SETS AND RELATIONS
Lesson 6

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Sets

A collection of objects in called a set .The objects that comprises of the set are called elements . Number of objects in a set can be finite or infinite.

For Example:

- A set of chairs,
- The set of nobel laureates in the worlds,the set of integers,
- The set of natural numbers less than 10,
- The set of points in the plane R2.

•The number of elements in a set is called the cardinality of the set. (If S is a set the cardinality is denoted by |S|)

Notations related to set

Usually a set is represented by its list of elements separated by comma, between two curly brackets.

Example

{1, 2, 3, 4, 5} is the list of integers bigger than 0 and lesser than or equal to 5.

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If S is a set and we want to denote that x is an element of the set we write as $x \in S$.

Some Special Sets

- **Finite and Infinite Sets**

A set is finite if it contains only a finite number of elements. Otherwise, the set is said to be an *infinite set.*

e.g., N = Set of all natural numbers = $\{1,2,3,\ldots..\}$

Z = Set of all integers

$= \{\ldots.., -2, -1, 0, 1, 2, \ldots\ldots\}$ are infinite sets.

and A = {a, e, i, o, u}

= Set of all vowels

B = Set of all students studying in a particular school are finite sets

Some Special Sets

- **Null Set**

A set which does not contain any element is called a null set. It is denoted by Θ. A null set is also called an empty set or a void set. Therefore, Θ = { }

e.g., A = {x : x is prime numbers between 90 and 96}= Θ

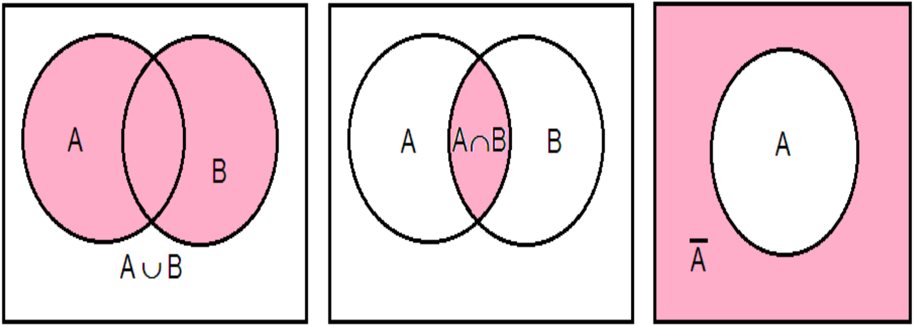
Operations on Sets

Union, ∪.
A ∪ B is the set of all elements that are in A OR B.

Intersection, ∩
A ∩ B is the set of all elements that are in A AND B.

Complement, A^c or $\overline{A}$
Ac is the set of elements NOT in A.

Operations on Sets



Some problems on set theory

- If $|A| = 5$ and $|B| = 8$ and $|A \cup B| = 11$ what is the size of $A \cap B$?

- If $|Ac \cap B| = 10$ and $|A \cap B^c| = 8$ and $|A \cap B| = 5$ then how many elements are there is $A \cup B$?

EXAMPLES

A travel agent surveyed 100 people to find out how many of them had visited the cities of Melbourne and Brisbane. Thirty-one people had visited Melbourne, 26 people had been to Brisbane, and 12 people had visited both cities. Draw a Venn diagram to find the number of people who had visited:

a) Melbourne or Brisbane

b) Brisbane but not Melbourne

c) only one of the two cities

d) neither city.

Relations

Relation is a definite manner or pattern which show how a set belongs to another one. It can be understand as an

Example i.e., Let A and B be two sets. Then, a relation R from A to B is a subset of $A \times B$. Thus, R is a relation from A to B $\leftrightarrow R \subseteq A \times B$. If R is a relation from a non-void set A to a non-void set B and if $(a, b) \in R$, then we write aRb which is read as ‘a is related to b’ by the relation R. If $(a, b) \notin R$, then we write $a \not R b$ and said that a is not related to b by the relation R.

Domain and Range of a Relation

- Let R be a relation from a set A to a set B. Then, the set of all first components or coordinates of the ordered pair belonging to R is called the domain of R, while the set of all second components or coordinates of the ordered pair of R is called the range of R.

