Intermediate Evaluation

LELEC2103

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Outline

Symbol timing recovery

Received signal in discrete time after the matched filter is given by:

$$y[n] = \sqrt{E_x}\alpha e^{j\theta} \sum_m s[m] g((n-m)T - \tau_d) + v[n] \quad (1)$$

$$\begin{aligned} y[n] =& \sqrt{E_x}\alpha s[n] g(\tau_d) \\ &+ \sqrt{E_x}\alpha e^{j\theta} \sum_{m \neq n} s[m] g((m-n)T - \tau_d) \\ &+ v[n] \end{aligned} \quad (2)$$

There is symbol interference due to τ_d.

Outline

Maximum energy

We define energy as

$$J(\tau) = E|(y(nT+\tau)|^2 = \alpha^2 E_x \sum_m |g(mT + \tau - \tau_d)|^2 + \sigma_v^2 \quad (3)$$

There is a maximum pour $\tau - \tau_d = 0$. We try to find $\hat{\tau}$ such that

$$\hat{\tau} = argmax_\tau J(\tau) \quad (4)$$

Direct maximization

In pratice we are in discrete time so:

$$\hat{\tau} = \frac{kT}{M} \quad k \in [0..M-1]$$

$$J[k] = E|r(nT + \frac{kT}{M})|^2 \tag{5}$$

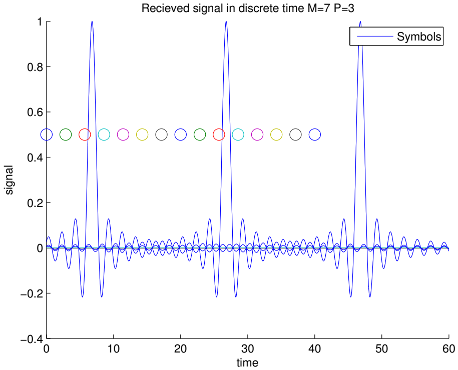
We can find the energy over P symbols and get

$$J_{appox}[k] = \frac{1}{P}\sum_{p=0}^{P-1} |r(pT + \frac{kT}{M})|^2 \tag{6}$$

Direct maximization

We find the maximum energy with

$$\hat{k} = argmax_{k[0..M-1]} J_{approx}[k] \quad (7)$$



Early gate

Now we will try to find the maximum by canceling the derivative.

$$\begin{aligned}\frac{d}{d\tau}J(\tau) &\simeq E(\frac{d}{d\tau}|y(nT+\tau)|^2) \\ &\simeq \frac{1}{P}\sum_{p=0}^{P-1} 2Re(y(pT+\tau)(y^*(pT+\tau+\delta)-y^*(pT+\tau-\delta)))\end{aligned} \tag{8}$$

In discrete time we get

$$J_\delta[k] = \frac{1}{P}\sum_{p=0}^{P-1} 2Re(r[pM+k](r^*[pM+k+\delta]-r^*[pM+k-\delta])) \tag{9}$$

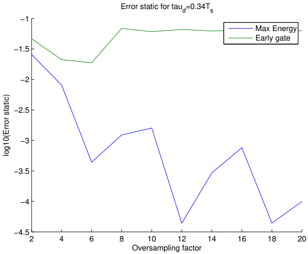
$$\hat{k} = argmin_{k\in[0..M-1]} J_\delta[k]$$

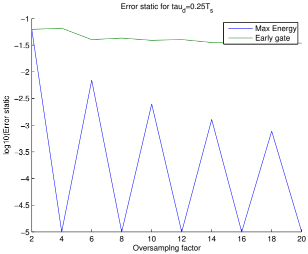
Outline

Error static

We define the error static as:

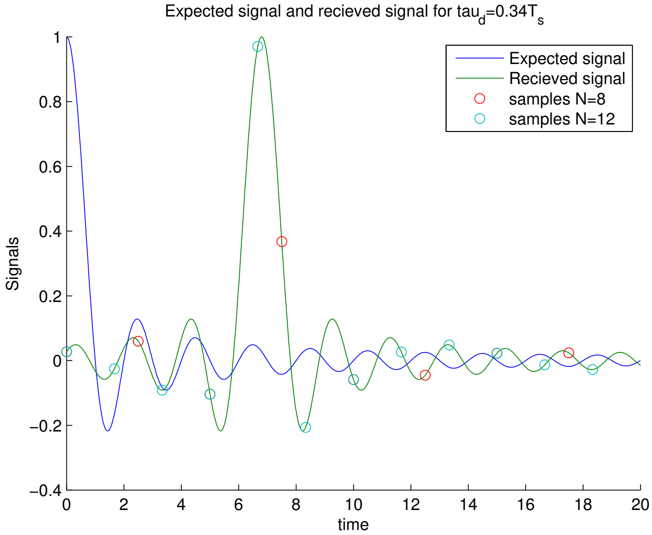
$$\epsilon[M] = E(\|\frac{\hat{\tau}(M) - \tau_d}{T_s}\|^2) \qquad (10)$$



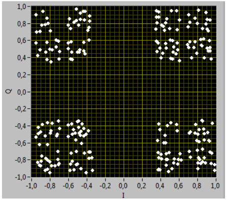


Error static

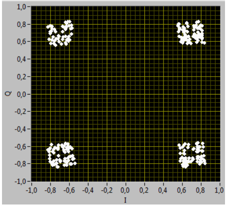
The shape of error static for direct maximization is due to the delay and the oversampling factor.



