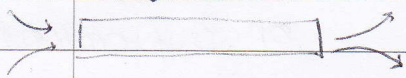


14 nov

# Parallelism

next week: lab  
take computer

## homework



Sem full = 0  
Sem free = max

$\$1 = 10$   
 $\$2 = \text{max}$

```
produce() {
    free.p();
    add;
    full.v();
}
```

```
consume() {
    full.p();
    cons remove();
    free.v();
}
```

How are exam graded

- is every semaphore's p & v called?
- every semaphore has a value?
- does it solve the problem?

## exercise & example

```
for
from(i from 1 to N) {
    v.v[i] = a + b.v[i]^2 + c.v[i]^3 + d.v[i]^4
    + e[i]^5 + f[i]^6 + g[i]^7
}
```

How to parallelize this code?

4) Divide the computation into small sub-computations

$$P_0 = b \cdot v[i]^2$$

$$P_1 = c \cdot v[i]^3$$

$$P_2 = g \cdot v[i]^7$$

note:  $P_n$  = processor n

problems: work is unbalanced between the processors

memory access to  $v[i]$  can be a problem as well

We could divide our work differently:

```
for i from 1 to N {
    // v.v[i] = a + b.v[i]^2 + c.v[i]^3 + d.v[i]^4 + v[i]^5 * (d + e.v[i] + f.v[i]^2 + g.v[i]^3)
    // do in parallel {
        x = a + b.v[i]^2 + c.v[i]^3
        y = v[i]^4
        z = d + e.v[i] + f.v[i]^2 + g.v[i]^3
    }
    300 end v.v[i] = x + y * z
```

↳ divisions

should be done in parallel

control parallelism: relies on the instructions to see what can be ||.



algorithmic complexity of this loop:  $O(N)$  for the first  
 $O(N)$  — second  
 on 3 processors

2) we take  $N$  processors

$$P_0 \rightarrow vv[1]$$

$$P_1 \rightarrow vv[2]$$

...

$$P_{N-1} \rightarrow vv[N]$$

Complexity:  $O(1)$   
 on  $N$  processors

from  $i$  from 1 to  $N$  in parallel  
 $vv[i] = \dots$

This is called Data parallelism  
 you do not look at the program but at the array of data.

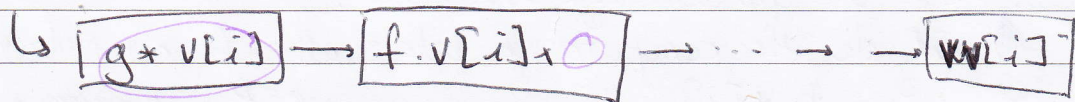
3) Horner sequences

$$vv[i] = a \times vv[i] (0 + b \cdot i^2 + d \cdot i^3 \dots)$$

$$= a \times vv[i] (0 + vv[i] (b + a \cdot i + d \cdot i^2 \dots))$$

$O(N)$  on 1 processor

This is a pipeline



If we don't have  $N$  processors but only  $p \ll N$  proces.

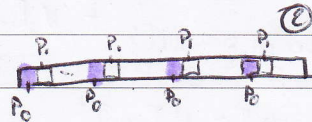


each processor  
 will have to compute  
 $N/p$  cells

In data parallelism, how do we associate some data to a processor?



$P_1$   $P_2$   $P_3$   $P_4$   
 parts  
 of consecutive cells  
 bloc distribution



cyclic distribution

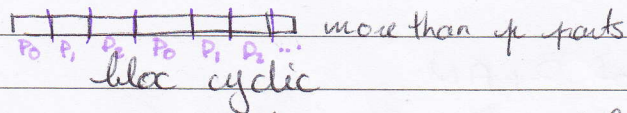
⊖ could be unbalanced

⊖ you are messing  
 with the cache



## Parallelism

③



you can play on the size of blocks to optimize for the cache while reducing the probability of having unbalanced computations.

note: look at map-reduce algorithm

## P-RAM

### Parallel Random Access Machine

- It has a set of processors  $P_1, P_2, \dots, P_n$ , and each processor knows its id.
- Each processor has a private memory and some registers. (Program counter is THE register we can't do without  $\rightarrow$  address of the next instruction to process)
- All processors have access to an unlimited shared memory.
- All processors have "classical" instructions +
  - halt: stops the processor
  - fork: starts a new program on a free processor
- Each instruction in the P-RAM takes 1 unit of time
- The execution model is synchronous: all processors perform 1 instruction at the same time (1 step in  $P_1$  takes 1 step in  $P_2$ ): lockstep
- Read-Write conflicts

it depends (concurrent reads or write)

