# VAR Stationary Example

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## Libraries

```
library(dplyr)
library(tsibble)
library(ggplot2)
library(feasts)
library(gridExtra)
library(MTS)
library(dse)
```

## Data Generation

The model I am going to simulate is as follows:

$ts1_t = 0.3 * ts1_{t-1} + \epsilon_1$

$ts2_t = 0.4 * ts2_{t-1} + ts1_{t-1} + \epsilon_2$

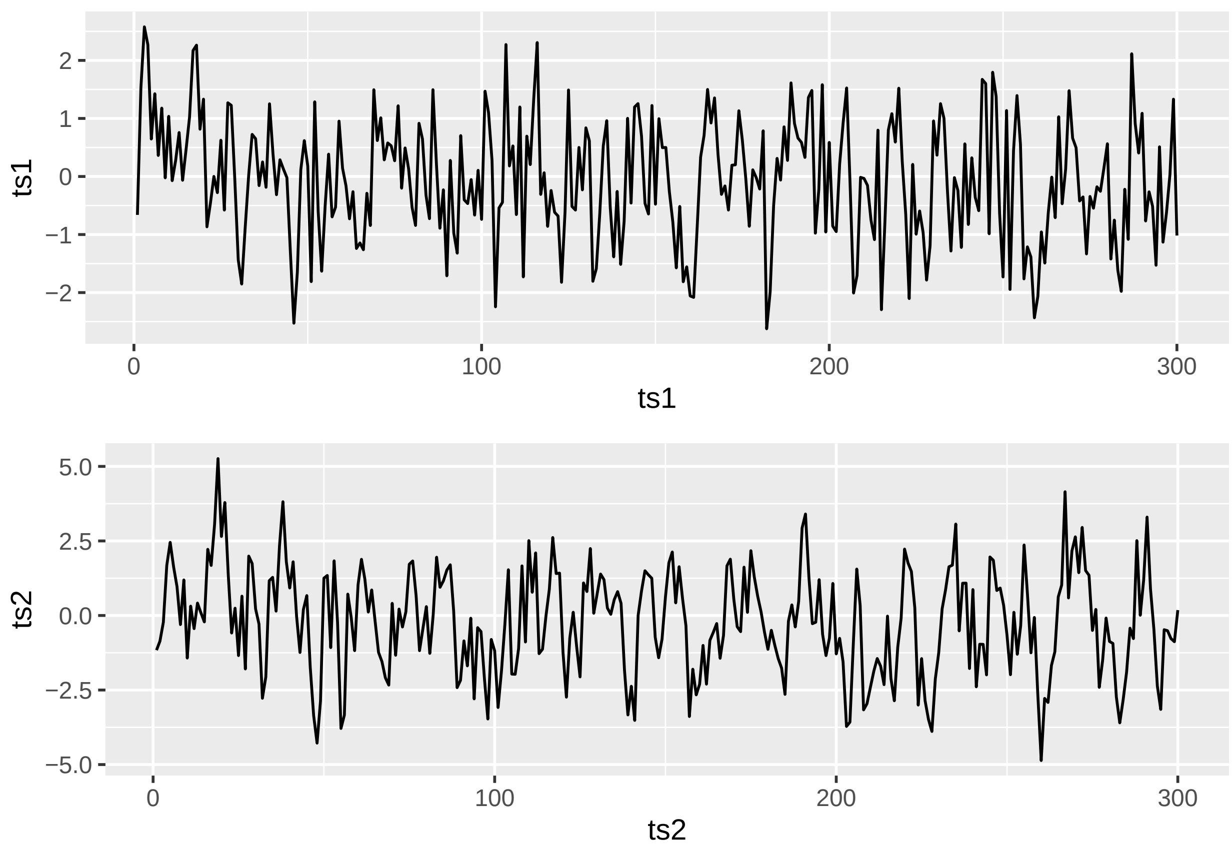
Which results in the following model

$\begin{pmatrix} ts1_t \\ ts2_t \end{pmatrix} = \begin{pmatrix} 0.3 & 0 \\ 1 & 0.4 \end{pmatrix} \begin{pmatrix} ts1_{t-1} \\ ts2_{t-1} \end{pmatrix} + \begin{pmatrix} \epsilon_1 \\ \epsilon_2 \end{pmatrix}$

