

# Programming in Vinyl

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# Structural Typing

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- ▶ Record types may be characterized *structurally*

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```
lens : s ::: t ∈ rs ⇒ s ::: t → Lens' (Rec rs) t
```

# Roll Your Own in Haskell



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```
f :: Rec ("foo" ::: A ': rs)
  → Rec ("bar" ::: B ': "foo" ::: A ': rs)
```

# Universes à la Tarski

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- ▶ A type  $\mathcal{U}$  of **codes** for types.

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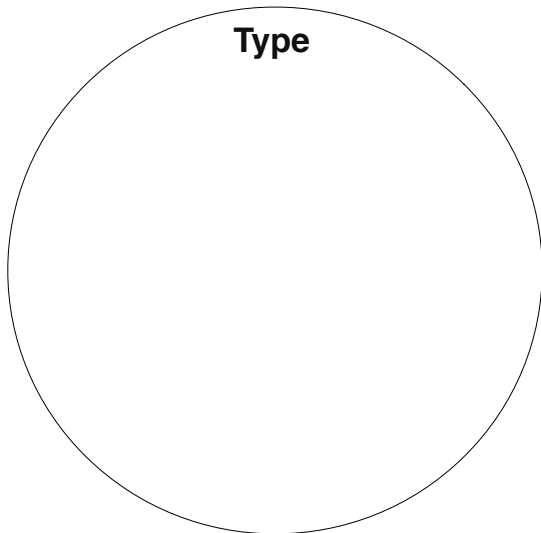
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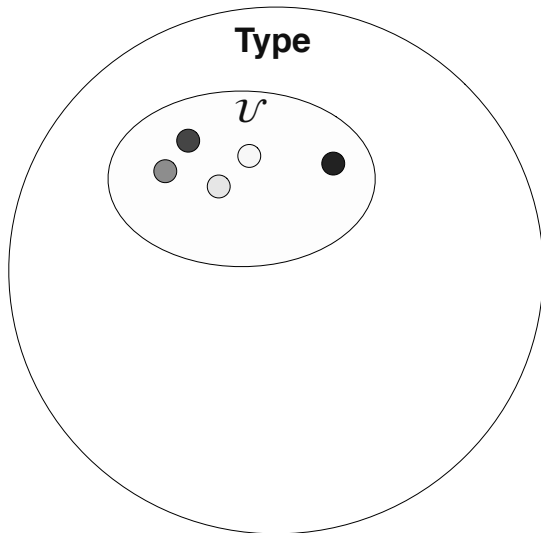
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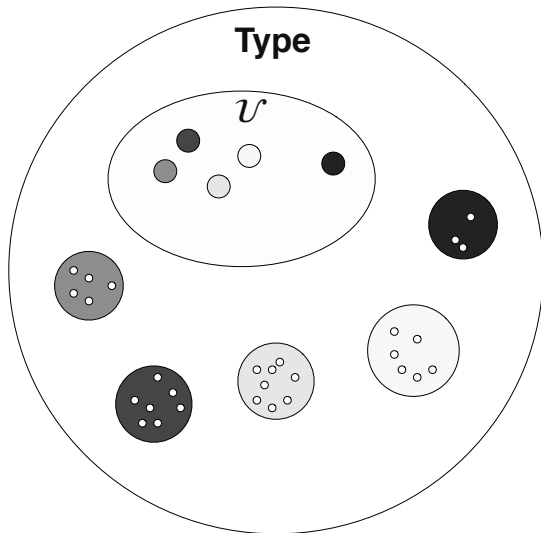
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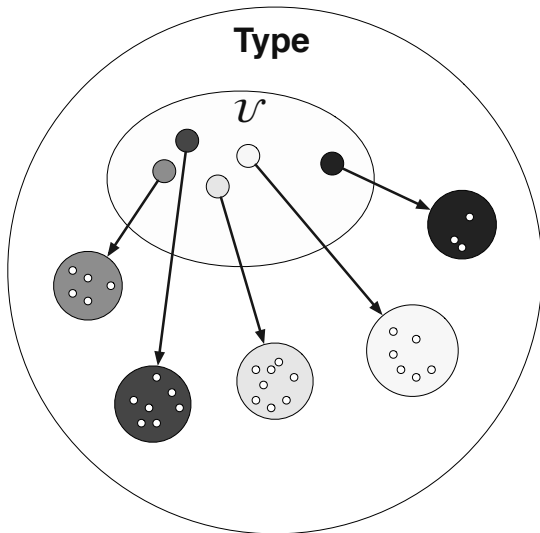


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$$\overline{\text{El}_{\mathcal{A}}(\text{Phone}[\ell]) \rightsquigarrow \text{list}(\mathbb{N})}$$

# Records as Products

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Records: the product of the image of  $El_{\mathcal{U}}$  in Type restricted to a subset of  $\mathcal{U}$ .

