Programming in Vinyl

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FOBO

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data R' = R' { x :: X } −− ˆ *Error*

Structural Typing

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- Record types may be characterized *structurally*

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class s ∈ (rs :: [∗])
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lens : s ::: t ∈ rs ⇒ s ::: t → Lens' (Rec rs) t
```

Roll Your Own in Haskell

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f :: Rec (”foo” ::: A ’: rs)
  $\rightarrow$ Rec (”bar” ::: B ’: ”foo” ::: A ’: rs)

Universes à la Tarski

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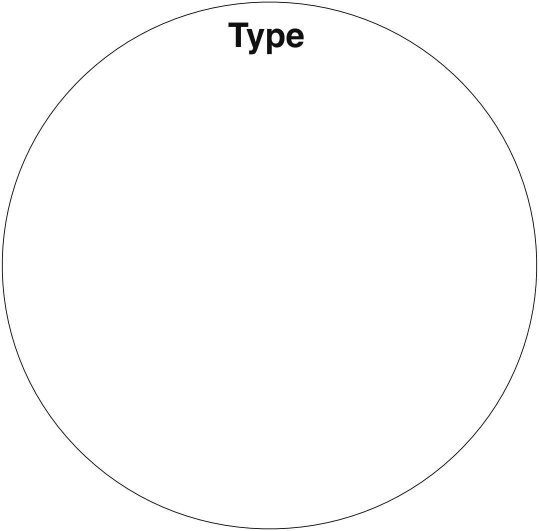
$$\frac{\Gamma \vdash s : \mathcal{U}}{\Gamma \vdash El_{\mathcal{U}}(s) : \text{Type}}$$

Universes à la Tarski



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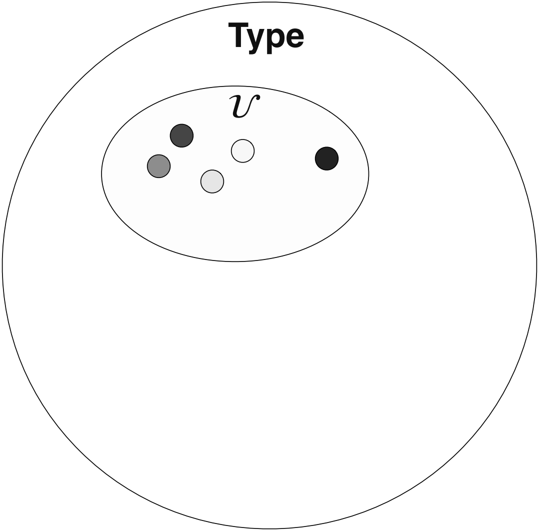

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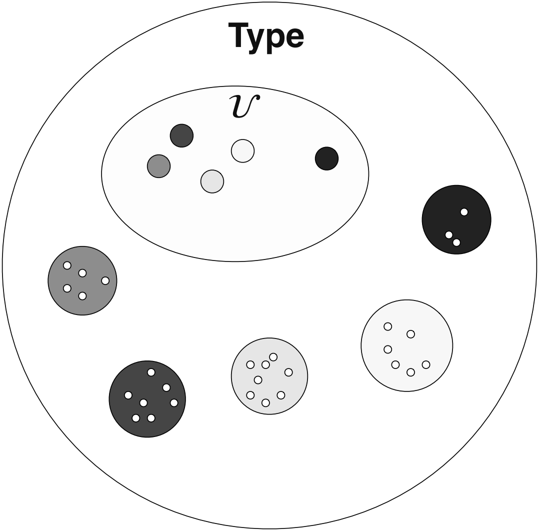

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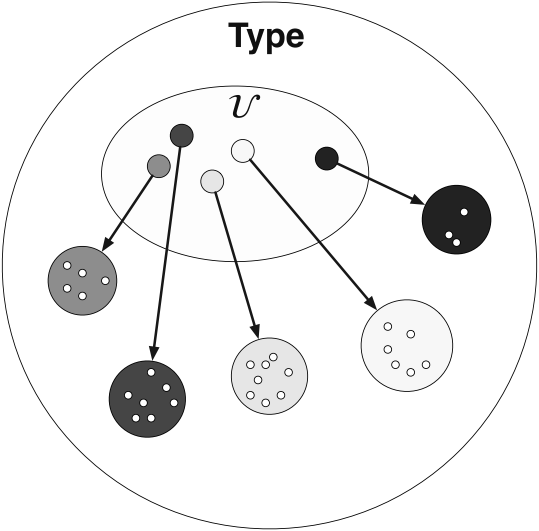

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$$\frac{}{El_{\mathcal{A}}(\mathsf{Phone}[\ell]) \rightsquigarrow \mathsf{list}(\mathbb{N})}$$

Records as Products

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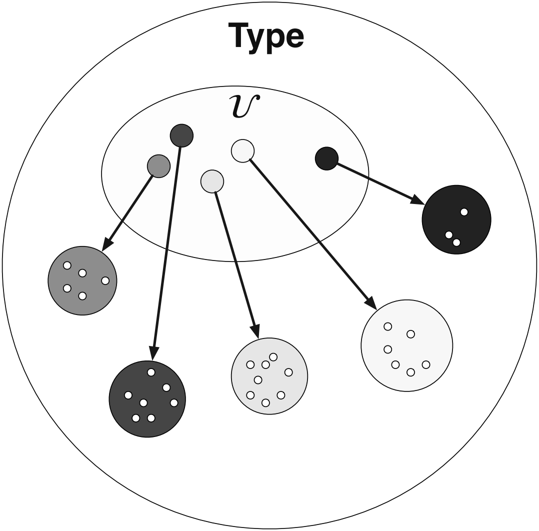
$$\mathsf{record}_{\mathcal{U}} \rightsquigarrow \sum_{\mathcal{V} \subseteq \mathcal{U}} \prod_{\mathcal{V}} El_{\mathcal{U}}|_{\mathcal{V}}$$

Records as Products



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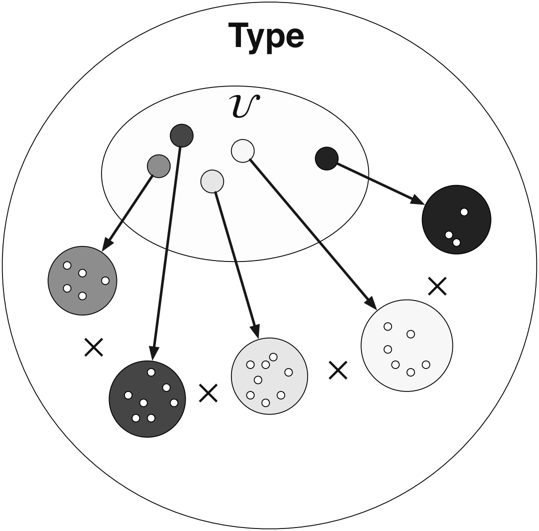

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$$ex : \mathsf{record}_{\mathcal{U}}$$

Example Record

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\begin{aligned}
\mathsf{record}_{\mathcal{U}} &\rightsquigarrow \sum_{\mathcal{V}\subseteq\mathcal{U}} \prod_{\mathcal{V}} El_{\mathcal{U}}|_{\mathcal{V}} \\
\mathcal{A}' &\rightsquigarrow \{\mathsf{Name}, \mathsf{Email}\ \textit{Work}\} \\
ex &: \mathsf{record}_{\mathcal{U}} \\
ex &\rightsquigarrow \langle \mathcal{A}', \lambda. \\
&\qquad \{\mathsf{Name} \mapsto \text{"Robert Harper"}; \\
&\qquad\ \mathsf{Email}\ \textit{Work} \mapsto \text{"rwh@cs.cmu.edu"}\}\rangle
