Summary of previous result

We have been interested in the evolution of a population of individuals on a structured network in the context of one mutant being introduced into the population. This is modeled with a Moran birth-death model. However, standard Moran model assumes the fitness of wildtype and mutant individuals are both fixed constant once the evolution begins, whereas in the real world, there could be environmental factors and incomplete penetrance that affect the correlation between genotype and phenotypic fitness. Specifically we have been looking at the fixation patterns when fitness of mutant is drawn from a probabilistic distribution rather than a fixed constant.

Previously we have found out that while the probability of fixation at high variance is linearly proportional to that at no variance for most graphs including geometric graphs, preferential attachment graphs and many others, there are certain families of graphs that demonstrated a deviation from this linear trend. Specifically, we noticed that detour graphs and some island graphs demonstrated interesting deviation from the line.

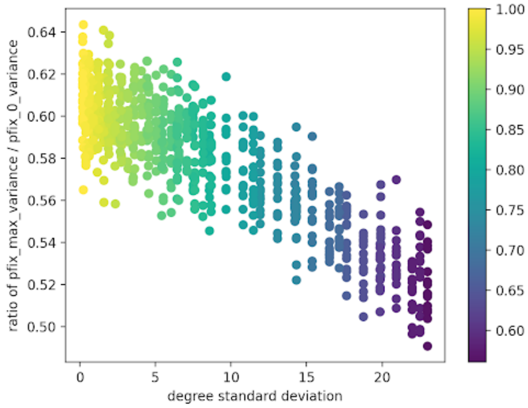
What's new

- Difference in degree between the 2 graph components is correlated with deviation behavior (which is - smaller than expected fixation probability at high variance). Size of components doesn't seem to matter (Section 3)
- The vertical differences could be due to noise, but might have other underlying factors (Section 4)
- Length of path to return to the "denser" component does not matter. (Section 5)
- Regular graph shows a stronger deviation than "wheel-like" graphs with same degree distribution. (Section 5)

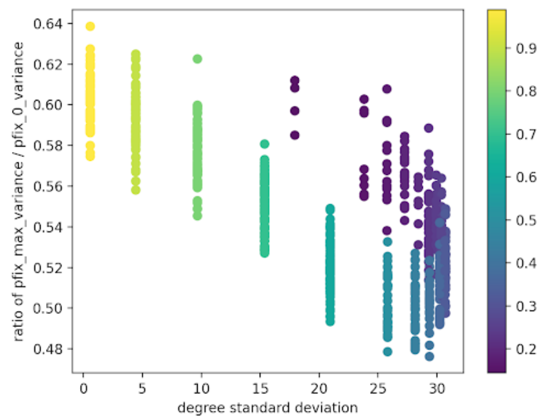
1 Deviation seems uncorrelated with degree mean, amplification factor, mixing pattern, fraction of triangles

For detour graphs, we noticed some variability with regard to the detour length and the degree standard variation, but it was hard to get more insights from it due to low variability among the detour graphs otherwise. Therefore we focused most of our attention on the two island graph families that showed strong deviation, island 2 and island 3 graphs.

Both island 2 and island 3 graphs are composition of 2 random regular graphs. In island graph 2, the size and mean degree of components are tuned. In island graph 3, the number / fraction of triangles are tuned. We see that graphs with same mean degree / standard deviation / mixing patterns etc show different degrees of deviations (those vertical lines) We have tried to plot the various factors that could possibly be a cause, but graphs in each



