Programming in Vinyl

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FOBO

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```
data R = R { x :: X }
data R' = R' { x :: X } -- ^ Error
```

Structural Typing

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- Sharing field names and accessors

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- Record types may be characterized *extensionsionally*

Row Polymorphism

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$$\frac{x : \{foo : A; \vec{rs}\}}{f(x) : \{foo : A, bar : B; \vec{rs}\}}$$

Roll Your Own in Haskell

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data (s :: Symbol) ::: (t :: *) = Field
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data Rec :: [$*$] $\rightarrow$ $*$ **where**

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data Rec :: [*] -> * where
  RNil :: Rec '[]
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class s ∈ (rs :: [*])
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class s ∈ (rs :: [∗])
class ss ⊆ (rs :: [∗]) where
  cast :: Rec rs → Rec ss
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Roll Your Own in Haskell

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class s ∈ (rs :: [∗])
class ss ⊆ (rs :: [∗]) where
  cast :: Rec rs → Rec ss
(=:) : s ::: t → t → Rec '[s ::: t]
(⊕) : Rec ss → Rec ts → Rec (ss ++ ts)
```

Roll Your Own in Haskell

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$$\text{f :: ”foo” ::: A} \in \text{rs} \Rightarrow \text{Rec rs} \rightarrow \text{Rec (”bar” ::: B ’: rs)}$$

Universes à la Tarski

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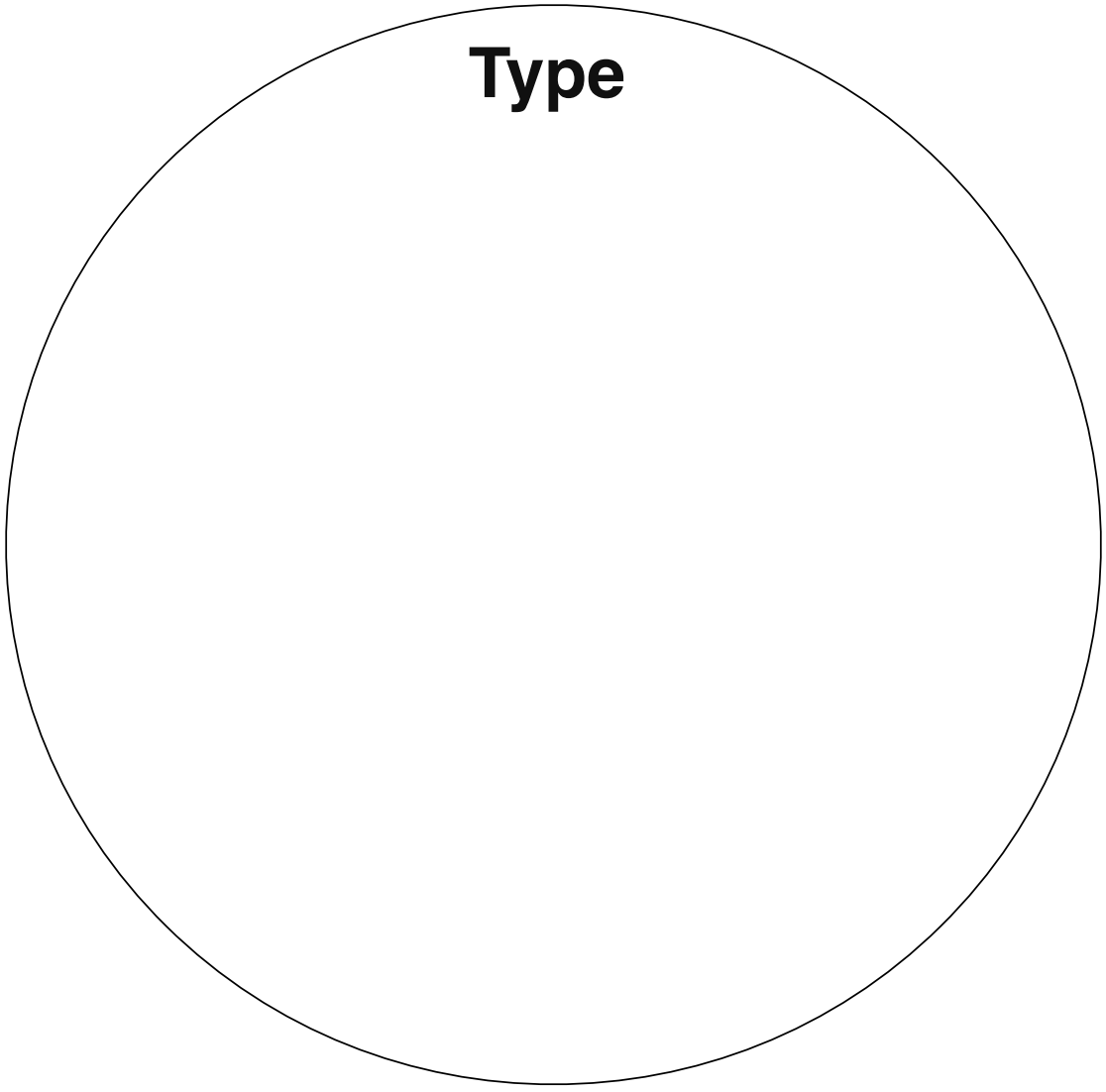
$$\frac{\Gamma \vdash s : \mathcal{U}}{\Gamma \vdash El_{\mathcal{U}}(s) : \text{Type}}$$