  Thus, Domain (R) = $\{a : (a, b) \in R\}$

  and Range (R) = $\{b : (a, b) \in R\}$

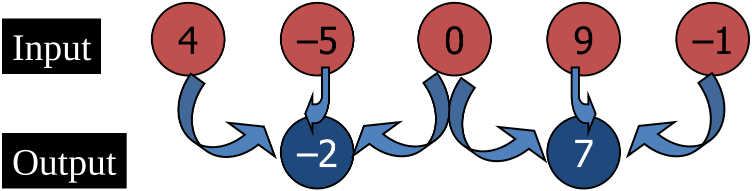
- It is evident from the definition that the domain of a relation from A to B is a subset of A and its range is a subset of B. e.g.,

  If $A = \{1,3,5,7\}$, $B = \{2,4,6,8,10\}$

  and $R = \{(1,8), (3,6), (5,2), (1,4)\}$ be a relation from A to B.

  Then, Dom (R) = $\{1,3,5\}$ and Range (R) = $\{8,6,2,4\}$.

Domain and Range of a Relation



•What is the **domain**?

{4, -5, 0, 9, -1}

•What is the **range**?

{-2, 7}

Is a relation a function?

What is a **function**?

A function is…a relation in which every input is paired with exactly one output"

Focus on the ***x*-coordinates**, when given a relation

If the set of ordered pairs have **different *x*-coordinates**,
it **IS A** function

If the set of ordered pairs have **same *x*-coordinates**,
it is **NOT** a function

- **Y-coordinates** have no bearing in determining functions

Example

$$\{(0,-5),(1,-4),(2,-3),(3,-2),(4,-1),(5,0)\}$$

- **Is this a function?**
  - **Hint:** Look only at the **x-coordinates YES**

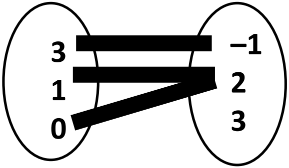
$$\{(-1,-7),(1,0),(2,-3),(0,-8),(0,5),(-2,-1)\}$$

- ***Is this a function?***
  - ***Hint:*** *Look only at the* ***x-coordinates NO***

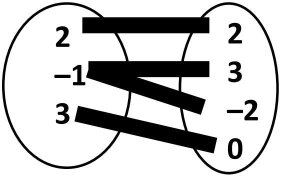
Example

Which mapping represents a function?

Choice One



Choice Two

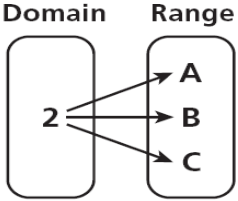


Choice 1

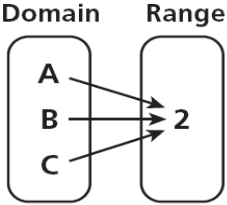
Example

Which mapping represents a function?

A.



B.



B

Example

Which situation represents a function?

a. The items in a store to their prices on a certain date

b. Types of fruits to their colors

There is only one price for each different item on a certain date. The relation from items to price makes it a function.

A fruit, such as an apple, from the domain would be associated with more than one color, such as red and green. The relation from types of fruits to their colors is not a function.

Example

- If R is a relation from set A = {2, 4, 5} to set B = {1, 2, 3, 4, 6,8}defined by xRy   x divides y.

  Find the domain and the range of R.

  (a) Dom (R) = {2}, Range (R) = {2, 4,6}

  (b) Dom (R) = {2, 4}, Range (R) = {2, 4,6,8}

  (c) Dom (R) = {4}, Range (R) = {2, 4,6}

  (d) None of the above

Sol. (b) Clearly, 2 R2, 2 R 4, 2 R6, 2 R8, 4R 4 and 4R8

R = {(2, 2), (2, 4), (2, 6), (2, 8), (4, 4), (4, 8)}

Clearly, Dom (R) = {2, 4}

and Range (R) = {2, 4, 6, 8}

Domain and Range in Real Life

Pete's Pizza Parlor charges \$5 for a large pizza with no toppings. They charge an additional \$1.50 for each of their 5 specialty toppings (tax is included in the price).

Jorge went to pick up his order. They said his total bill was \$9.50. Could this be correct? Why or why not?

