Assignment 1

Simulation of a Cellular Telephony Network

Problem Statement

The telecommunication company XPhone has been receiving complaints from its subscribers regarding quality of service (QoS) along a 40 km long highway connecting two major cities. The highway is covered by its cellular telephony network. The company needs to decide whether or not its system guarantees quality of service (QoS) in terms of percentages of dropped calls and blocked calls. Some measurements have been made of the traffic in the network on the highway. Our task is to model and simulate the system to determine whether the system can meet the quality of service requirements, and if so, which fixed channel allocation scheme offers the best service.

QoS Requirements

- blocked calls < 2%
- dropped calls < 1%

System Description

The two-way highway is 40 km long. The company uses 20 base stations, each covers a cell with 2 km diameter. There is no overlapping of cells. Where the reach of one base station ends, the reach of the next base station starts. Each base station has 10 channels so there are 10 channels available in each cell. When a subscriber initiates a call from within a cell, a channel in the cell will be allocated to the call. If no free channels are available in the base station, the call is blocked. When a subscriber making a call crosses a cell boundary before the end of the 40-km highway, the channel being used in the current cell is released and a new channel in the new cell has to be acquired: this is called a **Handover**. If a channel is not available in the new base station during a handover the call is dropped. When a subscriber making a call crosses the end of the 40-km highway (either end), the call will be terminated and the channel being used is released.

A Fixed Channel Allocation (FCA) scheme is used. The company wants us to test at least two FCA schemes:

No channel reservation

• 9 channels are allocated to each cell for new calls and handovers and 1 channel is reserved for handovers when the other 9 channels are not available. This means a new call will not be allocated a channel if there is only one free channel left.

The company has provided the following measurements:

- Call initiation times and their first base stations
- Call durations
- Car speeds

Assumptions

- The traffic volumes in the two directions are the same. This means the two directions of cars travelling along the highway have equal probabilities.
- A car maintains the same speed during a call.
- The position of the car initiating a call in a cell is uniformly distributed along the section of the highway covered by the base station.

Outline

In the following we are going to analyse the data to find what distributions the inter-arrival times of calls, the locations where calls are generated, the call durations, and car speeds follow respectively. In addition, we also need to find the parameter values of these distributions.

After that we are going to develop a discrete-event simulator.

In the end, we are going to run our simulator multiple times, each with a warm-up period for different FCA schemes to investigate how handover reservation scheme may affect the quality of service (i.e., blocking and dropping probabilities).

We are going to answer the following question:

• Is the current system able to meet the quality of service requirements and if so how many channels should be reserved for handover for best service?

```
In [1]: from heapq import *
    import xlrd
    import random
    import numpy as np
    import matplotlib.pyplot as plt
    from scipy.stats import norm
    from scipy.stats import poisson
    from scipy.stats import expon
file = ("PCS_TEST_DETERMINSTIC_19S2.xls") #file of the measurements provided
```

Investigate Data

Extract Data

```
In [2]: def extractData(file):
            initationTimes = []
            baseStations = []
            durations = []
            velocities = []
            wb = xlrd.open_workbook(file)
            sheet = wb.sheet_by_index(0)
            for i in range(1, sheet.nrows):
                 row=sheet.row_values(i)
                 initiationTime=row[1]
                 baseStation=int(row[2])-1 #We use indexing from 0 instead from 1
                 duration=row[3]
                speed=row[4]
                 initationTimes.append(initiationTime)
                 baseStations.append(baseStation)
                 durations.append(duration)
                 velocities.append(speed)
            return initationTimes, baseStations, durations, velocities
```

In [3]: initationTimes, baseStations, durations, velocities = extractData(file)

I) Call Interarrival Time Distribution

Step 1: Investigate Data Collection

At first we take a look at the measured interarrival times. Is the data IID? To proof that one call does not, for example, trigger other calls we use a scatter diagramm to asses whether the data points are independent. After that we plot the data to see if it is identically distributed.