\end{aligned}
$$

Presheaves

Presheaves

A presheaf on some category X is a functor $\mathcal{O}(X)^{\mathrm{op}} \to \mathbf{Type}$, where $\mathcal{O}$ is the category of open sets of X for whatever topology you have chosen.

Topologies on some space X

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- What are the open sets on X?

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- The empty set and X are open sets
- Finite intersections of open sets are open

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- Then records on a universe X give rise to a presheaf $\mathcal{R}$: subset inclusions are taken to casts from larger to smaller records

$$\text{for } V \subseteq X \quad \mathcal{R}(V) :\equiv \prod_{V} El_X|_V : \mathbf{Type}$$
$$\text{for } i : V \hookrightarrow U \quad \mathcal{R}(i) :\equiv \mathsf{cast} : \mathcal{R}(U) \to \mathcal{R}(V)$$

Records are Sheaves

Records are Sheaves

For a cover $U = \bigcup_i U_i$ on X, then:

$$\mathcal{R}(U) \xrightarrow{\quad e \quad} \prod_i \mathcal{R}(U_i) \underset{q}{\overset{p}{\rightrightarrows}} \prod_{i,j} \mathcal{R}(U_i \cap U_j)$$

is an equalizer, where

$$\begin{aligned} e &= \lambda r.\lambda i.\ \mathsf{cast}_{U_i}(r) \\ p &= \lambda f.\lambda i.\lambda j.\ \mathsf{cast}_{U_i \cap U_j}(f(i)) \\ q &= \lambda f.\lambda i.\lambda j.\ \mathsf{cast}_{U_i \cap U_j}(f(j)) \end{aligned}$$

Records are Sheaves

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\begin{tikzcd}
\mathcal{R}(U) \arrow[r, "e"] & \prod_i \mathcal{R}(U_i) \arrow[r, shift left, "p"] \arrow[r, shift right, "q"'] & \prod_{i,j} \mathcal{R}(U_i \cap U_j) \\
& \Gamma \arrow[ul, dashed, "!u"] \arrow[u, "m"'] &
\end{tikzcd}
$$

where

$$
\begin{aligned}
e &= \lambda r.\lambda i.\ \mathsf{cast}_{U_i}(r) \\
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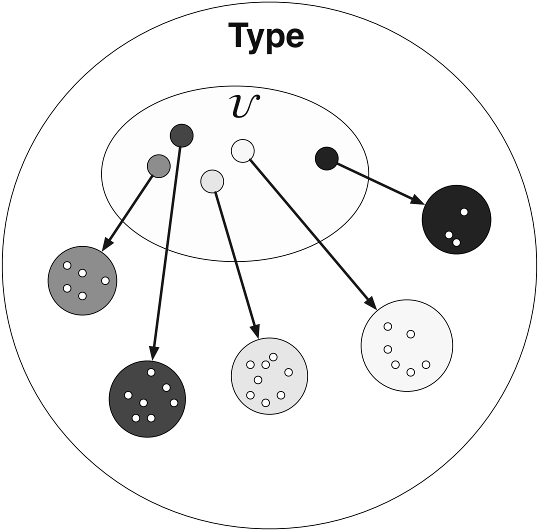

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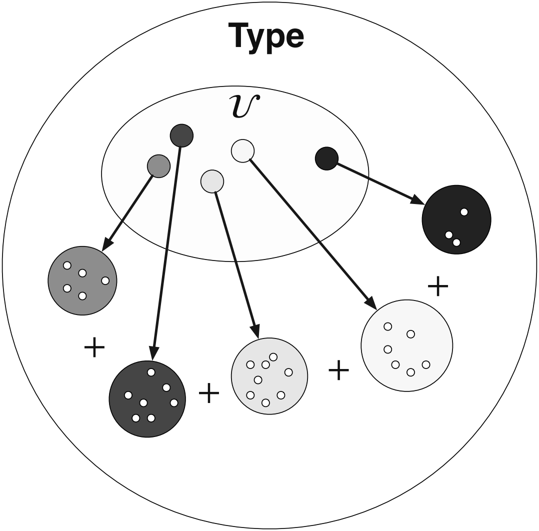

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- Parameterize Rec by $\mathcal{U}$, $El_{\mathcal{U}}$?

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type instance Id $ x = x

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ex :: HList [$\mathbb{Z}$, **Bool**, **String**]
ex = 34 :& **True** :& "vinyl" :& RNil

Validating Records