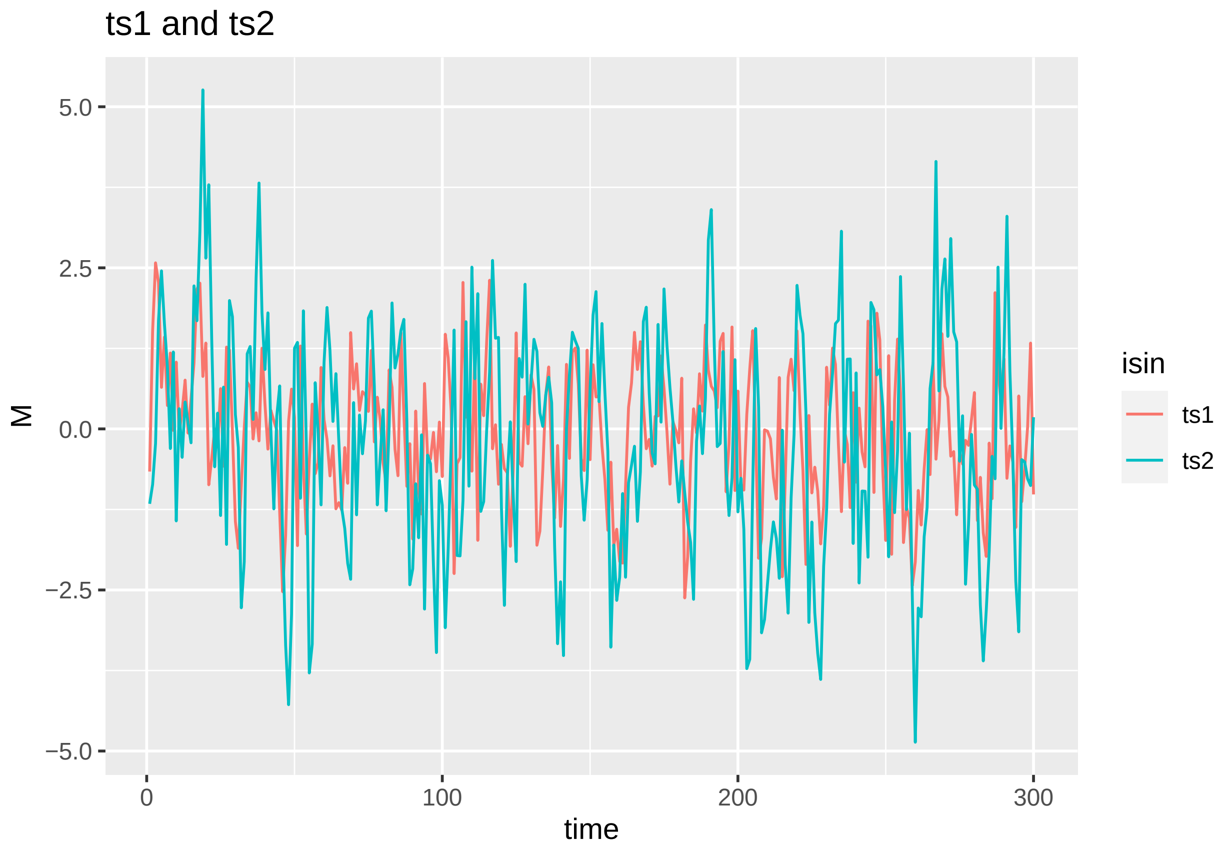
```
# Generates the two vectors
set.seed(14)
length = 320
testLength = 20
ts1 = vector("numeric", length)
noise = makeTSnoise(length, 1, 0)$w

# Simluates the model
ts1[1] = noise[1]
for(t in 2:length) {
  ts1[t] = 0.3*ts1[t - 1] + noise[t]
}

ts2 = vector("numeric", length)
noise = makeTSnoise(length, 1, 0)$w
ts2[1] = noise[1]
for(t in 2:length) {
  ts2[t] = 0.4*ts2[t - 1] + ts1[t - 1] + noise[t]
}

# Takes out the testing data
test1 = ts1[(length-testLength):(length-1)]
```

```
ts1 = ts1[1:(length-testLength)]
test2 = ts2[(length-testLength):(length-1)]
ts2 = ts2[1:(length-testLength)]

length = length - testLength

# Turns them into a time series object
ts = as_tibble(ts1)
ts = rename(ts, "ts1" = "value")
ts[,2] = ts2
ts = rename(ts, "ts2" = "...2")
ts[,3] = 1:length
ts = rename(ts, "index" = "...3")

ts = ts %>% as_tsibble(index = "index")
```

```
plot1 = ts %>% autoplot(ts1) + xlab("ts1")
plot2 = ts %>% autoplot(ts2) + xlab("ts2")
grid.arrange(plot1, plot2, nrow=2)
```




```
df1 = data.frame(time = ts$index, M = ts$ts1, isin = "ts1")
df2 = data.frame(time = ts$index, M = ts$ts2, isin = "ts2")
df = rbind(df1, df2)
ggplot(df, aes(x = time, y = M, color = isin)) + geom_line() + ggtitle("ts1 and ts2")
```


ts1 and ts2
5.0
2.5
0.0
−2.5
−5.0
M
0
100
200
300
time
isin
ts1
ts2

## Model Selection

We are going to try to select a $VAR(p)$ model. The model is:

$z_t = \phi_0 + \sum_{i=1}^{p} \phi_i z_{t-i} + a_t$

- $k$ is the number of time series we have
- $\phi_0$ is a $k$ dimensional constant vector
- $\phi_i$ is a $k$ by $k$ matrix
- $a_t$ is a sequence of independent and identically distributed random vectors with mean zero and covariance matrix $\Sigma_a$

### Order Selection with the Sequential likelihood ratio test:

What we are going to do for our order selection is compare $VAR(\ell)$ with $VAR(\ell - 1)$

$H_0$: $\phi_\ell = 0$

$H_A$: $\phi_\ell \neq 0$

```r
# Does the order test
VARorder(ts[,-3])
```

```
## selected order: aic =  1
## selected order: bic =  1
## selected order: hq =  1
## Summary table:
##        p    AIC    BIC     HQ     M(p) p-value
##  [1,]  0 1.1326 1.1326 1.1326   0.0000  0.0000
##  [2,]  1 0.1671 0.2165 0.1869 281.2862  0.0000
##  [3,]  2 0.1925 0.2913 0.2320   0.3591  0.9857
##  [4,]  3 0.2113 0.3595 0.2706   2.1968  0.6996
##  [5,]  4 0.1991 0.3967 0.2782  10.7857  0.0291
##  [6,]  5 0.2206 0.4675 0.3194   1.4218  0.8404
##  [7,]  6 0.2347 0.5310 0.3533   3.4420  0.4868
##  [8,]  7 0.2399 0.5856 0.3782   5.8303  0.2122
##  [9,]  8 0.2621 0.6572 0.4203   1.1907  0.8796
## [10,]  9 0.2770 0.7214 0.4549   3.1638  0.5308
## [11,] 10 0.2818 0.7757 0.4795   5.7926  0.2152
## [12,] 11 0.2945 0.8378 0.5119   3.6809  0.4509
## [13,] 12 0.2943 0.8869 0.5315   7.0347  0.1341
## [14,] 13 0.3159 0.9579 0.5728   1.3239  0.8573
```

As seen above, we should proceed with a $VAR(1)$ model

## Fitting the model

```r
# Does LS estimation of the model
m1 = VAR(ts[,-3], 1)
```

```
## Constant term:
## Estimates:  -0.0395207 -0.05098208
## Std.Error:  0.05796975 0.06336133
## AR coefficient matrix
## AR( 1 )-matrix
##       [,1]   [,2]
## [1,] 0.296 0.0289
## [2,] 0.929 0.4456
## standard error
##        [,1]   [,2]
## [1,] 0.0567 0.0338
## [2,] 0.0619 0.0370
##
## Residuals cov-mtx:
##             [,1]        [,2]
## [1,]  0.97981832 -0.01086404
## [2,] -0.01086404  1.17055340
##
## det(SSE) =  1.146812
## AIC =  0.1636523
## BIC =  0.213036
## HQ  =  0.1834157
```

From the output above, we get the following model:

$$z_t = \begin{pmatrix} -0.04 \\ -0.05 \end{pmatrix} + \begin{pmatrix} 0.30 & 0.03 \\ 0.929 & 0.44 \end{pmatrix} z_{t-1} + a_t$$

Some things to note:

- Granger Causality is low from ts2 to ts1 (with value of 0.04)
- Coefficients are really similar to our simulation

## Model Checking

### Stationarity

Its turns out that to test if a series is stationary, we can solve the following determinate:

$|I_k - \Phi_1 B| = 0$

and if the absolute value of the solutions are greater than 1, it is stationary!