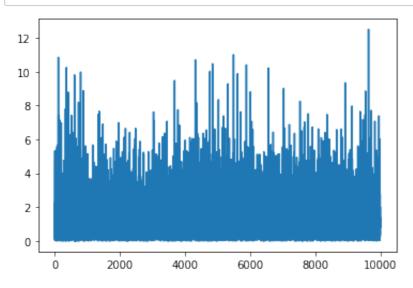
```
In [4]: def getInterarrivalTimes(initationTimes):
            interarrivalTimes = []
            for i in range(1,len(initationTimes)):
                 interarrivalTime = initationTimes[i]-initationTimes[i-1]
                 interarrivalTimes.append(interarrivalTime)
            return interarrivalTimes
In [5]: def createTupel(array):
            x = []
            y = []
            for i in range(1,len(array)):
                x.append(array[i-1])
                y.append(array[i])
            return x, y
In [6]: interarrivalTimes = getInterarrivalTimes(initationTimes)
        x, y = createTupel(interarrivalTimes)
In [7]: | x, y = createTupel(interarrivalTimes)
        plt.scatter(x,y)
        plt.show()
         12
```

12

2

The data points are scattered throughout the first quarter. This indicates that the data is independent.

In [8]: plt.plot(interarrivalTimes)
 plt.show()



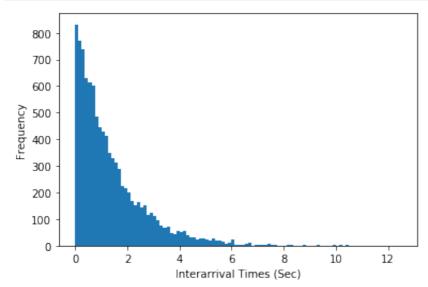
The data semes to be identical distributed as well. There are no major diffrences we can identify. We assume that the data is IID.

Step 2: Identify A Family Of Distributions

This step typically beginns with the development of a histogram. We break the range of data into k intervals of equal widths. Hines et al. [2002] state that k equal to the square root of the number of data items is a good approximation of k. Since we have 9999 data points we go with k = 100.

In [9]: k=100

```
In [10]: plt.hist(interarrivalTimes,k)
   plt.xlabel('Interarrival Times (Sec)')
   plt.ylabel('Frequency')
   plt.show()
```



We hypothesize the data has an exponential distribution.

Step 3: Estimation of the mean for the exponential distribution

```
In [11]: meanCallInterarrival = sum(interarrivalTimes)/len(interarrivalTimes)
print("Expo({})".format(meanCallInterarrival))
```

Expo(1.3698169264765245)

Step 4: Goodness-of-fit test of selected distribution

We use the chi-square test to test the hypothesis that these call interarrival times are exponentially distributed. To calculated the end points of the intervalls, we use the percent point function (ppf) of the scipy libary.

```
In [12]:
         def calExpInterall(mean,k):
             classIntervall = [0]
             for i in range(1,k):
                 ai=expon.ppf(i/k,scale=mean)
                 classIntervall.append(ai)
             return classIntervall
In [13]: def chiSquare(intervall,data,k):
             a = [0]*k
             for d in data:
                 for v in range(k-1,-1,-1):
                     if d>=intervall[v]:
                          a[v]=a[v]+1
                          break
             x2=0
             for i in range(k):
                 npj=(1/k)*len(data)
                 x2+=((a[i]-npj)*(a[i]-npj))/npj
             return x2
```

```
In [14]: expIntervall=calExpInterall(meanCallInterarrival,k)
    chiSquare(expIntervall,interarrivalTimes,k)
```

Out[14]: 111.66106610661065

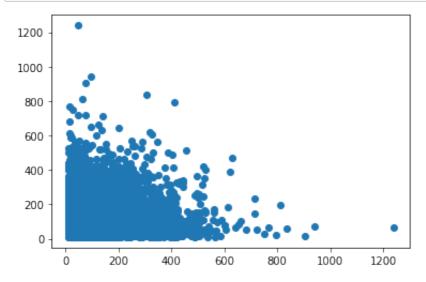
So the test statistic is X2=111.66. The degree of freedom of the chi-square distribution is k-s-1. s is the number of parameter of the hypothesized distribution estimated by the sample data. For the exponential distribution s=1. **Degree of Freedom: 98** We set our level of significance: **a=0.01**, the critical value for the x2 distribution is **133.476**. Since X2<133.476 the **hypothesis can be accepted**. Thus the hypothesised exponential distribution expo(1.3698169264765245) does fit the data well.