Universes à la Tarski



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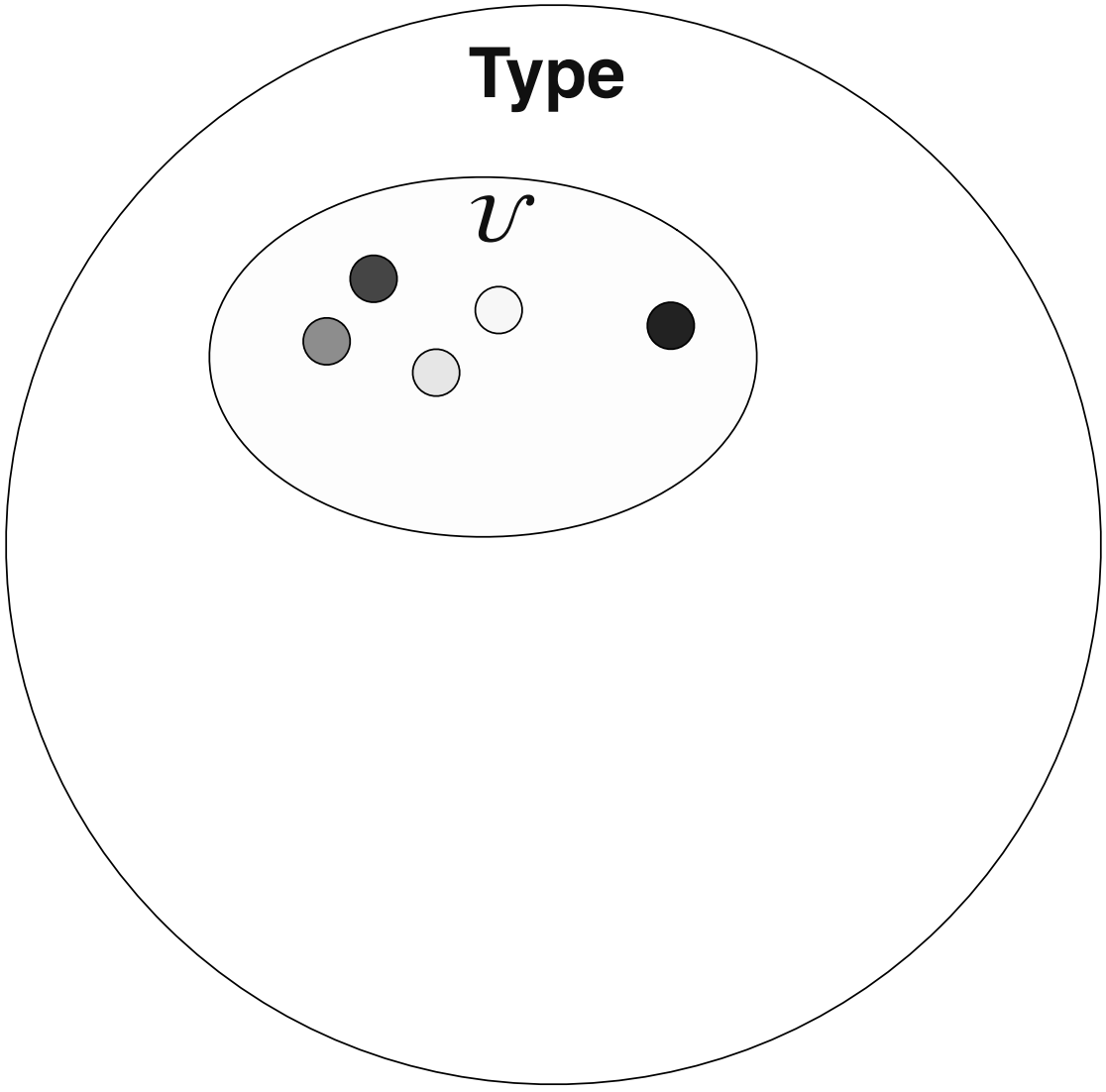

