

## Python code

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!pip install ortools

!pip install z3-solver
from z3 import *

from time import time as currenttime
from ortools.sat.python import cp_model
import re
import numpy

# helper function provided in Moodle
def transform_output(d):
    crlf = '\r\n'
    s = []
    s = ''.join(kk + crlf for kk in d['sol'])
    s = d['sat'] + crlf + s + d['mul_sol']
    s = crlf + s + crlf + str(d['exe_time']) if 'exe_time' in d else s
    return s

class VarArraySolutionPrinter(cp_model.CpSolverSolutionCallback):
    """Print intermediate solutions."""

    def __init__(self, variables, limit):
        cp_model.CpSolverSolutionCallback.__init__(self)
        self.__variables = variables
        self.__solution_count = 0
        self.__solution_limit = limit

    def on_solution_callback(self):
        self.__solution_count += 1

        for v in self.__variables:
            for user in range(len(v)):
                if self.Value(v[user]) == 1:
                    print(f'{v[user]}', end=' ')
            print()

        if self.__solution_count >= self.__solution_limit:
            print(f'\nStop searching after {self.__solution_limit} solutions found')
            self.StopSearch()

    def solution_count(self):
        return self.__solution_count

class Instance:
    def __init__(self):
        self.number_of_steps = 0
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    self.number_of_users = 0
self.number_of_constraints = 0
self.auth = []      # index: the user || element: steps authorised to the user
self.SOD = []       # the list of pairs of steps that must be assigned to the different users
self.BOD = []       # the list of pairs of steps that must be assigned to the same user
self.at_most_k = [] # the list of pairs of k and steps
self.one_team = []  # the list of pairs of steps and teams

def read_file(filename):
    def read_attribute(name):
        line = f.readline()
        match = re.match(f'{name}:\s*(\d+)$', line)
        if match:
            return int(match.group(1))
        else:
            raise Exception("Could not parse line {line}; expected the {name} attribute")

    instance = Instance()

    with open(filename) as f:
        instance.number_of_steps = read_attribute("#Steps")
        instance.number_of_users = read_attribute("#Users")
        instance.number_of_constraints = read_attribute("#Constraints")

        # initialise instance.auth with empty lists as elements
        instance.auth = [[] for u in range(instance.number_of_users)]

        for i in range(instance.number_of_constraints):
            l = f.readline()

            # 1st Constraint: Authorisations
            m = re.match(r"Authorisations u(\d+)(?: s\d+)*", l)
            if m:
                user_id = int(m.group(1))
                steps = [-1] # for users that are not authorised to perform any steps eg.
                Authorisations u1
                for m in re.finditer(r's(\d+)', l):
                    if -1 in steps:
                        steps.remove(-1) # if user has specified steps, then only store the steps
                authorised
                steps.append(int(m.group(1)) - 1) # -1 cuz list index starts from 0
                instance.auth[user_id - 1].extend(steps)
                continue

            # 2nd Constraint: Separation-of-duty
            m = re.match(r'Separation-of-duty s(\d+) s(\d+)', l)
            if m:
                steps = (int(m.group(1)) - 1, int(m.group(2)) - 1)
                instance.SOD.append(steps)
                continue

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# 3rd Constraint: Binding-of-duty
m = re.match(r'Binding-of-duty s(\d+) s(\d+)', l)
if m:
    steps = (int(m.group(1)) - 1, int(m.group(2)) - 1)
    instance.BOD.append(steps)
    continue

# 4th Constraint: At-most-k
m = re.match(r'At-most-k (\d+) (s\d+)(?: (s\d+))*', l)
if m:
    k = int(m.group(1))
    steps = []
    for m in re.finditer(r's(\d+)', l):
        steps.append(int(m.group(1)) - 1)
    instance.at_most_k.append((k, steps))
    continue

# 5th Constraint: One-team constraint
m = re.match(r'One-team\s+(s\d+)(?: s\d+)* (\(((u\d+)*\)))*', l)
if m:
    steps = []
    for m in re.finditer(r's(\d+)', l):
        steps.append(int(m.group(1)) - 1)

    teams = []
    for m in re.finditer(r'\(((u\d+|s*)+\')', l):
        team = []
        for users in re.finditer(r'u(\d+)', m.group(0)):
            team.append(int(users.group(1)) - 1)
        teams.append(team)

    instance.one_team.append((steps, teams))

    continue

else:
    raise Exception(f'Failed to parse this line: {l}')


return instance


def Solver(instance, filename, **kwargs):
    """
    :param filename:
    The constraint path
    :param kwargs:
    As you wish, you may supply extra arguments using the kwargs
    :return:
    A dict.
    """