For $VAR(1)$ models, the solutions of $B$ is simply the reciprocal of the eigenvalues of $\Phi_1$

```
eigen(m1$Phi)[1]
```

```
## $values
## [1] 0.5506652 0.1904566
```

So our series is stationary!

### Multivariate Portmanteau Statistics

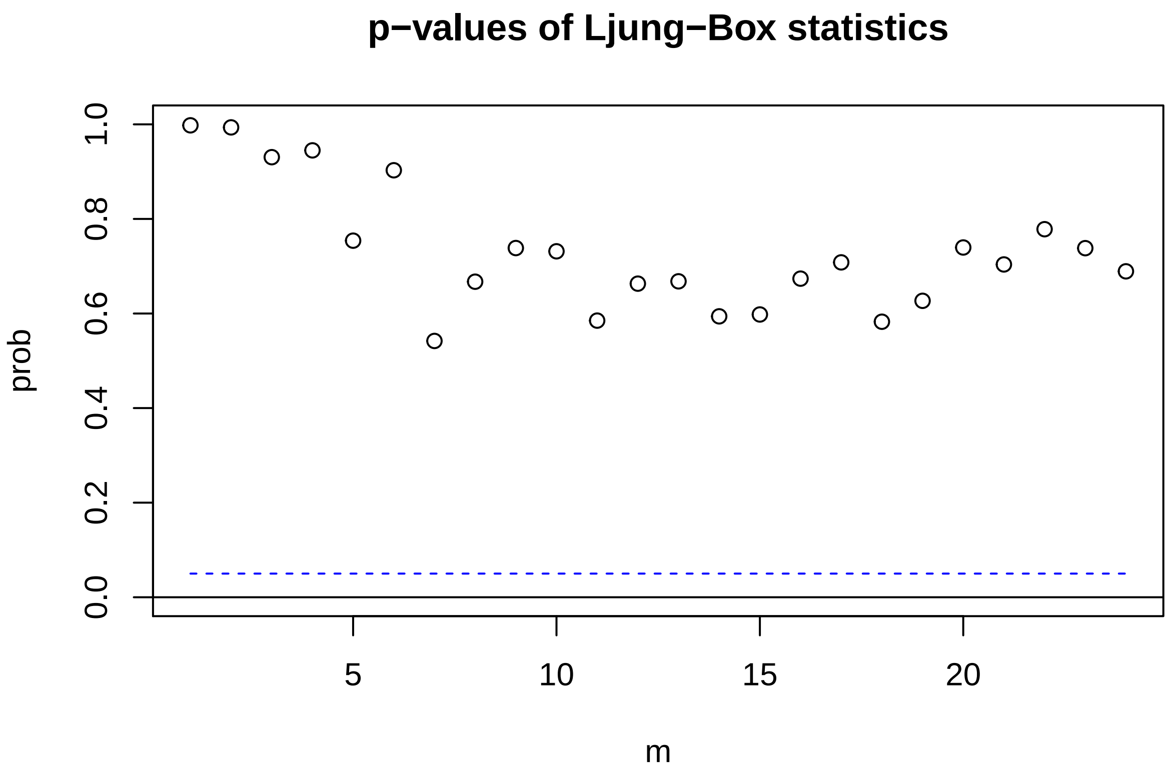
Let $R_\ell$ be the theoretical lag $\ell$ cross-correlation matrix of innovation $a_t$