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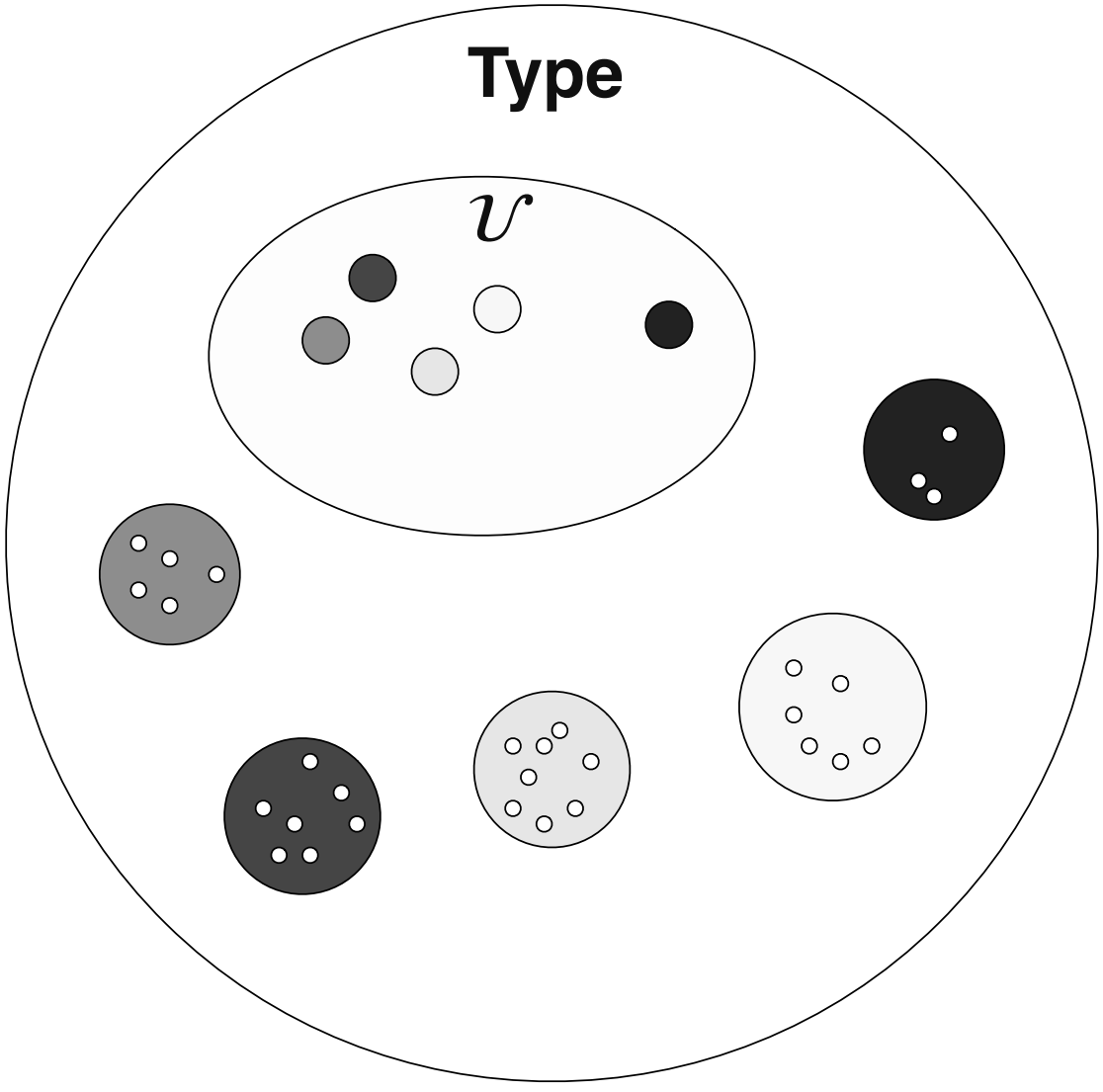

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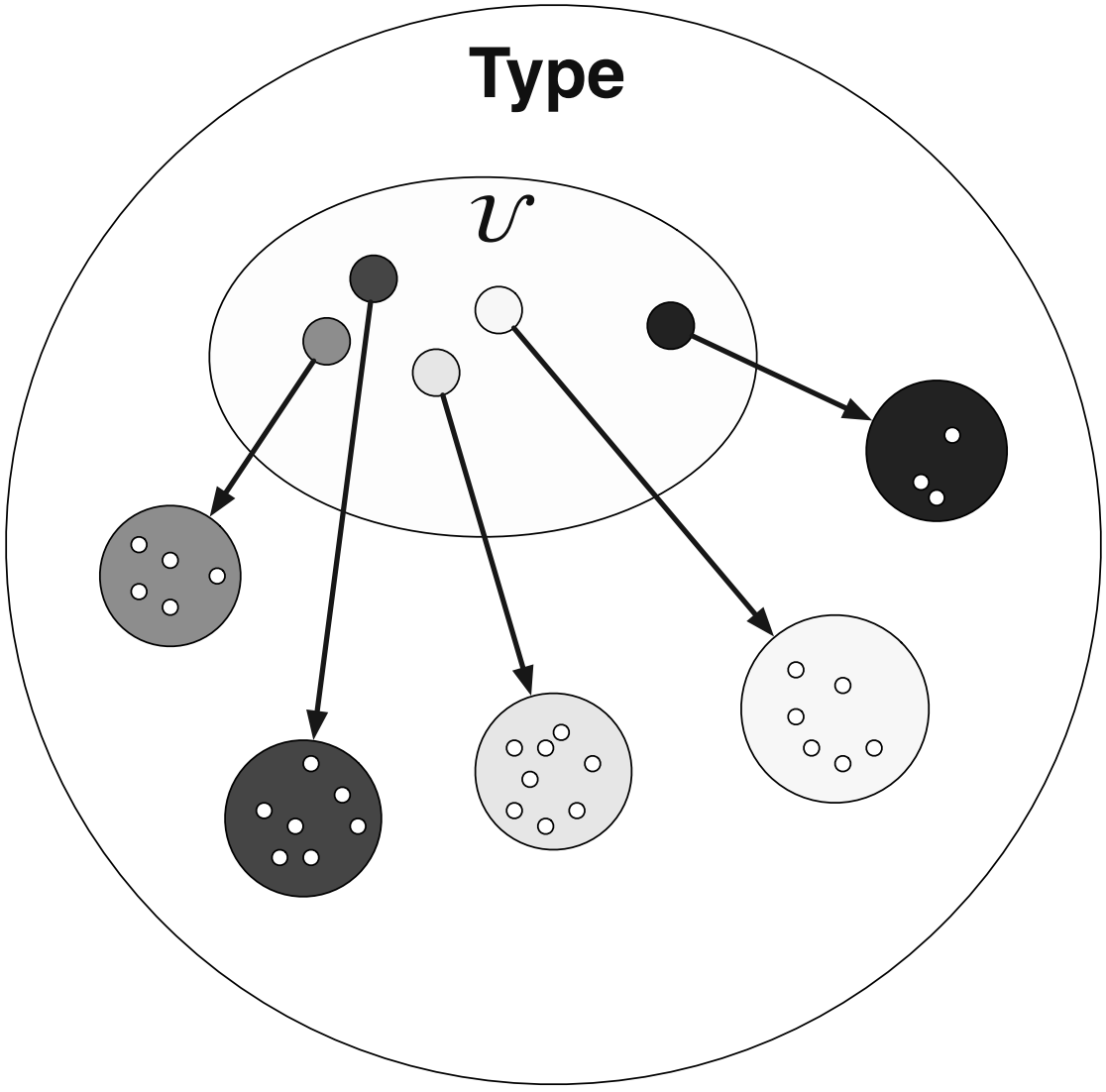