bob :: Rec $\mathsf{El}_{\mathcal{A}}$ [Name, Email Work]

Validating Records

$$\begin{aligned}&\text{bob} :: \text{Rec El}_{\mathcal{A}}\ [\text{Name, Email Work}]\\&\text{bob} = \text{Name} =: \text{"Robert␣Harper"}\\&\qquad \oplus\ \text{Email Work} =: \text{"rwh@cs.cmu.edu"}\end{aligned}$$

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$$\begin{array}{l}
\text{validateName} :: \textbf{String} \rightarrow \textbf{Either}\ \text{Error}\ \textbf{String} \\
\text{validateEmail} :: \textbf{String} \rightarrow \textbf{Either}\ \text{Error}\ \textbf{String} \\
\text{validatePhone} :: [\mathbb{N}] \rightarrow \textbf{Either}\ \text{Error}\ [\mathbb{N}]
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unnnnnhhh...

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unnnnnhhh...

validateContact
  :: Rec $\text{El}_{\mathcal{A}}$ [Name, Email Work]
  $\rightarrow$ **Either** Error (Rec $\text{El}_{\mathcal{A}}$ [Name, Email Work])

Welp.

Effects inside records

data Rec :: (TyFun $\mathcal{U}$ $* \rightarrow *$) $\rightarrow$ [$\mathcal{U}$] $\rightarrow *$ **where**
  RNil :: Rec $\text{el}_{\mathcal{U}}$ '[]
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(=:) : Applicative f $\Rightarrow$ sing r $\to$ $\mathrm{el}_{\mathcal{U}}$ \$ r $\to$ Rec $\mathrm{el}_{\mathcal{U}}$ f '[r]

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($\Lleftarrow$): sing r $\to$ f ($\text{el}_{\mathcal{U}}$ \$ r) $\to$ Rec $\text{el}_{\mathcal{U}}$ f '[r]

Effects inside records

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$$
\begin{array}{l}
(\Lleftarrow): \mathrm{sing}\ \mathrm{r} \to \mathrm{f}\ (\mathrm{el}_{\mathcal{U}}\ \$\ \mathrm{r}) \to \mathrm{Rec}\ \mathrm{el}_{\mathcal{U}}\ \mathrm{f}\ \text{'}[\mathrm{r}] \\
\mathrm{k} \Lleftarrow \mathrm{x} = \mathrm{x}\ :\&\ \mathrm{RNil}
\end{array}
$$

Compositional Validation

type $\text{Rec}_{\mathcal{A}}$ = $\text{Rec El}_{\mathcal{A}}$

Compositional Validation

type $\text{Rec}_{\mathcal{A}}$ = Rec $\text{El}_{\mathcal{A}}$
bob :: $\text{Rec}_{\mathcal{A}}$ Identity [Name, Email Work]

Compositional Validation

type $\text{Rec}_{\mathcal{A}}$ = Rec $\text{El}_{\mathcal{A}}$
bob :: $\text{Rec}_{\mathcal{A}}$ Identity [Name, Email Work]
bob = Name =: "Robert␣Harper"
    $\oplus$ Email Work =: "rwh@cs.cmu.edu"

Compositional Validation