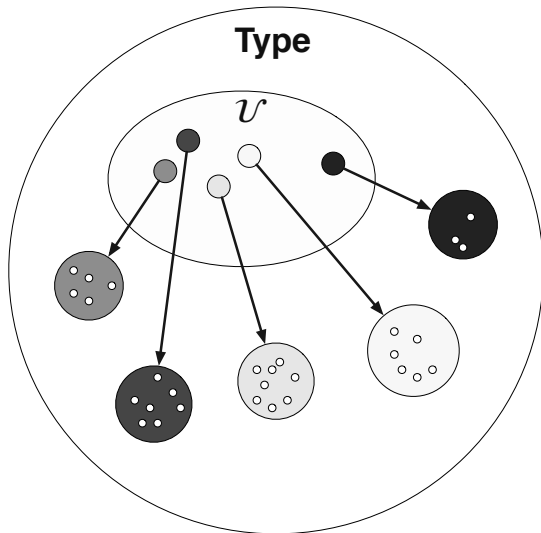


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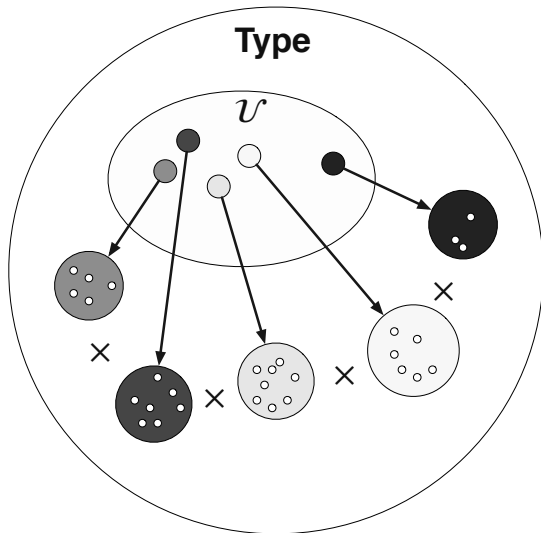
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$$ex \rightsquigarrow \langle \mathcal{A}', \lambda.$$

$$\{\text{Name} \mapsto \text{"Robert Harper"};$$

$$\text{Email } \textit{Work} \mapsto \text{"rwh@cs.cmu.edu"}\}\rangle$$



# Presheaves

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A presheaf on some category  $X$  is a functor  $\mathcal{O}(X)^{\text{op}} \rightarrow \mathbf{Type}$ , where  $\mathcal{O}$  is the category of open sets of  $X$  for whatever topology you have chosen.

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- ▶ The empty set and  $X$  are open sets
- ▶ Finite intersections of open sets are open

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$$\text{for } V \subseteq X \quad \mathcal{R}(V) := \prod_V El_X|_V : \mathbf{Type}$$

$$\text{for } i : V \hookrightarrow U \quad \mathcal{R}(i) := \text{cast} : \mathcal{R}(U) \rightarrow \mathcal{R}(V)$$

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For a cover  $U = \bigcup_i U_i$  on  $X$ , then:

$$\mathcal{R}(U) \xrightarrow{e} \prod_i \mathcal{R}(U_i) \begin{matrix} \xrightarrow{p} \\ \xrightarrow{q} \end{matrix} \prod_{i,j} \mathcal{R}(U_i \cap U_j)$$

is an equalizer, where

$$e = \lambda r. \lambda i. \text{cast}_{U_i}(r)$$

$$p = \lambda f. \lambda i. \lambda j. \text{cast}_{U_i \cap U_j}(f(i))$$

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where

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Corecords (extensible variants): the sum of the image of  $El_{\mathcal{U}}$  in Type restricted to a subset of  $\mathcal{U}$ .

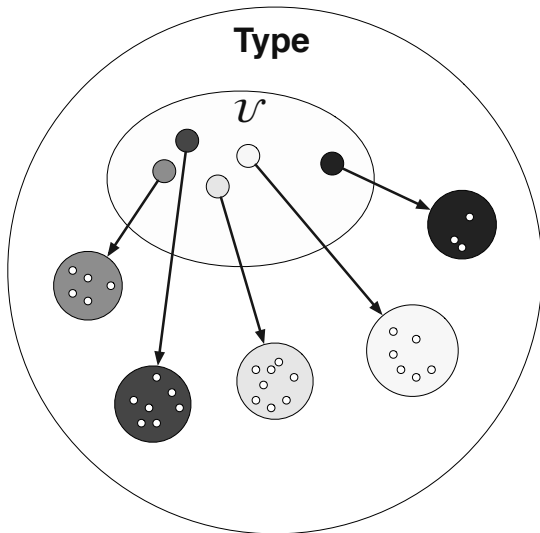
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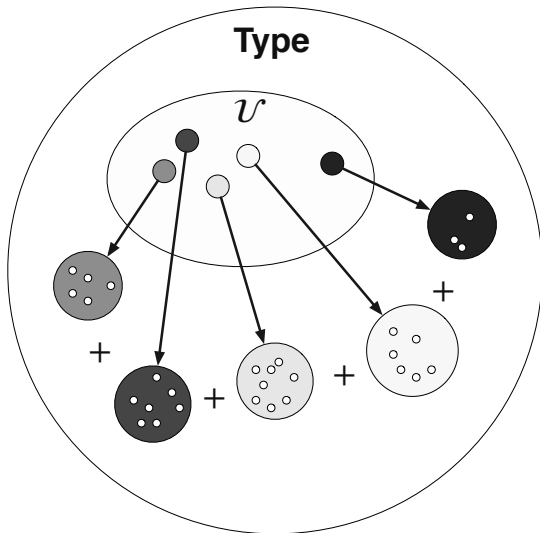
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- ▶ Use type families to approximate  $El_{\mathcal{U}}$
- ▶ Parameterize `Rec` by  $\mathcal{U}$ ,  $El_{\mathcal{U}}$ ?