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- Dynamics:

$$\frac{}{El_{\mathcal{A}}(\mathsf{Name}) \rightsquigarrow \mathbf{string}} \qquad \frac{}{El_{\mathcal{A}}(\mathsf{Email}[\ell]) \rightsquigarrow \mathbf{string}}$$

$$\frac{}{El_{\mathcal{A}}(\mathsf{Phone}[\ell]) \rightsquigarrow \mathsf{list}(\mathbb{N})}$$

Records as Products

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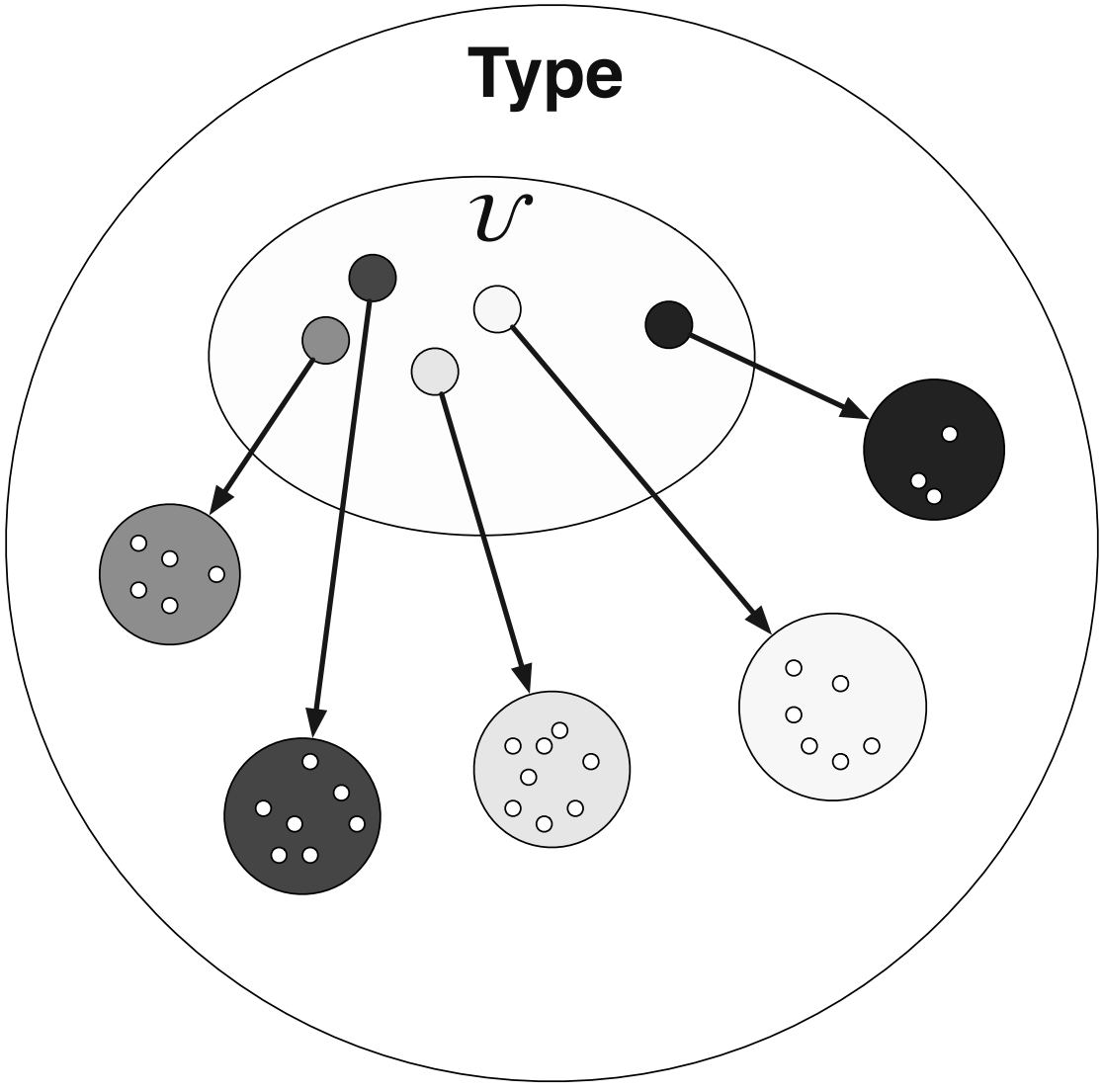
$$\mathsf{record}_{\mathcal{U}} \rightsquigarrow \sum_{\mathcal{V} \subseteq \mathcal{U}} \prod_{\mathcal{V}} El_{\mathcal{U}}|_{\mathcal{V}}$$