```
type Rec_A = Rec El_A
bob :: Rec_A Identity [Name, Email Work]
bob = Name =: "Robert Harper"
    ⊕ Email Work =: "rwh@cs.cmu.edu"
```

Compositional Validation

type Validator a = a $\rightarrow$ **Either** Error a

Compositional Validation

type Validator a = a $\rightarrow$ **Either** Error a
validateName :: Rec$_{\mathcal{A}}$ Validator '[Name]
validatePhone :: $\forall \ell$. Rec$_{\mathcal{A}}$ Validator '[Phone ℓ]
validateEmail :: $\forall \ell$. Rec$_{\mathcal{A}}$ Validator '[Email ℓ]

Compositional Validation

type Validator a = a $\rightarrow$ **Either** Error a
validateName :: Rec$_{\mathcal{A}}$ Validator '[Name]
validatePhone :: $\forall \ell$. Rec$_{\mathcal{A}}$ Validator '[Phone ℓ]
validateEmail :: $\forall \ell$. Rec$_{\mathcal{A}}$ Validator '[Email ℓ]

type TotalContact =
  [Name, Email Home, Email Work
  , Phone Home, Phone Work]

Compositional Validation

```
type Validator a = a → Either Error a
validateName :: Rec_A Validator '[Name]
validatePhone :: ∀ℓ. Rec_A Validator '[Phone ℓ]
validateEmail :: ∀ℓ. Rec_A Validator '[Email ℓ]

type TotalContact =
  [ Name, Email Home, Email Work
  , Phone Home, Phone Work ]

validateContact :: Rec_A Validator TotalContact
validateContact = validateName
                ⊕ validateEmail
                ⊕ validateEmail
                ⊕ validatePhone
                ⊕ validatePhone
```

Record Operators

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newtype Lift o f g x = Lift { runLift :: f x ‘o‘ g x }

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($\circledast$) :: Rec$_{\mathcal{U}}$ (Lift $(\rightarrow)$ f g) rs $\rightarrow$ Rec$_{\mathcal{U}}$ f rs $\rightarrow$ Rec$_{\mathcal{U}}$ g rs

Record Operators

newtype Lift o f g x = Lift $\{$ runLift :: f x ‘o‘ g x $\}$

type Validator = Lift $(\rightarrow)$ Identity (**Either** Error)

($\circledast$) :: Rec$_{\mathcal{U}}$ (Lift $(\rightarrow)$ f g) rs $\rightarrow$ Rec$_{\mathcal{U}}$ f rs $\rightarrow$ Rec$_{\mathcal{U}}$ g rs

rdist :: Applicative f $\Rightarrow$ Rec$_{\mathcal{U}}$ f rs $\rightarrow$ f (Rec$_{\mathcal{U}}$ Identity rs)

Compositional Validation

newtype Lift o f g x = Lift $\{$ runLift :: f x ‘o‘ g x $\}$
type Validator = Lift $(\to)$ Identity (**Either** Error)
($\circledast$) :: Rec$_{\mathcal{U}}$ (Lift $(\to)$ f g) rs $\to$ Rec$_{\mathcal{U}}$ f rs $\to$ Rec$_{\mathcal{U}}$ g rs
rdist :: Applicative f $\Rightarrow$ Rec$_{\mathcal{U}}$ f rs $\to$ f (Rec$_{\mathcal{U}}$ Identity rs)

Compositional Validation

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validateContact :: Rec$_{\mathcal{A}}$ Validator TotalContact

Compositional Validation

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$(\circledast)$ :: Rec$_{\mathcal{U}}$ (Lift $(\rightarrow)$ f g) rs $\rightarrow$ Rec$_{\mathcal{U}}$ f rs $\rightarrow$ Rec$_{\mathcal{U}}$ g rs
rdist :: Applicative f $\Rightarrow$ Rec$_{\mathcal{U}}$ f rs $\rightarrow$ f (Rec$_{\mathcal{U}}$ Identity rs)

validateContact :: Rec$_{\mathcal{A}}$ Validator TotalContact

bobValid :: Rec$_{\mathcal{A}}$ (**Either** Error) [Name, Email Work]

Compositional Validation

newtype Lift o f g x = Lift $\{$ runLift :: f x ‘o‘ g x $\}$
type Validator = Lift $(\rightarrow)$ Identity (**Either** Error)
($\circledast$) :: Rec$_{\mathcal{U}}$ (Lift $(\rightarrow)$ f g) rs $\rightarrow$ Rec$_{\mathcal{U}}$ f rs $\rightarrow$ Rec$_{\mathcal{U}}$ g rs
rdist :: Applicative f $\Rightarrow$ Rec$_{\mathcal{U}}$ f rs $\rightarrow$ f (Rec$_{\mathcal{U}}$ Identity rs)

validateContact :: Rec$_{\mathcal{A}}$ Validator TotalContact

bobValid :: Rec$_{\mathcal{A}}$ (**Either** Error) [Name, Email Work]
bobValid = cast validateContact $\circledast$ bob

Compositional Validation

newtype Lift o f g x = Lift $\{$ runLift :: f x ‘o‘ g x $\}$
type Validator = Lift $(\rightarrow)$ Identity (**Either** Error)
($\circledast$) :: Rec$_{\mathcal{U}}$ (Lift $(\rightarrow)$ f g) rs $\rightarrow$ Rec$_{\mathcal{U}}$ f rs $\rightarrow$ Rec$_{\mathcal{U}}$ g rs
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validateContact :: Rec$_{\mathcal{A}}$ Validator TotalContact

bobValid :: Rec$_{\mathcal{A}}$ (**Either** Error) [Name, Email Work]
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validBob :: **Either** Error (Rec$_{\mathcal{A}}$ Identity [Name, Email Work])

Compositional Validation

newtype Lift o f g x = Lift $\{$ runLift :: f x ‘o‘ g x $\}$
type Validator = Lift $(\rightarrow)$ Identity (**Either** Error)
($\circledast$) :: Rec$_{\mathcal{U}}$ (Lift $(\rightarrow)$ f g) rs $\rightarrow$ Rec$_{\mathcal{U}}$ f rs $\rightarrow$ Rec$_{\mathcal{U}}$ g rs
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validateContact :: Rec$_{\mathcal{A}}$ Validator TotalContact

bobValid :: Rec$_{\mathcal{A}}$ (**Either** Error) [Name, Email Work]
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validBob :: **Either** Error (Rec$_{\mathcal{A}}$ Identity [Name, Email Work])
validBob = rdist bobValid

Laziness as an effect

Laziness as an effect

newtype Identity a = Identity { runIdentity :: a }

Laziness as an effect