$H_0$: $R_1 = \cdots = R_m = 0$

$H_A$: $R_j \neq 0$ for some $1 \leq j \leq m$

```
mq(m1$residuals)
```

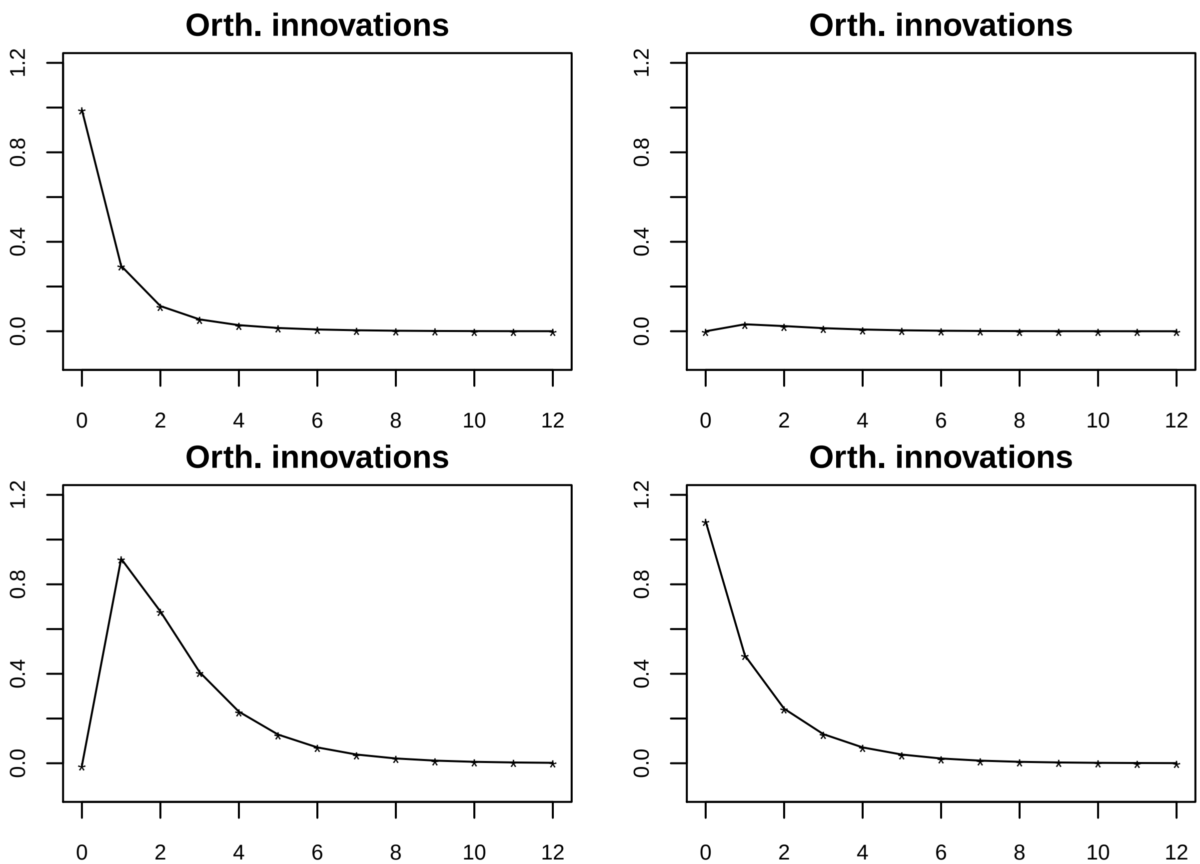
```
## Ljung-Box Statistics:
##            m       Q(m)     df    p-value
##  [1,]   1.000     0.131   4.000     1.00
##  [2,]   2.000     1.446   8.000     0.99
##  [3,]   3.000     5.696  12.000     0.93
##  [4,]   4.000     8.127  16.000     0.95
##  [5,]   5.000    15.384  20.000     0.75
##  [6,]   6.000    15.571  24.000     0.90
##  [7,]   7.000    26.566  28.000     0.54
##  [8,]   8.000    28.042  32.000     0.67
##  [9,]   9.000    30.244  36.000     0.74
## [10,]  10.000    34.118  40.000     0.73
## [11,]  11.000    41.370  44.000     0.58
## [12,]  12.000    43.360  48.000     0.66
## [13,]  13.000    47.061  52.000     0.67
## [14,]  14.000    52.872  56.000     0.59
## [15,]  15.000    56.676  60.000     0.60
## [16,]  16.000    58.405  64.000     0.67
## [17,]  17.000    61.180  68.000     0.71
## [18,]  18.000    68.874  72.000     0.58
## [19,]  19.000    71.436  76.000     0.63
## [20,]  20.000    71.523  80.000     0.74
## [21,]  21.000    76.618  84.000     0.70
## [22,]  22.000    77.599  88.000     0.78
## [23,]  23.000    82.988  92.000     0.74
## [24,]  24.000    88.684  96.000     0.69
```



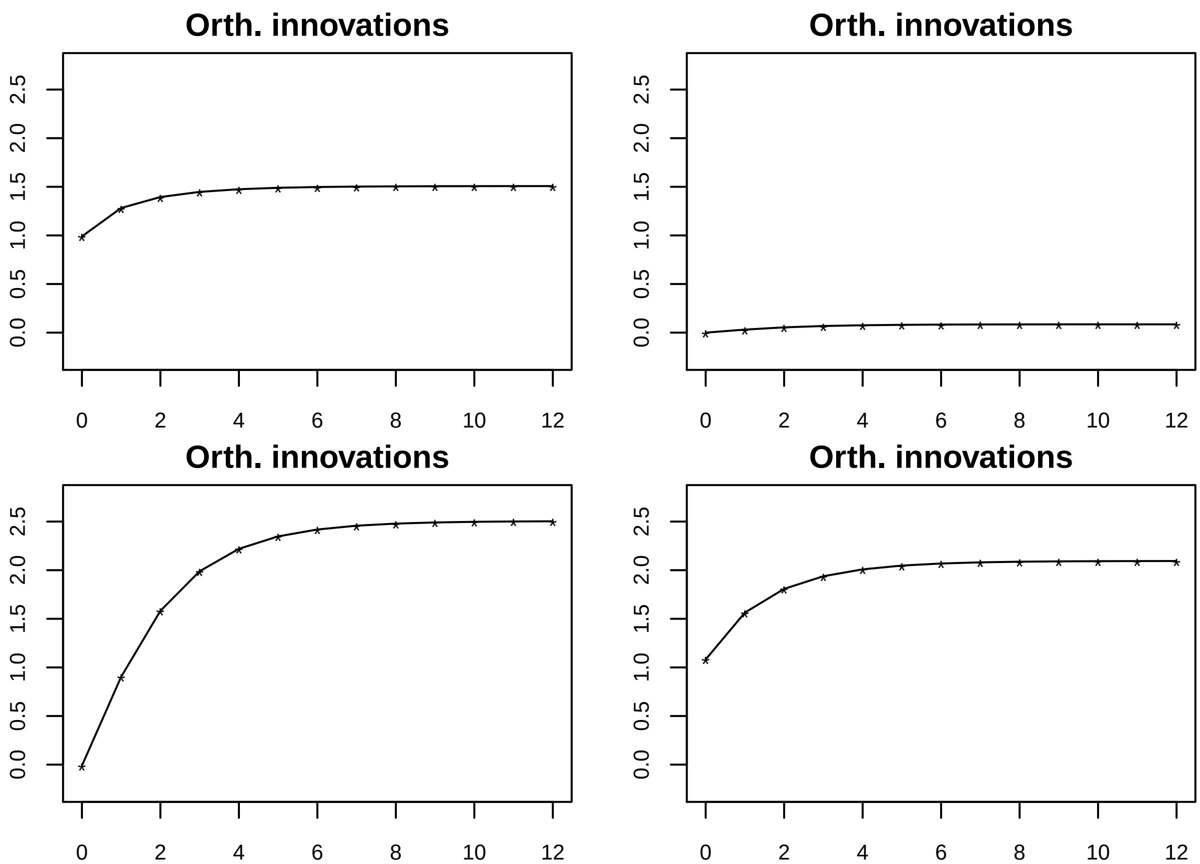

### Impulse

The first graph is the impulse response, while the second is accumulated response.

```
VARirf(m1$Phi, m1$Sigma)
```




```
## Press return to continue
```