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**data** Rec :: ( $\mathcal{U} \rightarrow *$ )  $\rightarrow$  [ $\mathcal{U}$ ]  $\rightarrow *$  **where**

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```
data TyFun :: * → * → *
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```
type family (f :: TyFun k l → *) $ (x :: k) :: l
```

# Records in Haskell (Actually)

**data** TyFun ::  $*$   $\rightarrow$   $*$   $\rightarrow$   $*$

**type** family (f :: TyFun k l  $\rightarrow$   $*$ ) \$ (x :: k) :: l

**data** Rec :: (TyFun  $\mathcal{U}$   $*$   $\rightarrow$   $*$ )  $\rightarrow$  [ $\mathcal{U}$ ]  $\rightarrow$   $*$  **where**

RNil :: Rec el <sub>$\mathcal{U}$</sub>  '[]

(:&) :: !(el <sub>$\mathcal{U}$</sub>  \$ r)  $\rightarrow$  !(Rec el <sub>$\mathcal{U}$</sub>  rs)  $\rightarrow$  Rec el <sub>$\mathcal{U}$</sub>  (r ': rs)

# Recovering HList



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type instance Id $ x = x
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type HList rs = Rec Id rs
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```
ex :: HList [ℤ, Bool, String]  
ex = 34 :& True :& "vinyl" :& RNil
```

# Validating Records

bob :: Rec El<sub>A</sub> [Name, Email Work]

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bob = Name =: "Robert\_Harper"

⊕ Email Work =: "rwh@cs.cmu.edu"

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bob :: Rec ElA [Name, Email Work]  
bob = Name =: "Robert_LHarper"  
      ⊕ Email Work =: "rwh@cs.cmu.edu"
```

```
validateName :: String → Either Error String  
validateEmail :: String → Either Error String  
validatePhone :: [N] → Either Error [N]
```

# Validating Records

bob :: Rec El<sub>A</sub> [Name, Email Work]

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validateName :: **String** → **Either** Error **String**

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*\*unnnnnnhhh...\**



# Validating Records

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bob :: Rec ElA [Name, Email Work]
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```
validateName :: String → Either Error String
validateEmail :: String → Either Error String
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```

*\*unnnnnnhhh...\**

```
validateContact
  :: Rec ElA [Name, Email Work]
  → Either Error (Rec ElA [Name, Email Work])
```

**Welp.**

# Effects inside records

```
data Rec :: (TyFun  $\mathcal{U}$   $*$   $\rightarrow$   $*$ )  $\rightarrow$  [ $\mathcal{U}$ ]  $\rightarrow$   $*$  where  
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  (:&) :: !( $\text{el}_{\mathcal{U}}$  $ r)  $\rightarrow$  !(Rec  $\text{el}_{\mathcal{U}}$  rs)  $\rightarrow$  Rec  $\text{el}_{\mathcal{U}}$  (r ': rs)
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# Effects inside records

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  RNil :: Rec  $\text{el}_{\mathcal{U}}$  f '[]  
  (:&)amp :: !(f (el $_{\mathcal{U}}$  $ r))  $\rightarrow$  !(Rec el $_{\mathcal{U}}$  f rs)  $\rightarrow$  Rec el $_{\mathcal{U}}$  f (r ': rs)
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(=:) : Applicative f  $\Rightarrow$  sing r  $\rightarrow$  el $_{\mathcal{U}}$  $ r  $\rightarrow$  Rec el $_{\mathcal{U}}$  f '[r]
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k =: x = pure x :& RNil

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(=:) : Applicative f  $\Rightarrow$  sing r  $\rightarrow$  el <sub>$\mathcal{U}$</sub>  \$ r  $\rightarrow$  Rec el <sub>$\mathcal{U}$</sub>  f '[r]

k =: x = pure x :& RNil

( $\Leftarrow$ ) : sing r  $\rightarrow$  f (el <sub>$\mathcal{U}$</sub>  \$ r)  $\rightarrow$  Rec el <sub>$\mathcal{U}$</sub>  f '[r]

# Effects inside records

**data** Rec :: (TyFun  $\mathcal{U}$   $*$   $\rightarrow$   $*$ )  $\rightarrow$  ( $*$   $\rightarrow$   $*$ )  $\rightarrow$  [ $\mathcal{U}$ ]  $\rightarrow$   $*$  **where**

