TUGAS TEORI PELUANG

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26.

x	0	1	2	3	4
$f(x)$	.1	.3	.3	.2	.1

$a.\ E(X) = \sum_x xf(x)$

$= 0(0{,}1) + 1(0{,}3) + 2(0{,}3) + 3(0{,}2) + 4(0{,}1)$

$= 1{,}9$

$b.\ Var(X) = E(X^2) - (E(X))^2$

$= 0(0{,}1) + 1(0{,}3) + 4(0{,}3) + 9(0{,}2) + 16(0{,}1) - 1{,}9^2$

$= 0{,}3 + 1{,}2 + 1{,}8 + 1{,}6 - 3{,}61$

$= 4.9 - 3{,}61$

$= 1{,}29$

$c.\ Y = 35X - 40$

$E(Y) = E(35X - 40)$

$= 35E(X) - E(40)$

$= 35(1.9) - 40$

$= 26{,}5$

$Var(Y) = E(Y^2) - (E(Y))^2$

$= E(1225X^2 - 2800X + 1600) - (26{,}5)^2$

$= 1225E(X^2) - 2800E(X) + 1600 - 702{,}25$

$= 1225(4.9) - 2800(1.9) + 897.75$

$= 6002{,}5 - 5320 + 897{,}75$

$= 1580{,}25$

27. $f(r) = 6r(1-r), 0 < r < 1$

$a.\ E(R) = \int_0^1 r(6r - 6r^2)\, dr$

$= \int_0^1 (6r^2 - 6r^3)\, dr$

$= \left[2r^3 - \frac{3}{2}r^4\right]_0^1$

$= 2 - \frac{3}{2}$

$= \frac{1}{2}$

$b.\ K = 2\pi R$

$$E(K) = E(2\pi R)$$
$$= 2\pi E(R)$$
$$= 2\pi(\frac{1}{2})$$
$$= \pi$$

$c.\ L = \pi R^2$

$$E(L) = E(\pi R^2)$$
$$= \pi E(R^2)$$
$$= \pi \int_0^1 r^2 f(r)\, dr$$
$$= \pi \int_0^1 r^2\,(6r - 6r^2)\, dr$$
$$= \pi \int_0^1 (6r^3 - 6r^4)\, dr$$
$$= \pi\left[\frac{3}{2} r^4 - \frac{6}{5} r^5\right]_0^1$$
$$= \pi\left(\frac{3}{2} - \frac{6}{5}\right)$$
$$= \pi\left(\frac{3}{10}\right)$$

36. $f(x) = exp\,[-(x+2)],\ -2 < x < \infty\ dan\ 0\ untuk\ yang\ lain$

$a.\ M_X(t) = \int_{-2}^{\infty} e^{tx} f(x)\, dx$
$$= \int_{-2}^{\infty} e^{tx}\,(e^{-x-2}) dx$$
$$= \int_{-2}^{\infty} e^{(t-1)x-2}\, dx$$
$$= \left[\frac{e^{(t-1)x-2}}{t-1}\right]_{-2}^{\infty}$$
$$= -\frac{e^{-2t}}{t-1}$$
$$= \frac{e^{-2t}}{1-t}$$

$b.\ M_X'(t) = \frac{e^{-2t}(2t-1)}{(1-t)^2}$

$$M_X''(t) = \frac{(-2e^{-2t}(2t-1) + 2e^{-2t})(1-t)^2 + 2e^{-2t}(2t-1)(1-t)}{(1-t)^4}$$

$$E(X) = M_x'(0)$$
$$= -1$$

$$
\begin{aligned}
E(X^2) &= M_X''(0) \\
&= \frac{(-2(-1)+2)1 + 2(-1)(1)}{1} \\
&= 4 - 2 \\
&= 2
\end{aligned}
$$