```
newtype Identity a = Identity { runIdentity :: a }
data Thunk a = Thunk { unThunk :: a }
```

Laziness as an effect

```
newtype Identity a = Identity { runIdentity :: a }
data Thunk a = Thunk { unThunk :: a }

type PlainRec𝒰 rs = Rec𝒰 Identity rs
```

Laziness as an effect

```
newtype Identity a = Identity { runIdentity :: a }
data Thunk a = Thunk { unThunk :: a }

type PlainRec_U rs = Rec_U Identity rs
type LazyRec_U rs = Rec_U Thunk rs
```

Concurrent Records with Async

Concurrent Records with Async

fetchName :: $\text{Rec}_{\mathcal{A}}$ **IO** '[Name]

Concurrent Records with Async

fetchName :: $\text{Rec}_{\mathcal{A}}$ **IO** '[Name]
fetchName = Name $\Lleftarrow$ someOperation

Concurrent Records with Async

fetchName :: $\text{Rec}_{\mathcal{A}}$ **IO** '[Name]
fetchName = Name $\Lleftarrow$ someOperation

fetchWorkEmail :: $\text{Rec}_{\mathcal{A}}$ **IO** '[Email Work]

Concurrent Records with Async

fetchName :: $\text{Rec}_{\mathcal{A}}$ **IO** '[Name]
fetchName = Name $\Lleftarrow$ someOperation

fetchWorkEmail :: $\text{Rec}_{\mathcal{A}}$ **IO** '[Email Work]
fetchWorkEmail = Email Work $\Lleftarrow$ anotherOperation

Concurrent Records with Async

```
fetchName :: Rec_A IO '[Name]
fetchName = Name ⇚ someOperation

fetchWorkEmail :: Rec_A IO '[Email Work]
fetchWorkEmail = Email Work ⇚ anotherOperation

fetchBob :: Rec_A IO [Name, Email Work]
```

Concurrent Records with Async

fetchName :: $\text{Rec}_{\mathcal{A}}$ **IO** '[Name]
fetchName = Name $\Lleftarrow$ someOperation

fetchWorkEmail :: $\text{Rec}_{\mathcal{A}}$ **IO** '[Email Work]
fetchWorkEmail = Email Work $\Lleftarrow$ anotherOperation

fetchBob :: $\text{Rec}_{\mathcal{A}}$ **IO** [Name, Email Work]
fetchBob = fetchName $\oplus$ fetchWorkEmail

Concurrent Records with Async

Concurrent Records with Async