Records as Products



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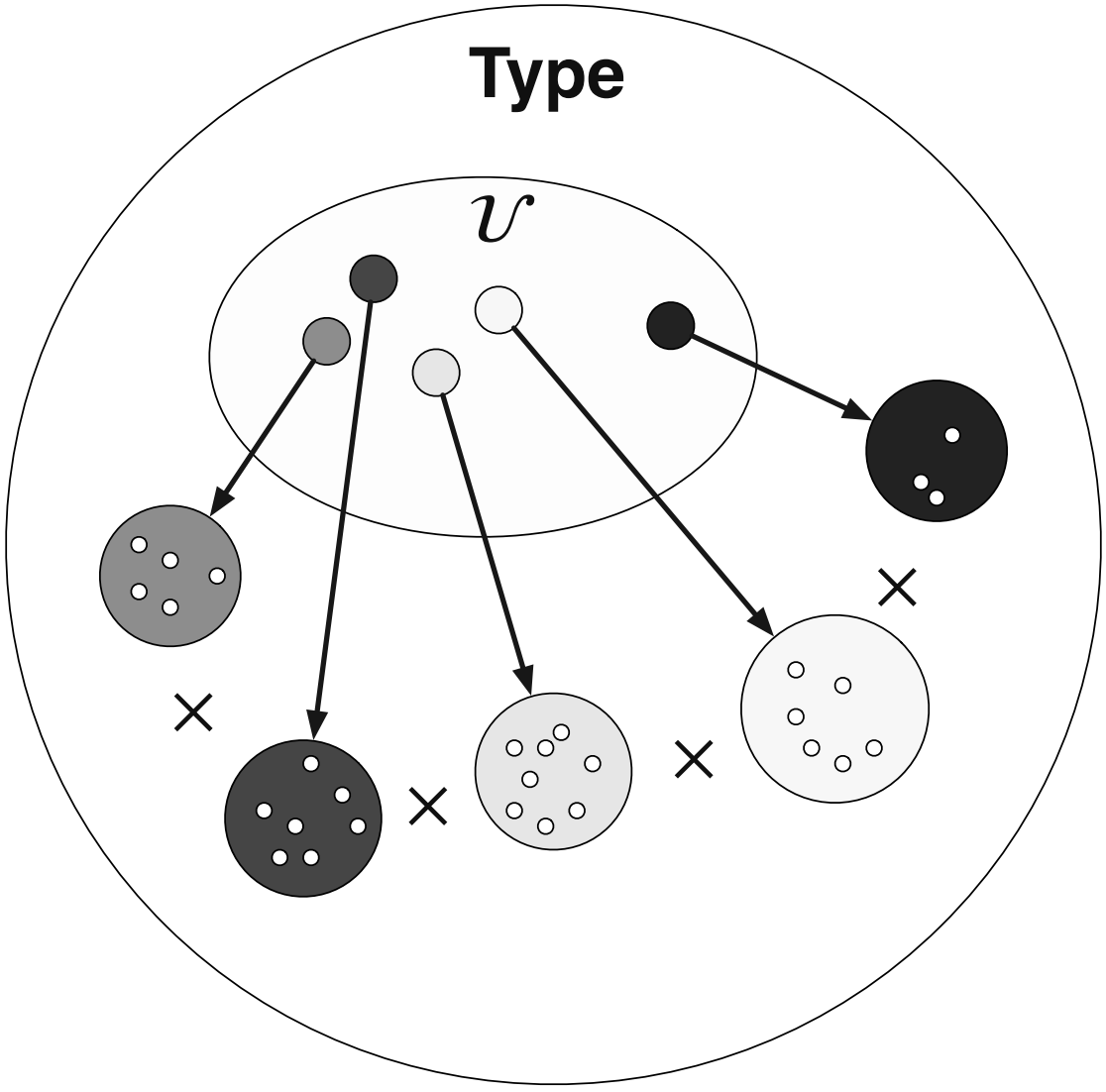

Records as Products

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$$ex : \mathsf{record}_{\mathcal{U}}$$

$$ex \rightsquigarrow \langle \mathcal{A}', \lambda.$$

$$\{\mathsf{Name} \mapsto \text{"Robert Harper"};$$

$$\mathsf{Email}\ \textit{Work} \mapsto \text{"rwh@cs.cmu.edu"}\}\rangle$$

Presheaves

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A presheaf on some category X is a functor $\mathcal{O}(X)^{\mathrm{op}} \to \mathbf{Type}$, where $\mathcal{O}$ is the category of open sets of X for whatever topology you have chosen.

Topologies on some space X

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- What are the open sets on X?