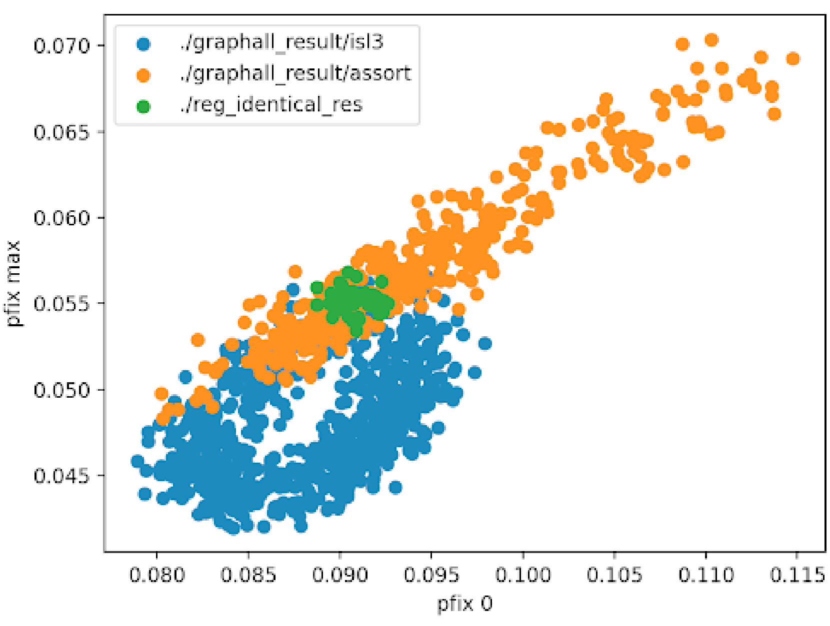
(a) island 2, color=amplification factor



(b) island 3, color=amplification factor

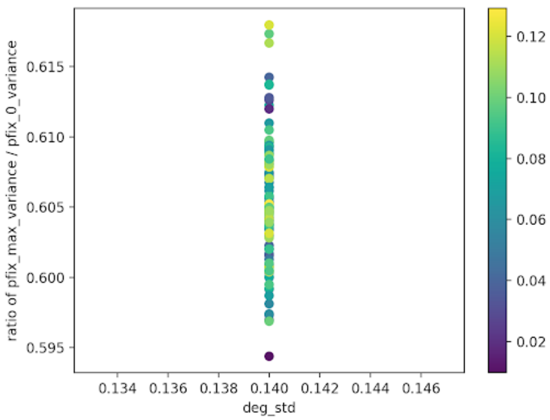
vertical bar seems to always have identical traits.

2 Composite graphs with two identical components of regular graphs are not interesting

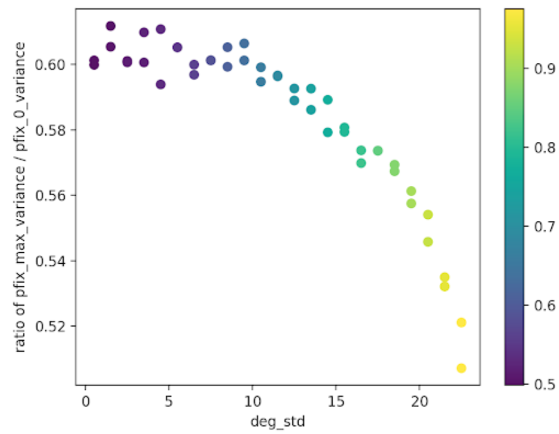


One observation we had from previous results is that all graphs that exhibit interesting patterns are composite graphs. Here, I experimented with the simplest composite graph - a graph with 2 identical regular subgraphs connected by 1 or 2 bridges. The assort graphs are showing a strong linear correlation, whereas the island 3 graphs - which we consider as "interesting", showed a clear deviation. In the figure above, we see that graphs with 2 identical regular component lie on the "line", and thus are most likely to be uninteresting.