RNil :: Rec  $\text{el}_{\mathcal{U}}$  f '[]

(:&) :: !(f (el <sub>$\mathcal{U}$</sub>  \$ r))  $\rightarrow$  !(Rec el <sub>$\mathcal{U}$</sub>  f rs)  $\rightarrow$  Rec el <sub>$\mathcal{U}$</sub>  f (r ': rs)

(=:) : Applicative f  $\Rightarrow$  sing r  $\rightarrow$  el <sub>$\mathcal{U}$</sub>  \$ r  $\rightarrow$  Rec el <sub>$\mathcal{U}$</sub>  f '[r]

k =: x = pure x :& RNil

( $\Leftarrow$ ) : sing r  $\rightarrow$  f (el <sub>$\mathcal{U}$</sub>  \$ r)  $\rightarrow$  Rec el <sub>$\mathcal{U}$</sub>  f '[r]

k  $\Leftarrow$  x = x :& RNil



# Compositional Validation

**type**  $\text{Rec}_{\mathcal{A}} = \text{Rec El}_{\mathcal{A}}$

# Compositional Validation

**type**  $\text{Rec}_{\mathcal{A}} = \text{Rec El}_{\mathcal{A}}$

$\text{bob} :: \text{Rec}_{\mathcal{A}} \text{ Identity } [\text{Name}, \text{Email Work}]$

# Compositional Validation

```
type RecA = Rec ElA  
bob :: RecA Identity [Name, Email Work]  
bob = Name =: "Robert_Harper"  
      ⊕ Email Work =: "rwh@cs.cmu.edu"
```

# Compositional Validation

```
type RecA = Rec ElA  
bob :: RecA Identity [Name, Email Work]  
bob = Name =: "Robert_Harper"  
      ⊕ Email Work =: "rwh@cs.cmu.edu"
```

# Compositional Validation

**type** Validator a = a  $\rightarrow$  **Either** Error a

# Compositional Validation

```
type Validator a = a → Either Error a  
validateName :: RecA Validator '[Name]  
validatePhone :: ∀ℓ. RecA Validator '[Phone ℓ]  
validateEmail :: ∀ℓ. RecA Validator '[Email ℓ]
```

# Compositional Validation

```
type Validator a = a → Either Error a  
validateName :: RecA Validator '[Name]  
validatePhone :: ∀ℓ. RecA Validator '[Phone ℓ]  
validateEmail :: ∀ℓ. RecA Validator '[Email ℓ]
```

```
type TotalContact =  
  [ Name, Email Home, Email Work  
    , Phone Home, Phone Work ]
```

# Compositional Validation

**type** Validator a = a  $\rightarrow$  **Either** Error a  
validateName :: Rec <sub>$\mathcal{A}$</sub>  Validator '[Name]  
validatePhone ::  $\forall \ell$ . Rec <sub>$\mathcal{A}$</sub>  Validator '[Phone  $\ell$ ]  
validateEmail ::  $\forall \ell$ . Rec <sub>$\mathcal{A}$</sub>  Validator '[Email  $\ell$ ]

**type** TotalContact =  
[ Name, Email Home, Email Work  
 , Phone Home, Phone Work ]

validateContact :: Rec <sub>$\mathcal{A}$</sub>  Validator TotalContact  
validateContact = validateName  
                   $\oplus$  validateEmail  
                   $\oplus$  validateEmail  
                   $\oplus$  validatePhone  
                   $\oplus$  validatePhone



# Record Operators

# Record Operators

```
newtype Lift o f g x = Lift { runLift :: f x 'o' g x }
```

# Record Operators

```
newtype Lift o f g x = Lift { runLift :: f x 'o' g x }
```

```
type Validator = Lift (→) Identity (Either Error)
```

# Record Operators

**newtype** Lift o f g x = Lift { runLift :: f x 'o' g x }

**type** Validator = Lift (→) Identity (**Either** Error)

( $\odot$ ) :: Rec<sub>U</sub> (Lift (→) f g) rs → Rec<sub>U</sub> f rs → Rec<sub>U</sub> g rs

# Record Operators

**newtype** Lift o f g x = Lift { runLift :: f x 'o' g x }

**type** Validator = Lift (→) Identity (**Either** Error)

( $\odot$ ) :: Rec<sub>U</sub> (Lift (→) f g) rs → Rec<sub>U</sub> f rs → Rec<sub>U</sub> g rs

rdist :: Applicative f ⇒ Rec<sub>U</sub> f rs → f (Rec<sub>U</sub> Identity rs)

# Compositional Validation

```
newtype Lift o f g x = Lift { runLift :: f x 'o' g x }  
type Validator = Lift (→) Identity (Either Error)  
(⊛) :: RecU (Lift (→) f g) rs → RecU f rs → RecU g rs  
rdist :: Applicative f ⇒ RecU f rs → f (RecU Identity rs)
```

