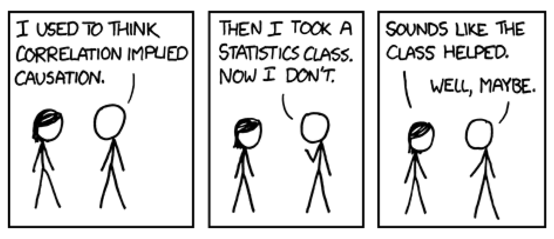

Random bits of M1S

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Outline

"Lies, damn lies and statistics." – Mark Twain. Luckily, this course is about probability, so I can leave the lies to the second year lecturers.... Assessment of uncertainty in such real-life problems is a complex issue which requires a rigorous mathematical treatment. M1S develops the probability framework in which questions of practical interest can be posed and resolved.

Contents

The course will cover the following topics

1. SAMPLE SPACES AND EVENTS
2. PROBABILITY: DEFINITIONS, INTERPRETATIONS
3. CONDITIONAL PROBABILITY
4. COMBINATORICS
5. DISCRETE RANDOM VARIABLES
6. CONTINUOUS RANDOM VARIABLES
7. TRANSFORMATIONS
8. JOINT DISTRIBUTIONS

I will cut and paste various bits of LaTeXfrom the coursenotes and some plots from my MSc course in nonparametric regressions to produce this poster.

Sample Space and Events

The set of **all** possible outcomes is called the **SAMPLE SPACE**, Ω.

If ω is a possible outcome, $\omega \in \Omega$.

$$\Omega = \{\omega_1, \omega_2, \ldots\}$$

$\omega_1, \omega_2, \ldots$ are **elements** of Ω.

Subsets of Ω are called **EVENTS**.
DE MORGAN'S LAWS

1. $(E \cup F)' = E' \cap F'$
2. $(E \cap F)' = E' \cup F'$

This can be extended to countably infinite events.

Axioms of Probability

Axioms of Probability
For events $E, F \subseteq \Omega$

(I) $0 \le \mathrm{P}(E) \le 1$
(II) $\mathrm{P}(\Omega) = 1$
(III) If $E \cap F = \phi$, then $\mathrm{P}(E \cup F) = \mathrm{P}(E) + \mathrm{P}(F)$ (Addition rule)

References

[1] D. Stirzaker. Elementary Probability. Cambridge University Press, 2003.
[2] S. Ross. A First Course in Probability. Prentice Hall, 2001.
[3] G. Grimmett. Probability and Random Processes. Oxford University Press, 2001.

Combinatorics

Definition
An **ordered** arrangement of r items from n is a **PERMUTATION**.
The number of ordered sample of r items from a population of size n ($r \le n$) is

$$\begin{aligned} {}^nP_r &= \frac{n!}{(n-r)!} = (n)_r \\ &= n(n-1)\ldots(n-r+1) \end{aligned}$$

Definition
An **unordered** arrangement of r items from n is a **COMBINATION**.
The number of unordered samples of size r items from a population of size n ($r \le n$) is

$$^nC_r = \frac{n!}{r!(n-r)!} = \binom{n}{r}$$

SUMMARY
total number of samples of r from n:

	WITHOUT REPLACEMENT	WITH REPLACEMENT
ORDERED	$(n)_r$	n^r
UNORDERED	$\binom{n}{r}$	$\binom{n+r-1}{r}$

Discrete Random Variables

Bernoulli distribution:

$$\begin{aligned} f_X(x) &= \begin{cases} 1-\theta & x = 0 \\ \theta & x = 1 \end{cases} \\ &= \theta^x(1-\theta)^{1-x} \quad x \in \{0,1\} \end{aligned}$$

$0 \le \theta \le 1$.

Binomial distribution

$$f_X(x) = \mathrm{P}(X = x) = \binom{n}{x}\theta^x(1-\theta)^{n-x}$$

for $0 \le x \le n$, and zero otherwise.

Continuous Random Variables

Exponential Distribution

$$f_X(x) = \lambda \mathrm{e}^{-\lambda x} \quad x > 0$$

for $\lambda > 0$.