	Read		Write	
Concurrent	<del>RR</del>	CR	<del>WW</del>	CW
Exclusive	RR	ER	WE	EW

ER: No more than 1 access to a memory address

CR: Multiple accesses are allowed at the same time

EW: Exclusive write

CW: Allowed when all processors write the same value

other conditions: (1) consistency  $\rightarrow$

(2) arbitrary: only one value will be written, but we don't know which one.

(3) priority: a processor has priority over others.



④ Fusion: applying a function to all values and the written value is the result. (Max, +, ...)

#### 4 main types of P-RAM:

EREW for the rest of this course,  
 CREW we will study algo on these 4 P-RAMs  
 ERCW | 4 subtypes  
 CRCW

A parallel algorithm is defined as a couple (algorithm, P-RAM), P-RAM being the most restrictive.

Exercice: Max Value on P-RAM

input: array of size  $n = 2^m$

output: max value

1) Sequential version:

V1

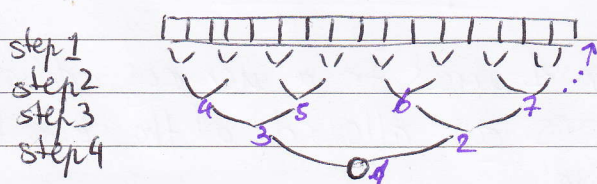
```

max = a[1];
for i in a {
  if a[i] > max {
    max = a[i];
  }
}
return max;
  
```

V2

```

for l from m-1 to 0
  for j = 2^l to 2^{l+1}-1 in parallel
    b[j] = max(b[2j], b[2j+1]);
  
```



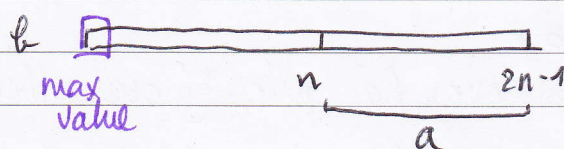
How to represent a tree as an array?

1) number the nodes from ~~the~~ 1 from the root, breadth first

2) given a node j:

- childs are  $2j$ ;  $2j+1$
- parent is  $\lfloor \frac{j}{2} \rfloor$ .

Size of b: ( $b > a$ )  
 b has a size of  $2^m - 1$





## Parallelism

On which P-RAM does it work?

- No processor writes in the same cell
- No processor reads in the same cell
- ↳ so we need an EREW P-RAM.

Complexity:  $O(\log(n))$  on  $\frac{n}{2}$  processors  
↳  $O(n)$  (big O notation)

For next week

- 1) ↳ complexity of the sequential version of the algorithm
- 2) You are free to use the P-RAM you want, what is the fastest algorithm to compute the max value?

For lab session

OPEN MP → library (ext. of the compiler)

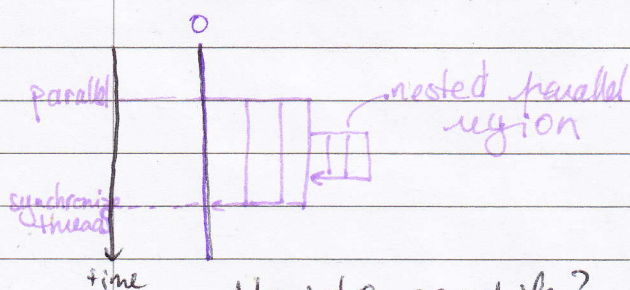
↳ indicate where shit's parallel and it deals with threads for you.

An open MP program is "single process with multiple threads" (simulates a P-RAM)

Threads are created when encountering a parallel region.

The number of threads is decided at ~~one time~~ runtime by OpenMP, but can be set by developers.

Each thread has an id and the master thread has an id of 0.



```
#pragma omp parallel for  
for (i=0; i<10000; i++) {  
    a[i] = b[i] + c[i]  
}
```

How to compile?

gcc -o test-para test.c -fopenmp

install gcc: Linux Subsystem  
(installed last year)