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ETH

Eidgenössische Technische Hochschule Zürich Swiss Federal Institute of Technology Zurich

Acoustics I: sound field calculations

Reto Pieren 2024

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calculation of a situation specific location- and time dependent sound field (often p)

- conditions for a valid solution:
 - fulfillment of the wave equation or Helmholtz equation
 - fulfillment of the boundary conditions
 - sources
 - boundaries (borders of space)
- analytical solutions for special geometries only
- numerical solutions in the general case:
 - finite elements
 - boundary elements
 - time domain methods such as FDTD

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Green's theorem:

$$\check{p}(x, y, z, \omega) = \frac{1}{4\pi} \int_{S} \left(j\omega \rho_0 \check{v}_S(\omega) \frac{e^{-j\omega r/c}}{r} + \check{p}_S(\omega) \frac{\partial}{\partial n} \frac{e^{-j\omega r/c}}{r} \right) dS$$

S: closed surface

 \check{v}_S : sound particle velocity on and normal to S

 \check{p}_S : sound pressure on S

r: distance of the surface point to the receiver point (x, y, z)

lacktriangle Kirchhoff-Helmholtz integral ightarrow wave field synthesis

Kirchhoff - Helmholtz integral

Kirchhoff -Helmholtz integral

- Kirchhoff-Helmholtz integral KHI is valid:
 - in the interior of S
 - in the exterior of S
 - on the surface S with a correction factor of 2

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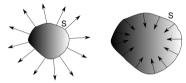
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Boundary Elements Method

Boundary Elements Method

Boundary Elements Method

typical radiation problem:



- surface velocity is given as boundary condition
- search for sound pressure field inside or outside of S
- solution with the Boundary Elements Method:
 - discretisation of the radiator surface in n elements
 - with KHI: $p_{S,i} = \sum_{i=1}^{n} f(p_{S,i}, v_{S,i})$
 - \triangleright solve the system of equations with *n* unknowns $\rightarrow p_{S,i}$
 - calculate sound pressure at any point in space with the KHI

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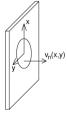
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Rayleigh Integral

Rayleigh Integral

Rayleigh Integral

- \triangleright radiation of an oscillating piston \rightarrow Kirchhoff-Helmholtz Integral
- special case: oscillating piston mounted in a large and rigid wall
 - \triangleright wall introduces boundary condition: $v_n = 0$



Rayleigh Integral

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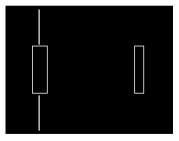
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- replace the effect of the wall by a mirror source
- lacktriangle oscillating piston ightarrow pulsating piston

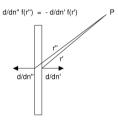


Rayleigh Integral

Rayleigh Integral

evaluation of the Kirchhoff Helmholtz Integral:

$$\check{p}(x, y, z, \omega) = \frac{1}{4\pi} \int_{S} \left(j\omega \rho_0 \check{v}_S(\omega) \frac{e^{-j\omega r/c}}{r} + \check{p}_S(\omega) \frac{\partial}{\partial n} \frac{e^{-j\omega r/c}}{r} \right) dS$$



contribution of sound pressure = 0!

Rayleigh Integral

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Kirchhoff Helmholtz Integral simplifies to the Rayleigh Integral:

$$\check{p}(x, y, z, \omega) = \frac{j\omega\rho_0}{2\pi} \int_{S} \check{v}_n(x, y, \omega) \frac{e^{-jkr}}{r} dS$$

S: visible piston surface (front) v_n : piston velocity

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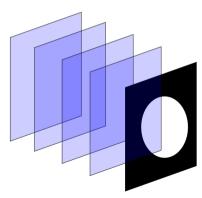
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Kirchhoff's approximations: diffraction problems

- screen with aperture:
 - plane wave hits the aperture in a hard screen
 - sound pressure field behind the screen?



Kirchhoff's approximations: diffraction problems

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- solution: application of the Rayleigh Integral
 - needed: sound particle velocity in the aperture
 - Kirchhoff's approximation:
 - assume sound particle velocity as if no screen is present
 - ightharpoonup ightharpoonup ignore boundaries
 - error decreases with decreasing ratio wavelength / diameter

Kirchhoff's approximations: diffraction problems

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example: sound field of a plane wave behind an aperture of 25 cm diameter

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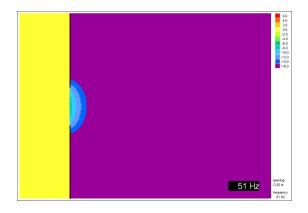
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sound field behind aperture with Kirchhoff's aproximation