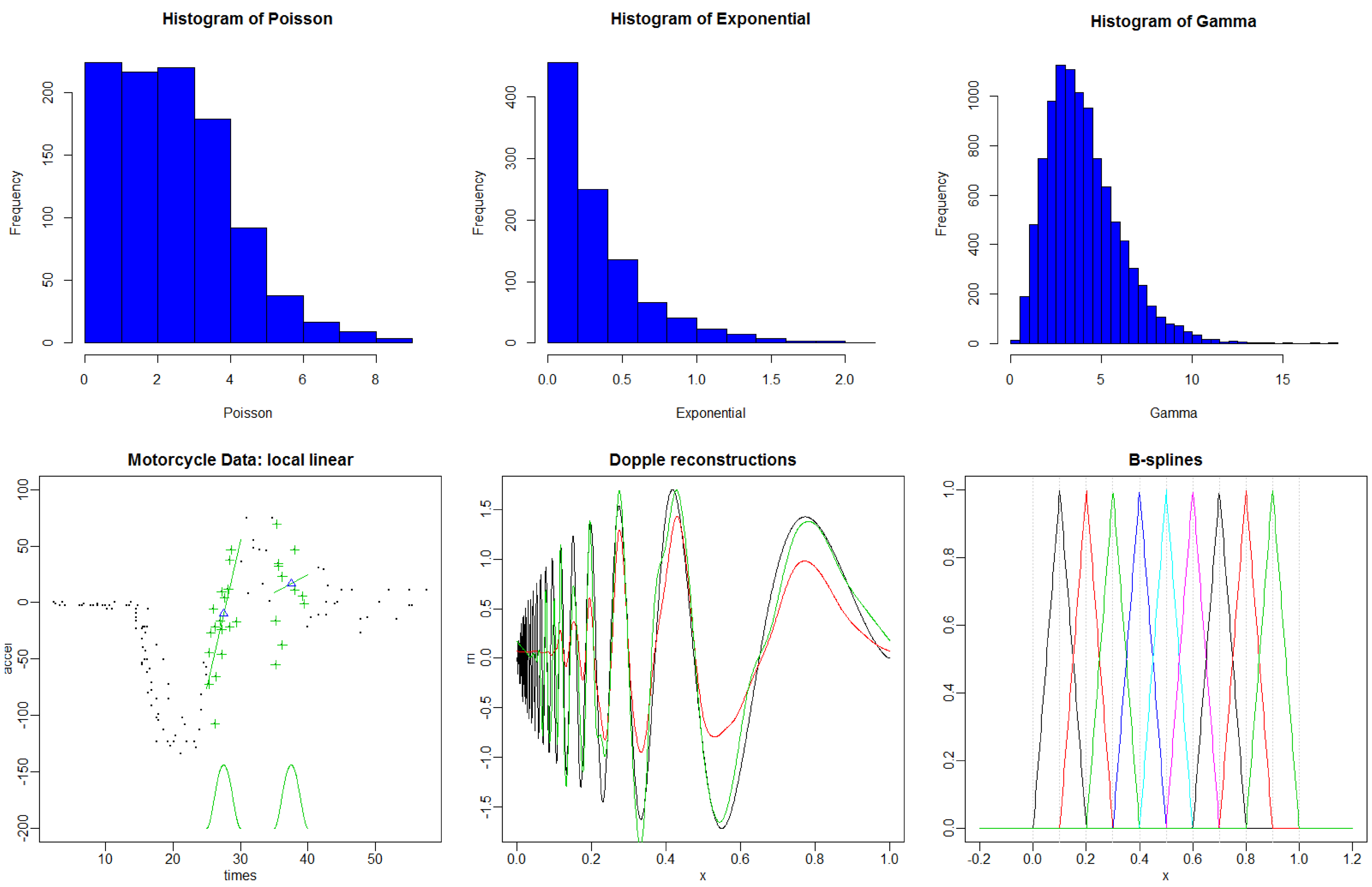
Gamma Disbribution:

$$f_Y(y) = \frac{\beta^\alpha}{\Gamma(\alpha)} y^{\alpha-1} e^{-\beta y} \quad y > 0$$

where α = shape
β = scale

Some Graphics

Here are some random plots:



Conclusions

Pretty easy to put boxes exactly where you like!