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## Model Forecasting

### Static Forecasting

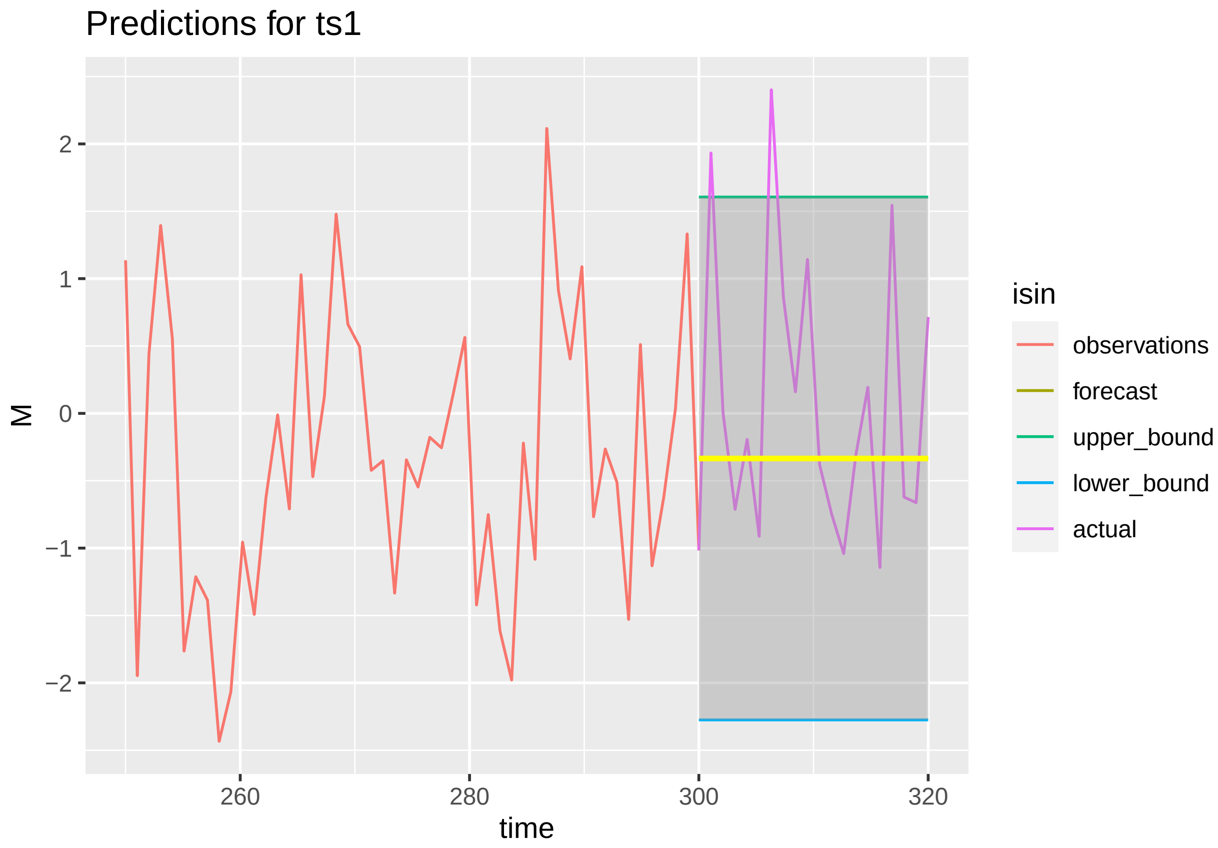
```
pred = VARpred(m1, h = testLength)
```

```
## orig  300
## Forecasts at origin:  300
##            ts1     ts2
##  [1,] -0.33489 -0.9148
##  [2,] -0.16490 -0.7696
##  [3,] -0.11047 -0.5470
##  [4,] -0.08796 -0.3973
##  [5,] -0.07699 -0.3097
##  [6,] -0.07121 -0.2605
##  [7,] -0.06809 -0.2332
##  [8,] -0.06637 -0.2181
##  [9,] -0.06543 -0.2098
## [10,] -0.06491 -0.2052
## [11,] -0.06463 -0.2027
## [12,] -0.06447 -0.2013
## [13,] -0.06439 -0.2006
## [14,] -0.06434 -0.2001
## [15,] -0.06431 -0.1999
## [16,] -0.06430 -0.1998
## [17,] -0.06429 -0.1997
## [18,] -0.06429 -0.1997
## [19,] -0.06428 -0.1996
## [20,] -0.06428 -0.1996
## Standard Errors of predictions:
##         [,1]  [,2]
##  [1,] 0.9899 1.082
##  [2,] 1.0326 1.496
##  [3,] 1.0390 1.661
##  [4,] 1.0404 1.715
##  [5,] 1.0408 1.732
##  [6,] 1.0409 1.737
##  [7,] 1.0409 1.739
##  [8,] 1.0409 1.739
##  [9,] 1.0409 1.739
## [10,] 1.0409 1.739
## [11,] 1.0409 1.739
## [12,] 1.0409 1.739
## [13,] 1.0409 1.739
## [14,] 1.0409 1.739
## [15,] 1.0409 1.739
## [16,] 1.0409 1.739
## [17,] 1.0409 1.739
## [18,] 1.0409 1.739
## [19,] 1.0409 1.739
## [20,] 1.0409 1.739
## Root mean square errors of predictions:
##         [,1]  [,2]
##  [1,] 0.9948 1.087
##  [2,] 1.0343 1.511
```