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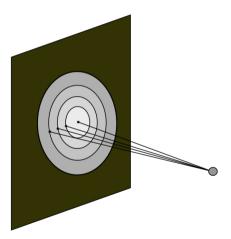
Rayleigh Integral:

$$\check{p}(x,y,z,\omega) = \frac{j\omega\rho_0}{2\pi} \int_{S} \check{v}_n(x,y,\omega) \frac{e^{-jkr}}{r} dS$$

- approximation with Fresnel zones for receivers not too close:
 - ignore small changes of *r*
 - ▶ differentiate phase in classes + (0 degrees) and (180 degrees) only
 - corresponding regions in the aperture: Fresnel zones

Kirchhoff's approximations: diffraction problems

Fresnel zones in case of circular aperture:



Kirchhoff's approximations

Kirchhoff's approximations: diffraction problems

Kirchhoff's approximations

$$p \sim \frac{A_1}{r_1} - \frac{A_2}{r_2} + \frac{A_3}{r_3} - \frac{A_4}{r_4} \dots$$

A_i: area of the *i*-th Fresnel zone

r_i: average distance to the i-th Fresnel zone

for large apertures:

$$p\sim \frac{A_1}{2r_1}$$

if aperture = 1. Fresnel zone \rightarrow amplification of +6 dB re. free field

Fresnel zones for reflection problems

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- reflection at inhomogeneous or finite surfaces:
 - half of the 1. Fresnel zone defines the relevant region on a reflector
 - concept allows for the estimation of situations with:
 - ▶ small reflectors $F < \frac{A_1}{2} \to p_{\mathsf{refl}} \approx \frac{2F}{A_1} p_{\mathsf{refl}\infty}$
 - inhomogeneous reflectors

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- common method to solve differential equations by discretization of the field volume
- well suited for:
 - bounded field regions such as vehicle interiors
 - coupled structure/fluid systems, e.g. simulation of airborne sound insulation in the laboratory
 - ightharpoonup simulation of inhomogeneous properties of the medium (c, density)
- not well suited for:
 - radiation in unbounded space

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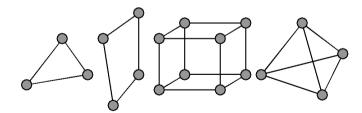
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finite elements

- discretization of the field volume in finite elements
- establish one equation per element and node
- assembly of the system of equations
- solve the system of equation for each frequency of interest



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FDTD:

finite differences in the time domain

Finite Differences in the Time Domain (FDTD)

FDTD

- standard method to find solutions of differential equations numerically

$$ightharpoonup$$
 grad $(p) = -\rho \frac{\partial v}{\partial t}$

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standard method to find solutions of differential equations numerically

usage of the fundamental acoustical partial differential equations in the time domain:

•
$$grad(p) = -\rho \frac{\partial \vec{v}}{\partial t}$$

Newton

$$-\frac{\partial p}{\partial t} = \kappa P_0 \operatorname{div}(\vec{v})$$

Poisson, mass conservation

- strategy:
 - discretisation of simulation domain in space and time
 - replacement of derivatives by differences (space and time)

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- standard method to find solutions of differential equations numerically
- usage of the fundamental acoustical partial differential equations in the time domain:

•
$$grad(p) = -\rho \frac{\partial \vec{v}}{\partial t}$$

Newton

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Poisson, mass conservation

- strategy:
 - discretisation of simulation domain in space and time
 - replacement of derivatives by differences (space and time)
 - ightharpoonup updating equations in time

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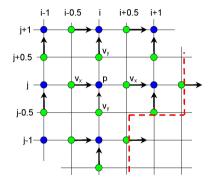
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finite differences in the time domain (FDTD)

2D-formulation:



$$v_x^{\text{new}} = v_x^{\text{old}} - \alpha (p_{\text{right}} - p_{\text{left}})$$
 $p^{\text{new}} = p^{\text{old}} - \beta (v_{\text{xright}} - v_{\text{xleft}}) - \beta (v_{\text{ytop}} - v_{\text{ybottom}})$

finite differences in the time domain (FDTD)

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- example: road traff
- example: road traff
- situation
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- example: railway cutting
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- typical simulation / calculation:
 - impulse-like pressure distribution as starting condition
 - time-stepwise updating of the field variables at the grid points
- advantages:
 - no system of equation that has to be solved
 - impulse response as a result contains information about all frequencies
- disadvantage:
 - implementation of frequency domain boundary conditions is not straight forward

finite differences in the time domain (FDTD)

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finite differences in the time domain (FDTD)

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 - impulse response as a result contains information about all frequencies
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 - implementation of frequency domain boundary conditions is not straight forward

finite differences in the time domain (FDTD)