Yes One pizza with 3 toppings cost \$9.50

Susan went to pick up her order. They said she owed \$10.25. Could this be correct? Why or why not?

No One pizza with 4 toppings cost \$11

Functions

Functions are mappings from a set (called domain) to another set (called range).

$f : D \rightarrow R$

For all element $x \in D$, $f(x) \in R$.

For every element in D there is a uniquely defined element $f(x)$.

Algebra of real functions

(i)Addition of two real functions

Let f : X →R and g : X → R be any two real functions, where X Ɛ R.
Then we define (f + g) : X → R by (f + g) (x) = f (x) + g (x), for all x Ɛ X.

(ii) Subtraction of a real function from another

Let f : X → R and g : X → R be any two real functions, where X $\subseteq$ R.
Then, we define (f – g) : X → R by (f – g) (x) = f (x) – g (x), for all x Ɛ X.

(iii) Multiplication by a Scalar

Let f : X → R be a real function and a be any scalar belonging to R. Then the product αf is function from X to R defined by (α f) (x) = α f (x), x Ɛ X.

Example

Find x and y if:

(i) $(4x + 3, y) = (3x + 5, -2)$ (ii) $(x - y, x + y) = (6, 10)$

Solution

(i) Since $(4x + 3, y) = (3x + 5, -2)$, so

$4x + 3 = 3x + 5$

or $x = 2$

and $y = -2$

(ii) $x - y = 6$

$x + y = 10$

$2x = 16$

Or $x = 8$

$8 - y = 6$

$y = 2$

Example

What is the size of the truth-table for a function
$f : \{0, 1\}^n \to \{0, 1\}$?

What is the size of the truth-table for a function
f : {0, 1}n → {0, 1}?

How many different functions are there from {1, 2, 3, 4, 5, 6}
to {a, e, i, o, u}?

How many different Boolean functions of the for
{0, 1}n → {0, 1} are possible?

Function Notation

$f(x)$ means function of x and is read "f of x."

$f(x) = 2x + 1$ is written in function notation.

The notation $f(1)$ means to replace x with 1 resulting in the function value.

$f(1) = 2x + 1$
$f(1) = 2(1) + 1$
$f(1) = 3$

Function Notation

Given $g(x) = x^2 - 3$, find $g(-2)$.

$$g(-2) = x^2 - 3$$
$$g(-2) = (-2)^2 - 3$$
$$g(-2) = 1$$

Function Notation

Given $f(x) = 2x^2 - 3x$, the following.

a. $f(3)$

$f(3) = 2x^2 - 3x$
$f(3) = 2(3)^2 - 3(3)$
$f(3) = 2(9) - 9$
$f(3) = 9$

b. $3f(x)$

$3f(x) = 3(2x^2 - 3x)$
$3f(x) = 6x^2 - 9x$

c. $f(3x)$

$f(3x) = 2x^2 - 3x$
$f(3x) = 2(3x)^2 - 3(3x)$
$f(3x) = 2(9x^2) - 3(3x)$
$f(3x) = 18x^2 - 9x$

Example of a Function

$f : \{0, 1, 2, 3, 4, 5\} \rightarrow R$
$f(0) = 0$
$f(1) = 1$
$f(2) = 4$
$f(3) = 9$
$f(4) = 16$
$f(5) = 25$

How to represent the function f ?
Either explicitly give the function OR say $f(x) = x2$

Example of a Function

1) Let A={-1,2,3} and B={1,3}

a) AxB b)BxA c) Is AxB =BxA d) Is N(AxB)=N(BxA)

2) If A = {2, 4, 6, 9} and B = {4, 6, 18, 27, 54}, a ∈ A, b ∈ B, find the set of ordered pairs such that 'a' is factor of 'b' and a < b.

3) Find the domain and range of the relation R given by R = {(x, y) : y = 6x + x ; where x, y ∈ N and x < 6}.

4) Is the following relation a function? Justify your answer (6 Remarks)

(a) R1 = {(2, 3), (1 2 , 0), (2, 7), (– 4, 6)}

(b) R2 = {(x, |x |) | x is a real number}

(c) R3 = {(4, 6), (3, 9), (– 11, 6), (3, 11)}

THANK YOU