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- The empty set and X are open sets
- Finite intersections of open sets are open

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- Then records on a universe X give rise to a presheaf $\mathcal{R}$: subset inclusions are taken to casts from larger to smaller records

$$\text{for } V \subseteq X \quad \mathcal{R}(V) :\equiv \prod_{V} El_X|_V : \mathbf{Type}$$

$$\text{for } i : V \hookrightarrow U \quad \mathcal{R}(i) :\equiv \mathsf{cast} : \mathcal{R}(U) \to \mathcal{R}(V)$$

Records are Sheaves

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Corecords as Sums

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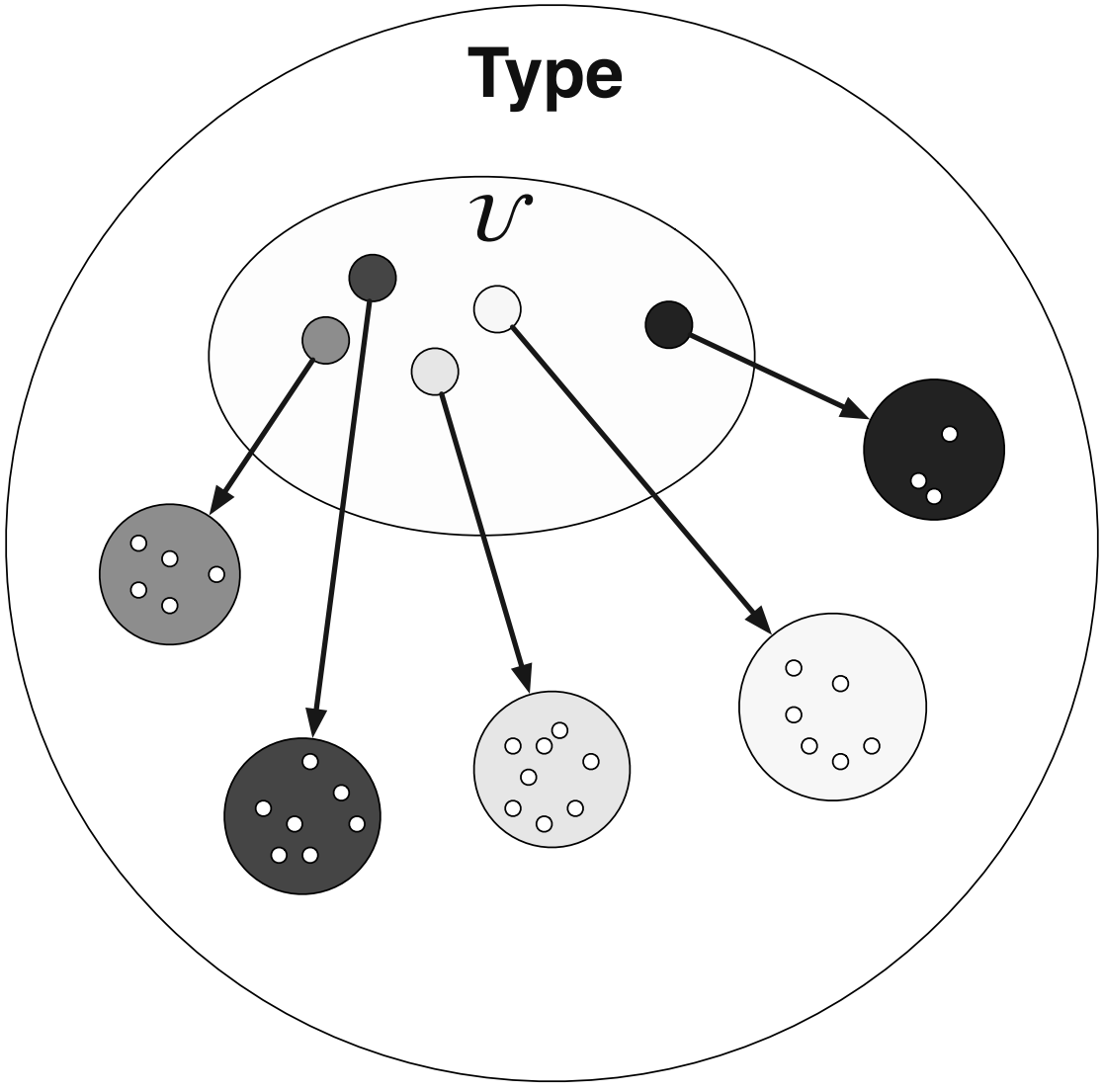

