =2, k_i =1	6.40	0	12.90	Small
k_p =4, k_i =1	7.21	0	16.90	Large
k_p =1, k_i =2	2.65	0	5.36	Reduced
k_p =1, k_i =4	0.513	1.75	3.41	Eliminated

5.0 Conclusion

The goal of a PI controller is to take an input value and maintain it at a given set-point over time within a tolerable limit of steady-state error. Unlike other control options, PI controllers do not require the user to have an extensive background in mathematics, control theory or electrical engineering to understand them. If PI controllers are properly tuned, they will outperform almost any other control options. It is in tuning the controllers that the greatest gains in performance may be found. It can be concluded that, k_p = 1, k_i = 4 tuning has the best response. For processes that require quicker responses and faster steady-state operations, like flow control and fast temperature control, the k_p =1, k_i =4 tuning can be employed.

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