II) Call Duration Distribution

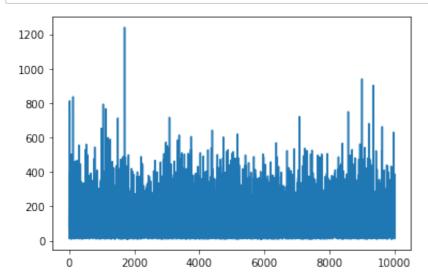
We will take the same steps as before:

Step 1: Investigate Data Collection

```
In [15]: x, y = createTupel(durations)
    plt.scatter(x,y)
    plt.show()
```



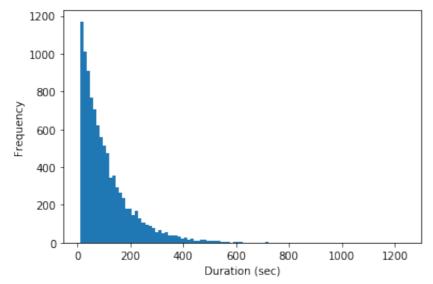
```
In [16]: plt.plot(durations)
  plt.show()
```



The data points are scattered throughout the first quarter. This indicates that the data is independent. The data sems to be identical distributed as well. We assume that the data is IID.

Step 2: Identify A Family Of Distributions

```
In [17]: c = plt.hist(durations,k)
    plt.xlabel('Duration (sec)')
    plt.ylabel('Frequency')
    plt.show()
```



```
In [18]: print("Minimum: {}".format(min(durations)))
```

Minimum: 10.003951603252272

From the histogram we notice that there are no calls close to 0. The minimal time for a call is 10 sec. To fit a distribution to the data we assume that **a call lasts at least 10 sec** and that the remaining time has an exponential distribution.

Step 3: Estimation of the mean for the exponential distribution

```
In [19]: remainingTime=[x-10 for x in durations]
```

```
In [20]: meanCallDuration = sum(remainingTime)/len(remainingTime)
    print("Expo({})".format(meanCallDuration))
```

Expo(99.83590073874737)

Step 4: Goodness-of-fit test of selected distribution

We use the chi-square test to test the hypothesis that these call interarrival times are exponentially distributed.

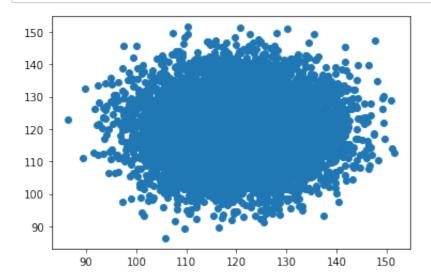
Out[21]: 95.91999999999999

So the test statistic is X2=95.9. The degree of freedom of the chi-square distribution **98**. We set our level of significance: **a=0.01**, the critical value is **134.642**. Since X2<134.642 the **hypothesis can be accepted**. Thus the hypothesised exponential distribution does fit the data well.

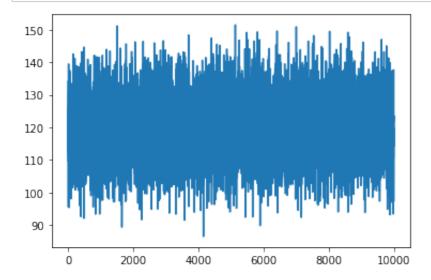
III) Velocity Distribution

Step 1: Investigate Data Collection

```
In [22]: x, y = createTupel(velocities)
plt.scatter(x,y)
plt.show()
```



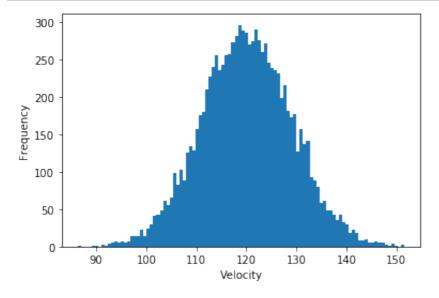
In [23]: plt.plot(velocities)
 plt.show()



By looking at the scatter plot and the plot of the data we see that most of the data is around 120. We assume that the data is IID.