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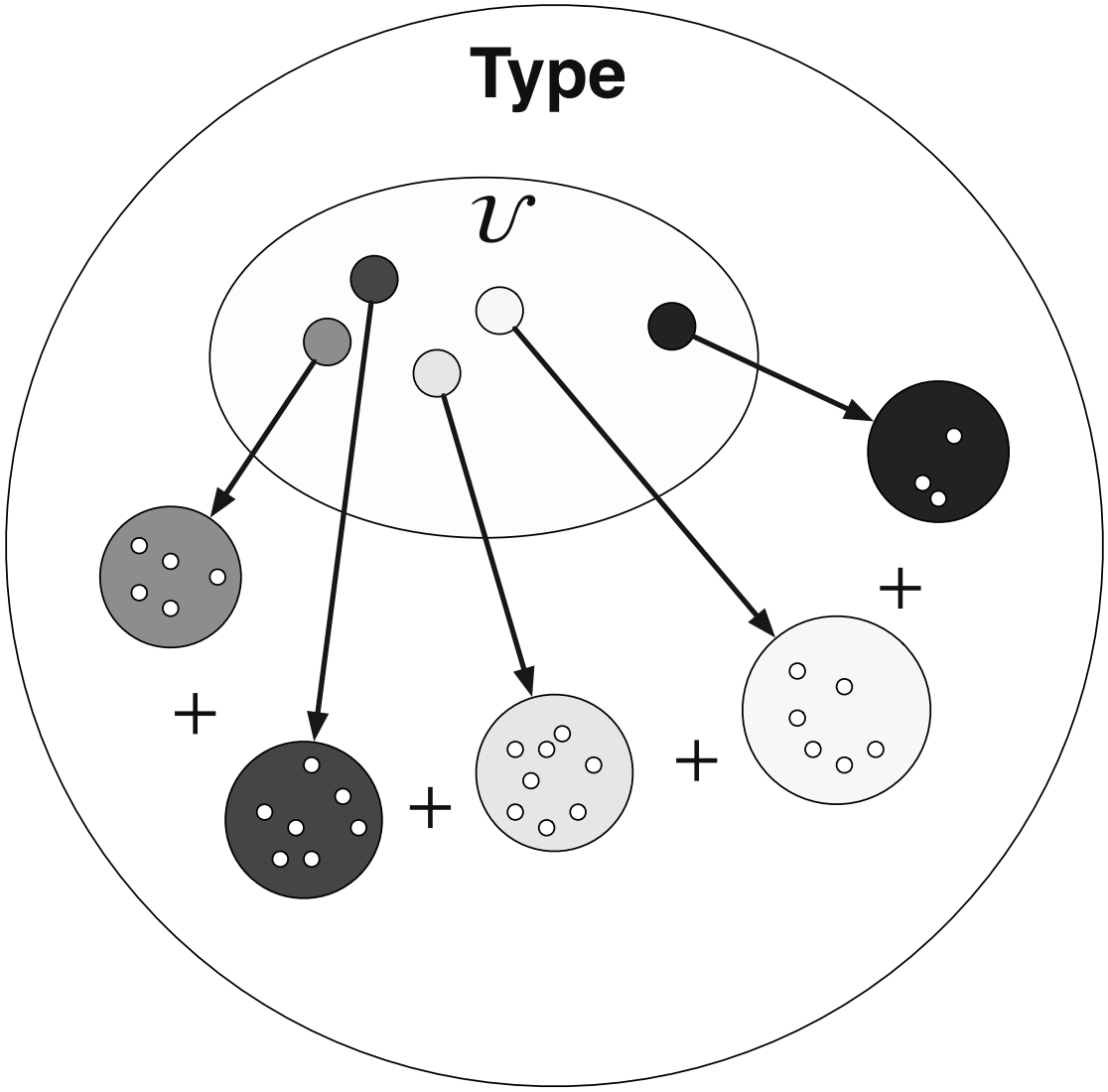

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- Parameterize Rec by $\mathcal{U}$, $El_{\mathcal{U}}$?

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Recovering HList

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ex = 34 :& **True** :& ”vinyl” :& RNil

Validating Records

bob :: Rec $\mathrm{El}_{\mathcal{A}}$ [Name, Email Work]

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validatePhone :: [$\mathbb{N}$] $\rightarrow$ **Either** Error [$\mathbb{N}$]

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unnnnnhhh...

validateContact
:: Rec El$_{\mathcal{A}}$ [Name, Email Work]
$\rightarrow$ **Either** Error (Rec El$_{\mathcal{A}}$ [Name, Email Work])

Welp.

Effects inside records

data Rec :: $(\mathcal{U} \to *) \to [\,\mathcal{U}\,] \to *$ **where**
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($\Leftarrow\!\shortmid$): sing r $\to$ f ($\mathrm{el}_{\mathcal{U}}$ r) $\to$ Rec $\mathrm{el}_{\mathcal{U}}$ f '[r]

Effects inside records

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  RNil :: Rec el𝒰 f '[]
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(⇚): sing r → f (el𝒰 r) → Rec el𝒰 f '[r]
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Compositional Validation

type $\mathsf{Rec}_{\mathcal{A}} = \mathsf{Rec}\ \mathsf{El}_{\mathcal{A}}$

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type TotalContact =
  [Name, Email Home, Email Work
  , Phone Home, Phone Work]

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type TotalContact =
  [Name, Email Home, Email Work
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validateContact :: $\text{Rec}_{\mathcal{A}}$ Validator TotalContact
validateContact = validateName
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rdist :: Applicative f $\Rightarrow$ $\text{Rec}_{\mathcal{U}}$ f rs $\rightarrow$ f ($\text{Rec}_{\mathcal{U}}$ Identity rs)

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type Validator = Lift $(\rightarrow)$ Identity (**Either** Error)
$(\circledcirc)$ :: $\mathrm{Rec}_{\mathcal{U}}$ (Lift $(\rightarrow)$ f g) rs $\rightarrow$ $\mathrm{Rec}_{\mathcal{U}}$ f rs $\rightarrow$ $\mathrm{Rec}_{\mathcal{U}}$ g rs
rdist :: Applicative f $\Rightarrow$ $\mathrm{Rec}_{\mathcal{U}}$ f rs $\rightarrow$ f ($\mathrm{Rec}_{\mathcal{U}}$ Identity rs)

validateContact :: $\mathrm{Rec}_{\mathcal{A}}$ Validator TotalContact

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bobValid :: Rec_A (Either Error) [Name, Email Work]
bobValid = cast validateContact ⊛ bob
```

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Laziness as an effect

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newtype Identity a = Identity { runIdentity :: a }
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type LazyRec_U rs = Rec_U Thunk rs
```