```
newtype Concurrently a
  = Concurrently { runConcurrently :: IO a }
```

Concurrent Records with Async

newtype Concurrently a
  = Concurrently { runConcurrently :: **IO** a }

$(\circledS) :: (\forall\ a.\ f\ a \rightarrow g\ a) \rightarrow Rec_{\mathcal{U}}\ f\ rs \rightarrow Rec_{\mathcal{U}}\ g\ rs$

Concurrent Records with Async

newtype Concurrently a
  = Concurrently $\{$ runConcurrently :: **IO** a $\}$

($) :: ($\forall$ a. f a $\rightarrow$ g a) $\rightarrow$ $\text{Rec}_{\mathcal{U}}$ f rs $\rightarrow$ $\text{Rec}_{\mathcal{U}}$ g rs

bobConcurrently :: $\text{Rec}_{\mathcal{A}}$ Concurrently [Name, Email Work]

Concurrent Records with Async

newtype Concurrently a
  = Concurrently { runConcurrently :: **IO** a }

$(\circledS) :: (\forall\ a.\ f\ a \rightarrow g\ a) \rightarrow \mathrm{Rec}_{\mathcal{U}}\ f\ rs \rightarrow \mathrm{Rec}_{\mathcal{U}}\ g\ rs$

bobConcurrently :: $\mathrm{Rec}_{\mathcal{A}}$ Concurrently [Name, Email Work]
bobConcurrently = Concurrently $\circledS$ fetchBob

Concurrent Records with Async

newtype Concurrently a
  = Concurrently { runConcurrently :: **IO** a }

$(\circledS) :: (\forall\ a.\ f\ a \to g\ a) \to \mathrm{Rec}_{\mathcal{U}}\ f\ rs \to \mathrm{Rec}_{\mathcal{U}}\ g\ rs$

bobConcurrently :: $\mathrm{Rec}_{\mathcal{A}}$ Concurrently [Name, Email Work]
bobConcurrently = Concurrently $\circledS$ fetchBob

concurrentBob :: Concurrently ($\mathrm{Rec}_{\mathcal{A}}$ Identity [...])

Concurrent Records with Async

newtype Concurrently a
  = Concurrently { runConcurrently :: **IO** a }

(⊚$⊚) :: (∀ a. f a → g a) → $\text{Rec}_{\mathcal{U}}$ f rs → $\text{Rec}_{\mathcal{U}}$ g rs

bobConcurrently :: $\text{Rec}_{\mathcal{A}}$ Concurrently [Name, Email Work]
bobConcurrently = Concurrently ⊚$⊚ fetchBob

concurrentBob :: Concurrently ($\text{Rec}_{\mathcal{A}}$ Identity [...])
concurrentBob = rdist bobConcurrently

Concurrent Records with Async

Concurrent Records with Async

fetchBob :: $\text{Rec}_{\mathcal{A}}$ **IO** [Name, Email Work]
bobConcurrently :: $\text{Rec}_{\mathcal{A}}$ Concurrently [Name, Email Work]
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Concurrent Records with Async

fetchBob :: Rec$_{\mathcal{A}}$ **IO** [Name, Email Work]
bobConcurrently :: Rec$_{\mathcal{A}}$ Concurrently [Name, Email Work]
concurrentBob :: Concurrently (Rec$_{\mathcal{A}}$ Identity [...])

bob :: **IO** (Rec$_{\mathcal{A}}$ Identity [Name, Email Work])

Concurrent Records with Async

fetchBob :: $\text{Rec}_{\mathcal{A}}$ **IO** [Name, Email Work]
bobConcurrently :: $\text{Rec}_{\mathcal{A}}$ Concurrently [Name, Email Work]
concurrentBob :: Concurrently ($\text{Rec}_{\mathcal{A}}$ Identity [...])

bob :: **IO** ($\text{Rec}_{\mathcal{A}}$ Identity [Name, Email Work])
bob = runConcurrently concurrentBob

Containers: The Syntax for Data Types

$$\frac{}{\mathsf{container} : \mathsf{Type}}$$

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$$\frac{}{\textsf{container} : \textsf{Type}} \qquad \frac{\mathcal{U} : \textsf{Type} \quad El_{\mathcal{U}} : \mathcal{U} \to \textsf{Type}}{\mathcal{U} \lhd El_{\mathcal{U}} : \textsf{container}}$$

Containers: The Syntax for Data Types