Step 2: Identify A Family Of Distributions

```
In [24]: plt.hist(velocities,k)
    plt.xlabel('Velocity')
    plt.ylabel('Frequency')
    plt.show()
```



We hypothesize the data has a normal distribution.

Step 3: Estimation of the parameters of a normal distribution

```
In [25]: meanVelocity =np.mean(velocities)
    stdVelocity=np.std(velocities)
    print("N({}, {})".format(meanVelocity,stdVelocity**2))

N(120.07209801685764, 81.33527102508032)
```

Step 4: Goodness-of-fit test of selected distribution¶

```
In [26]: def calNormInterall(mean,std,k):
    classIntervall = [0]
    for i in range(1,k):
        ai=norm.ppf(i/k,mean,std)
        classIntervall.append(ai)
    return classIntervall
```

```
In [27]: normIntervall=calNormInterall(meanVelocity,stdVelocity,k)
    chiSquare(normIntervall,velocities,k)
```

Out[27]: 96.44

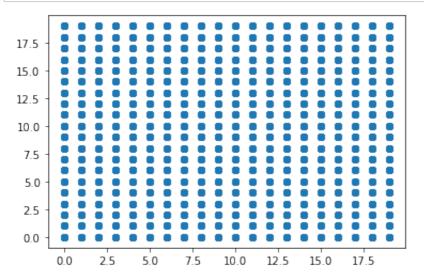
So the test statistic is X2=96.44. The degree of freedom of the chi-square distribution is k-s-1. For a normal distribution s=2. So the degree of freedom is 97. We set our level of significants at a=0.01, the critical value is 132.309. Since X2<132.309 the hypothesis can be accepted. Thus the hypothesised normal distribution does fit the data well.

IV) Base Station Distribution

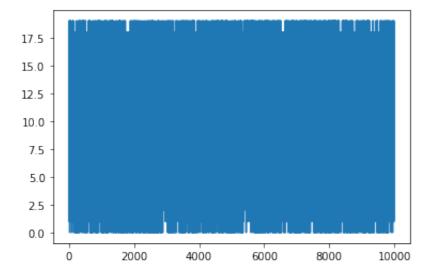
Step 1: Investigate Data Collection

We know that we have 20 diffrent stations and that calls are handovered to the next cell left or right.

In [28]: x, y = createTupel(baseStations)
plt.scatter(x,y)
plt.show()



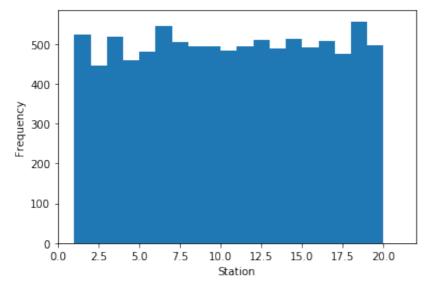
In [29]: plt.plot(baseStations)
 plt.show()



By looking at the scatter plot and the plot of the data we see already that the data has a normal distribution. We assume that the data is IID.

Step 2: Identify A Family Of Distributions

```
In [30]: plt.hist(baseStations,20,[1,21])
  plt.xlabel('Station')
  plt.ylabel('Frequency')
  plt.show()
```



We hypothesize the data has an uniform distribution U(1,20). We use the chi-square test to test the hypothesis. Since we only have 20 stations it is clear that k=20.

Step 3: Goodness-of-fit test of selected distribution

Out[31]: 25,656

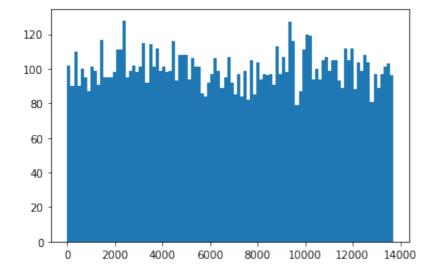
So the test statistic is X2=25.656. The degree of freedom of the chi-square distribution is k-s-1. For a uniform distribution s=1. So the degree of freedom is **18**. We set our level of significants at **a=0.01**, the critical value is **34.805**. Since X2<34.805 the **hypothesis can be accepted**. Thus the hypothesised normal distribution U(1,20) does fit the data well.