# Compositional Validation

```
newtype Lift o f g x = Lift { runLift :: f x 'o' g x }  
type Validator = Lift (→) Identity (Either Error)  
(⊙) :: RecU (Lift (→) f g) rs → RecU f rs → RecU g rs  
rdist :: Applicative f ⇒ RecU f rs → f (RecU Identity rs)  
  
validateContact :: RecA Validator TotalContact
```

# Compositional Validation

```
newtype Lift o f g x = Lift { runLift :: f x 'o' g x }  
type Validator = Lift (→) Identity (Either Error)  
(⊙) :: RecU (Lift (→) f g) rs → RecU f rs → RecU g rs  
rdist :: Applicative f ⇒ RecU f rs → f (RecU Identity rs)  
  
validateContact :: RecA Validator TotalContact  
  
bobValid :: RecA (Either Error) [Name, Email Work]
```



# Compositional Validation

```
newtype Lift o f g x = Lift { runLift :: f x 'o' g x }  
type Validator = Lift (→) Identity (Either Error)  
(⊛) :: RecU (Lift (→) f g) rs → RecU f rs → RecU g rs  
rdist :: Applicative f ⇒ RecU f rs → f (RecU Identity rs)
```

```
validateContact :: RecA Validator TotalContact
```

```
bobValid :: RecA (Either Error) [Name, Email Work]  
bobValid = cast validateContact ⊛ bob
```

# Compositional Validation

```
newtype Lift o f g x = Lift { runLift :: f x 'o' g x }  
type Validator = Lift (→) Identity (Either Error)  
(⊛) :: RecU (Lift (→) f g) rs → RecU f rs → RecU g rs  
rdist :: Applicative f ⇒ RecU f rs → f (RecU Identity rs)
```

```
validateContact :: RecA Validator TotalContact
```

```
bobValid :: RecA (Either Error) [Name, Email Work]  
bobValid = cast validateContact ⊛ bob
```

```
validBob :: Either Error (RecA Identity [Name, Email Work])
```

# Compositional Validation

```
newtype Lift o f g x = Lift { runLift :: f x 'o' g x }  
type Validator = Lift (→) Identity (Either Error)  
(⊛) :: RecU (Lift (→) f g) rs → RecU f rs → RecU g rs  
rdist :: Applicative f ⇒ RecU f rs → f (RecU Identity rs)
```

```
validateContact :: RecA Validator TotalContact
```

```
bobValid :: RecA (Either Error) [Name, Email Work]  
bobValid = cast validateContact ⊛ bob
```

```
validBob :: Either Error (RecA Identity [Name, Email Work])  
validBob = rdist bobValid
```

# Laziness as an effect

# Laziness as an effect

```
newtype Identity a = Identity { runIdentity :: a }
```

# Laziness as an effect

```
newtype Identity a = Identity { runIdentity :: a }  
data Thunk a = Thunk { unThunk :: a }
```

# Laziness as an effect

**newtype** Identity a = Identity { runIdentity :: a }

**data** Thunk a = Thunk { unThunk :: a }

**type** PlainRec <sub>$\mathcal{U}$</sub>  rs = Rec <sub>$\mathcal{U}$</sub>  Identity rs

# Laziness as an effect

```
newtype Identity a = Identity { runIdentity :: a }
```

```
data Thunk a = Thunk { unThunk :: a }
```

```
type PlainRecU rs = RecU Identity rs
```

```
type LazyRecU rs = RecU Thunk rs
```



# Concurrent Records with Async

# Concurrent Records with Async

`fetchName :: RecA IO '[Name]`

# Concurrent Records with Async

```
fetchName :: RecA IO '[Name]  
fetchName = Name  $\Leftarrow$  someOperation
```

# Concurrent Records with Async

`fetchName :: RecA IO '[Name]`

`fetchName = Name  $\Leftarrow$  someOperation`

`fetchWorkEmail :: RecA IO '[Email Work]`

# Concurrent Records with Async

$\text{fetchName} :: \text{Rec}_{\mathcal{A}} \text{IO } '[\text{Name}]$

$\text{fetchName} = \text{Name} \Leftarrow \text{someOperation}$

$\text{fetchWorkEmail} :: \text{Rec}_{\mathcal{A}} \text{IO } '[\text{Email Work}]$

$\text{fetchWorkEmail} = \text{Email Work} \Leftarrow \text{anotherOperation}$

# Concurrent Records with Async

$\text{fetchName} :: \text{Rec}_{\mathcal{A}} \text{IO } '[\text{Name}]$

$\text{fetchName} = \text{Name} \Leftarrow \text{someOperation}$

$\text{fetchWorkEmail} :: \text{Rec}_{\mathcal{A}} \text{IO } '[\text{Email Work}]$

$\text{fetchWorkEmail} = \text{Email Work} \Leftarrow \text{anotherOperation}$

$\text{fetchBob} :: \text{Rec}_{\mathcal{A}} \text{IO } [\text{Name}, \text{Email Work}]$