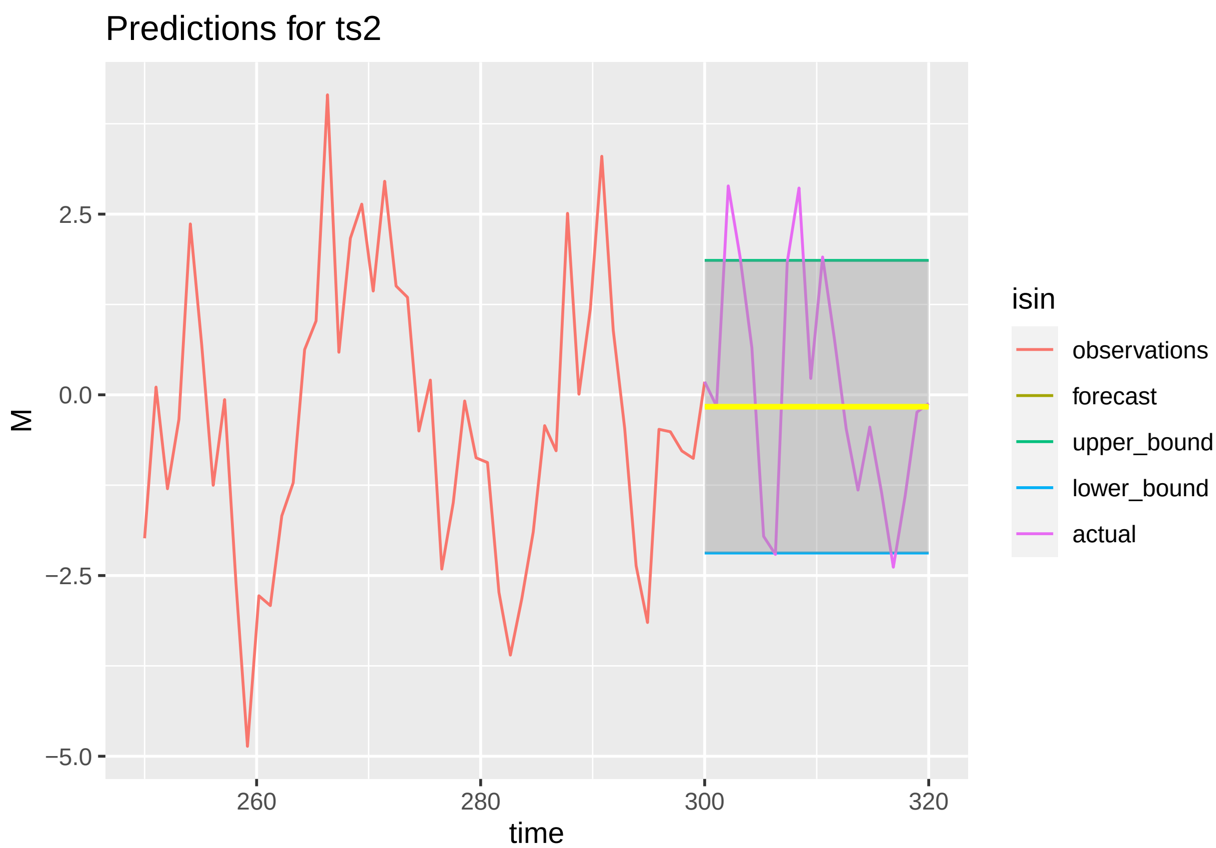
```
##  [3,] 1.0392 1.667
##  [4,] 1.0405 1.717
##  [5,] 1.0408 1.733
##  [6,] 1.0409 1.737
##  [7,] 1.0409 1.739
##  [8,] 1.0409 1.739
##  [9,] 1.0409 1.739
## [10,] 1.0409 1.739
## [11,] 1.0409 1.739
## [12,] 1.0409 1.739
## [13,] 1.0409 1.739
## [14,] 1.0409 1.739
## [15,] 1.0409 1.739
## [16,] 1.0409 1.739
## [17,] 1.0409 1.739
## [18,] 1.0409 1.739
## [19,] 1.0409 1.739
## [20,] 1.0409 1.739
```

```
# Calculates the confidence interval
upperConf = pred$pred + 1.96 * pred$se.err
lowerConf = pred$pred - 1.96 * pred$se.err
```

```
drawLength = 50
drawStart = length - drawLength

## wrap data into a data.frame
df1 = data.frame(time = seq(drawStart,length,length=drawLength), M = ts$ts1[(drawStart+1):length], isin
df2 = data.frame(time = seq(length,length+testLength,length=testLength), M = pred$pred[1] , isin = "for
df3 = data.frame(time = seq(length,length+testLength,length=testLength), M = upperConf[1],isin = "upper
df4 = data.frame(time = seq(length,length+testLength,length=testLength), M = lowerConf[1], isin = "lowe
df5 = data.frame(time = seq(length,length+testLength,length=testLength), M = test1 , isin = "actual")
df = rbind(df1, df2, df3, df4, df5)

## ggplot object
ggplot(df, aes(x = time, y = M, color = isin)) + geom_line() + ggtitle("Predictions for ts1") + geom_sm
```




```r
## wrap data into a data.frame
df1 = data.frame(time = seq(drawStart,length,length=drawLength), M = ts$ts2[(drawStart+1):length], isin
df2 = data.frame(time = seq(length,length+testLength,length=testLength), M = pred$pred[2] , isin = "fore
df3 = data.frame(time = seq(length,length+testLength,length=testLength), M = upperConf[2],isin = "upper_
df4 = data.frame(time = seq(length,length+testLength,length=testLength), M = lowerConf[2], isin = "lowe
df5 = data.frame(time = seq(length,length+testLength,length=testLength), M = test2, isin = "actual")
df = rbind(df1, df2, df3, df4, df5)

## ggplot object
ggplot(df, aes(x = time, y = M, color = isin)) + geom_line() + ggtitle("Predictions for ts2") + geom_smo
```