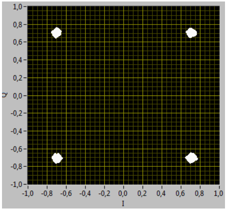
Constellations



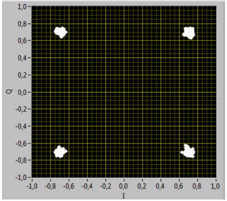
(a) M = 2



(b) M = 4



(c) M = 10



(d) M = 20

Figure: Constellation for direct maximization

Channel estimation and equalization

Channel distortion ($h(t) \neq \delta(t)$) so received signal suffers from ISI :

$$y[n] = h[0]s[n] + \sum_{m \neq n} s[m]h[n-m] + v[n] \qquad (11)$$

- Goal : reduce effect of channel (apply "inverse filter") = *equalization*.
- Equalization needs the channel response before = channel *estimation*.

Use a *training sequence* to estimate the channel.
Both methods use a *least-squares approximation* method : expensive!

Direct least-squares equalization

Direct method : apply channel estimation and equalization at once! Only 1 LLS.

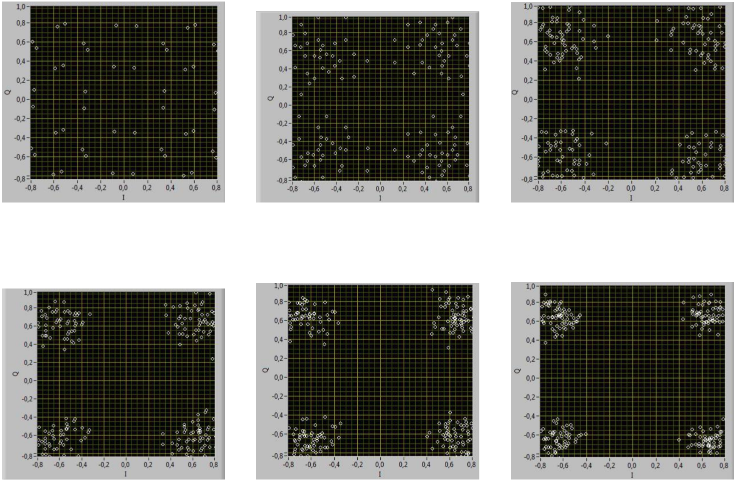
$$\sum_{l=0}^{L_f} f[l]y[n+n_d-l] = t[n] \ , \quad n = 0 \ldots N_t \tag{12}$$

Estimate filter parameters $f[0] \ldots f[L_f]$ by creating a filter that matches training sequence t from the received signal y.
Note :

- n_d = filter delay
- L_f = filter length

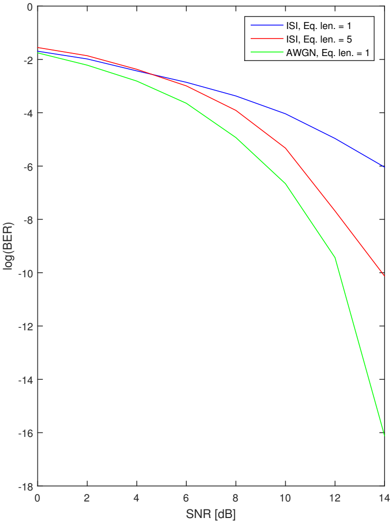
Simulation : influence of channel length

Increasing L_f from 1 to 6 (better estimations)



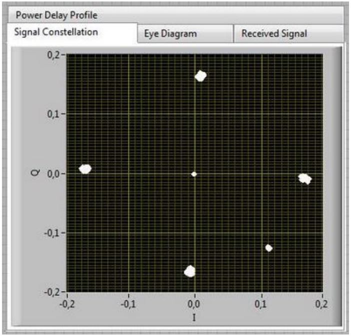
Simulation : influence of noise

Noise on the training sequence corrupts the equalizer and propagates to all symbols!



Experiment : effect of channel

No equalization! Constellation shifted + scaled.



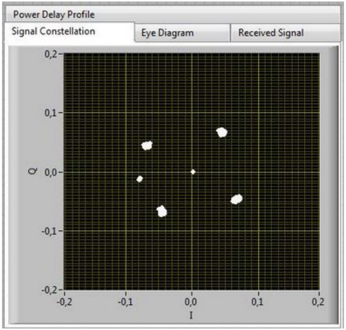


Figure: Received constellation without equalizer.