Warm up periode of simulation

To determine the length of the warm up periode of the simulation, we want to find out the average number of new calls in a short time periode from the measured data.

```
In [32]: a=plt.hist(initationTimes,100)
print("Mean: {}".format(np.mean(a[0])))
```

Mean: 100.0



We seperate the initation times of the calls in k=100 buckets. Since the last call initation time is at second 13697, the intervalls have a length of about 137 seconds. When we now calculate the mean amount of calls in one bucket we get 100. In avergae there are about 100 initation calls in 137 sec.

For our simulation the warm up periode will end, when we reach an interval i * 137 sec where in its time periode appear more than 100 calls.

Events:

There are 3 types of Events that are handled by the simulator:

Call initiation [time, speed, station, position, duration, direction]

Call termination [time, station]

Call handover [time, speed, station, duration, direction]

```
In [34]: class Event:
             def __gt__(self, other):
                  return self.time > other.time
          class InitiationEvent(Event):
             def __init__(self, time, speed, station, position, duration, direction):
                  self.time = time
                  self.speed = speed
                  self.station = station
                  self.position = position
                  self.duration = duration
                  self.direction = direction
         class TerminationEvent(Event):
             def __init__(self,time,station):
                  self.time = time
                  self.station = station
          class HandoverEvent(Event):
             def __init__(self, time, speed, station, duration, direction):
                  self.time = time
                  self.speed = speed
                  self.station = station
                  self.duration = duration
                  self.direction = direction
```

Code for generation random calls

Assumptions:

- (a) The traffic volumes in the two directions are the same. This means the two directions of cars travelling along the highway have equal probabilities.
- (b) A car maintains the same speed during a call.
- (c) The position of the car initiating a call in a cell is uniformly distributed along the section of the highway covered by the base station.

```
In [35]: def genRandomCall():
    callInterarrivalTime=np.random.exponential(meanCallInterarrival)
    initiationTime=currentTime+callInterarrivalTime

    duration=10.0+np.random.exponential(meanCallDuration)

    speed=np.random.normal(meanVelocity, stdVelocity)

    baseStation=random.randint(1,20)-1 #returns integer from the discret uniform distribution
    #we use indexing from 0 insted from 1

    position=random.uniform(0,2)
        direction=int((-1)**random.randint(-1,0)) #-1 for going west, 1 for going east

    callInitiationEvent=InitiationEvent(initiationTime, speed, baseStation, position, duration, direction)
    heappush(eventHeap, callInitiationEvent)
```

Event Handling Functions

```
In [36]: def initiationEventHandler(event):
             if stations[event.station]<channelsForNewCalls:</pre>
                 stations[event.station]+=1
                 report.write("Event Time: " + str(event.time) + " Call started in station " + str(event.station
                 computeNextEvent(event)
             else:
                 report.write("Event Time: " + str(event.time) + " Call blocked in station " + str(event.station
                 if not warmUp: #If the warm up periode is over, the blocked calls are counted
                     global callsBlocked
                     callsBlocked+=1
             report.write(str(stations)+"\n")
             CheckWarmUp(event)
             if not warmUp:
                 global callsTotal
                 callsTotal+=1
             genRandomCall()
```

In this function the math for computing the time of the next event happens and the new event is created and pushed onto the heap

report.write(str(stations)+"\n")

```
In [39]: def computeNextEvent(event):
             if isinstance(event, HandoverEvent):
                 distnanceToNextCell=2
             else:
                 #the position relative to the station is computed
                 if event.direction == -1:
                     distnanceToNextCell=event.position
                 else:
                     distnanceToNextCell=2-event.position
             timeToNextCell=(distnanceToNextCell/event.speed)*60*60 #time is meassured in sec
             nextStation=event.station+event.direction
             time=currentTime+timeToNextCell
             if timeToNextCell >= event.duration: #call ends in this cell
                 teminationTime=currentTime+event.duration
                 callTerminationEvent = TerminationEvent(teminationTime,event.station)
                 heappush(eventHeap, callTerminationEvent)
             elif nextStation<0 or nextStation > 19: #car leaves the highway
                 callTerminationEvent = TerminationEvent(time, event.station)
                 heappush(eventHeap, callTerminationEvent)
             else: #call handover
                 speed = event.speed
                 duration = event.duration-timeToNextCell
                 direction = event.direction
                 callHandoverEvent = HandoverEvent(time, speed, nextStation, duration, direction)
                 heappush(eventHeap, callHandoverEvent)
```