Concurrent Records with Async

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fetchName :: $\text{Rec}_{\mathcal{A}}$ **IO** '[Name]

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$$\text{fetchName} :: \text{Rec}_{\mathcal{A}}\ \mathbf{IO}\ '[\text{Name}]$$
$$\text{fetchName} = \text{Name} \Leftarrow\!\dashv \text{someOperation}$$

$$\text{fetchWorkEmail} :: \text{Rec}_{\mathcal{A}}\ \mathbf{IO}\ '[\text{Email Work}]$$

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fetchName :: Rec_A IO '[Name]
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fetchWorkEmail = Email Work <=| anotherOperation
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fetchBob :: $\text{Rec}_{\mathcal{A}}$ **IO** [Name, Email Work]

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fetchBob :: $\text{Rec}_{\mathcal{A}}$ **IO** [Name, Email Work]
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Concurrent Records with Async

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```
newtype Concurrently a
  = Concurrently { runConcurrently :: IO a }
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```

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bob :: **IO** ($\text{Rec}_{\mathcal{A}}$ Identity [Name, Email Work])

Concurrent Records with Async

fetchBob :: $\text{Rec}_{\mathcal{A}}$ **IO** [Name, Email Work]
bobConcurrently :: $\text{Rec}_{\mathcal{A}}$ Concurrently [Name, Email Work]
concurrentBob :: Concurrently ($\text{Rec}_{\mathcal{A}}$ Identity [...])

bob :: **IO** ($\text{Rec}_{\mathcal{A}}$ Identity [Name, Email Work])
bob = runConcurrently concurrentBob

Containers: The Syntax for Data Types

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Restricting Containers

$$\frac{C : \mathsf{container} \quad \mathcal{V} \subseteq C.\mathsf{Sh}}{C|_{\mathcal{V}} : \mathsf{container}}$$

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Container Lifting

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$$\frac{C \rightsquigarrow \mathcal{U} \triangleleft El_{\mathcal{U}}}{C \uparrow F \rightsquigarrow \mathcal{U} \triangleleft F \circ El_{\mathcal{U}}}$$

A Menagerie of Quantifiers

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Dependent Products:

$$\frac{\Gamma \vdash A : \mathsf{Type} \quad \Gamma, x : A \vdash B : \mathsf{Type}}{\Gamma \vdash \prod_A B : \mathsf{Type}}$$

$$\frac{\Gamma, x : A \vdash e : B[x]}{\Gamma \vdash \lambda x.e : \prod_A B}$$

A Menagerie of Quantifiers

Dependent Sums:

$$\frac{\Gamma \vdash A : \mathsf{Type} \quad \Gamma, x : A \vdash B : \mathsf{Type}}{\Gamma \vdash \sum_A B : \mathsf{Type}}$$

$$\frac{\Gamma \vdash a : A \quad \Gamma \vdash b : B[a]}{\Gamma \vdash \langle a, b \rangle : \sum_A B}$$

A Menagerie of Quantifiers

Inductive Types:

$$\frac{\Gamma \vdash A : \mathsf{Type} \quad \Gamma, x : A \vdash B : \mathsf{Type}}{\Gamma \vdash \mathrm{W}_A\, B : \mathsf{Type}}$$

$$\frac{\Gamma \vdash a : A \quad \Gamma, v : B[a] \vdash b : \mathrm{W}_A\, B}{\Gamma \vdash \mathsf{sup}(a; v.\, b) : \mathrm{W}_A\, B}$$

A Menagerie of Quantifiers

Coinductive Types:

$$\frac{\Gamma \vdash A : \mathsf{Type} \quad \Gamma, x : A \vdash B : \mathsf{Type}}{\Gamma \vdash \mathrm{M}_A\, B : \mathsf{Type}}$$

$$\frac{\Gamma \vdash a : A \quad \Gamma, v : B[a] \vdash b : \infty\,(\mathrm{M}_A\, B)}{\Gamma \vdash \mathsf{inf}(a; v.\, b) : \mathrm{M}_A\, B}$$

A Scheme for Quantifiers

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$$\frac{\Gamma, A : \mathsf{Type}, (x : A \vdash B : \mathsf{Type}) \vdash Q_A B : \mathsf{Type}}{\Gamma \vdash Q \text{ quantifier}}$$

Quantifiers Give Containers Semantics

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$$\frac{\Gamma, A : \mathsf{Type}, (x : A \vdash B : \mathsf{Type}) \vdash Q_A B : \mathsf{Type}}{\Gamma \vdash Q \text{ quantifier}}$$

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$$\frac{C \rightsquigarrow \mathcal{U} \triangleleft El_{\mathcal{U}}}{[\![C]\!]_Q \rightsquigarrow Q_{\mathcal{U}} El_{\mathcal{U}}}$$

Vinyl Records as Containers

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Records and corecords are finite products and sums respectively.

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Questions