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"""
# Printing (accessing or output) values from test instances
print("====")
print(f"\tFile: {filename}")
print(f"\tNumber of Steps: {instance.number_of_steps}")
print(f"\tNumber of Users: {instance.number_of_users}")
print(f"\tNumber of Constraints: {instance.number_of_constraints}")
print(f"\tAuthorisations: {instance.auth}")
print(f"\tSeparation-of-duty: {instance.SOD}")
print(f"\tBinding-of-duty: {instance.BOD}")
print(f"\tAt-most-k: {instance.at_most_k}")
print(f"\tOne-team: {instance.one_team}")
print("====")

"" Start of Solver """
model = cp_model.CpModel()

user_assignment = [[model.NewBoolVar(f's{s + 1}: u{u + 1}') for u in range(instance.number_of_users)] for s in range(instance.number_of_steps)]

# each step is assigned to exactly one user
for step in range(instance.number_of_steps):
    model.AddExactlyOne(user_assignment[step][user] for user in range(instance.number_of_users))
"""

----- Authorisations constraint: -----
if user u is not authorised to step s, then step s cannot be assigned to user u
"""

# apply constraint ONLY to users with specified authorisations
for user in range(instance.number_of_users):
    if instance.auth[user]:
        for step in range(instance.number_of_steps):
            if step not in instance.auth[user]:
                model.Add(user_assignment[step][user] == 0)
"""

----- Separation-of-duty constraint: -----
separated steps must be assigned to the different user
"""

for (separated_step1, separated_step2) in instance.SOD:
    for user in range(instance.number_of_users):
        model.Add(user_assignment[separated_step2][user] == 0).OnlyEnforceIf(user_assignment[separated_step1][user])

"""

----- Binding-of-duty constraint: -----
bound steps cannot be assigned to the same user
"""

for (bound_step1, bound_step2) in instance.BOD:
    for user in range(instance.number_of_users):

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model.Add(user_assignment[bound_step2][user]) ==

1).OnlyEnforceIf(user_assignment[bound_step1][user])

"""

----- At-most-k constraint: -----
number of users assigned to the group of steps must not be greater than k
"""

# takes long time to solve when number of users is large
# """
for (k, steps) in instance.at_most_k:
    # print(f'{k} --- {steps}')
    user_assignment_flag = [model.NewBoolVar(f'at-most-k_u{u}') for u in range(instance.number_of_users)]
    for user in range(instance.number_of_users):
        for step in steps:
            model.Add(user_assignment_flag[user])
1).OnlyEnforceIf(user_assignment[step][user])
    model.Add(sum(user_assignment[step][user] for step in steps) >= user_assignment_flag[user])
    model.Add(sum(user_assignment_flag) <= k)
# """

# much longer time in large instances (eg. Examples 16 and 17) because more variables
# and more constraints are declared

"""

----- One-team constraint: -----
steps can only be assigned to team with flag = 1,
steps cannot be assigned to users with no team
"""

for (steps, teams) in instance.one_team:
    team_flag = [model.NewBoolVar(f'team{t}') for t in range(len(teams))]
    model.AddExactlyOne(team_flag) # only one team can be chosen
    for team_index in range(len(teams)):
        for step in steps:
            for user in teams[team_index]:
                model.Add(user_assignment[step][user])
0).OnlyEnforceIf(team_flag[team_index].Not())
    # steps cannot be assigned to users that is not listed in teams too
    users_in_teams = list(numpy.concatenate(teams).flat)
    for step in steps:
        for user in range(instance.number_of_users):
            if user not in users_in_teams:
                model.Add(user_assignment[step][user] == 0)

""" End of Solver """

starttime = float(currenttime() * 1000)
solver = cp_model.CpSolver()
solution_printer = VarArraySolutionPrinter(user_assignment, 1000)

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solver.parameters.enumerate_all_solutions = True
# solver.parameters.max_time_in_seconds = 10.0
status = solver.Solve(model, solution_printer)
endtime = float(currenttime()) * 1000

d = dict(
    sat='unsat',
    sol="",
    mul_sol="",
    exe_time=str(endtime - starttime) + 'ms'
)
if status == cp_model.OPTIMAL or status == cp_model.FEASIBLE:
    d['sat'] = 'sat'
    dsol = []

    for s in range(instance.number_of_steps):
        for u in range(instance.number_of_users):
            solver_value = solver.Value(user_assignment[s][u])
            if solver_value:
                dsol.append(f's{s + 1}: u{u + 1}')

d['sol'] = dsol

print("\nStatus: %s" % solver.StatusName(status))
print('Number of solutions found: %i' % solution_printer.solution_count())
if solution_printer.solution_count() > 1:
    d['mul_sol'] = f'other solutions exist, {solution_printer.solution_count()}'
    solutions found'
else:
    d['mul_sol'] = "this is the only solution"
return d
else:
    print("\nStatus: %s" % solver.StatusName(status))
    return d

if __name__ == '__main__':
    dpath = 'instances/example14.txt'
    inst = read_file(dpath)
    d = Solver(inst, dpath, silent=False)
    s = transform_output(d)
    print(s)
```

----- End of CW2 Sample Guidance -----