**Dynamic Forecasting**

```
currData = ts
currPred1 = vector("numeric", testLength)
currPred2 = vector("numeric", testLength)
upperConf1 = vector("numeric", testLength)
upperConf2 = vector("numeric", testLength)
lowerConf1 = vector("numeric", testLength)
lowerConf2 = vector("numeric", testLength)
for (h in (length + 1):(length+testLength)) {
  # build the model
  m2 = VAR(currData, 1, output = F)

  i = h - length

  # predicts
  currPred = VARpred(m2, output = T)
  currPred1[i] = currPred$pred[1]
  currPred2[i] = currPred$pred[2]
  upperConf1[i] = currPred1[i] + 1.96*currPred$se.err[1]
  upperConf2[i] = currPred2[i] + 1.96*currPred$se.err[2]
  lowerConf1[i] = currPred1[i] - 1.96*currPred$se.err[1]
  lowerConf2[i] = currPred2[i] - 1.96*currPred$se.err[2]
```

```r
  oldData = as_tibble(currData)
  currData = as_tibble(currData$ts1)
  currData[h,1] = test1[i]
  oldData = oldData[,-1]
  oldData[h, 1] = test2[i]
  currData[,2] = oldData$ts2
  currData[,3] = 1:h
  currData = rename(currData, "ts1" = "value")
  currData = rename(currData, "ts2" = "...2")
  currData = rename(currData, "index" = "...3")
  currData = currData %>% as_tsibble(index = "index")
}
```