# Concurrent Records with Async

$\text{fetchName} :: \text{Rec}_{\mathcal{A}} \text{IO } '[\text{Name}]$

$\text{fetchName} = \text{Name} \Leftarrow \text{someOperation}$

$\text{fetchWorkEmail} :: \text{Rec}_{\mathcal{A}} \text{IO } '[\text{Email Work}]$

$\text{fetchWorkEmail} = \text{Email Work} \Leftarrow \text{anotherOperation}$

$\text{fetchBob} :: \text{Rec}_{\mathcal{A}} \text{IO } [\text{Name}, \text{Email Work}]$

$\text{fetchBob} = \text{fetchName} \oplus \text{fetchWorkEmail}$

# Concurrent Records with Async



# Concurrent Records with Async

```
newtype Concurrently a  
  = Concurrently { runConcurrently :: IO a }
```

# Concurrent Records with Async

**newtype** Concurrently a  
= Concurrently { runConcurrently :: **IO** a }

$(\textcircled{\$}) :: (\forall a. f\ a \rightarrow g\ a) \rightarrow \text{Rec}_{\mathcal{U}}\ f\ rs \rightarrow \text{Rec}_{\mathcal{U}}\ g\ rs$

# Concurrent Records with Async

**newtype** Concurrently a  
= Concurrently { runConcurrently :: **IO** a }

$(\textcircled{\$}) :: (\forall a. f\ a \rightarrow g\ a) \rightarrow \text{Rec}_{\mathcal{U}}\ f\ rs \rightarrow \text{Rec}_{\mathcal{U}}\ g\ rs$

bobConcurrently ::  $\text{Rec}_{\mathcal{A}}\ \text{Concurrently}\ [\text{Name}, \text{Email Work}]$

# Concurrent Records with Async

**newtype** Concurrently a  
= Concurrently { runConcurrently :: **IO** a }

$(\textcircled{\$}) :: (\forall a. f\ a \rightarrow g\ a) \rightarrow \text{Rec}_{\mathcal{U}}\ f\ rs \rightarrow \text{Rec}_{\mathcal{U}}\ g\ rs$

bobConcurrently ::  $\text{Rec}_{\mathcal{A}}$  Concurrently [Name, Email Work]

bobConcurrently = Concurrently  $(\textcircled{\$})$  fetchBob

# Concurrent Records with Async

**newtype** Concurrently a  
= Concurrently { runConcurrently :: **IO** a }

$(\textcircled{\$}) :: (\forall a. f\ a \rightarrow g\ a) \rightarrow \text{Rec}_{\mathcal{U}}\ f\ rs \rightarrow \text{Rec}_{\mathcal{U}}\ g\ rs$

bobConcurrently ::  $\text{Rec}_{\mathcal{A}}$  Concurrently [Name, Email Work]

bobConcurrently = Concurrently  $(\textcircled{\$})$  fetchBob

concurrentBob :: Concurrently ( $\text{Rec}_{\mathcal{A}}$  Identity [...])

# Concurrent Records with Async

**newtype** Concurrently a  
= Concurrently { runConcurrently :: **IO** a }

$(\textcircled{\$}) :: (\forall a. f\ a \rightarrow g\ a) \rightarrow \text{Rec}_{\mathcal{U}}\ f\ rs \rightarrow \text{Rec}_{\mathcal{U}}\ g\ rs$

bobConcurrently ::  $\text{Rec}_{\mathcal{A}}$  Concurrently [Name, Email Work]

bobConcurrently = Concurrently  $(\textcircled{\$})$  fetchBob

concurrentBob :: Concurrently ( $\text{Rec}_{\mathcal{A}}$  Identity [...])

concurrentBob = rdist bobConcurrently

# Concurrent Records with Async

# Concurrent Records with Async

```
fetchBob :: RecA IO [Name, Email Work]  
bobConcurrently :: RecA Concurrently [Name, Email Work]  
concurrentBob :: Concurrently (RecA Identity [...])
```



# Concurrent Records with Async

```
fetchBob :: RecA IO [Name, Email Work]
bobConcurrently :: RecA Concurrently [Name, Email Work]
concurrentBob :: Concurrently (RecA Identity [...])

bob :: IO (RecA Identity [Name, Email Work])
```

# Concurrent Records with Async

```
fetchBob :: RecA IO [Name, Email Work]
bobConcurrently :: RecA Concurrently [Name, Email Work]
concurrentBob :: Concurrently (RecA Identity [...])

bob :: IO (RecA Identity [Name, Email Work])
bob = runConcurrently concurrentBob
```

# Containers: The Syntax for Data Types

container : Type

# Containers: The Syntax for Data Types

$$\frac{}{\text{container} : \text{Type}} \qquad \frac{\mathcal{U} : \text{Type} \quad El_{\mathcal{U}} : \mathcal{U} \rightarrow \text{Type}}{\mathcal{U} \triangleleft El_{\mathcal{U}} : \text{container}}$$