Main simulation function

```
In [40]:
    def simulate(simulationTime):
        global currentTime
        global warmUpTime
        genRandomCall()

    while (len(eventHeap)>0 and simulationTime+warmUpTime>currentTime):
        event = heappop(eventHeap)
        currentTime=event.time

    if isinstance(event,InitiationEvent):
        initiationEventHandler(event)
    elif isinstance(event,TerminationEvent):
        terminationEventHandler(event)
    else:
        handoverEventHandler(event)
    report.close()
```

Running the simulation

```
In [41]: def refresh(): #Used to define and clear all simulation related variables for one run
             qlobal eventHeap
             global stations
             global currentTime
             global callsDropped
             global callsBlocked
             qlobal callsTotal
             qlobal report
             global warmUp
             global initiatedCalls
             global warmUpTime
             eventHeap = [] #We use a heap to shedule the order of the events
             stations = [0]*20 #Each element in the list represents one station (index 0-19) with 0-10 free chann
             currentTime = 0.0
             callsDropped=0 #Calls droped after warm up
             callsBlocked=0 #Calls blocked after warm up
             callsTotal=0 #Total calls after warm up
             report = open("report.txt", "w") #file for the report
             warmUp=True
             initiatedCalls=[] #intervall of 137 to count calls
             warmUpTime = 0 #The time is unknown so far and will be determined by the WarmUp Methode
```

```
In [42]: def doNruns(n):
    allDroppedCalls=[]
    allBlockedCalls=[]

while n>0:
        refresh()
        simulate(simulationTime)
        allDroppedCalls.append((callsDropped/callsTotal)*100)
        allBlockedCalls.append((callsBlocked/callsTotal)*100)
        n-=1
    return allDroppedCalls, allBlockedCalls
```

FCA 10 - No reservation

- Simulation Time: 24 hours plus warm-up time
- 10 runs

FCA 9 - One channel reservated for Handover

- Simulation Time: 24 hours plus warm-up time
- 10 runs

FCA 8 - Two channel reservated for Handover

- Simulation Time: 24 hours plus warm-up time
- 10 runs

Output Analysis

```
In [46]: def calcS2(array):
    xj=np.array(array)
    s2=np.sum((xj-np.mean(xj))**2)/(len(xj)-1)
    return s2

def calcConfidenceInterval(array,t):
    mean=np.mean(array)
    s2=calcS2(array)
    l=t*(np.sqrt(s2)/np.sqrt(n))
    return [mean-l,mean+l], mean
```

Confidence interval

We calculate the 95% confidence interval with a statistical significants of a = 0.05 to give recommendations for the FCA scheme. We expect that with a proper bility of 95%, our interval captures the true population parameter.

```
In [47]: t=2.262
    ConfidenceIntervalDropped10, meanDropped10=calcConfidenceInterval(ToalPercentDroppedCalls10,t)
    ConfidenceIntervalBlocked10, meanBlocked10=calcConfidenceInterval(ToalPercentBlockedCalls10,t)
    ConfidenceIntervallTotalIncidents10, meanTotal10=calcConfidenceInterval(TotalPercentBlockedAndDropped10,
    ConfidenceIntervalDropped9, meanDropped9=calcConfidenceInterval(ToalPercentDroppedCalls9,t)
    ConfidenceIntervalBlocked9, meanBlocked9=calcConfidenceInterval(ToalPercentBlockedCalls9,t)
    ConfidenceIntervallTotalIncidents9, meanTotal9=calcConfidenceInterval(ToalPercentBlockedAndDropped9,t)
    ConfidenceIntervalBlocked8, meanBlocked8=calcConfidenceInterval(ToalPercentBlockedCalls8,t)
    ConfidenceIntervallTotalIncidents8, meanTotal8=calcConfidenceInterval(TotalPercentBlockedAndDropped8,t)
```