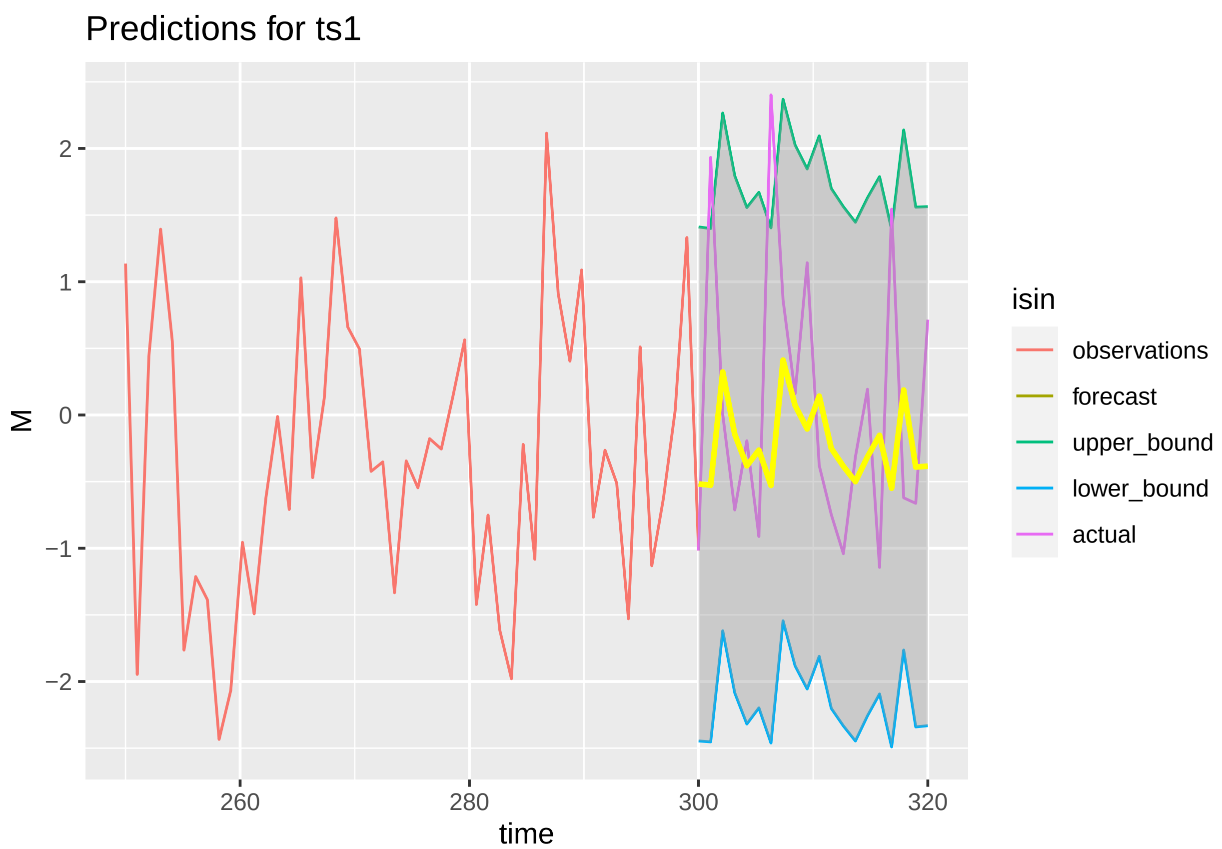
```
## orig  300
## Forecasts at origin:  300
##      ts1      ts2    index
##  -0.5175  -0.9147 301.0000
## Standard Errors of predictions:
## [1] 9.839e-01 1.082e+00 2.470e-14
## Root mean square errors of predictions:
## [1] 9.904e-01 1.089e+00 2.486e-14
## orig  301
## Forecasts at origin:  301
##      ts1      ts2    index
##  -0.5268  -0.8971 302.0000
## Standard Errors of predictions:
## [1] 9.827e-01 1.082e+00 2.282e-13
## Root mean square errors of predictions:
## [1] 9.892e-01 1.089e+00 2.297e-13
## orig  302
## Forecasts at origin:  302
##      ts1      ts2    index
##   0.3233   1.6897 303.0000
## Standard Errors of predictions:
## [1] 9.911e-01 1.081e+00 8.535e-14
## Root mean square errors of predictions:
## [1] 9.976e-01 1.088e+00 8.591e-14
## orig  303
## Forecasts at origin:  303
##      ts1      ts2    index
##  -0.1456   1.2896 304.0000
## Standard Errors of predictions:
## [1] 9.896e-01 1.081e+00 1.211e-13
## Root mean square errors of predictions:
## [1] 9.961e-01 1.088e+00 1.219e-13
## orig  304
## Forecasts at origin:  304
##      ts1      ts2    index
##  -0.3812   0.1961 305.0000
## Standard Errors of predictions:
## [1] 9.885e-01 1.080e+00 1.236e-13
## Root mean square errors of predictions:
## [1] 9.950e-01 1.087e+00 1.244e-13
## orig  305
```

```
## Forecasts at origin:  305
##      ts1      ts2    index
##  -0.2635   0.1233 306.0000
## Standard Errors of predictions:
## [1] 9.869e-01 1.079e+00 1.221e-13
## Root mean square errors of predictions:
## [1] 9.934e-01 1.086e+00 1.229e-13
## orig  306
## Forecasts at origin:  306
##      ts1      ts2    index
##  -0.5286  -1.7473 307.0000
## Standard Errors of predictions:
## [1] 9.860e-01 1.083e+00 8.223e-14
## Root mean square errors of predictions:
## [1] 9.924e-01 1.090e+00 8.276e-14
## orig  307
## Forecasts at origin:  307
##      ts1      ts2    index
##   0.4123   1.2213 308.0000
## Standard Errors of predictions:
## [1] 9.983e-01 1.082e+00 1.169e-13
## Root mean square errors of predictions:
## [1] 1.005e+00 1.089e+00 1.177e-13
## orig  308
## Forecasts at origin:  308
##       ts1       ts2     index
##   0.07193   1.61455 309.00000
## Standard Errors of predictions:
## [1] 9.970e-01 1.081e+00 6.426e-14
## Root mean square errors of predictions:
## [1] 1.003e+00 1.088e+00 6.467e-14
## orig  309
## Forecasts at origin:  309
##      ts1      ts2    index
##  -0.1048   1.4408 310.0000
## Standard Errors of predictions:
## [1] 9.954e-01 1.081e+00 2.683e-14
## Root mean square errors of predictions:
## [1] 1.002e+00 1.088e+00 2.701e-14
## orig  310
## Forecasts at origin:  310
##      ts1      ts2    index
##   0.1417   1.1603 311.0000
## Standard Errors of predictions:
## [1] 9.962e-01 1.082e+00 1.184e-13
## Root mean square errors of predictions:
## [1] 1.003e+00 1.089e+00 1.191e-13
## orig  311
## Forecasts at origin:  311
##      ts1      ts2    index
##  -0.2511   0.4883 312.0000
## Standard Errors of predictions:
## [1] 9.951e-01 1.081e+00 1.881e-13
## Root mean square errors of predictions:
```

```
## [1] 1.001e+00 1.088e+00 1.893e-13
## orig  312
## Forecasts at origin:  312
##      ts1      ts2    index
##  -0.3849  -0.3588 313.0000
## Standard Errors of predictions:
## [1] 9.938e-01 1.079e+00 1.250e-13
## Root mean square errors of predictions:
## [1] 1.000e+00 1.086e+00 1.258e-13
## orig  313
## Forecasts at origin:  313
##      ts1      ts2    index
##  -0.4999  -1.1923 314.0000
## Standard Errors of predictions:
## [1] 9.929e-01 1.077e+00 3.228e-14
## Root mean square errors of predictions:
## [1] 9.993e-01 1.084e+00 3.249e-14
## orig  314
## Forecasts at origin:  314
##      ts1      ts2    index
##  -0.3137  -0.8877 315.0000
## Standard Errors of predictions:
## [1] 9.914e-01 1.076e+00 2.336e-13
## Root mean square errors of predictions:
## [1] 9.977e-01 1.082e+00 2.351e-13
## orig  315
## Forecasts at origin:  315
##       ts1       ts2     index
##  -0.15242  -0.01348 316.00000
## Standard Errors of predictions:
## [1] 9.902e-01 1.074e+00 1.802e-13
## Root mean square errors of predictions:
## [1] 9.965e-01 1.081e+00 1.814e-13
## orig  316
## Forecasts at origin:  316
##      ts1      ts2    index
##  -0.5496  -1.6941 317.0000
## Standard Errors of predictions:
## [1] 9.902e-01 1.075e+00 1.663e-13
## Root mean square errors of predictions:
## [1] 9.965e-01 1.082e+00 1.673e-13
## orig  317
## Forecasts at origin:  317
##      ts1      ts2    index
##   0.1876   0.3694 318.0000
## Standard Errors of predictions:
## [1] 9.955e-01 1.074e+00 8.738e-14
## Root mean square errors of predictions:
## [1] 1.002e+00 1.081e+00 8.793e-14
## orig  318
## Forecasts at origin:  318
##      ts1      ts2    index
##  -0.3904  -1.2570 319.0000
## Standard Errors of predictions:
```

```
## [1] 9.950e-01 1.077e+00 3.745e-14
## Root mean square errors of predictions:
## [1] 1.001e+00 1.084e+00 3.769e-14
## orig  319
## Forecasts at origin:  319
##       ts1       ts2    index
##  -0.3839  -0.7588 320.0000
## Standard Errors of predictions:
## [1] 9.935e-01 1.077e+00 1.849e-13
## Root mean square errors of predictions:
## [1] 9.997e-01 1.083e+00 1.860e-13
```

```
## wrap data into a data.frame
df1 = data.frame(time = seq(drawStart,length,length=drawLength), M = ts$ts1[(drawStart+1):length], isin
df2 = data.frame(time = seq(length,length+testLength,length=testLength), M = currPred1 , isin = "foreca
df3 = data.frame(time = seq(length,length+testLength,length=testLength), M = upperConf1,isin = "upper_b
df4 = data.frame(time = seq(length,length+testLength,length=testLength), M = lowerConf1, isin = "lower_
df5 = data.frame(time = seq(length,length+testLength,length=testLength), M = test1, isin = "actual")
df = rbind(df1, df2, df3, df4, df5)

## ggplot object
ggplot(df, aes(x = time, y = M, color = isin)) + geom_line() + ggtitle("Predictions for ts1") + geom_sm
```




```
## wrap data into a data.frame
df1 = data.frame(time = seq(drawStart,length,length=drawLength), M = ts$ts1[(drawStart+1):length], isin
df2 = data.frame(time = seq(length,length+testLength,length=testLength), M = currPred2, isin = "forecas
df3 = data.frame(time = seq(length,length+testLength,length=testLength), M = upperConf2,isin = "upper_b
```

```
df4 = data.frame(time = seq(length,length+testLength,length=testLength), M = lowerConf2, isin = "lower_
df5 = data.frame(time = seq(length,length+testLength,length=testLength), M = test2, isin = "actual")
df = rbind(df1, df2, df3, df4, df5)

## ggplot object
ggplot(df, aes(x = time, y = M, color = isin)) + geom_line() + ggtitle("Predictions for ts2") + geom_smo
```