# Containers: The Syntax for Data Types

$$\frac{}{\text{container} : \text{Type}} \qquad \frac{\mathcal{U} : \text{Type} \quad El_{\mathcal{U}} : \mathcal{U} \rightarrow \text{Type}}{\mathcal{U} \triangleleft El_{\mathcal{U}} : \text{container}}$$

$$\frac{C : \text{container}}{C.\text{Sh} : \text{Type}}$$

# Containers: The Syntax for Data Types

$$\frac{}{\text{container} : \text{Type}} \qquad \frac{\mathcal{U} : \text{Type} \quad El_{\mathcal{U}} : \mathcal{U} \rightarrow \text{Type}}{\mathcal{U} \triangleleft El_{\mathcal{U}} : \text{container}}$$

$$\frac{C : \text{container}}{C.\text{Sh} : \text{Type}} \qquad \frac{C \rightsquigarrow \mathcal{U} \triangleleft El_{\mathcal{U}}}{C.\text{Sh} \rightsquigarrow \mathcal{U}}$$

# Containers: The Syntax for Data Types

$$\frac{}{\text{container} : \text{Type}} \qquad \frac{\mathcal{U} : \text{Type} \quad El_{\mathcal{U}} : \mathcal{U} \rightarrow \text{Type}}{\mathcal{U} \triangleleft El_{\mathcal{U}} : \text{container}}$$

$$\frac{C : \text{container}}{C.\text{Sh} : \text{Type}} \qquad \frac{C \rightsquigarrow \mathcal{U} \triangleleft El_{\mathcal{U}}}{C.\text{Sh} \rightsquigarrow \mathcal{U}}$$

$$\frac{C : \text{container}}{C.\text{Po} : C.\text{Sh} \rightarrow \text{Type}}$$

# Containers: The Syntax for Data Types

$$\frac{}{\text{container} : \text{Type}} \qquad \frac{\mathcal{U} : \text{Type} \quad El_{\mathcal{U}} : \mathcal{U} \rightarrow \text{Type}}{\mathcal{U} \triangleleft El_{\mathcal{U}} : \text{container}}$$

$$\frac{C : \text{container}}{C.\text{Sh} : \text{Type}} \qquad \frac{C \rightsquigarrow \mathcal{U} \triangleleft El_{\mathcal{U}}}{C.\text{Sh} \rightsquigarrow \mathcal{U}}$$

$$\frac{C : \text{container}}{C.\text{Po} : C.\text{Sh} \rightarrow \text{Type}} \qquad \frac{C \rightsquigarrow \mathcal{U} \triangleleft El_{\mathcal{U}}}{C.\text{Po} \rightsquigarrow El_{\mathcal{U}}}$$



# Restricting Containers

$$\frac{C : \text{container} \quad \mathcal{V} \subseteq C.\text{Sh}}{C|_{\mathcal{V}} : \text{container}}$$

# Restricting Containers

$$\frac{C : \text{container} \quad \mathcal{V} \subseteq C.\text{Sh}}{C|_{\mathcal{V}} : \text{container}}$$

$$\frac{C \rightsquigarrow \mathcal{U} \triangleleft El_{\mathcal{U}}}{C|_{\mathcal{V}} \rightsquigarrow \mathcal{V} \triangleleft El_{\mathcal{U}}|_{\mathcal{V}}}$$

# Container Lifting

$$\frac{C : \text{container} \quad F : \text{Type} \rightarrow \text{Type}}{C \uparrow F : \text{container}}$$

# Container Lifting

$$\frac{C : \text{container} \quad F : \text{Type} \rightarrow \text{Type}}{C \uparrow F : \text{container}}$$

$$\frac{C \rightsquigarrow \mathcal{U} \triangleleft El_{\mathcal{U}}}{C \uparrow F \rightsquigarrow \mathcal{U} \triangleleft F \circ El_{\mathcal{U}}}$$

# A Menagerie of Quantifiers

# A Menagerie of Quantifiers

*Dependent Products:*

$$\frac{\Gamma \vdash A : \mathbf{Type} \quad \Gamma, x : A \vdash B : \mathbf{Type}}{\Gamma \vdash \prod_A B : \mathbf{Type}}$$

$$\frac{\Gamma, x : A \vdash e : B[x]}{\Gamma \vdash \lambda x. e : \prod_A B}$$

# A Menagerie of Quantifiers

*Dependent Sums:*

$$\frac{\Gamma \vdash A : \mathbf{Type} \quad \Gamma, x : A \vdash B : \mathbf{Type}}{\Gamma \vdash \sum_A B : \mathbf{Type}}$$

$$\frac{\Gamma \vdash a : A \quad \Gamma \vdash b : B[a]}{\Gamma \vdash \langle a, b \rangle : \sum_A B}$$

# A Menagerie of Quantifiers

*Inductive Types:*

$$\frac{\Gamma \vdash A : \mathbf{Type} \quad \Gamma, x : A \vdash B : \mathbf{Type}}{\Gamma \vdash W_A B : \mathbf{Type}}$$

$$\frac{\Gamma \vdash a : A \quad \Gamma, v : B[a] \vdash b : W_A B}{\Gamma \vdash \mathbf{sup}(a; v. b) : W_A B}$$