- computational effort:
 - \triangleright 2D-simulation of a region of 200 m \times 40 m
 - $f_{\text{max}} = 2 \text{ kHz} \rightarrow \text{discretization in space: } 0.02 \text{ m}$
 - ightharpoonup mesh size $10'000 \times 2'000 = 20 \cdot 10^6$ grid points
 - ightharpoonup calculation time \rightarrow one hour

2-/3-D simulations

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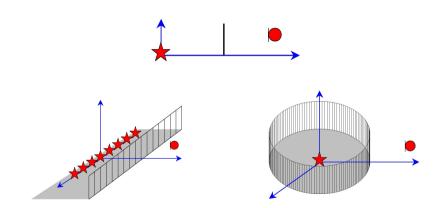
example: railway line

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- ▶ mapping of 3-dimensional geometries onto 2 independent coordinates:
 - translation invariant situation
 - rotation invariant situation

2-/3-D simulations

FDTD



2-/3-D simulations

FDTD

- translation invariant situation
 - cartesian coordinate system
 - situation geometry does not change in y-direction
 - lack all derivatives of the sound field equations with respect to y-direction are set to 0
 - \triangleright simulated source = coherent line source with extension in v-direction
 - coherent incoherent line source??

2-/3-D simulations

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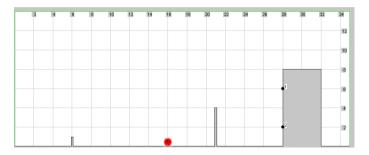
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- rotation invariant situation
 - cylindrical coordinate system
 - lacktriangle situation geometry does not change with angle ϕ
 - \blacktriangleright all derivatives of the sound field equations with respect to $\phi\text{-direction}$ are set to 0
 - simulated source = point source in the origin
 - lacktriangle caution: reflections lead to focusing effects at the source position ightarrow only strictly propagating waves allowed

finite differences in the time domain (FDTD)

example: road traffic situation

example: road traffic situation



road traffic noise situation

reflection at noise barrier

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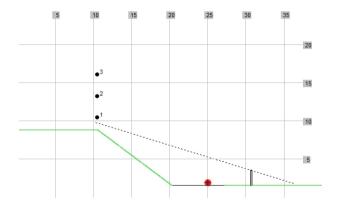
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reflection at noise barrier

finite differences in the time domain (FDTD)

example: Hardbrücke, effect of absorbing layer at the bottom of bridge



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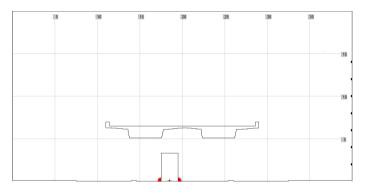
Hardbrücke example: railway lin

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reflecting bridge:

Hardbrücke

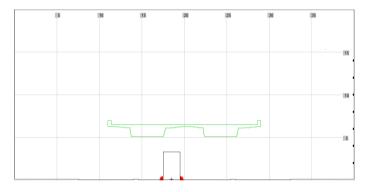


Hardbrücke - reflecting

finite differences in the time domain (FDTD)

absorbing bridge:

Hardbrücke



Hardbrücke - absorbing

finite differences in the time domain (FDTD)

example: railway line cutting



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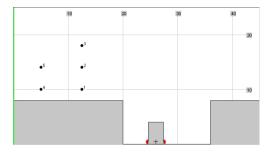
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example: railway line cutting



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Kirchhoff-Helmholtz integral is valid for arbitrary surfaces

$$\check{p}(x,y,z,\omega) = \frac{1}{4\pi} \int_{S} \left(j\omega \rho_0 \check{v}_S(\omega) \frac{e^{-j\omega r/c}}{r} + \check{p}_S(\omega) \frac{\partial}{\partial n} \frac{e^{-j\omega r/c}}{r} \right) dS$$

▶ for specifically designed surfaces further simplifications are possible

acoustical holography

for a plane S that closes in infinity

acoustical holography



sound pressure in the right half space is given as:

$$\check{p}(x, y, z, \omega) = j \int_{S} \check{p}_{S}(\omega) \cos \phi \left(1 - \frac{j}{kr}\right) \frac{e^{-jkr}}{\lambda r} dS$$

acoustical holography

acoustical holography

- \triangleright equation from above describes $p \neq 0$ in 3D-space by a $p \neq 0$ representation on a 2D-plane
- ightharpoonup ightharpoonup principle of holography
- holography in practical applications:
 - simultaneous determination of sound pressure distribution (amplitude and phase) at discrete grid points on a suitable plane
 - usage of microphone arrays
 - sequential sampling by using a fixed reference (phase)
 - ightharpoonup ightharpoonup complete information about the 3D field

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