

Homework #3

1. $y = \frac{1}{v+x}$, $v = \sin(w)$, $x = e^w$, $w=0$

$w=0$, $x=1$, $v=0$

$$\begin{array}{c} x \rightarrow y \\ w \rightarrow v \rightarrow y \end{array} \quad \frac{\partial y}{\partial w} = \frac{\partial y}{\partial x} \frac{\partial x}{\partial w} + \frac{\partial y}{\partial v} \frac{\partial v}{\partial w}$$

$$= -\frac{1}{(v+x)^2} \cdot e^w + -\frac{1}{(v+x)^2} \cdot \cos(w)$$

$$\left. \frac{\partial y}{\partial w} \right|_{w=0} = -\frac{1}{(0+1)^2} \cdot e^0 + -\frac{1}{(0+1)^2} \cdot \cos(0)$$

$$= -1 - \cos(0) = -2$$

$$\therefore \left. \frac{\partial y}{\partial w} \right|_{w=0} = -2$$

2. $E_1(w) = \frac{1}{2} \sum_{k=1}^3 (t_{1k} - O_{1k})^2 = \frac{1}{2} \{ (0.4-0.5)^2 + (0.6-0.3)^2 + (0.9-0.4)^2 \}$

$$= \frac{1}{2} (0.01 + 0.09 + 0.25) = \frac{1}{2} \cdot 0.35 = 0.175$$

$$E_2(w) = \frac{1}{2} \sum_{k=1}^3 (t_{2k} - O_{2k})^2 = \frac{1}{2} \{ (0.5-0.5)^2 + (0.5-0.4)^2 + (0.7-0.5)^2 \}$$

$$= \frac{1}{2} (0 + 0.01 + 0.04) = \frac{1}{2} \cdot 0.05 = 0.025$$

$$E_3(w) = \frac{1}{2} \sum_{k=1}^3 (t_{3k} - O_{3k})^2 = \frac{1}{2} \{ (0.7-0.7)^2 + (0.8-0.8)^2 + (0.1-0.1)^2 \}$$

$$= \frac{1}{2} \cdot 0 = 0$$

$$E(w) = \sum_{n=1}^3 E_n(w) = 0.175 + 0.025 + 0 = 0.2$$

$$\therefore E(w) = 0.2$$

$$3. \frac{\partial E}{\partial W_{kj}} = \frac{\partial E}{\partial \text{net}_{nk}} \frac{\partial \text{net}_{nk}}{\partial W_{kj}} = \left(\frac{\partial E}{\partial h_{nk}} \frac{\partial h_{nk}}{\partial \text{net}_{nk}} \right) \frac{\partial \text{net}_{nk}}{\partial W_{kj}} = \delta_{nk} h_{nj}$$

$$\frac{\partial E}{\partial W_{ji}} = \frac{\partial E}{\partial \text{net}_{nj}} \frac{\partial \text{net}_{nj}}{\partial W_{ji}} = \left(\frac{\partial E}{\partial h_{nj}} \frac{\partial h_{nj}}{\partial \text{net}_{nj}} \right) \frac{\partial \text{net}_{nj}}{\partial W_{ji}}$$

$$= \left(\left(\sum_{k=1}^K \frac{\partial E}{\partial \text{net}_{nk}} \frac{\partial \text{net}_{nk}}{\partial h_{nj}} \right) \frac{\partial h_{nj}}{\partial \text{net}_{nj}} \right) \frac{\partial \text{net}_{nj}}{\partial W_{ji}}$$

$$= \left(\left(\sum_{k=1}^K \delta_{nk} W_{kj} \right) \frac{\partial h_{nj}}{\partial \text{net}_{nj}} \right) h_{ni} = \delta_{nj} h_{ni}$$

$$\frac{\partial E}{\partial W_{ip}} = \frac{\partial E}{\partial \text{net}_{ni}} \frac{\partial \text{net}_{ni}}{\partial W_{ip}} = \left(\frac{\partial E}{\partial h_{ni}} \frac{\partial h_{ni}}{\partial \text{net}_{ni}} \right) \frac{\partial \text{net}_{ni}}{\partial W_{ip}}$$

$$= \left(\left(\sum_{j=1}^J \frac{\partial E}{\partial \text{net}_{nj}} \frac{\partial \text{net}_{nj}}{\partial h_{ni}} \right) \frac{\partial h_{ni}}{\partial \text{net}_{ni}} \right) \frac{\partial \text{net}_{ni}}{\partial W_{ip}}$$

$$= \left(\left(\sum_{j=1}^J \delta_{nj} W_{ji} \right) \frac{\partial h_{ni}}{\partial \text{net}_{ni}} \right) h_{np} = \delta_{ni} h_{np}$$

$$\delta_{nk} = \frac{\partial E}{\partial h_{nk}} \frac{\partial h_{nk}}{\partial \text{net}_{nk}} = -(t_n - h_{nk}) h_{nk} (1 - h_{nk})$$

$$\delta_{nj} = \left(\sum_{k=1}^K \delta_{nk} W_{kj} \right) \frac{\partial h_{nj}}{\partial \text{net}_{nj}} = \left(\sum_{k=1}^K \delta_{nk} W_{kj} \right) h_{nj} (1 - h_{nj})$$

$$\delta_{ni} = \left(\sum_{j=1}^J \delta_{nj} W_{ji} \right) \frac{\partial h_{ni}}{\partial \text{net}_{ni}} = \left(\sum_{j=1}^J \delta_{nj} W_{ji} \right) h_{ni} (1 - h_{ni})$$

다음장에 계속 >>

$$4. x=1, y=1$$

$$w_1^0, w_2^0 = 1$$

$$f(x) = \frac{1}{1+e^{-x}}$$

$$h = f(w_1 x) = f(1) = \frac{1}{1+e^{-1}}$$

$$0 = f(w_2 h) = f(h) = f\left(\frac{1}{1+e^{-1}}\right)$$

$$w_2' = w_2^0 - \eta (y - 0) o (1 - 0) h$$

$$w_1' = w_1^0 - \eta x h (1 - h) w_2^0 (y - 0) o (1 - 0)$$

$$w_2' = 1 - 0.1 (1 - f(h)) f(h) (1 - f(h)) h = 1 - 0.1 (1 - f(h))^2 h f(h)$$

$$w_1' = 1 - 0.1 \cdot 1 \cdot h (1 - h) \cdot 1 \cdot (1 - f(h)) f(h) (1 - f(h)) = 1 - 0.1 (1 - f(h))^2 h f(h) (1 - h)$$

$$w_2' = 1 - 0.1 \left(1 - f\left(\frac{1}{1+e^{-1}}\right)\right)^2 \cdot \frac{1}{1+e^{-1}} \cdot f\left(\frac{1}{1+e^{-1}}\right) = 0.9948$$

$$w_1' = 1 - 0.1 \left(1 - f\left(\frac{1}{1+e^{-1}}\right)\right)^2 \cdot \frac{1}{1+e^{-1}} \cdot f\left(\frac{1}{1+e^{-1}}\right) \left(1 - \frac{1}{1+e^{-1}}\right) = 0.9986$$

```
import numpy as np
def sigmoid(x):
    return 1/(1+np.exp(-x))
```

```
w2 = 1-0.1*((1-sigmoid(sigmoid(1)))**2)*sigmoid(1)*sigmoid(sigmoid(1))
w1 = 1-0.1*((1-sigmoid(sigmoid(1)))**2)*sigmoid(1)*sigmoid(sigmoid(1))*(1-sigmoid(1))
```

```
print(w2)
print(w1)
```

```
0.9947886947063983
0.9985984641471458
```