# A Menagerie of Quantifiers

*Coinductive Types:*

$$\frac{\Gamma \vdash A : \mathbf{Type} \quad \Gamma, x : A \vdash B : \mathbf{Type}}{\Gamma \vdash M_A B : \mathbf{Type}}$$

$$\frac{\Gamma \vdash a : A \quad \Gamma, v : B[a] \vdash b : \infty (M_A B)}{\Gamma \vdash \mathbf{inf}(a; v. b) : M_A B}$$

# A Scheme for Quantifiers

# A Scheme for Quantifiers

$$\frac{\Gamma, A : \mathbf{Type}, (x : A \vdash B : \mathbf{Type}) \vdash Q_A B : \mathbf{Type}}{\Gamma \vdash Q \text{ quantifier}}$$

# Quantifiers Give Containers Semantics

$$\frac{\Gamma, A : \mathbf{Type}, (x : A \vdash B : \mathbf{Type}) \vdash Q_A B : \mathbf{Type}}{\Gamma \vdash Q \text{ quantifier}}$$

# Quantifiers Give Containers Semantics

$$\frac{\Gamma, A : \mathbf{Type}, (x : A \vdash B : \mathbf{Type}) \vdash Q_A B : \mathbf{Type}}{\Gamma \vdash Q \text{ quantifier}}$$

$$\frac{C : \text{container} \quad Q \text{ quantifier}}{\llbracket C \rrbracket_Q : \mathbf{Type}}$$

# Quantifiers Give Containers Semantics

$$\frac{\Gamma, A : \mathbf{Type}, (x : A \vdash B : \mathbf{Type}) \vdash Q_A B : \mathbf{Type}}{\Gamma \vdash Q \text{ quantifier}}$$

$$\frac{C : \text{container} \quad Q \text{ quantifier}}{\llbracket C \rrbracket_Q : \mathbf{Type}}$$

$$\frac{C \rightsquigarrow \mathcal{U} \triangleleft El_{\mathcal{U}}}{\llbracket C \rrbracket_Q \rightsquigarrow Q_{\mathcal{U}} El_{\mathcal{U}}}$$

# Vinyl Records as Containers

# Vinyl Records as Containers

Records and corecords are finite products and sums respectively.



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$$\text{Rec } El_{\mathcal{U}} F \text{ rs} \cong \llbracket (\mathcal{U} \triangleleft El_{\mathcal{U}}) |_{rs \ni -} \uparrow F \rrbracket_{\Pi}$$

# Vinyl Records as Containers

Records and corecords are finite products and sums respectively.

$$\begin{aligned}\text{Rec } El_{\mathcal{U}} F rs &\cong \llbracket (\mathcal{U} \triangleleft El_{\mathcal{U}}) |_{rs \ni -} \uparrow F \rrbracket_{\Pi} \\ \text{CoRec } El_{\mathcal{U}} F rs &\cong \llbracket (\mathcal{U} \triangleleft El_{\mathcal{U}}) |_{rs \ni -} \uparrow F \rrbracket_{\Sigma}\end{aligned}$$

# Vinyl Records as Containers

Records and corecords are finite products and sums respectively.

$$\text{Rec } El_{\mathcal{U}} F rs \cong \llbracket (\mathcal{U} \triangleleft El_{\mathcal{U}}) |_{rs \ni -} \uparrow F \rrbracket_{\Pi}$$

$$\text{CoRec } El_{\mathcal{U}} F rs \cong \llbracket (\mathcal{U} \triangleleft El_{\mathcal{U}}) |_{rs \ni -} \uparrow F \rrbracket_{\Sigma}$$

$$??? El_{\mathcal{U}} F rs \cong \llbracket (\mathcal{U} \triangleleft El_{\mathcal{U}}) |_{rs \ni -} \uparrow F \rrbracket_{\mathbf{W}}$$

# Vinyl Records as Containers

Records and corecords are finite products and sums respectively.

$$\begin{aligned}\text{Rec } El_{\mathcal{U}} F rs &\cong \llbracket (\mathcal{U} \triangleleft El_{\mathcal{U}}) |_{rs \ni -} \uparrow F \rrbracket_{\Pi} \\ \text{CoRec } El_{\mathcal{U}} F rs &\cong \llbracket (\mathcal{U} \triangleleft El_{\mathcal{U}}) |_{rs \ni -} \uparrow F \rrbracket_{\Sigma} \\ \\ ??? El_{\mathcal{U}} F rs &\cong \llbracket (\mathcal{U} \triangleleft El_{\mathcal{U}}) |_{rs \ni -} \uparrow F \rrbracket_{\mathbf{W}} \\ ??? El_{\mathcal{U}} F rs &\cong \llbracket (\mathcal{U} \triangleleft El_{\mathcal{U}}) |_{rs \ni -} \uparrow F \rrbracket_{\mathbf{M}}\end{aligned}$$

## Questions