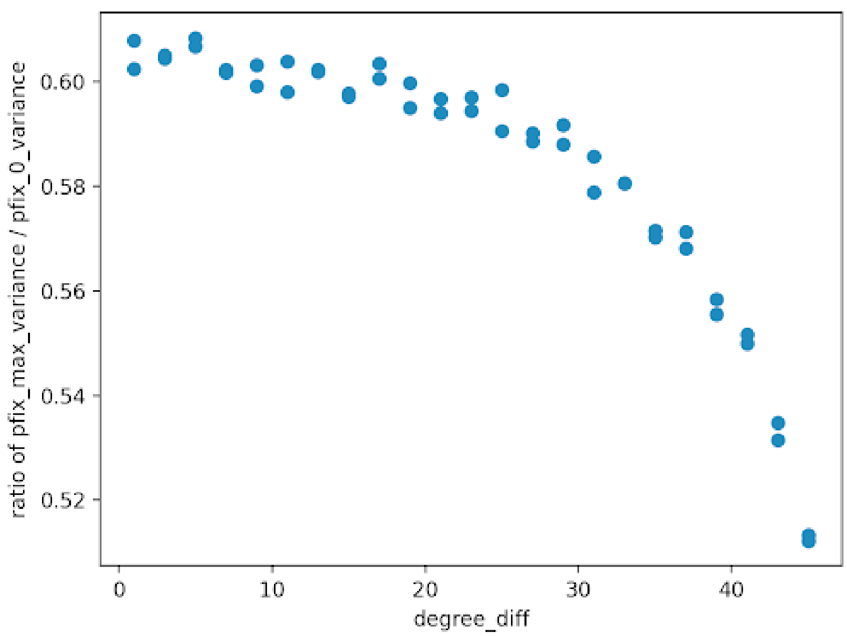
3 Degree differential between the two components likely to be what makes it interesting, not size



(a) both components being regular graphs with same degree, but different sizes



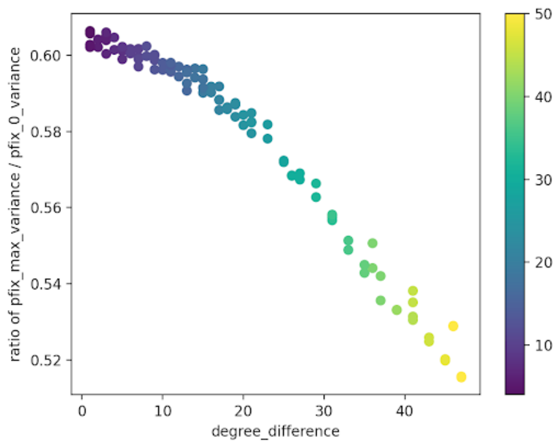
(b) both regular subgraphs have same size but different degrees, color: transitivity



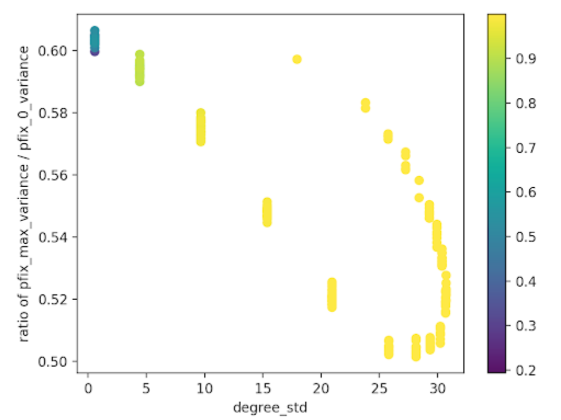
This figure further illustrated that the deviation correlates to the degree difference between the two components, but this correlation is not linear.

4 it could be noise, but not always, more confirmation needed

I took some advice from YP regarding how many simulations I should be running, and now I am running on the order of millions simulations. It seems like with a larger number of



(a) island 2 graph with 8 million simulations, color: size of dense component



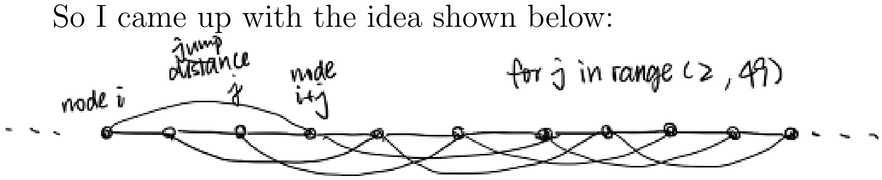
(b) island 3 graph with 4 million simulations, color: transitivity

simulations, the vertical bars diminishes for island 2 graphs so it could really be me not running enough simulations. However, when I ran island 3 graphs with the same order of magnitude of simulations, there are still vertical "bars".

5 Some little failed explorations

While visually examining structures of island 3 graphs, I saw an interesting pattern where there are some back-to-back triangles in a linear pattern which doesn't seem to contribute to the "connectivity" of the graph component. I thought perhaps the number of "steps" it takes to get back to the denser component of graph may be a factor, since detour graphs are showing strong deviation.