$$\frac{}{\text{container} : \mathsf{Type}} \qquad \frac{\mathcal{U} : \mathsf{Type} \quad El_{\mathcal{U}} : \mathcal{U} \to \mathsf{Type}}{\mathcal{U} \lhd El_{\mathcal{U}} : \text{container}}$$

$$\frac{C : \text{container}}{C.\mathsf{Sh} : \mathsf{Type}}$$

Containers: The Syntax for Data Types

$$\frac{}{\textsf{container} : \textsf{Type}} \qquad \frac{\mathcal{U} : \textsf{Type} \quad El_{\mathcal{U}} : \mathcal{U} \to \textsf{Type}}{\mathcal{U} \lhd El_{\mathcal{U}} : \textsf{container}}$$

$$\frac{C : \textsf{container}}{C.\textsf{Sh} : \textsf{Type}} \qquad \frac{C \leadsto \mathcal{U} \lhd El_{\mathcal{U}}}{C.\textsf{Sh} \leadsto \mathcal{U}}$$

Containers: The Syntax for Data Types

$$\frac{}{\textsf{container} : \textsf{Type}} \qquad \frac{\mathcal{U} : \textsf{Type} \quad El_{\mathcal{U}} : \mathcal{U} \to \textsf{Type}}{\mathcal{U} \lhd El_{\mathcal{U}} : \textsf{container}}$$

$$\frac{C : \textsf{container}}{C.\textsf{Sh} : \textsf{Type}} \qquad \frac{C \leadsto \mathcal{U} \lhd El_{\mathcal{U}}}{C.\textsf{Sh} \leadsto \mathcal{U}}$$

$$\frac{C : \textsf{container}}{C.\textsf{Po} : C.\textsf{Sh} \to \textsf{Type}}$$

Containers: The Syntax for Data Types

$$\frac{}{\mathsf{container} : \mathsf{Type}} \qquad \frac{\mathcal{U} : \mathsf{Type} \quad El_{\mathcal{U}} : \mathcal{U} \to \mathsf{Type}}{\mathcal{U} \lhd El_{\mathcal{U}} : \mathsf{container}}$$

$$\frac{C : \mathsf{container}}{C.\mathsf{Sh} : \mathsf{Type}} \qquad \frac{C \rightsquigarrow \mathcal{U} \lhd El_{\mathcal{U}}}{C.\mathsf{Sh} \rightsquigarrow \mathcal{U}}$$

$$\frac{C : \mathsf{container}}{C.\mathsf{Po} : C.\mathsf{Sh} \to \mathsf{Type}} \qquad \frac{C \rightsquigarrow \mathcal{U} \lhd El_{\mathcal{U}}}{C.\mathsf{Po} \rightsquigarrow El_{\mathcal{U}}}$$

Restricting Containers

$$\frac{C : \mathsf{container} \quad \mathcal{V} \subseteq C.\mathsf{Sh}}{C|_{\mathcal{V}} : \mathsf{container}}$$

Restricting Containers

$$\frac{C : \mathsf{container} \quad \mathcal{V} \subseteq C.\mathsf{Sh}}{C|_{\mathcal{V}} : \mathsf{container}}$$

$$\frac{C \rightsquigarrow \mathcal{U} \lhd El_{\mathcal{U}}}{C|_{\mathcal{V}} \rightsquigarrow \mathcal{V} \lhd El_{\mathcal{U}}|_{\mathcal{V}}}$$

Container Lifting

$$\frac{C : \mathsf{container} \quad F : \mathsf{Type} \to \mathsf{Type}}{C \uparrow F : \mathsf{container}}$$

Container Lifting

$$\frac{C : \textsf{container} \quad F : \textsf{Type} \to \textsf{Type}}{C \uparrow F : \textsf{container}}$$

$$\frac{C \rightsquigarrow \mathcal{U} \lhd El_{\mathcal{U}}}{C \uparrow F \rightsquigarrow \mathcal{U} \lhd F \circ El_{\mathcal{U}}}$$

A Menagerie of Quantifiers

A Menagerie of Quantifiers

Dependent Products:

$$\frac{\Gamma \vdash A : \textsf{Type} \quad \Gamma, x : A \vdash B : \textsf{Type}}{\Gamma \vdash \prod_A B : \textsf{Type}}$$

$$\frac{\Gamma, x : A \vdash e : B[x]}{\Gamma \vdash \lambda x.e : \prod_A B}$$

A Menagerie of Quantifiers

Dependent Sums:

$$\frac{\Gamma \vdash A : \mathsf{Type} \quad \Gamma, x : A \vdash B : \mathsf{Type}}{\Gamma \vdash \sum_A B : \mathsf{Type}}$$

$$\frac{\Gamma \vdash a : A \quad \Gamma \vdash b : B[a]}{\Gamma \vdash \langle a, b \rangle : \sum_A B}$$

A Menagerie of Quantifiers

Inductive Types:

$$\frac{\Gamma \vdash A : \mathsf{Type} \quad \Gamma, x : A \vdash B : \mathsf{Type}}{\Gamma \vdash \mathrm{W}_A\, B : \mathsf{Type}}$$

