Sample Problems, Part 1

1. Let the set of machine numbers be $M = M(t, k^-, k^+) = M(6, -2, 5)$.

a) Give the numbers ε_0, and M_∞ of this set.

b) Convert $\frac{1}{16}$, 0.67, and 4.23 to this set, i.e. give $fl\left(\frac{1}{16}\right)$, $fl(0.67)$ and $fl(4.23)$.

c) Perform the machine addition $fl(0.67) \oplus fl(4.23)$.

d) Give error bounds for $fl(0.67)$, $fl(4.23)$ and the result. *(2 points)*

2. Prove that for $A \in \mathbb{K}^{n\times n}$ we have $\|A\|_2 \leq \sqrt{\|A\|_1 \|A\|_\infty}$. *(1 points)*

3. Let $A = \begin{bmatrix} -2 & -3 \\ 0 & -4 \\ -3 & 3 \end{bmatrix}$. Give $||A||_1$, $||A||_2$, $||A||_\infty$, and $||A||_F$. *(1 points)*

4. Let $A = \begin{bmatrix} 3 & 1 & -1 \\ 6 & 5 & 0 \\ -3 & 0 & 4 \end{bmatrix}$ and $b = \begin{bmatrix} 2 \\ 16 \\ 9 \end{bmatrix}$.

a) Solve $Ax = b$ for $x \in \mathbb{R}^3$.

b) Give the LU-decomposition $A = LU$.

c) Give A^{-1}, $\det(A)$, and $\text{cond}_\infty(A)$. *(2 points)*

5. Let $A = \begin{bmatrix} 4 & 12 & -16 \\ 12 & 37 & -43 \\ -16 & -43 & 98 \end{bmatrix}$. Give the Cholesky-decomposition $A = LL^T$. *(1 points)*

6. Implement an algorithm for computing the inverse matrix, using floating point arithmetics. The matrix should be read from the standard input. Scoring: 1 point if your solution finds the existing inverse of a 300×300 matrix in less than a second; 2 points if your solution also gives a warning when the matrix is practically not invertible. Preferred languages: C/C++, Java.

7. Implement an algorithm for deciding if a symmetric input matrix is positive definite (hint: Cholesky decomposition). Use floating point arithmetics. The matrix should be read from the standard input. Scoring: 1 point if your solution gives the correct answer for a 500×500 matrix in less than a second. Preferred languages: C/C++, Java.