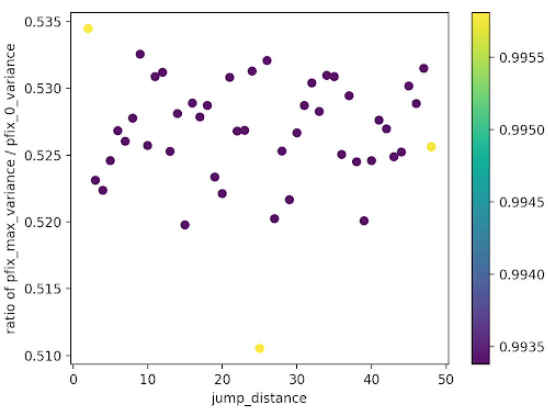
So I came up with the idea shown below:



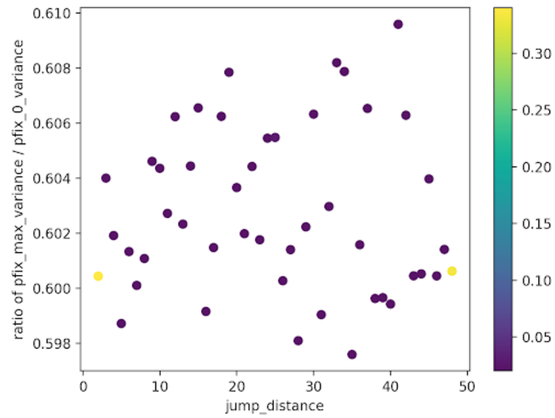
The first experiment I did was basically a "detour" graph with a "chain" with variable jump distance and a fully connected component.

Ok it is not very interesting. Then I thought maybe it is because all the graphs are already showing such great deviations (the average here is 0.52 while the value without deviation is about 0.6).

In the second experiment, instead of using a complete graph as the other component, I used a 3 regular graph – this way both components have the same degree. Well, the result is still not very interesting.

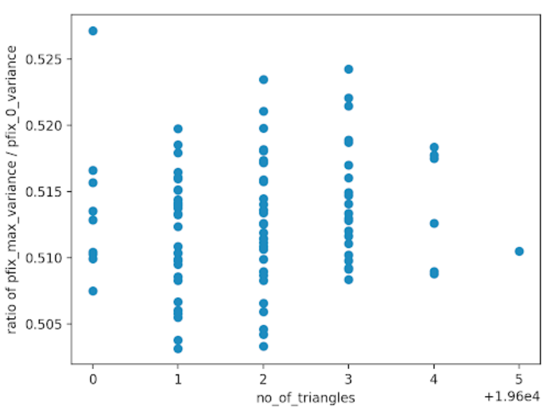


(a) composite graphs with a complete graph and a "chain" of nodes with variable jump distance

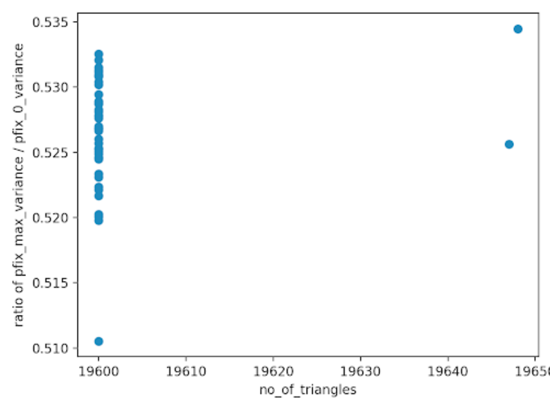


(b) composite graphs with a 3 regular graph and a "chain" of nodes with variable jump distance

What might be a little bit interesting is that a 3-regular graph seems to show a stronger deviation than such "wheel-ly" graphs. For figure (a), the values ranges from 0.505 to 0.525,



(a) result for composite graphs consisting a complete graph and a 3-regular graph



(b) result for composite graphs consisting a complete graph and a constructed "wheel" graph with variable jump distance

whereas for figure (b), the values are mostly above 0.520. There is little overlap and I think there is some statistical significance. (stats test TBD)