$$\frac{\Gamma \vdash a : A \quad \Gamma, v : B[a] \vdash b : \mathrm{W}_A\, B}{\Gamma \vdash \mathsf{sup}(a; v.\, b) : \mathrm{W}_A\, B}$$

A Menagerie of Quantifiers

Coinductive Types:

$$\frac{\Gamma \vdash A : \textsf{Type} \quad \Gamma, x : A \vdash B : \textsf{Type}}{\Gamma \vdash \mathrm{M}_A\, B : \textsf{Type}}$$

$$\frac{\Gamma \vdash a : A \quad \Gamma, v : B[a] \vdash b : \infty\,(\mathrm{M}_A\, B)}{\Gamma \vdash \textsf{inf}(a; v.\, b) : \mathrm{M}_A\, B}$$

A Scheme for Quantifiers

A Scheme for Quantifiers

$$\frac{\Gamma, A : \mathsf{Type}, (x : A \vdash B : \mathsf{Type}) \vdash Q_A B : \mathsf{Type}}{\Gamma \vdash Q \text{ quantifier}}$$

Quantifiers Give Containers Semantics

$$\frac{\Gamma, A : \mathsf{Type}, (x : A \vdash B : \mathsf{Type}) \vdash Q_A B : \mathsf{Type}}{\Gamma \vdash Q \text{ quantifier}}$$

Quantifiers Give Containers Semantics

$$\frac{\Gamma, A : \mathsf{Type}, (x : A \vdash B : \mathsf{Type}) \vdash Q_A B : \mathsf{Type}}{\Gamma \vdash Q \text{ quantifier}}$$

$$\frac{C : \text{container} \quad Q \text{ quantifier}}{[\![C]\!]_Q : \mathsf{Type}}$$

Quantifiers Give Containers Semantics

$$\frac{\Gamma, A : \mathsf{Type}, (x : A \vdash B : \mathsf{Type}) \vdash Q_A B : \mathsf{Type}}{\Gamma \vdash Q \text{ quantifier}}$$

$$\frac{C : \text{container} \quad Q \text{ quantifier}}{[\![C]\!]_Q : \mathsf{Type}}$$

$$\frac{C \rightsquigarrow \mathcal{U} \lhd El_{\mathcal{U}}}{[\![C]\!]_Q \rightsquigarrow Q_{\mathcal{U}} El_{\mathcal{U}}}$$

Vinyl Records as Containers

Vinyl Records as Containers

Records and corecords are finite products and sums respectively.

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$$\mathsf{Rec}\ El_{\mathcal{U}}\ F\ rs \cong [\![(\mathcal{U} \triangleleft El_{\mathcal{U}})|_{rs \ni -} \uparrow F]\!]_{\Pi}$$

Vinyl Records as Containers

Records and corecords are finite products and sums respectively.

$$\begin{aligned}
\mathsf{Rec}\ El_{\mathcal{U}}\ F\ rs &\cong [\![(\mathcal{U} \triangleleft El_{\mathcal{U}})|_{rs\ni -} \uparrow F]\!]_{\Pi} \\
\mathsf{CoRec}\ El_{\mathcal{U}}\ F\ rs &\cong [\![(\mathcal{U} \triangleleft El_{\mathcal{U}})|_{rs\ni -} \uparrow F]\!]_{\Sigma}
\end{aligned}$$

Vinyl Records as Containers

Records and corecords are finite products and sums respectively.

$$\mathsf{Rec}\ El_{\mathcal{U}}\ F\ rs \cong [\![(\mathcal{U} \lhd El_{\mathcal{U}})|_{rs \ni -} \uparrow F]\!]_{\Pi}$$
$$\mathsf{CoRec}\ El_{\mathcal{U}}\ F\ rs \cong [\![(\mathcal{U} \lhd El_{\mathcal{U}})|_{rs \ni -} \uparrow F]\!]_{\Sigma}$$

$$???\ El_{\mathcal{U}}\ F\ rs \cong [\![(\mathcal{U} \lhd El_{\mathcal{U}})|_{rs \ni -} \uparrow F]\!]_{\mathrm{W}}$$

Vinyl Records as Containers

Records and corecords are finite products and sums respectively.

$$\mathsf{Rec}\ El_{\mathcal{U}}\ F\ rs \cong [\![(\mathcal{U} \lhd El_{\mathcal{U}})|_{rs \ni -} \uparrow F]\!]_{\Pi}$$
$$\mathsf{CoRec}\ El_{\mathcal{U}}\ F\ rs \cong [\![(\mathcal{U} \lhd El_{\mathcal{U}})|_{rs \ni -} \uparrow F]\!]_{\Sigma}$$

$$???\ El_{\mathcal{U}}\ F\ rs \cong [\![(\mathcal{U} \lhd El_{\mathcal{U}})|_{rs \ni -} \uparrow F]\!]_{\mathrm{W}}$$
$$???\ El_{\mathcal{U}}\ F\ rs \cong [\![(\mathcal{U} \lhd El_{\mathcal{U}})|_{rs \ni -} \uparrow F]\!]_{\mathrm{M}}$$

Questions