FCA₁₀

```
In [48]: print("Confidence Interval for dropped calls: "+str(ConfidenceIntervalDropped10))
print("Confidence Interval for blocked calls: "+str(ConfidenceIntervalBlocked10))
print("Confidence Interval for dropped and blocked calls: "+str(ConfidenceIntervallTotalIncidents10))

Confidence Interval for dropped calls: [0.5184293047322447, 0.5935820862361804]
Confidence Interval for blocked calls: [0.30668651398447, 0.34485324909635884]
```

Confidence Interval for dropped and blocked calls: [0.8274567004499656, 0.9360944535992881]

FCA9

```
In [49]: print("Confidence Interval for dropped calls: "+str(ConfidenceIntervalDropped9))
    print("Confidence Interval for blocked calls: "+str(ConfidenceIntervalBlocked9))
    print("Confidence Interval for dropped and blocked calls: "+str(ConfidenceIntervallTotalIncidents9))
```

```
Confidence Interval for dropped calls: [0.31539170859027493, 0.3608868143927652]
Confidence Interval for blocked calls: [1.063944014694628, 1.126980532536468]
Confidence Interval for dropped and blocked calls: [1.3982658333165858, 1.4689372368975504]
```

FCA8

```
In [50]: print("Confidence Interval for dropped calls: "+str(ConfidenceIntervalDropped8))
print("Confidence Interval for blocked calls: "+str(ConfidenceIntervalBlocked8))
print("Confidence Interval for dropped and blocked calls: "+str(ConfidenceIntervallTotalIncidents8))

Confidence Interval for dropped calls: [0.16469675119825472, 0.20873496813455522]
Confidence Interval for blocked calls: [2.629827654213221, 2.7775649395672515]
Confidence Interval for dropped and blocked calls: [2.811677962435635, 2.9691463506776485]
```

Summary of simulation conclusions and recommendations

- FCA10 and FCA9 both fullfill the service requirements
- FCA 8 does not fullfill the requirements, since Blocked Calls > 2%

We now have to compare FCA10 against FCA9

Paired-t confidence interval

We calculate the 95% confidence interval with a statistical significants of a = 0.05 to give recommendations for the FCA scheme. We expect that with a properbility of 95%, our interval captures the true population parameter. With approximately 95% confidence, we can say which strategy is better.

```
In [51]: t=2.262
DiffrenceDropped = np.array(ToalPercentDroppedCalls10)-np.array(ToalPercentDroppedCalls9)
ConfidenceIntervalDropped10_9, meanDropped10_9 = calcConfidenceInterval(DiffrenceDropped,t)

DiffrenceBlocked = np.array(ToalPercentBlockedCalls10)-np.array(ToalPercentBlockedCalls9)
ConfidenceIntervalBlocked10_9, meanBlocked10_9 = calcConfidenceInterval(DiffrenceBlocked,t)

DiffrenceTotal = np.array(ConfidenceIntervallTotalIncidents10)-np.array(ConfidenceIntervallTotalIncident ConfidenceIntervalTotal10_9, meanTotal10_9 = calcConfidenceInterval(DiffrenceTotal,t)
```

In [52]: print("The amount of dropped calls decreases by {}% - {}% with FCA9)".format(ConfidenceIntervalDropped10 ConfidenceIntervalDropped10 print("the amount of blocked calls increases by {}% - {}% with FCA9)".format(abs(ConfidenceIntervalBlock abs(ConfidenceIntervalBlock print("In total there are between {}% - {}% more calls blocked and dropped with FCA9".format(abs(ConfidenceIntervalBlock abs(ConfidenceIntervalBlock)).

The amount of dropped calls decreases by 0.17487317549525777% - 0.26085969249012725% with FCA9) the amount of blocked calls increases by 0.8022793572366836% - 0.7371054269135834% with FCA9) In total there are between 0.571029283649415% - 0.5326226325154676% more calls blocked and dropped with FCA9

The company has to decide between FCA10 and FCA9 depending on their policy

- If we care more about dropped calls and can accept more blocked calls, we can use FCA9
- If we want to increase the overall performance of the system (less blocked and dropped calls) we should stay with FCA10